Technische Universität München
T31 SFB 1258 Projekt D04
„Neutrinos and Dark Matter in Astro- and Particle Physics“

How BSM physics puts pNR EFT to the test

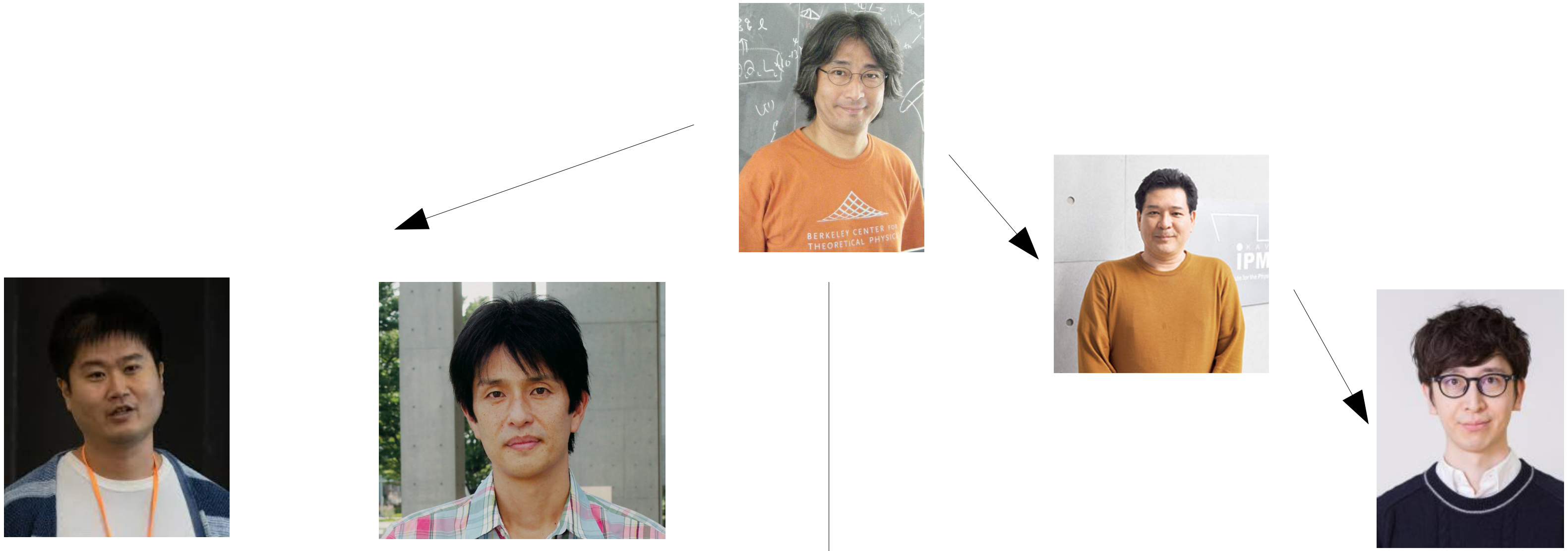
Tobias Binder

18th December 2024

Hitoshi Fest, Kavli IPMU

How I met Hitoshi

- PhD summer school **2015**, „Invisibles Network“, Madrid
- Asked Hitoshi one question
- A month later I visited Kavli IPMU…



Structure formation on small scales,
see talk by Ayuki Kamada

Dark Matter long-range force effects:
Sommerfeld + bound states

How BSM physics puts pNR EFT to the test

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Based on:

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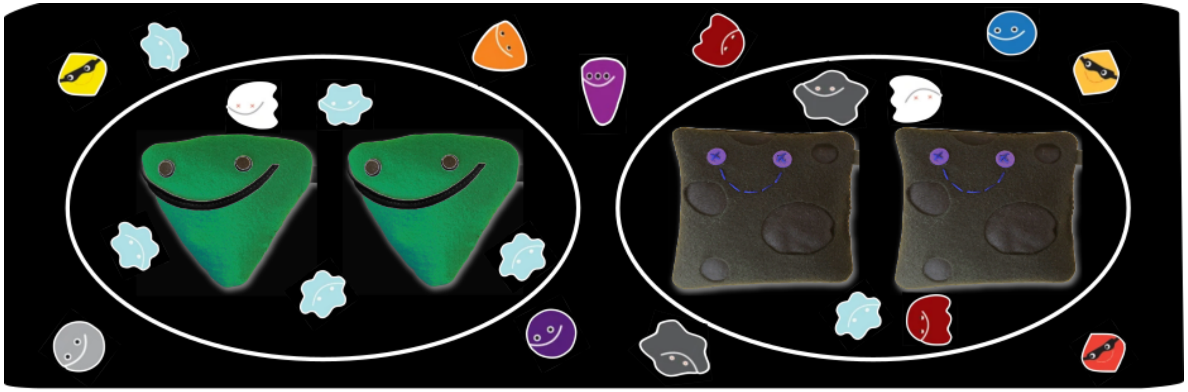
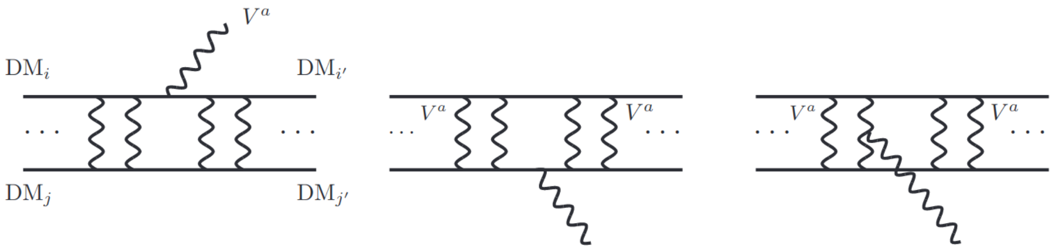
Hitoshi Fest, Kavli IPMU

Heavy Quarkonia and Dark Matter



- Share common non-relativistic description (pNR EFT)
- Valid for $\frac{\alpha^2 m}{n^2} >> \Lambda$
- Within validity regime, large n only possible in BSM physics → testing pNR EFT



Potential non-relativistic EFT (pNR EFT)

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

- non-relativistic description of pairs, e.g., heavy quarkonia
- Hierarchy of scales: $m >> \alpha m >> \alpha^2 m >> \Lambda$

$$\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots$$

$$\mathcal{L}_{\text{pNREFT}} \supset \int \mathrm{d}^3 r \ \mathrm{Tr}\left[\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger (iD_0 - H_{\text{adj}})\mathrm{Adj} \right.$$
$$\left. - V_A(\underline{\mathrm{Adj}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}}\mathrm{S} + \text{h.c.}) - \frac{V_B}{2}\mathrm{Adj}^\dagger \{\boldsymbol{r} \cdot g\boldsymbol{E}, \mathrm{Adj}\} + \cdots \right].$$

e.g. quarkonium, squark:

$$3 \otimes \bar{3} = 1 \oplus 8 \qquad \mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g \qquad (\sigma v)_{nl} = \frac{C_F}{N_c^2}\frac{4\alpha}{3}\Delta E^3 \underline{|\langle \psi^1_{nl}| \, \mathbf{r} \, |\psi^8_{\mathbf{p}}\rangle|^2}$$

Matrix elements

$$\langle \psi_{nl} | \, r^X \, | \psi_{\mathbf{p}} \rangle$$

Our analytic results systematize, simplifies and generalize previous results:

- **Monopole (X=0)** [Oncala & Petraki 19,21]
- **Dipole (X=1):**
    - Minimal DM, Colored co-annihilation [Ellis et al. 15, Mitridate et al. 17, Harz & Petraki 18, ...]
    - Dark U(1), SU(N) [Harling et al. 14, ..., Asadi 21, Biondini et al. 23]
- **Quadrupole (X=2)** [Wise et al. 14,16, Petraki et al. 15, Biondini 21,22]

This talk: formation of Rydberg states via dipole interaction in gauge theories

Bound-state formation in U(1) gauge theory

$$\mathcal{S}(\chi\bar{\chi}) \rightarrow \mathcal{B}(\chi\bar{\chi})_{nl} + \gamma$$

$$(\sigma v)_{nl} = \tfrac{4\alpha}{3}\Delta E^3 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_{\mathbf{p}}\rangle|^2$$

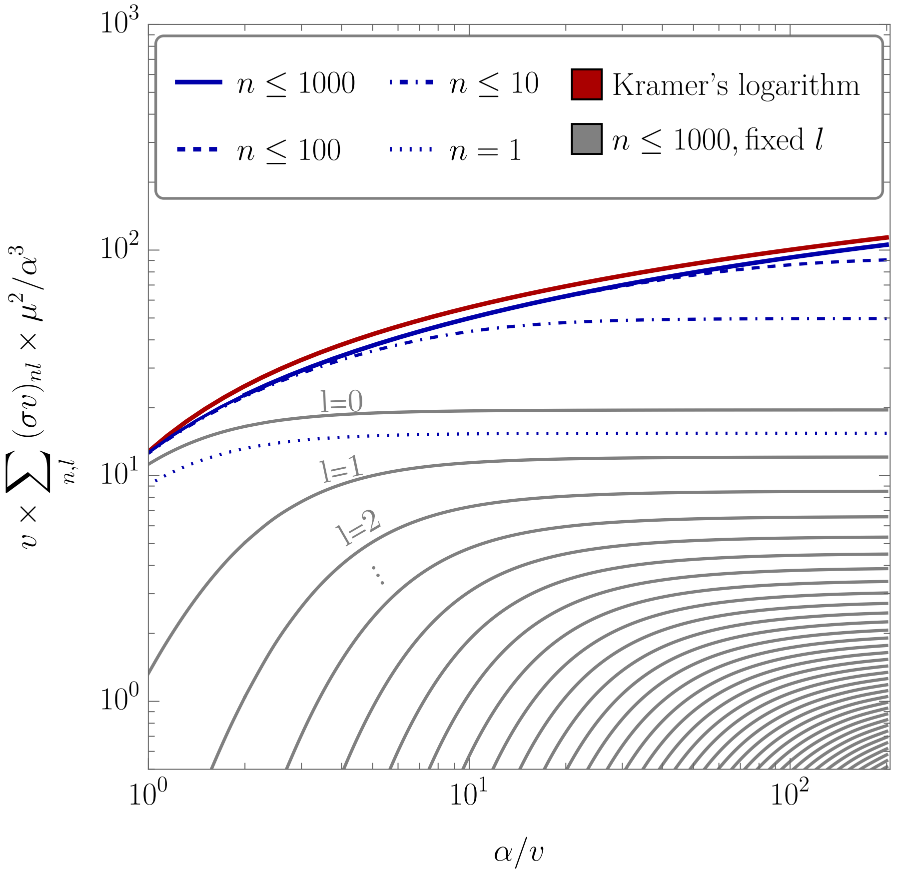
(e.g. hydrogen, (dark) positronium,
complex scalars)

- Up to half a million bound states:

  all n $\leq 1000, l \leq n-1.$

- Confirm Kramer‘s logarithm within expected error as a check:

$$\sum_{n,\ell}(\sigma v)_{n\ell} \simeq \frac{32\pi}{3\sqrt{3}}\frac{\alpha^2}{\mu^2}\frac{\alpha}{v}\left[\log(\alpha/v)+\gamma_E\right], \text{ for } v \ll \alpha.$$



Summed BSF cross section, $U(1)$

Bound-state formation in SU(3) gauge theory

$3 \otimes \bar{3} = 1 \oplus 8$

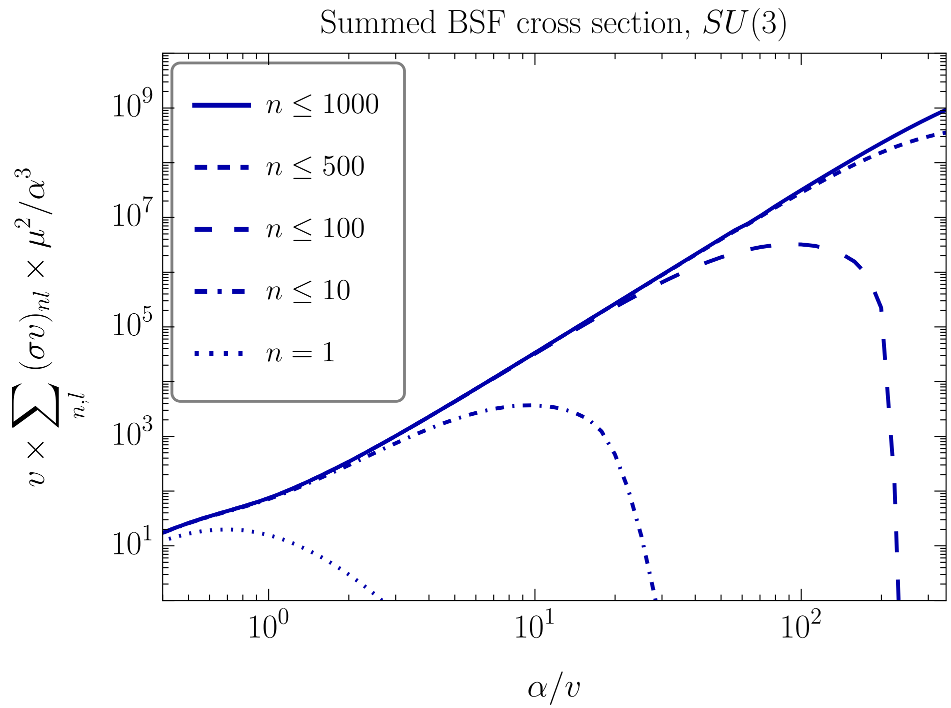
$\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$

$(\sigma v)_{nl} = \frac{C_F}{N_c^2} \frac{4\alpha}{3} \Delta E^3 |\langle \psi^1_{nl} | \mathbf{r} | \psi^8_{\mathbf{p}} \rangle|^2$

(e.g. quarkonium, squark)

- Assume constant coupling
- Low velocity scaling much stronger:

  $\sum_{nl} (\sigma v)_{nl} \propto v^{-4}$ for $v \ll \alpha$
- Raises concerns about partial wave-unitarity violation



Bound-state formation in SU(3) gauge theory

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$\mathcal{S}(\chi\bar{\chi})^{8} \rightarrow \mathcal{B}(\chi\bar{\chi})^{1}_{nl} + g$$

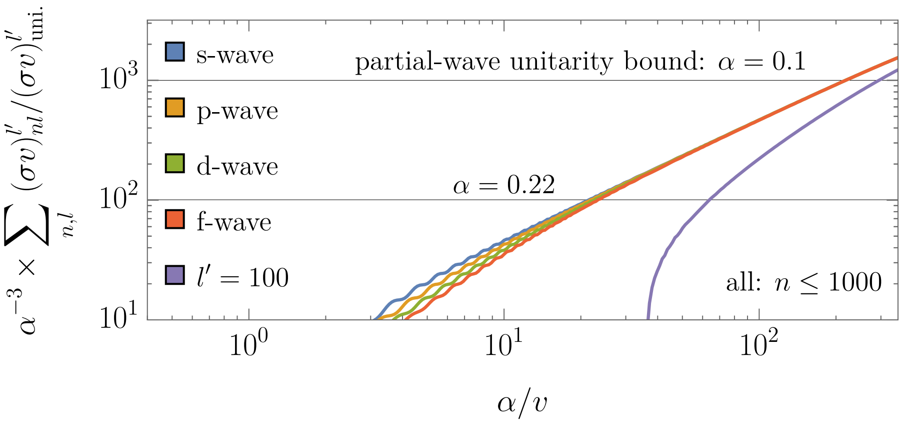
$$(\sigma v)_{nl} = \frac{C_F}{N_c^2}\frac{4\alpha}{3}\Delta E^3 \left| \langle \psi^1_{nl} | \, \mathbf{r} \, | \psi^8_{\mathbf{p}} \rangle \right|^2$$

(e.g. quarkonium, squark)

- Unitarity condition:

  $$\sum_{nl} (\sigma v)^{l'}_{nl} \leq (\sigma v)^{l'}_{\text{uni.}} = \frac{\pi(2l'+1)}{\mu^2 v}$$

- Observe partial-wave unitarity violation in the perturbative regime

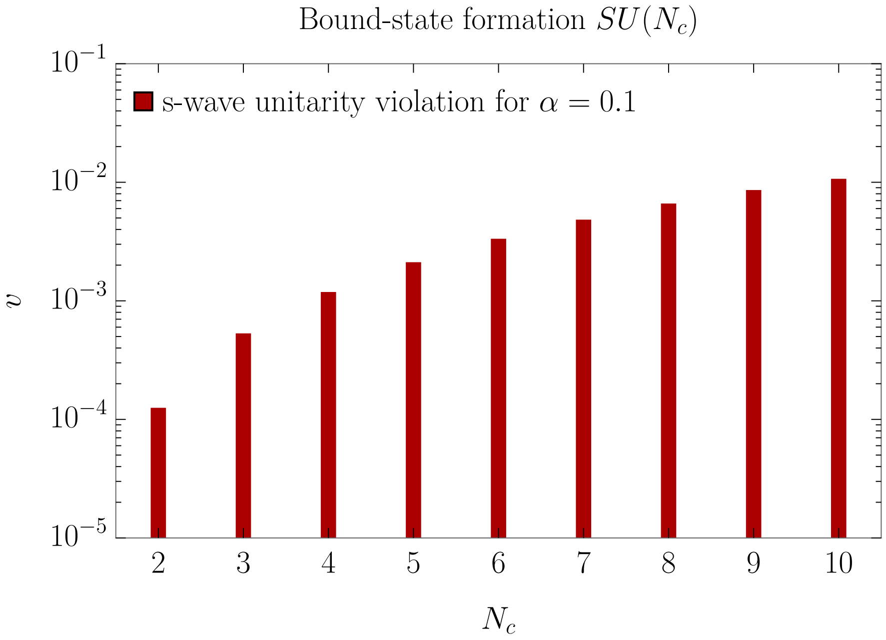


Partial wave unitarity violation in SU(N)

- We observe partial wave unitarity violation in SU(N) gauge theories for perturbatively small couplings

- More generally: if initial state is less attractive than final state, partial wave unitarity will be violated at a finite velocity

- Unitarization?

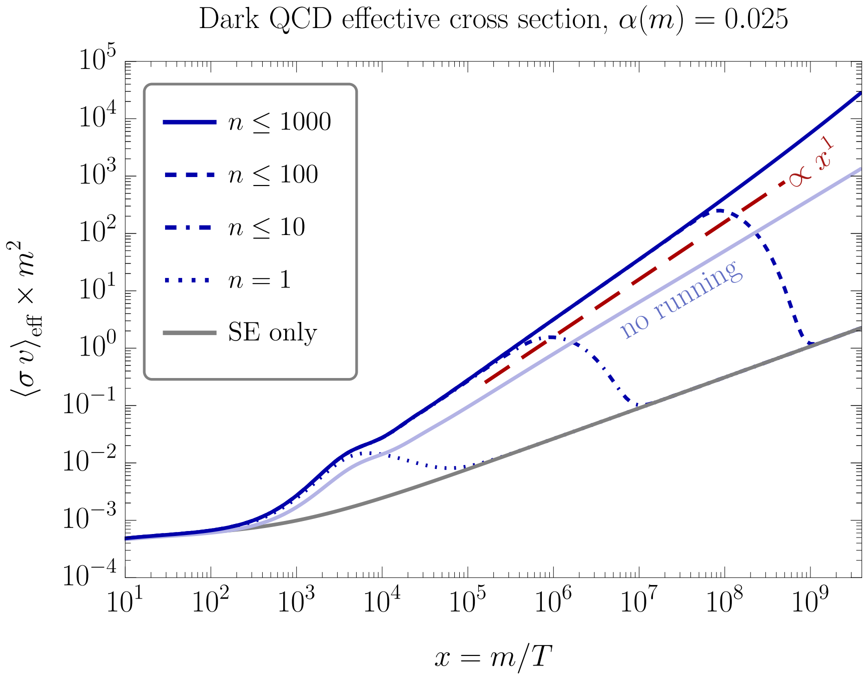


In the following, focussing on the regime consistent with perturbativity and unitarity

Effective cross section: Dark QCD sector

$$\dot{n} + 3Hn = -\langle \sigma v \rangle_{\text{eff}} (n^2 - n_{\text{eq}}^2)$$

- s-wave bound states only, dominant decay mode
- „eternal depletion" in the perturbative regime, i.e. no freeze-out
- Slope increases with N in SU(N)



SM SU(3) and U(1) charged mediator model

„t-channel“ simplified toy model:

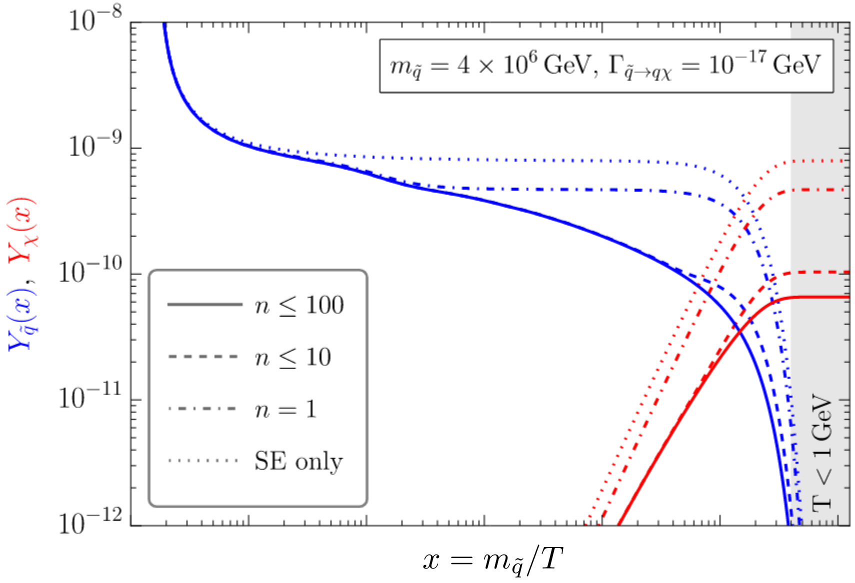
$$\mathcal{L} \supset \lambda_\chi \tilde{q} \bar{q}_R \chi + h.c.$$

- $\tilde{q}$: scalar mediator, carries SM electric and color charge
- q_R: right handed SM quark
- χ : Majorana Fermion Dark Matter
- Possible DM production scenarios:

$$\begin{aligned} \Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\gg H(m_{\tilde{q}}) && \text{coannihilation}\,, \\ \Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\sim H(m_{\tilde{q}}) && \text{conversion-driven}\,, \\ \Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\ll H(m_{\tilde{q}}) && \text{superWIMP}/\text{freeze-in}\,. \end{aligned}$$

SuperWIMP regime

- superWIMP mechanism: late decay of mediator into DM
- Usually: final DM yield independent of actual size of the decay rate.
- Bound state effects introduce a *dependence* of the DM yield on the decay rate as a novel feature.



Summary & Conclusion

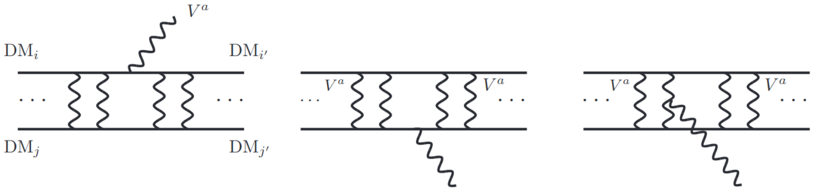
- Rydberg states important in unbroken regime of non-abelian gauge theories
- SuperWIMP regime:

  - *novel feature*: bound state effects can introduce a dependence of the DM yield on the mediator lifetime

  - *DM mass corrections*: by up to an order of magnitude

- unitarization of bound-state formation in unbroken non-Abelian gauge theories within the regime of perturbatively small couplings (?)

Happy 60th-birthday, Hitoshi!



Colored co-annihilation examples

- **I.e., co-annihilating partner charged under SM SU(3)**
- **Longe-range effects impact** $(\Delta m_\chi, m_\chi)$**- plane**
  - Squark (scalar triplet)
  - Gluino (fermion octet)

[Ellis *et al.* 15, Liew & Luo 16,
Mitridate *et al.* 17]

[Harz & Petraki 18,19]

- **+ Higgs**
  - Additional attractive contribution
  - (squark) octet can be bounded

[Gross *et al.* 18, Fukuda & Luo & Shirai 18]

- **Non-perturbative regime**
  (for mass splitting below confinging scale)

DM production scenarios

„t-channel“ simplified model:

$$\mathcal{L} \supset \lambda_\chi \tilde{q} \bar{q}_R \chi + h.c.$$

DM production can be classified into:

$$\begin{aligned}
\Gamma^{\chi\to\tilde{q}}_{\mathrm{conv}} &\gg H(m_{\tilde{q}}) & \text{coannihilation}\,, \\
\Gamma^{\chi\to\tilde{q}}_{\mathrm{conv}} &\sim H(m_{\tilde{q}}) & \text{conversion-driven}\,, \\
\Gamma^{\chi\to\tilde{q}}_{\mathrm{conv}} &\ll H(m_{\tilde{q}}) & \text{superWIMP}/\text{freeze-in}\,.
\end{aligned}$$

$$\frac{\mathrm{d}Y_{\tilde{q}}}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\left[\frac{1}{2}\langle\sigma_{\tilde{q}\tilde{q}^\dagger}v\rangle_{\mathrm{eff}}\left(Y_{\tilde{q}}^2 - Y_{\tilde{q}}^{\mathrm{eq}\,2}\right)\right. \tag{19}$$

$$\left. + \langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\mathrm{eq}}Y_{\tilde{q}}^{\mathrm{eq}}\right) + \frac{\Gamma^{\tilde{q}\to\chi}_{\mathrm{conv}}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\mathrm{eq}}}{Y_\chi^{\mathrm{eq}}}\right)\right],$$

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\left[\langle\sigma_{\chi\chi}v\rangle\left(Y_\chi^2 - Y_\chi^{\mathrm{eq}\,2}\right)\right. \tag{20}$$

$$\left. + \langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\mathrm{eq}}Y_{\tilde{q}}^{\mathrm{eq}}\right) - \frac{\Gamma^{\tilde{q}\to\chi}_{\mathrm{conv}}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\mathrm{eq}}}{Y_\chi^{\mathrm{eq}}}\right)\right],$$

pNREFT

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

$$\mathcal{L} \xrightarrow{m} \mathcal{L}^{\mathrm{NR}} \xrightarrow{\alpha m} \mathcal{L}^{\mathrm{pNR}}$$

Non-relativistic effective field theory for the ultra-soft scale $\alpha^2 m$

- *potential* non-relativistic (pNR) QED:

$$\mathcal{L}^{\mathrm{pNRQED}} \supset \int \mathrm{d}^3 r\; S^\dagger(\mathbf{x},\mathbf{r},t)\left[i\partial_t + \frac{\nabla^2_{\mathbf{x}}}{4m} + \frac{\nabla^2_{\mathbf{r}}}{m} - V(r) + i2\frac{\pi\alpha^2}{m^2}\delta^3(\mathbf{r}) + \;\mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t)\right] S(\mathbf{x},\mathbf{r},t)$$

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$

$$(\sigma v)_{nl} = \frac{4\alpha}{3}|\left\langle\psi_{nl}\right|\mathbf{r}\left|\psi_v\right\rangle|^2 \Delta E^3$$

pNREFT

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

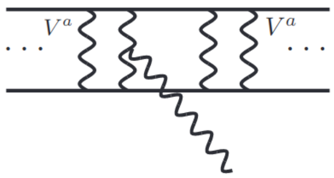
$$\mathcal{L} \overset{m}{\rightarrow} \mathcal{L}^{\mathrm{NR}} \overset{\alpha m}{\rightarrow} \mathcal{L}^{\mathrm{pNR}}$$

Non-relativistic effective field theory for the ultra-soft scale $\alpha^2 m$

- *potential* non-relativistic (pNR) SU(N) in the weakly coupled regime:

$$\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots$$

$$\mathcal{L}_{\mathrm{pNREFT}} \supset \int \mathrm{d}^3 r \; \mathrm{Tr} \Big[\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger (iD_0 - H_{\mathrm{adj}})\mathrm{Adj}$$
$$- V_A(\mathrm{Adj}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) - \frac{V_B}{2} \mathrm{Adj}^\dagger \{\boldsymbol{r} \cdot g\boldsymbol{E}, \mathrm{Adj}\} + \cdots \Big] .$$



Included, e.g. [Binder et al. 2021]

e.g. quarkonium, squark:

$3 \otimes \bar{3} = 1 \oplus 8$

$\mathcal{S}(\chi\bar{\chi})^8 \rightarrow \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$

$(\sigma v)_{nl} = \frac{C_F}{N_c^2} \frac{4\alpha}{3} \Delta E^3 |\, \langle \psi^1_{nl} |\, \mathbf{r} \,| \psi^8_{\mathbf{p}} \rangle \,|^2$

Positronium example

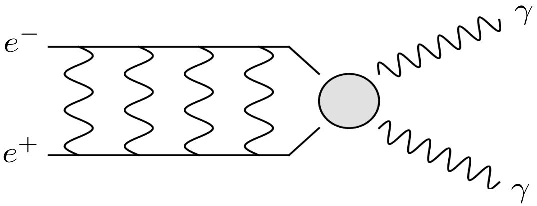
Bound-state decay and Sommerfeld enhancement:

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

Pirenne & Wheeler 1946

$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha/v)\,, \text{ for } v \lesssim \alpha.$$

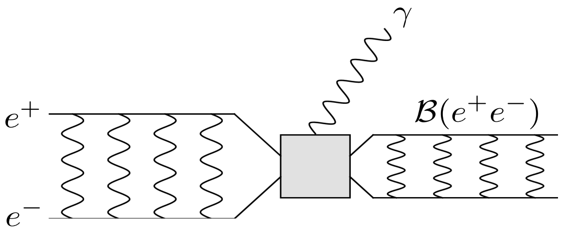
Sakharov 1948
(Sommerfeld 1931)



Bound-state formation (recombination):

$$(\sigma v)_{nl} = \frac{4\alpha}{3} |\langle \psi_{nl} | \mathbf{r} | \psi_v \rangle|^2 \Delta E^3$$
$$\sim 3 \times \text{annihilation}, \text{ for } v \lesssim \alpha.$$

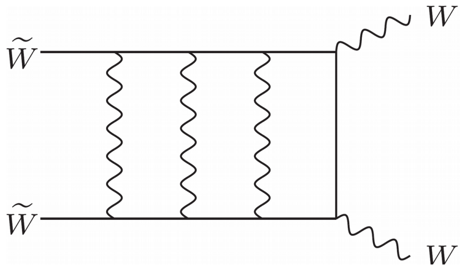
(and n=1,l=0.)



(originates from the Electric Dipole Operator „gr.E“, see e.g. Landau&Lifshitz)

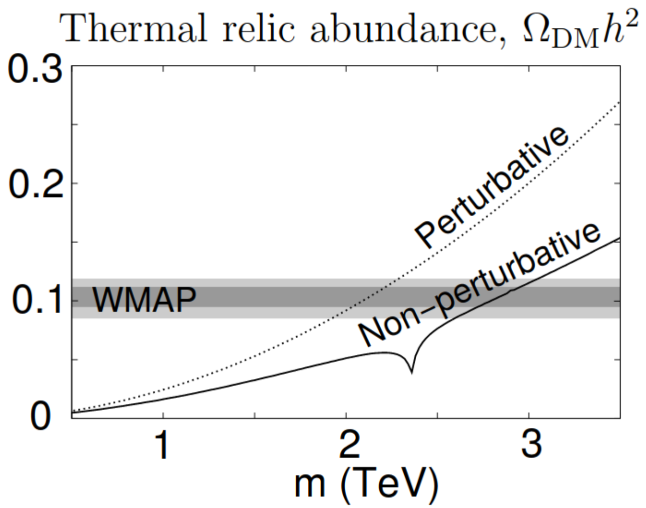
Wino Dark Matter example

$$\begin{pmatrix}\widetilde{W}^{+}\\ \widetilde{W}^{0}\\ \widetilde{W}^{-}\end{pmatrix}$$



- Majorana Fermion, SU(2) Triplet, zero Hypercharge („most minimal WIMP“)
- Sommerfeld-enhanced annihilation allows for heavier Wino masses
- ID signal mass sensitive, see e.g.

[Rinchiuso, Slatyer et al. 20]



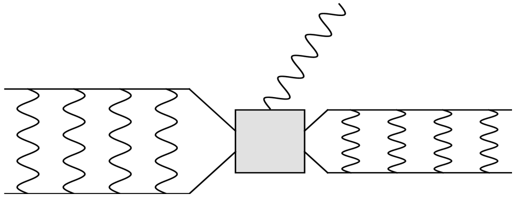
[Hisano et al. 03,05,06]

General dipole transition matrix elements

- „gr.E“ leads to matrix elements of the form:

$$\langle\psi_f|\,\mathbf{r}\,|\psi_i\rangle = \int \mathrm{d}^3r\ \psi_f^\star(\mathbf{r})\ \mathbf{r}\ \psi_i(\mathbf{r}).$$

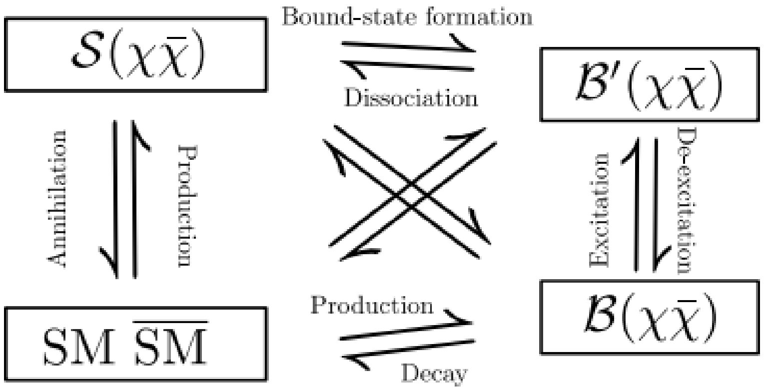
$$\mathrm{V}_{i/f} = -\frac{\alpha_{i/f}^{\mathrm{eff}}}{r}$$



- E.g.: (chromo-) electric dipole transitions of pairs in unbroken U(1) and SU(N) gauge theories
- Analytic result in terms of recurrence relations* allows for efficient and numerically stable evaluation.
- Tested against know results for low excitations

*) in QED limit consistent with
[W. Gordon, Zur Berechnung der Matrizen beim Wasserstoffatom, Annalen der Physik 394 (1929)]

Effective cross section



Quasi steady-state

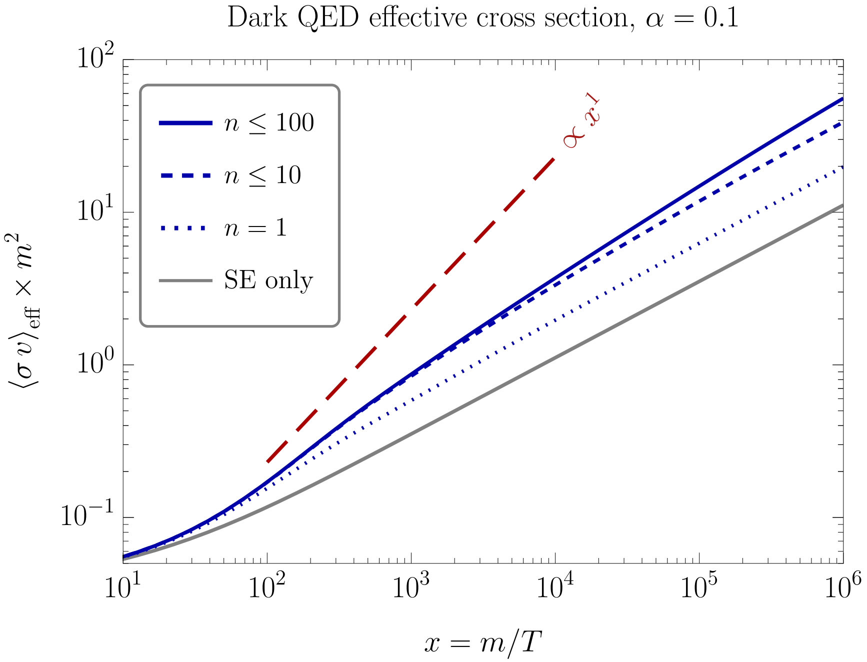
$$\dot{n} + 3Hn = -\langle\sigma v\rangle_{\text{eff}}(n^2 - n_{\text{eq}}^2)$$

$$\langle\sigma v\rangle_{\text{eff}} = \langle\sigma v\rangle_{\text{ann}} + \sum_{\mathcal{B}} \langle\sigma v\rangle_{\mathcal{B}} R_{\mathcal{B}}$$

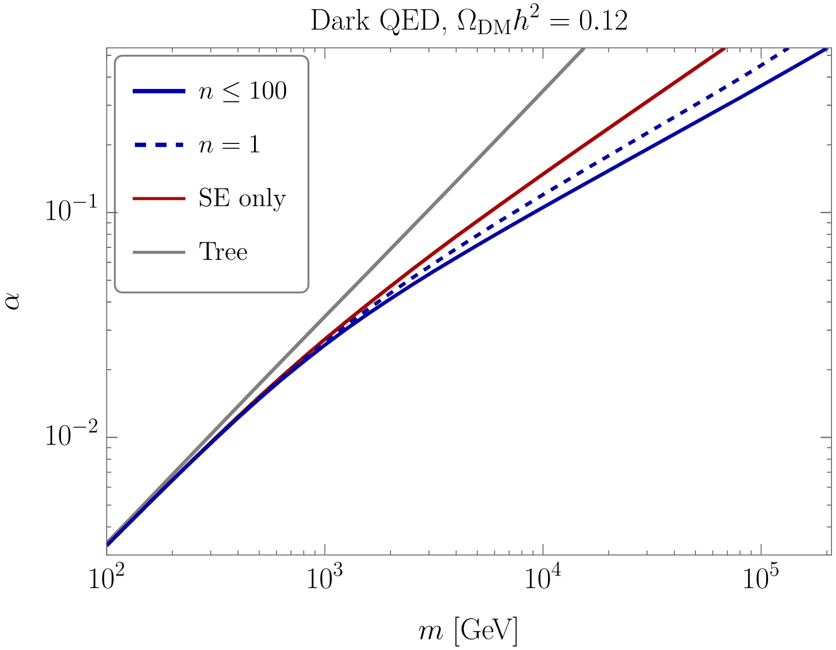
[Ellis et al. 15, Mitridate et al. 17,
Binder et al. 21, Garny & Heisig 21]

- Effective cross section encodes complex interplay between annihilation, bound-state formation, excitation, bound-state decay and reverse processes

Effective cross section: Dark QED sector



Dark QED effective cross section, $\alpha = 0.1$



Dark QED, $\Omega_{\text{DM}}h^2 = 0.12$

- Includes about 5000 bound states and all possible dipole transitions (~10^6).
- Dark QED indeed freezes out.
- Upper bound on DM mass consistent with perturbative unitarity is 0.2 PeV.

Constraints

- DM produced relativistically from heavy mediator decay
- DM can be „too hot“, i.e., substructure can be erased by free-streaming effect
- Substructure probed by Ly-alpha observations
- Bound state effects open up parameter space
- Corrections to the DM mass up to an order of magnitude