Strong EM Waves and Fast Radio Bursts

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Maxwell equations

$$\nabla \times \underset{\sim}{B} = \frac{4\pi \underset{\sim}{j}}{c} + \frac{1}{c}\frac{\partial E}{\partial t} \qquad \nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

$$\nabla \cdot \underset{\sim}{E} = 4\pi\rho \qquad \nabla \cdot \underset{\sim}{B} = 0 \quad \rightarrow \underset{\sim}{B} = \nabla \times \underset{\sim}{A} \ ; \ A^{\mu} = (\phi, \underset{\sim}{A})$$

Are linear : superposition of charges → superposition of $\underset{\sim}{E}, \underset{\sim}{B}$ fields

- However the response of charges $(\rho, \underset{\sim}{j})$ to $(\underset{\sim}{E}, \underset{\sim}{B})$ <u>need not</u> be linear, though it often is for weak waves.

In Lorenz gauge $\partial_{\mu} A^{\mu} = 0$, Maxwell eqs → $\Box A^{\mu} = \frac{4\pi}{c} J^{\mu}$ $\quad J^{\mu} = (c\rho, \underset{\sim}{j})$

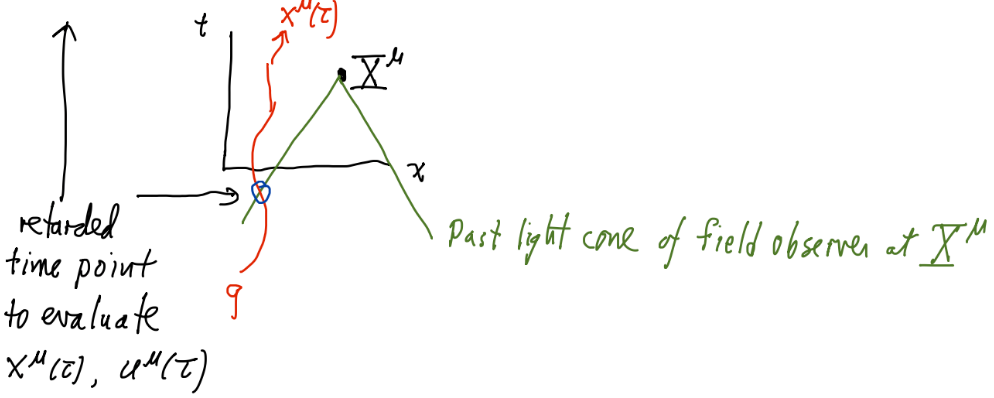
In 1-D, 3-D (but not even dimensions like 2-D!) the Green's function solution depends only on charges where they cross the past lightcone of an observer:

$$A^{\mu} = \int \frac{[J^{\mu}]\, d^3 r'}{|\underset{\sim}{r} - \underset{\sim}{r}'|} \qquad [J^{\mu}] = J^{\mu}\left(\underset{\sim}{r}', t - \frac{1}{c}|\underset{\sim}{r} - r'|\right)$$

is retarded time potential

For a single charge q on worldline $x^\mu(\tau)$, 4-veloc $u^\mu = \frac{dx^\mu}{d\tau}$ seen by observer at X^μ

$$A^\mu(X^\mu) = q\left[\frac{u^\mu}{R_\alpha u^\alpha}\right]_{ret} \qquad R^\alpha = X^\mu - x^\mu(\tau)$$



Differentiating at fixed retarded time gives the fields

$$\mathbf{E} = q\left(\frac{1-\beta^2}{R^2}\frac{(\hat{n}-\boldsymbol{\beta})}{(1-\hat{n}\cdot\boldsymbol{\beta})^3} + \frac{\hat{n}\times[(\hat{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{R(1-\hat{n}\cdot\boldsymbol{\beta})^3}\right)$$

$$\mathbf{B} = \hat{n}\times\mathbf{E},$$

“near field”

“radiation”

For $\boldsymbol{\beta} \ll 1$, $\mathbf{E} \to q\,\mathbf{a}_\perp/R$

Roger Y. Tsien 1972 *Am. J. Phys.* **40**, 46. [Harvard junior project w/ E.M. Purcell; 2008 Nobel Prize Chemistry for Green Fluorescent Protein to track gene expression.

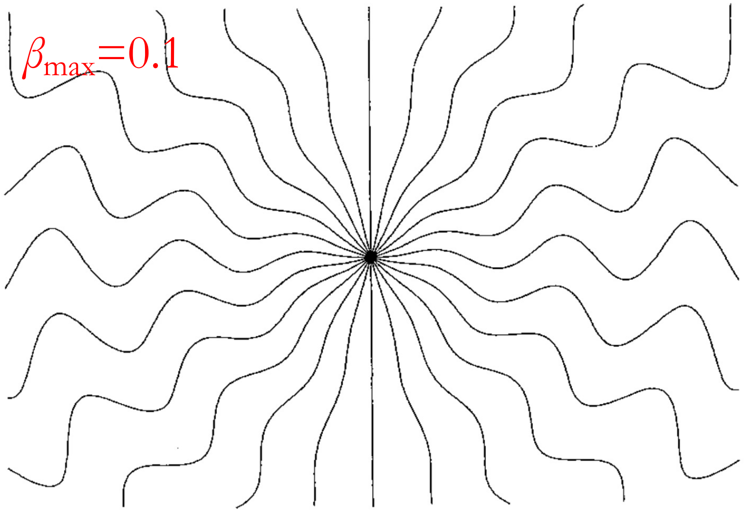


FIG. 13. Electric field lines of a charge undergoing one-dimensional simple harmonic motion with $\beta_{\max}=0.10$, $t_0=0$.

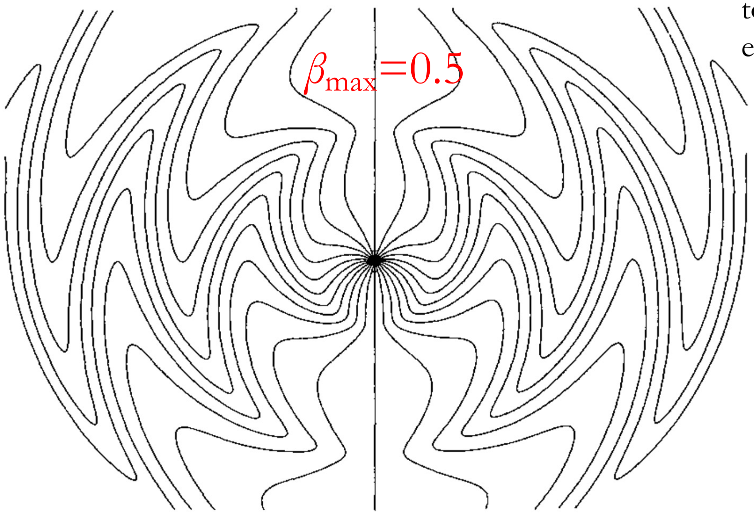


FIG. 14. Electric field lines of a charge undergoing one-dimensional simple harmonic motion with $\beta_{\max}=0.50$, $t_0=0$.

full exact field lines of charges in simple harmonic motion, of frequency ω, amplitude a along y-axis, with $\beta_{\text{max}}=a\omega$

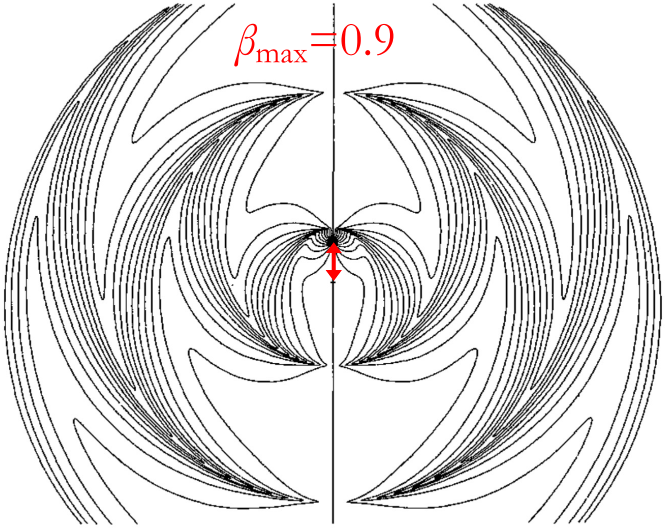
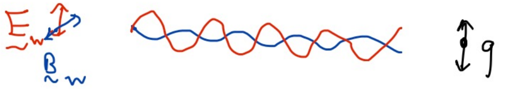


FIG. 15. Electric field lines of a charge undergoing one-dimensional simple harmonic motion with $\beta_{\max}=0.90$, $t_0=\frac{1}{2}\pi/\omega$ so that $y(t_0)=a$.

Response of charges to an imposed EM wave:



For a free charge (plasma) $\frac{d\underset{\sim}{p}}{dt} = q\left(\underset{\sim}{E}_w + \underbrace{\frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}_w}\right)$

For EM wave in (near) vacuum, $|\underline{E}| = |\underline{B}|$

For weak EM waves (usual experience) accelerated charge has $\frac{|v|}{c} \ll 1$ ⇒ Neglect $\frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}$ term.

(*) So $\underset{\sim}{a} = \frac{q}{m}\underset{\sim}{E}_w$ - linear response for $\frac{|v|}{c} \ll 1$

And each charge radiates

$$\underset{\sim}{E}_{rad} = q\frac{a_\perp}{R} = \frac{q^2}{m}\frac{E_{\perp w}}{R}\Big|_{ret}$$

- linearly proportional to the incident field, with phase-shift depending on distance (due to retarded time)

However note from (*) that $\frac{|v|}{c} \ll 1$ will start to break down for wave of frequency ω when

$$\frac{v}{c} \sim \frac{1}{c}\frac{a}{\omega} = \frac{q}{cm\omega}|E_w| \sim 1. \qquad \omega = 2\pi\nu$$ ← freq in Hz.

Define dimensionless strong wave parameter f for $e^\pm$ ($q=e$, $m=m_e$) $\boxed{f = \frac{e|E_w|}{cm_e 2\pi\nu}}$

$\frac{v}{c} = f$ for $f \ll 1$.

Note: $f = \frac{eA}{m_e c^2} = \frac{P_\perp}{m_e c}$ conserved canonical ⊥ momentum / $m_e c$

Relation of strong wave parameter to energy flux in EM wave:

$$F \doteq c\frac{E_w^2}{4\pi} = f^2 c\pi\left(\frac{m_e c\nu}{e}\right)^2 = f^2 \cdot 3\times10^{14}\,\nu_{GHz}^2 \text{ erg cm}^{-2}\text{s}^{-1}$$

$$= f^2\ 0.3\,\nu_{GHz}^2 \text{ TW m}^{-2}$$

$\lambda = 1\mu m \rightarrow$

$\nu = 3\,10^5\,GHz$

$\Rightarrow F = 3\,10^{18}\frac{W}{cm^2} f^2\left(\frac{1\mu m}{\lambda}\right)^2$

Achievable in CPA (Chirp pulse Amplification lasers!

Other literature notations

$f \equiv a \equiv a_0$

Some examples:

- Tokyo Skytree: ERP = 200kW near $\nu \sim 100$ MHz (FM and TV) at 10km distance

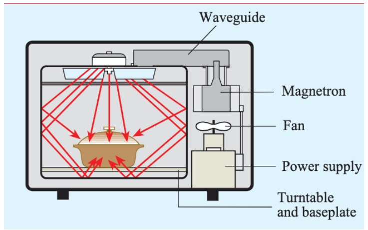
$$F \simeq \frac{P_{ERP}}{4\pi r^2} = 0.2 \text{ erg cm}^{-2}\text{s}^{-1} \rightarrow f = 2\times10^{-7}\left(\frac{10\,km}{r}\right)$$

- Inside your Microwave oven:

$V \sim (30\,cm)^3$. Standing waves energy $U = \frac{E^2}{4\pi}V$. Power required to maintain

$P = U\frac{\omega}{Q}$ $\quad \omega = 2\pi\nu$, $\nu = 2.4\,GHz$, $\lambda = c/\nu = 12.2$ cm) $\quad Q \sim 10^2 - 10^4 \equiv 10^2 Q_2$

lots of food $\uparrow$ $\quad \nwarrow$ empty

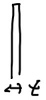


Typical oven 1.2kW, 80% efficient magnetron

$\Rightarrow P = 1kW \Rightarrow E = \sqrt{\frac{2QP}{\nu V}} = 0.2\,Q_2^{\frac{1}{2}}$ statvolt/cm $\Rightarrow f = 2\times10^{-4}\,Q_2^{\frac{1}{2}}$

$\uparrow$

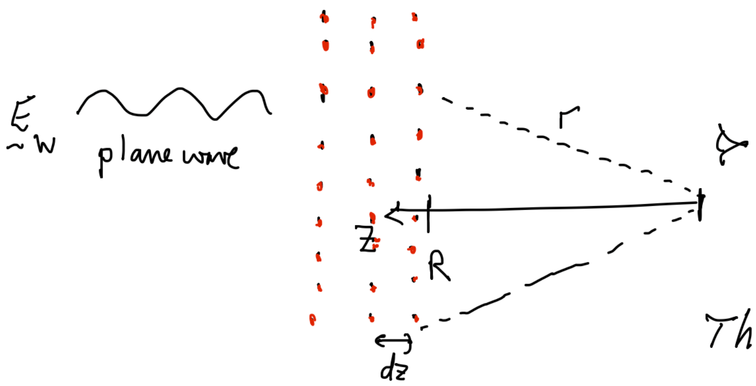
50-500 V/cm - but don't put Al foil ($t \sim 10^{-2}$ cm)!



Air breakdown $E_{break} \sim 30{,}000\,V/cm = 300\,V/10^{-2}\,cm$!

For many charges (e^- neutralized by slowly moving ions, or by equal and oppositely moving e^+), with

$f = \frac{v}{c} << 1$ each contributes $\frac{e^2}{m_e} \frac{\underset{\sim}{E}_{\perp w}}{r} \Big|_{ret} = \frac{e^2}{m_e} \frac{\underset{\sim}{E}_w e^{-ikz}}{r} e^{i\omega t}$



(cf Feynman Lectures I ch31 §1-2)

$\Rightarrow$ total $\underset{\sim}{E}_r$ from electrons in dz, density n_e is

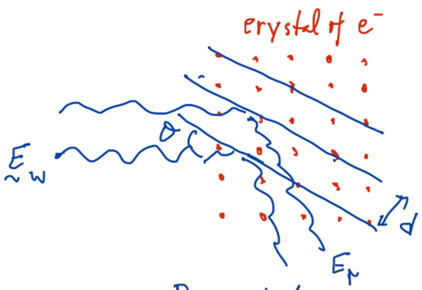
$$\underset{\sim}{E}_r = i \left(n_e dz\right) \frac{2\pi e^2}{\omega \; m_e} e^{i\omega(t - z/c)} \underset{\sim}{E}_w$$

This radiation in the forward direction (all coherent, since radiated waves move along z at same speed as $\underset{\sim}{E}_w$) shifts the phase of $E_{tot} = \underset{\sim}{E}_w + \underset{\sim}{E}_r$ by an amount $\propto \Delta z$ i.e. changes the phase speed to $\frac{c}{n}$, where

$$n = 1 - \underbrace{\frac{2\pi n_e e^2}{m_e \omega^2}}_{\text{assumed} << 1 \text{ in this simple argument w/ } \frac{\omega}{k} = c + O(\epsilon)}$$ is the index of refraction

What about E_r in other directions?

Unlike forward direction, E_r not all in phase with each other!



Bragg's law

$n\lambda = 2d\sin\theta$

successive layers constructively add with phase difference $2\pi n$.

Diffraction spots

with intensity $\propto E_r^2 \propto N_e^2$

and solid angular width

$\propto \frac{1}{N_e}$

So total power in diffraction spots $\propto N_e$

Random phases

total $E_r \propto \sqrt{N_e}$

No preferred angles

Intensity $\propto |E_r^2| \propto N_e$

- same as adding power from individual scattering particles (Thomson for e^-)

Att: Scattering is due to index of refraction variations due to Poisson variations in n_e.

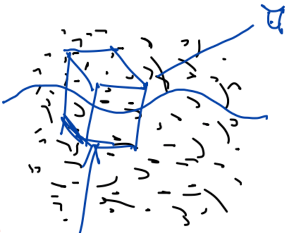
Why is the sky blue?

$n_{air} = 2\times10^{19}\ \mathrm{cm}^{-3}$

blue light $\lambda = 0.4\,\mu\mathrm{m}$

$\left(\frac{\lambda}{2}\right)^3 n$ = # coherent particles

$= 2\times10^5$!



they do in an isolated grain!

All 2×10^5 particles in $\left(\frac{\lambda}{2}\right)^3$ box radiate coherently

- so why isn't the Rayleigh scattered power $(2\times10^5)^2\times$ individual power?

Usual derivation for dispersion relation in a medium or plasma:

Assume response j, ρ are linear in $\underset{\sim}{E}$, $\underset{\sim}{B}$, e.g. $j_l = \sum_m \sigma_{lm} E_m$

So can Fourier decompose general $E(t)$, $B(t)$ into $E(t) = \int E(\omega) e^{i(\underset{\sim}{k}\cdot\underset{\sim}{x} - \omega t)} \, d\omega$ etc.

Cold plasma, $B_0 = 0$ (no background field)

Linearized equation of motion for e^- fluid = same as single e^-:

$$\underset{\sim}{V} = \frac{-ie}{\omega m} \underset{\sim}{E}$$

$$\Rightarrow j = -nev = \frac{ine^2}{\omega m_e} \underset{\sim}{E}$$

Substitute in Maxwell eqs: → dispersion relation

$$\omega^2 = c^2 k^2 + \omega_{pe}^2 \qquad \omega_{pe}^2 = \frac{4\pi n_e e^2}{m_e} \qquad \left(\text{or for } e^\pm: \frac{8\pi n_e e^2}{m_e}\right)$$

$$(\hbar\omega)^2 = c^2(\hbar k)^2 + (\hbar\omega_{pe})^2$$

Photon gains effective rest-mass energy due to interaction with the medium

Dispersion of radio wave pulses (pulsars, fast radio bursts) in the universe

$$\omega^2 = c^2k^2 + \omega_{pe}^2$$

$$V_{ph} = \frac{\omega}{k} > c$$

$$V_{grp} = \frac{d\omega}{dk} = \frac{c^2}{V_{ph}} = c\sqrt{1-\frac{\omega_{pe}^2}{\omega^2}} < c$$

$$\nu_{pe} = \frac{\omega_{pe}}{2\pi} = 9000\left(\frac{n_e}{1\,cm^{-3}}\right)^{\frac{1}{2}} Hz$$

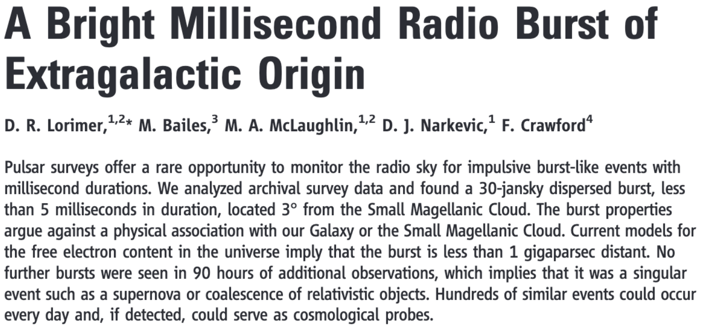
$$\Delta t(\omega) - \Delta t(\omega\to\infty) = \int_0^D \frac{dl}{V_g} \underset{\substack{\uparrow \\ \omega \gg \omega_{pe}}}{\simeq} \frac{D}{c} - \frac{1}{2c}\underbrace{\frac{\int \omega_{pe}^2\, dl}{\omega^2}}_{\propto \frac{\int n_e\, dl}{\nu^2} \leftarrow \text{DM } (cm^{-3}pc)}$$

$$= \frac{4.1488\ ms}{\nu_{GHz}^2} \times \frac{DM}{cm^{-3}pc}$$

Typical FRB: DM = 375 $cm^{-3}pc$: $\Delta t \simeq 790\,ms$ at 1.4 GHz

$\Delta t \simeq 4.3\,s$ at 600 MHz !

Fast Radio Bursts



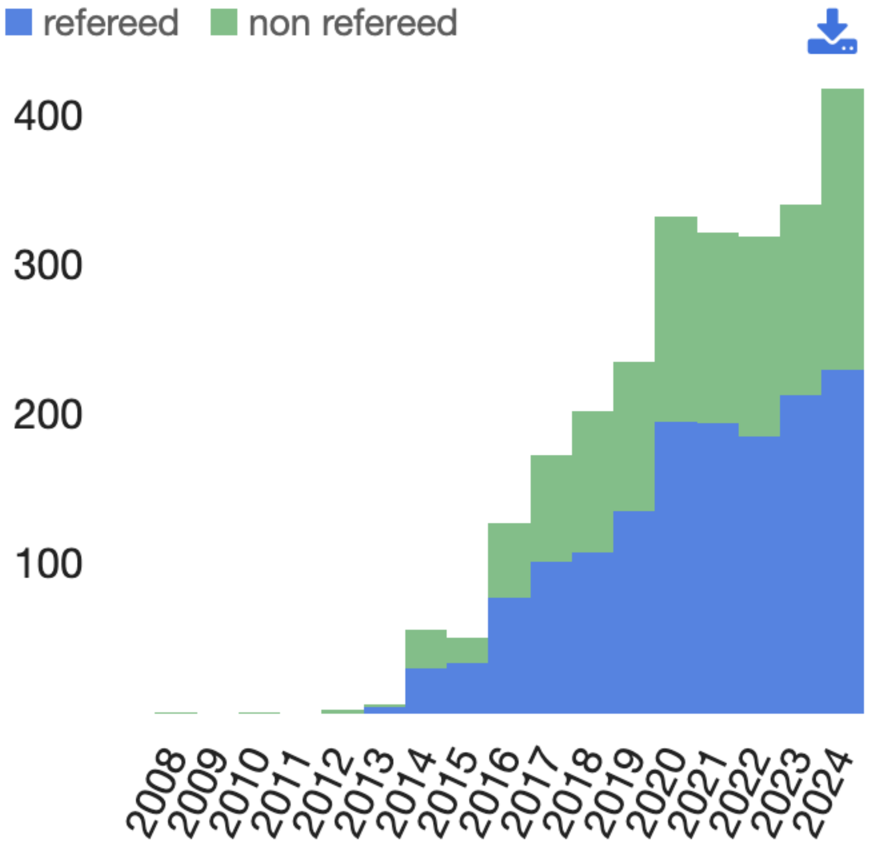
A Bright Millisecond Radio Burst of Extragalactic Origin

D. R. Lorimer,[1,2]* M. Bailes,[3] M. A. McLaughlin,[1,2] D. J. Narkevic,[1] F. Crawford[4]

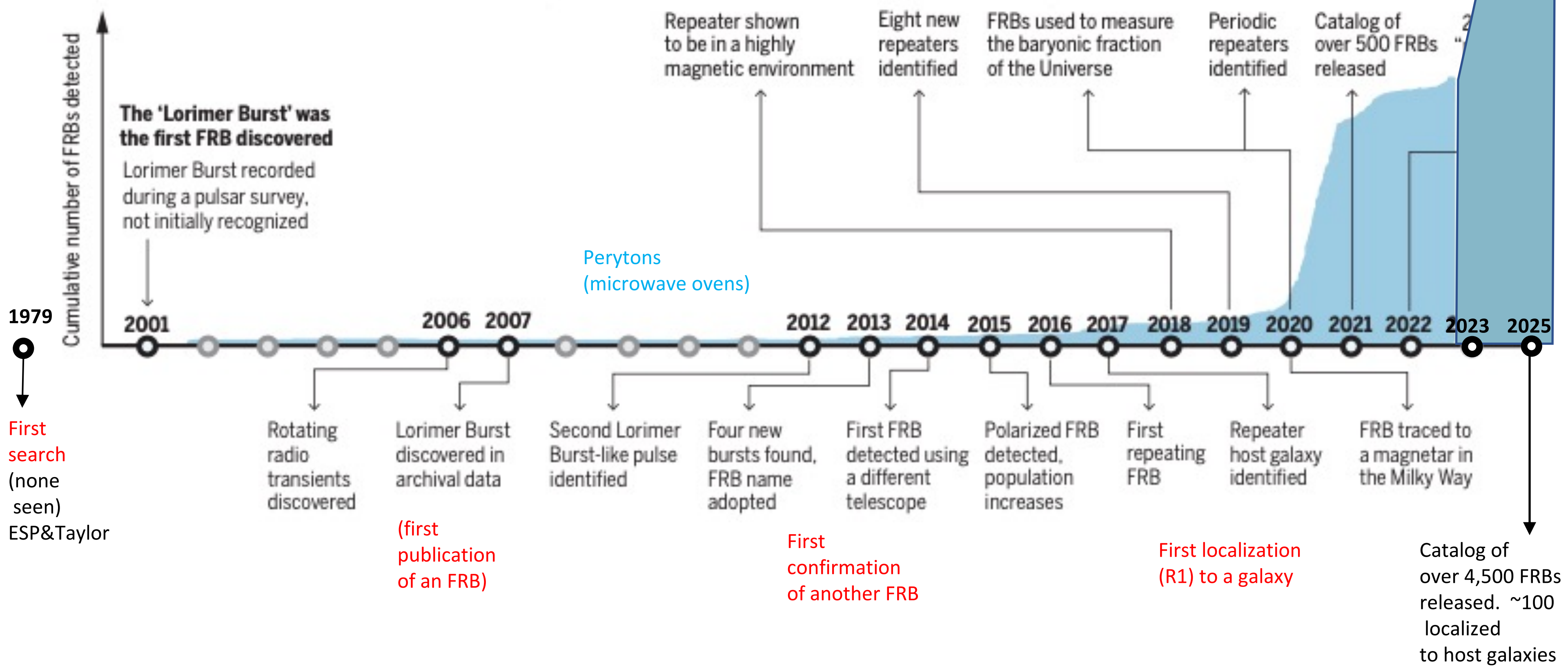
Pulsar surveys offer a rare opportunity to monitor the radio sky for impulsive burst-like events with millisecond durations. We analyzed archival survey data and found a 30-jansky dispersed burst, less than 5 milliseconds in duration, located 3° from the Small Magellanic Cloud. The burst properties argue against a physical association with our Galaxy or the Small Magellanic Cloud. Current models for the free electron content in the universe imply that the burst is less than 1 gigaparsec distant. No further bursts were seen in 90 hours of additional observations, which implies that it was a singular event such as a supernova or coalescence of relativistic objects. Hundreds of similar events could occur every day and, if detected, could serve as cosmological probes.

Data *collected* 2001.
Not noticed until
2007 discovery!
Science **318**, 777

Papers with “Fast Radio Burst” in abstract



Timeline of FRB discoveries



The 'Lorimer Burst' was the first FRB discovered

Lorimer Burst recorded during a pulsar survey, not initially recognized

Perytons (microwave ovens)

Repeater shown to be in a highly magnetic environment

Eight new repeaters identified

FRBs used to measure the baryonic fraction of the Universe

Periodic repeaters identified

Catalog of over 500 FRBs released

First search (none seen) ESP&Taylor

Rotating radio transients discovered

Lorimer Burst discovered in archival data

(first publication of an FRB)

Second Lorimer Burst-like pulse identified

Four new bursts found, FRB name adopted

First FRB detected using a different telescope

First confirmation of another FRB

Polarized FRB detected, population increases

First repeating FRB

Repeater host galaxy identified

First localization (R1) to a galaxy

FRB traced to a magnetar in the Milky Way

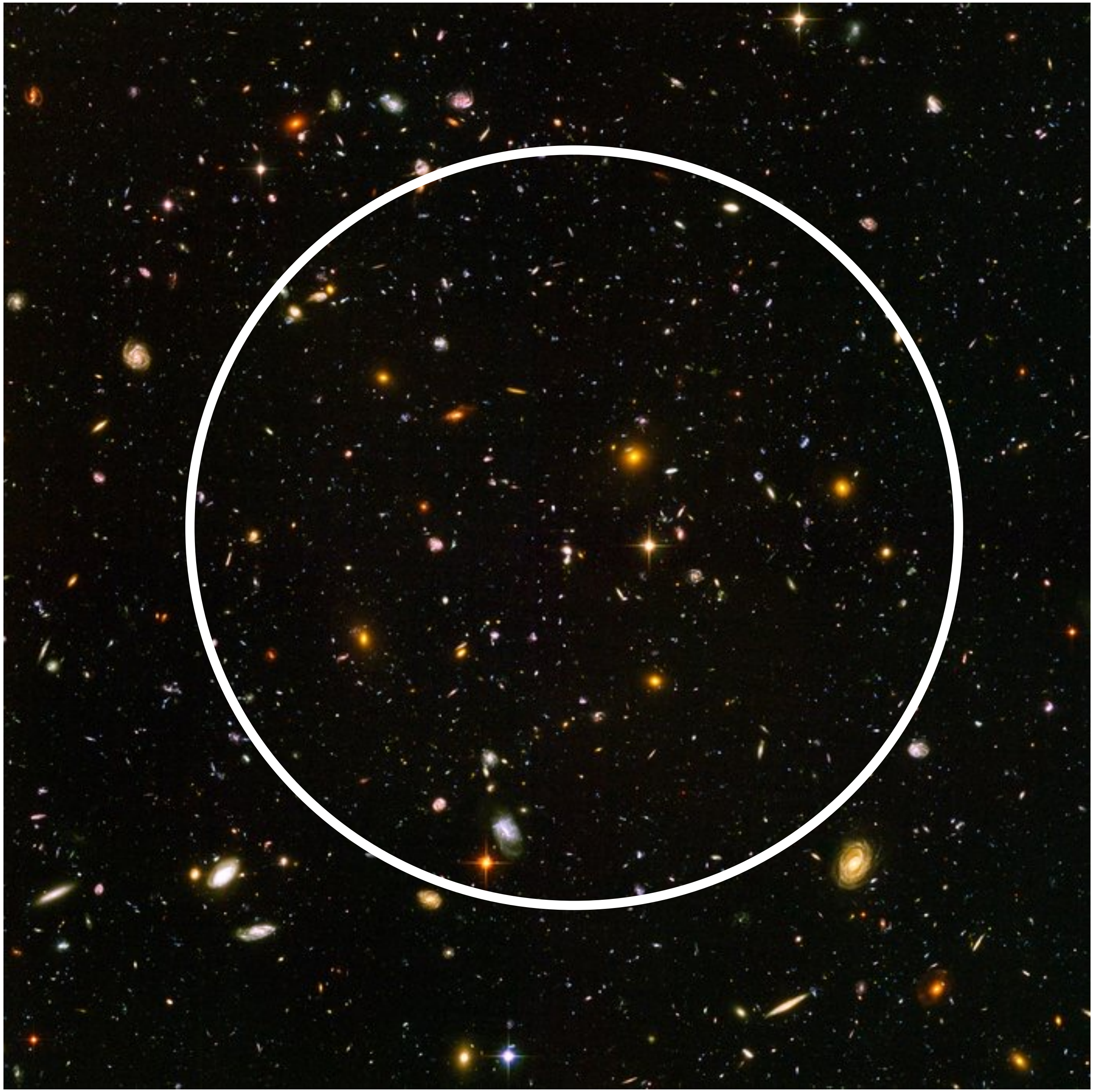
Catalog of over 4,500 FRBs released. ~100 localized to host galaxies

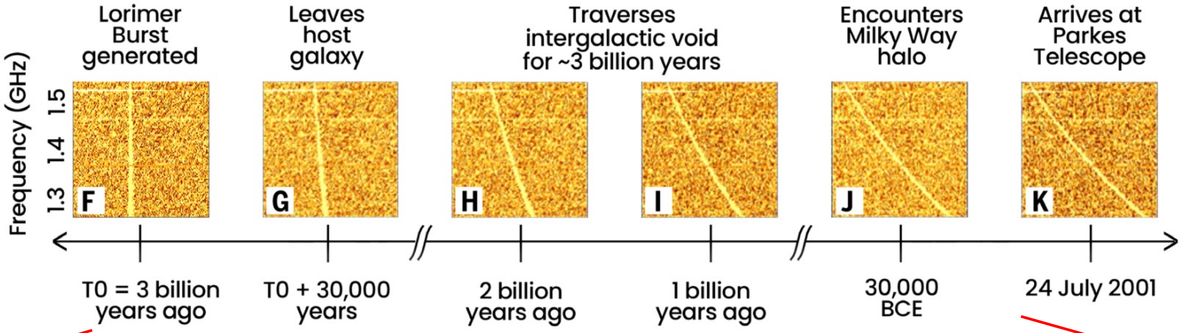
The localization problem.

Hubble UDF image contains about 10,000 galaxies at all distances in 3’x3’.

Circle is ~beam of Parkes or CHIME telescope (70-100m). 1’ radius.

Need an interferometer with >3km (DSA-110, ASKAP) baseline to identify a host galaxy. >1000 km (EVN, CHIME outriggers) to say where in the galaxy it is!





$$\mathrm{DM} = \frac{\mathrm{DM_{source}}}{1+z} + \frac{\mathrm{DM_{host}}}{1+z} + \overbrace{\langle \mathrm{DM_{IGM}}(z)\rangle + \left(\mathrm{DM_{IGM}}(z) - \langle \mathrm{DM_{IGM}}(z)\rangle\right)}{} + \sum_i \frac{\mathrm{DM}_{i,CGM}}{1+z_i} + \mathrm{DM_{MW\ halo}} + \mathrm{DM_{MW\ disk}}$$

mean IGM large-scale structure galaxy halos

$\downarrow$

Comoving electron density for fully ionized IGM:

$$n_{e0} = \frac{(1-Y)\rho_b}{m_H} + \frac{2Y\rho_b}{m_{He}} = 2.20 \times 10^{-7}\mathrm{cm}^{-3}\left(\frac{\Omega_b h^2}{0.0222}\right)$$

(For z<3, He^{++}.
$\times$ 0.93 for 3<z<6, He^{+})
Should subtract
~7% fraction of baryons
in stars, neutral gas, etc.

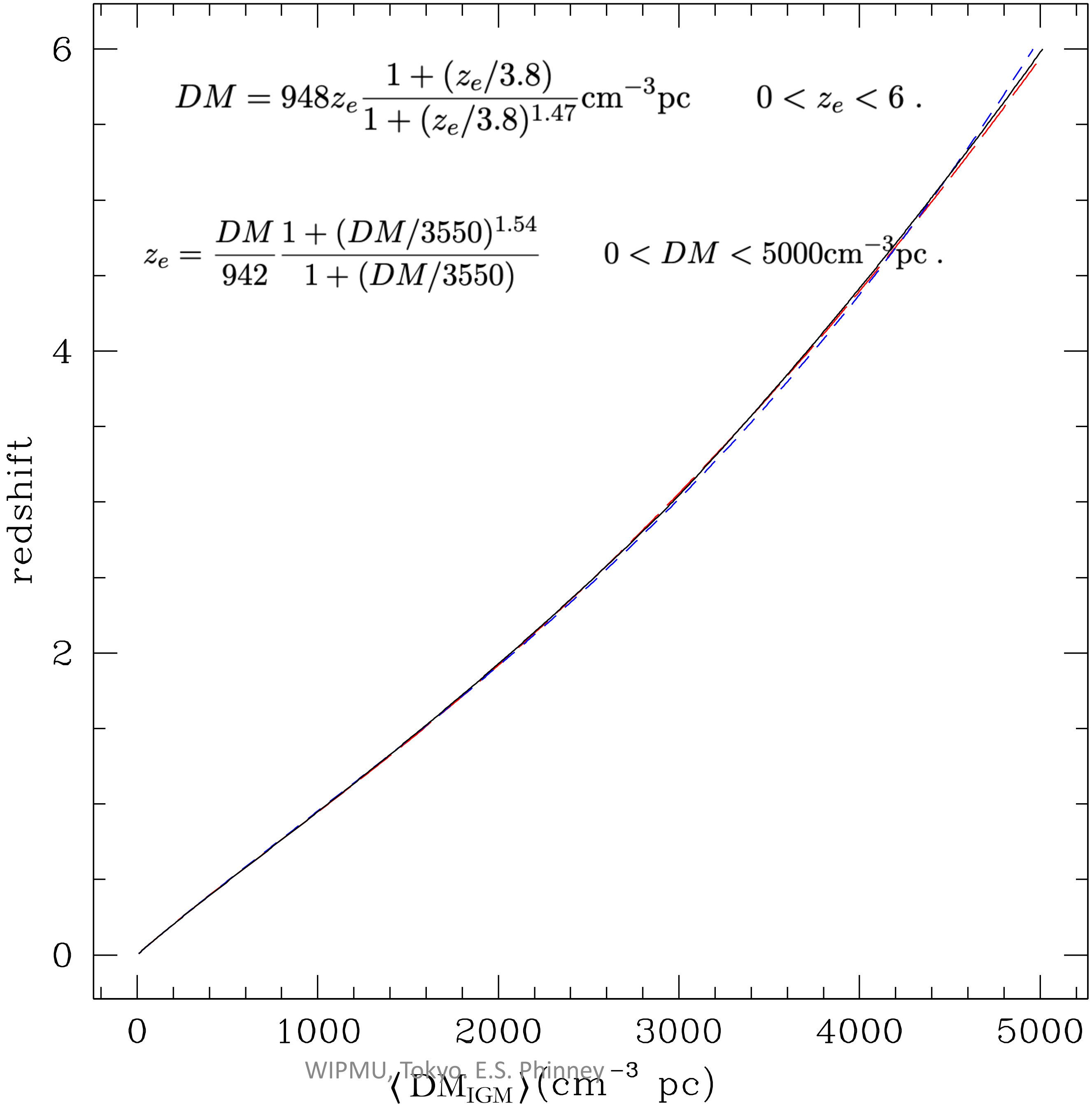
Time delay at frequency ν:

$$\Delta t(\nu) = \frac{e^2}{2\pi m_e c}\frac{1}{\nu^2}DM$$

Dispersion measure

$$\langle \mathrm{DM_{IGM}} \rangle = \frac{c}{H_0}\int_0^{z_e} dz \frac{(1+z)n_{e0}}{E(z)}$$

Ffits & Plot from
Phinney 2016
Good to 1%



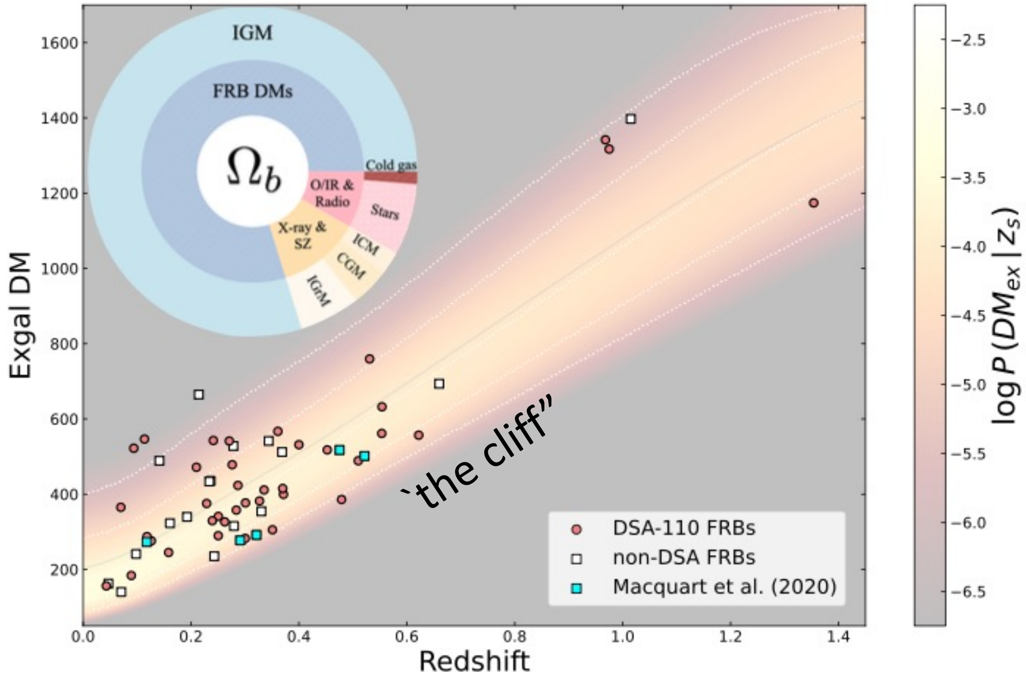
$$DM = 948 z_e \frac{1+(z_e/3.8)}{1+(z_e/3.8)^{1.47}} \mathrm{cm}^{-3}\mathrm{pc} \qquad 0 < z_e < 6 \ .$$

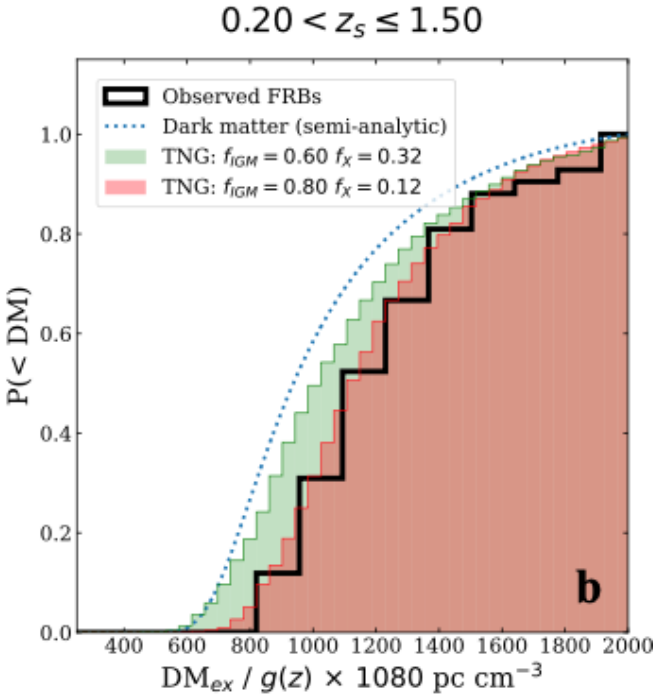
$$z_e = \frac{DM}{942} \frac{1+(DM/3550)^{1.54}}{1+(DM/3550)} \qquad 0 < DM < 5000 \mathrm{cm}^{-3}\mathrm{pc} \ .$$

- Before FRBs:
  - Stars and gas of galaxies are only 7% of baryons inferred from Big Bang Nucleosynthesis and cosmic microwave background.
  - Hot X-ray emitting gas in the intergalactic medium of galaxy groups and clusters are ~13% of baryons. c.f. Fukugita, Hogan & Peebles 1998, 2004!
- Where were the remaining ~80% of baryons ?!
- From FRB dispersion measures: the missing baryons are in a rather smoothly distributed ionized gas.
- The baryons near galaxies are less clustered than dark matter: *feedback* from massive stars and AGN must have ejected >50% into the intergalactic medium.

M81 starburst galaxy with outflows:
starlight (HST): yellow
Hα (HST): orange
Dust (Spitzer): red
X-ray (Chandra): blue

Connor, Ravi+ 2025 *Nature Astronomy* (arXiv:2409.16952):
DSA-110 sample. Evidence that baryons must be expelled from galaxy halos, groups and clusters, or DM$-z$ would have more dispersion than observed.

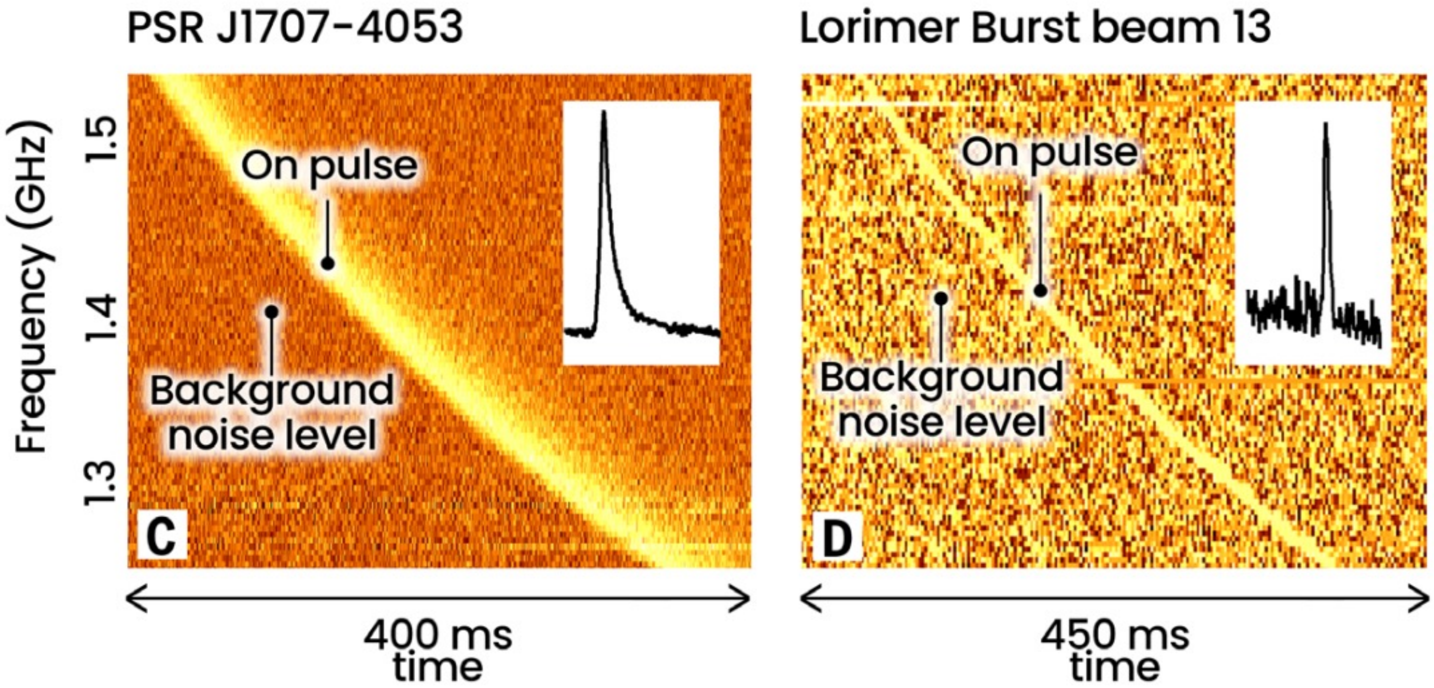


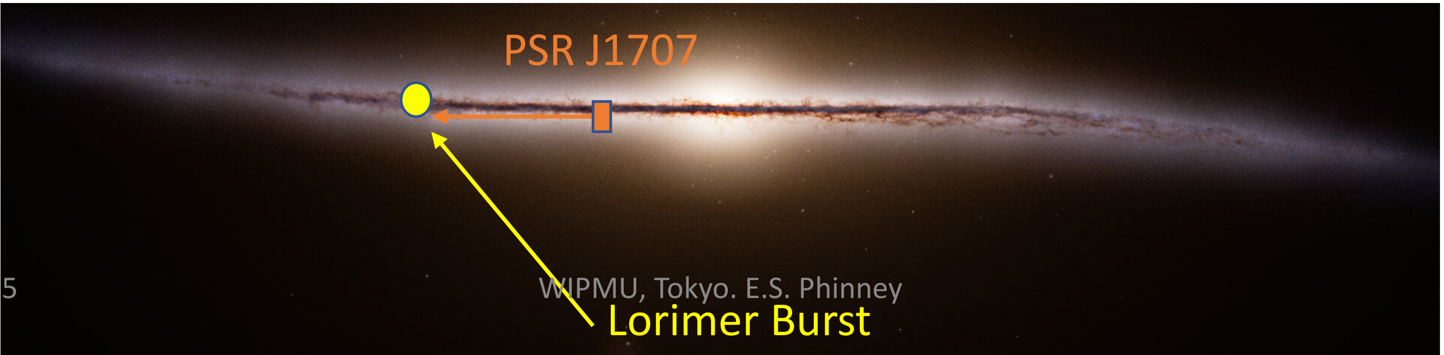


Guesstimate (statistically removed host galaxy, halo, Milky way halo, Milky Way DM's,
Those, plus cluster and group IGM, can only increase DM above the ``cliff'' of smooth IGM.

An ordinary pulsar
at D=5kpc exactly in the plane of the Milky Way,
$b=-0.2°$.
DM =360 cm^{-3}pc.

The first discovered Fast Radio Burst
At high galactic latitude, $b=-42°$
Other high latitude pulsars have DM<25.
But Lorimer burst had
DM = 375 cm^{-3}pc: extragalactic (z ~0.25)

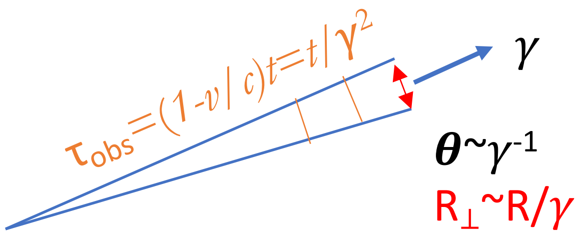




Fast Radio Burst properties

Physical properties

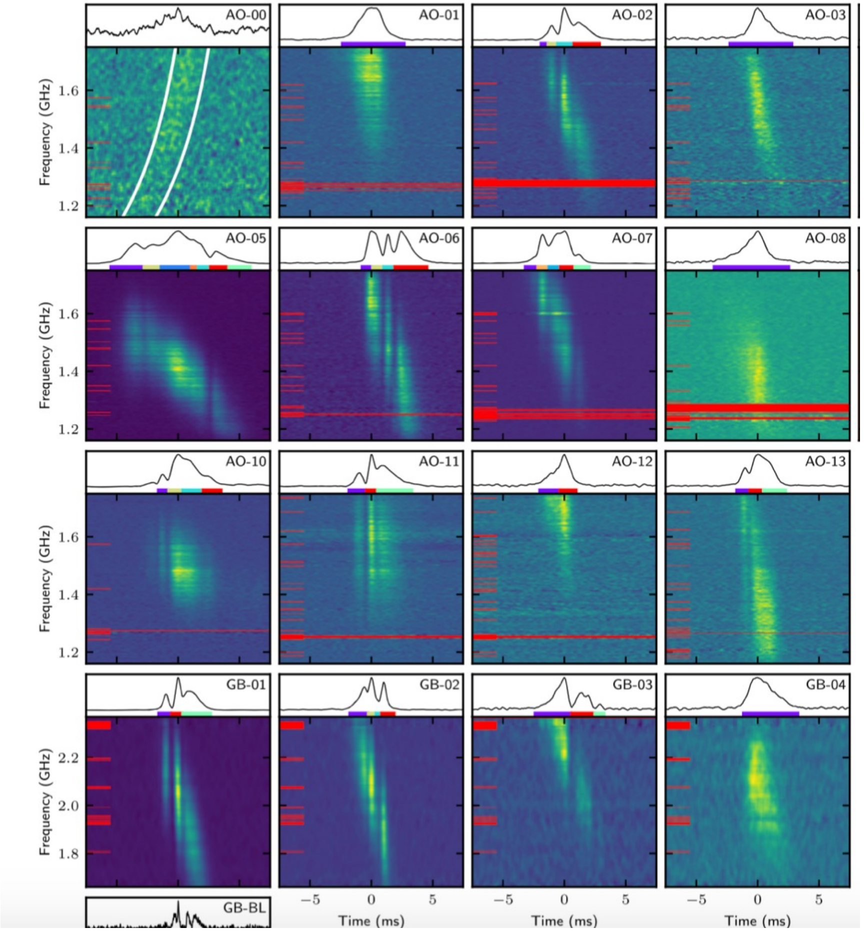
- 0.1-10ms (obs selection). $c\tau$ ~3km
- 0.001-1,600Jy (obs selection)
- Some have substructure on 100ns. $c\Delta t$=30m.
- mostly *part* of 0.4 -2GHz.
- High >90% linear polarizations common; PA can swing with time during burst.
- A few bursts have high circular polarization (91%: FRB 20201124A)
- Scintillation from host galaxies ISM sometimes observed →Projected size<100km.



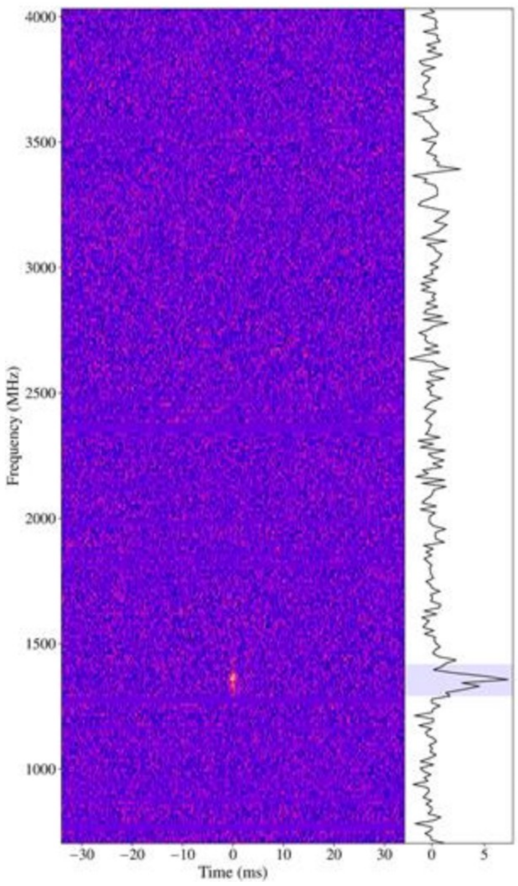
Astrophysical sources

- One FRB from a magnetar in center of Milky Way, associated with one unremarkable of hundreds of X-ray/gamma-ray bursts from SGR 1935+2154.
- Most FRBs from non-star-forming regions of galaxies similar to Milky-Way.
- A few FRBs from star forming regions (2 in dwarf starburst galaxies).
- One FRB from a globular cluster (last massive star died 13Gyr ago!)
- A few FRBs from outskirts of elliptical galaxies (last massive star died 13Gyr ago)
- Large fraction of FRBs probably from NS formed in late mergers (accretion-induced collapse –cf SN Ia), not mostly from young supernovae/magnetars.

Dynamic spectra of FRBs



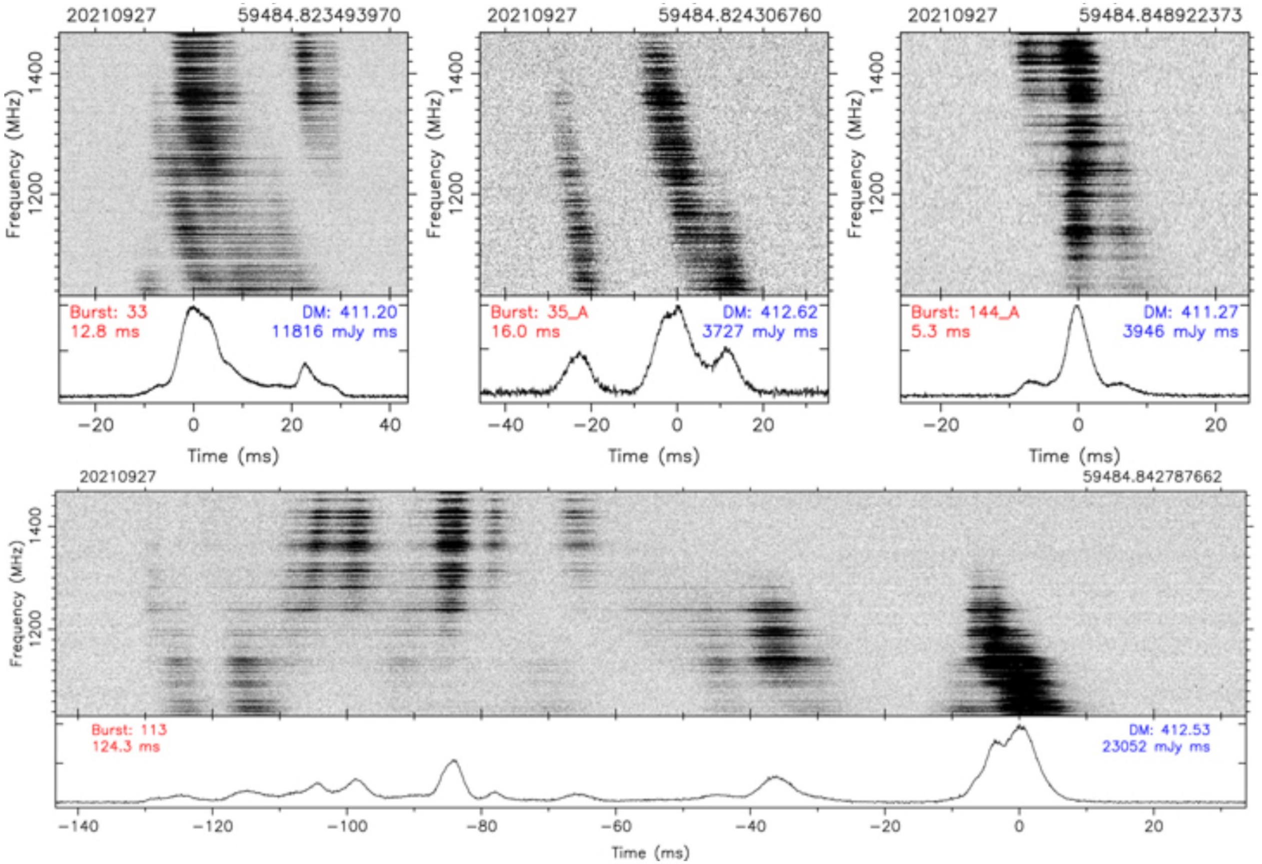
"sad trombone" ubiquitous



Bursts of rFRB20121102A
Hessels+ 2019

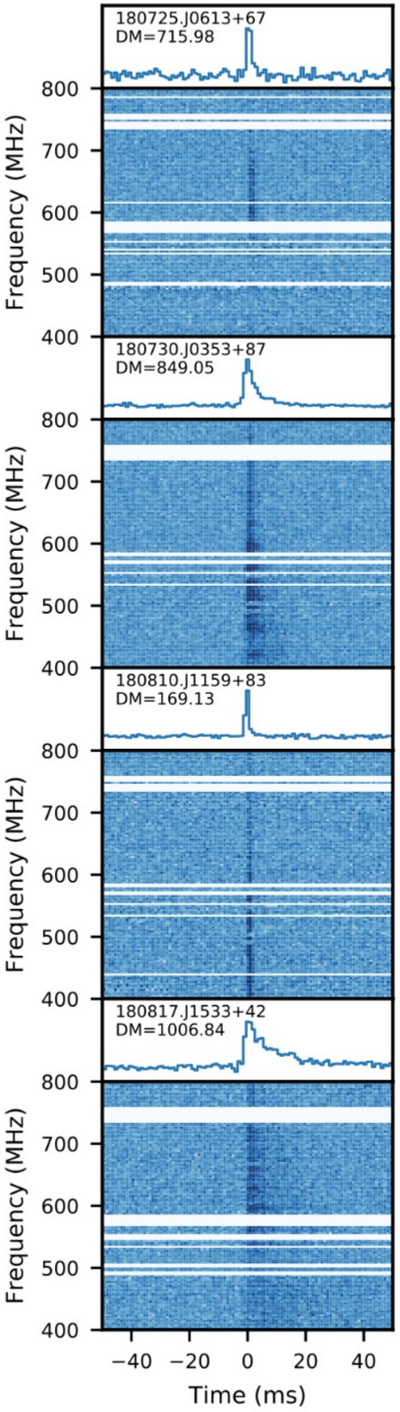
FRB 190711
Kumar+ 2020

Dynamic spectra of FRBs



Bursts of rFRB20201124A
Zhou+ 2022

4 CHIME FRBs
CHIME/FRB 2019b



$1\ \mathrm{Jy} = 10^{-23}\ \mathrm{erg\ cm^{-2}\ s^{-1}\ Hz^{-1}}$

FRB at distance D from earth
Has flux density S_ν at freq $\nu \to \nu + \Delta\nu$.
At distance r from that FRB:

$$\begin{cases} F = \dfrac{S_\nu\, \Delta\nu\ 4\pi D^2}{4\pi r^2} = f^2\, c\pi \left(\dfrac{m_e c\nu}{e}\right)^2 \\ \Rightarrow f = \sqrt{\dfrac{S\Delta\nu}{c\pi}} \left(\dfrac{D}{r}\right)\left(\dfrac{e}{m_e c\nu}\right) \end{cases}$$

Typical FRB:

$S_\nu = 1\ \mathrm{Jy}$ at $\nu = 1.4\,\mathrm{GHz}$, $\Delta\nu = 0.3\,\mathrm{GHz}$, $D \sim 1\,\mathrm{Gpc}$

$$f = \frac{10^{13}\,\mathrm{cm}}{r}$$

$$= 10^{-3}\ \frac{10^{16}\,\mathrm{cm}}{r}$$

$\uparrow$

Powerful FRB:

$S_\nu = 10^3\,\mathrm{Jy}$ at $\nu = 0.6\,\mathrm{GHz}$ $\Delta\nu = 0.3\,\mathrm{GHz}$, $D \sim 3\,\mathrm{Gpc}$

$$f = \frac{5\times10^{16}\,\mathrm{cm}}{r}$$

$$= 10^{-3}\ \frac{15\,\mathrm{pc}}{r}$$

$1\,\mathrm{pc} = 3.1\times10^{18}\,\mathrm{cm}$

$\uparrow$

at $f = 10^{-3}$, electron quiver velocity exceeds thermal velocity $\sqrt{\frac{kT}{m_e}}$ in warm interstellar medium ($T \sim 10^4\,\mathrm{K}$)

Thus free-free (Bremsstrahlung) absorption modified! (Lu & ESP 2020).

at $f > 1$, propagation modified: $\omega_p^2 = \dfrac{4\pi n e^2}{m_e c} \to \dfrac{4\pi n e^2}{\underset{''\gamma}{f}\, m_e c}$ and more!

How do these radii compare to the likely environment?

- FRB $c\Delta t$ and scattering size limits suggest neutron star ($R \sim 12$ km) or its wind nebula.
- Bursty nature of FRB's suggest magnetic reconnection and/or neutron star quakes.

✓ SGR 1935+2154 FRB from Galactic Center magnetar soft gamma repeater.

Adequate energy

surface field of NS

$$\dot{E}_B \simeq \frac{\frac{1}{6} B_s^2 R_{NS}^3}{\tau_{decay}} \simeq 3\, 10^{35} \mathrm{erg\, s^{-1}} \left(\frac{B_s}{10^{15} G}\right)^2 \left(\frac{10^4 y}{\tau_{decay}}\right)$$

← $10^4 y$ is time for ambipolar diffusion $10^{15}G$ to escape from NS core.

NB $B_s > \frac{m_e c^2}{\hbar e} = 4 \times 10^{13}$ G

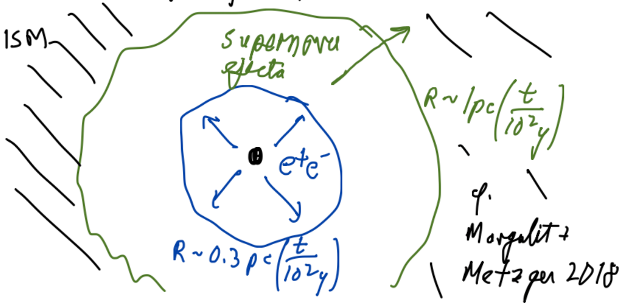
- quantum field - QED vacuum index of refraction $n - 1 > 1$
- Landau levels relativistic ...

"magnetar"

Two possibilities

A] Young $\lesssim 10^3 y$ magnetar plerion

in a young supernova remnant



cf. Margalit + Metzger 2018

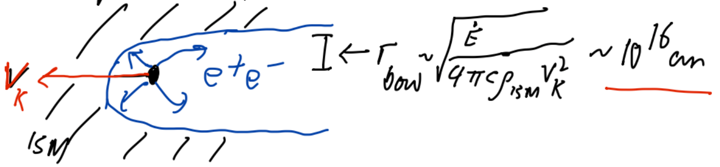
B] Older $\gtrsim 10^4 y$ magnetar wind nebula

in ISM.

- Neutron star natal kicks $V_K \sim 200 - 1{,}000$ km/s

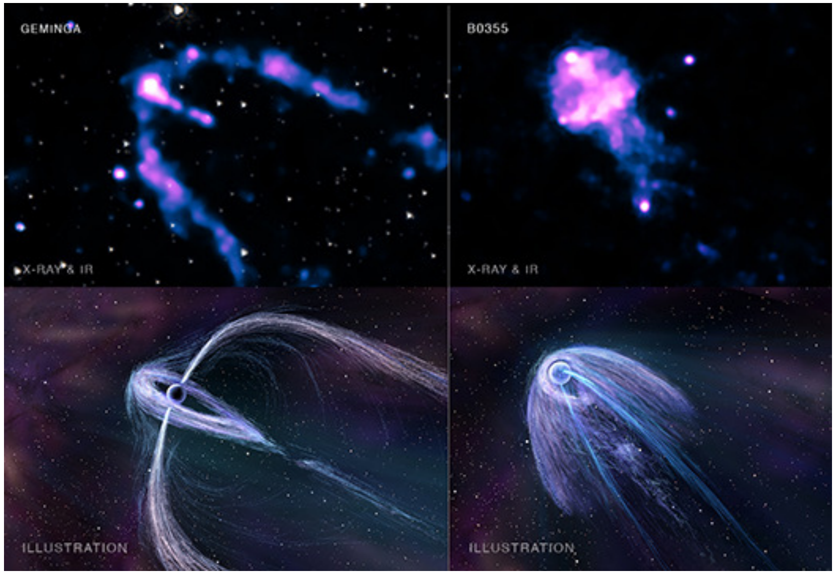
∃ star escapes SNR at $t \sim 10^3 - 10^4 y$

- Magnetar wind ram pressure balances ISM ram:

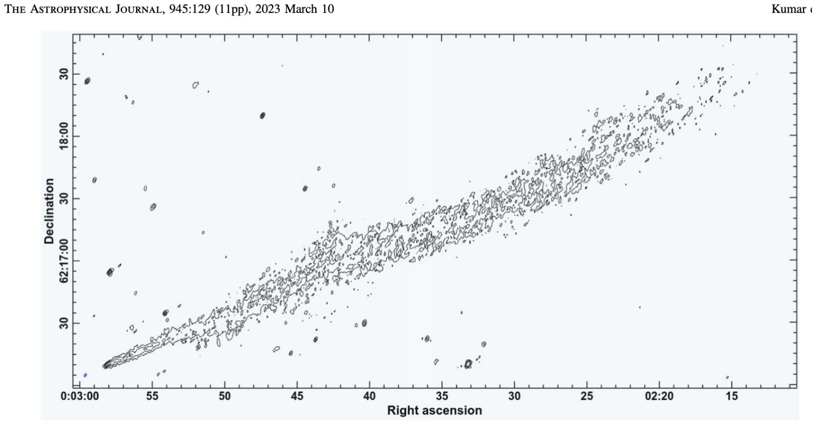


$$\leftarrow r_{bow} \sim \sqrt{\frac{\dot{E}}{4\pi c \rho_{ISM} V_K^2}} \sim 10^{16} \mathrm{cm}$$

Chandra X-ray images of
Geminga and B0355+54
Romani+



PSR J0002+6216 4-12GHz radio image
dE/dt~10^{35}erg/s
Kumar+ 2023


The Astrophysical Journal, 945:129 (11pp), 2023 March 10 Kumar

Length > 3pc
Standoff and width at head 0.003pc
=10^{16}cm.

So for FRBs, two cases of strong wave propagation are of interest:

A) In relativistic ($\gamma \sim 10^3$?) e^+e^- plasma bubble $R \sim 10^{18}$ cm around a young $< 10^3$ y magnetar

- Never a strong wave in ISM

B) In the interstellar gas around the cometary trail around a middle-aged $\gtrsim 10^4$ y magnetar with a substantial kick velocity.

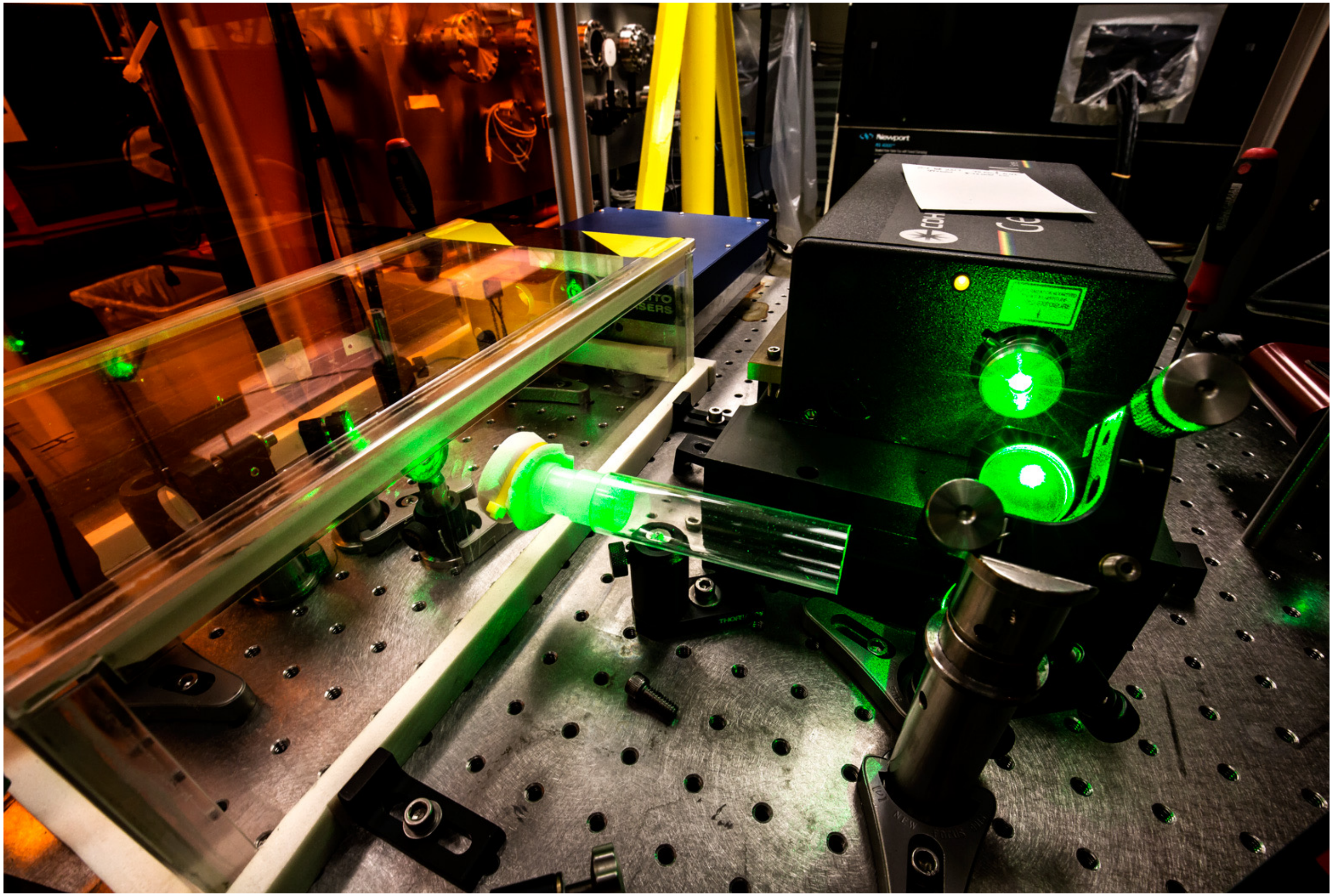
- Powerful pulses can be relativistic in the ISM.

very different physics in the two cases!

Other applications of theory of strong EM wave propagation in plasma

- Propagation of pulses in Chirped pulse amplification lasers ($\underset{\sim}{E}$ ionizes atoms) (Morou, Strickland, Nobel 2018)
- Laser-driven plasma wave next-generation particle accelerators, laser fusion...

see reviews by $\{$ Morou + 2006 Rev Mod Phys 78, 309
Esaray + 2009 Rev Mod Phys 81, 1229



Hercules laser (U. Michigan), $F=2\times10^{22}$ W cm^{-2}, $\lambda=0.5\mu$m, f =40 !!

The challenge of understanding strong EM waves:

$$F = q\Big(\underset{\sim}{E} + \underset{\uparrow}{\frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}}\Big) = \frac{d}{dt}\,\frac{m\underset{\sim}{v}}{\sqrt{1-\underset{\sim}{v}\cdot\underset{\sim}{v}/c^2}}$$

- can no longer neglect for $f \geq 0.1$, since quiver $v \sim c$
- since $\underset{\sim}{v}$ depends on $\underset{\sim}{E}$ and $\underset{\sim}{B}$ (and RHS has $\underset{\sim}{v}\cdot\underset{\sim}{v}$) response is nonlinear.
  - Can no longer Fourier decompose waves into superposition of indep monochromati solutions
  - waves no longer sinusoidal

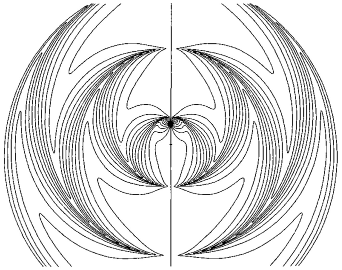


FIG. 15. Electric field lines of a charge undergoing one-dimensional simple harmonic motion with $\beta_{\max}=0.90$, $t_0=\frac{1}{2}\pi/\omega$ so that $y(t_0)=a$.

- Radiation from relativistic particles is dominated by high harmonics of the driving power.
- Nonlinearities → mode coupling, instabilities, stochastic plasma heating
  - So what actually escapes?
  - From what radius do "conventional" radio waves begin?
  - What features of FRB spectra and polarization are set by this?

What can you do to start? some suggested student projects

1) Plot particle trajectories (cold plasma)

   - for strong monochromatic harmonic input EM waves

     (there are exact analytical solutions for linear + circular polariz)

   - what are conditions for chaos? Stochastic heating?

   - What about general elliptical polarization?

   - What about finite wave train?

   - What about broad band, non harmonic EM wave, $\Delta\nu \sim \frac{1}{4}\nu$ & harmonics

2) Find cold plasma dispersion relation for plane wave of all polarizations

   - Group velocity for a strong wave pulse

     wave packet (for e.g. $f \sim 10^{-2}$, $f \sim 10$)

3) The radiation from quivering particles for $f \gtrsim 1$ is no longer principally at the input frequency. Harmonic generation, not a simple dispersion relation

- Relation to induced scattering, absorption
- Strong wave means particles in a drive wave are moving coherently

  ⇒ N^2 not N

  classical "maser" emission, induced scattering

4) Effect on plasma heating, free-free absortion

- What is final state after the FRB pulse has gone through?

v × **B** force accelerates electron to $\gamma \sim f$ along direction of wave propagation.
Currents and filamentation!

Infinite plane wave: electron at rest accelerated to $\gamma_{\max} \approx f^{\,2}$
(not f, due to phase locking). Single pulse: $\gamma_{\max} \approx f$ if $B_{\rm ext}$=0.
In large $B_{\rm ext}$ get c ($E \times B/B_{\rm ext}^{\,2}$) drift instead.

Some references to start with:

Particle motion in strong waves:

PHYSICAL REVIEW E VOLUME 62, NUMBER 3 SEPTEMBER 2000

Dynamics of a charged particle in a circularly polarized traveling electromagnetic wave

A. Bourdier and S. Gond*

PHYSICAL REVIEW E, VOLUME 63, 036609

Dynamics of a charged particle in a linearly polarized traveling electromagnetic wave

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Physica D 206 (2005) 1–31

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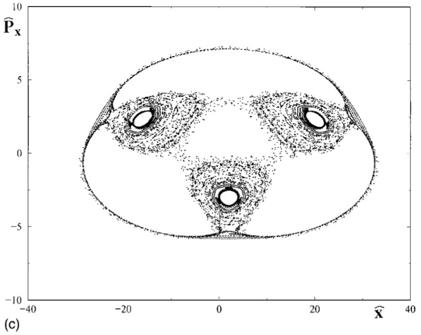
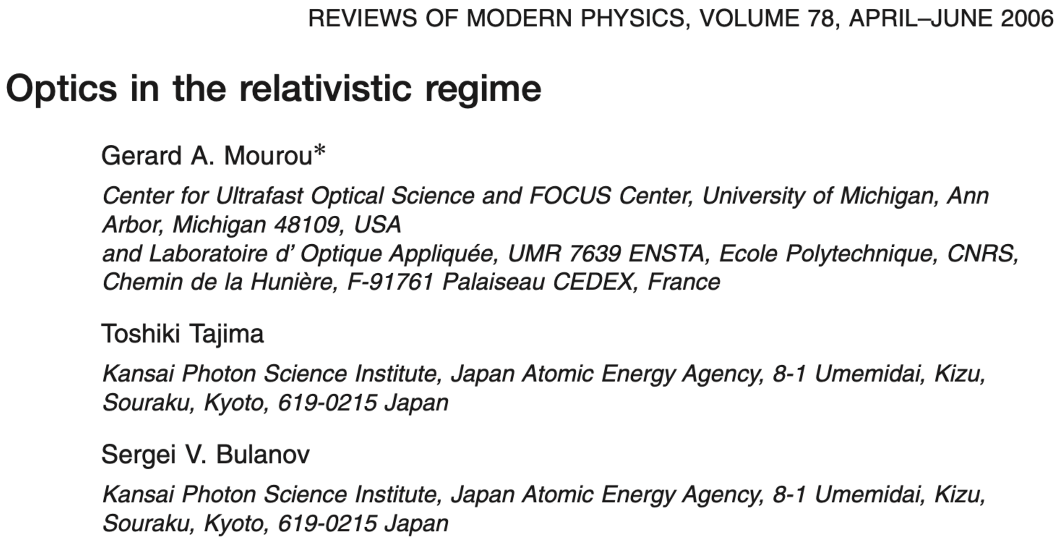
www.elsevier.com/locate/physd



Stochastic heating in ultra high intensity laser-plasma interaction

A. Bourdier*, D. Patin, E. Lefebvre

Strong wave theory and applications of Chirped Pulse Amplified lasers, wake fields



REVIEWS OF MODERN PHYSICS, VOLUME 78, APRIL–JUNE 2006

Optics in the relativistic regime

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Center for Ultrafast Optical Science and FOCUS Center, University of Michigan, Ann Arbor, Michigan 48109, USA
and Laboratoire d' Optique Appliquée, UMR 7639 ENSTA, Ecole Polytechnique, CNRS, Chemin de la Hunière, F-91761 Palaiseau CEDEX, France

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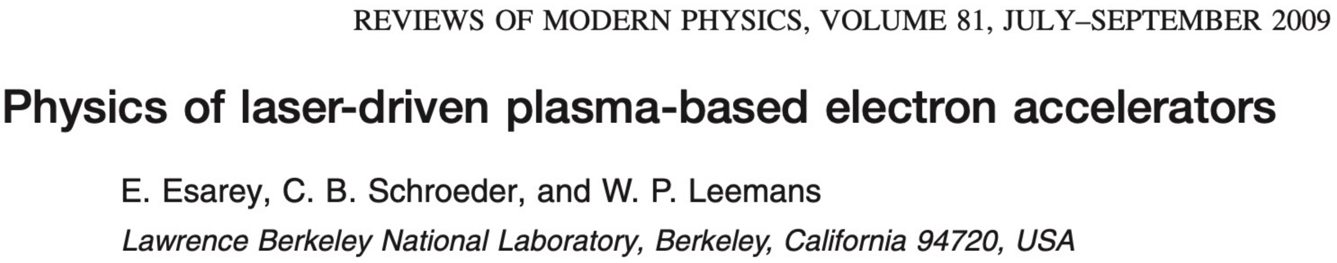
Kansai Photon Science Institute, Japan Atomic Energy Agency, 8-1 Umemidai, Kizu, Souraku, Kyoto, 619-0215 Japan

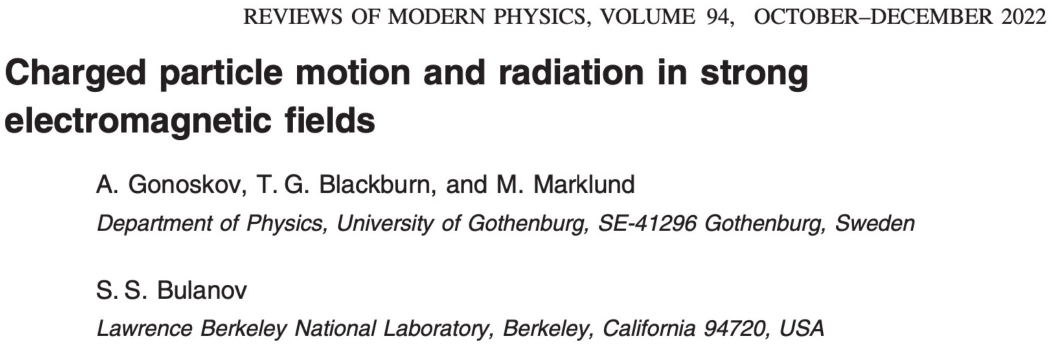
Sergei V. Bulanov
Kansai Photon Science Institute, Japan Atomic Energy Agency, 8-1 Umemidai, Kizu, Souraku, Kyoto, 619-0215 Japan



REVIEWS OF MODERN PHYSICS, VOLUME 81, JULY–SEPTEMBER 2009

Physics of laser-driven plasma-based electron accelerators

E. Esarey, C. B. Schroeder, and W. P. Leemans
Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA



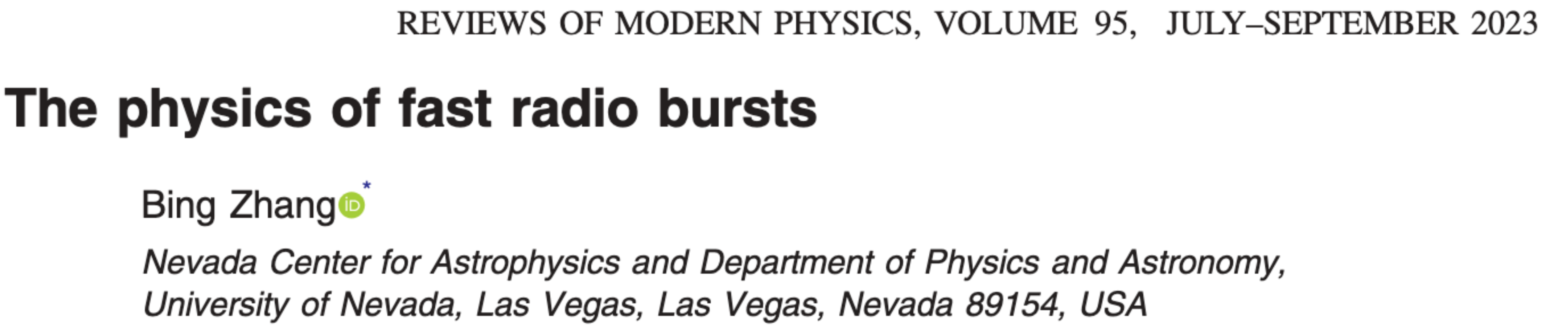
REVIEWS OF MODERN PHYSICS, VOLUME 94, OCTOBER–DECEMBER 2022

Charged particle motion and radiation in strong electromagnetic fields

A. Gonoskov, T. G. Blackburn, and M. Marklund
Department of Physics, University of Gothenburg, SE-41296 Gothenburg, Sweden

S. S. Bulanov
Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

Fast Radio Bursts

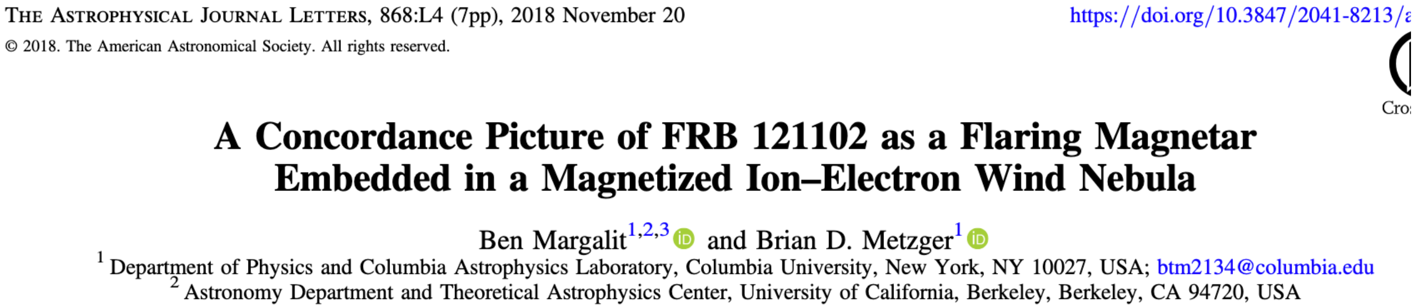


REVIEWS OF MODERN PHYSICS, VOLUME 95, JULY–SEPTEMBER 2023

The physics of fast radio bursts

Bing Zhang*

Nevada Center for Astrophysics and Department of Physics and Astronomy,
University of Nevada, Las Vegas, Las Vegas, Nevada 89154, USA



THE ASTROPHYSICAL JOURNAL LETTERS, 868:L4 (7pp), 2018 November 20 https://doi.org/10.3847/2041-8213/

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A Concordance Picture of FRB 121102 as a Flaring Magnetar Embedded in a Magnetized Ion–Electron Wind Nebula

Ben Margalit[1,2,3] and Brian D. Metzger[1]

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[2] Astronomy Department and Theoretical Astrophysics Center, University of California, Berkeley, Berkeley, CA 94720, USA

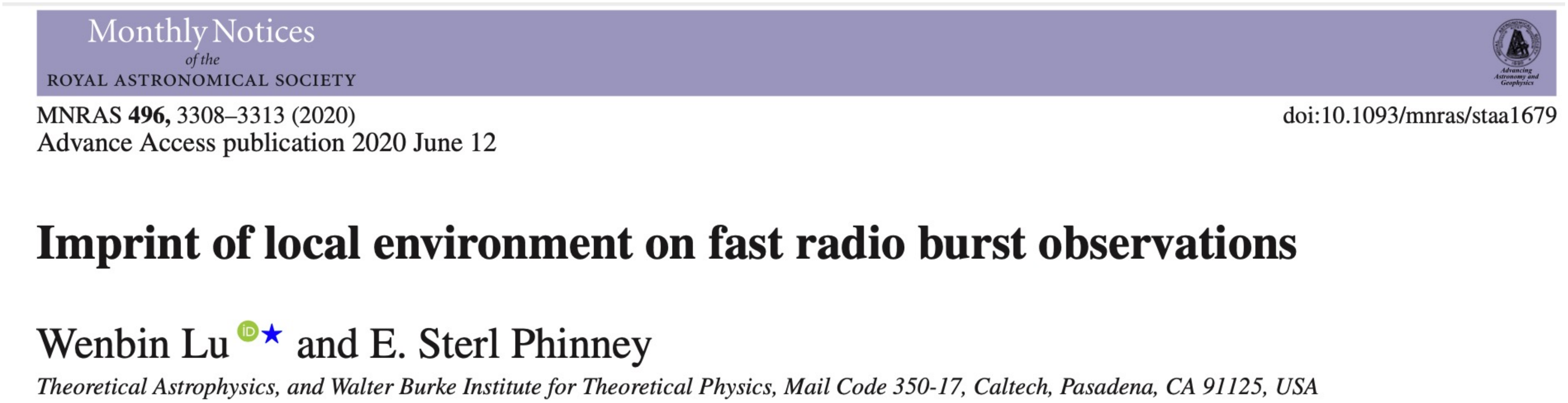
Article

Emission Mechanisms of Fast Radio Bursts

Yuri Lyubarsky

Physics Department, Ben-Gurion University, P.O.B. 653, Beer-Sheva 84105, Israel; e-mail: lyub@bgu.ac.il

arxiv:2103.00470



Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

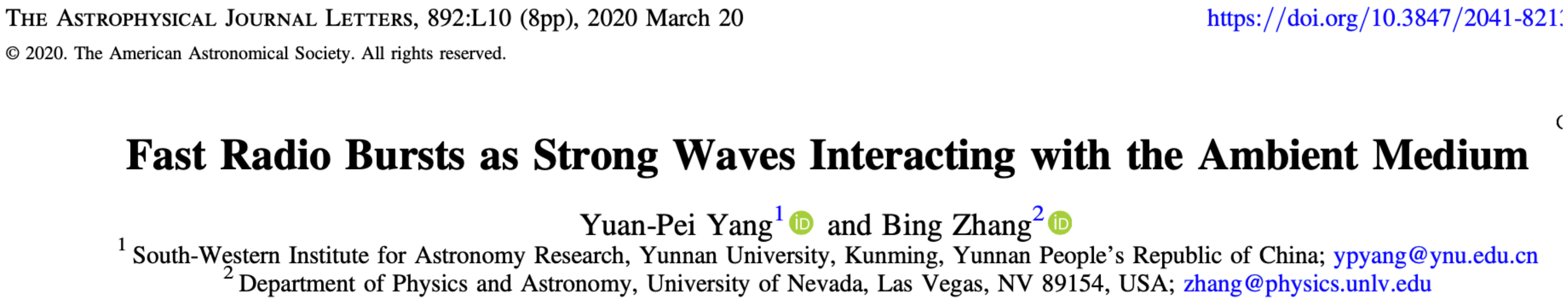
MNRAS **496,** 3308–3313 (2020)
Advance Access publication 2020 June 12

doi:10.1093/mnras/staa1679

Imprint of local environment on fast radio burst observations

Wenbin Lu ★ and E. Sterl Phinney

Theoretical Astrophysics, and Walter Burke Institute for Theoretical Physics, Mail Code 350-17, Caltech, Pasadena, CA 91125, USA



The Astrophysical Journal Letters, 892:L10 (8pp), 2020 March 20

https://doi.org/10.3847/2041-821

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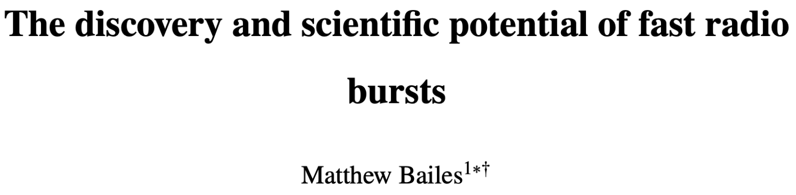
Fast Radio Bursts as Strong Waves Interacting with the Ambient Medium

Yuan-Pei Yang[1] and Bing Zhang[2]

[1] South-Western Institute for Astronomy Research, Yunnan University, Kunming, Yunnan People's Republic of China; ypyang@ynu.edu.cn
[2] Department of Physics and Astronomy, University of Nevada, Las Vegas, NV 89154, USA; zhang@physics.unlv.edu

FRB observations and cosmology:

arXiv:2211.06048

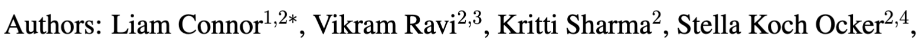


The discovery and scientific potential of fast radio bursts

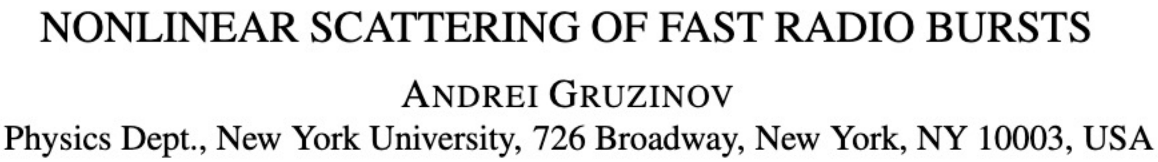
Matthew Bailes[1*†]

A gas-rich cosmic web revealed by the partitioning of the missing baryons

https://arxiv.org/pdf/2409.16952



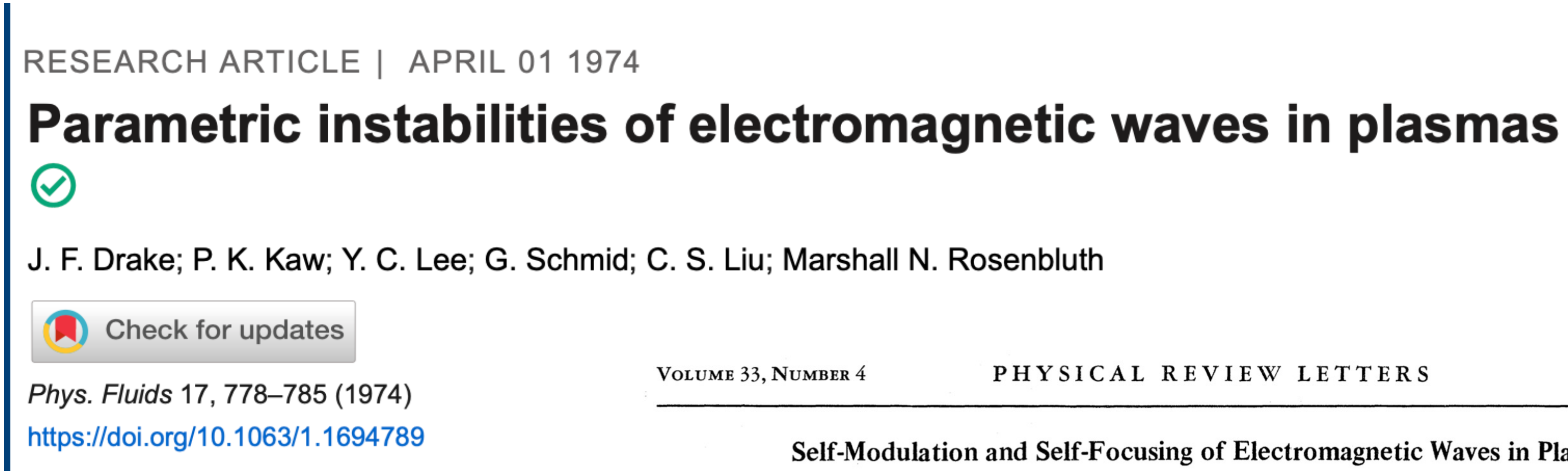
Authors: Liam Connor[1,2*], Vikram Ravi[2,3], Kritti Sharma[2], Stella Koch Ocker[2,4],



NONLINEAR SCATTERING OF FAST RADIO BURSTS

ANDREI GRUZINOV
Physics Dept., New York University, 726 Broadway, New York, NY 10003, USA

https://arxiv.org/pdf/1912.08150



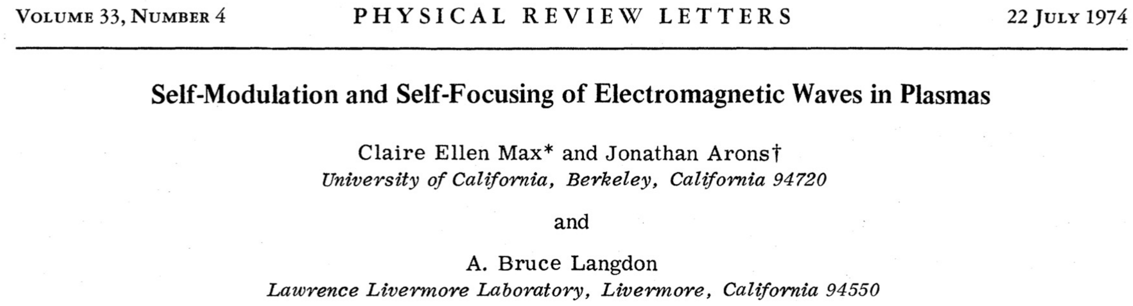
RESEARCH ARTICLE | APRIL 01 1974

Parametric instabilities of electromagnetic waves in plasmas

J. F. Drake; P. K. Kaw; Y. C. Lee; G. Schmid; C. S. Liu; Marshall N. Rosenbluth

Check for updates

Phys. Fluids 17, 778–785 (1974)
https://doi.org/10.1063/1.1694789



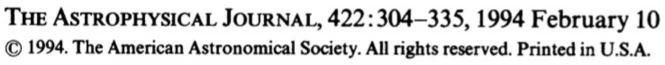
VOLUME 33, NUMBER 4 PHYSICAL REVIEW LETTERS 22 JULY 1974

Self-Modulation and Self-Focusing of Electromagnetic Waves in Plasmas

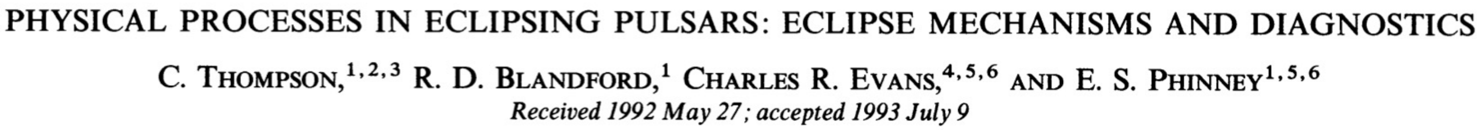
Claire Ellen Max* and Jonathan Arons†
University of California, Berkeley, California 94720

and

A. Bruce Langdon
Lawrence Livermore Laboratory, Livermore, California 94550



THE ASTROPHYSICAL JOURNAL, 422:304–335, 1994 February 10



PHYSICAL PROCESSES IN ECLIPSING PULSARS: ECLIPSE MECHANISMS AND DIAGNOSTICS

C. THOMPSON,[1,2,3] R. D. BLANDFORD,[1] CHARLES R. EVANS,[4,5,6] AND E. S. PHINNEY[1,5,6]
Received 1992 May 27; accepted 1993 July 9

Article | Published: 04 November 2020

A fast radio burst associated with a Galactic magnetar

C. D. Bochenek, V. Ravi, K. V. Belov, G. Hallinan, J. Kocz, S. R. Kulkarni & D. L. McKenna

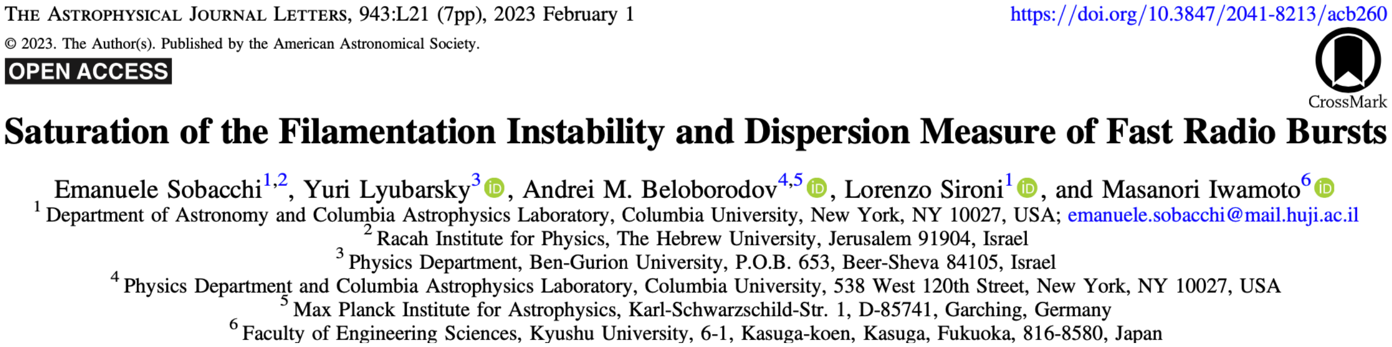
Article | Published: 23 February 2022

A repeating fast radio burst source in a globular cluster

F. Kirsten, B. Marcote, K. Nimmo, J. W. T. Hessels, M. Bhardwaj, S. P. Tendulkar, A. Keimpema, J. Yang, M. P. Snelders, P. Scholz, A. B. Pearlman, C. J. Law, W. M. Peters, M. Giroletti, Z. Paragi, C. Bassa, D. M. Hewitt, U. Bach, V. Bezrukovs, M. Burgay, S. T. Buttaccio, J. E. Conway, A. Corongiu, R. Feiler, … W. Vlemmings

Nature **602**, 585–589 (2022) | Cite this article

Numerical PIC simulations. Note that these, like laser experiments, have boxes only a few wavelengths in size. Not directly relevant to FRBs, where region is
~ 10^5 to 10^{15} wavelengths in size!



THE ASTROPHYSICAL JOURNAL LETTERS, 943:L21 (7pp), 2023 February 1
https://doi.org/10.3847/2041-8213/acb260

Saturation of the Filamentation Instability and Dispersion Measure of Fast Radio Bursts

Emanuele Sobacchi[1,2], Yuri Lyubarsky[3], Andrei M. Beloborodov[4,5], Lorenzo Sironi[1], and Masanori Iwamoto[6]
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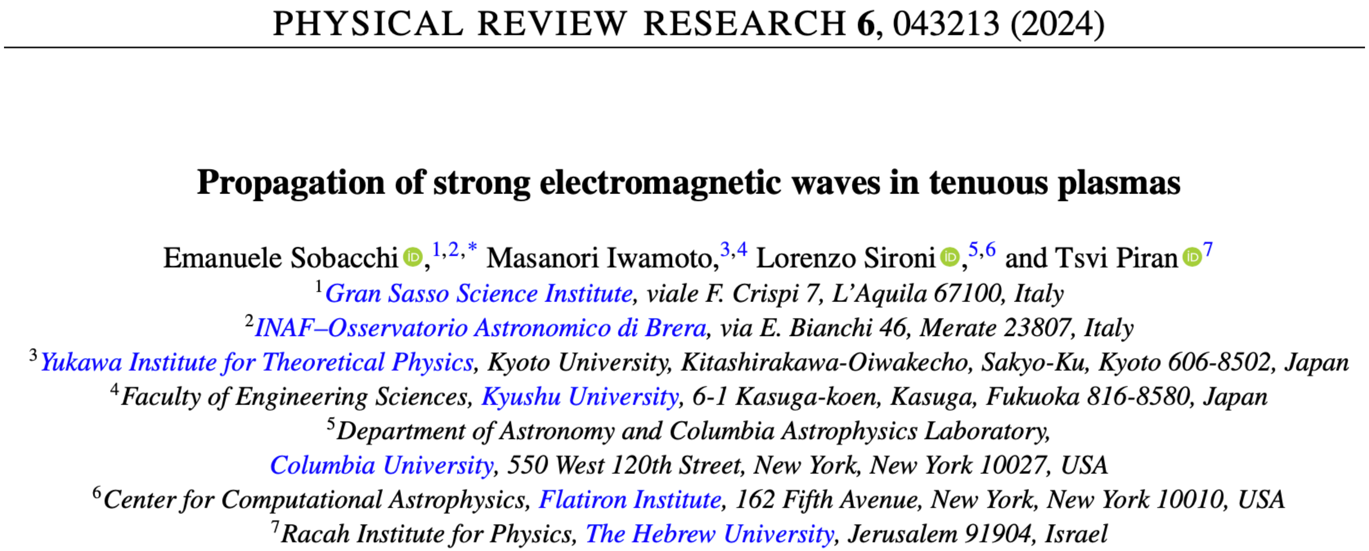

PHYSICAL REVIEW RESEARCH **6**, 043213 (2024)

Propagation of strong electromagnetic waves in tenuous plasmas

Emanuele Sobacchi,[1,2,*] Masanori Iwamoto,[3,4] Lorenzo Sironi,[5,6] and Tsvi Piran[7]
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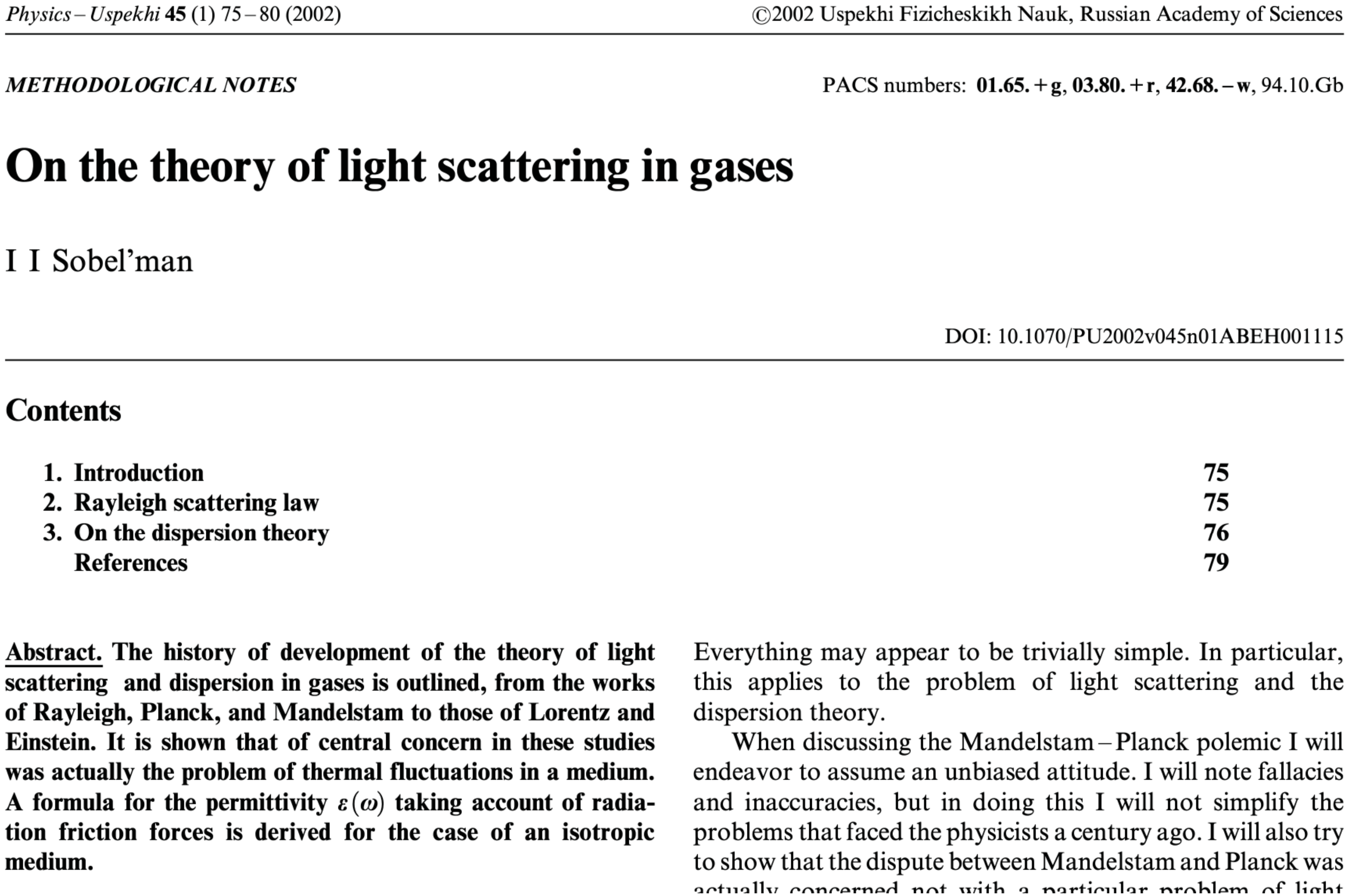
[6] *Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, New York 10010, USA*
[7] *Racah Institute for Physics, The Hebrew University, Jerusalem 91904, Israel*

Historical bonus, on coherent vs incoherent scattering and index of refraction!



Physics – Uspekhi **45** (1) 75 – 80 (2002)

METHODOLOGICAL NOTES

PACS numbers: **01.65. + g**, **03.80. + r**, **42.68. – w**, 94.10.Gb

On the theory of light scattering in gases

I I Sobel'man

DOI: 10.1070/PU2002v045n01ABEH001115

Contents

Abstract. The history of development of the theory of light scattering and dispersion in gases is outlined, from the works of Rayleigh, Planck, and Mandelstam to those of Lorentz and Einstein. It is shown that of central concern in these studies was actually the problem of thermal fluctuations in a medium. A formula for the permittivity $\varepsilon(\omega)$ taking account of radiation friction forces is derived for the case of an isotropic medium.

Everything may appear to be trivially simple. In particular, this applies to the problem of light scattering and the dispersion theory.

When discussing the Mandelstam – Planck polemic I will endeavor to assume an unbiased attitude. I will note fallacies and inaccuracies, but in doing this I will not simplify the problems that faced the physicists a century ago. I will also try to show that the dispute between Mandelstam and Planck was actually concerned not with a particular problem of light