# Categorical and analytic invariants in algebraic, symplectic and complex geometry

2025 3-7 FEB.

KAVLI IPMU
東京大学 国際高等研究所 カブリ数物連携宇宙研究機構
KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

## INVITED SPEAKERS

- RINA ANNO (Kansas State University)
- HUIJUN FAN (Wuhan University)
- WAHEI HARA (Kavli IPMU)
- LUTZ HILLE (University of Münster)
- OSAMU IYAMA (University of Tokyo)
- MIKHAIL KAPRANOV (Kavli IPMU)
- DOGANCAN KARABAS (Kavli IPMU)
- TATSUKI KUWAGAKI (Kyoto University)
- TIMOTHY LOGVINENKO (Cardiff University)
- ALEXEY LVOV (St. Petersburg University)
- KATHERINE MAXWELL (Kavli IPMU)
- TODOR MILANOV (Kavli IPMU)
- SHINNOSUKE OKAWA (Osaka University)
- KYOJI SAITO (RIMS)
- ATSUSHI TAKAHASHI (Osaka University)
- RYO TAKAHASHI (Nagoya University)
- SOFIA TIRABASSI (Stockholm University)
- YUKINOBU TODA (Kavli IPMU)

## ORGANIZING COMMITTEE

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https://indico.ipmu.jp/event/457/overview

# Program

*Red: Changed on 2nd Feb.

**3rd February (Monday)**

10:00-11:00 Osamu Iyama
Cohen-Macaulay representations of Artin-Schelter Gorenstein algebras of dimension one

11:30-12:30 Shinnosuke Okawa
On the derived category of Ekedahl's fake projective planes

14:00-15:00 Kyoji Saito
Semi-infinite Hodge structure associated with hyperbolic root systems

16:00-17:00 Ryo Takahashi
Finiteness of Orlov spectra of singularity categories

**4th February (Tuesday)**

10:00-11:00 Lutz Hille
Polynomial Invariants for Full Exceptional Sequences

11:30-12:30 Atsushi Takahashi
Set of full exceptional collections and mirror symmetry

14:00-15:00 Huijun Fan
Fukaya category of Landau-Ginzburg model via Witten equation

16:00-17:00 Todor Milanov
Genus-0 permutation-equivariant KGW invariants of the point

**5th February (Wednesday)**

10:00-11:00 Sofia Tirabassi
Effective characterizations of semi-abelian varieties

11:30-12:30 Wahei Hara
Derived equivalence for the simple flop of type $G_2^\dagger$

Free afternoon

**6th February (Thursday)**

10:00-11:00 Mikhail Kapranov
Perverse sheaves on h/W and constant term of Eisenstein series

11:30-12:30 Rina Anno
Barr-Beck theorem for enhanced triangulated categories

14:00-15:00 Yukinobu Toda
Dolbeault Geometric Langlands conjecture via quasi-BPS categories

16:00-17:00 Timothy Logvinenko
The Heisenberg algebra of a vector space and Hochschild homology

**7th February (Friday)**

10:00-11:00 Katherine Maxwell
The Neveu-Schwarz group and applications to superstring theory

11:30-12:30 Alexey Lvov
Coherent sheaves on maximally singular varieties

14:00-15:00 Dogancan Karabas
Wrapped and compact Fukaya categories of plumbings

16:00-17:00 Tatsuki Kuwagaki
Hodge microsheaves on cotangent bundles and plumbings

# Abstracts

**Osamu Iyama**, The University of Tokyo

Title: **Cohen-Macaulay representations of Artin-Schelter Gorenstein algebras of dimension one**

We study Cohen-Macaulay representations over (not necessarily commutative) Gorenstein rings by using the tilting theory of singularity categories. We study Artin-Schelter Gorenstein algebras A of dimension one including singular Calabi-Yau algebras and classical Gorenstein orders. We prove that the generically projective Z-graded singularity category of A always admits an (explicitly described) silting object, and admits a tilting object if and only if either A is regular or a certain homological invariant g of A (called the average Gorenstein parameter) is non-positive. These results recover our previous results for commutative Gorenstein rings with Buchweitz and Yamaura. We explain our results by giving some examples. This is a joint work with Yuta Kimura and Kenta Ueyama.

**Shinnosuke Okawa**, Osaka University

Title: **On the derived category of Ekedahl's fake projective planes**

Ekedahl constructed a family of smooth projective surfaces in characteristic 2 with ample canonical bundles which are purely inseparable finite covers of the projective plane. We prove that the derived category of such a surface admits an exceptional collection of line bundles of length 3. A remarkable feature is that the semiorthogonal complement of the collection has 2-torsion Grothendieck group whereas its Hochschild homology is non-trivial, which suggests that the definition of quasi-phantom categories should be modified in positive characteristics. This is a joint work with Koshiro Murai.

**Kyoji Saito**, RIMS

Title: **Semi-infinite Hodge structure associated with hyperbolic root systems**

It is well-known that there exist semi-infinite Hodge structure associated to finite or elliptic root systems (which describes the lattice of vanishing cycles for either simple or elliptic root systems). Recently, we found that the semi-infinite Hodge structure exist for hyperbolic root systems of rank 2. This is a surprise, since the hyperbolic root systems do not have geometric origin so the they behaves quite differently than the above classical cases (e.g. some eigenvalues of monodromy are not root of unity but real). In the present talk, we will describe the construction down to the earth.

**Ryo Takahashi**, Nagoya University

Title: **Finiteness of Orlov spectra of singularity categories**

The Orlov spectrum of a triangulated category is the set of generation times of strong generators. Ballard, Favero and Katzarkov proved that the singularity category of a hypersurface isolated singularity has finite Orlov spectrum. In this talk, we will introduce the new notion of uniformly dominant local rings. We will show that the singularity category of a uniformly dominant isolated singularity has finite Orlov spectrum, and consider when a given local ring is uniformly dominant.

**Lutz Hille**, University of Münster

Title: **Polynomial Invariants for Full Exceptional Sequences**

For a full exceptional sequence on the projective plane there is the famous Markov equation, which can be generalized to full exceptional sequences of length three in any triangulated category. There is also an application to cluster mutations in joint work with Beineke and Brüstle. In this talk we define polynomial invariants for full exceptional sequences of length n. It turns out that such polynomial invariants define invariants of the triangulated category, so it is of interest to find them all. In this talk we determine generators for ring of polynomial invariants and study the connection to the natural braid group action. Moreover, we prove several properties and present examples. It turns out that polynomial invariants are closely related to the Coxeter transformation and the properties of the Grothendieck group together with its Euler form.

**Atsushi Takahashi**, Osaka Univeristy

Title: **Set of full exceptional collections and mirror symmetry**

For ADE singularities, Deligne gave a characterization of sets of distinguished bases and a recursion relation for their cardinalities, and proved Looijenga's conjecture on their coincidence with the degrees of Lyashko-Looijenga maps which capture topological information of bifurcation sets. Categorifying distinguished bases into full exceptional collections in derived directed Fukaya categories motivated by the idea of homological mirror symmetry, I'll explain how Deligne's recursion and comparison of degrees of Lyashko-Looijenga map with numbers of full exceptional collections can be naturally generalized.

**Huijun Fan**, Wuhan University

Title: **Fukaya category of Landau-Ginzburg model via Witten equation**

Landau-Ginzburg model has become a cornerstone theory of global mirror symmetry. The closed string A-theory of a LG model has already been built, and is well-known as the quantum singularity theory (or FJRW theory). An open string theory of a LG model has also been treated in the paper " Fukaya Category of Landau-Ginzburg model, arXiv:18012.11748v1 ", but with not much attention. In this talk, I will recall the construction in this paper, which is

related to the boundary value problem of the Witten equations arising from Landau-Ginzburg model, and mention the Maurer- Cartan element conjecture proposed by Gaiotto-Moore-Witten (or Kapranov- Kontsevich-Soibelman).

**Todor Milanov**, Kavli IPMU

Title: **Genus-0 permutation-equivariant KGW invariants of the point**

K-theoretic Gromov–Witteh (KGW) theory was introduced by Givental and Y.P. Lee as a generalization of Gromov–Witten theory. Recently, Givental realised that if we want to compute KGW invariants via fixed-point localization methods, we have to consider a more general theory, i.e., the permutation equivariant version of KGW theory. I would like to give an introduction to this topic and to explain how to compute the invariants in genus-0 for the simplest possible target – the point.

**Sofia Tirabassi**, Stockholm University

Title: **Effective characterizations of semi-abelian varieties**

I will show how three logaritmic plurigenera and the logarithic irregularity are enough to characterize semi-abelian surfaces among the quasi-projective surfaces. I will also present some results for higher dimensional varieties in a very special case. This is joint work with Mendes Lopes and Pardini and a work in progress with J. Baudin.

**Wahei Hara**, Kavli IPMU

Title: **Derived equivalence for the simple flop of type $G_2^\dagger$**

In this talk we discuss an example of a simple flop that was found by Kanemitsu, from the point of view of derived categories. A simple flop is a flop between two smooth varieties that is connected by one smooth blow-up and one smooth blow-down, and those flops were partially classified by Kanemitsu, using Dynkin data. The exceptional divisor of the blow-ups has two projective bundle structures of the same rank, and is called a roof. The simple flop of type $G_2^\dagger$ , which we discuss in this talk, is the only known example of a simple flop that has the non- homogeneous roof. The main theorem of the talk is that the simple flop of type $G_2^\dagger$ gives a derived equivalence. The proof is done by using tilting bundles, and hence it also produces a noncommutative crepant resolution that is derived equivalent to both sides of the flop. Despite its Dynkin label, the construction of the tilting bundles is related to rational homogeneous manifolds of Dynkin type $B_3$ and $D_4$ .

**Mikhail Kapranov**, Kavli IPMU

Title: **Perverse sheaves on h/W and constant term of Eisenstein series**

Graded braided Hopf algebras (such as the Hall algebra of a curve over $F_q$) correspond to factorizing systems of perverse sheaves on the symmetric products of the complex line. The talk will present an analog of this correspondence for an arbitrary complex reductive group G where the role of the symmetric product is played by the quotient h/W. We exhibit an algebra C so that C-Mod = Perv(h/W) with respect to the natural stratification. The relations in C include the Langlands formula for the constant term of Eisenstein series in the theory of automorphic forms. This formula generalizes the compatibilty between multiplication and comultiplication in a graded braided Hopf algebra (obtained for G=$GL_n$. The algebra C is the W-invariant subalgebra in the algebra B describing perverse sheaves on h. This matches nicely the description of h/W as the spectrum of the algebra of invariants. Joint work with V. Schechtman, O. Schiffmann and J. Yuan.

**Rina Anno**, Kansas State University

Title: **Barr-Beck theorem for enhanced triangulated categories**

We establish a technical framework that includes a bicategory of enhanced triangulated categories with certain enhanceable functors and natural transformations, and a theory of A-infinity algebra and module objects in this setting. This allows us to prove the analogue of the Barr-Beck theorem for enhanced triangulated categories. Moreover, we provide a DG category whose derived category of modules is equivalent to our analogue of the Eilenberg-Moore category of the monad RF, where (F,R) are adjoint enhanced functors between enhanced triangulated categories. We also develop a notion of the derived category of comodules over an A-infinity coalgebra satisfying some restrictions in our formalism, equipped with module-comodule correspondence, and stable under coalgebra homotopy equivalences. Together, this implies a version of descent for derived categories of sheaves. This is joint work with Timothy Logvinenko.

**Yukinobu Toda**, Kavli IPMU

Title: **Dolbeault Geometric Langlands conjecture via quasi-BPS categories**

In this talk, I will introduce the notion of 'limit category' for cotangents of smooth stacks, which is expected to give a categorical degeneration of the category of D-modules on them. I show that the limit category for the moduli stack of Higgs bundles admits a semiorthogonal decomposition into products of quasi-BPS categories, which are categorifications of BPS invariants of some non-compact Calabi-Yau 3-folds. I propose the formulation of Dolbeault Geometric Langlands conjecture using the limit category, which is regarded as a classical limit of Geometric Langlands correspondence. I also show that the limit category admits Hecke operators. This is a joint work in progress with Tudor Padurariu.

**Timothy Logvinenko**, Cardiff University

Title: **The Heisenberg algebra of a vector space and Hochschild homology**

In arXiv:2105.13334, Gyenge, Koppensteiner and Logvinenko constructed a 2-categorification of the Heisenberg algebra of a smooth and proper DG category, and decategorified it via Grothendieck group. In this talk, I will explain the ongoing effort to make this work with the Hochschild homology $HH_*$, instead. Effectively, this means extending it from a lattice in $HH_0$ to the whole Hochschild homology. This first raises a question of what is the Heisenberg algebra of a graded vector space. Then, one has to construct the crucial map from the Heisenberg algebra of $HH_*$ of a DG category to the $HH_*$ of the Heisenberg 2-category. The payoff is a direct generalisation of Nakajima ' s original result on the Heisenberg algebra acting on the cohomology of Hilbert schemes of points on a surface.

**Katherine Maxwell**, Kavli IPMU

Title: **The Neveu-Schwarz group and applications to superstring theory**

The super Mumford form is a section over the moduli space of super Riemann surfaces, characterized by invariance under the action of the Neveu-Schwarz action. In light of difficulties in performing integrals in superstring theory arising from the super Mumford form, it was suggested in the 80s that the relationship of the moduli space of super Riemann surfaces to the super Sato Grassmannian may be fruitful. Based on joint work with A. Voronov, I will discuss possible approaches to extending the super Mumford form, including our results on the proposed formula by A. Schwarz.

**Alexey Lvov**, St. Petersburg University

Title: **Coherent sheaves on maximally singular varieties**

We define and study the category $Coh(C_X)$ associated with an algebraic variety X. The category $Coh(C_X)$ is an inductive limit of categories of coherent sheaves on singular models of X. It can be thought of as a category of coherent sheaves on a maximally singular model of X. We will discuss the description and various properties of $Coh(C_X)$, in particular it's global dimension.

**Dogancan Karabas**, Kavli IPMU

Title: **Wrapped and compact Fukaya categories of plumbings**

Given any finite quiver Q, where each vertex corresponds to a fixed Lagrangian $L_v$ , I will describe an associated symplectic manifold known as the plumbing of $T^*L_v$'s along Q. Using a local-to-global approach, I will explain how their

wrapped Fukaya category can be expressed as a Ginzburg dg algebra with based loop space coefficients or a derived multiplicative preprojective algebra. In the second part of my talk, I will demonstrate that microlocal sheaves on the union of $L_v$'s recover the compact Fukaya category of the plumbing, generalising the Nadler-Zaslow correspondence for cotangent bundles. The first part is joint work with Sangjin Lee (arXiv:2405.10783), and the second part is ongoing work with Sangjin Lee and Wonbo Jeong.

**Tatsuki Kuwagaki**, Kyoto University

Title: **Hodge microsheaves on cotangent bundles and plumbings**

The theory of Hodge microsheaves aims at generalizing the theory of mixed Hodge modules in twofold: (1) "infinite-dimensional" like wrapped sheaves of Nadler, (2) "microlocal" in the style of Bezrukavnikov-Kapranov. In this talk, I'll explain some background philosophy and some nontrivial computational results in the theory, based on joint work with Takahiro Saito.