Can we prove inflation?

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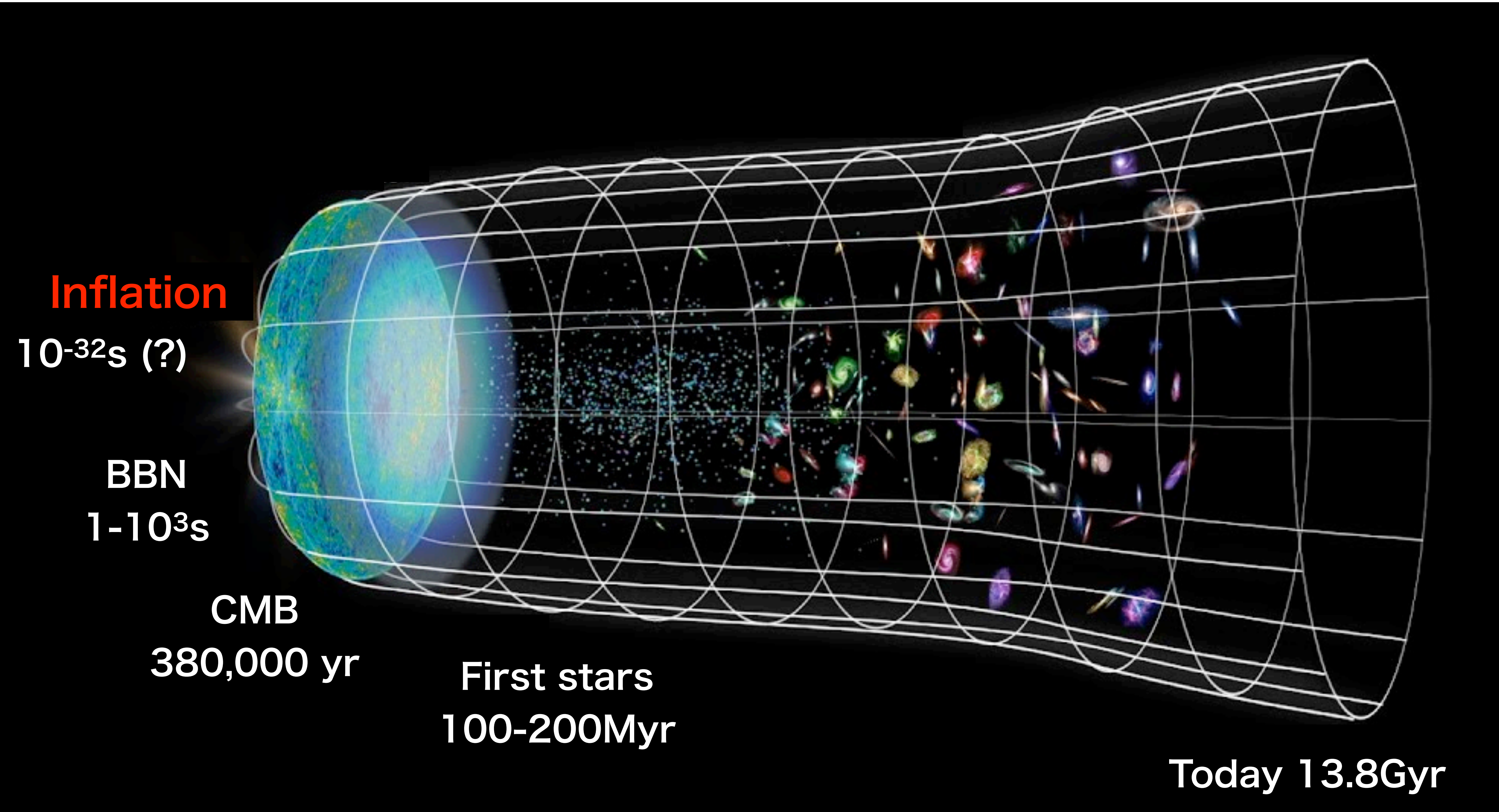
Inflation

Early accelerated expansion of the Universe



No direct evidence, but supported by many observational facts.

How can we prove inflation?

prove

Dictionary.com

verb (used with object)

proved, proved, proven, proving.

1 to establish the truth or genuineness of, as by evidence or argument.

> *to prove one's claim.*

Synonyms: verify, substantiate, confirm, demonstrate

Antonyms: disprove

2 *Law.* to establish the authenticity or validity of (a will); probate.

3 to give demonstration of by action.

4 to subject to a test, experiment, comparison, analysis, or the like, to determine quality, amount, acceptability, characteristics, etc..

> *to prove ore.*

5 to show (oneself) to have the character or ability expected of one, especially through one's actions.

6 *Mathematics.* to verify the correctness or validity of by mathematical demonstration or arithmetical proof.

How can we ~~prove~~ confirm/test inflation?

Predictions of Inflation Theory

1. Accelerated expansion of the Universe

$$\ddot{a} > 0$$

$a(t)$: scale factor of the Universe

t : cosmic time

2. Nearly scale-invariant SCALAR perturbations

$$\mathcal{P}_{\mathrm{S,prim}}(k) \sim \mathrm{const.}$$

k : wavenumber

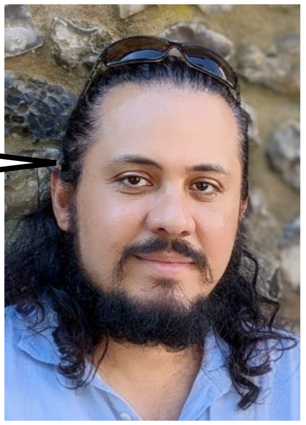
3. Nearly scale-invariant TENSOR perturbations

$$\mathcal{P}_{\mathrm{T,prim}}(k) \sim \mathrm{const.}$$

How can we ~~prove~~ confirm/test inflation?

<u>Predictions of Inflation Theory</u>

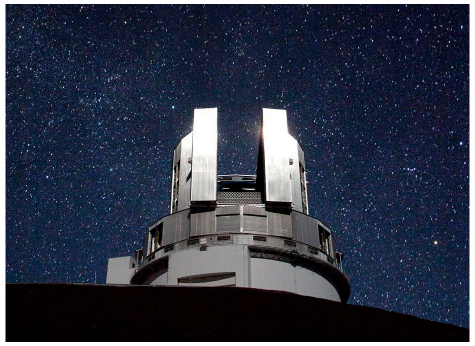
Special Thanks to
Yashar Akrami (IFT)



1. Accelerated expansion of the Universe

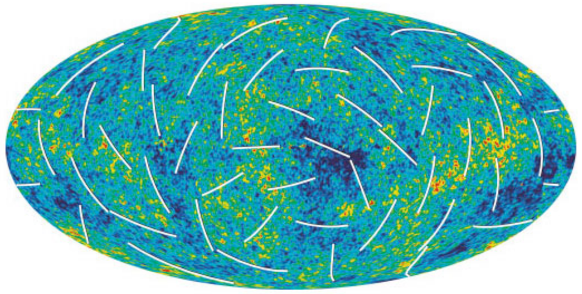
Only a direct measurement of the accelerated expansion could serve as proof of cosmic inflation!

2. Nearly scale-invariant SCALAR perturbations ✔



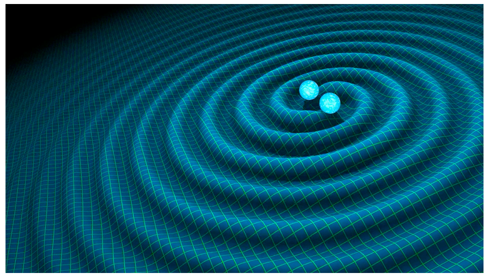
- Cosmic microwave background (CMB)
- Galaxy survey

3. Nearly scale-invariant TENSOR perturbations



- CMB B-mode polarization
- Direct detection of gravitational waves (GWs)

→ This talk



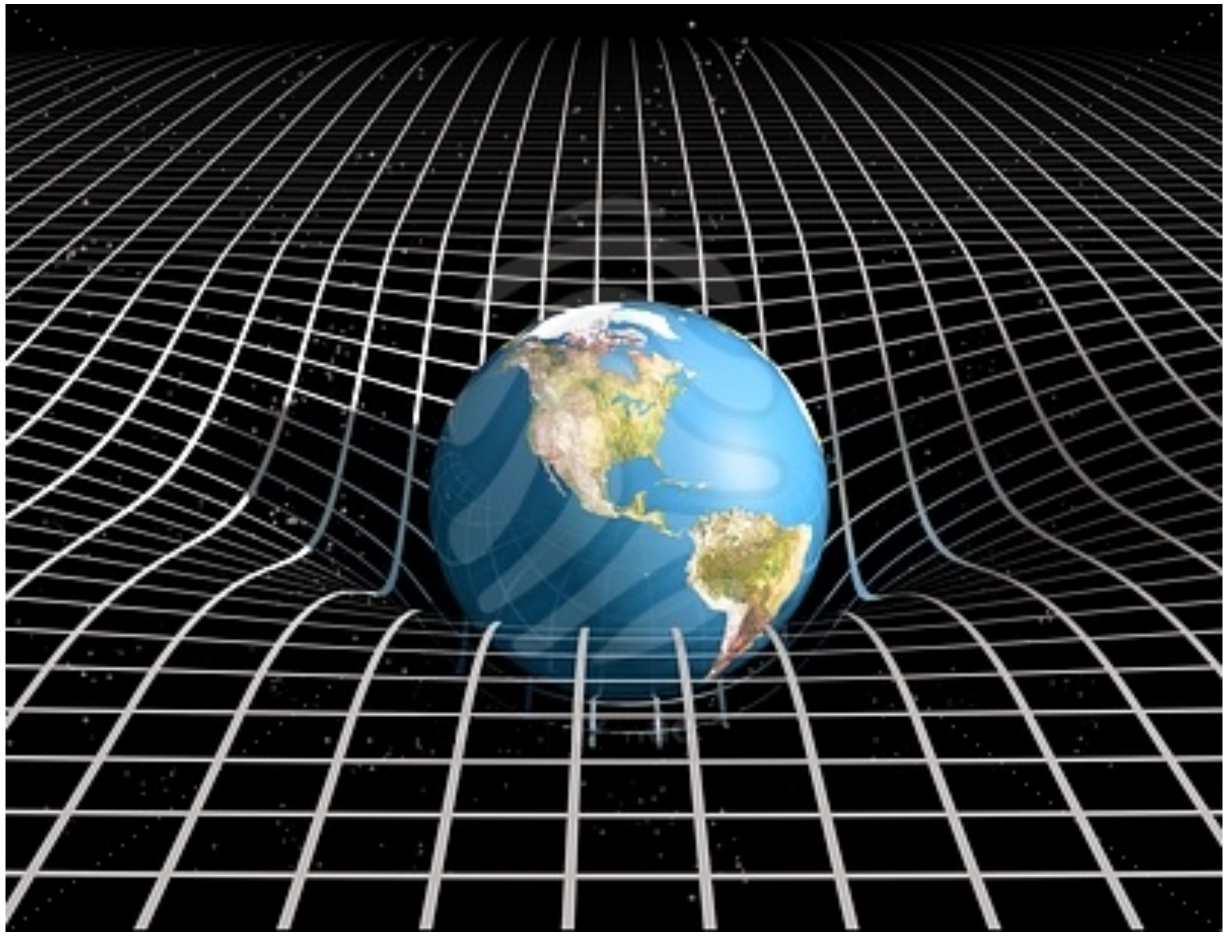
Basics of cosmology

Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Geometry
= Gravity

Matter



Equation for an expanding Universe

Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

ρ : density p: pressure

Friedmann–Lemaître–Robertson–Walker (FLRW) metric)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$a(t)$: scale factor K : curvature $<10^{-2}$ from observation

Friedmann equation: $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

Hubble expansion rate

Friedmann equation: $H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$

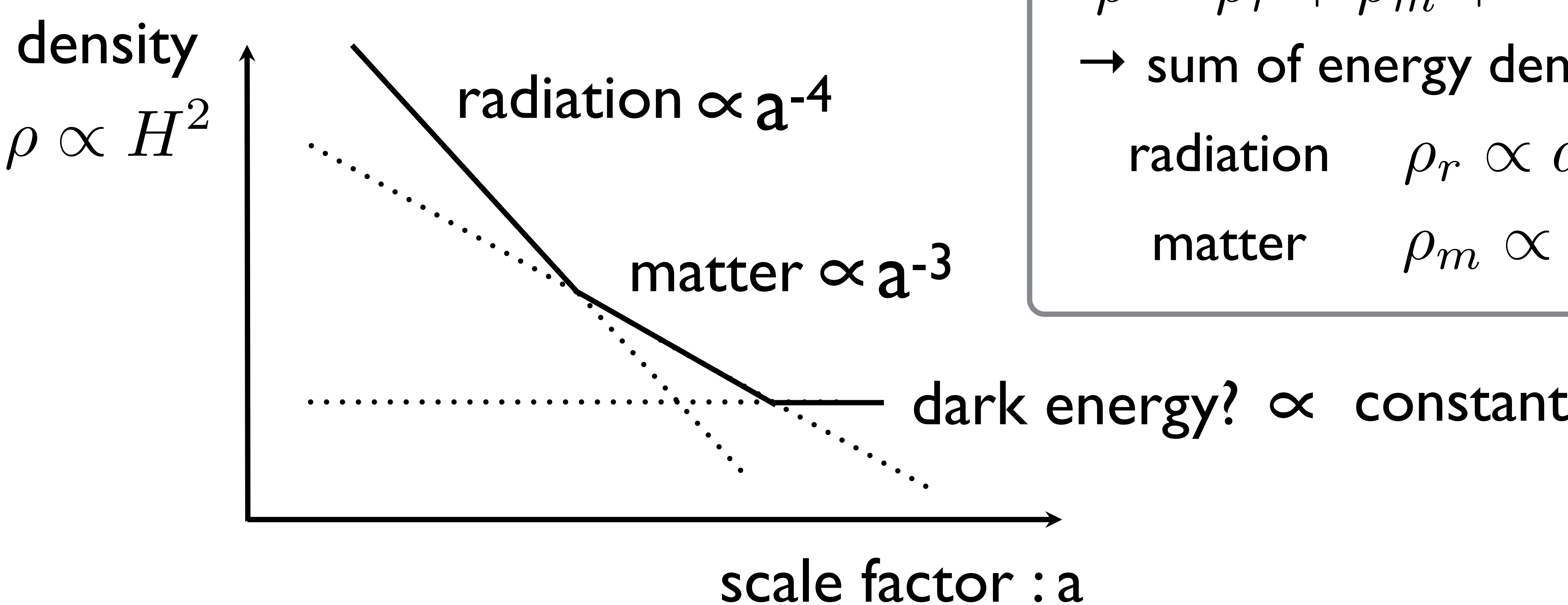
$H \equiv \frac{\dot{a}}{a}$: Hubble parameter

$\rho = \rho_r + \rho_m + \cdots$

→ sum of energy densities

radiation $\rho_r \propto a^{-4}$

matter $\rho_m \propto a^{-3}$



Motivation to consider inflation

Horizon / Flatness / Monopole problems

Inflation explains why…

the Universe is homogeneous

the Universe is flat

we do not observe magnetic monopole

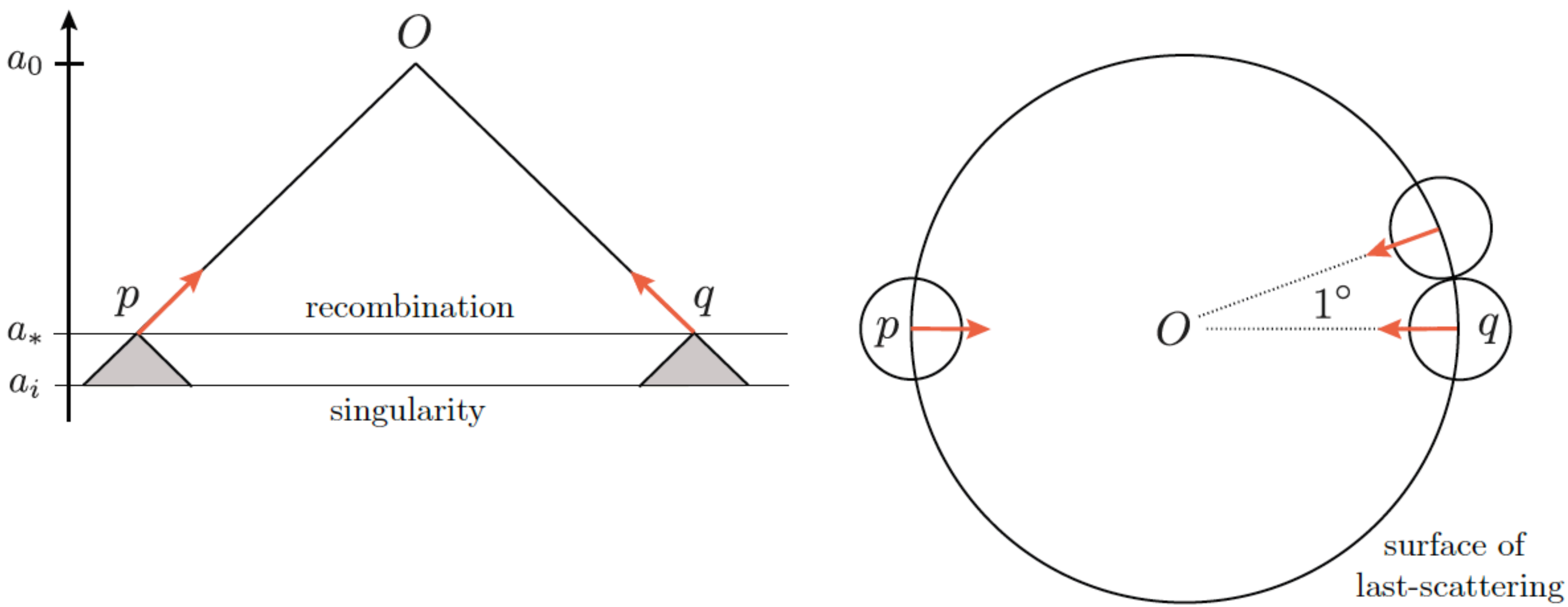
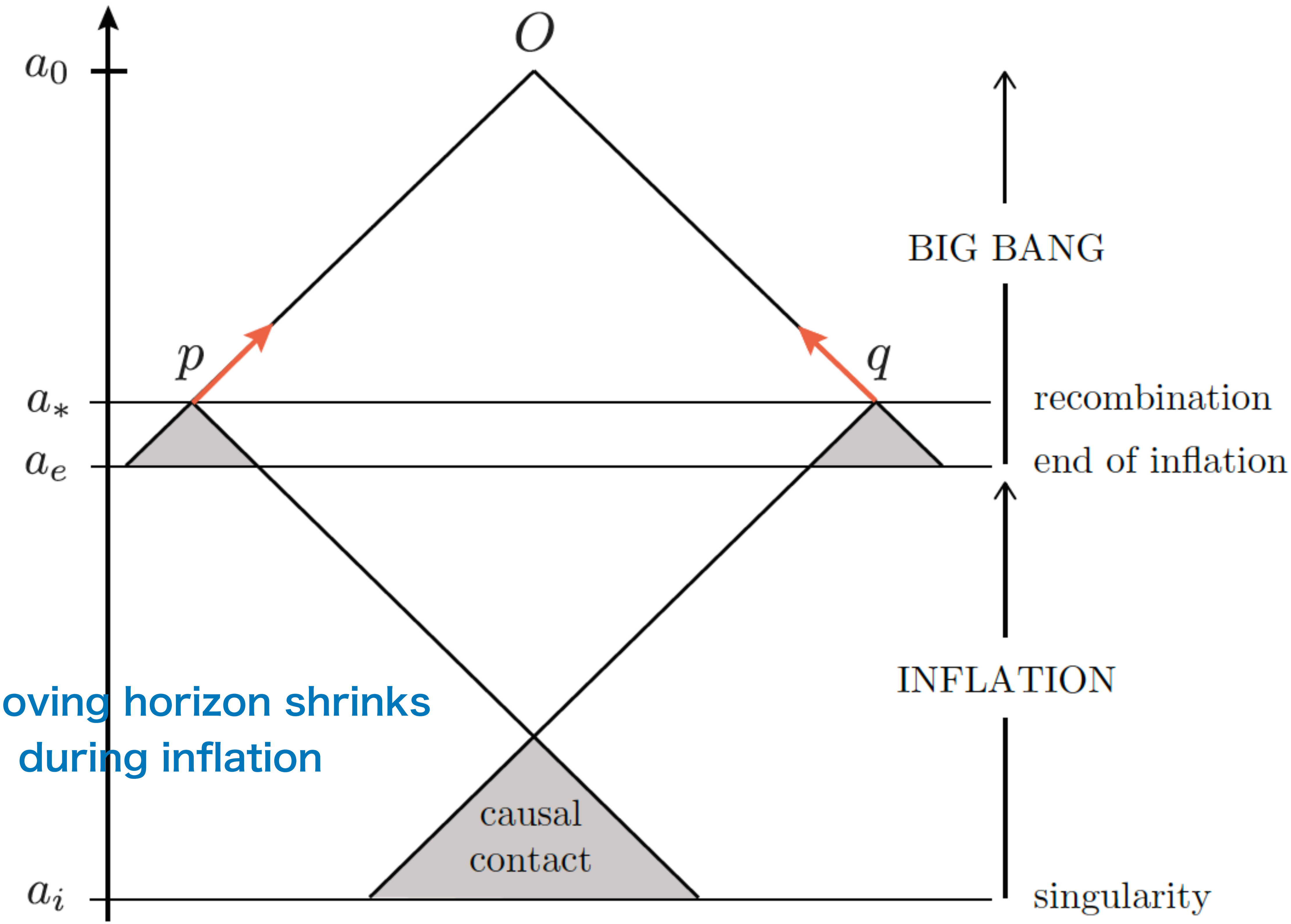


Figure from Baumman, "TASI Lectures on Primordial Cosmology", arXiv:1807.03098

Horizon problem



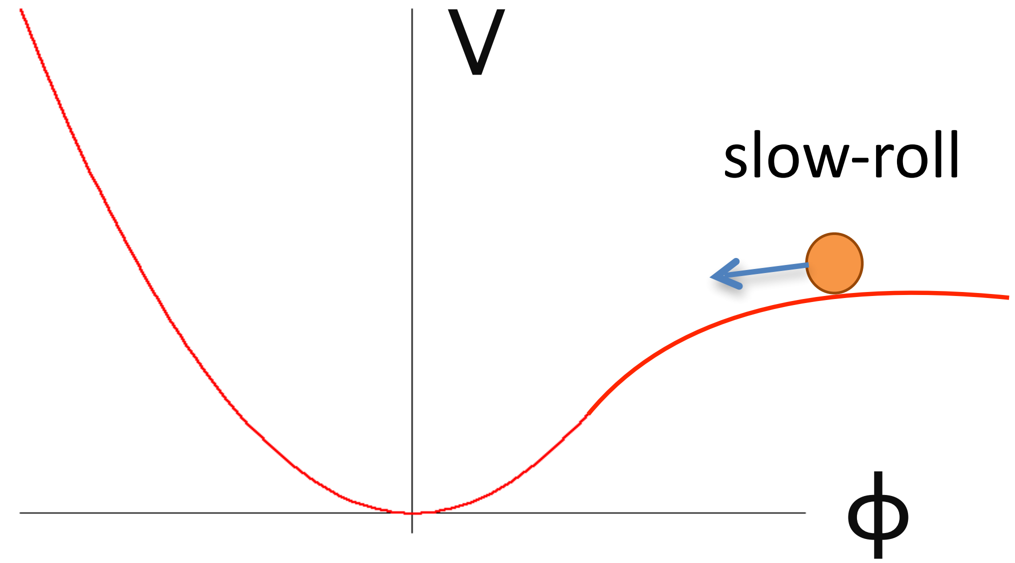
Comoving horizon shrinks during inflation

Figure from Baumman, “TASI Lectures on Primordial Cosmology”, arXiv:1807.03098

What drives inflation?

Most popular scenario

Inflation is driven by a scalar field slowly rolling down in its potential



Why use a scalar field?

→ Because it easily produces an isotropic universe.

Energy density of a scalar field

$$\rho_\phi = \dot{\phi}^2/2 + V$$

Equation of Motion

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Friedmann equation

$$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

slow-roll approximation

$$\dot{\phi}^2/2 \ll V$$

$$\longrightarrow \quad H = \text{const.} \quad \propto V$$

$$\rightarrow \quad a \propto \exp(Ht)$$

Exponential expansion

Why do we want to confirm inflation?

because it's about the origin of the universe
and could be linked to physics beyond standard models

What causes the accelerated expansion?

Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Geometry = Gravity

Matter

modification of gravity

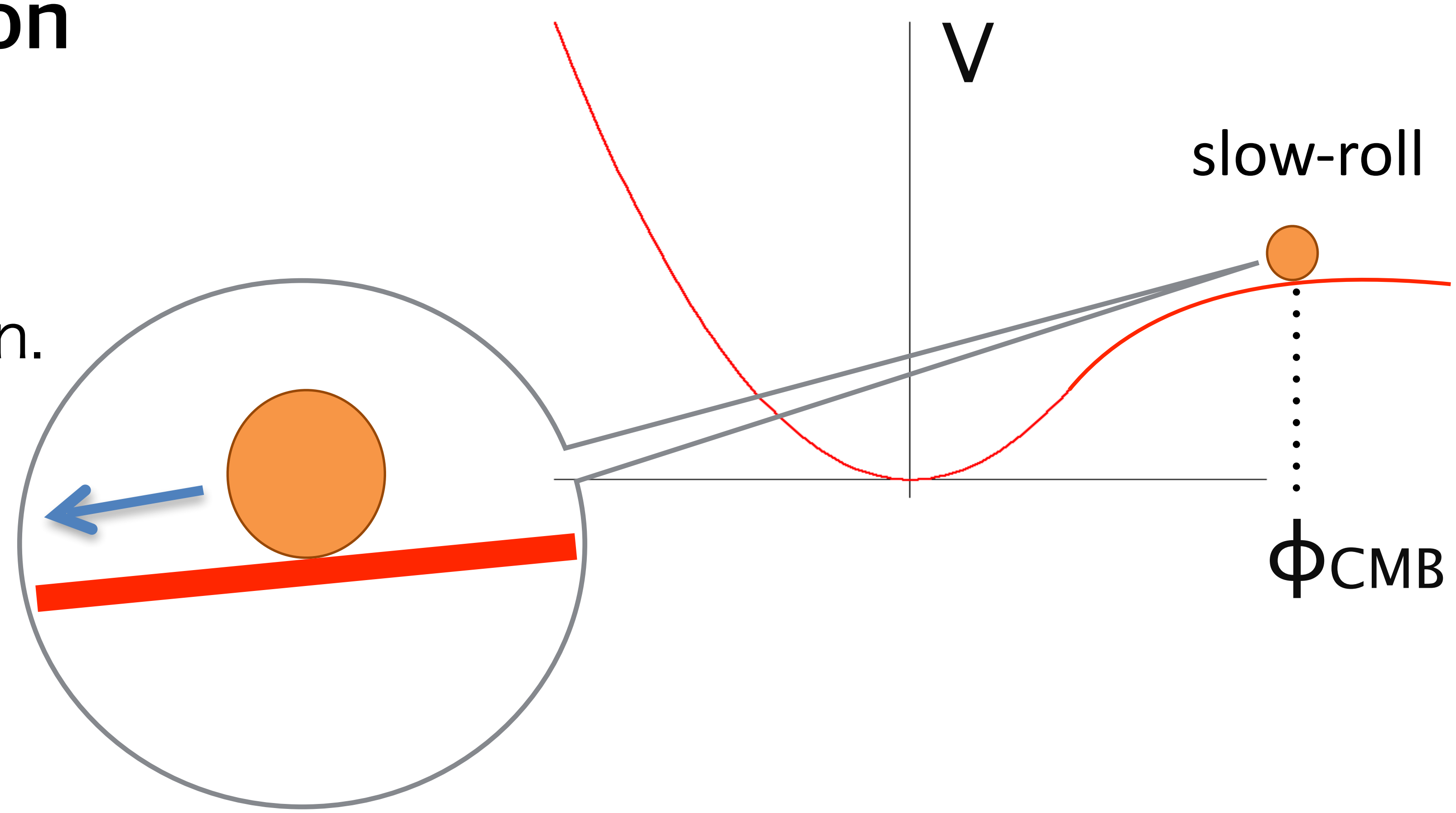
new (scalar) particle

→ new hint for the fundamental law of the universe

Predictions of inflation theory

Slow-roll parametrization

Since the scalar field rolls slowly, its position does not change much during inflation.



slow-roll parameters << 1

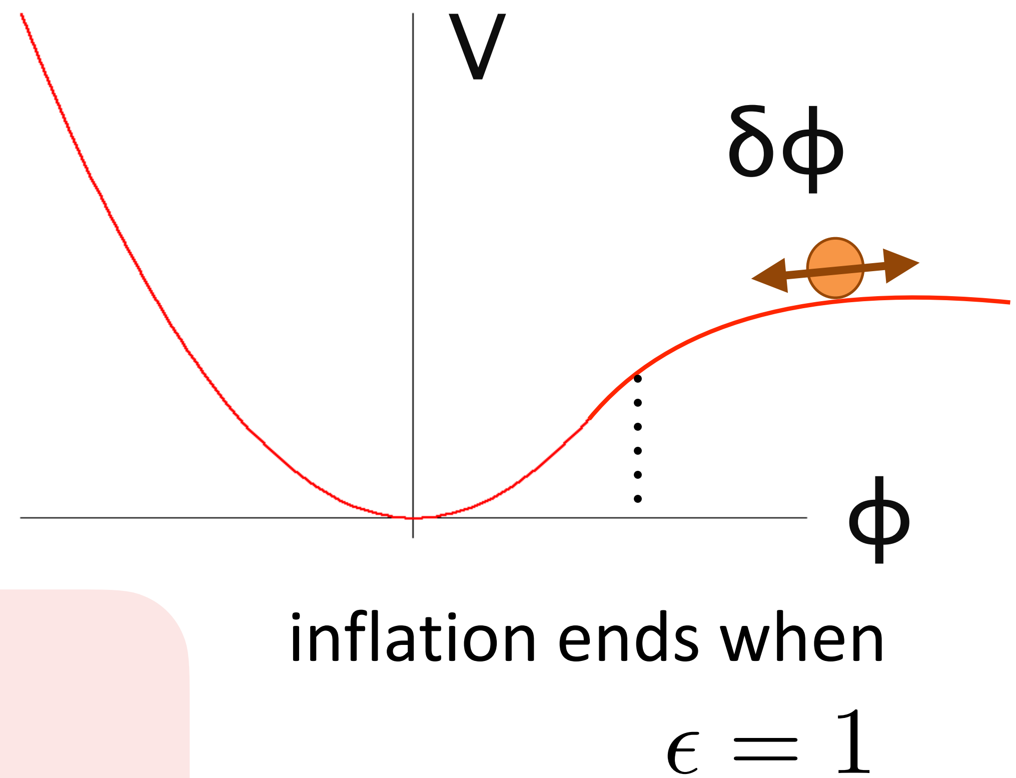
$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv M_{\rm Pl}^2\frac{V''}{V} \qquad \xi_V^2 \equiv M_{\rm Pl}^4\frac{V'V'''}{V^2}$$

→ A quantity that characterizes the potential V around $\phi=\phi_{\rm CMB}$ (' denotes a derivative with respect to ϕ)

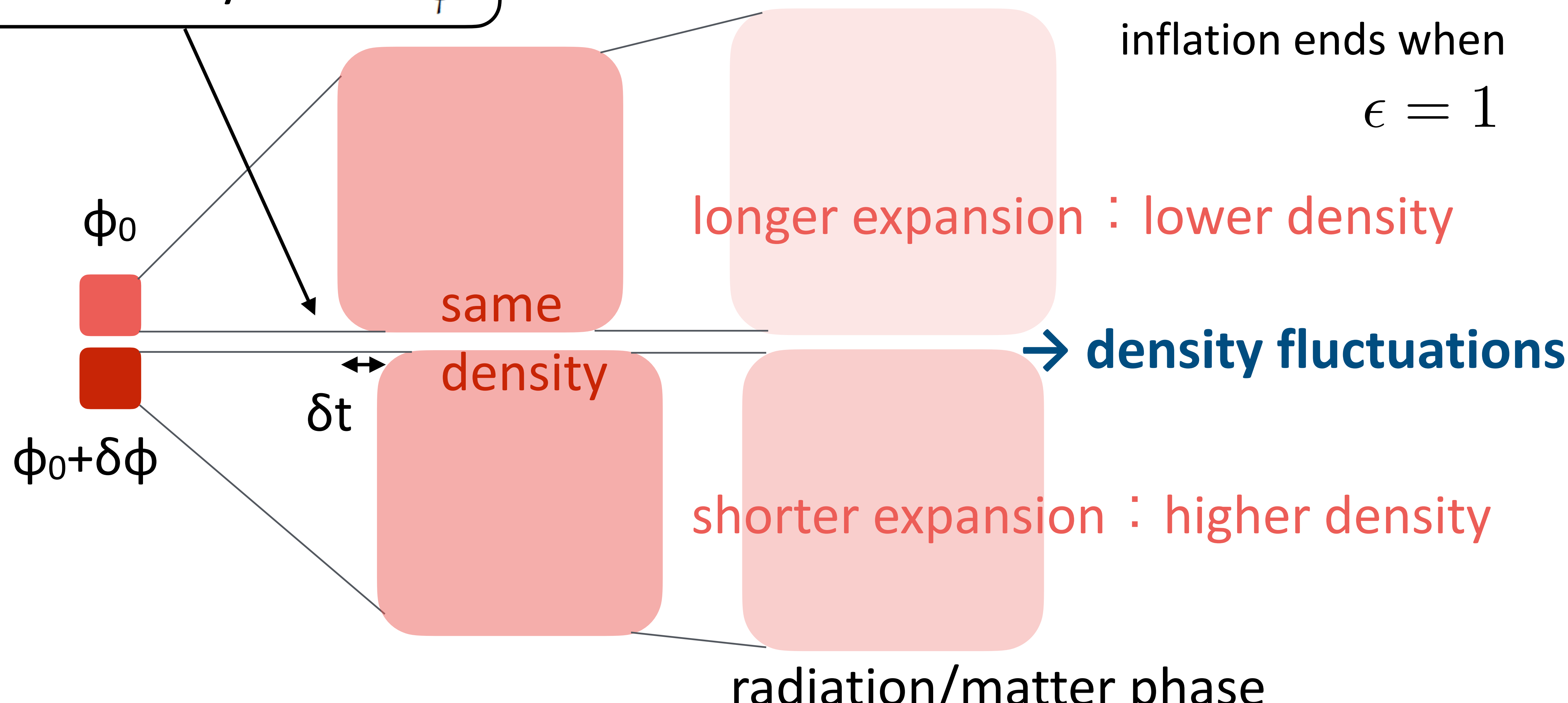
Creation of scalar perturbations

Inflaton scalar field has quantum fluctuations

→ Average ϕ_0 , Fluctuation $\delta\phi$

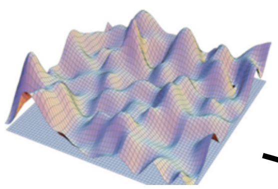


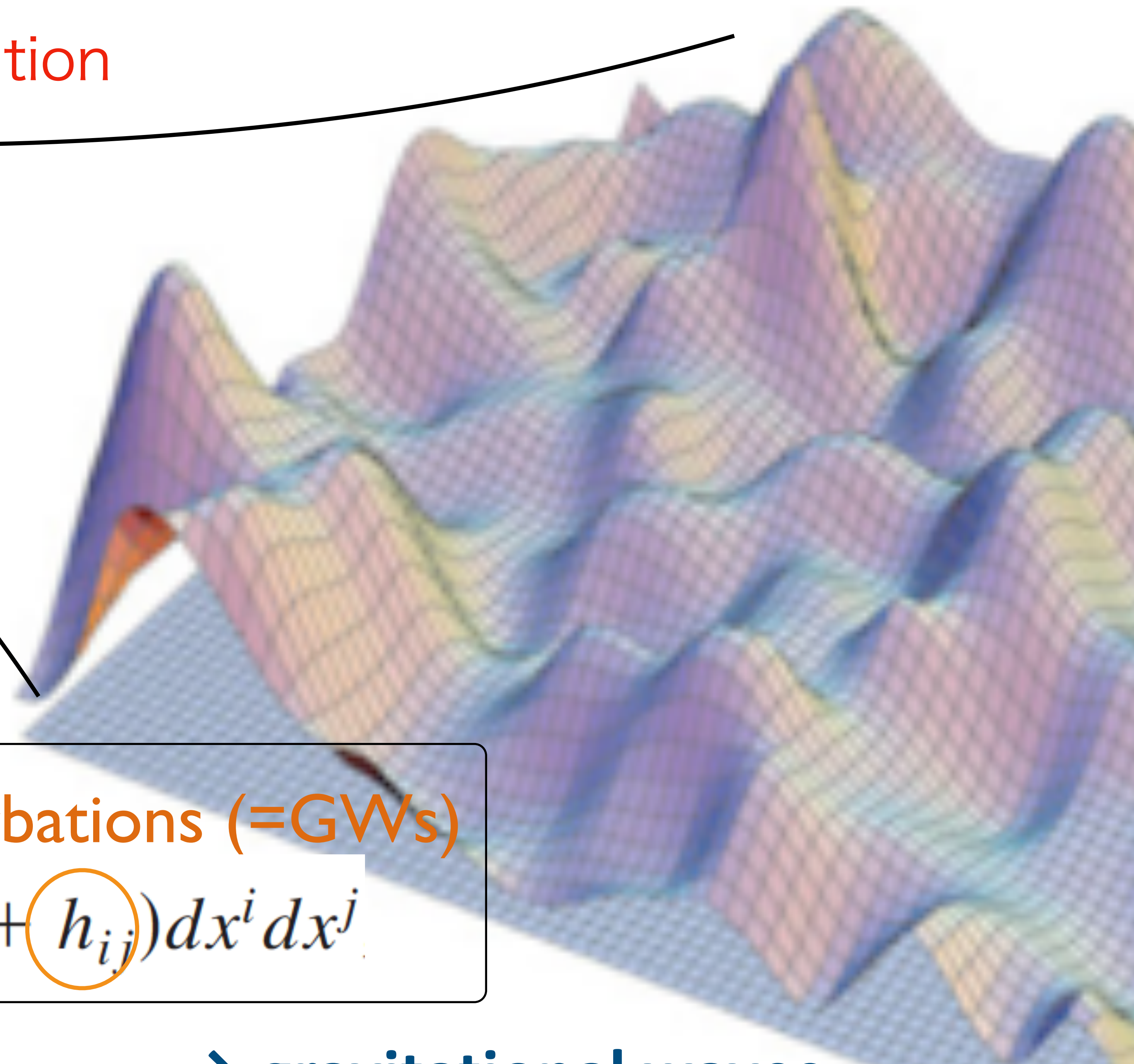
End of inflation is deviated by $\delta t = \frac{\delta\phi}{\dot{\phi}}$



Creation of tensor perturbations

Space-time itself exhibits
quantum fluctuations





Flat Universe + tensor perturbations (=GWs)

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

→ **gravitational waves**

Predictions of inflation theory

Scalar power spectrum

$$\mathcal{P}_{S,\mathrm{prim}}(k) = \frac{1}{\pi\epsilon}\left(\frac{H}{m_{\mathrm{Pl}}}\right)^2\Bigg|_{k=aH}$$

→ observed as density perturbations

Spectral tilt

$$n_S(k) - 1 \equiv \frac{d\ln\mathcal{P}_{S,\mathrm{prim}}(k)}{d\ln k} \simeq -6\epsilon + 2\eta$$

Running index

$$\alpha_S(k) \equiv \frac{dn_S(k)}{d\ln k} \simeq -16\epsilon\eta + 24\epsilon^2 + 2\xi^2$$

Tensor power spectrum

$$\mathcal{P}_{T,\mathrm{prim}}(k) = \frac{16}{\pi}\left(\frac{H}{m_{\mathrm{Pl}}}\right)^2\Bigg|_{k=aH}$$

→ observed as gravitational waves

Tensor-to-scalar ratio

$$r \equiv \frac{\mathcal{P}_{T,\mathrm{prim}}(k)}{\mathcal{P}_{S,\mathrm{prim}}(k)} \simeq 16\epsilon$$

Spectral tilt

$$n_T(k) \equiv \frac{d\ln\mathcal{P}_{T,\mathrm{prim}}(k)}{d\ln k} \simeq -2\epsilon$$

Predictions of inflation theory

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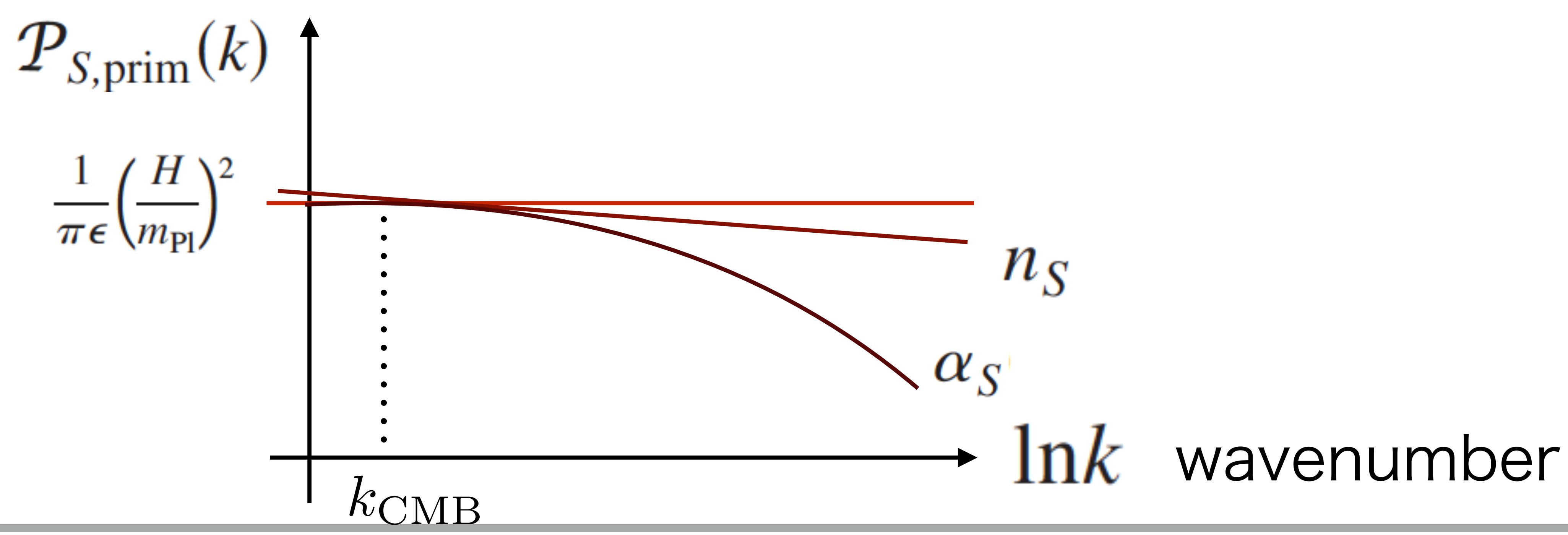
supressed

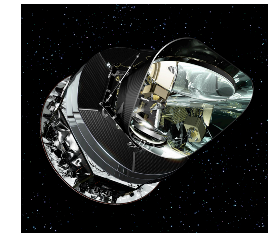


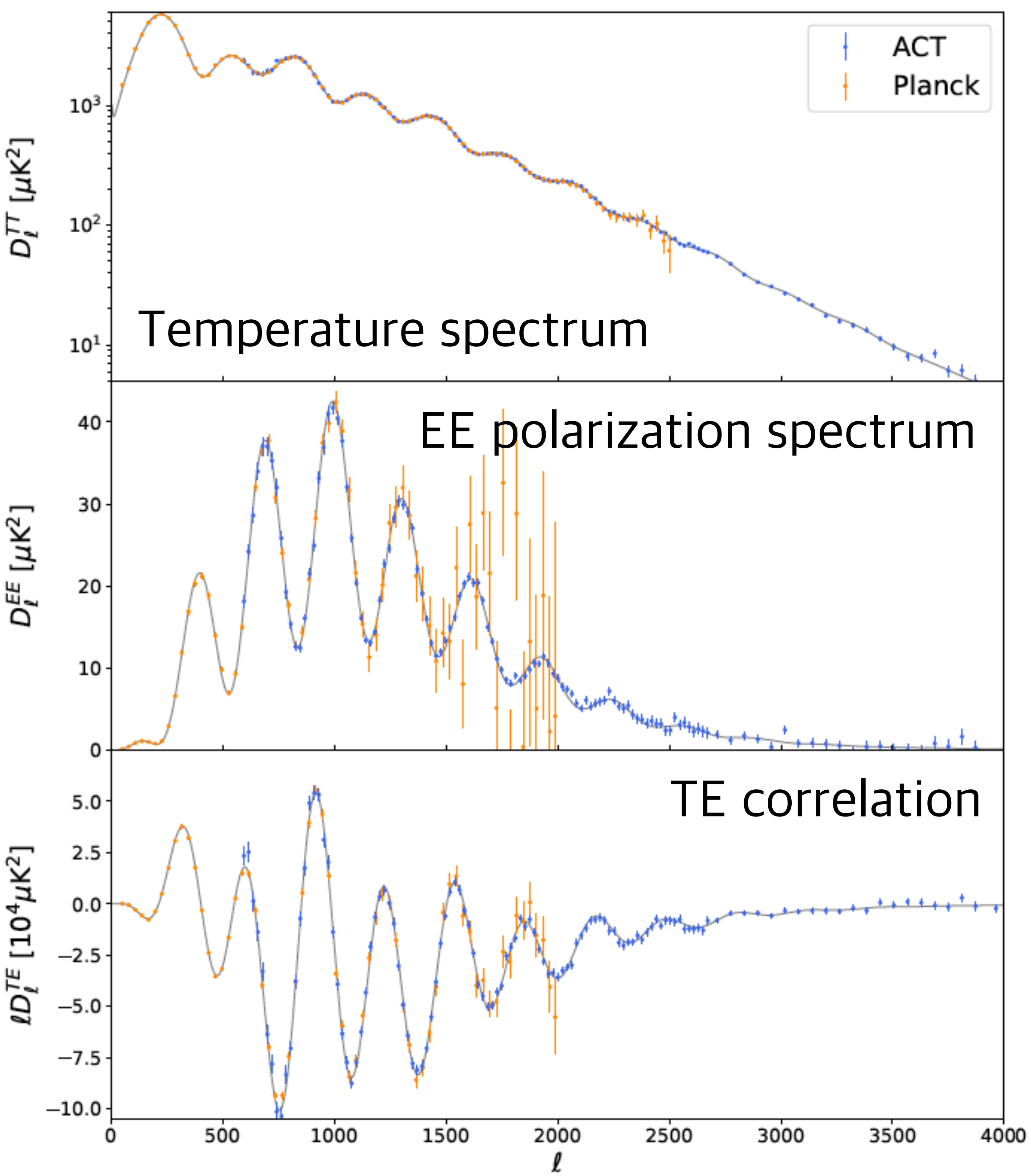
Observational support so far

2. Nearly scale-invariant SCALAR perturbations

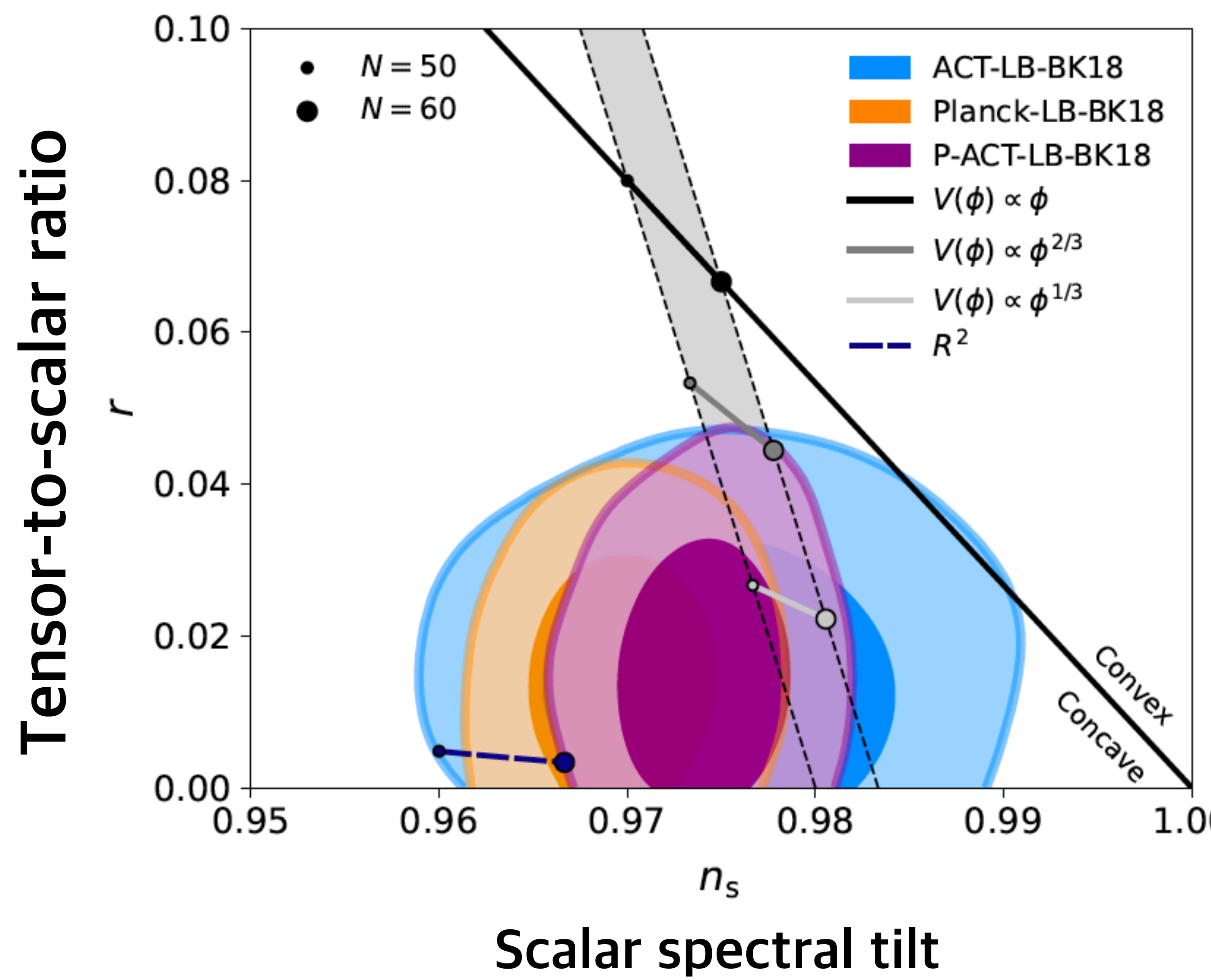
strongly supported by the the CMB observations

ACT Collaboration, arXiv:2503.14452, arXiv:2503.14454





Small deviation from the scale invariant spectrum (n_s=1)



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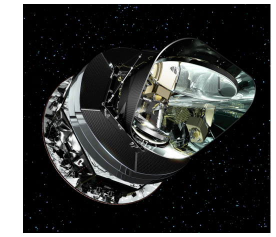
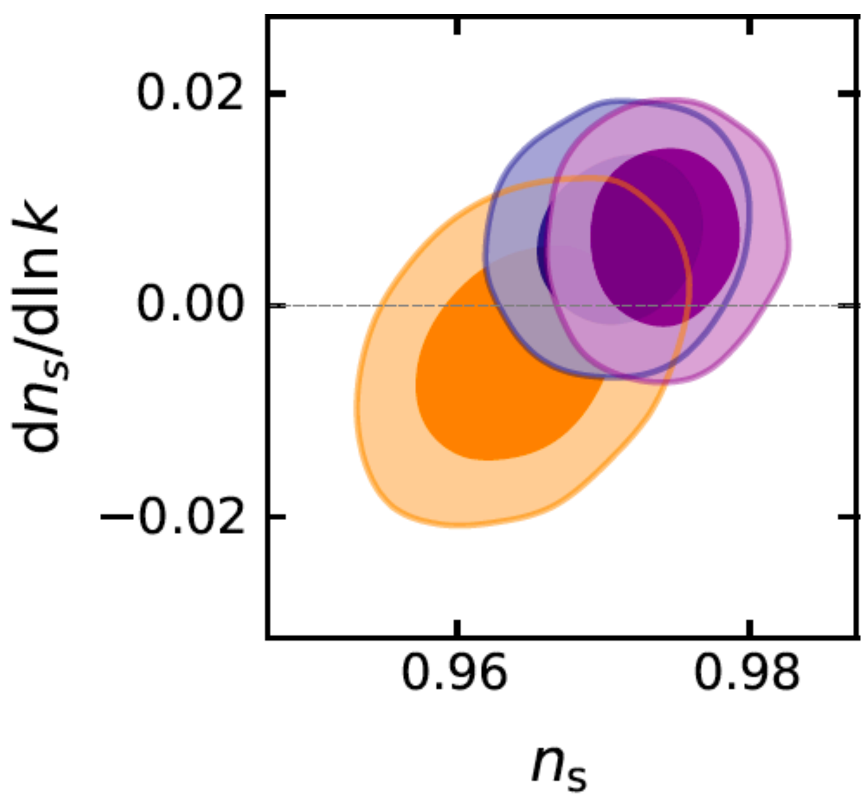


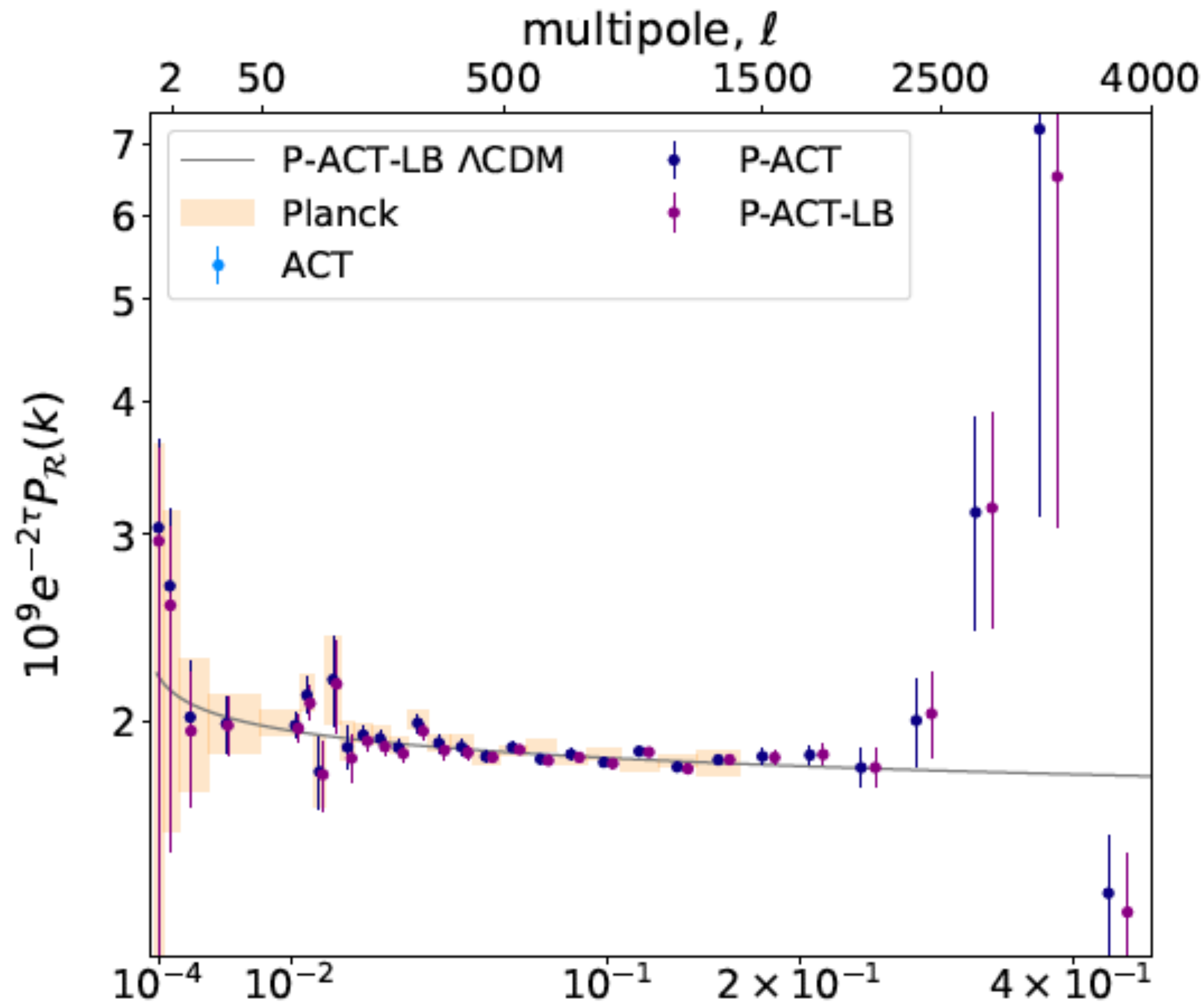
Reconstruction of the power spectrum

Scalar running is consistent with zero

Running index



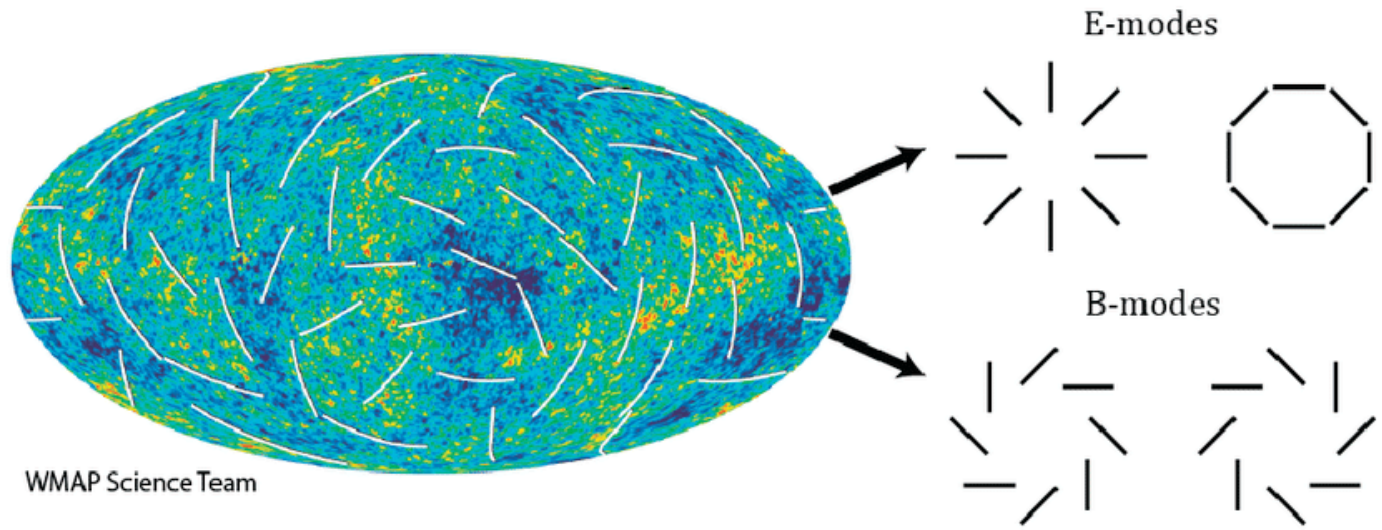
Scalar spectral tilt



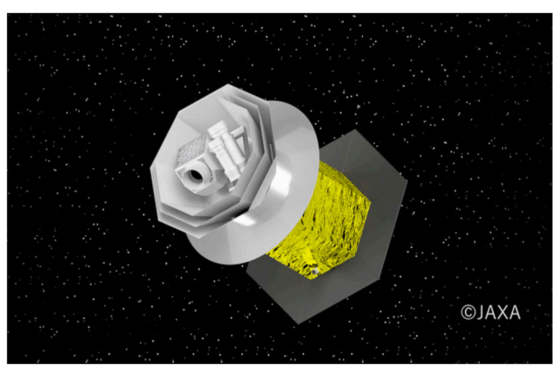
Future observations

3. Nearly scale-invariant TENSOR perturbations

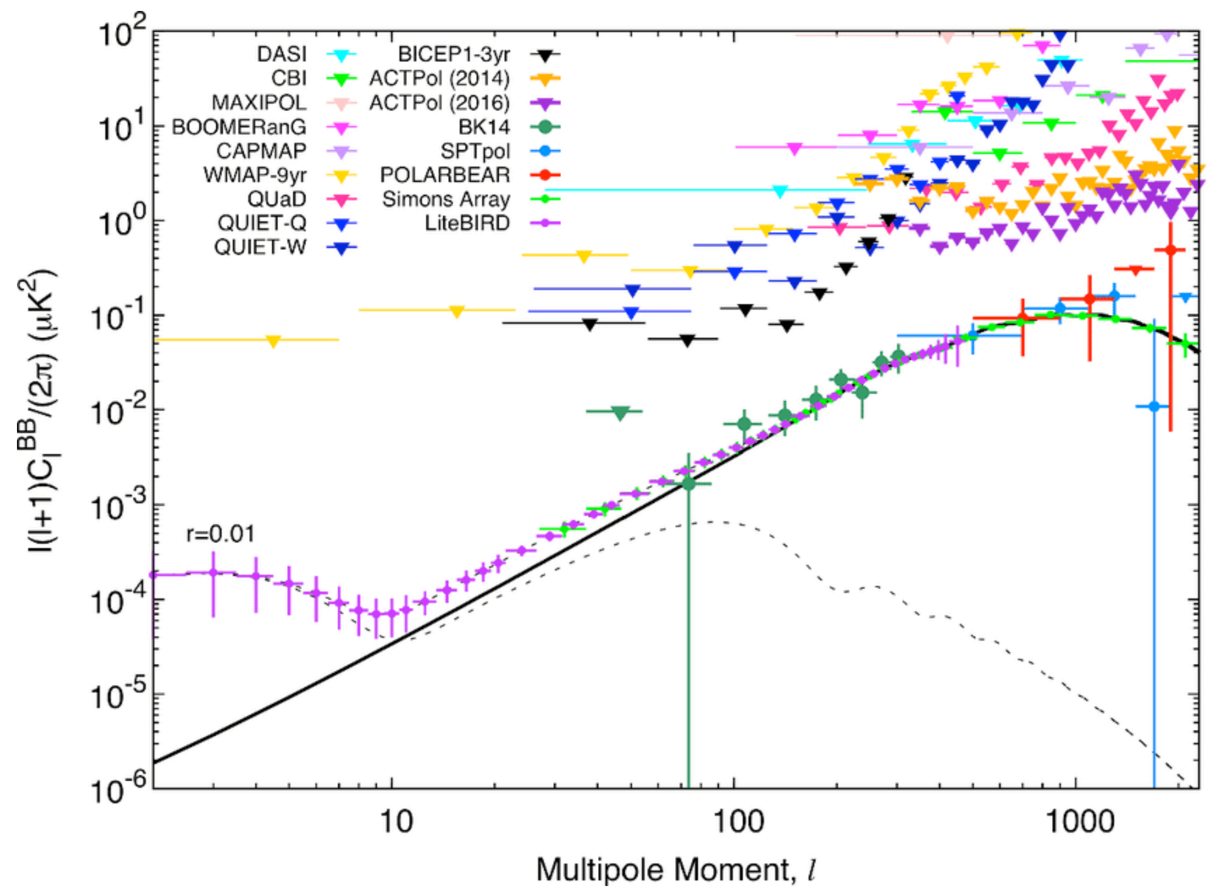
CMB B-mode polarization is a unique probe for inflation!



LiteBIRD



Simons Observatory

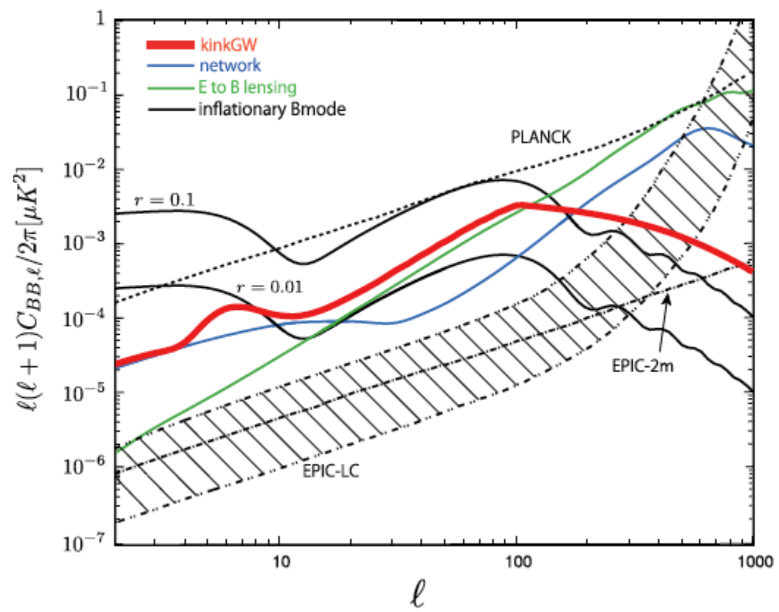


But... Is this enough?

No, because not only inflation theory predicts B-mode polarization!

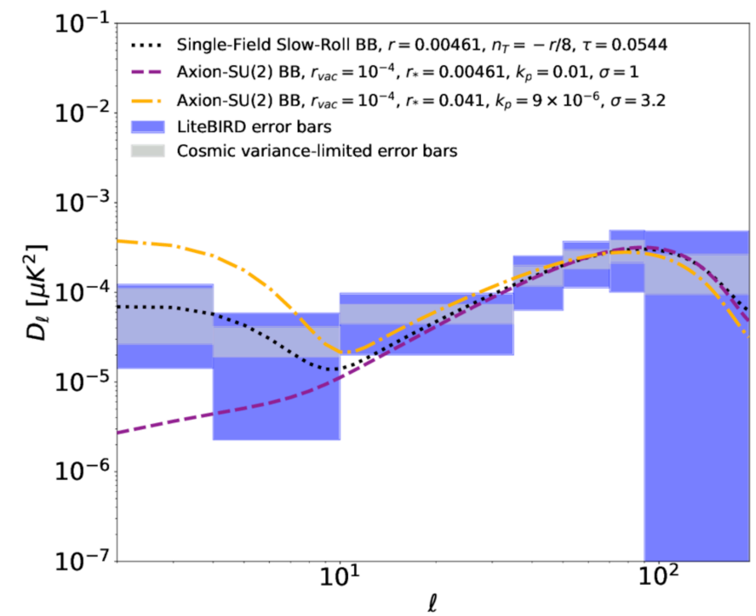
- **Cosmic strings**

Kawasaki et al., PRD 82, 103504 (2010)



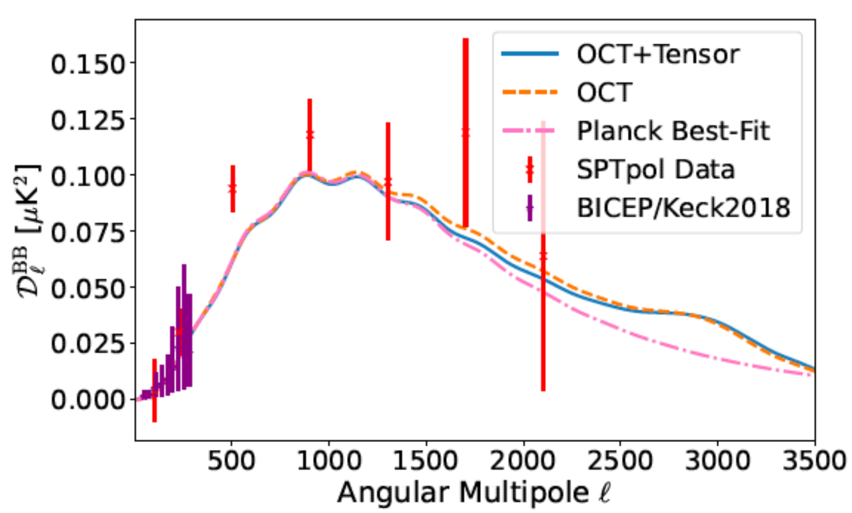
- **SU(2) gauge fields**

LiteBIRD Collaboration, PTEP 042F01 (2023)



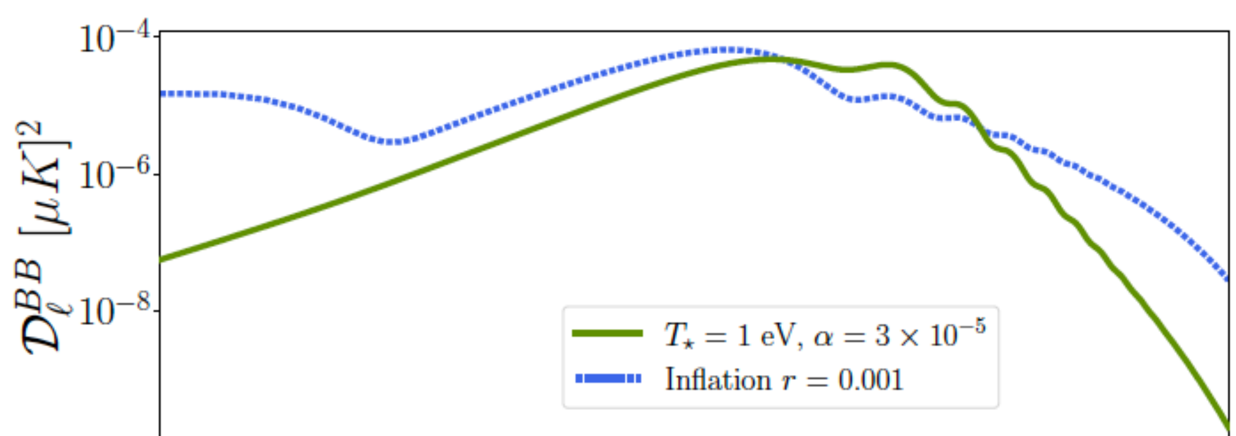
- **Magnetic fields**

Khalife & Pitrou, arXiv:2410.03612



- **Phase transitions**

Greene et al., arXiv:2410.23348



Any further tests? → **Consistency relation!** $r = -8n_T$

Predictions of inflation theory

Scalar power spectrum

$$\mathcal{P}_{S,\mathrm{prim}}(k) = \frac{1}{\pi\epsilon}\left(\frac{H}{m_{\mathrm{Pl}}}\right)^2\Bigg|_{k=aH}$$

→ observed as density perturbations

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→ observed as gravitational waves

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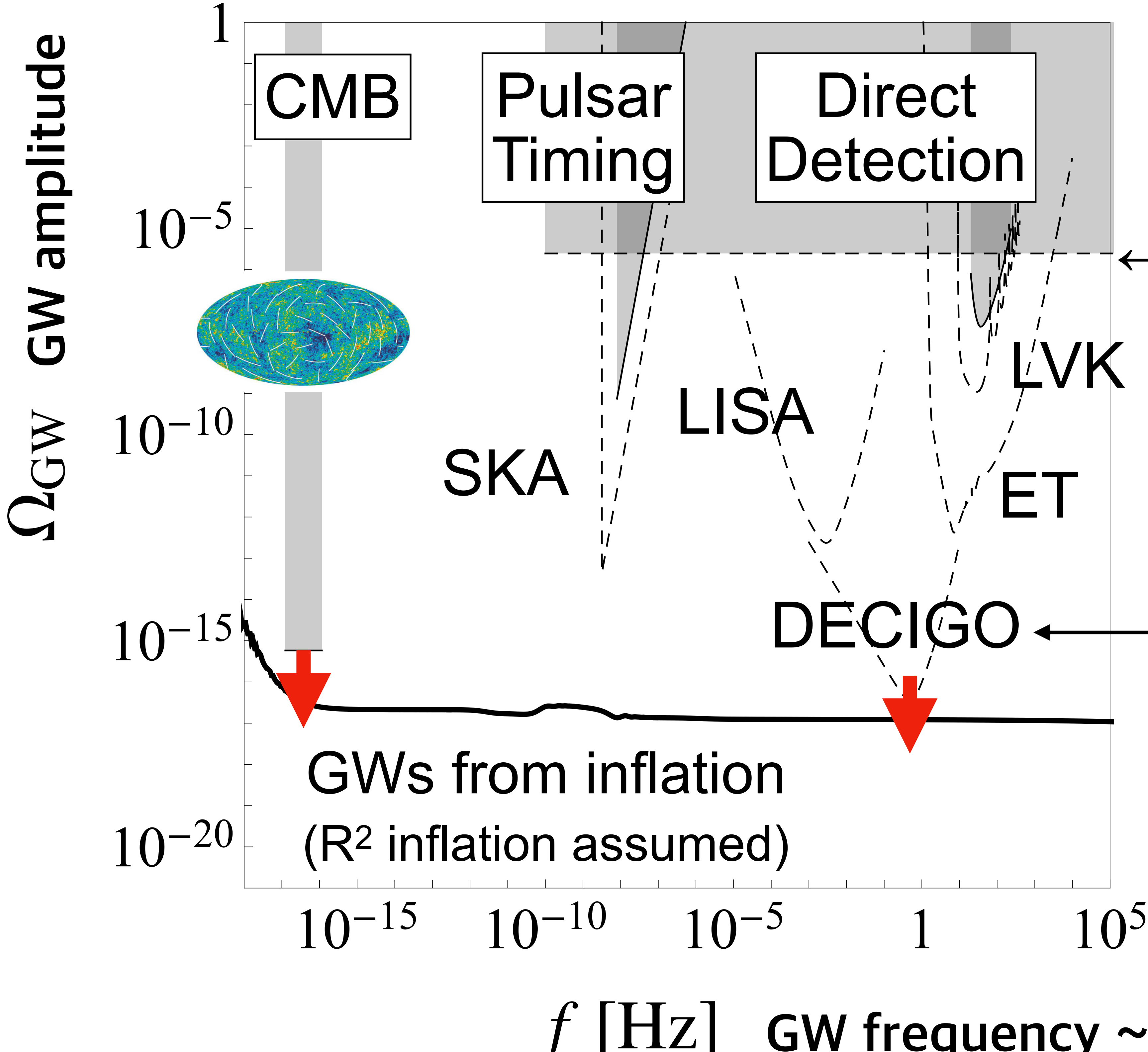
$$r = -8n_T$$

Spectral tilt

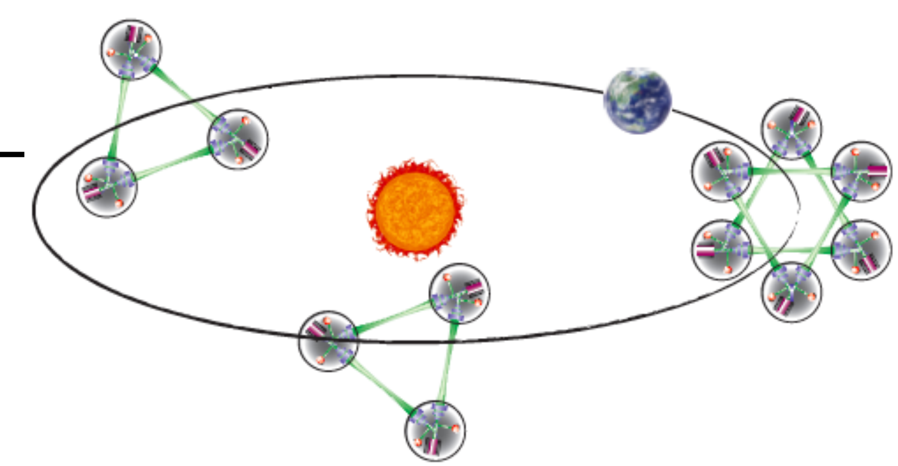
$$n_T(k) \equiv \frac{d\ln\mathcal{P}_{T,\mathrm{prim}}(k)}{d\ln k} \simeq -2\epsilon$$

Direct detection of GWs

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k}$$

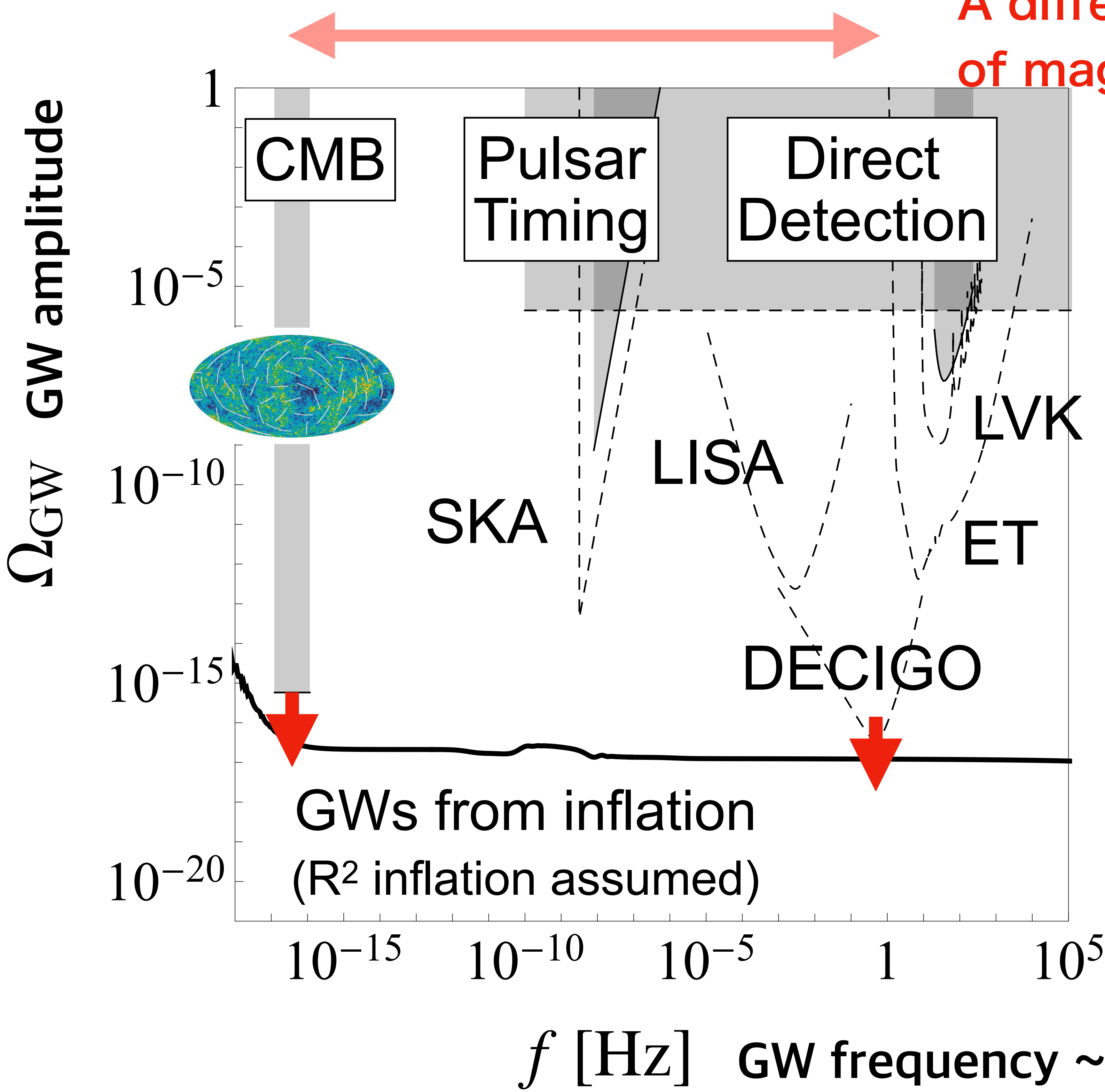


Satellite experiment plan in Japan



Seto, Kawamura, Nakamura, PRL 87, 22 (2001)

Measuring GWs at two different scales!



GW frequency ~ (scale)$^{-1}$

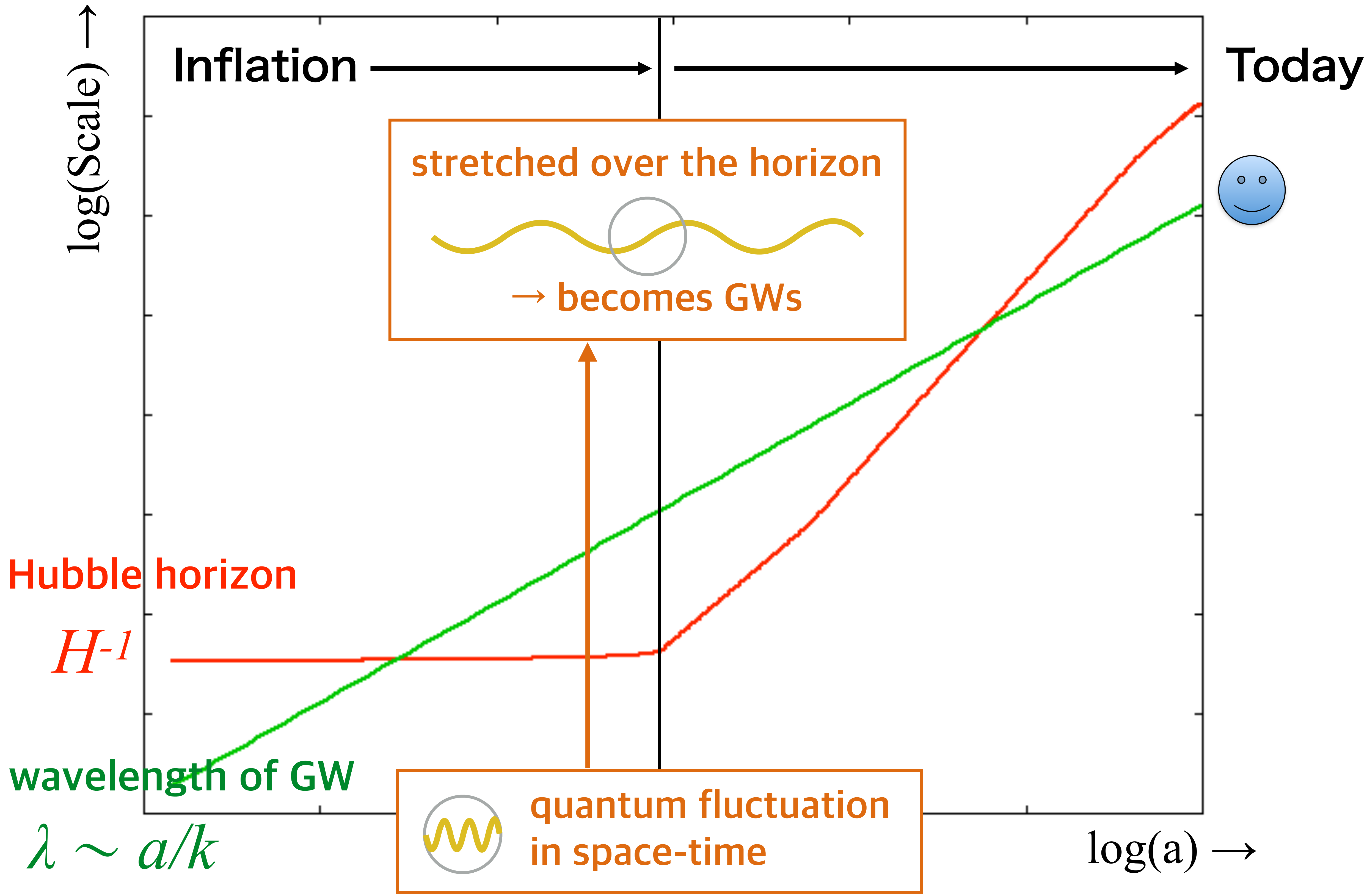
A difference of 15 orders of magnitude in scale!

→ It helps to test the consistency relation!

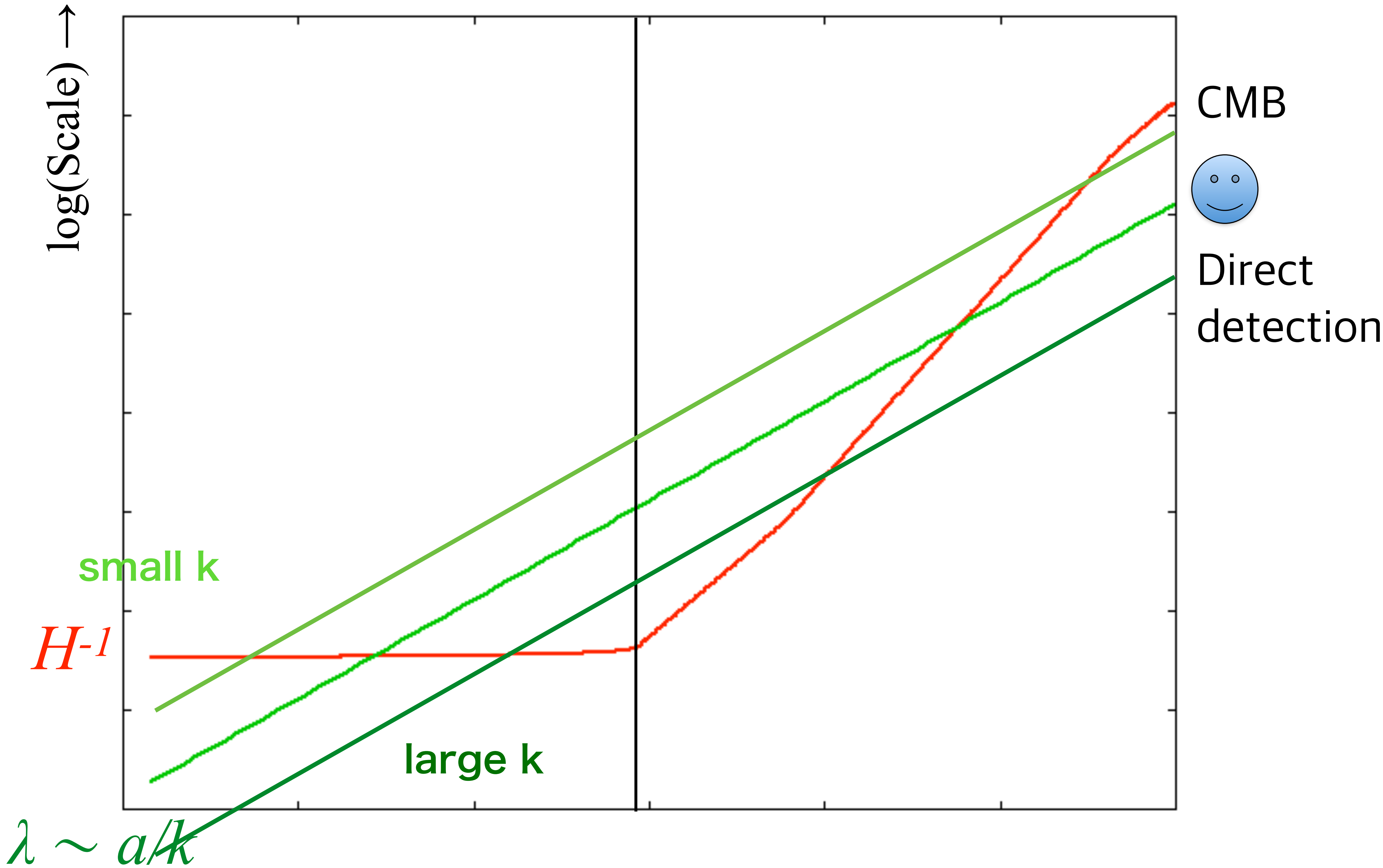
$$r = -8 n_T$$

amplitude (r) tilt (n_T)

Primordial GWs



Importance of measuring at two scales



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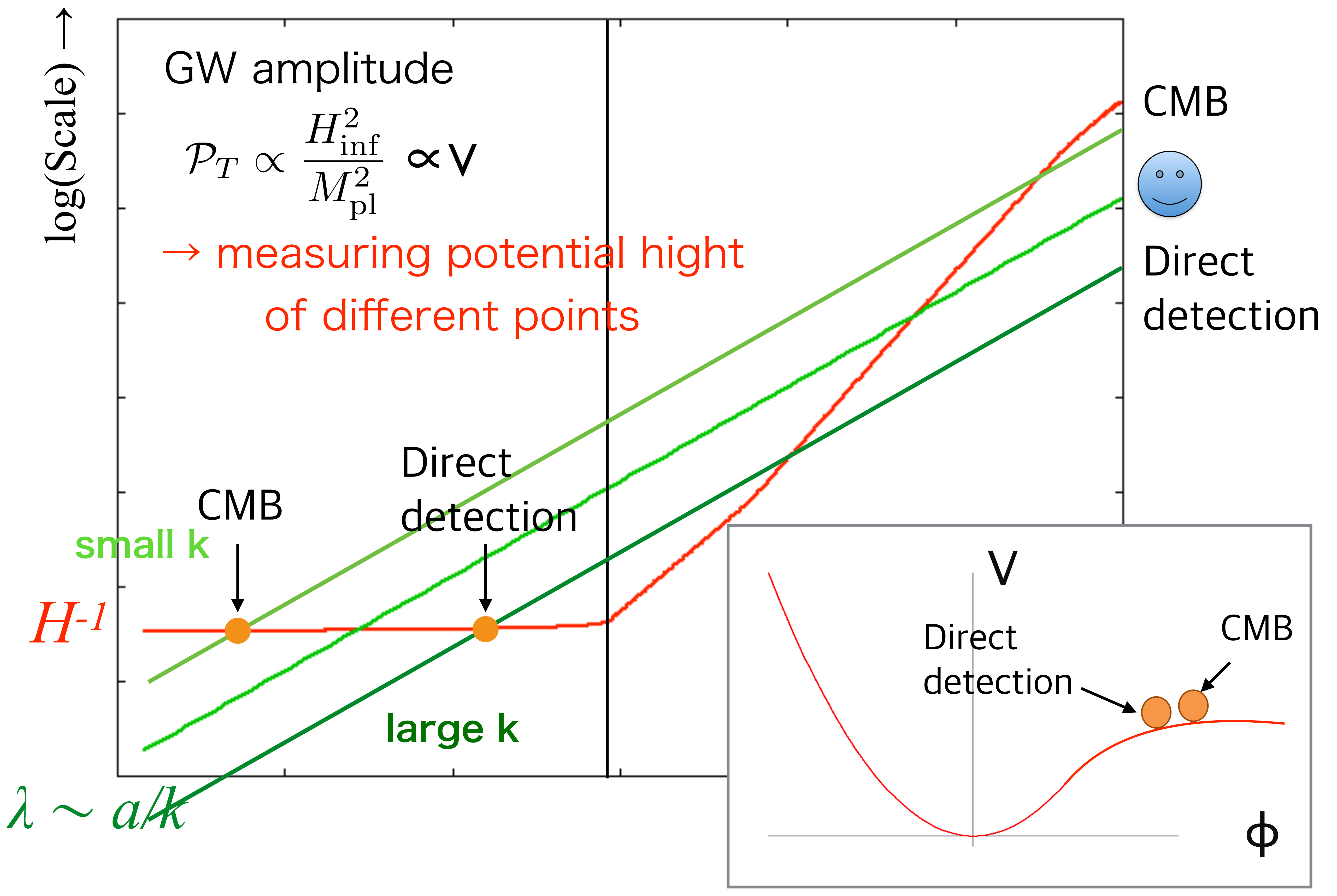
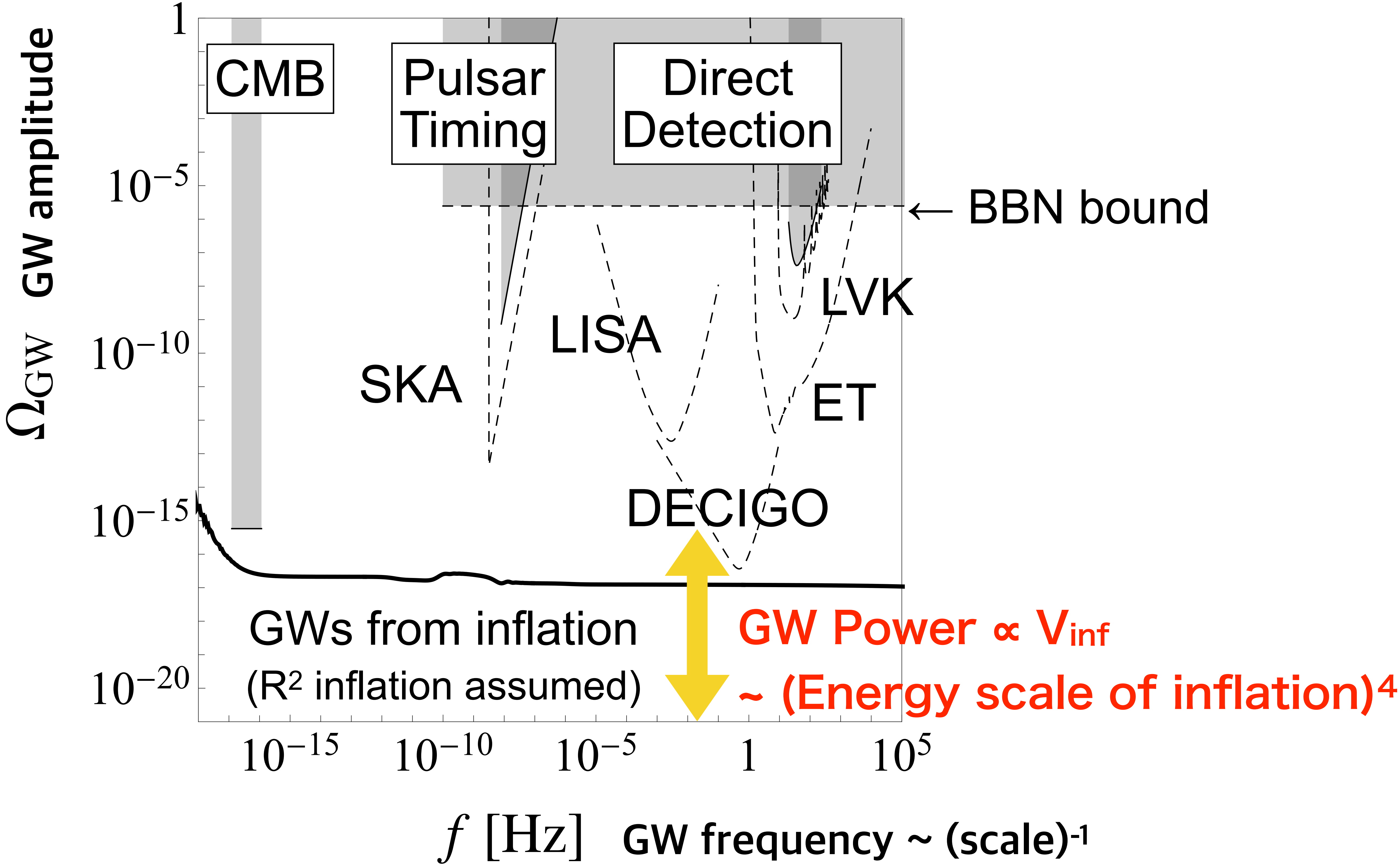


Difficulty I

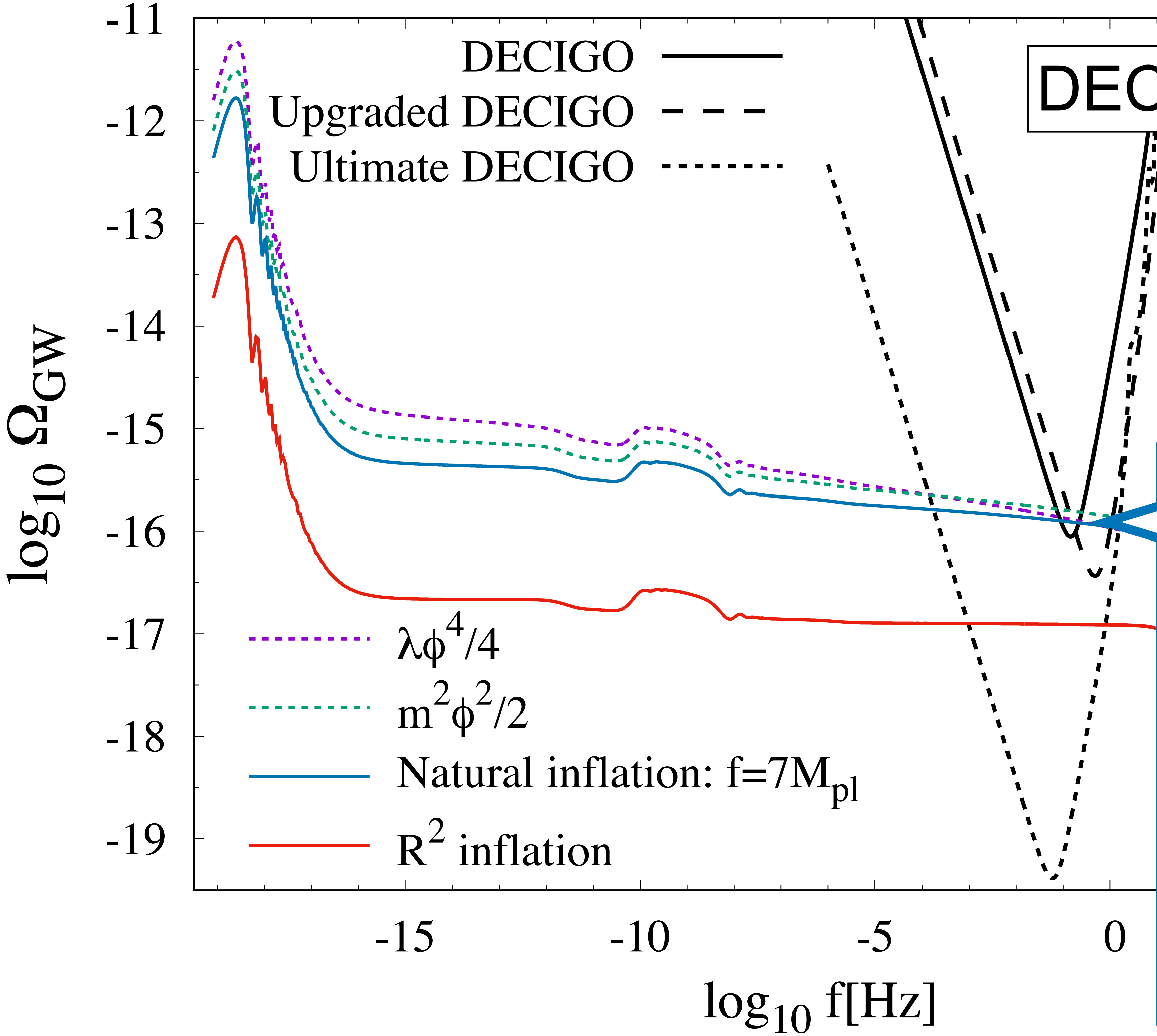
Amplitude is typically very small



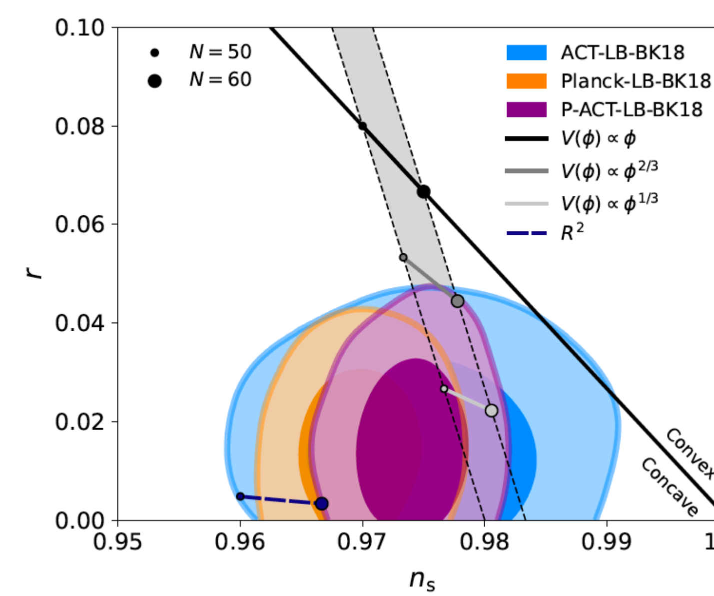
Dependence on inflation models

Kuroyanagi et al. PRD 79, 103501 (2009)
Kuroyanagi et al. PRD 90, 063513 (2014)

CMB



Too large r
→ ruled out by the CMB observation!



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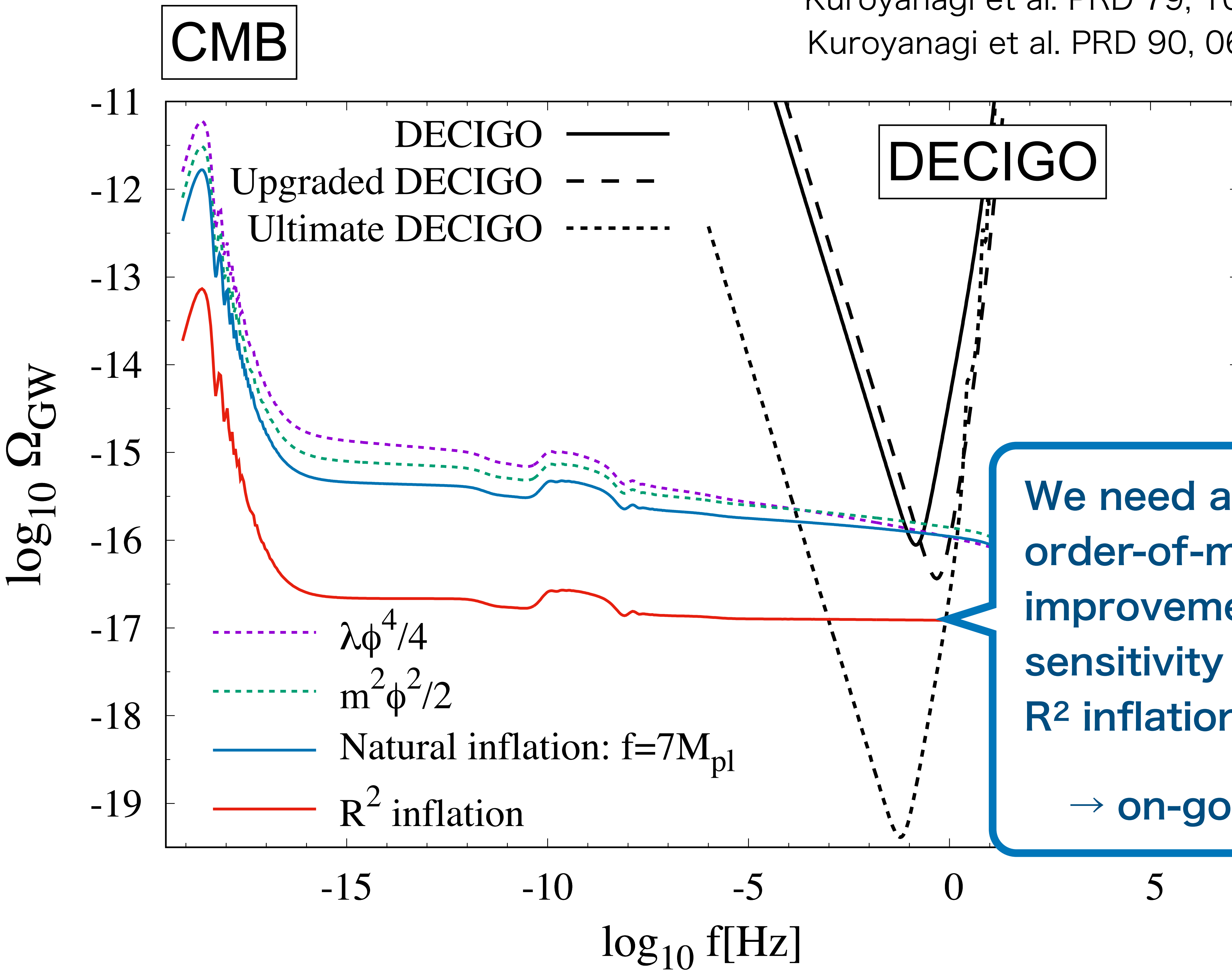


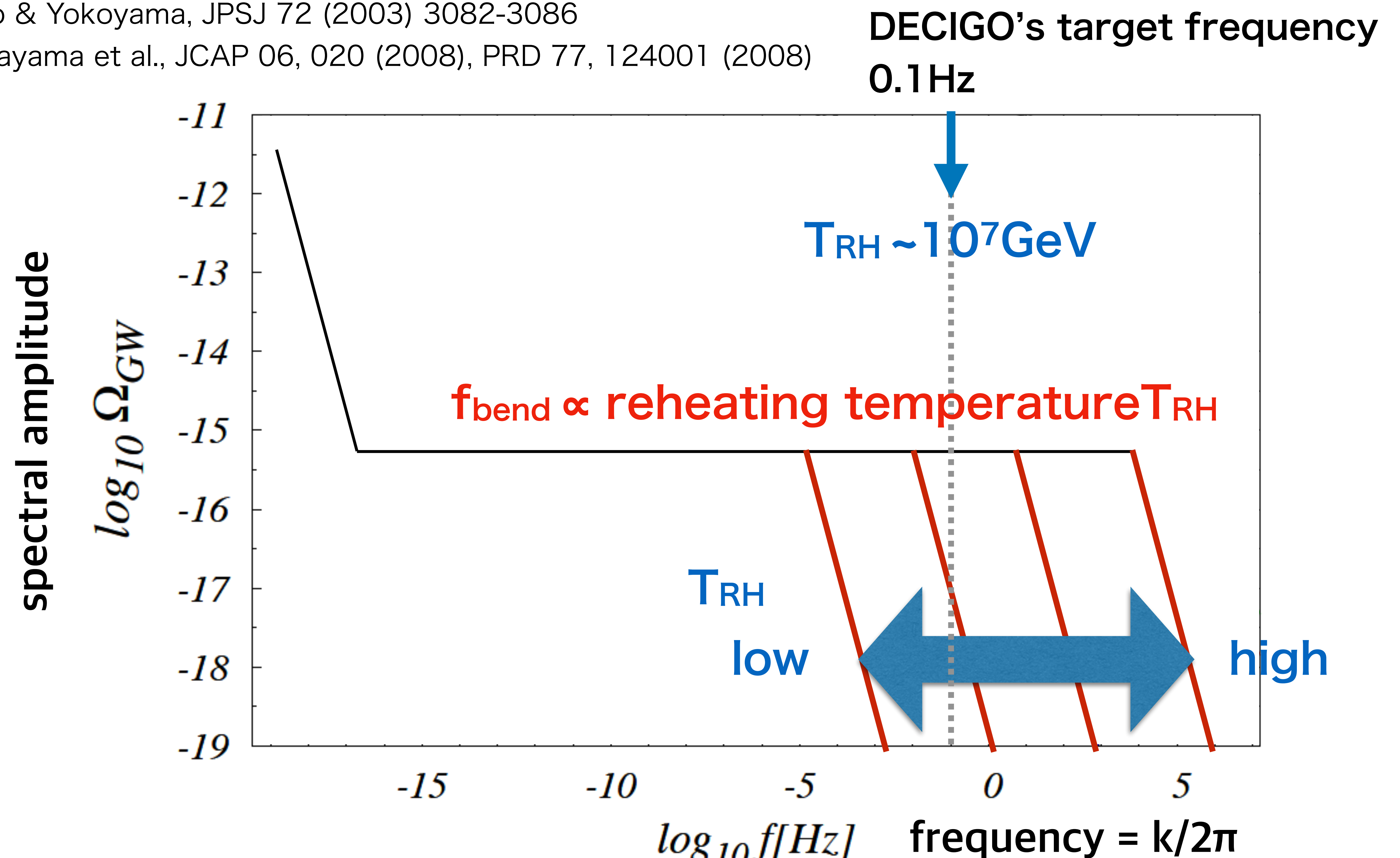
We need a one-order-of-magnitude improvement in sensitivity to reach R^2 inflation.

→ on-going effort

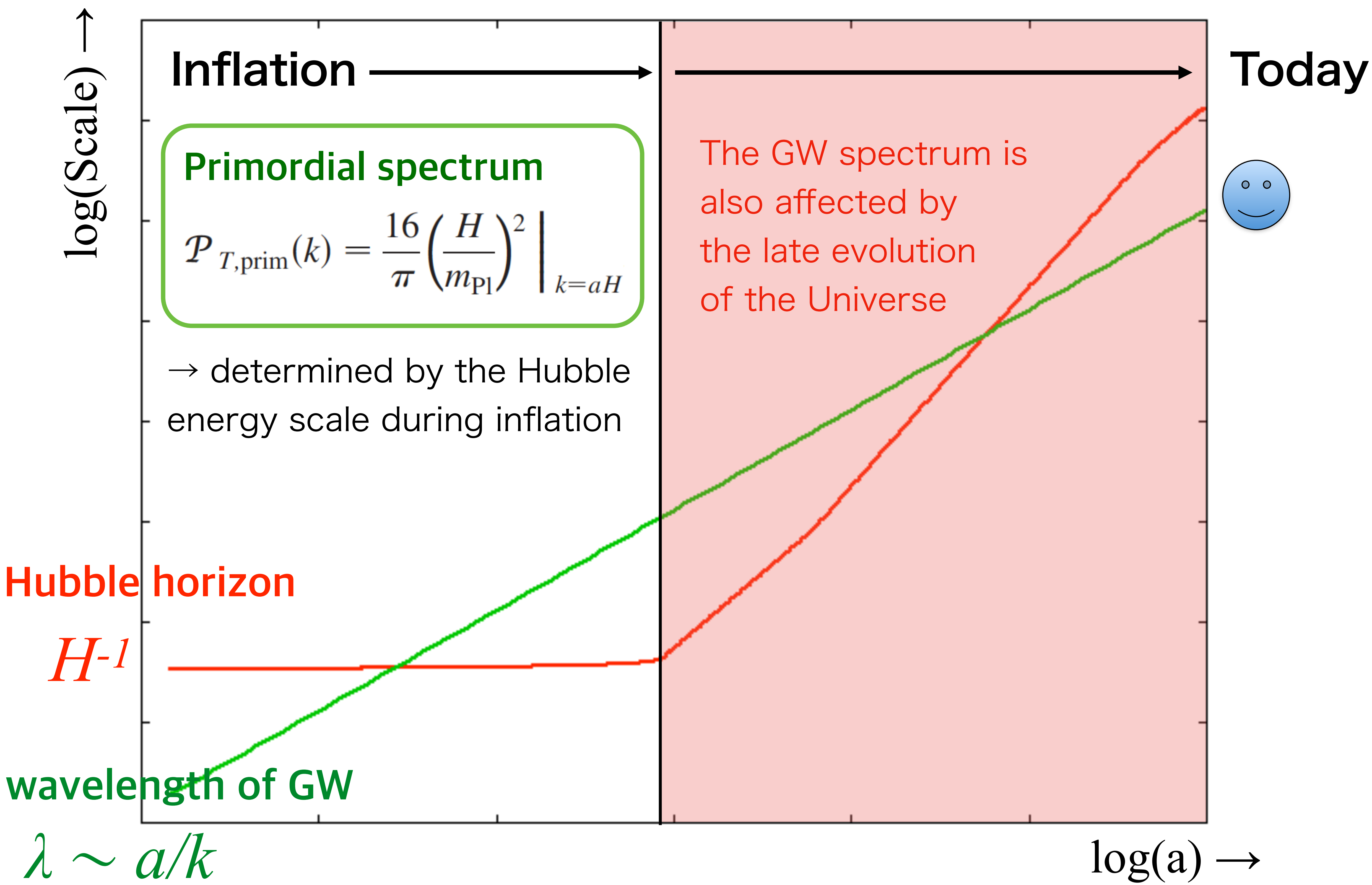
Difficulty 2

High frequency modes may be suppressed by reheating

Seto & Yokoyama, JPSJ 72 (2003) 3082-3086
Nakayama et al., JCAP 06, 020 (2008), PRD 77, 124001 (2008)



Early history of the Universe



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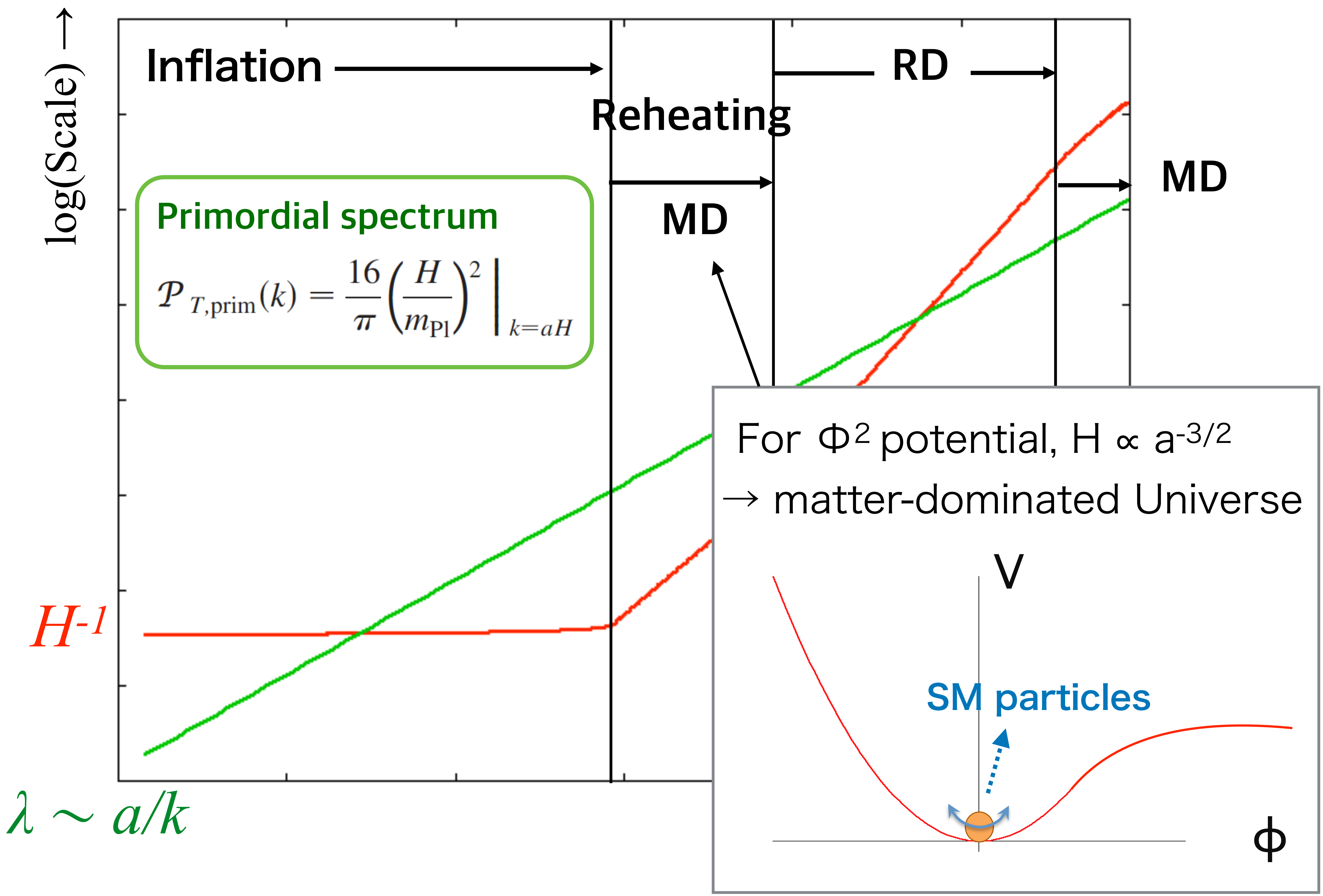

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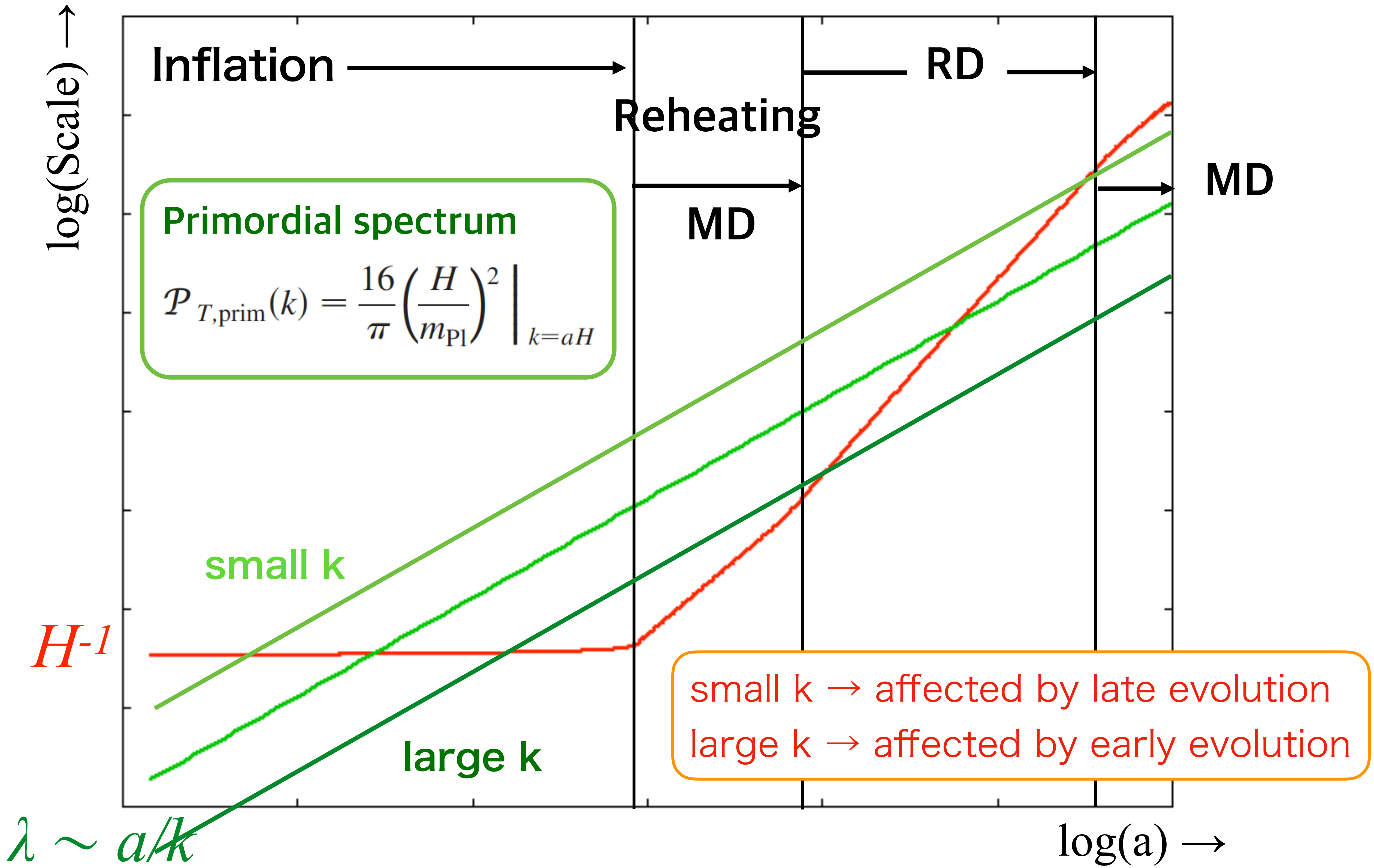
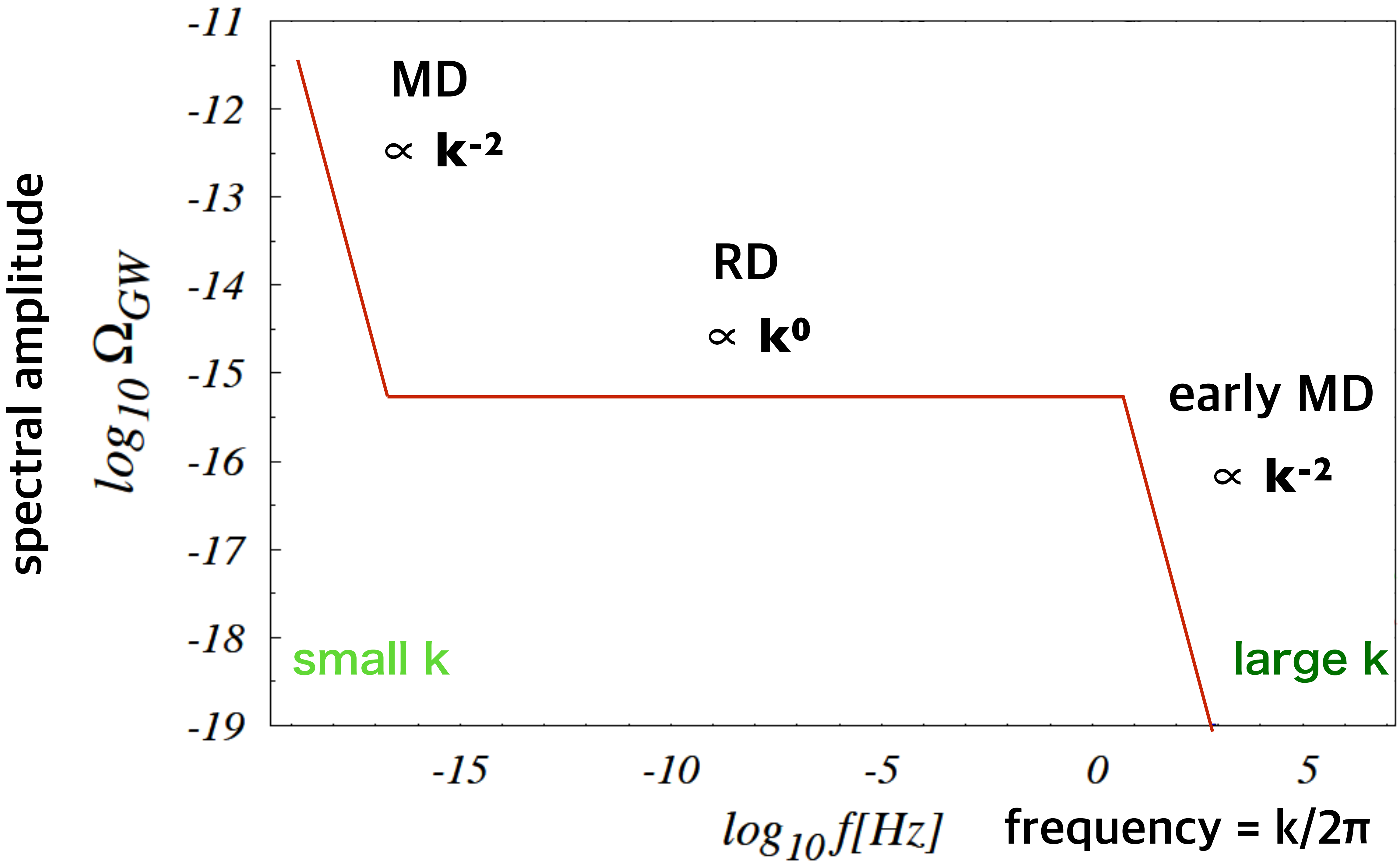


Effect of the early matter phase

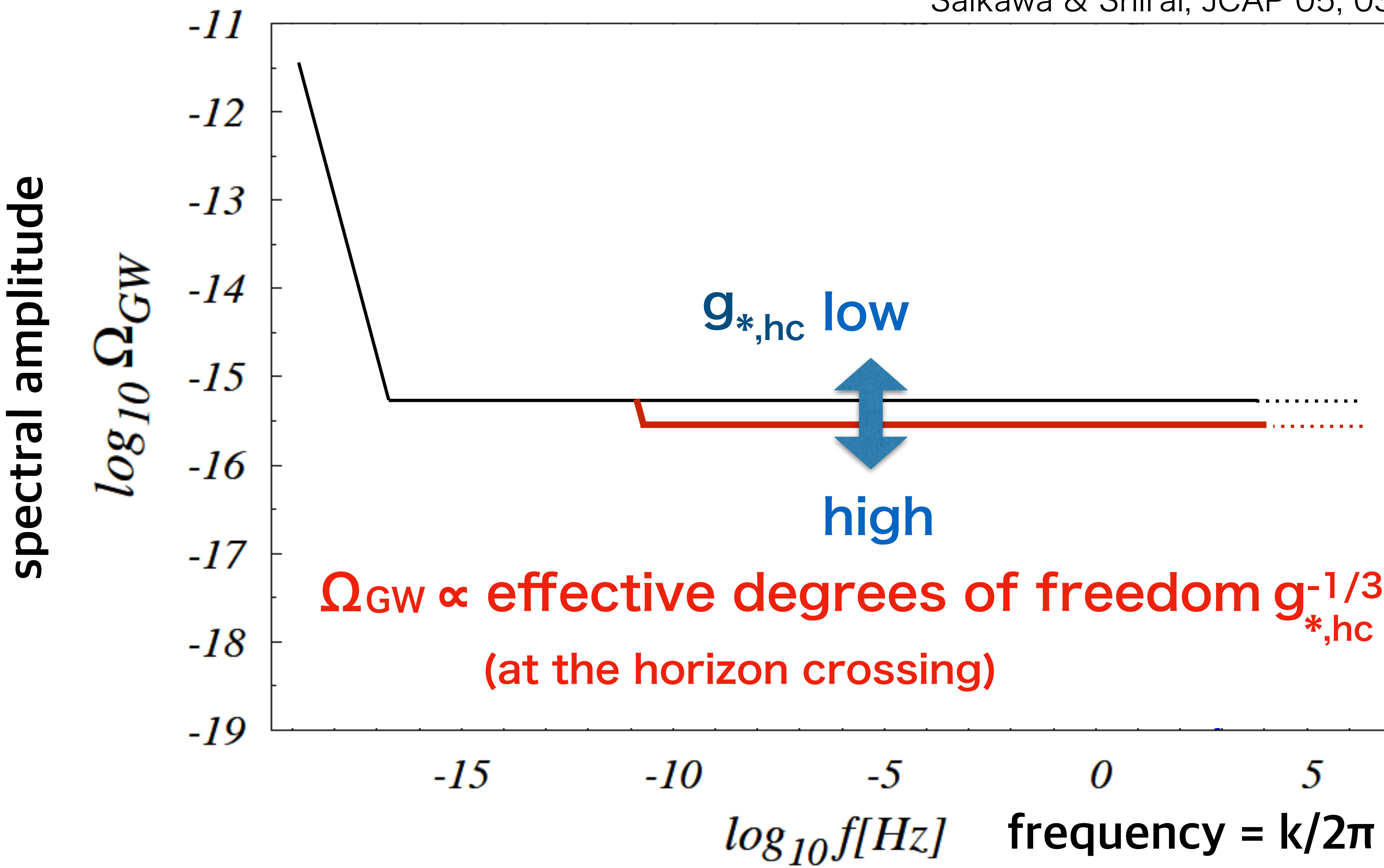
We need $T_{RH} > 10^7$GeV to have the signal at the DECIGO frequency!



Difficulty 3

Ambiguity in predicting the amplitude due to the effective degrees of freedom

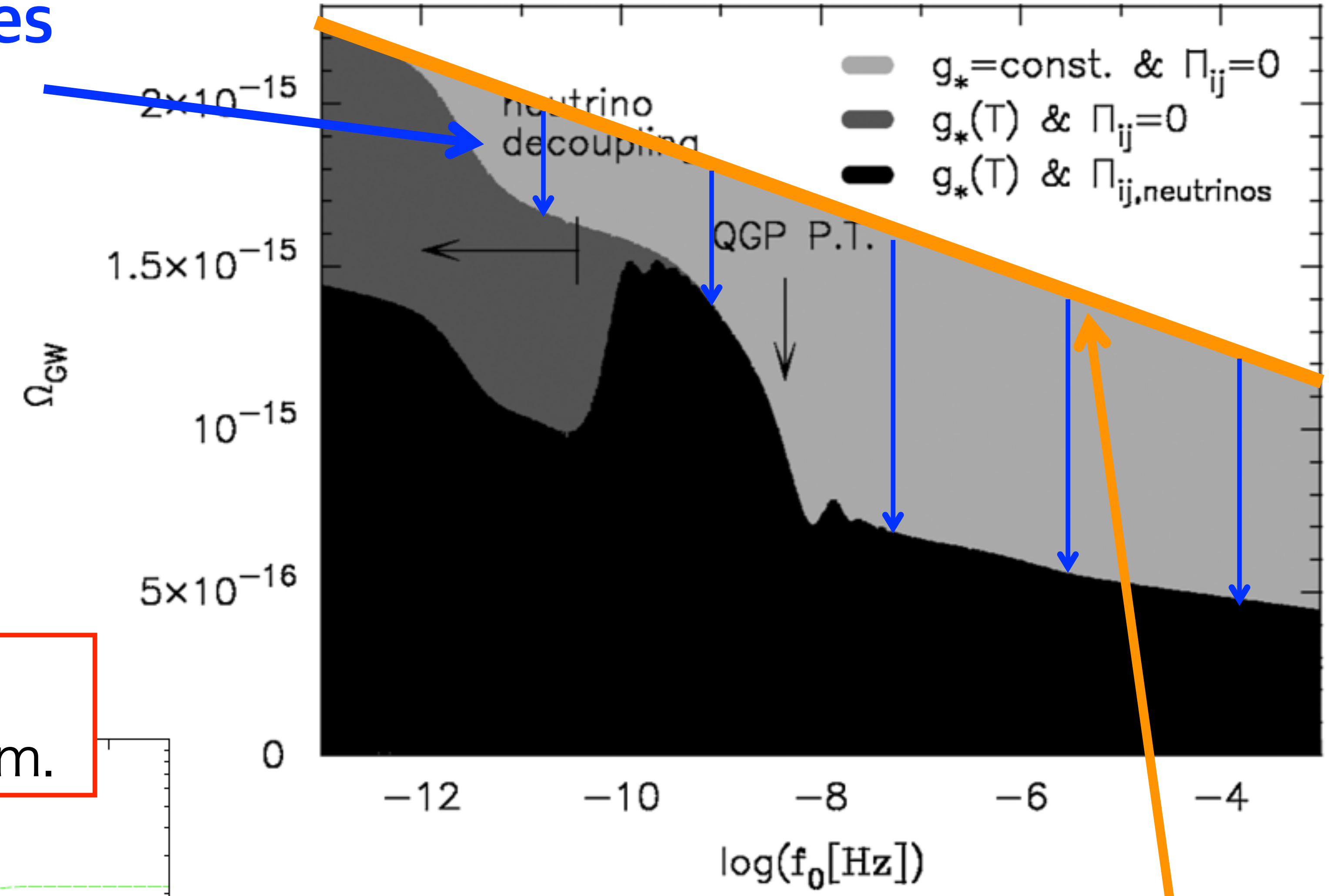
Watanabe & Komatsu, PRD 73, 123515 (2006)
Saikawa & Shirai, JCAP 05, 035 (2018)



Effective number of degrees of freedom

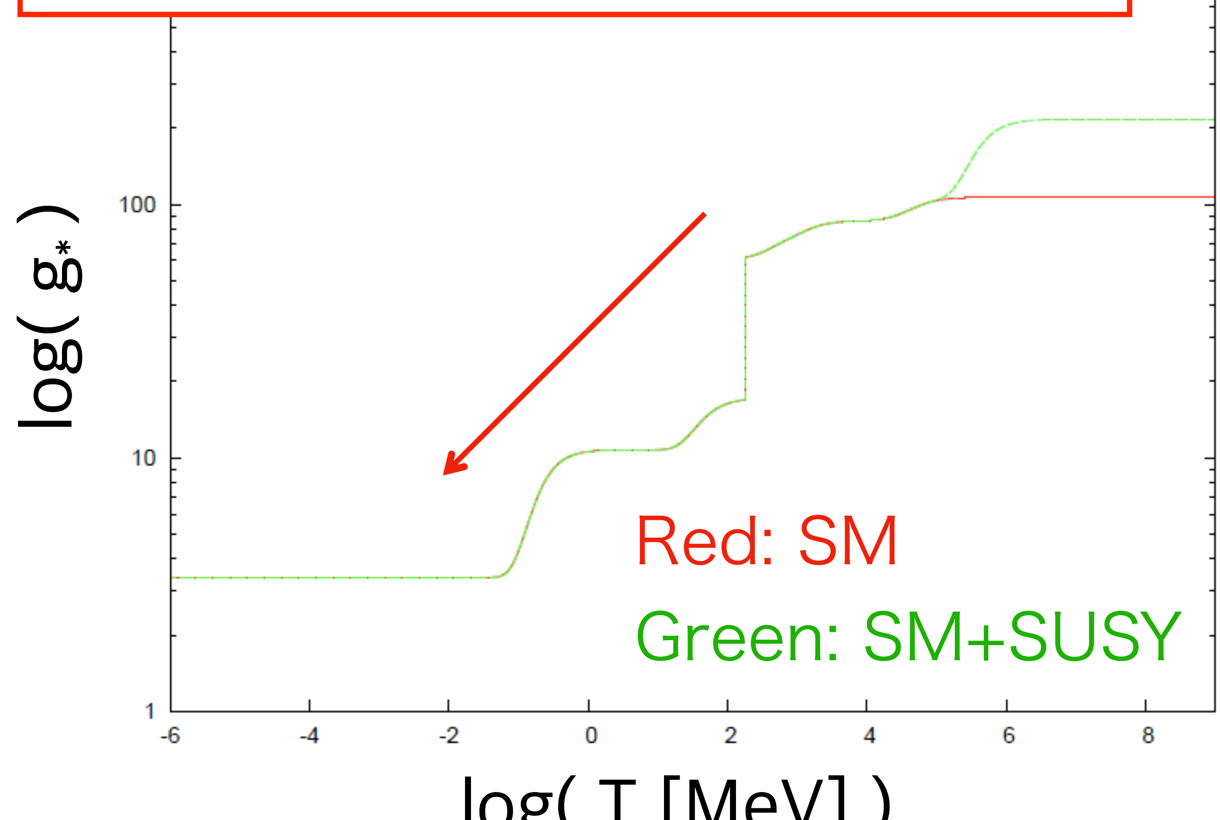
Damping due to the changes in effective number of degrees of freedom g_*

density	$\rho(T) = \dfrac{\pi^2}{30} g_*(T) T^4,$
entropy	$s(T) = \dfrac{2\pi^2}{45} g_{*s}(T) T^3$



primordial spectrum with tilt

As T decreases, each particles become non-relativistic when T~m.



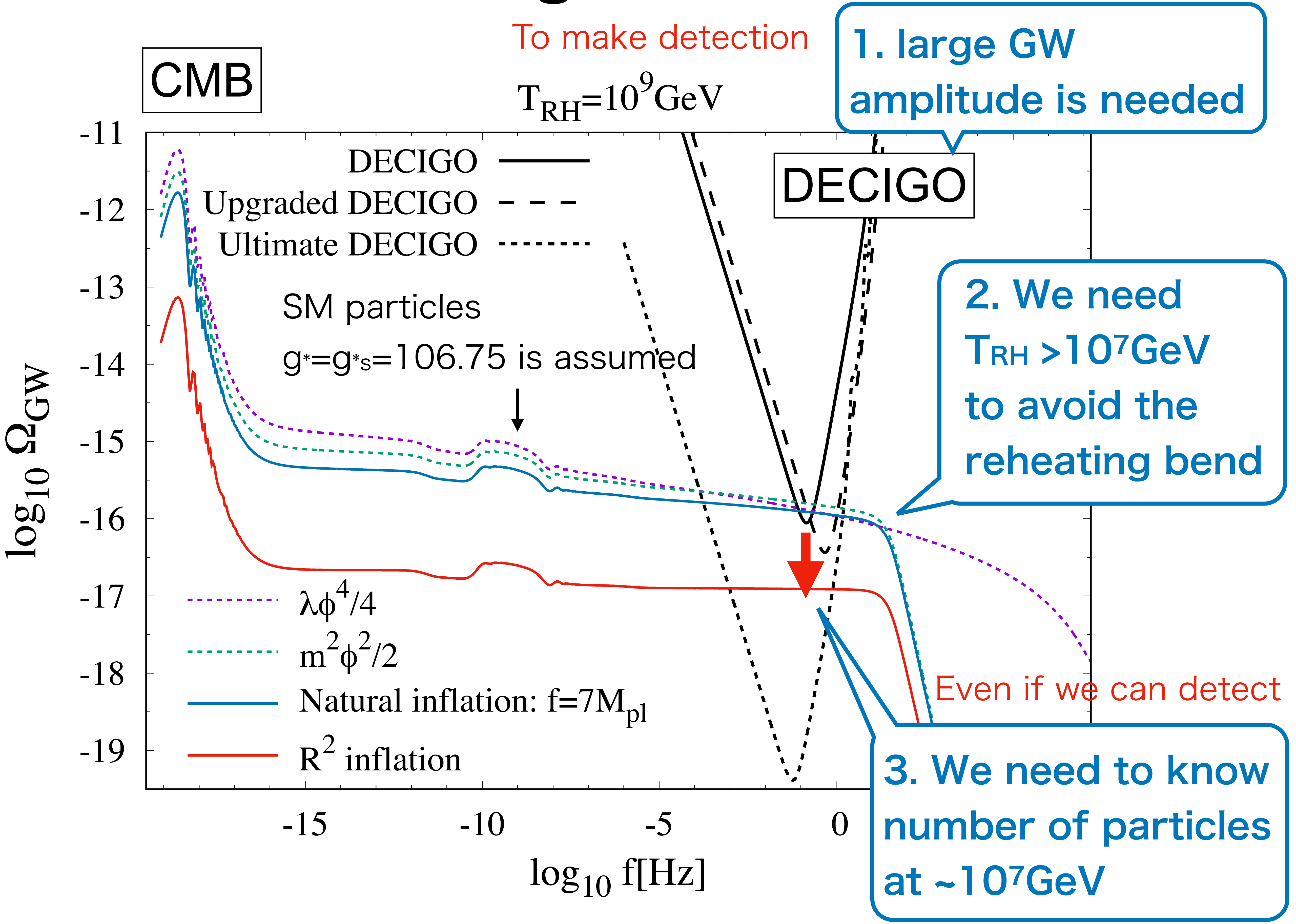
For SM particles, $g_*=g_{*s}=106.75$

If we have non-SM particles,

g_*, $g_{*s} > 106.75$

→ **More damping in the GW amplitude**

Three challenges to measure n_T



Summary

Several observations can be used to test inflation

1. Accelerated expansion of the Universe

・Required to solve the horizon / flatness / monopole problems

2. Nearly scale-invariant SCALAR perturbations

・Almost confirmed by CMB & large scale observations!

3. Nearly scale-invariant TENSOR perturbations

・Key observations to confirm inflation

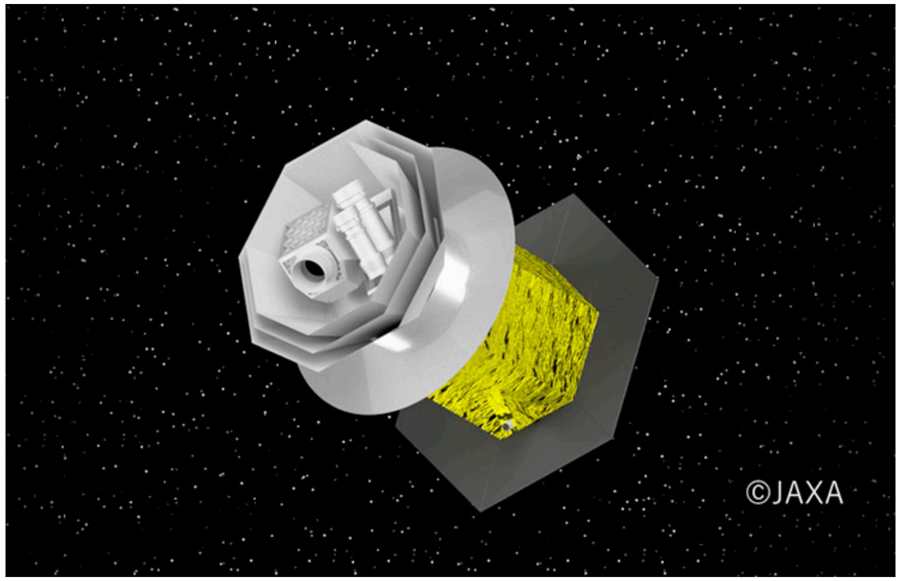
・Testing the consistency relation through CMB B-modes and direct gravitational wave detection would offer further, stronger evidence

Summary

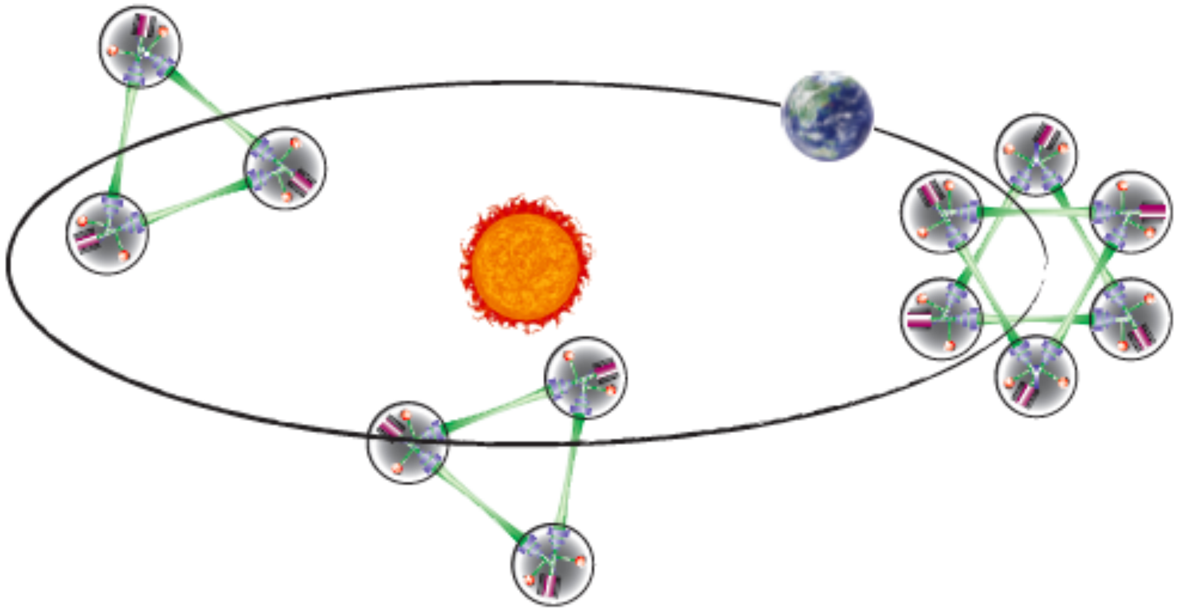
Two Japanese experiments are the key to confirm inflation

Let's push it forward!

LiteBIRD



DECIGO



3. Nearly scale-invariant TENSOR perturbations

・Key observations to confirm inflation

・Testing the consistency relation through CMB B-modes and direct gravitational wave detection would offer further, stronger evidence

Possible discussion

Further observational tests?

1. Accelerated expansion of the Universe

- Any way to measure the expansion directly???

2. Nearly scale-invariant SCALAR perturbations

- Gaussianity (Non-Gaussianity) of perturbations
- Quantum nature of perturbations

3. Nearly scale-invariant TENSOR perturbations

- Gaussianity (Non-Gaussianity) of perturbations
- Quantum nature of perturbations

Possible discussion

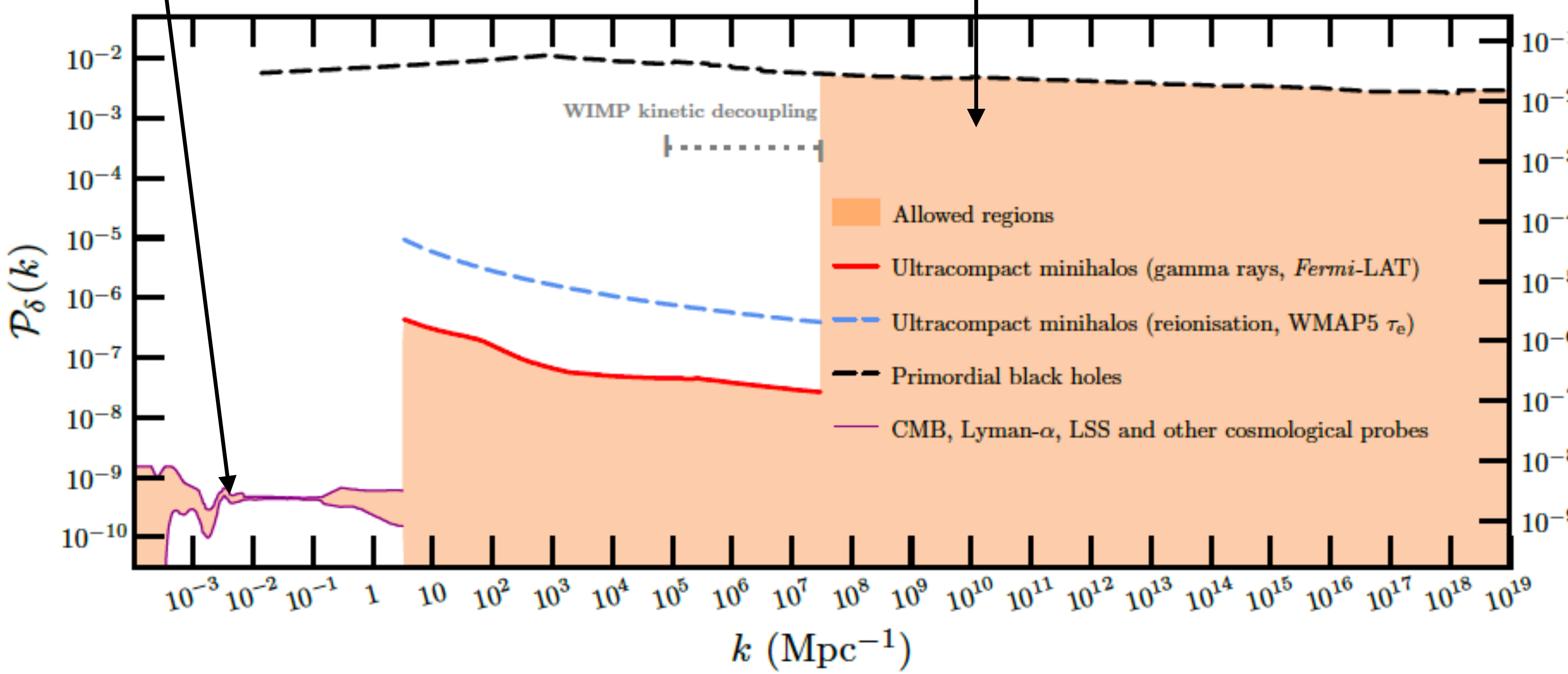
Non-linearity appears when we add extra ingredients

- Gauge field during inflation
- Formation of primordial black holes

$P_S \sim 2 \times 10^{-9}$ @ CMB scale

can be much larger at small scales

2nd order GWs is a unique probe

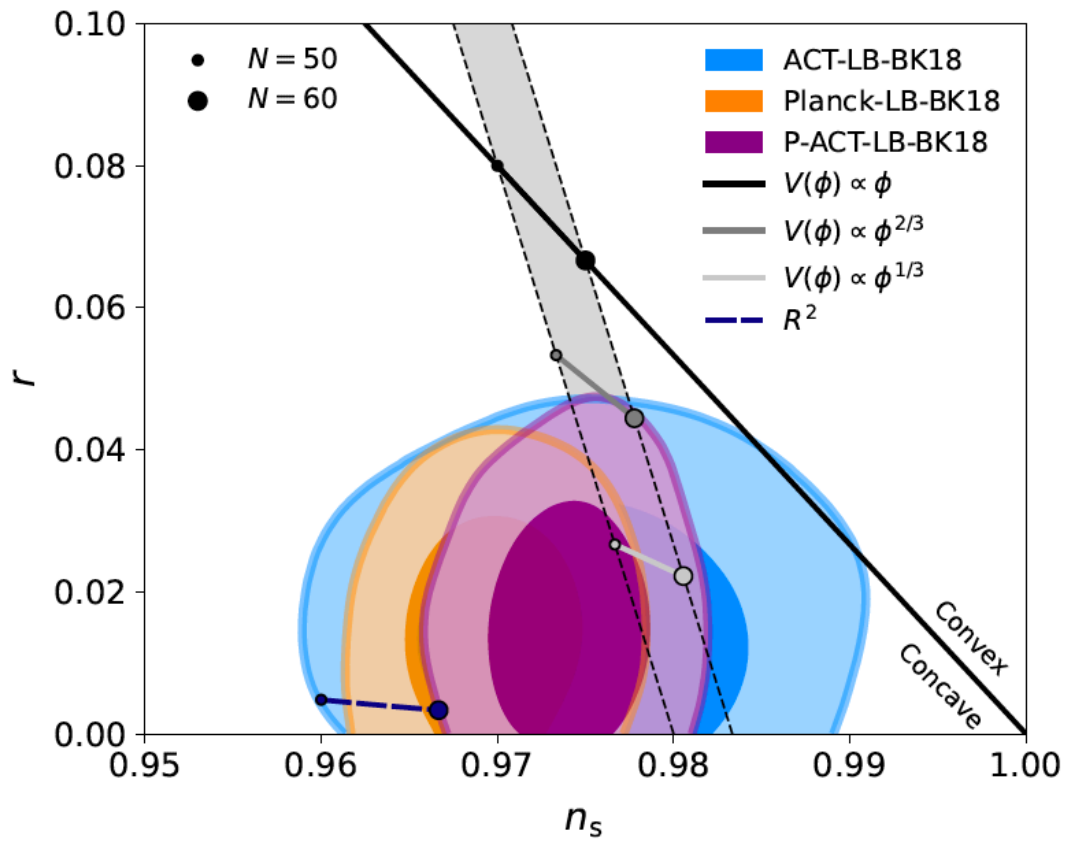


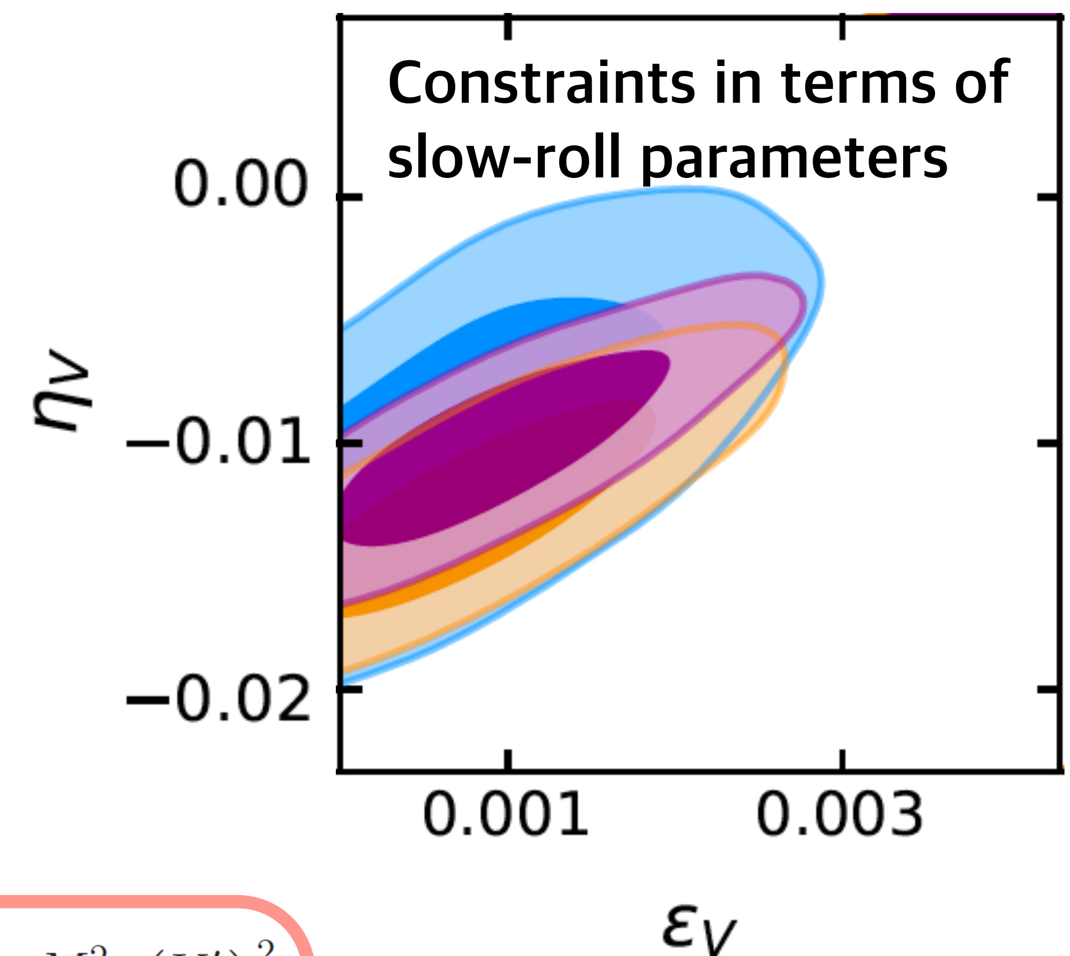
→ developing a precise analytic formalism is still an active area of research

Figure by T. Bringmann et al, PRD 85, 125027 (2012)

How is it likely to have large r?

(added after a comment from Yanagida-san)





Spectral tilt $\quad n_S(k) - 1 \equiv \dfrac{d \ln \Delta^2_{\zeta,\mathrm{prim}}(k)}{d \ln k} \simeq -6\epsilon + 2\eta \qquad \epsilon_V \equiv \dfrac{M_{\mathrm{Pl}}^2}{2}\left(\dfrac{V'}{V}\right)^2$

Tensor-to-scalar ratio $\quad r \equiv \dfrac{\mathcal{P}_{T,\mathrm{prim}}(k)}{\mathcal{P}_{S,\mathrm{prim}}(k)} \simeq 16\epsilon \qquad \eta_V \equiv M_{\mathrm{Pl}}^2 \dfrac{V''}{V}$

A small ε (small r) indicates a large negative η, which leads $\varepsilon \ll |\eta|$

<u>My personal optimistic view</u>

The idea of 'naturalness' can vary from person to person, but if we assume that it is natural to have $\varepsilon \sim |\eta|$, then we would not expect ε to be extremely small.

→ **Hope for observable tensor mode!**