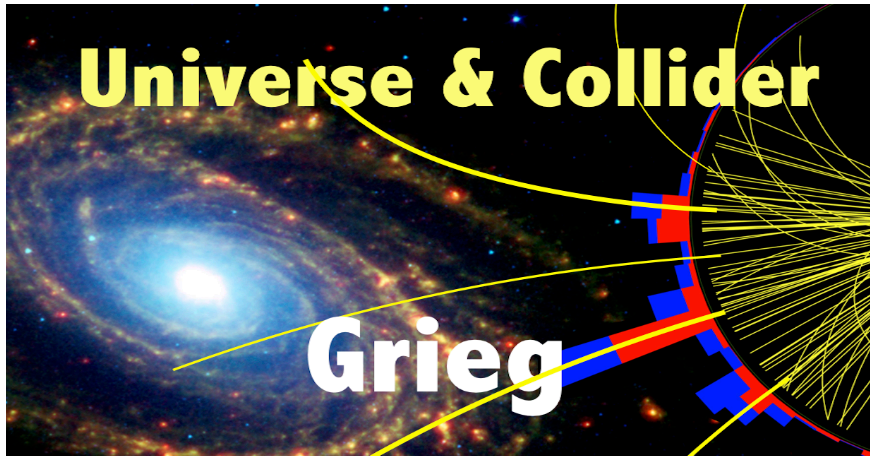
Imprints of dark matter interaction with plasma in the early Universe

Ayuki Kamada (University of Warsaw)



UNIVERSITY OF WARSAW

Based on
Binder, Covi, AK, Murayama, Takahashi and Yoshida, JCAP, 2016
AK and Takahashi, JCAP, 2018
Ando, AK, Sekiguchi and Takahashi, PRD, 2019

Dec. 20, 2024 @ Hitoshi Fest

Murayama-san as a guru

Not frequent interaction

- Murayama-san was super busy
as a director of IPMU

Student still needs a thesis subject

- Murayama-san's suggestion "Start with this"

A theory of dark matter

Nima Arkani-Hamed,[1] Douglas P. Finkbeiner,[2] Tracy R. Slatyer,[3] and Neal Weiner[4]
[1]*School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540, USA*
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(Received 31 October 2008; published 27 January 2009)

- my master study started from 2009-

- I was negative about studying dark matter physics for a while

- people studied dark matter as a by-product
of their new-physics models at that time

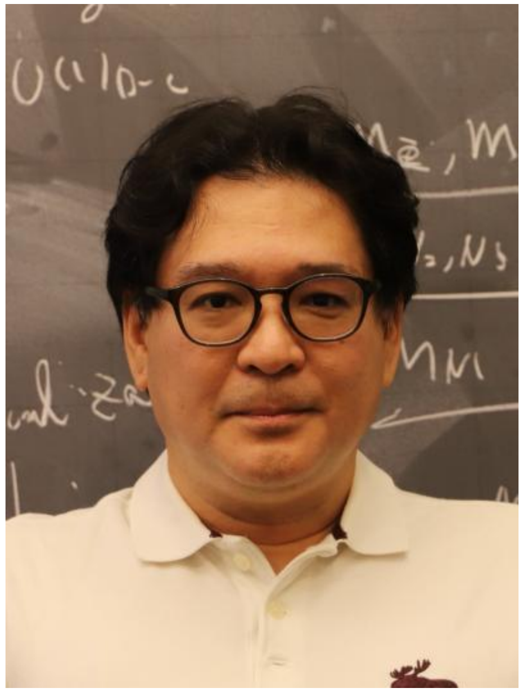
- you know what people study nowadays

Murayama-san as a guru

Moreover

- there are experts in IPMU about two phenomenologies discussed in the paper

  - Sommerfeld enhancement in dark matter physics



Explosive Dark Matter Annihilation

Junji Hisano,[1] Shigeki Matsumoto,[1] and Mihoko M. Nojiri[2]

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[2]*YITP, Kyoto University, Kyoto 606-8502, Japan*

(Received 25 July 2003; published 22 January 2004)

  - simulation of self-interacting dark matter

WEAKLY SELF-INTERACTING DARK MATTER AND THE STRUCTURE OF DARK HALOS

Naoki Yoshida,[1] Volker Springel,[1,2] Simon D. M. White,[1] and Giuseppe Tormen[3]

Received 2000 June 9; accepted 2000 July 25; published 2000 November 9

Murayama-san as a guru

Dancing in the palm

- self-interacting dark matter in view of diversity problem

Self-Interacting Dark Matter Can Explain Diverse Galactic Rotation Curves

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(Received 23 November 2016; revised manuscript received 23 June 2017; published 13 September 2017)

- relation between Sommerfeld enhancement and self-interaction through analytic continuation of the amplitude

Quantum theory of dark matter scattering

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Murayama-san as a guru

Lecture notes

- extremely useful (even for my own teaching)

221A Lecture Notes
Landau Levels

A Precise Counting of the Degeneracy

This appendix is only for the mathematically inclined, and you can skip it entirely if not interested. The degeneracy of the ground states can be studied exactly if the system is not an open plane but rather a compact Riemann surface. We set $e = c = \hbar = 1$ to simplify notation. Then the conclusion from Section 3.2 is that the number of states is approximately $N \simeq \frac{1}{2\pi}\Phi$, where $\Phi = B\pi R^2$ is the total magnetic flux.

Let us reformulate the problem of solving for the ground state wave functions so that it can be generalized to Riemann surfaces. We can regard the electromagnetism with a background magnetic field in two-dimensions as a complex line bundle over a complex plane. The gauge connection in the symmetric gauge is

$$\begin{aligned} A &= \frac{B}{2}(-ydx + xdy) \\ &= \frac{B}{2}\left(\frac{i(z-\bar{z})}{2}\frac{dz+d\bar{z}}{2} + \frac{z+\bar{z}}{2}\frac{-i(dz-d\bar{z})}{2}\right) = \frac{B}{4}(-i\bar{z}dz + izd\bar{z}). \end{aligned} \tag{A.6}$$

Using a "complexified" gauge transformation, *i.e.*, extending the structure group from U(1) to GL(1,$\mathbb{C}$), we can transform the gauge connection to $A' = A + (\partial + \bar{\partial})\Lambda$,

$$A' = -i\frac{B}{2}\bar{z}dz \tag{A.7}$$

with the gauge parameter $\Lambda = -i\frac{B}{4}\bar{z}z$. Namely, $A_z = -i\frac{B}{2}\bar{z}$, $A_{\bar{z}} = 0$. Note that this gauge transformation changes the inner product of the wave functions to $\int dxdy\psi^*\psi e^{-B\bar{z}z/2}$ because GL(1,$\mathbb{C}$) is not unitary. Then the ground state equation in this gauge is simply $(\bar{\partial} + A_{\bar{z}})\psi = \bar{\partial}\psi = 0$. Therefore, the question is to find a complete set of holomorphic functions. As seen

221A Lecture Notes
Supplemental Material on Harmonic Oscillator

Let us see how one can calculate expectation values of operators using the coherent states. Note that any operator made up of x and p can be rewritten in terms of a and $a^\dagger$. Furthermore an operator can be brought to the form that all annihilation operators are moved to the left, and creation operators to the right using their commutation relations. Therefore we can cast any operators to the form $\mathcal{O} = a^n a^{\dagger m}$ without a loss of generality.* Then its expectation value can be calculated as

$$\begin{aligned} \langle\psi|\mathcal{O}|\psi\rangle &= \langle\psi|a^n a^{\dagger m}|\psi\rangle \\ &= \int\frac{d^2f}{\pi}\langle\psi|a^n|f\rangle e^{-f^*f}\langle f|a^{\dagger m}|\psi\rangle \\ &= \int\frac{d^2f}{\pi}f^n f^{*m}\langle\psi|f\rangle e^{-f^*f}\langle f|\psi\rangle \\ &= \int\frac{d^2f}{\pi}f^n f^{*m}|\langle f|\psi\rangle|^2 e^{-f^*f}. \end{aligned} \tag{49}$$

Therefore, the combination $|\langle f|\psi\rangle|^2 e^{-f^*f}$ can be viewed as the probability density on the phase space, where the operator $a^n a^{\dagger m}$ is simply brought to the numbers $f^n f^{*m}$.

This observation allows us to "view" a state as a probability density on the phase space. First of all, the classical motion is along a zero-thickness circle on the phase space. It is always at a point at a given moment, and the point moves along the circle as time evolves. This is shown as the first picture in Fig. 1. Note that the time corresponds to the phase, while the energy to the number.

*The operators of the form $a^{\dagger m}a^n$ are said to be "normal ordered." Maybe I should call those in the order we use here "abnormally ordered."

Contents

Interaction of dark matter with plasma

- thermal freeze-out (chemical decoupling)
- kinetic decoupling
- Fokker-Planck approximation

Imprints on small-scale structures

- dark acoustic oscillation
- Landau damping
- overshooting of density perturbation

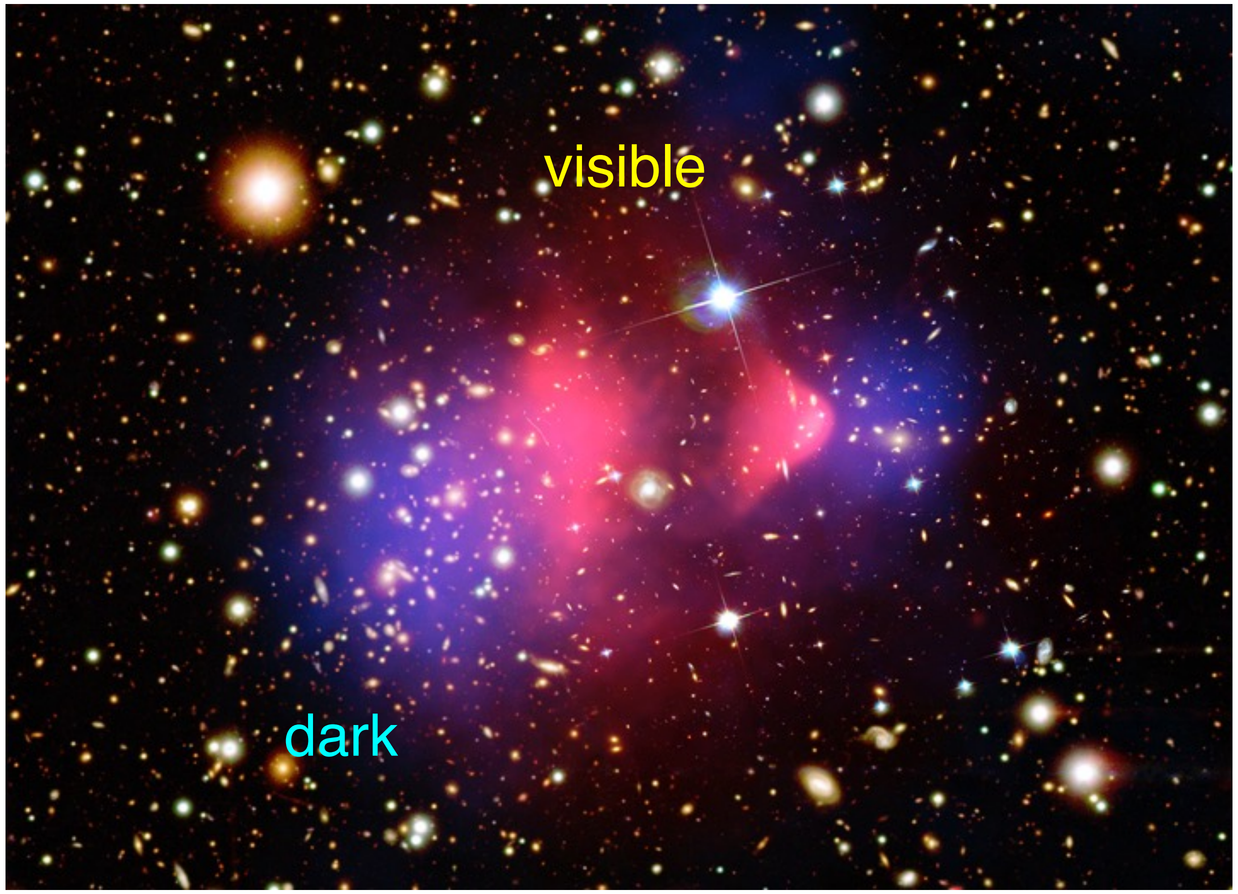
Wino dark matter

- enhanced formation of Earth-size halos

Dark matter

Dark matter

- evident from cosmological observations
    - cosmic microwave background (CMB)…
- essential to form galaxies in the Universe
- one of the biggest mysteries
    - astronomy, cosmology, particle physics…



bullet cluster

Weakly interactive massive particles

Attractive features of WIMPs

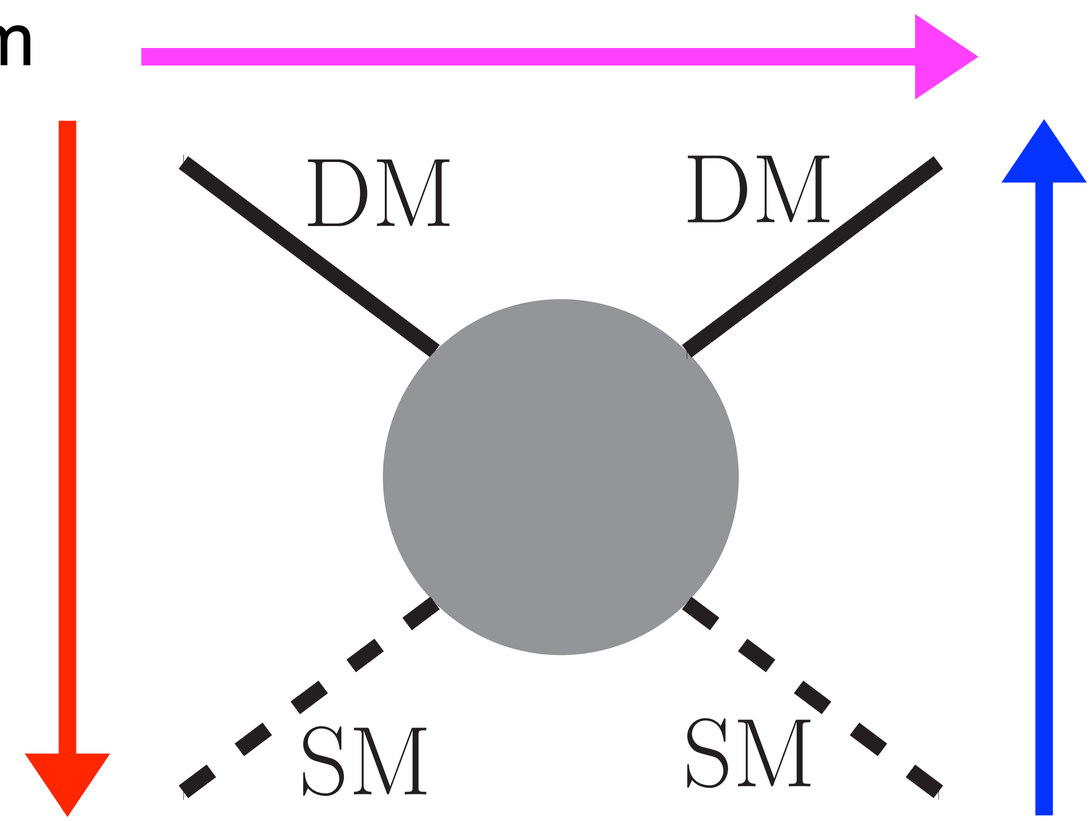
- thermal freeze-out (annihilation in the early Universe)

$$\Omega h^2 = 0.1 \times \frac{3 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

- weak-scale annihilation cross section $\langle \sigma_{\mathrm{ann}} v \rangle \simeq 1\,\mathrm{pb} \times c$

- well motivated by hierarchy problem and TeV-scale new physics

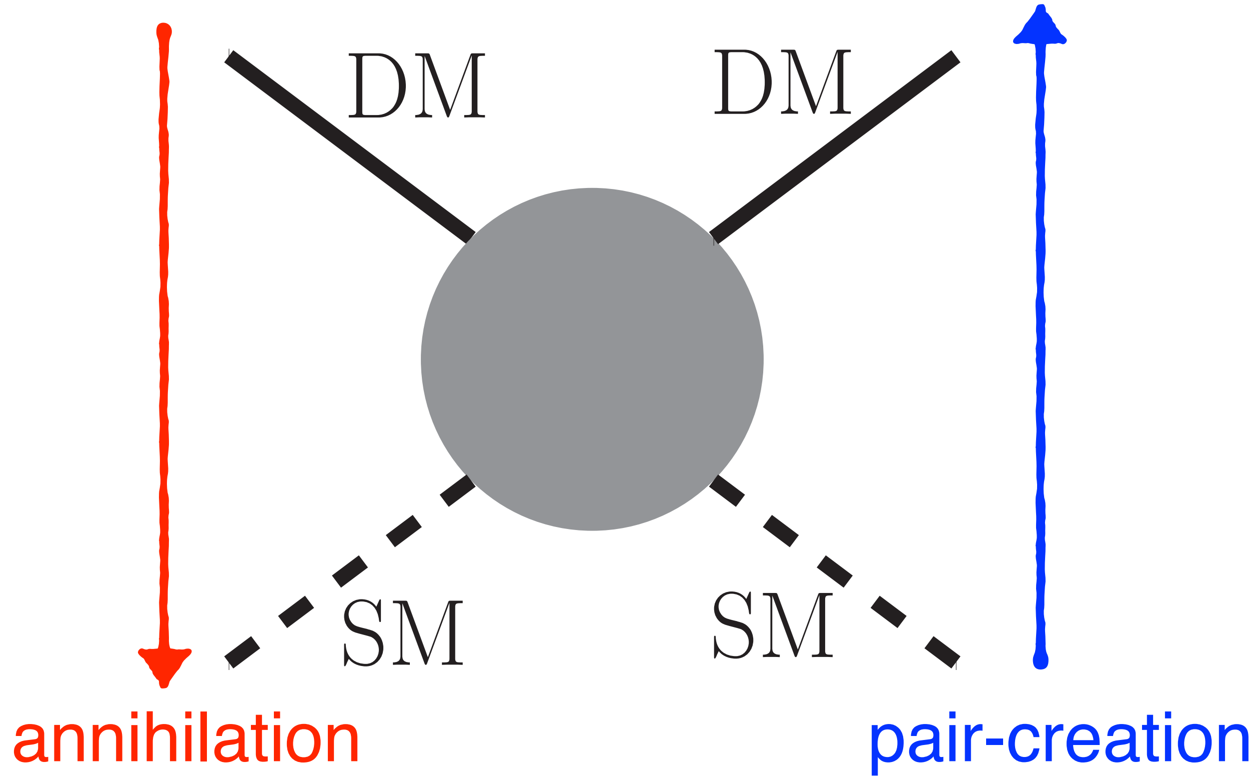
- various search strategies
  - direct detection
  - indirect detection
  - collider

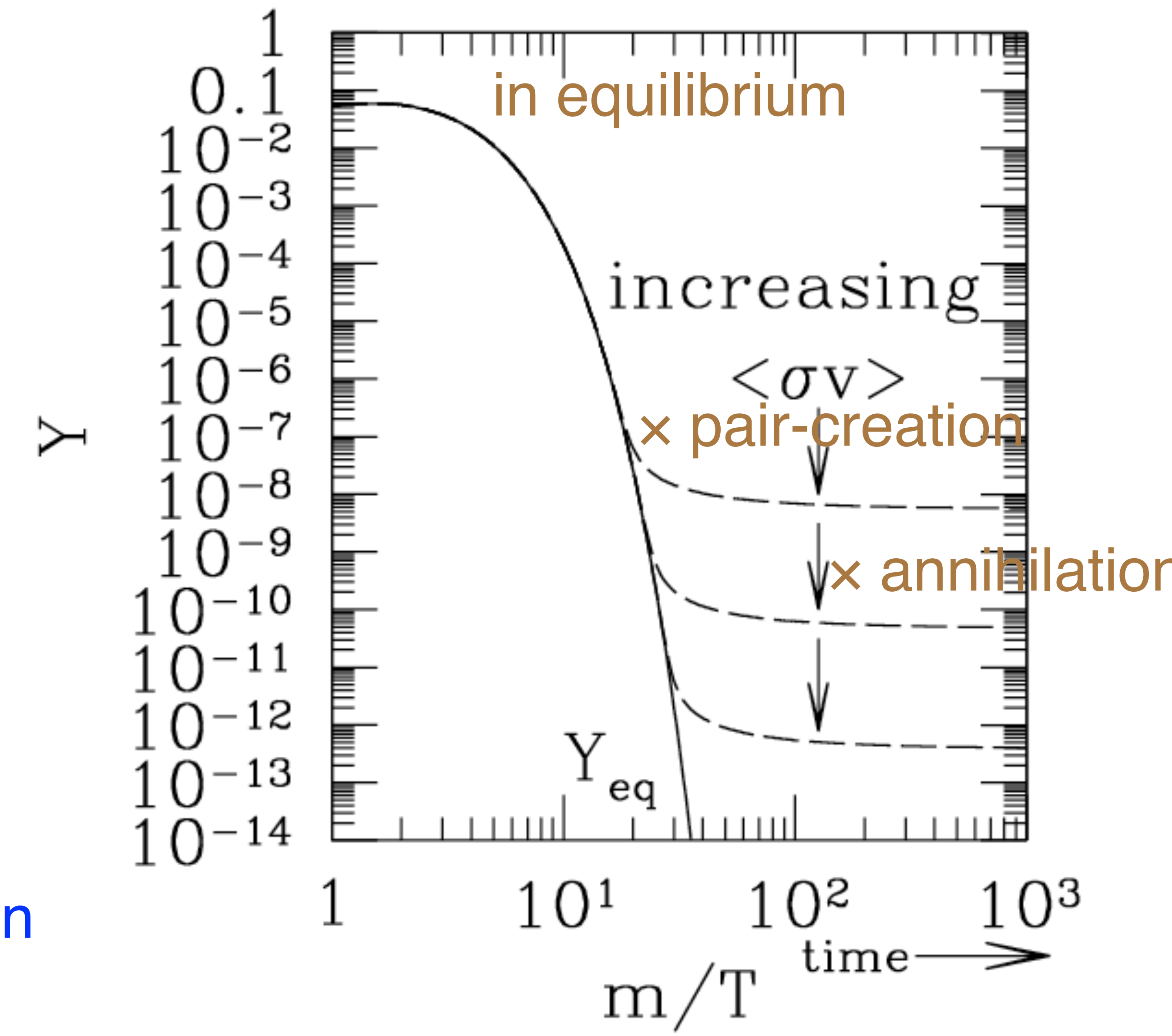


Thermal freeze-out

In a textbook

$$\dot{n}_{\rm dm} + 3Hn_{\rm dm} = -\langle\sigma v\rangle(\underline{n_{\rm dm}^2} - \underline{n_{\rm dm}^{\rm eq2}})$$





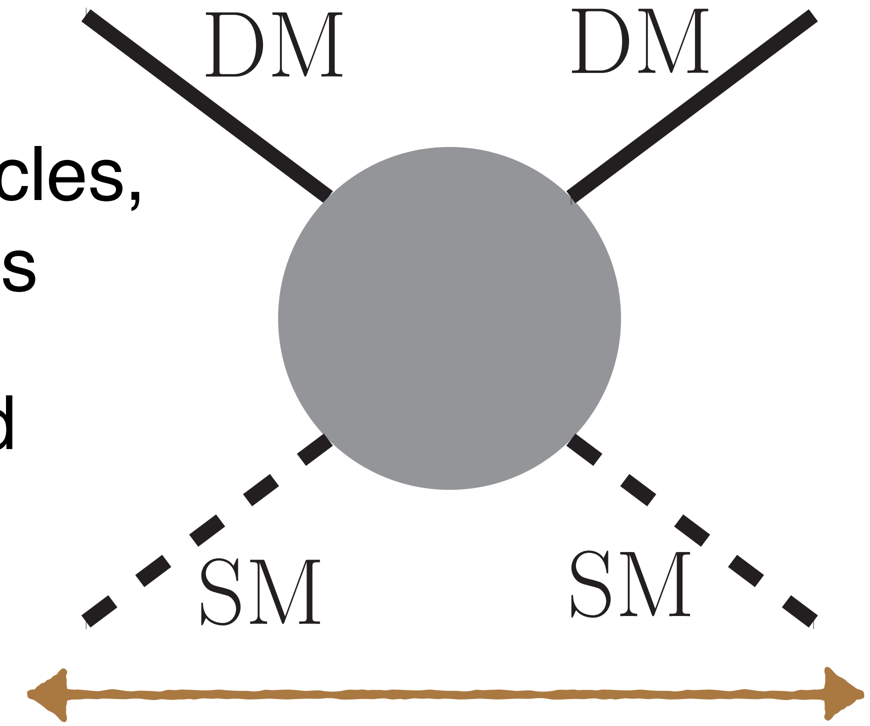
Underlying assumption

- detailed balance (T-invariance)

- kinetic equilibrium (distribution function is proportional to thermal distribution)

Kinetic equilibrium

Elastic scattering



- does not change the number of DM particles, but redistribute energy among DM particles

- if frequent, kinetic equilibrium is achieved

  - how to quantify it?

Our approach

Binder, Covi, AK, Murayama, Takahashi, and Yoshida, JCAP, 2016

- Fokker-Planck approximation (momentum transfer per collision << typical momentum of DM); diffusion in momentum space

$$\frac{1}{E}C_{\rm el}[f] = \frac{\partial}{\partial \vec{p}} \cdot \left[\gamma\left(mT\frac{\partial f}{\partial \vec{p}} + (\vec{p} - m\vec{u})f\right)\right]$$

$$\gamma = \frac{1}{6mT}\sum_s \int \frac{d^3\vec{k}}{(2\pi)^3} f^{\rm eq}(1 \mp f^{\rm eq}) \int_{-4\vec{k}^2}^{0} (-t)\frac{d\sigma}{dt} v$$

T - temperature of plasma

u - bulk velocity of plasma

- momentum transfer rate

t - Mandelstam variable

Kinetic decoupling

Gamow criterion

- if γ >> H, kinetic equilibrium is justified
- in a large class of WIMPs, kinetic decoupling happens after chemical decoupling (or freeze-out)
  - exceptions **Binder, Bringmann, Gustafsson, and Hryczuk, PRD, 2017**

Impact of kinetic decoupling (background level)

- by assuming Maxwell distribution of DM, from the Fokker-Planck equation, we derived the temperature evolution equation

$$\dot{T}_{\rm dm} + 2HT_{\rm dm} = -2\gamma(T_{\rm dm} - T)$$

  - if γ >> H, $T_{\rm dm} = T \propto 1/a$
  - if γ << H, $T_{\rm dm} \propto 1/a^2$

Binder, Gustafsson, AK, Sandner, and Wiesner, PRD, 2018

- temperature evolution is important for strongly-velocity dependent (say, Sommerfeld enhanced) annihilation cross section

Contents

Imprints on small-scale structures

- dark acoustic oscillation
- Landau damping
- overshooting of density perturbation

Wino dark matter

- enhanced formation of Earth-size halos

Kinetic decoupling

Imprint of kinetic decoupling (perturbation level)

- since Fokker-Planck equation is linear to phase space distribution, it is straightforward (but lengthy) to derive perturbation evolution equations, by using eigenfunction of Fokker-Planck operator in momentum space

$$\dot{\delta} = -\theta - \frac{1}{2}\dot{h}\,, \tag{2.50}$$

$$\dot{\theta} = -\frac{\dot{a}}{a}\theta - k^2\sigma + k^2(c_\chi^2\delta + \pi) + \gamma_0 a(\theta_{\rm TP} - \theta)\,, \tag{2.51}$$

$$\dot{\sigma} = -2\frac{\dot{a}}{a}\sigma - k\left(\frac{2T_0}{m_\chi}\right)^{3/2}\left(\frac{21}{4}f_{03} + f_{11}\right) + \frac{4}{3}\frac{T_0}{m_\chi}\theta + \frac{2}{3}\frac{T_0}{m_\chi}(\dot{h} + 6\dot{\eta}) - 2\gamma_0 a\sigma\,, \tag{2.52}$$

$$\begin{aligned}\dot{\pi} = & -2\frac{\dot{a}}{a}\pi + \frac{5}{4}k\left(\frac{2T_0}{m_\chi}\right)^{3/2} f_{11} - \frac{1}{a^2}\frac{d(a^2c_\chi^2)}{d\tau}\delta - \left(\frac{5}{3}\frac{T_0}{m_\chi} - c_\chi^2\right)\theta - \frac{1}{2}\left(\frac{5}{3}\frac{T_{\chi 0}}{m_\chi} - c_\chi^2\right)\dot{h} \\ & -2\gamma_0 a\left[\pi - \frac{T_1}{m_\chi} - \left(\frac{T_0}{m_\chi} - c_\chi^2\right)\delta\right] + 2\gamma_0 a\left(\frac{T_0}{T_{\chi 0}} - 1\right)\frac{T_{\chi 0}}{m_\chi}\frac{\gamma_1}{\gamma_0}\,.\end{aligned} \tag{2.53}$$

Binder, Covi, <u>AK</u>, Murayama, Takahashi, and Yoshida, JCAP, 2016

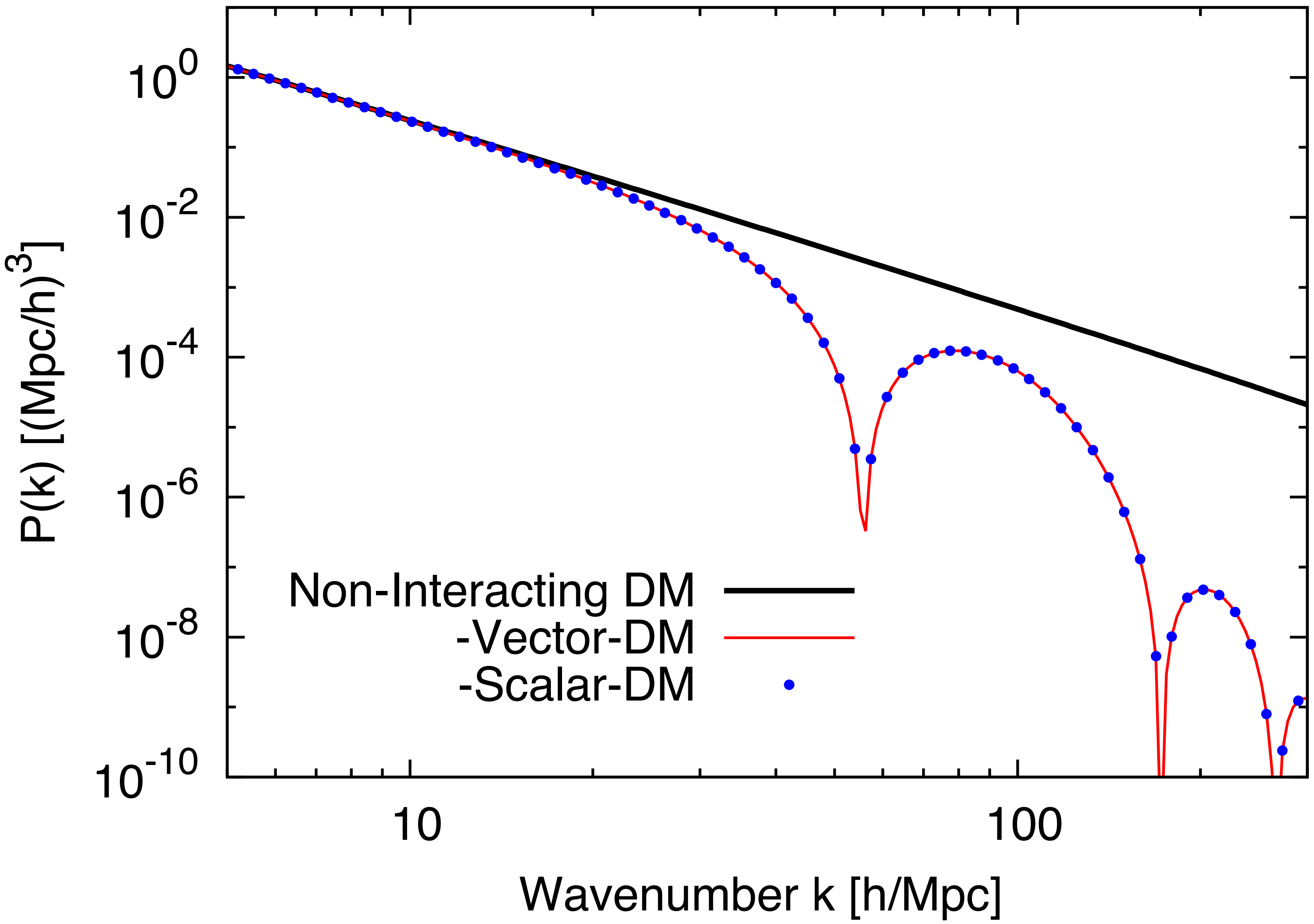
- in synchronous gauge (notation of Ma and Bertschinger, 1995); also in conformal Newtonian gauge; gauge equivalence is established

- only lower orders of Boltzmann hierarchy

Small-scale matter power spectrum

Dark acoustic oscillation

- because of plasma pressure



Binder, Covi, AK, Murayama, Takahashi, and Yoshida, JCAP, 2016

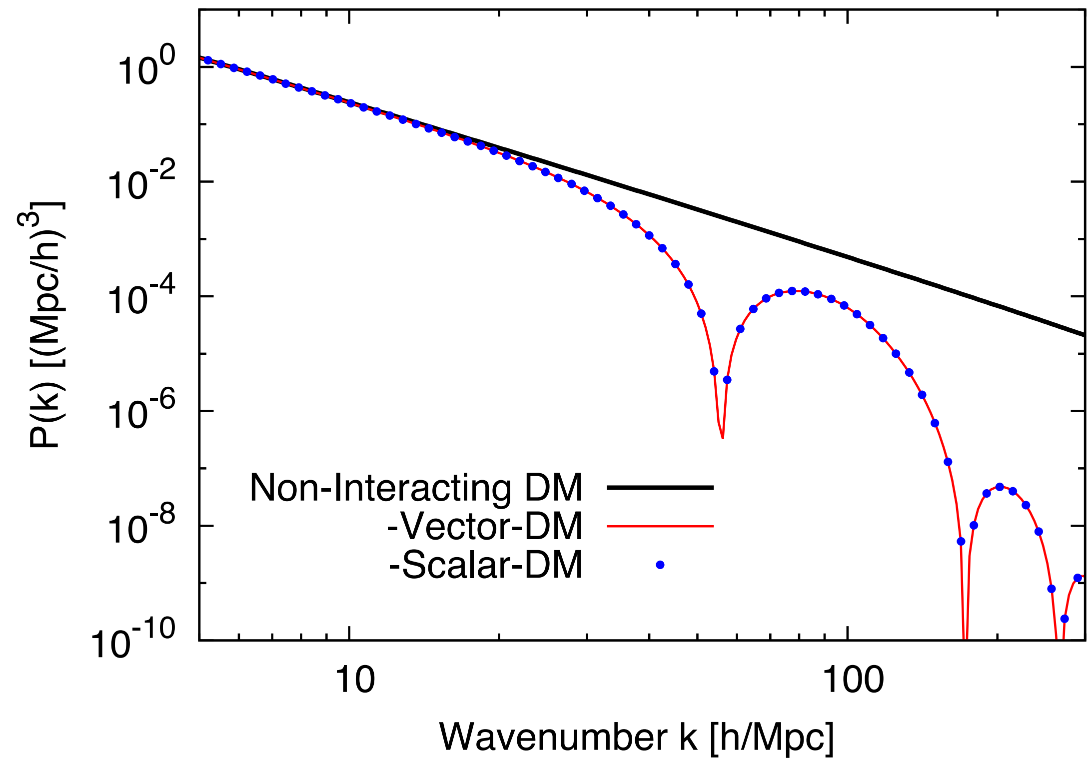
- oscillation scale is given by the Hubble horizon at the kinetic decoupling

Dark acoustic oscillation

Damped oscillation

- often attributed to plasma pressure preventing DM particles from clustering

  - but CDM clustering in radiation domination is just logarithmic

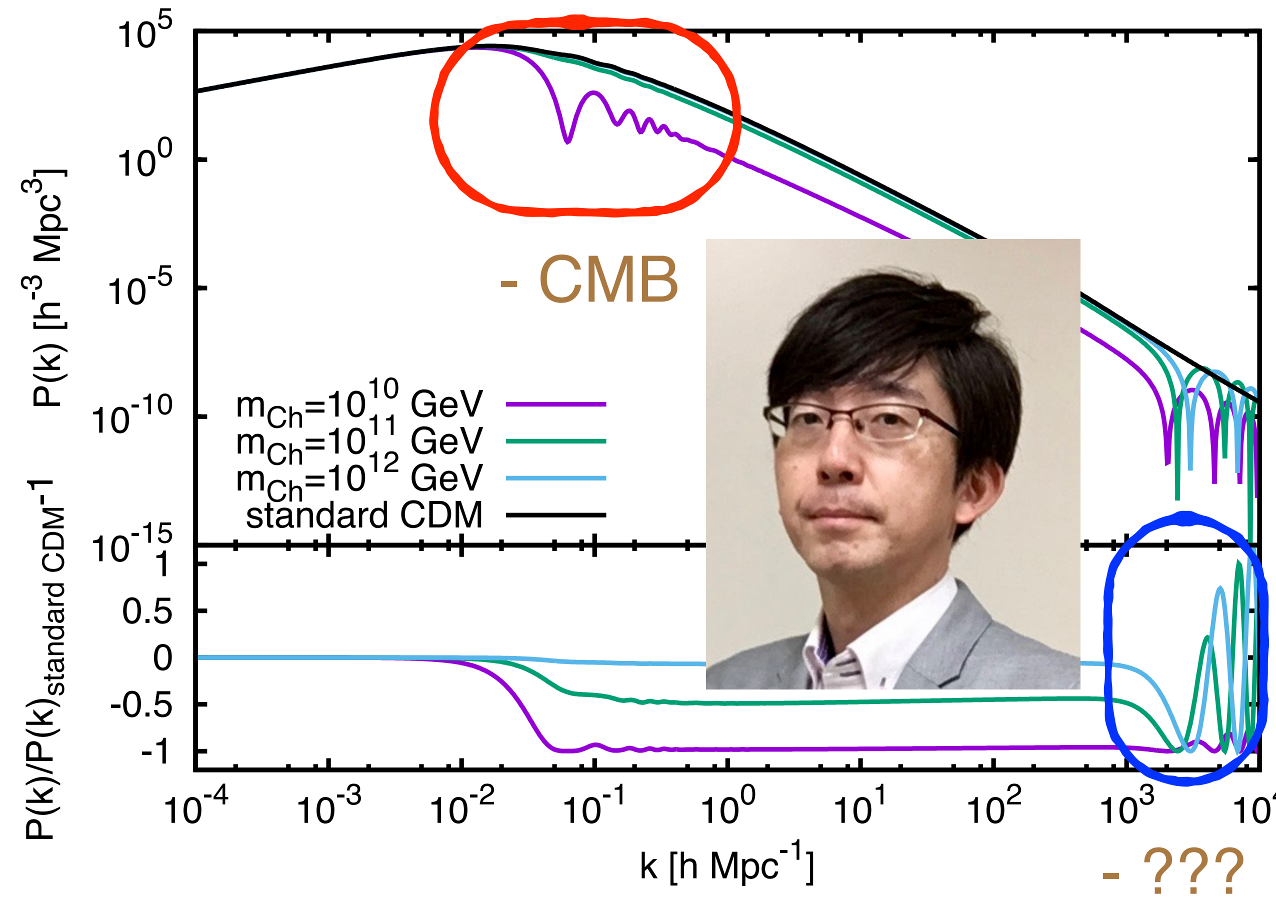


AK, Kohri, Takahashi, and Yoshida, PRD, 2016

Historical digression

- while studying imprints of charged massive particles on CMB, we by accident found undamped dark acoustic oscillation at very small scales

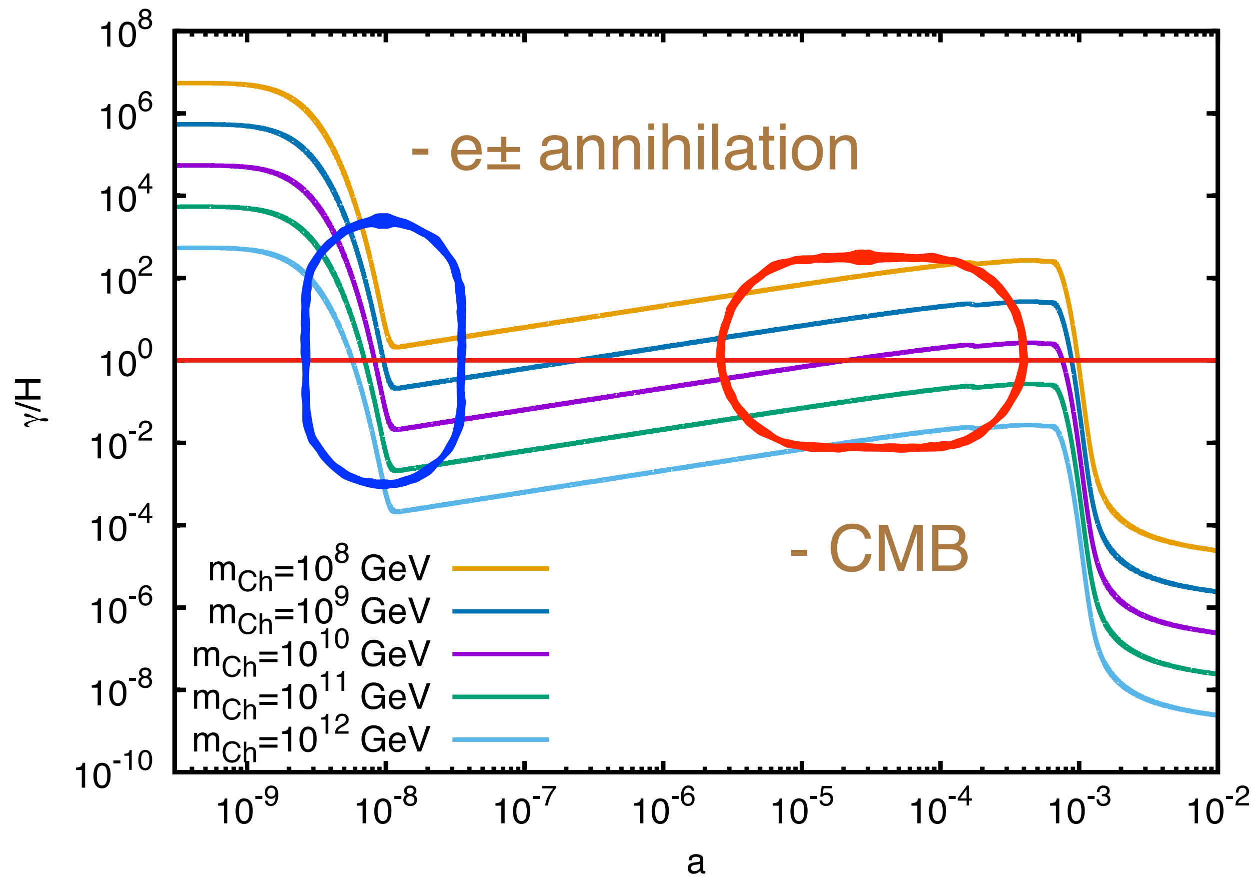
- first encounter with Tomo-san's strong opinion "We have to identify why"



Sudden kinetic decoupling

CHAMPs'γ

- sudden kinetic decoupling through e± annihilation



AK, Kohri, Takahashi, and Yoshida, PRD, 2016

damped oscillation

- is so-called Landau damping (phase mixing); finite duration of kinetic decoupling mixes different phases of oscillations

- plays a role in CMB spectrum though subdominant compared to Silk damping (by Komatsu-san)

Historical digression

- second encounter with Tomo-san's strong opinion "We have to develop analytical understanding"

Analytic understanding

In the end of the day

- done with analytic continuation, proper choice of integration path, and steepest descent approximation

We are interested in the resultant late time density perturbations and thus let us take $\tau \to \infty$ in eqs. (3.16) and (3.17). Then f_1 and f_2 become functions of k. In general we need to rely on numerical methods to evaluate the integrals in eqs. (3.16) and (3.17). On the other hand, if the momentum transfer rate per Hubble time follows a simple scaling law (corresponding to eq. (2.2)),

$$\gamma_0 a \tau = \left(\frac{\tau_d}{\tau}\right)^{n+4}, \tag{3.20}$$

we can analytically evaluate the integrals with the help of the steepest descent method as done in ref. [26]. There n is set to be 0, but we can generalize the result to an arbitrary value of n as

$$f_1 \simeq \left(\frac{4\pi}{5+n}q\right)^{1/2} \exp(-R) \left[\cos\left(I - \frac{4+n}{5+n}\frac{\pi}{4}\right) + q\cos\left(I - \frac{4+n}{5+n}\frac{3\pi}{4}\right)\right], \tag{3.21}$$

where R and I are respectively defined as the real and imaginary parts of

$$R + iI = 2\frac{5+n}{4+n} q \exp\left(i\frac{4+n}{5+n}\frac{\pi}{2}\right), \tag{3.22}$$

with

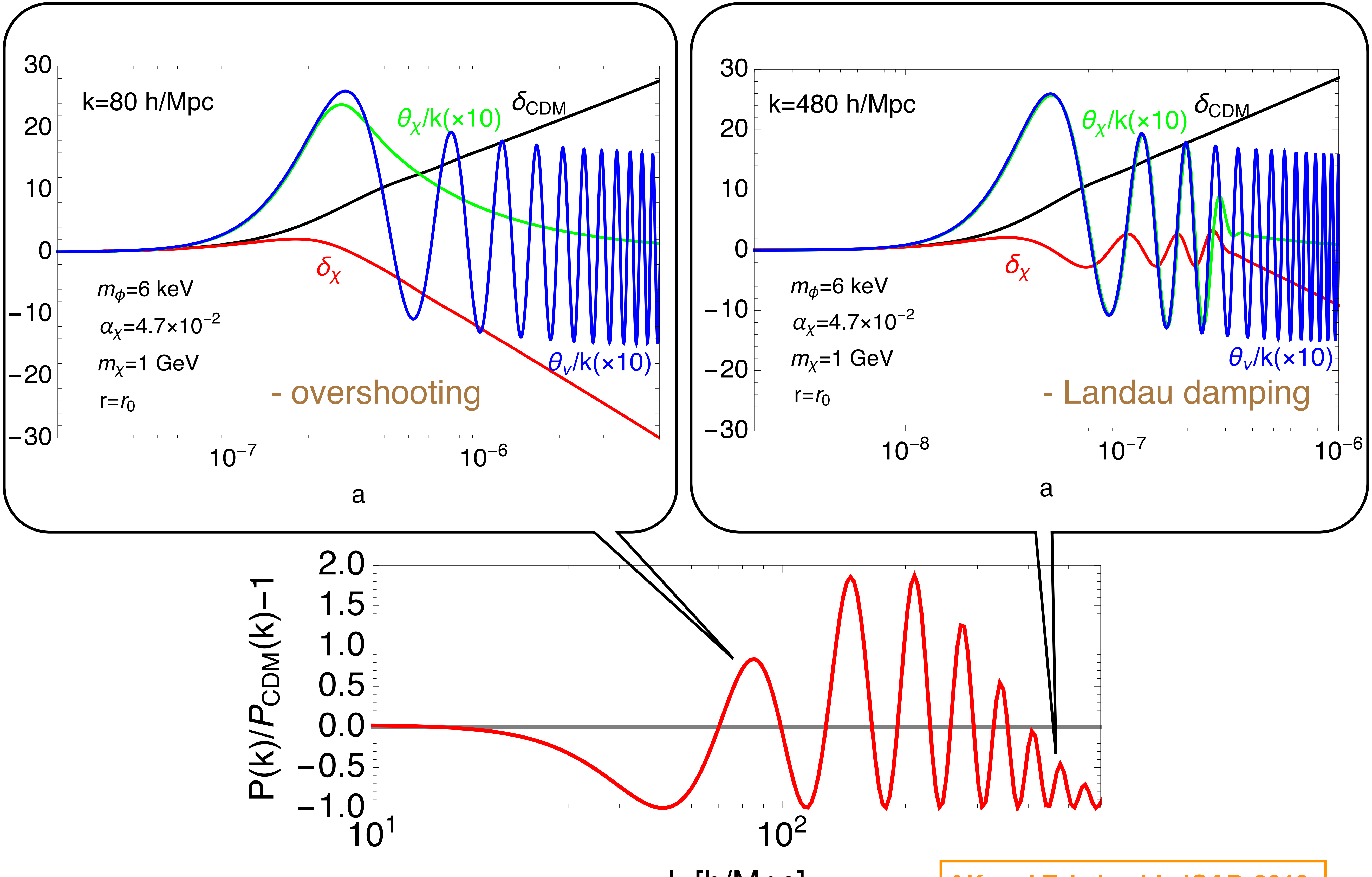
$$q = \frac{1}{2}\left(\frac{k\tau_d}{\sqrt{3}}\right)^{(4+n)/(5+n)}. \tag{3.23}$$

Bertschinger, PRD, 2006

AK and Takahashi, JCAP, 2018

- confirm no Landau damping in the instantaneous limit of kinetic decoupling

Overshooting



AK and Takahashi, JCAP, 2018

Contents

Wino dark matter

- enhanced formation of Earth-size halos

Electroweakino dark matter

One multiplet before electroweak symmetry breaking

- neutral component (dark matter) is associated with slightly heavier charged components

- kinetic decoupling of dark matter is determined by inelastic scattering to/from charged components rather than elastic scattering

  - electroweakino dark matter evades direct-detection experiments since inelastic scattering is kinematically not accessible

Take wino dark matter as an example

Hisano-san's talk

neutralino (χ^0) and charginos ($\chi^\pm$) $\Delta m_\chi \simeq 160\,\mathrm{MeV}$

- supersymmetric partner of weak gauge boson

Randall and Sundrum, NPB, 1999

- natural dark matter candidates in split-susy scenarios; anomaly mediation by Murayama-san!

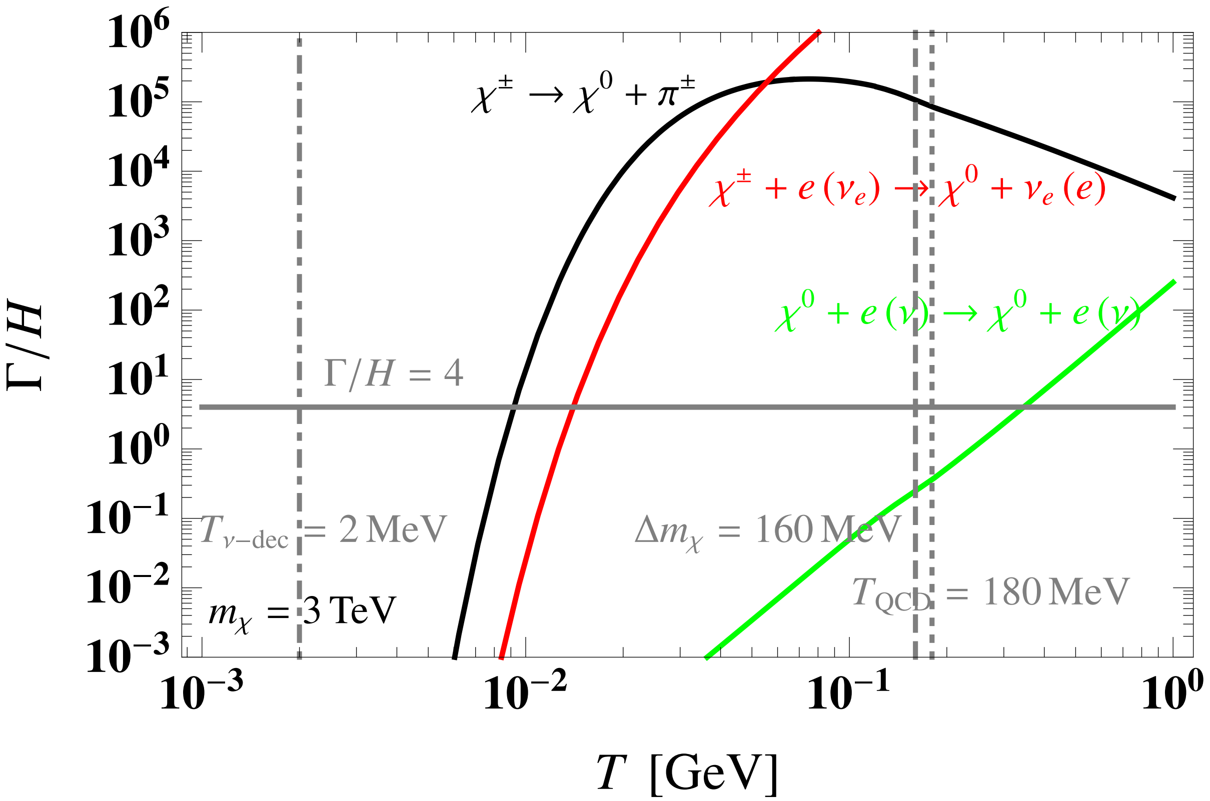
Giudice, Luty, Murayama, and Rattazzi, JHEP, 1998

Imprints on matter power spectrum

Ando, AK, Sekiguchi, and Takahashi, PRD, 2019

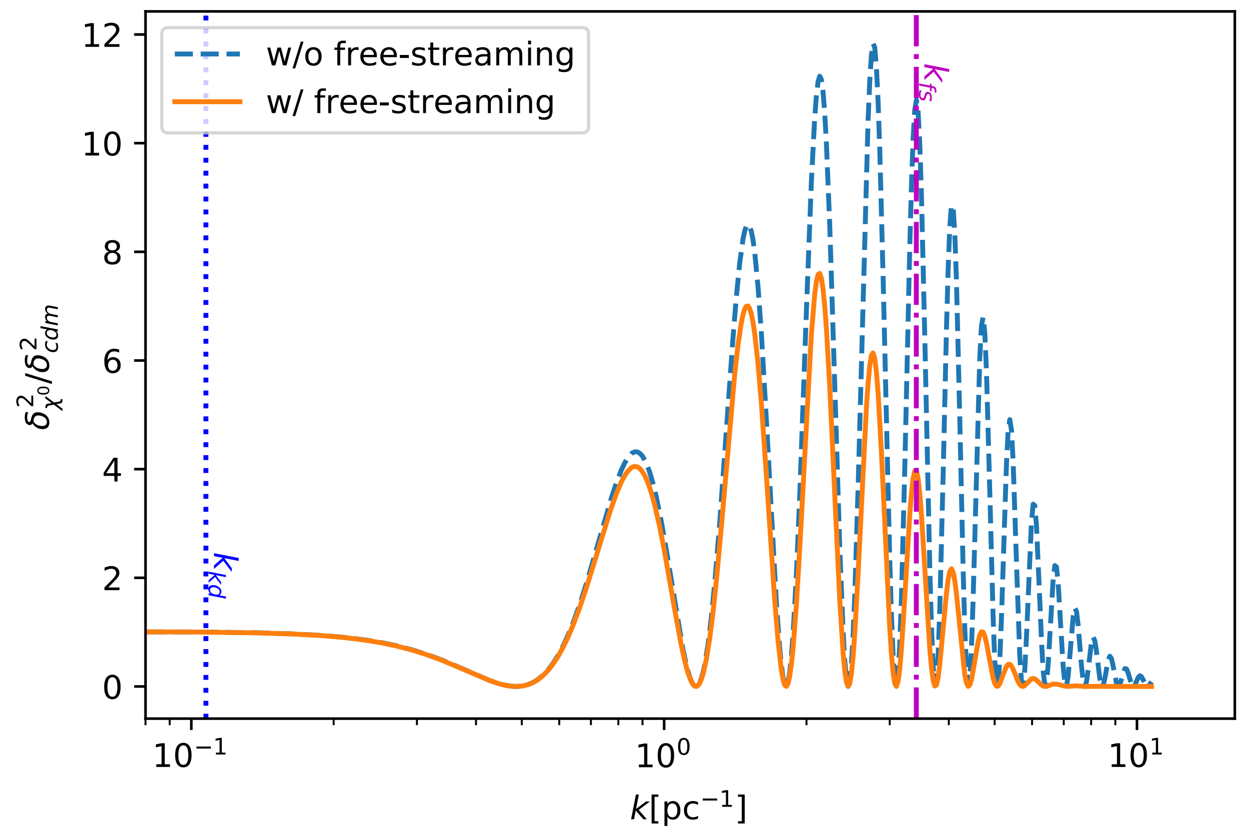
Sudden kinetic decoupling

- around $T \simeq \Delta m_\chi/20 \simeq 8\,\mathrm{MeV}$
- sharp T-dependence because of Boltzmann factor



Overshooting in matter power spectrum

- enhancement around pc scale
- not only Landau damping but free-streaming damping also suppress smaller-scale clustering

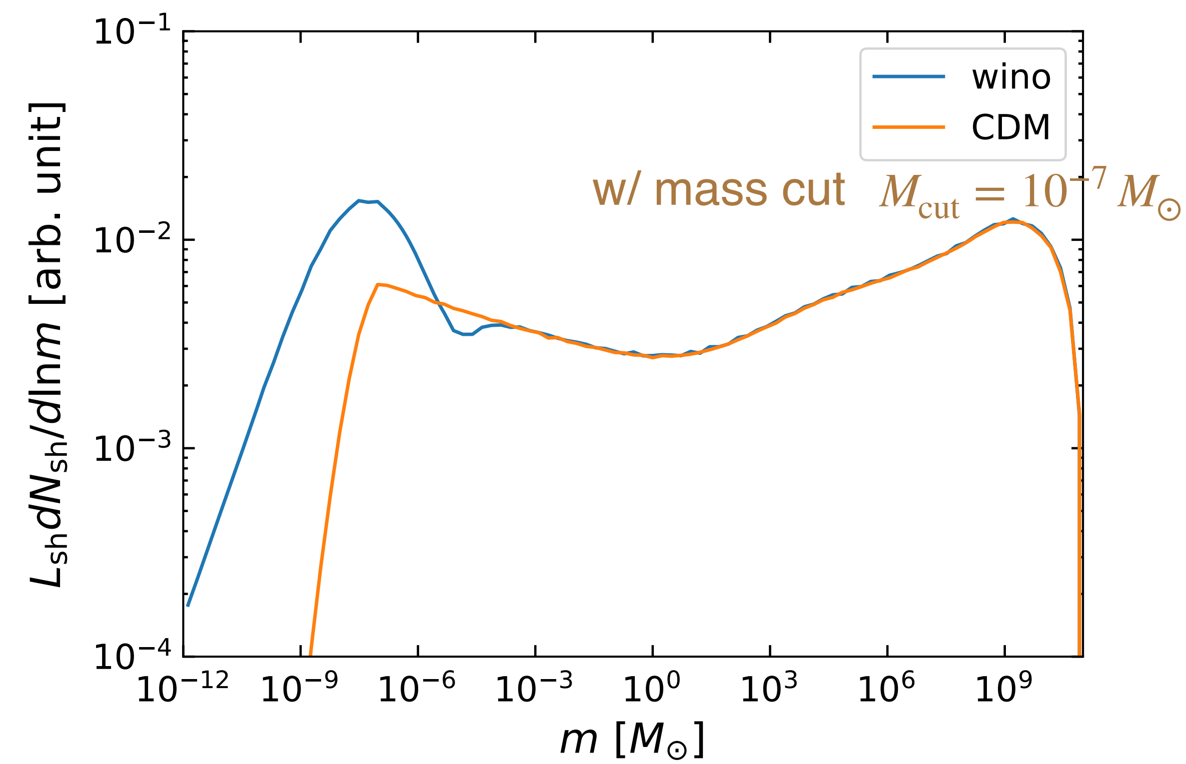


Enhance late-time annihilation rate

Ando, AK, Sekiguchi, and Takahashi, PRD, 2019

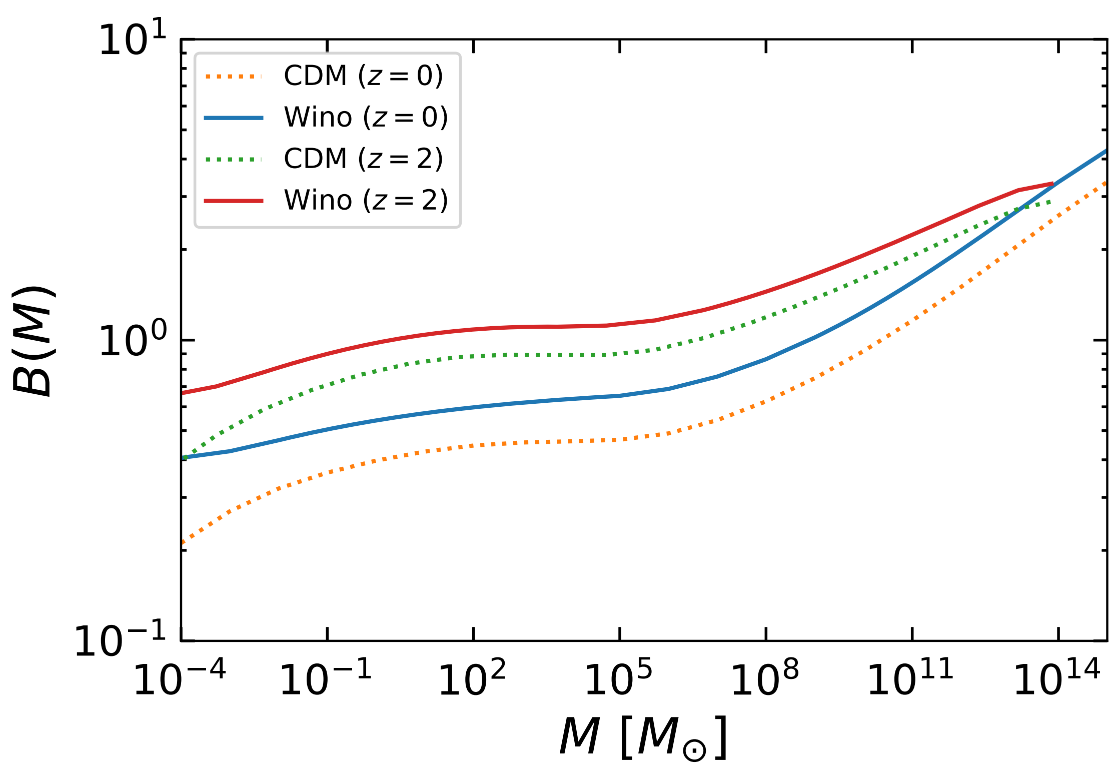
Enhance formation of Earth-size halos

- twice more compared to CDM
- also sub-halos in a halo



Boost factor

- DM particles in sub-halos have bigger annihilation rate than those in diffuse opponents even if total number is the same
- results in enhancement of (1+B)
- important for indirect detection experiments concerning galaxies and galaxy clusters



Summary

Murayama-san as a guru

- I have to learn how to propose projects with good prospects to students
- also how to provide good chances and environments
- I wish Murayama-san further success

Kinetic equilibration

- assumed in standard freeze-out
- Fokker-Planck approach to reduce collision term in a tractable form
- leaves dark acoustic oscillations in density perturbation
- not necessarily damped; overshooting

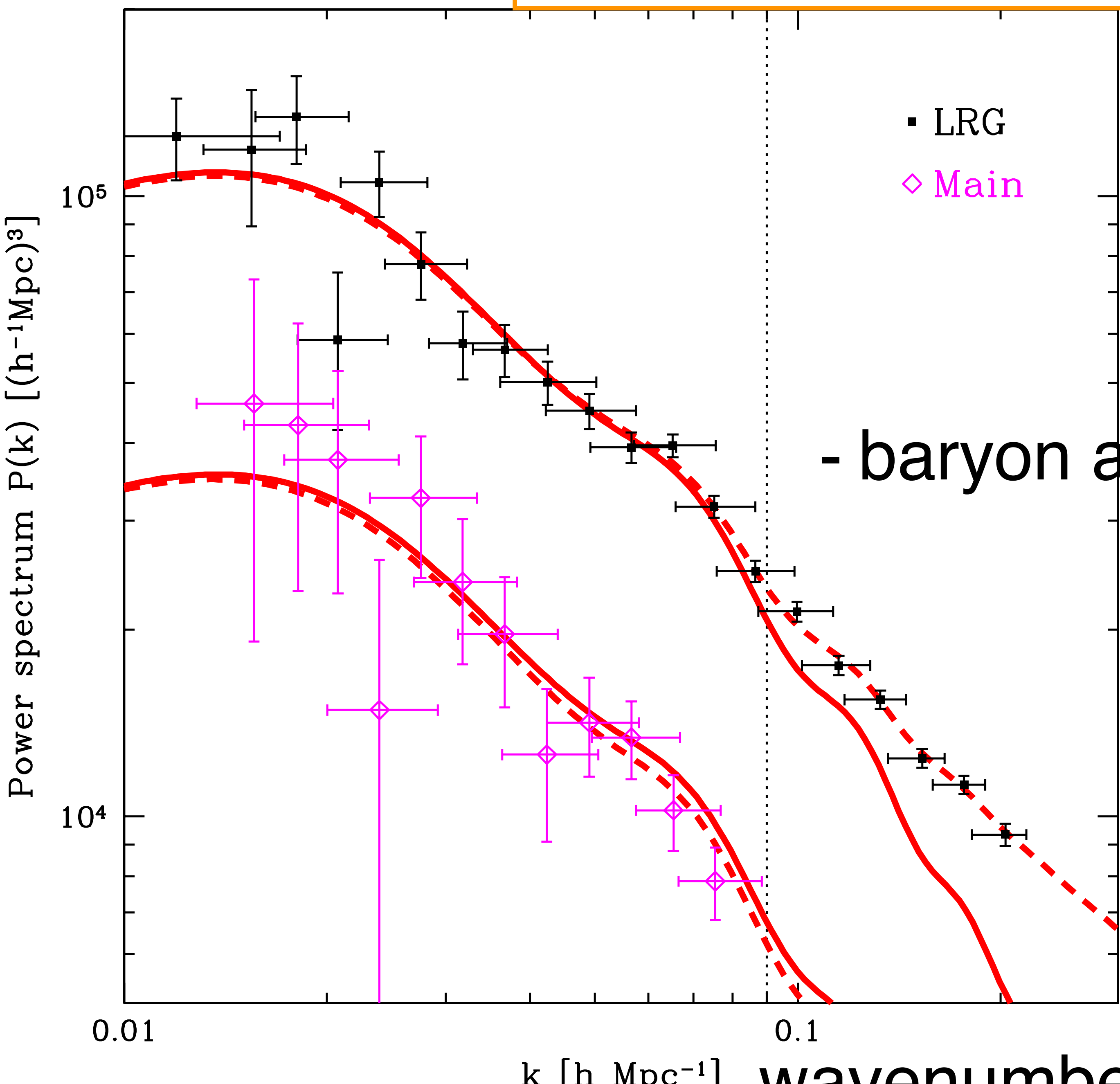
Thank you

Baryon acoustic oscillation

Baryons are involved in plasma acoustic oscillation until decoupling (recombination)

SDSS collaboration, PRD, 2006

matter power spectrum



- baryon acoustic oscillation

wiggles on the horizon scale at decoupling are imprinted on the matter power spectrum

observed in galaxy clustering

wavenumber [h/Mpc]

Part 1: (Mini-) split supersymmetry

Arkani-Hamed and Dimopoulos, JHEP, 2005 | Giudice and Romanino, NPB, 2004 | Wells, PRD, 2005 ...

LHC discovery of 125 GeV Higgs and null-detection of top partners

- something wrong in naturalness and postulated solutions (including but not only TeV-scale SUSY)

Mini-split SUSY: pragmatic SUSY mass spectrum

- sfermions, heavy Higgses > 100 TeV; gravitino > 100 TeV
  - 125 GeV Higgs although naturalness unanswered
  - no experimental (e.g., flavor) or cosmological (e.g., gravitino) problem
- gauginos ~ TeV; higgsino ~ ???: experimental window
  - provide a dark matter candidate $R\text{-parity} = (-1)^{3(B-L)+2s}$
  - precise grand unification of gauge couplings

Dark matter candidate

gravitino ~ 100 TeV, gauginos ~ TeV - anomaly mediation

→ (likely) wino (or higgsino) dark matter

Randall and Sundrum, NPB, 1999

Giudice, Luty, Murayama, and Rattazzi, JHEP, 1998

Thermal pure wino dark matter

Ibe and Yanagida, PLB, 2012 | Ibe, Matsumoto and Yanagida, PRD, 2012 ...

higgsino > 100 TeV

neutralino (χ^0) and charginos ($\chi^\pm$) $\Delta m_\chi \simeq 160\,\mathrm{MeV}$

Ibe, Matsumoto, and Sato PLB, 2013 ...

perturbative annihilation $\chi^0\chi^0 \to W^+W^-$

+ co-annihilation

+ Sommerfeld enhancement

→ $\Omega_\chi h^2 = \Omega_{\rm dm} h^2$ for $m_\chi \simeq 3\,\mathrm{TeV}$

Hisano, Matsumoto, Nagai, Saito, and Senami, PLB, 2007 ...

- no free parameter left

- accompanied by slightly heavier charged component (EWIMP or electroweakino)

Elastic processes for neutral wino

Elastic scattering is subdominant!

- $\chi^0 W^\pm \to \chi^0 W^\pm$: related with perturbative annihilation

  inefficient after $T \sim m_W$

- $\chi^0 L \to \chi^0 L \quad (L = \nu,\ e)$: loop suppressed **Ibe, AK, and Matsumoto, PRD, 2013**

$$\gamma_{\rm ela} = 8\frac{100}{\pi^3} g_{\rm loop}^2 G_F^4 m_W^4 \frac{T^6}{m_\chi}$$

$$g_{\rm loop} = \frac{1}{3\pi^2}\left(2(8-\omega-\omega^2)\sqrt{\frac{\omega}{4-\omega}}\arctan\left(\sqrt{\frac{4-\omega}{\omega}}\right) - \omega\left(2-(3+\omega)\ln\omega\right)\right)$$

$$\omega = m_W^2/m_\chi^2$$

inefficient after $T \sim 1\,\mathrm{GeV}$

Relevant processes for charged wino

$\chi^{\pm}$ is in kinetic equilibrium with primordial plasma $\gamma,\ e,\ \nu,\ \pi,\ \ldots$

$$f_{\chi^{\pm}}(\mathbf{p}) \approx \frac{n_{\chi^{\pm}}}{g_{\chi^{\pm}}}\left(\frac{2\pi}{m_{\chi}T}\right)^{3/2} \exp\left(-\frac{(\mathbf{p}-m_{\chi}\mathbf{u})^2}{2m_{\chi}T}\right)$$

$\mathbf{u}$: bulk velocity

$\chi^{\pm}$ and χ^{0} are in chemical equilibrium through inelastic processes

Arcadi and Ullio, PRD, 2011

- $\chi^{\pm} \to \chi^{0} + \pi^{\pm}$: decay

$$\Gamma_{\mathrm{dec}} \approx \frac{f_{\pi}^2 G_F^2 |V_{ud}|^2}{\pi} \Delta m_{\chi}^3 \sqrt{1-\frac{m_{\pi^{\pm}}^2}{\Delta m_{\chi}^2}}$$

- $\chi^{\pm} L' \to \chi^{0} L$: inelastic scattering

$$\Gamma_{\mathrm{inela}} = 2\frac{8G_F^2}{\pi^3}T^3\left(\Delta m_{\chi}^2 + 6\Delta m_{\chi}T + 12T^2\right)$$

$$n_{\chi^{\pm}} \approx \frac{g_{\chi^{\pm}}}{g_{\chi^{0}}} n_{\chi^{0}} \exp\left(-\frac{\Delta m_{\chi}}{T}\right)$$

Relevant processes for neutral wino

Inelastic processes keep χ^0 in kinetic equilibrium and are dominant

- $\chi^0 + \pi^\pm \to \chi^\pm, \quad \chi^0 L \to \chi^\pm L'$: conversion approximation

Arcadi and Ullio, PRD, 2011

$\Delta m_\chi, T << \sqrt{m_\chi T}$

$$\frac{1}{E} C_{\chi^0,\mathrm{inela}} \approx g_{\chi^\pm}(\Gamma_{\mathrm{dec}} + \Gamma_{\mathrm{inela}}) \left(f_{\chi^\pm} - f_{\chi^0} \exp\left(-\frac{\Delta m_\chi}{T} \right) \right)$$

For χ^0, interaction rates are suppressed by $\exp\left(-\Delta m_\chi / T\right)$

- efficient until $T \sim \Delta m_\chi / 20$

Cosmological background

- synchronous gauge

$$ds^2 = a^2\left(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right)\ , \quad h_{ij} = \hat{k}_i \hat{k}_j h + \left(\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij}\right)\eta$$

- evolution of number density $\quad n_\chi = g_\chi \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_\chi$

$$\dot{n}_{\chi^0} + 3\frac{\dot{a}}{a}n_{\chi^0} = -\left(\dot{n}_{\chi^\pm} - 3\frac{\dot{a}}{a}n_{\chi^\pm}\right)(\approx 0)\,,$$

- total wino number conservation - chemical equilibrium

- evolution of temperature $\quad 3m_\chi T_\chi n_\chi = g_\chi \int \frac{d^3\mathbf{p}}{(2\pi)^3}\mathbf{p}^2 f_\chi$

$$\dot{T}_{\chi^0} + 2\frac{\dot{a}}{a}T_{\chi^0} \approx a\boxed{\left[g_{\chi^\pm}(\Gamma_{\rm dec} + \Gamma_{\rm inela})\exp\left(-\frac{\Delta m_\chi}{T}\right) + 2g_{\chi^0}\gamma_{\rm ela}\right]}\left(T - T_{\chi^0}\right)$$

- effective reaction rate

Cosmological perturbations

- evolution of density perturbation $n_\chi \delta_\chi = g_\chi \int \frac{d^3\mathbf{p}}{(2\pi)^3} \delta f_\chi$

$$\dot{\delta}_{\chi^0} + \theta_{\chi^0} + \frac{1}{2}\dot{h} = -\left(\frac{\dot{n}_{\chi^0}}{n_{\chi^0}} + 3\frac{\dot{a}}{a}\right)(\delta_{\chi^0} - \delta_{\chi^\pm}) - \frac{n_{\chi^\pm}}{n_{\chi^0}}\left(\dot{\delta}_{\chi^\pm} + \theta_T + \frac{1}{2}\dot{h}\right)(\approx 0)$$

- total wino number conservation

$$\delta_{\chi^\pm} \approx \delta_{\chi^0} + \frac{\Delta m_\chi}{T}\delta_T, \qquad \delta_T = \frac{\delta T}{T}, \quad \theta_T = i\mathbf{k}\cdot\mathbf{u}$$

- chemical equilibrium

- evolution of velocity perturbation $m_\chi n_\chi \theta_\chi = g_\chi \int \frac{d^3\mathbf{p}}{(2\pi)^3}(i\mathbf{k}\cdot\mathbf{p})\delta f_\chi$

$$\dot{\theta}_{\chi^0} + \frac{\dot{a}}{a}\theta_{\chi^0} \approx a\boxed{g_{\chi^\pm}(\Gamma_{\text{dec}} + \Gamma_{\text{inela}})\exp\left(-\frac{\Delta m_\chi}{T}\right)}\left(\theta_T - \theta_{\chi^0}\right)$$

- the same effective reaction rate as for temperature evolution

- sour term drives dark acoustic oscillation

Overshooting phenomenon

Seen when kinetic decoupling proceeds rapidly

$$\frac{1}{\Gamma/H}\frac{d(\Gamma/H)}{dt} \gg H$$ around kinetic decoupling $\Gamma/H \sim 1$

Discovered for Coulomb scattering of charged massive particle (CHAMP) with electrons $\Gamma \propto \exp\left(-m_e/T\right)$ $T_{\rm kd} \sim m_e/20$

AK, Kohri, Takahashi, and Yoshida, PRD, 2017

Analytic approximation can be derived for $\Gamma/H = \left(\tau/\tau_{\rm d}\right)^n$

Extension with fudge factors for $k \gg k_{\rm kd}$

AK and Takahashi, JCAP, 2018

$$\frac{\delta^2_{\chi^0}}{\delta^2_{\rm cdm}} \approx c_{\mathcal{N}}\frac{4\pi}{n}\exp\left(-\frac{k}{k_{\rm damp}}\right)\left(\frac{k}{2\sqrt{3}k_{\rm kd}}\right)^3\sin^2\left(\frac{k}{\sqrt{3}k_{\rm kd}}\right)$$

- damping - enhancement

$$k_{\rm damp} = c_{\rm damp}\frac{n}{\pi}\sqrt{3}k_{\rm kd}\,, \qquad k_{\rm kd} = \frac{c_{\rm kd}}{\tau_{\rm kd}}$$

Free-streaming

Dark matter thermal motion is neglected

- pressure, sound speed, …

Higher multipoles in the Boltzmann hierarchy are truncated

- anisotropic inertia, entropy perturbation, …

Free-streaming is taken into account as $\delta_\chi \to \delta_\chi \times \exp\left(-\frac{k^2}{2k_{\rm fs}(\tau)^2}\right)$

Bertschinger, PRD, 2006

$$k_{\rm fs}^{-1} = \sqrt{\frac{6T_{\rm kd}}{5m_{\chi^0}}} \int_{\tau_*}^{\tau} \frac{d\tau'}{a(\tau')/a_{\rm kd}} \approx \sqrt{\frac{6T_{\rm kd}}{5m_{\chi^0}}} \tau_{\rm kd} \ln\left(\frac{\tau_{\rm eq}}{\tau_*}\right), \quad \tau_* = 1.05\tau_{\rm kd}$$

- long after the matter radiation equality

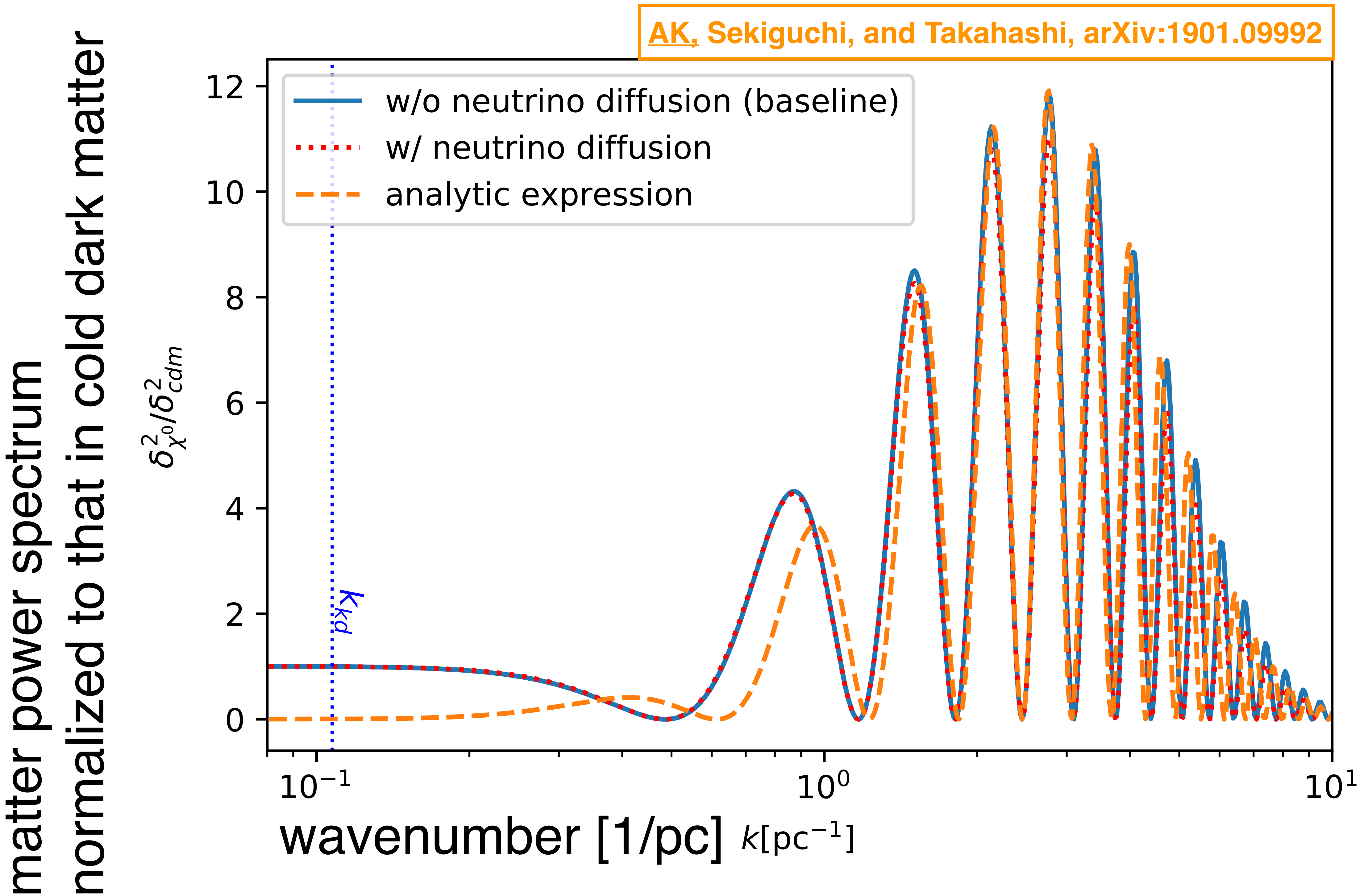
$$k_{\rm fs} \simeq 3.5\,/{\rm pc} > 1/\tau_{\rm kd} \simeq 0.11\,/{\rm pc}$$

$$M_{\rm fs} \simeq 1.0 \times 10^{-7}\, M_\odot < M_{\rm kd} \simeq 1.1 \times 10^{-4}\, M_\odot$$

$$M_{\rm kd} = \frac{4\pi}{3}\rho_{\chi,0}\tau_{\rm kd}^3$$

$$M_{\rm fs} = \frac{4\pi}{3}\rho_{\chi,0}\left(\frac{\pi}{k_{\rm fs}}\right)^3$$

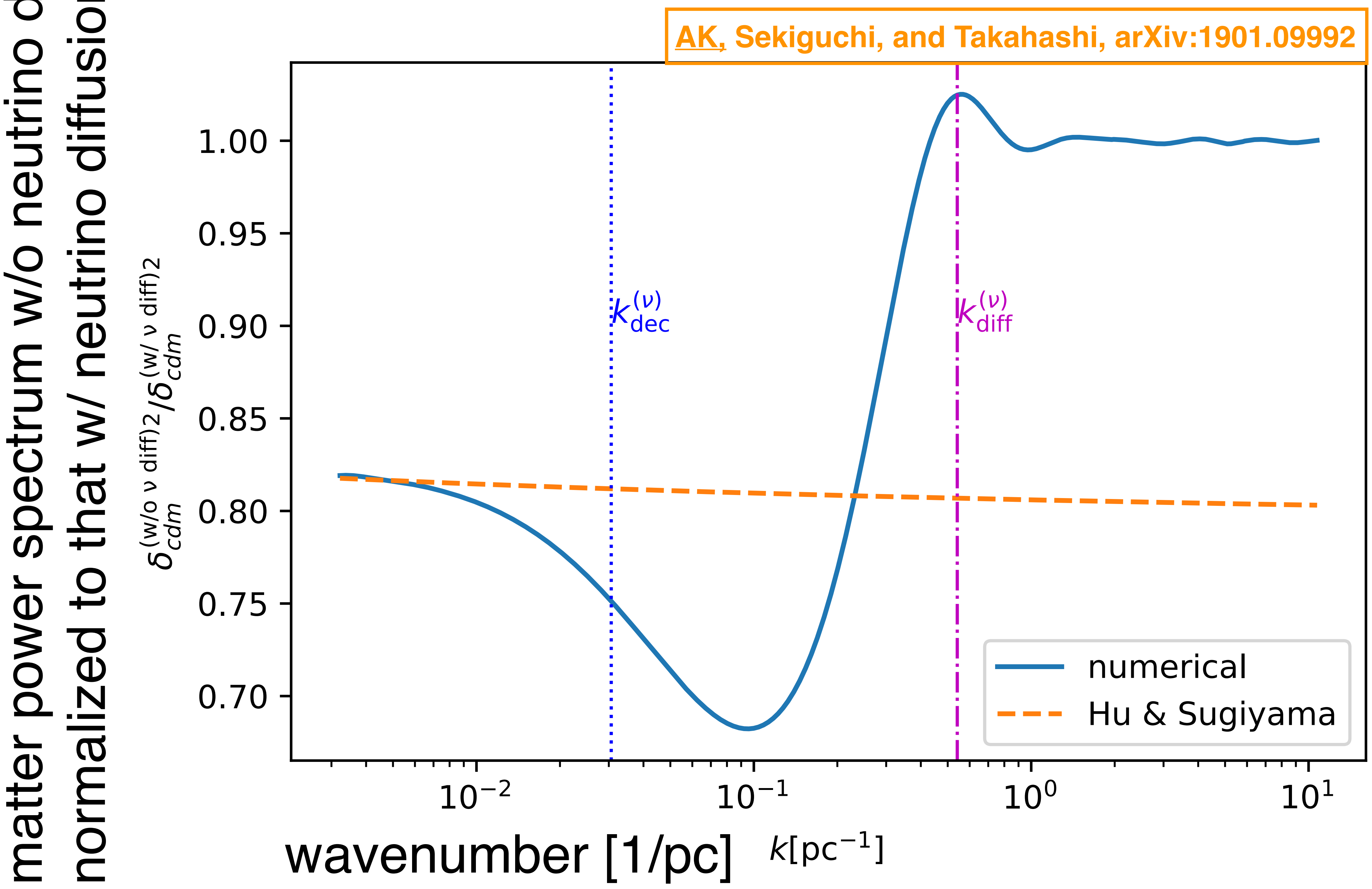
Neutrino diffusion



Neutrino diffusion is not important for the ratio $\delta^2_{\chi^0}/\delta^2_{\mathrm{cdm}}$

Neutrino diffusion

Cold dark matter w/ and w/o neutrino diffusion



Neutrino diffusion changes the power spectrum up to 30%

Part 3: Boost factor

Wino (or generically electroweakino) dark matter

- annihilation into gauge bosons are non-perturbatively enhanced toward lower velocity (Sommerfeld enhancement)

Hisano, Matsumoto, and Nojiri, PRD, 2003

→ good target of indirect detection experiments

Hisano, Matsumoto, and Nojiri, PRL, 2004

...

Boost factor: $L(M) = (1+B(M))\bar{L}(M)$ - total luminosity

$\bar{L}(M)$ - contribution from coarse-grained distribution squared $\langle\rho\rangle^2$

$\langle\rho^2\rangle/\langle\rho\rangle^2 \geq 1$ $\quad\langle\rho\rangle$ - e.g. Navarro-Frenk-White (NFW) profile

$B(M)$ - we estimate a subhalo contribution by using a halo model with extrapolations toward small scales

Strigari, Koushiappas, Bullock, and Kaplinghat, PRD, 2007

Kuhlen, Diemand, and Madau, ApJ, 2008

Boost factor

We adopt a halo model approach to estimate the boost factor with extrapolations

$$B(M) = \frac{1}{\bar{L}(M)} \int_{m_{\rm min}}^{M} dm \frac{dn_{\rm sub}}{dm} (1 + B_{\rm sub}(m)) \bar{L}_{\rm sub}(m)$$

$m_{\rm min}$: minimal halo mass

Hierarchical structure: $B = B_{\rm sub}$, $\bar{L} = \bar{L}_{\rm sub}$

Coarse-grained luminosity $\bar{L}$

- Navarro-Frenk-White (NFW) profile c-M relation

power-law or flattened?

Subhalo mass function: $dn_{\rm sub}/dm$ power-law index $\alpha = 1.9\text{-}2.0$

$$\times \left. \frac{(dn/d\ln M)_\chi}{(dn/d\ln M)_{\rm cdm}} \right|_{\rm PS}$$ for wino dark matter

Halo profile

Navarro-Frenk-White (NFW) profile: $\rho = \dfrac{\rho_s}{(r/r_s)(1+r/r_s)^2}$

Navarro, Frenk, and White, ApJ, 1996

Navarro, Frenk, and White, ApJ, 1997

$$\left(\rho_s,\ r_s\right) \leftrightarrow \left(c = r_{200}/r_s,\ M = M_{200}\right) \to \bar{L} = Mc^3/f(c)^2$$

$f(c) = \ln(1+c) - c/(1+c)$ - normalization does not matter

c-M relation

- case 1: power law - more sensitive to the smallest halos

$$c_{200} = 7.80\left(\frac{M_{200}}{10^{12}\,M_\odot/h}\right)^{-0.08}\left[1 + 0.2\left(\frac{M_{200}}{10^{15}\,M_\odot/h}\right)^{1/2}\right]$$

Gao, Frenk, Jenkins, Springel, and White, MNRAS, 2012

Prada, Klypin, Cuesta, BetancortRijo, and Primack, MNRAS, 2012

- case 2: flattening toward smaller halos

Diemand, Moore, and Stadel, Nature, 2005 … Sanchez-Conde and Prada, MNRAS, 2012

$$c_{200} = \sum_{i=0}^{5} c_i\left[\ln\left(\frac{M_{200}}{M_\odot/h}\right)\right]^i$$

$$c_i = \{37.5153,\ -1.5093,\ 1.636\times10^{-2},\ 3.66\times10^{-4},\ -2.89237\times10^{-5},\ 5.32\times10^{-7}\}$$

Subhalo mass function

Cold dark matter subhalo mass function:

$$\frac{dn_{\rm sub}}{dm} = \frac{A}{M}\left(\frac{m}{M}\right)^{-\alpha}, \quad A : \int_{10^{-5}M}^{10^{-2}M} dm\, m \frac{dn_{\rm sub}}{dm} = 0.1$$

Springel, Wang, Vogelsberger, Ludlow, Jenkins, Helmi, Navarro, Frenk, and White, MNRAS, 2008

$(\alpha, A) = (1.9,\ 0.0318)$ Diemand, Kuhlen, and Madau, ApJ, 2007 | Madau, Diemand, and Kuhlen, ApJ, 2008

$(2.0,\ 0.0145)$ - more sensitive to the smallest halos

Wino subhalo mass function:

$$\frac{dn_{\rm sub}}{dm} \to \frac{dn_{\rm sub}}{dm} \times \left.\frac{(dn/d\ln M)_\chi}{(dn/d\ln M)_{\rm cdm}}\right|_{\rm PS}$$ and $m_{\rm min} = 0$

(for comparison, $\frac{dn_{\rm sub}}{dm}$ as it is and $m_{\rm min} = M_{\rm kd} \simeq 1.1 \times 10^{-4} M_\odot$)

Subhalo mass function

Modified Press-Schechter: $$\frac{dn}{d\ln M} = \frac{\rho_{\chi,0}}{M} f(\nu) \frac{d\ln\nu}{d\ln M}$$

$$f(\nu) = A\sqrt{\frac{2q\nu^2}{\pi}}\left(1 + (q\nu^2)^{-p}\right)\exp\left(-\frac{q\nu^2}{2}\right), \qquad \nu = \frac{\delta_c}{D(z)\sigma(M)}$$

- growth factor $D(0) = 1$

Ellipsoidal collapse: $A = 0.3222\,,\ p = 0.3\,,\ q = 1$

Sheth and Tormen, MNRAS, 1999

Leo, Baugh, Li, and Pascoli, JCAP, 2018

Smooth-k filter: $$\sigma^2(M) = \int_0^\infty d\ln k \frac{k^3}{2\pi^2} P(k) W^2(k;M)$$

$$W(k;M) = (1 + (kR)^{\hat{\beta}})^{-1}\,,\ M = \frac{4\pi}{3}\rho_{m,0}(\hat{c}R)^3\,, \qquad \hat{\beta} = 4.8\,,\ \hat{c} = 3.30$$

- match to simulation results for matter power spectrum with dark acoustic oscillation

Subhalo mass function

Modified Press-Schechter: $$\frac{dn}{d\ln M} = \frac{\rho_{\chi,0}}{M} f(\nu) \frac{d\ln\nu}{d\ln M}$$

$$f(\nu) = A\sqrt{\frac{2q\nu^2}{\pi}}\left(1 + (q\nu^2)^{-p}\right)\exp\left(-\frac{q\nu^2}{2}\right), \quad \nu = \frac{\delta_c}{D(z)\sigma(M)}$$

- growth factor

Ellipsoidal collapse: $A = 0.3222\,,\ p = 0.3\,,\ q = 0.707$ $\quad D(0) = 1$

Sheth and Tormen, MNRAS, 1999

(Real-space) top-hat filter: $$\sigma^2(M) = \int_0^\infty d\ln k \frac{k^3}{2\pi^2} P(k) W^2(k; M)$$

$$W(k; M) = \frac{3(\sin(kR) - kR\cos(kR))}{(kR)^3}\,,\ M = \frac{4\pi}{3}\rho_{m,0}R^3$$

Estimated boost factor

Smooth-k filter:

Enhanced by a factor of 4.1 and 8.8 (1.5 and 2.9)
with $\alpha = 1.9$ and 2.0 in case 1 (2) when compared to $m_{\rm min} = M_{\rm kd}$

(Real-space) top-hat filter:

5.4 and 12.5 (1.7 and 3.5)

Without $B_{\rm sub}$

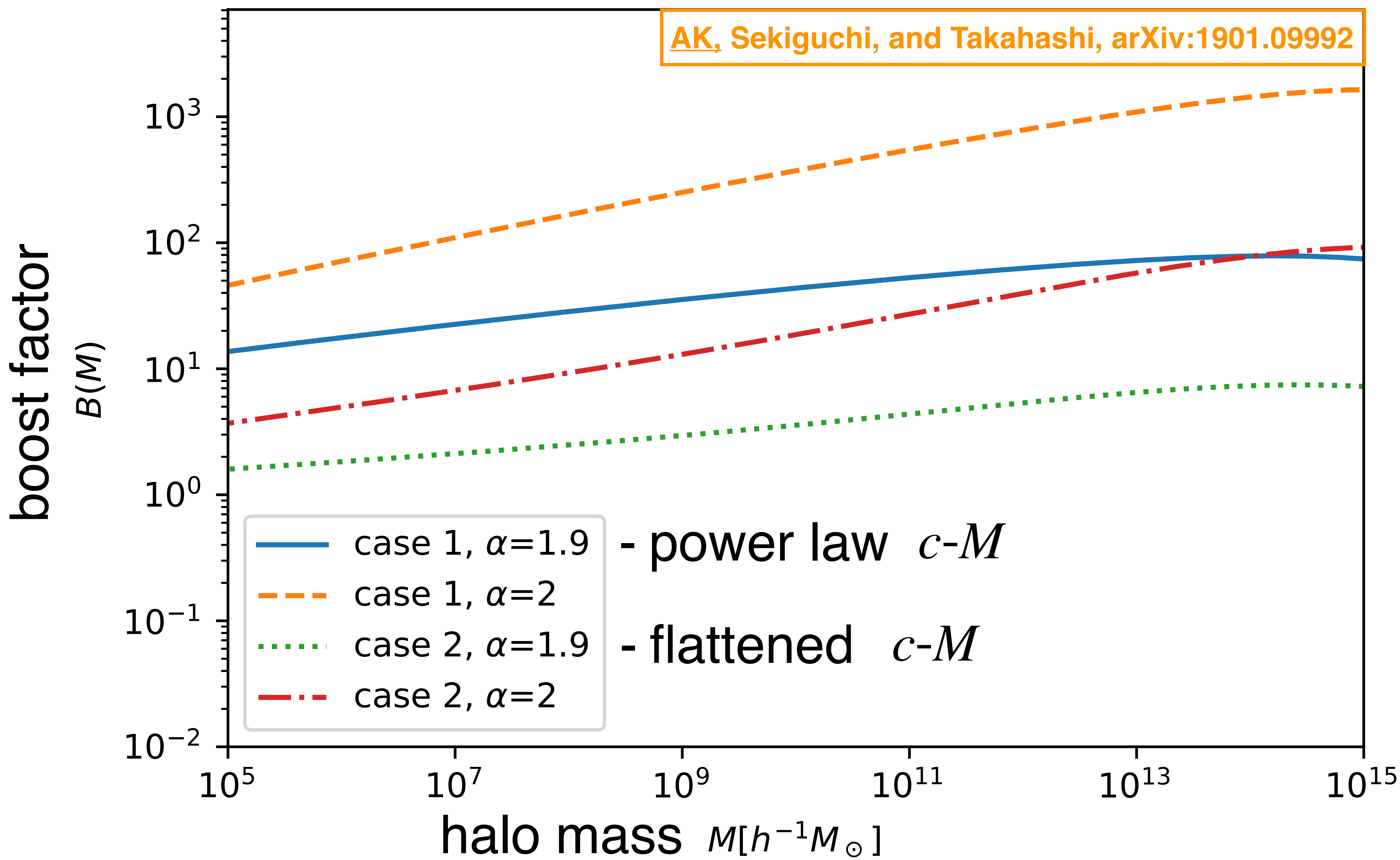
Smooth-k filter:

2.8 and 7.5 (1.2 and 2.5)

(Real-space) top-hat filter:

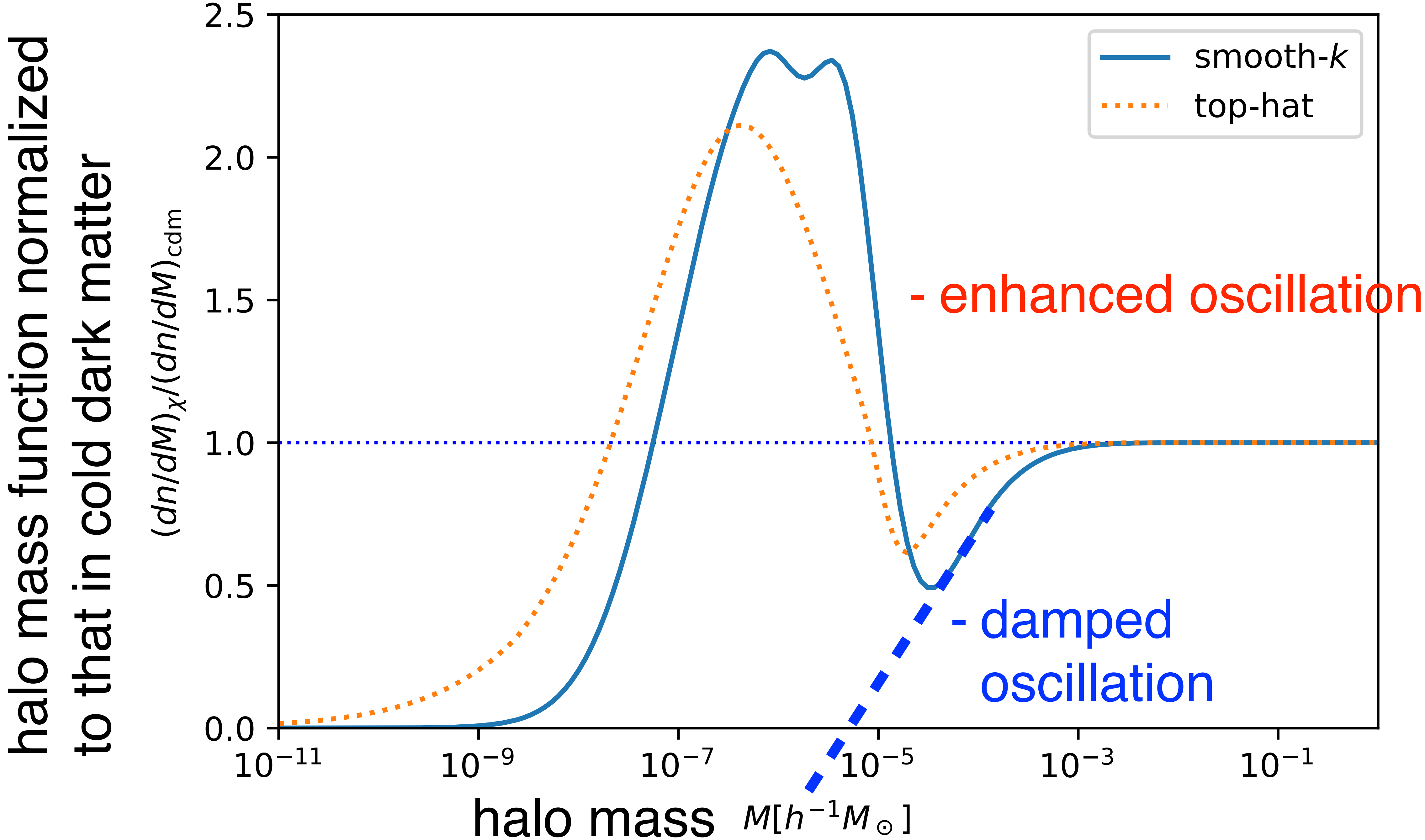
3.2 and 10.3 (1.2 and 2.7)

Estimated boost factor



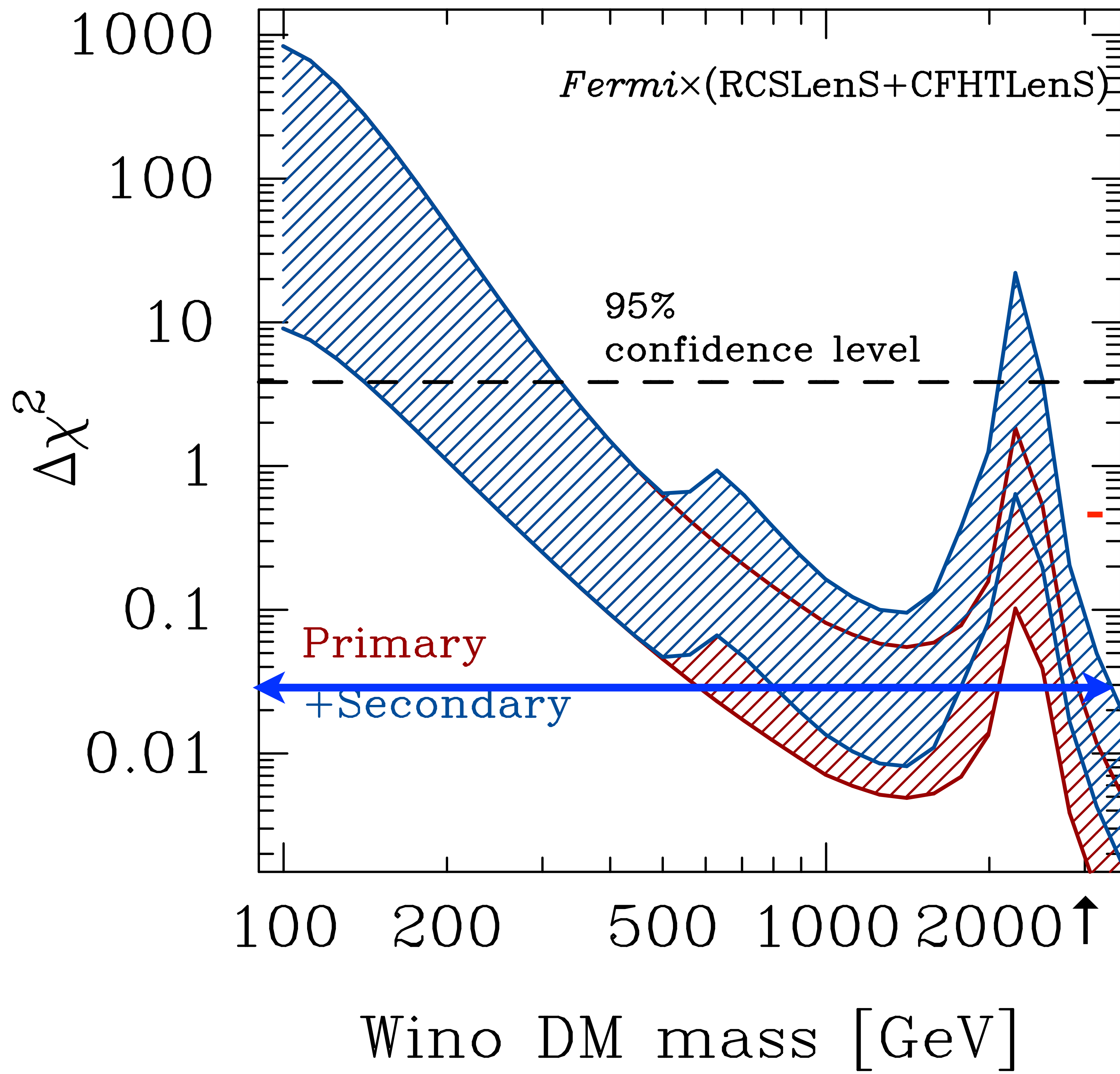
Boost factor depends on power-law index of the subhalo mass function α and the c-M relation

Estimated boost factor



Enhanced by a factor of 4.1 and 8.8 (1.5 and 2.9)
with $\alpha = 1.9$ and 2.0 in case 1 (2)
when compared to damped oscillation

Indirect detection



Enhanced abundance of Earth mass halos is not taken into account: to be updated!

- boost factor uncertainty

LSST-like survey

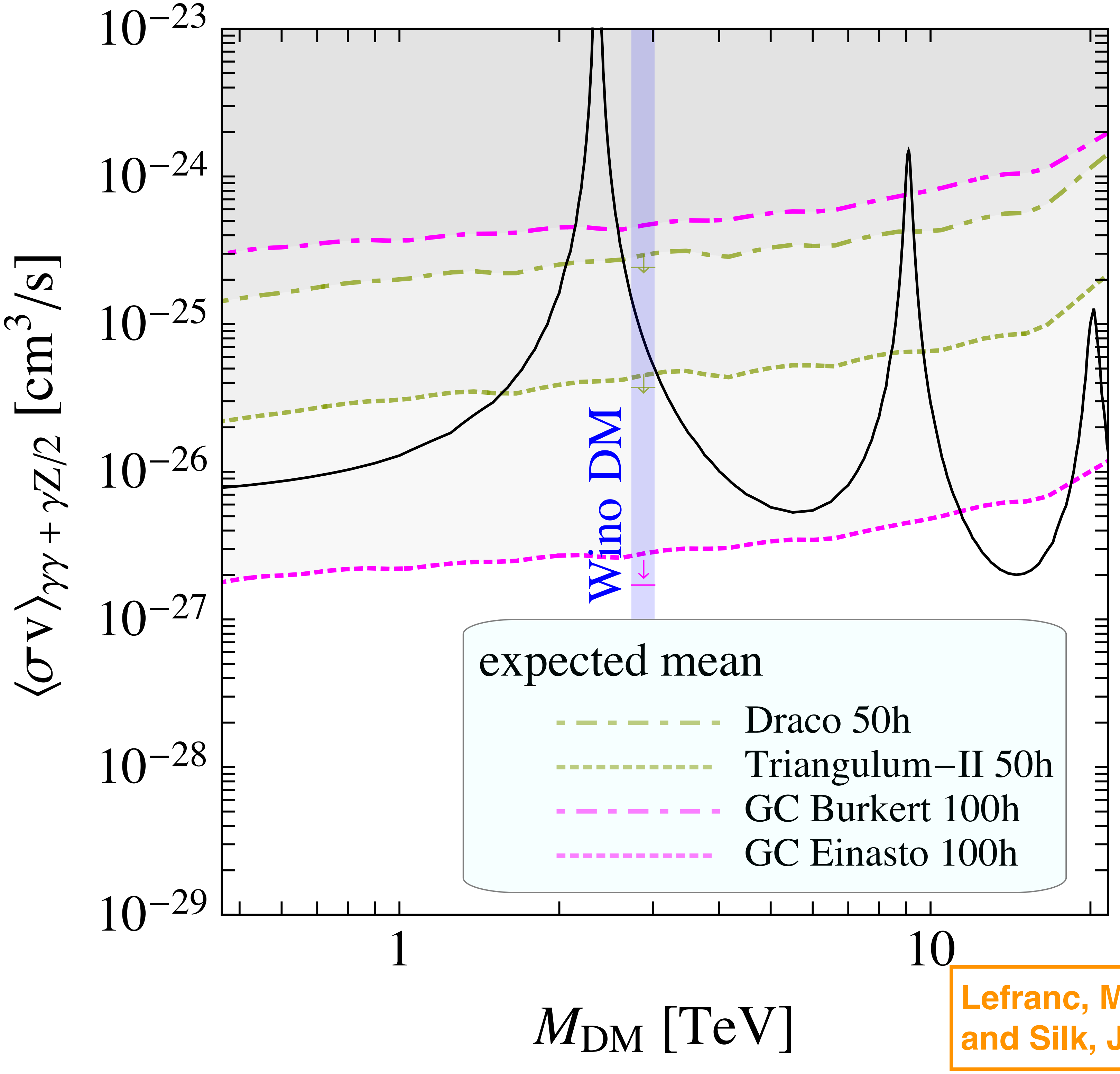
Shirasaki, Macias, Horiuchi, Shirai, and Yoshida, PRD, 2016

↑ thermal wino $m_\chi \simeq 3\,\mathrm{TeV}$

Cross-correlation of gamma-ray background w/ large-scale structure (e.g., weak lensing) will enjoy statistical improvement in near future!

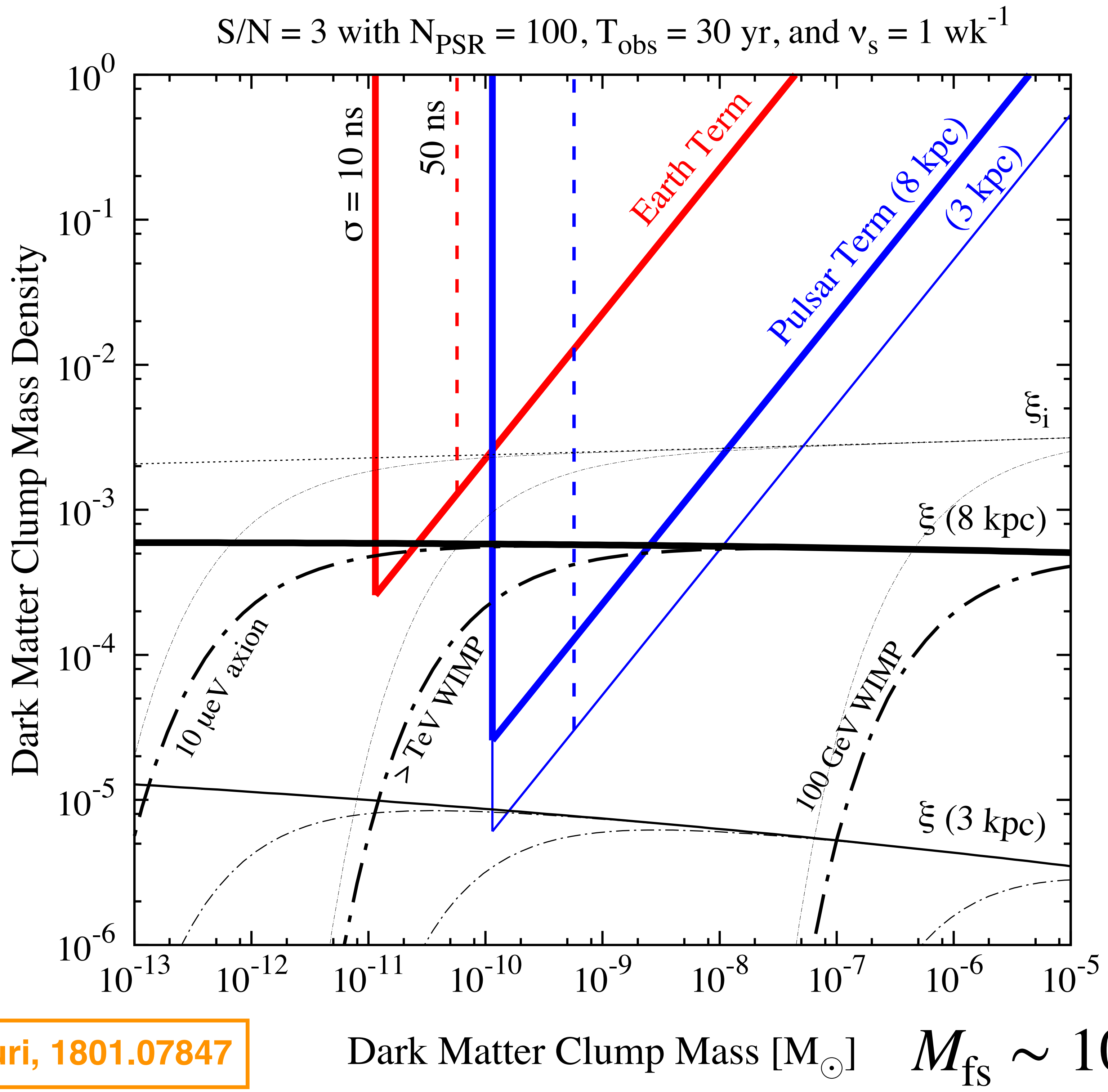
Indirect detection

CTA sensitivities



Lefranc, Moulin, Panci, Sala, and Silk, JCAP, 2016

Detectability of small-size clump



Kashiyama and Oguri, 1801.07847

$M_{\rm fs} \sim 10^{-7}\, M_\odot$

Detection of dark matter clump encounter with the Earth or a pulsar in pulsar timing array data