Did the universe have a begining?

What happened at early times?

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W(I)PMU

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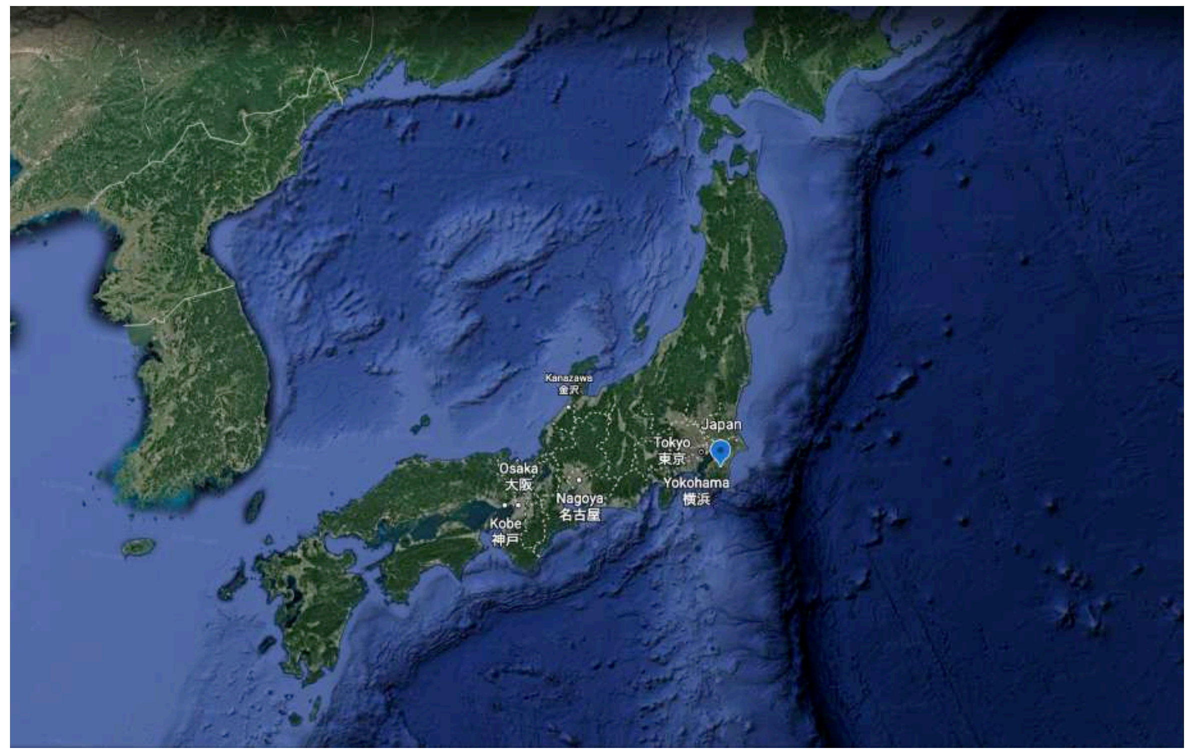
Image:NASA, ESA, CSA, and STScI

Google *drive*

You can find this presentation and notebooks for exercises in this Google drive link:

https://drive.google.com/drive/folders/1hAemYHTBHciRsI3PI1WIztu2pArwRMT6?usp=sharing

A little bit about me…



I am originally from Brazil

Undergrad and masters ⟶ University of São Paulo, Brazil

PhD ⟶ Universidade McGill, Canada

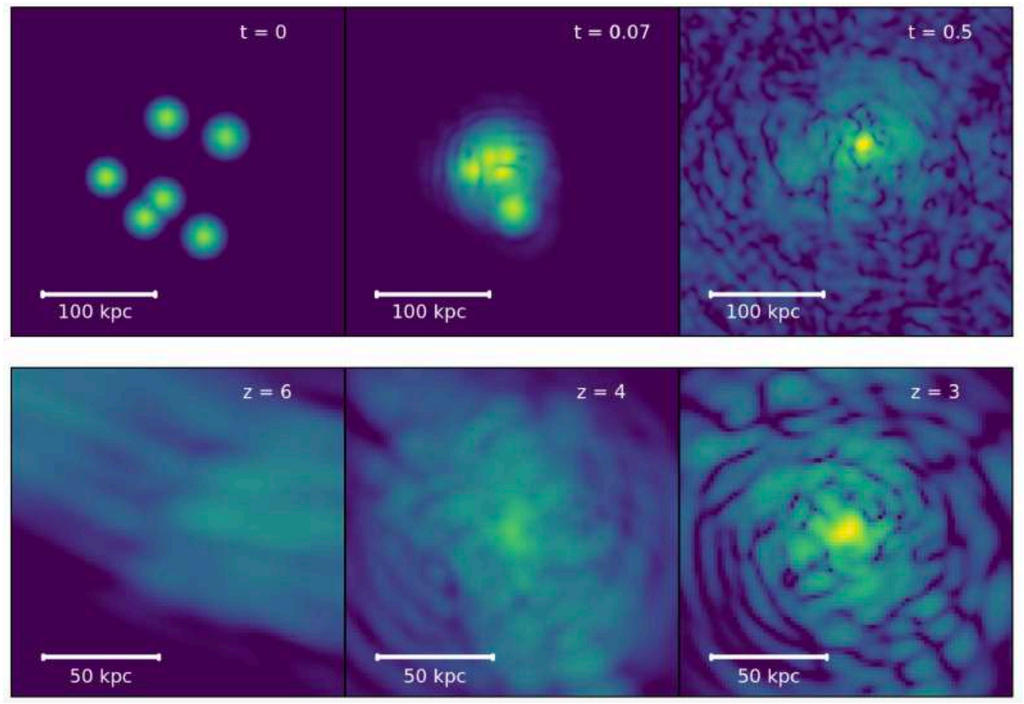
Postdoc ⟶ Max Planck Institute for Astrophysics, Germany

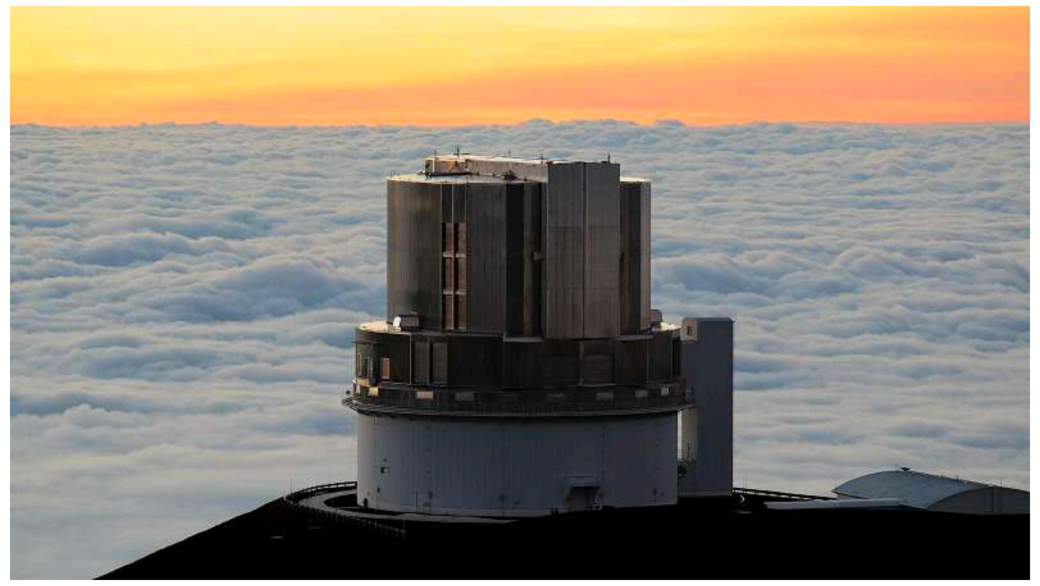
Currently: Assistant Professor at Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo

<ins>My research:</ins>

Theoretical cosmology

- Early universe
- Dark energy
- Dark matter
    - Ultra-light DM, axions





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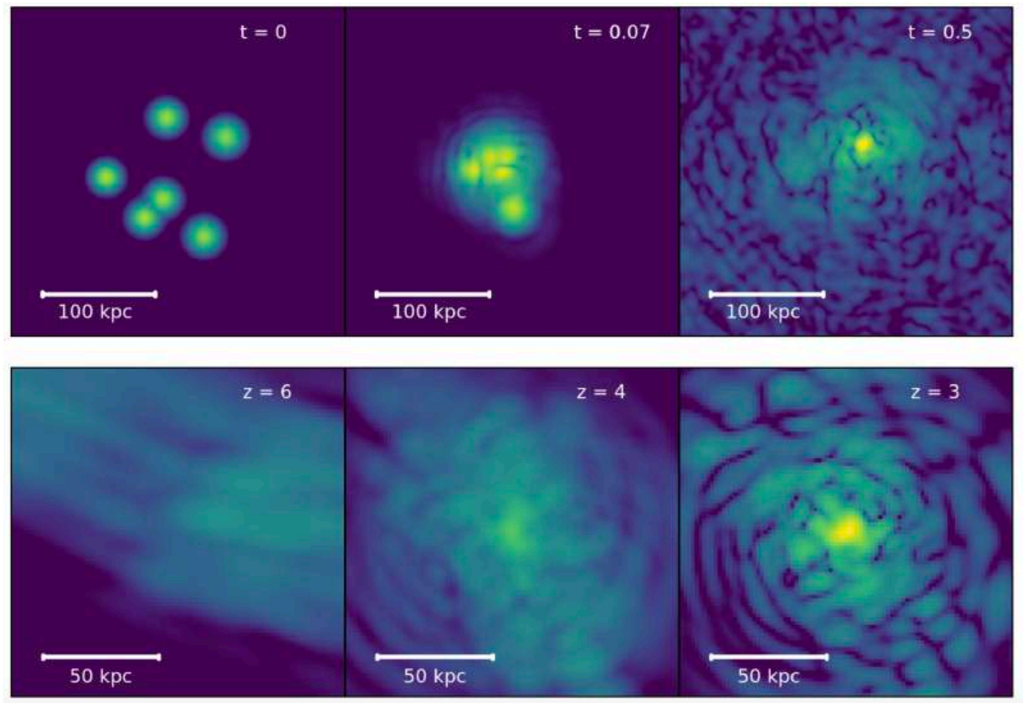
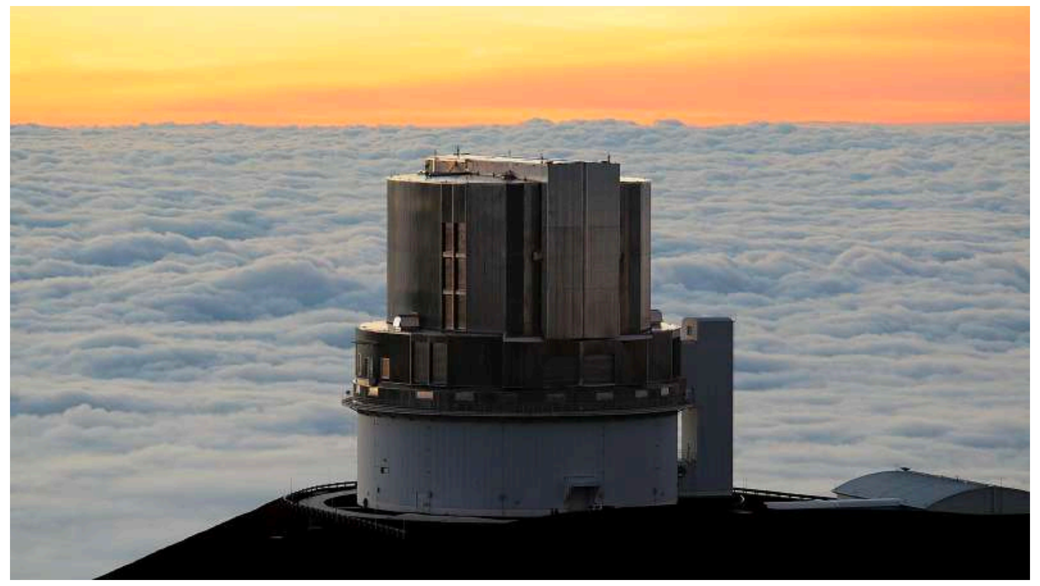




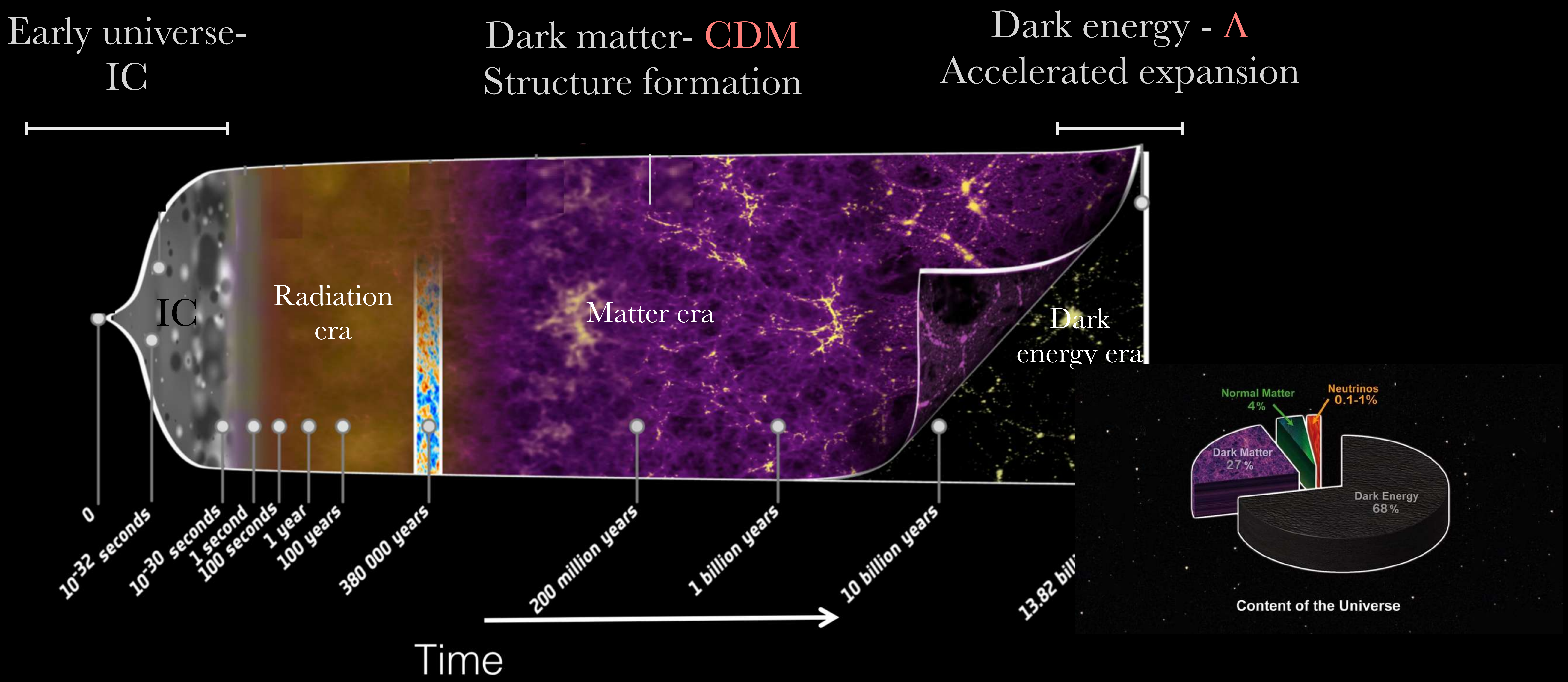
What we know!

Cosmology

- Cosmology studies the evolution and composition of the universe
- We try to understand:
  - How the universe was formed
  - How everything we see and are in the universe was formed
  - How did we get here → how did the universe evolved until today
  - Where is it going in the future?

Cosmology

- Cosmology studies the evolution and composition of the universe
- Cosmology became a precision science. (~30 years)
- Λ**CDM**: standard model, 6 parameters measured with precision ~1%. Huge success!



Cosmology

– Huge success! Cosmology became a precision science. (~30 years)
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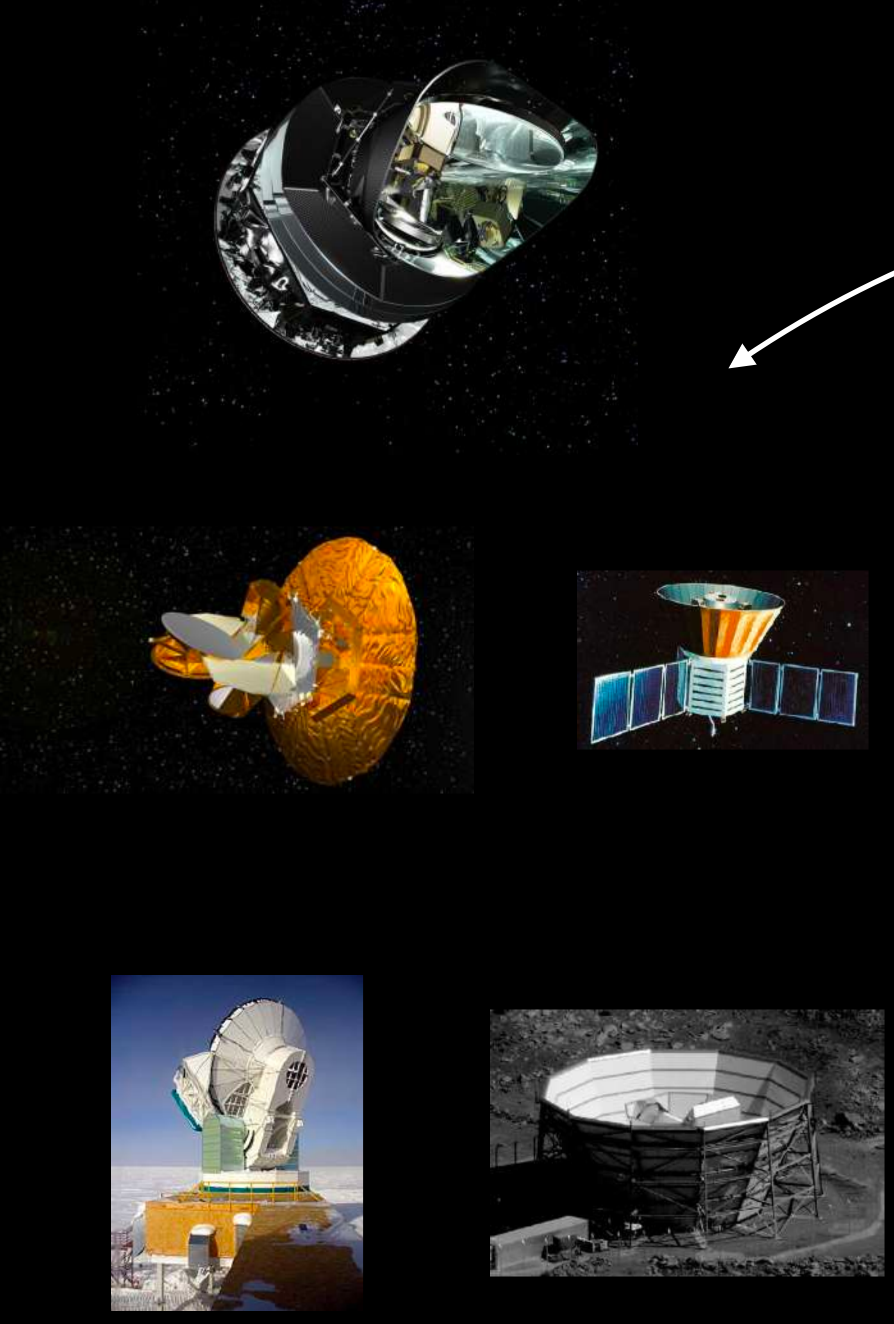
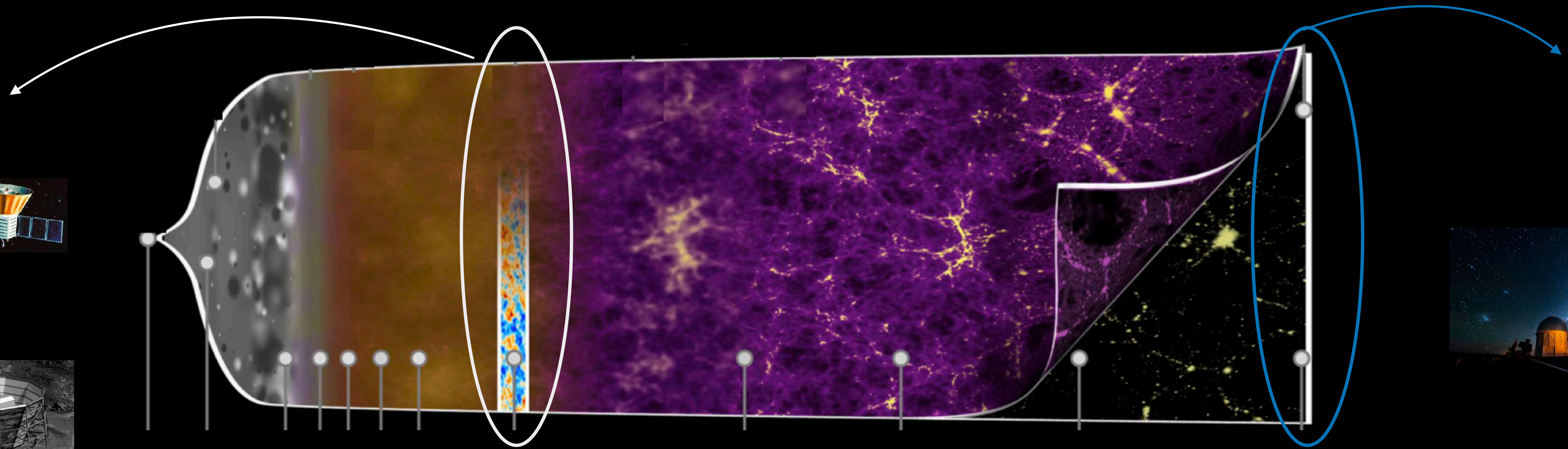
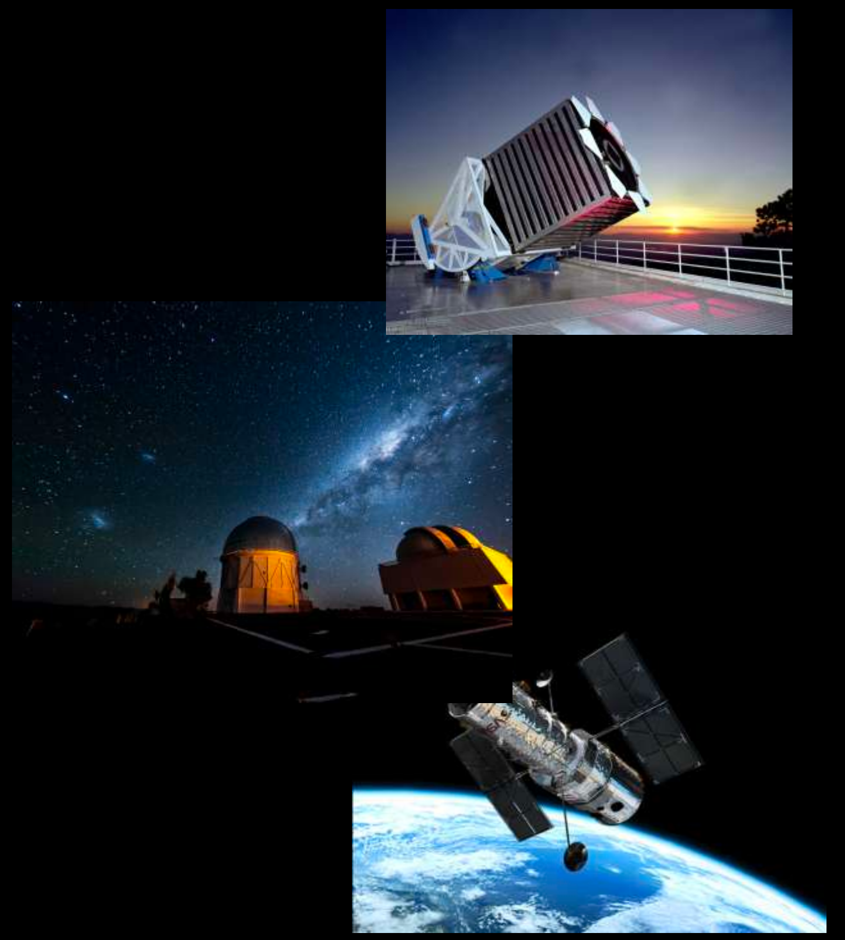
Early universe-
IC

Dark matter- CDM
Structure formation

Dark energy - Λ
Accelerated expansion

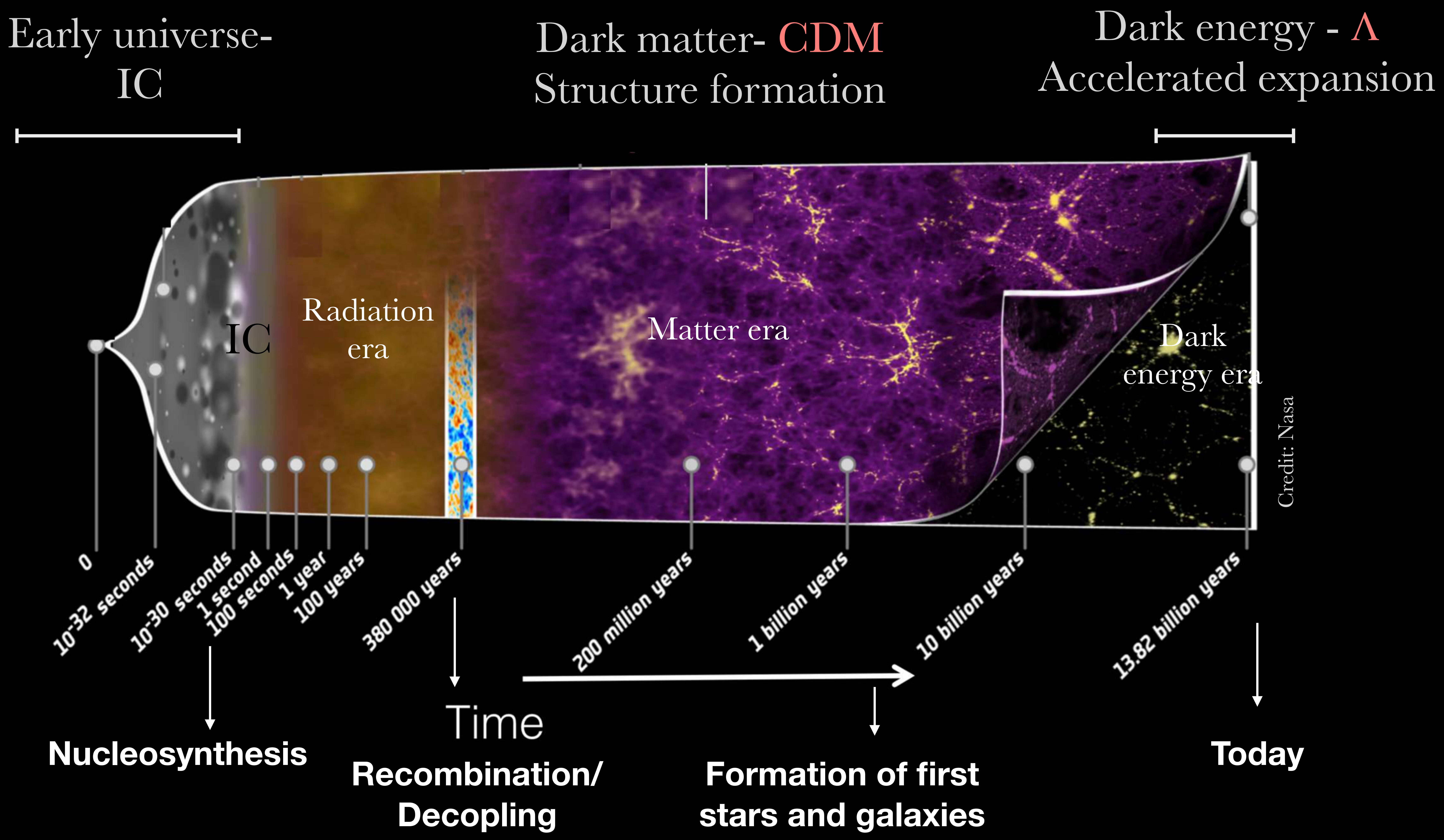






- Theoretical advances
- Observations with growing precision

ΛCDM: *standard model*

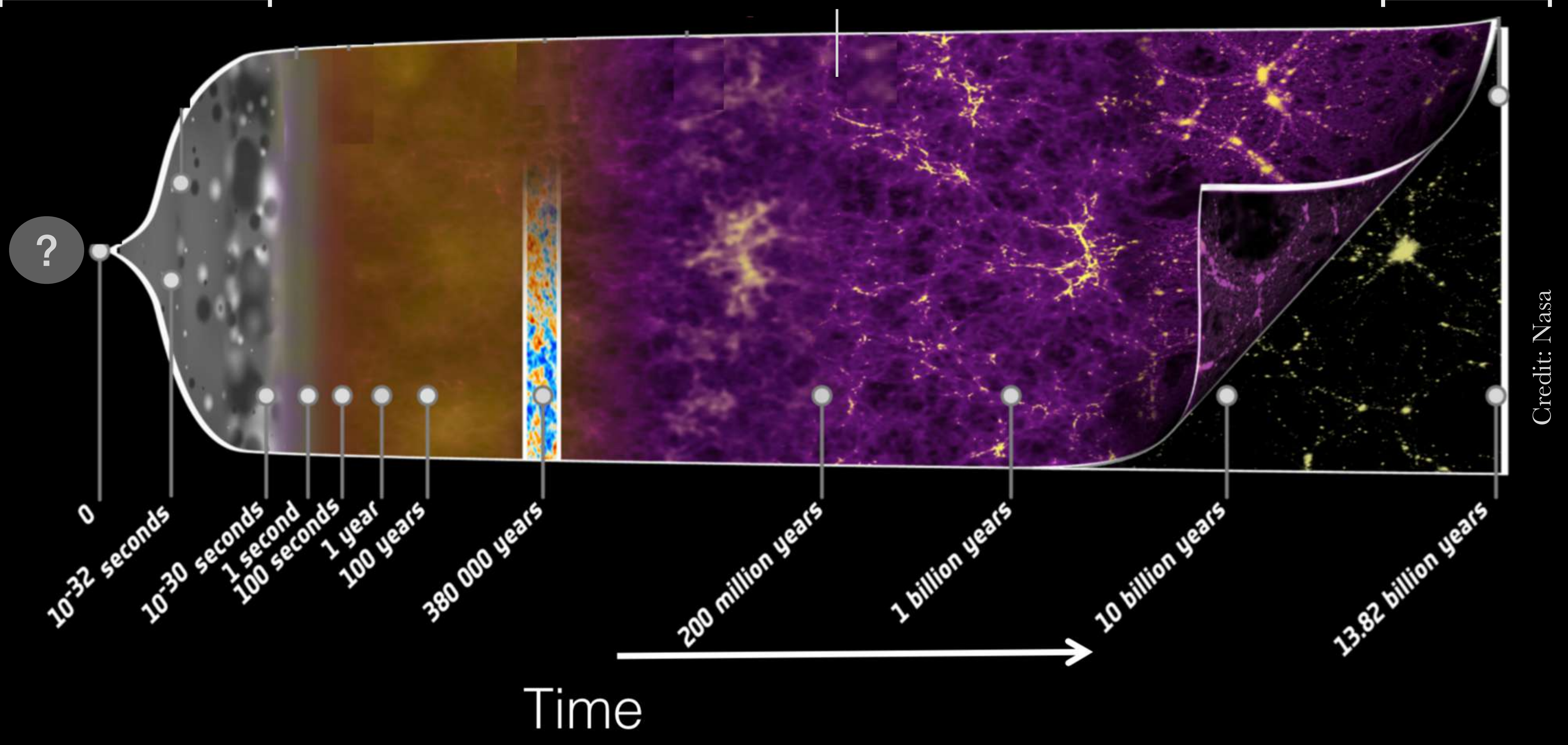


MANY fundamental open questions

Early universe
Big Bang? What is the physics of the early universe?

Dark matter p
What is the dark matter?

Dark energy -
What is the dark energy?



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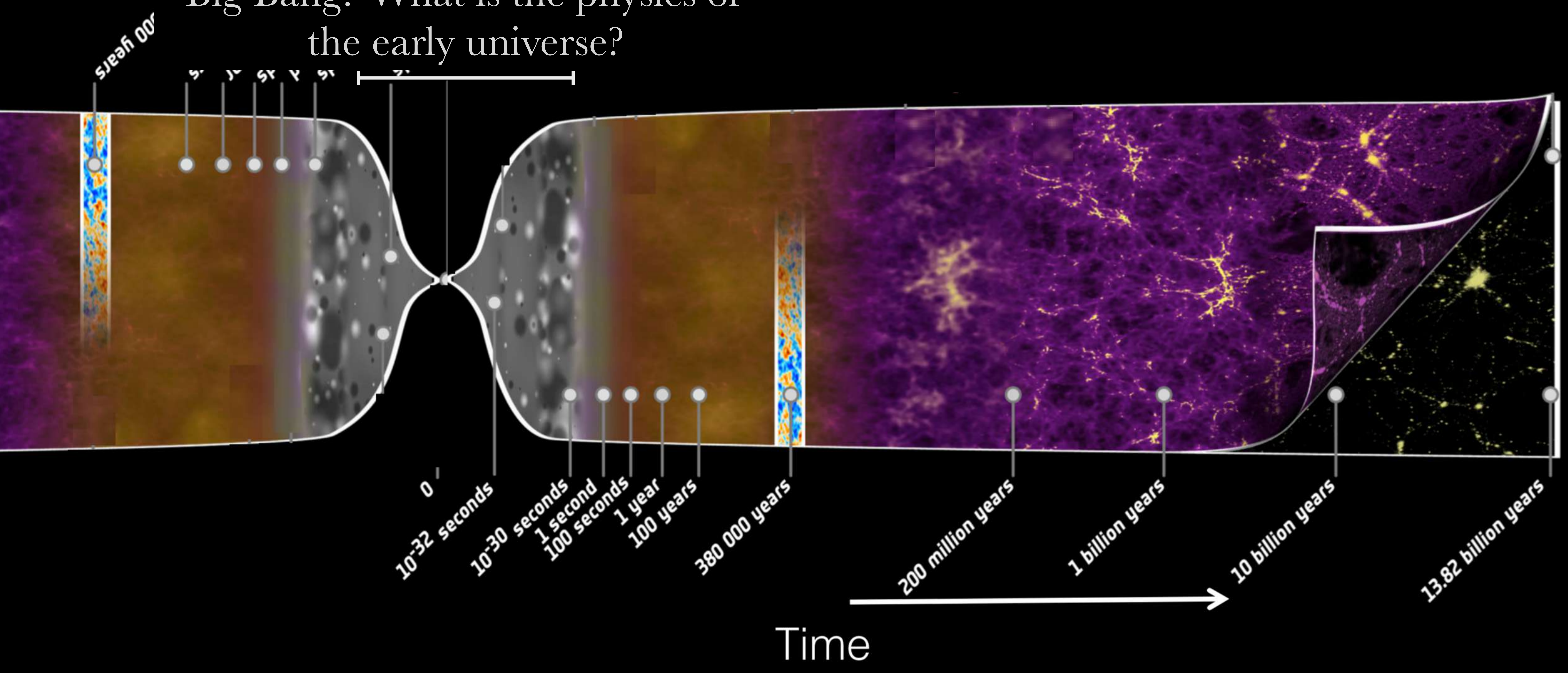
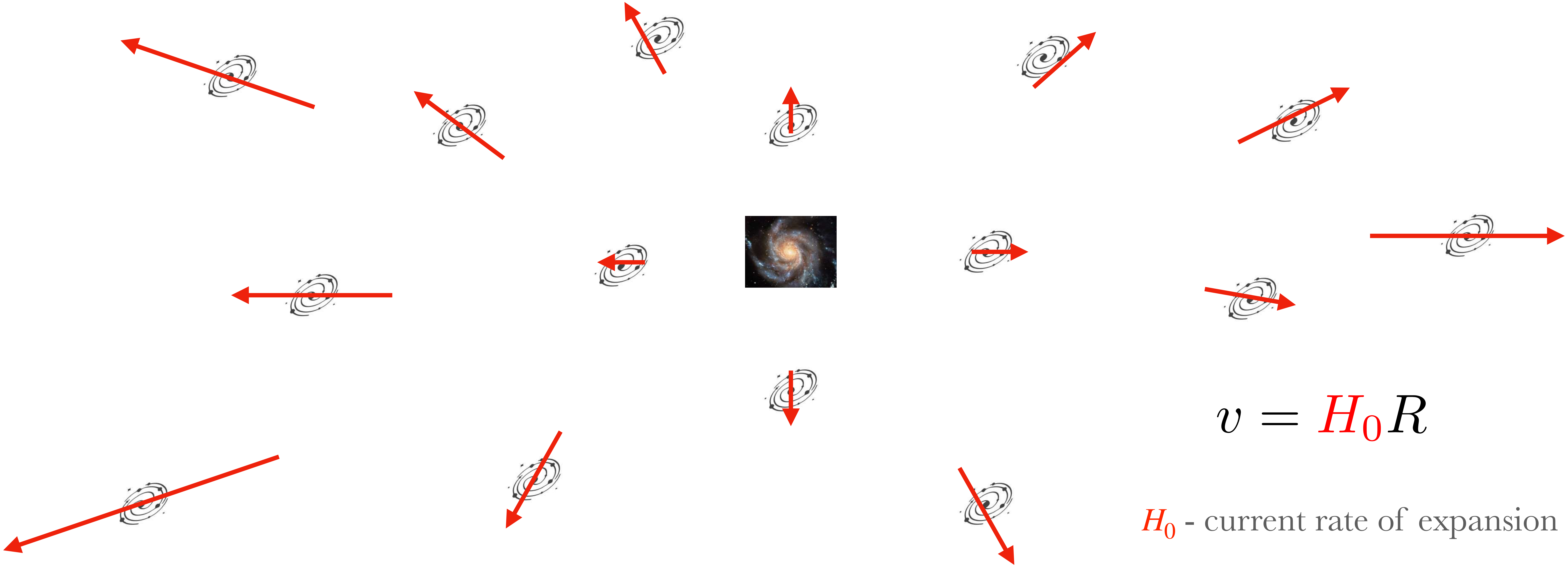


Credit: Nasa

General view of cosmology

Expanding universe: Hubble-Lemaitre law

Hubble, in 1929, and Lemaître, in 1927, discovered the relation between the recession velocity of galaxies and their distances.



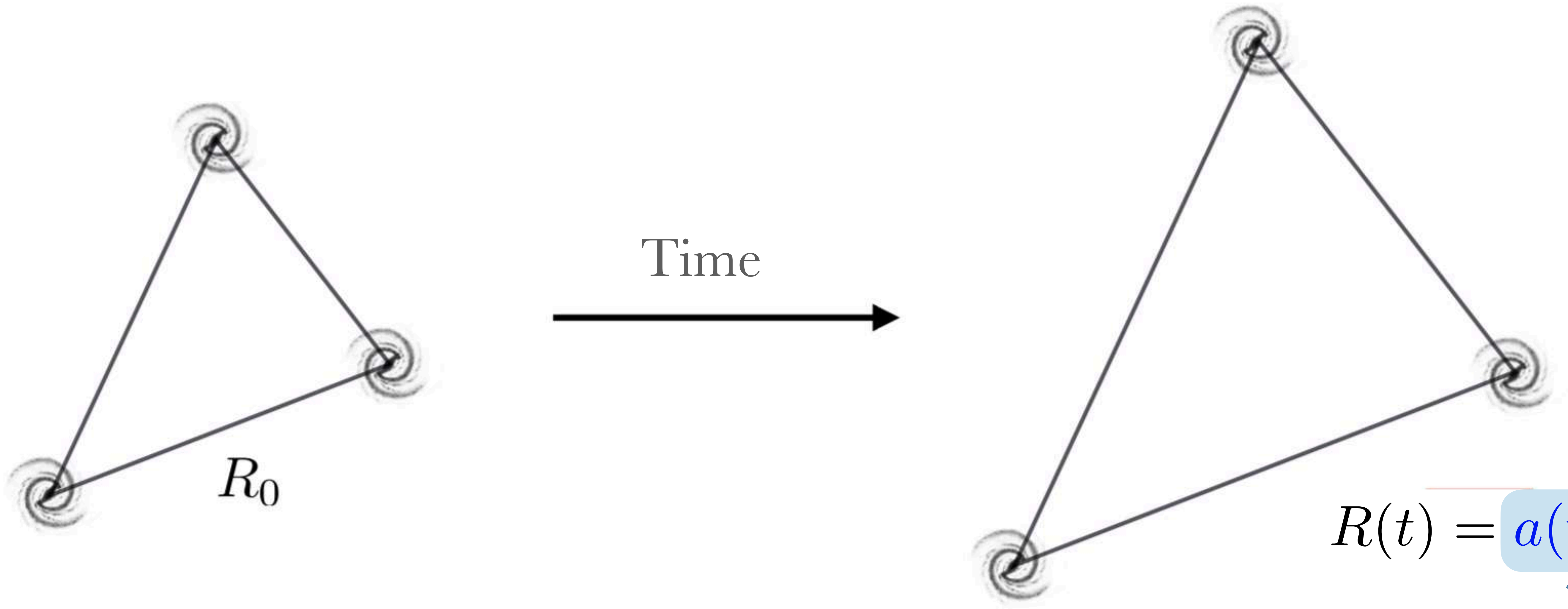
$$v = H_0 R$$

H_0 - current rate of expansion

Expanding universe: Hubble-Lemaitre law

In general relativity, we interpret this as the universe expanding. An expansion of the space between galaxies.

Crédito: D. Baumann



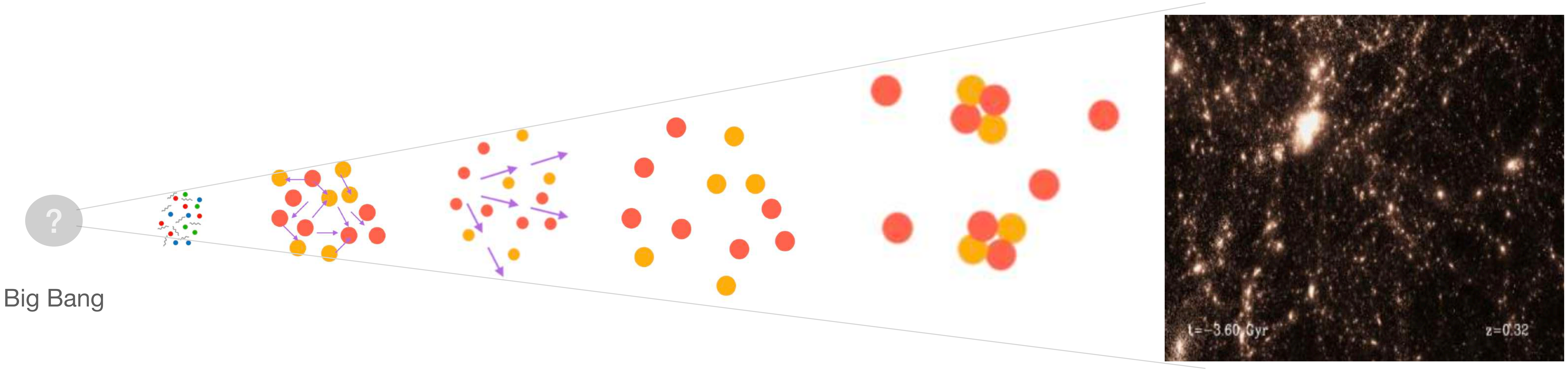
$$R(t) = a(t) R_0$$

Scale factor ← $a(t)$

$$v \equiv \dot{R} = \frac{\dot{a}}{a} \equiv H_0 R$$

H_0 → Hubble parameter (constant): current expansion rate of the universe

Standard cosmological model - Hot Big Bang model

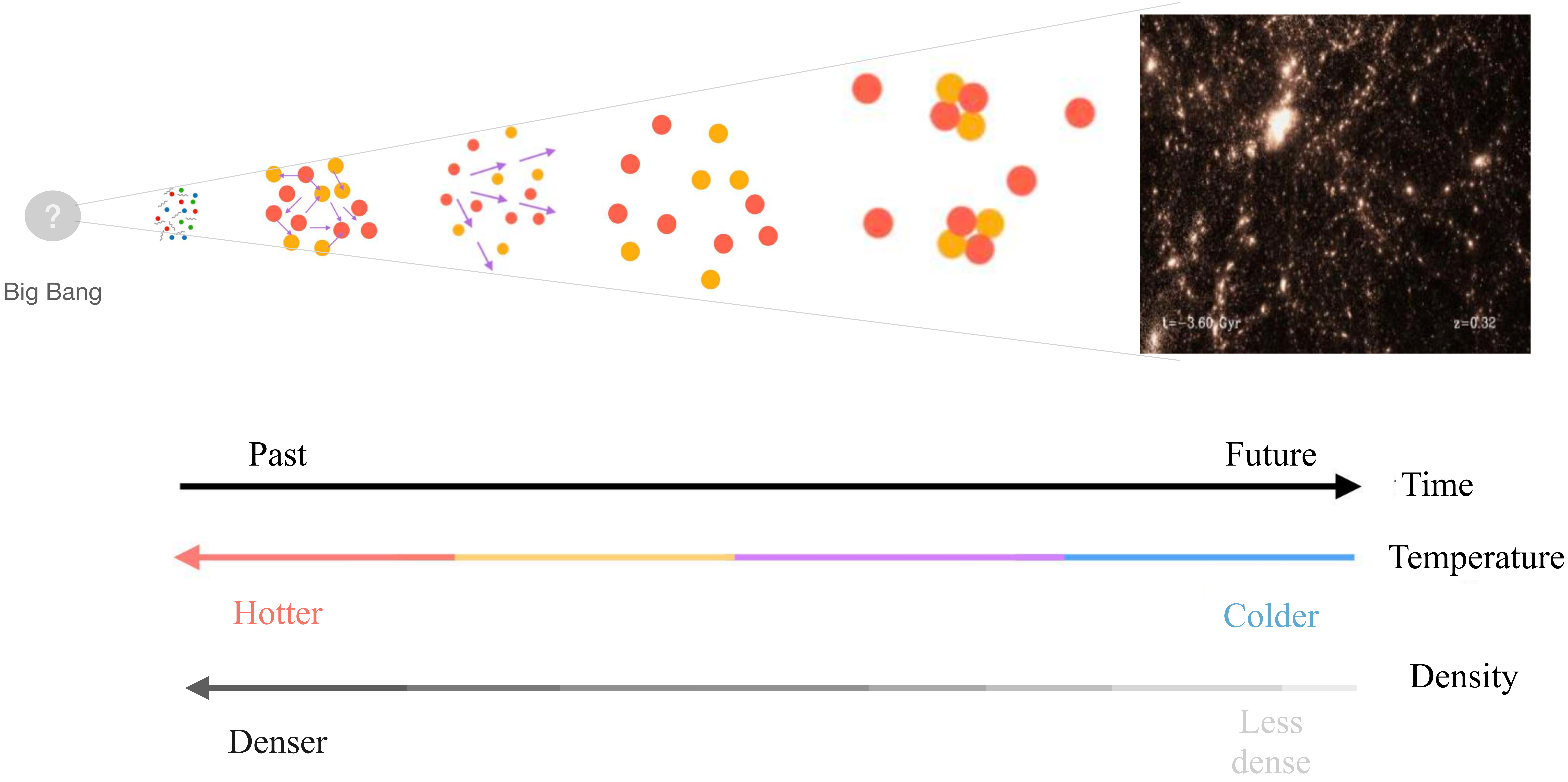


The universe is expanding!

Past —— radiation era → matter era → dark energy era → *Today*

If the universe is expanding, this means that before its energy was contained in a small, hot and **dense** region.

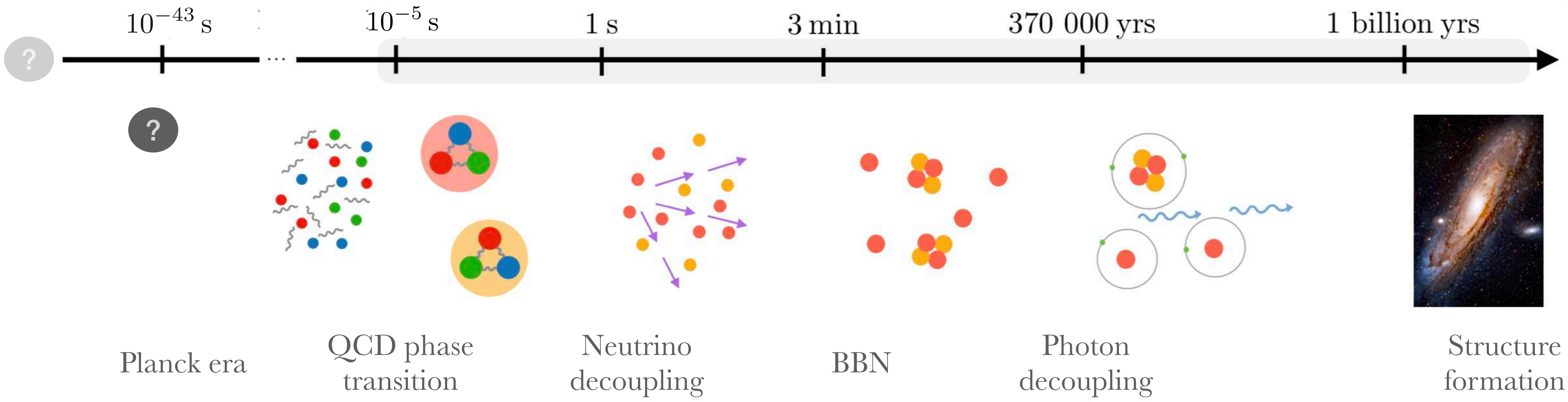
Standard cosmological model - Hot Big Bang model



Thermal history of the universe

The universe“started" hot e **dense** → As it cools, the structures we know start to form

SCM



Crédito: D. Baumann

How do we describe this mathematically?

Dynamics - Friedmann equations

The previous description of the universe is incomplete $\longrightarrow$ it does not provide any prediction about the scale factor, which is the only dynamical quantity present. We need to define the **evolution of the scale factor**.

That is determined by *content of the universe*

Dynamics - Friedmann equations

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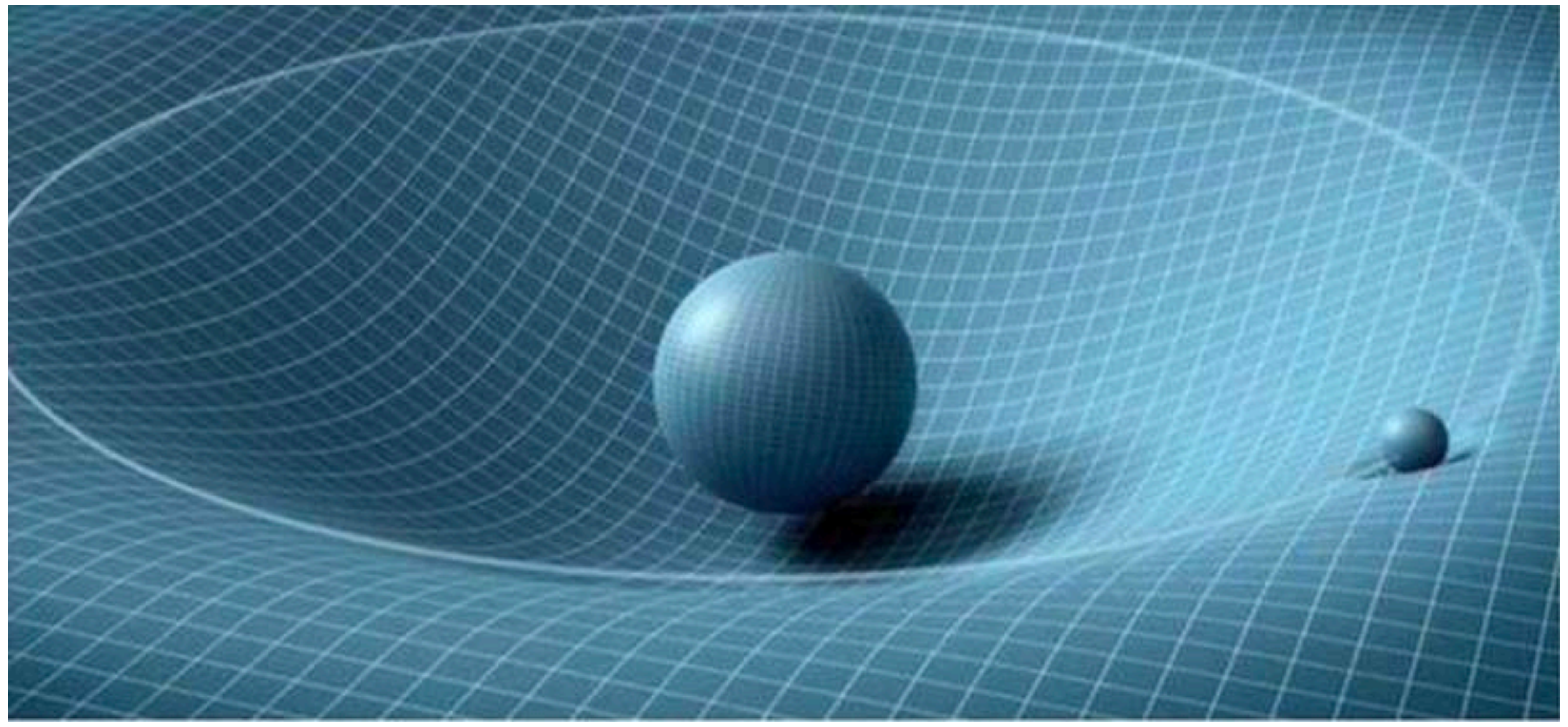
That is determined by *content of the universe*

This description is made using **general relativity**

The dynamics and kinematics of our universe are determined by Eisntein's general relativity, where its field equations, valid in all points of the universe, tell us how the content of the universe affects its dynamics.

Useful concepts of general relativity

General relativity: space-time is deformed by the presence of an object with mass.
The higher the mass, higher is the *local* curvature of the universe



General relativity: gravity is geometry!

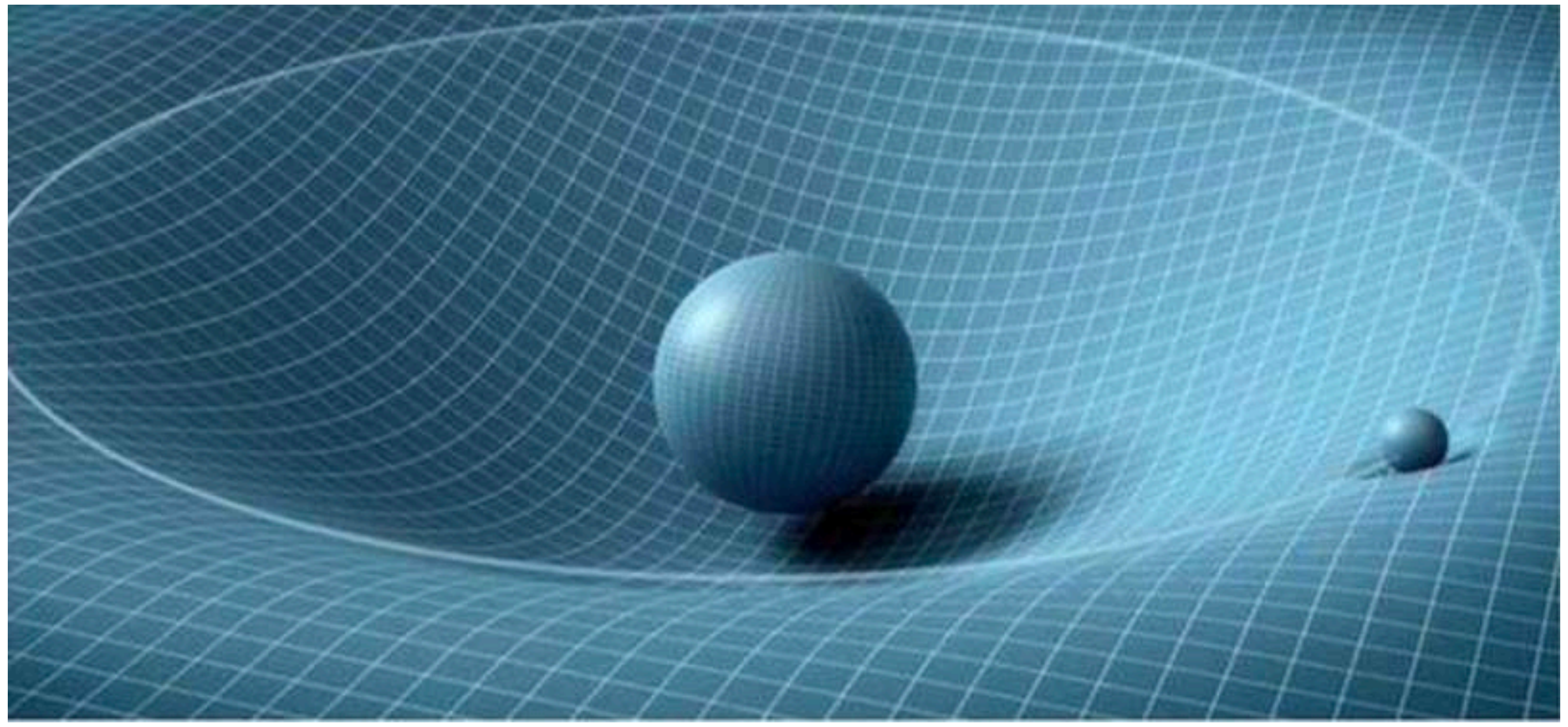
Mass/energy determines the geometry of the space-time
Space geometry determines the movement of mass/energy

Useful concepts of general relativity

General relativity: gravity is geometry!

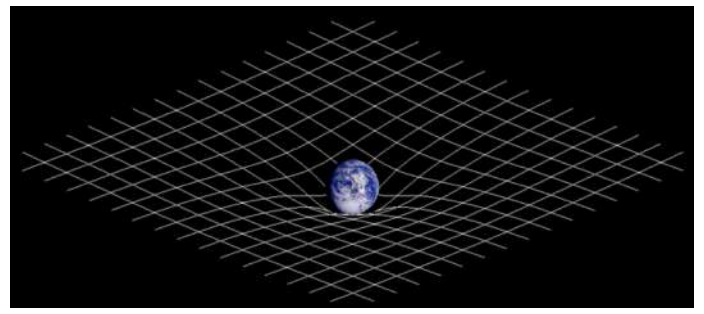
Mass/energy determines the geometry of the space-time
Space geometry determines the movement of mass/energy

<u>Dynamics</u>



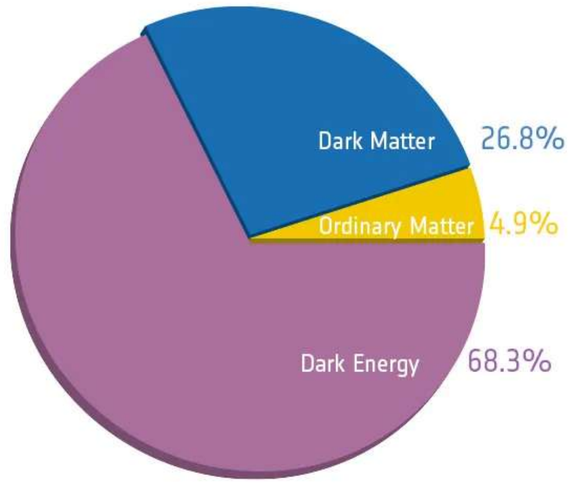
Einstein equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$



Geometry-
How universe expands

Components

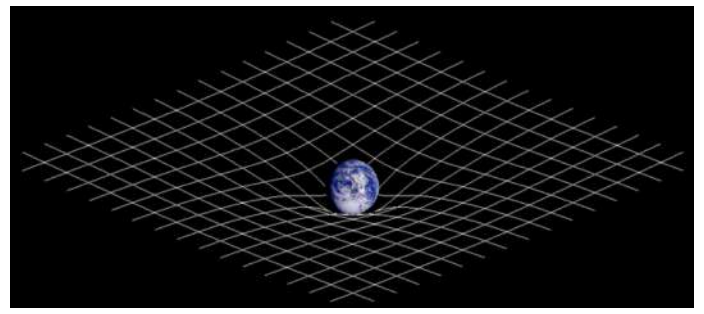


Questions?

Useful concepts of general relativity

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

$$\underbrace{G_{\mu\nu}}_{\text{Geometry- How universe expands}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{Components}}$$



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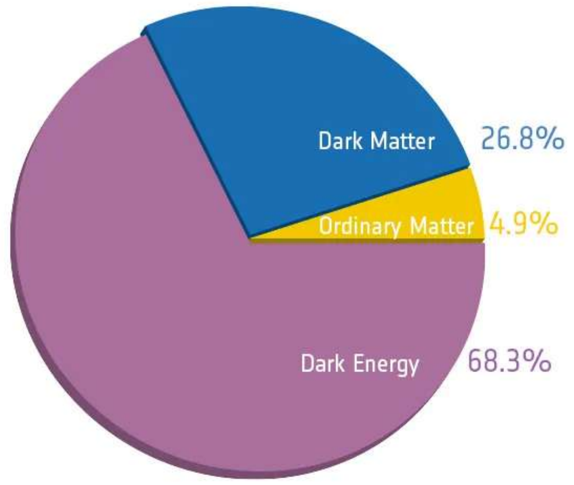

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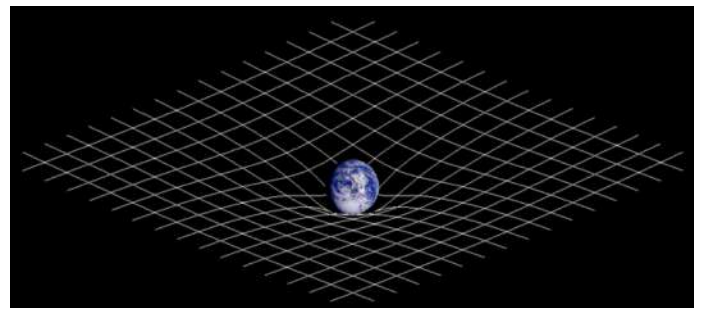

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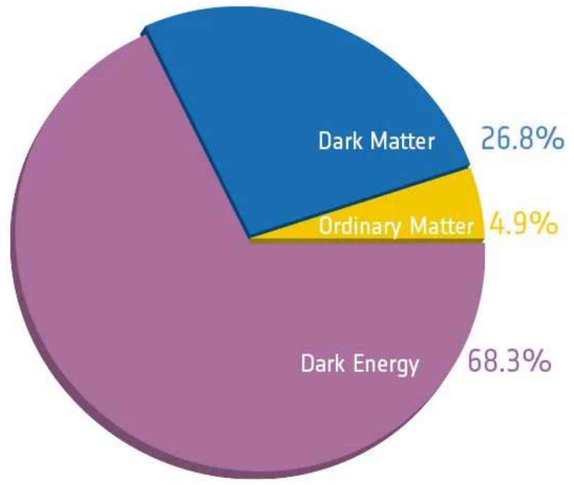


BUT only Einstein's equations are not enough to describe our universe

Structure and evolution of our universe

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

1.

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Geometry-
How universe expands

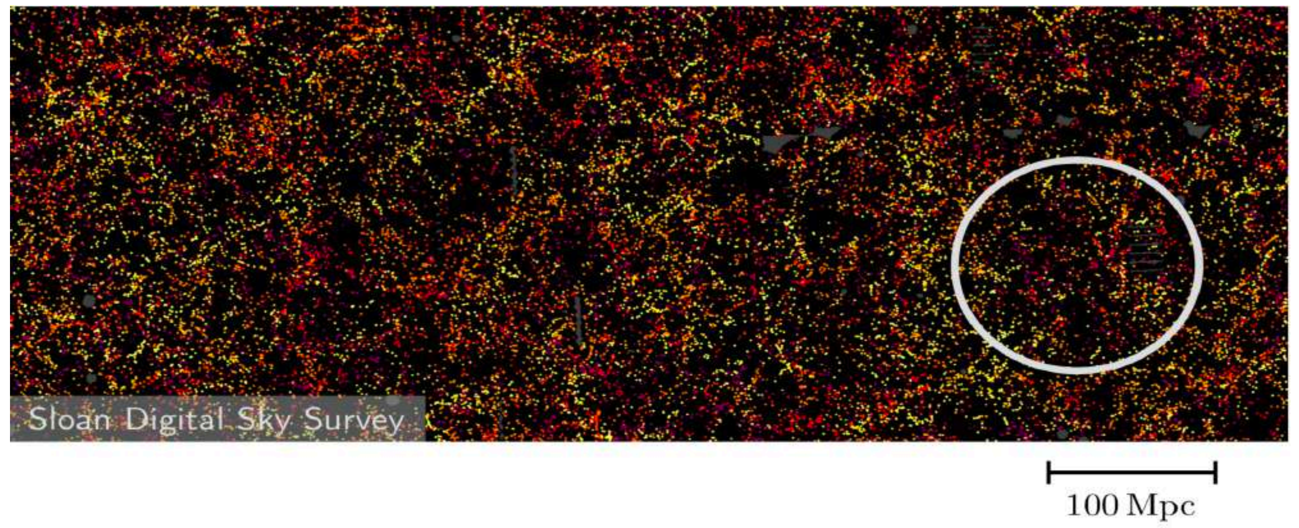
Components

BUT only Einstein's equations are not enough to describe our universe

2. Cosmological principle: the universe is homogeneous and isotropic on large scales

Translation invariance

Rotations invariance



At each time, the universe is the same in each place and direction; the dynamics is the same in all of the universe, except for local irregularities → spatial properties

Dynamics - Friedmann equations

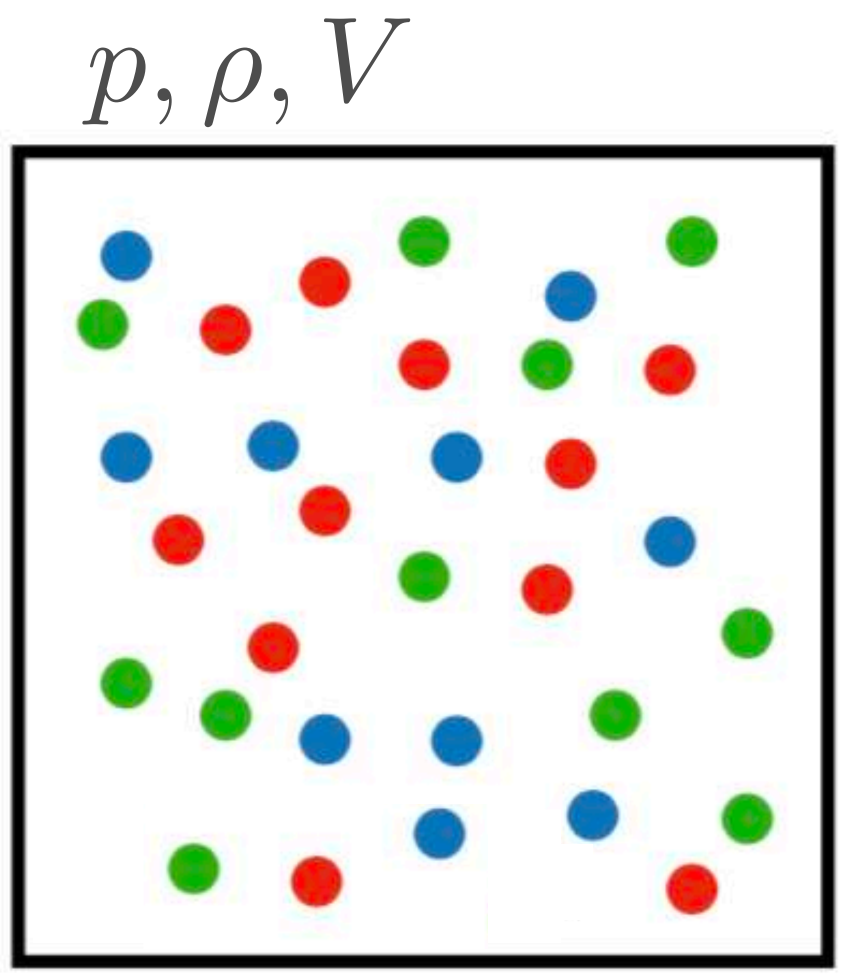
The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Components

Components of the universe

To describe a homogenous universe, we use perfect fluids, described by:



$$p = \text{pressure}$$
$$\rho = \text{energy density}$$

Cosmological fluids are described by a constant ***equation of state (EoS)***

$$\omega = \frac{P}{\rho}$$

$$\begin{cases} w = 0 & \text{matter} \\ w = 1/3 & \text{radiation} \\ \dots \end{cases}$$

Dynamics - Friedmann equations

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Using:

Geometry

Universe is homogeneous and isotropic

FRW metric

+

Components

Perfect fluid: p and ρ

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

Dynamics - Friedmann equations

Rate of the expansion of the universe

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Acceleration of the expansion

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
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This is the equation that describes the evolution of our universe!

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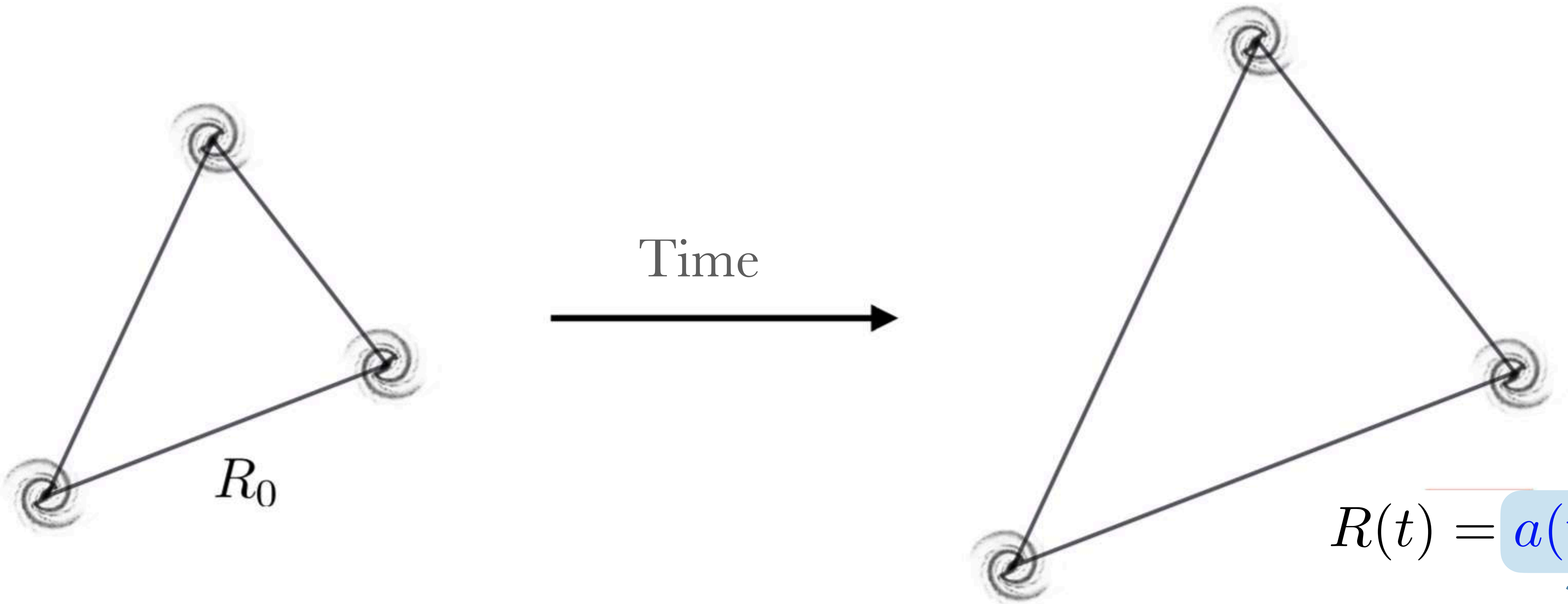
Crédito: D. Baumann



$$R(t) = a(t) R_0$$

Scale factor

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Hubble parameter (constant): current expansion rate of the universe

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

Combining these equations (taking the derivative of the first and using the second)

$$\dot{\rho} + 3H(\rho + P) = 0$$

Continuity equation:

conservation of the energy density

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

ρ and P here are actually the sum of all the components in the universe $\Longrightarrow$ ρ_{tot}, p_{tot}

We can also rewrite the 1st Friedmann equation as:

$$1 = \Omega_{tot} - \frac{k}{a^2 H^2}$$

Density parameter

$$\Omega_{tot} = \sum_i \Omega_i \,, \qquad \text{onde } \Omega_i = \frac{\rho_i}{\rho_{crit}}$$

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
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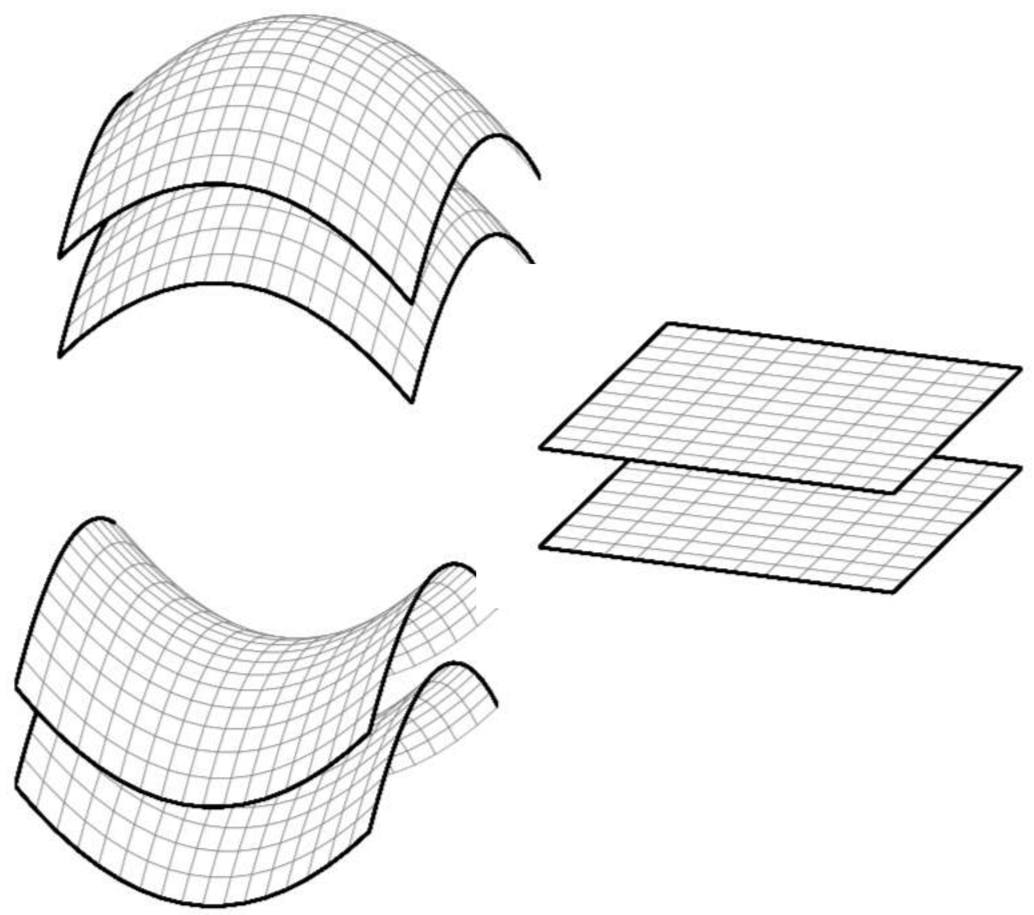
$\Omega_{total} > 1 \Leftrightarrow k = +1\,,$ Closed universe

$\Omega_{total} = 1 \Leftrightarrow k = 0\,,$ Flat universe

$\Omega_{total} < 1 \Leftrightarrow k = -1\,,$ Open universe



Questions?

Components of the universe

To describe a homogenous universe, we use perfect fluids, following the equation:

$$\dot{\rho} + 3H\left(\rho + P\right) = 0$$

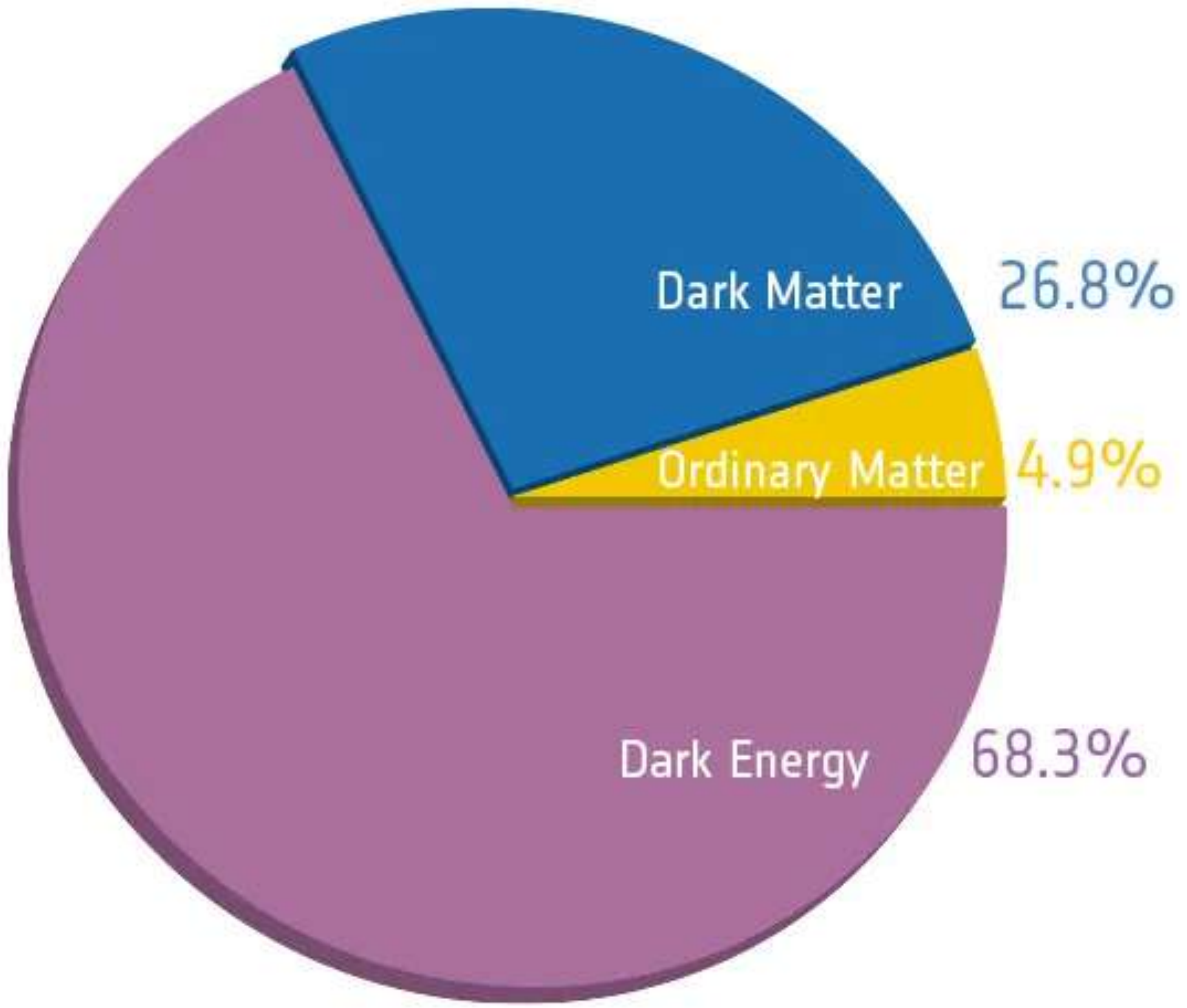
Cosmological fluids are described by a constant ***equation of state (EoS)***

$$\omega = \frac{P}{\rho}$$

Leading to:

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \longrightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

Components of the universe



Credit: ESA and the Planck Collaboration

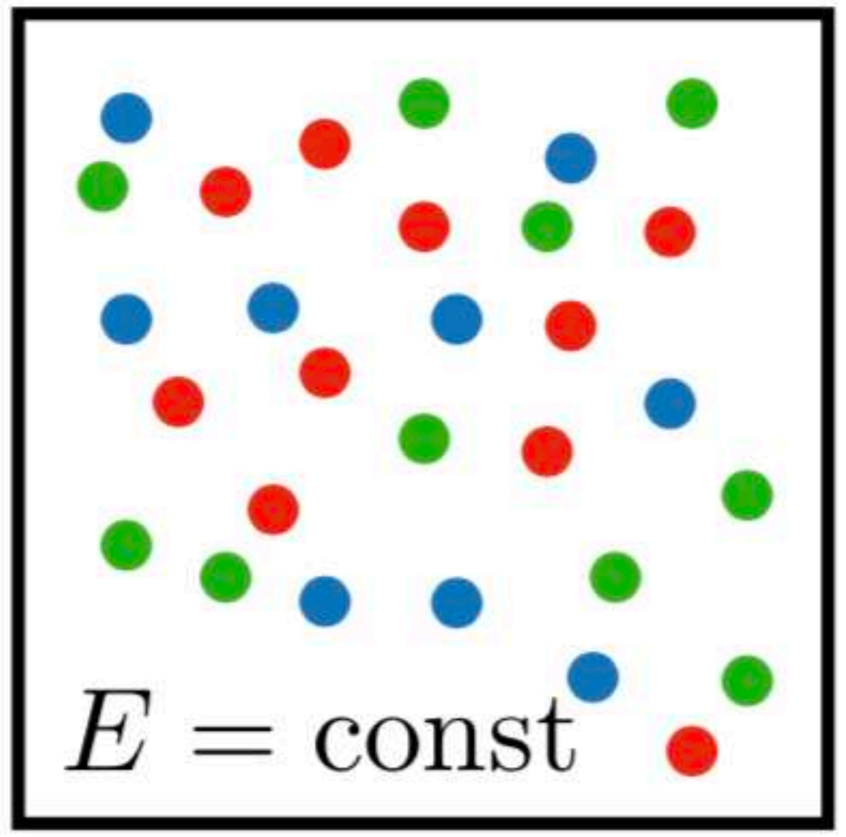
Each component evolves and leads to a different expansion of the universe.

Lets study how each component evolves

Matter

Matter is a fluid with zero pressure ($\omega = 0$):

$$P = 0 \longleftrightarrow \boxed{\rho \equiv \frac{E}{V} \propto a^{-3}}$$



Credit: D. Baumann

Inserting in Friedmann's eq, matter evolves as:

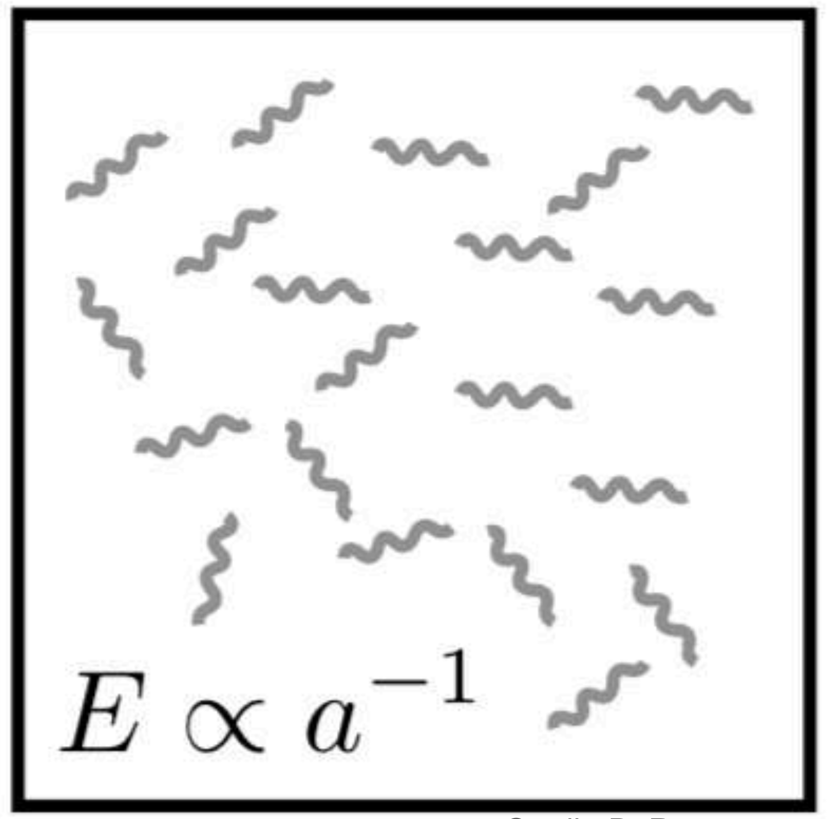
$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho \propto a^{-3} \longrightarrow \boxed{a \propto t^{2/3}}$$

In our universe, 2 components behave as matter:
dark matter (25%) e a ordinary matter - baryons (5%).

Radiation

Radiation is a relativistic fluid ($\omega = 1/3$):

$$P = \frac{1}{3}\rho \quad \longleftrightarrow \quad \boxed{\rho \equiv \frac{E}{V} \propto a^{-4}}$$



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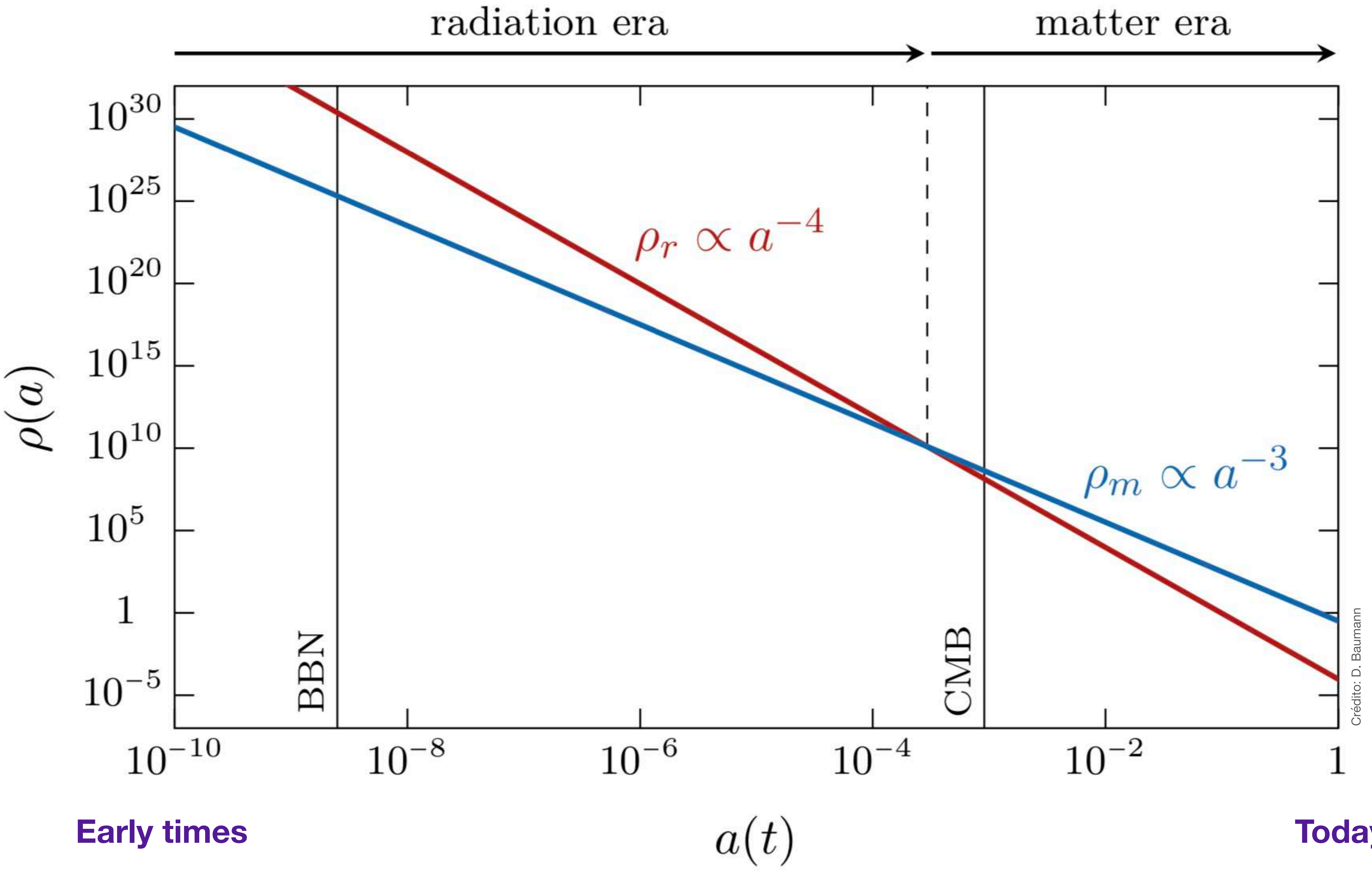
Inserting in Friedmann's eq, raditaion evolves as:

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho \propto a^{-4} \quad \longrightarrow \quad \boxed{a \propto t^{1/2}}$$

Radiation dominates the evolution of the universe at early stages, before matter.

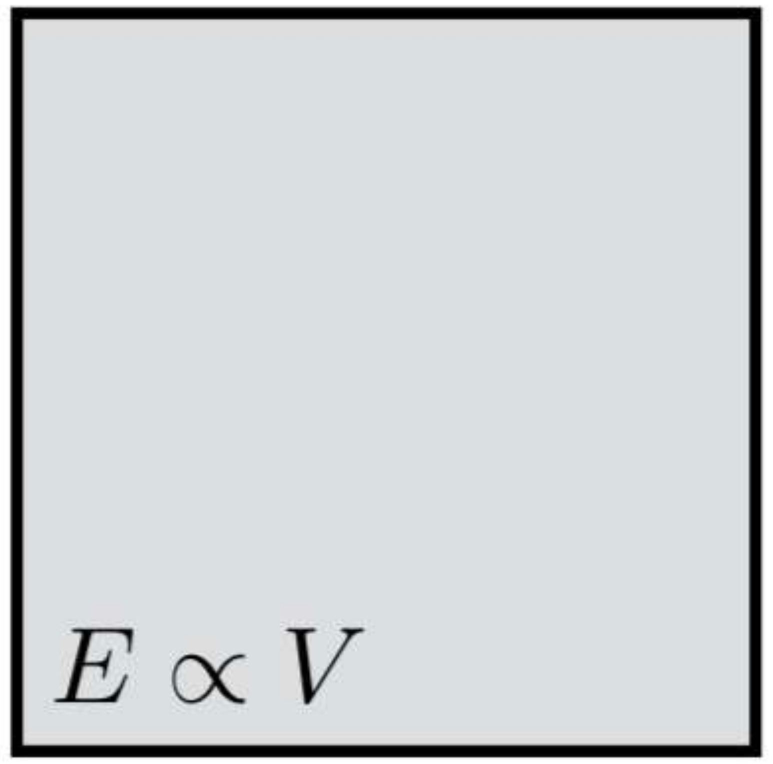
Matter and radiation



Early times **Today**

Dark energy

Observational data indicates that the universe is expanding in an **accelerated** way $\ddot{a} > 0$



Credit: D. Baumann

Acceleration

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

Diceleration

$$w = \frac{p}{\rho} < -\frac{1}{3}$$

The component which is the source of this accelerated expansion we call dark energy

$\ddot{a} < 0$

Decelerated expansion

$$P > -\frac{1}{3}\rho$$

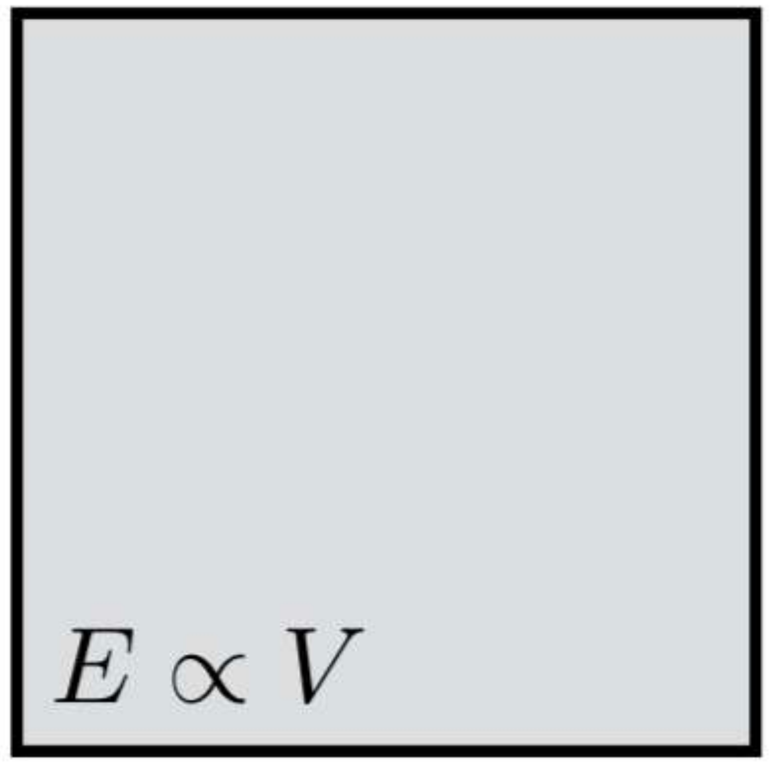
$\ddot{a} > 0$

Accelerated expansion

$$P < -\frac{1}{3}\rho$$

Dark energy

Observational data indicates that the universe is expanding in an **accelerated** way $\ddot{a} > 0$



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Diceleration

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The component which is the source of this accelerated expansion we call dark energy

<u>Cosmological constant</u>

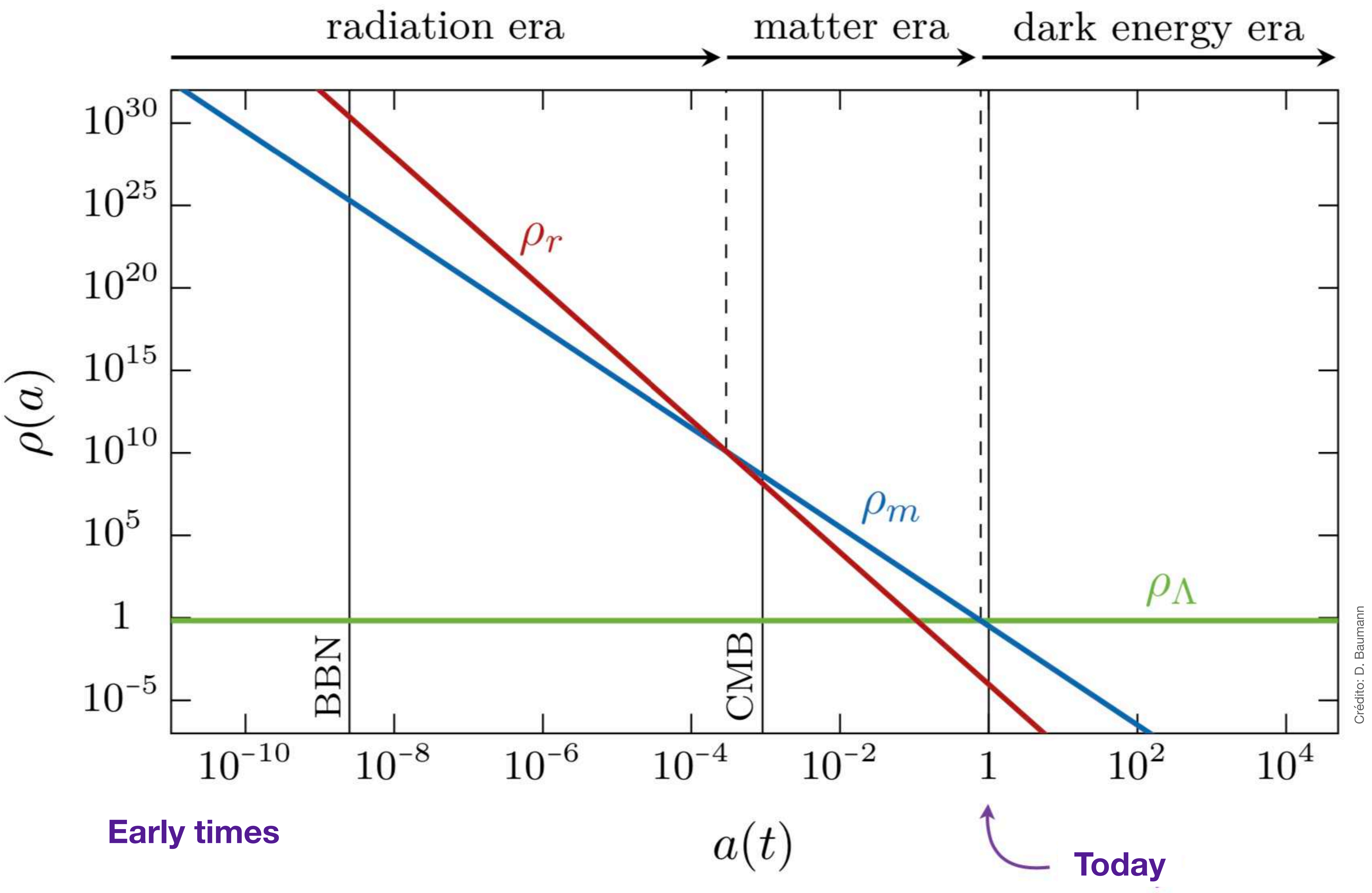
Cosmological const.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad + \quad w = -1 \quad \Longrightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 \propto \rho = \text{const}$$

accelerates expansion

$$a \propto e^{H_0 t}$$

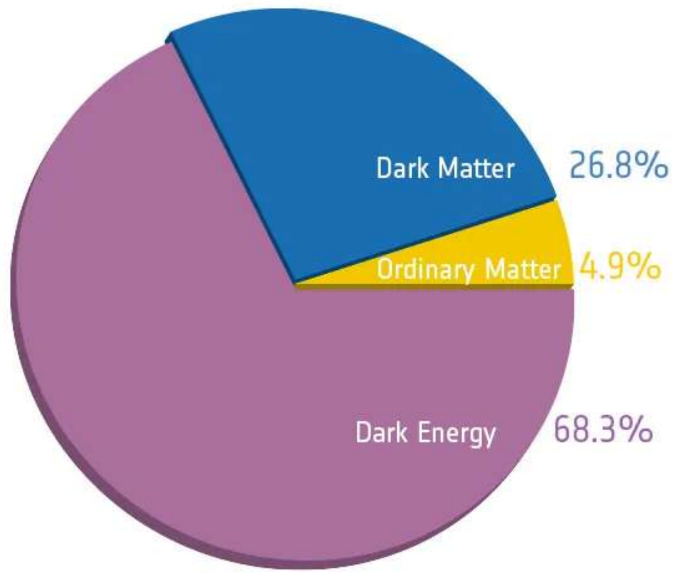
Matter, radiation and dark energy



Summary

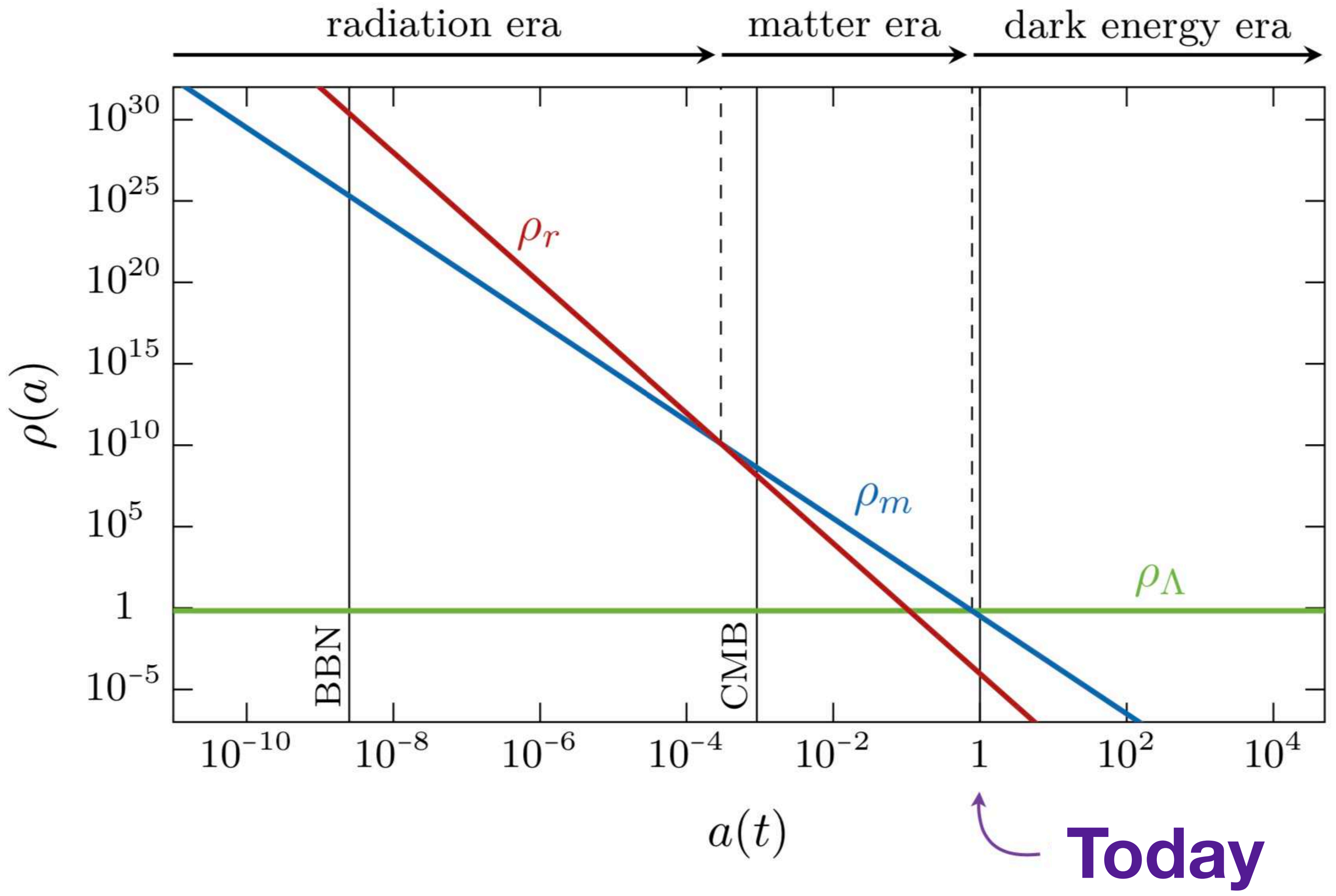
Expansion of the universe → Rate of expansion depends on the component

Components of the universe



Credit: ESA and the Planck Collaboration

$$\Rightarrow \quad \frac{\dot\rho}{\rho} = -3(1+w)\frac{\dot a}{a} \longrightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

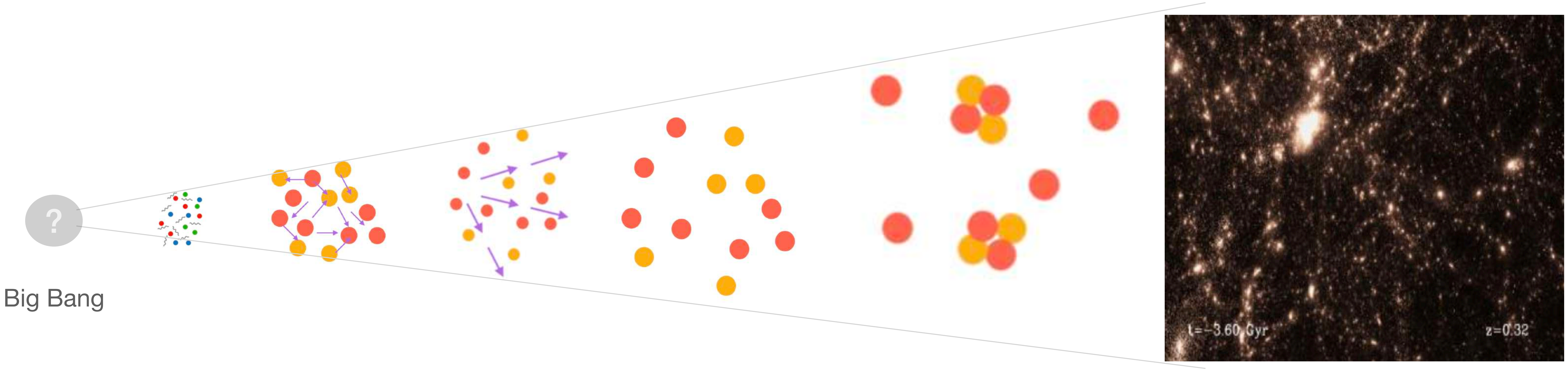


With that, we know that:

Past radiation era → matter era → dark energy era → *Today*

$$\Omega_{tot} = \Omega_m + \Omega_{rad} + \Omega_{de}$$

Standard cosmological model - Hot Big Bang model



The universe started from a region that was hot and **dense** $\longrightarrow$ *expanding* and cooling after that

Composition: radiation, matter, dark matter and dark energy

Today dominated by dark energy, expanding in an accelerated way

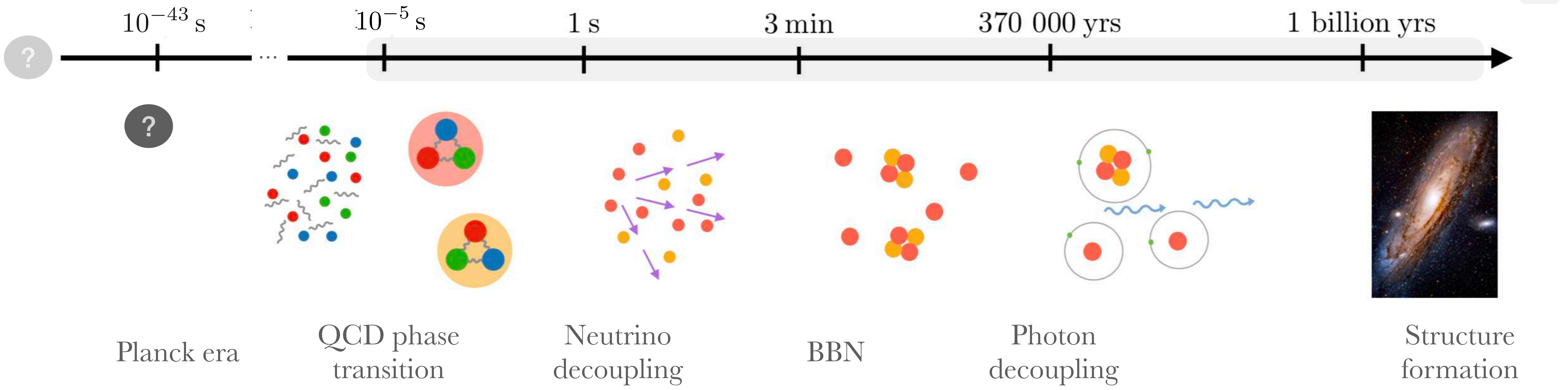
The ΛCDM explains the evolution of the structures in the universe (from small initial perturbations) and the formation and abundance of all the elements in our universe (standard model of the elementary particles)

ΛCDM: parametrizes this entire evolution with 6 parameters!

Thermal history of the universe

The universe“started" hot e **dense** → As it cools, the structures we know start to form

SCM



Crédito: D. Baumann

Standard cosmological model

Cosmological parameters

Standard cosmological model - LCDM model

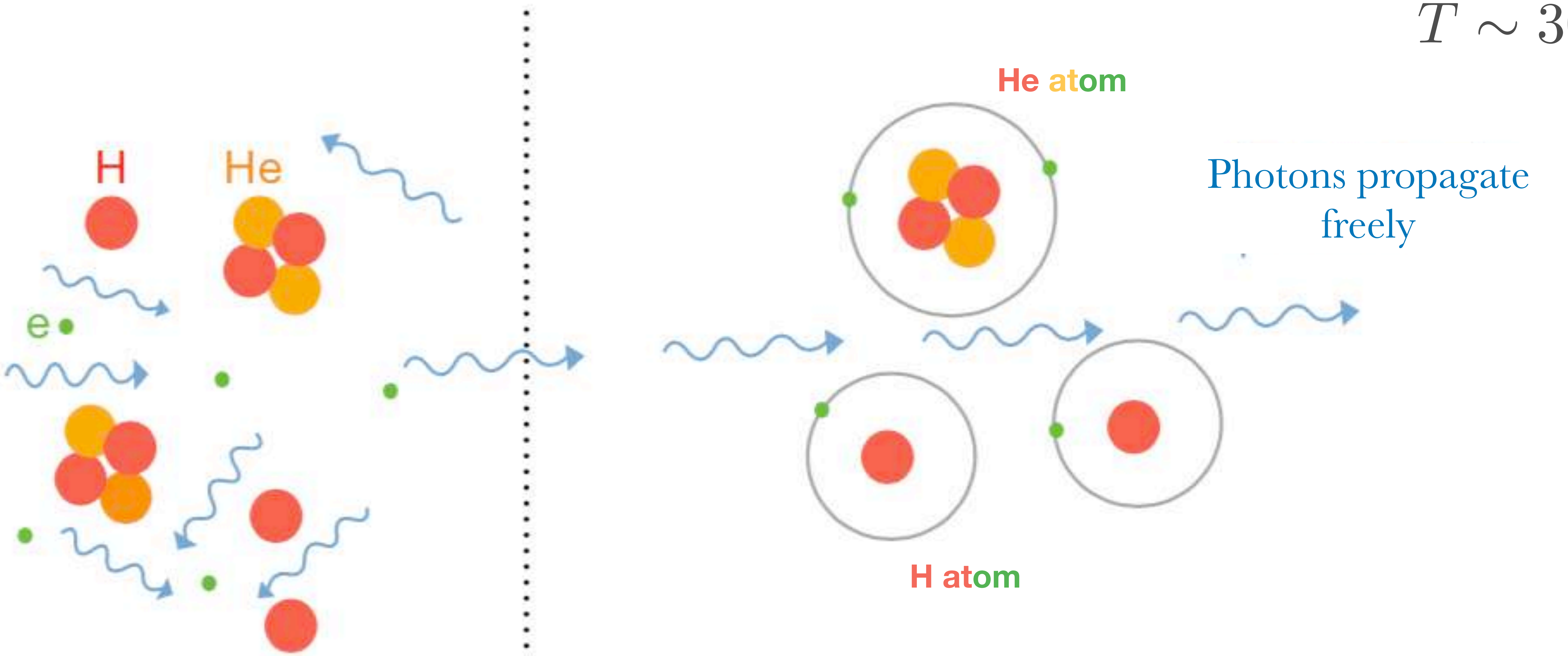
$$\{\Omega_b, \, \Omega_m \, , \Omega_\Lambda \, , n_s \, , A_s \, , \tau\}$$

We parametrize this entire evolution and composition of the universe using 6 parameters!

Recombination and photon decoupling

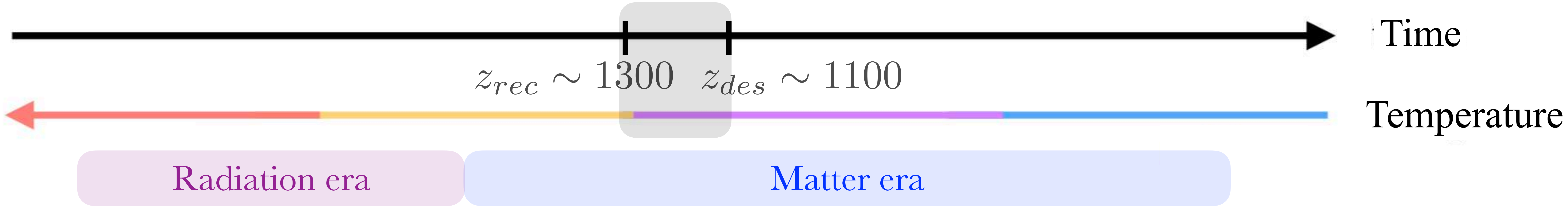
$t \sim 370000\,\text{yrs}$

$T \sim 3000\,\text{K}$



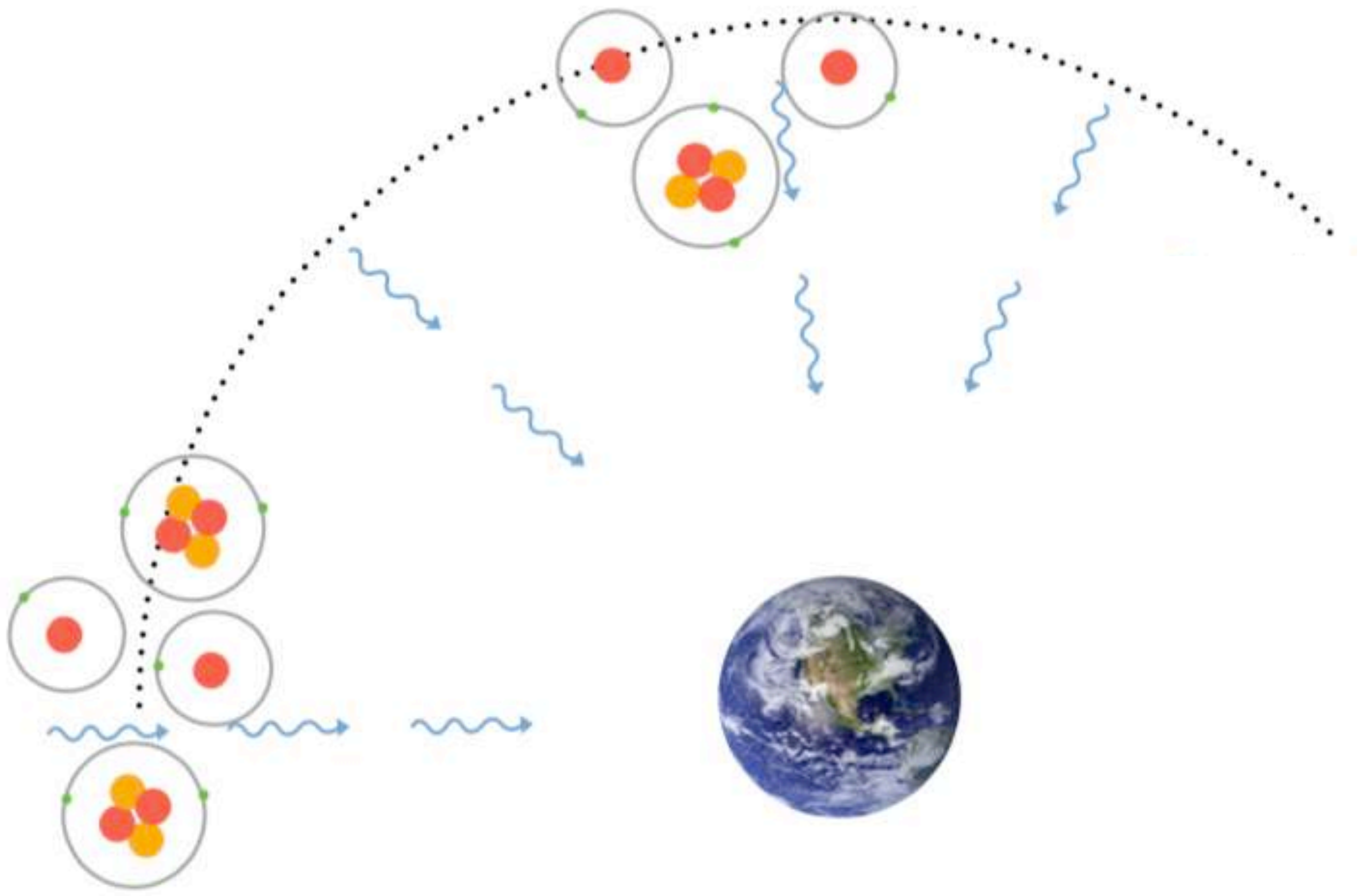
Plasma ("soup") of coupled H, He, elétrons and radiation - thermal equilibrium
- universe is opaque: radiation cannot scape!

Atoms are formed!
Charged electrons bound with n H and He nucleus



Crédito: D. Baumann

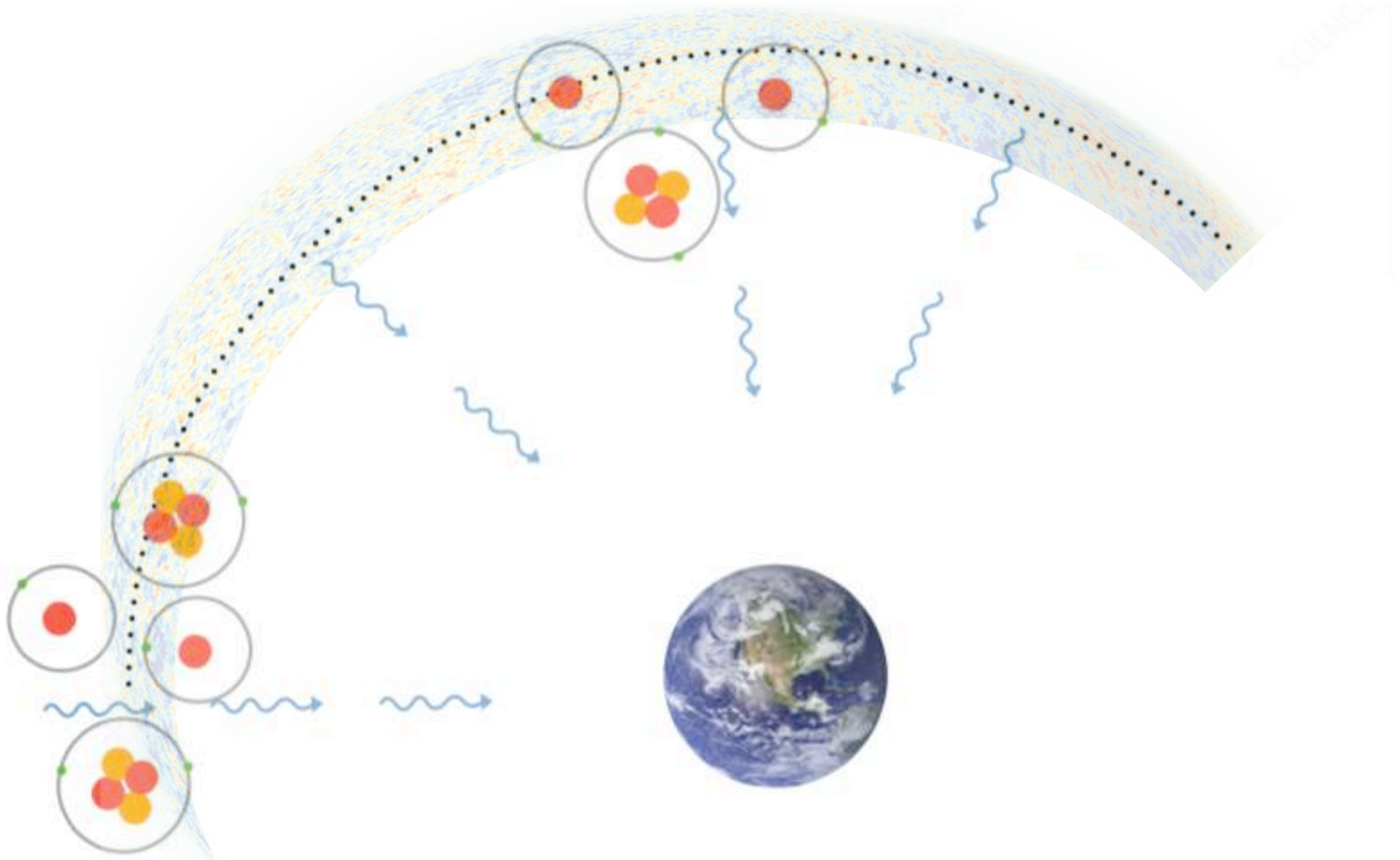
These photons are the first light of our universe…



Crédito: D. Baumann

… e tell us how the universe was at early times.

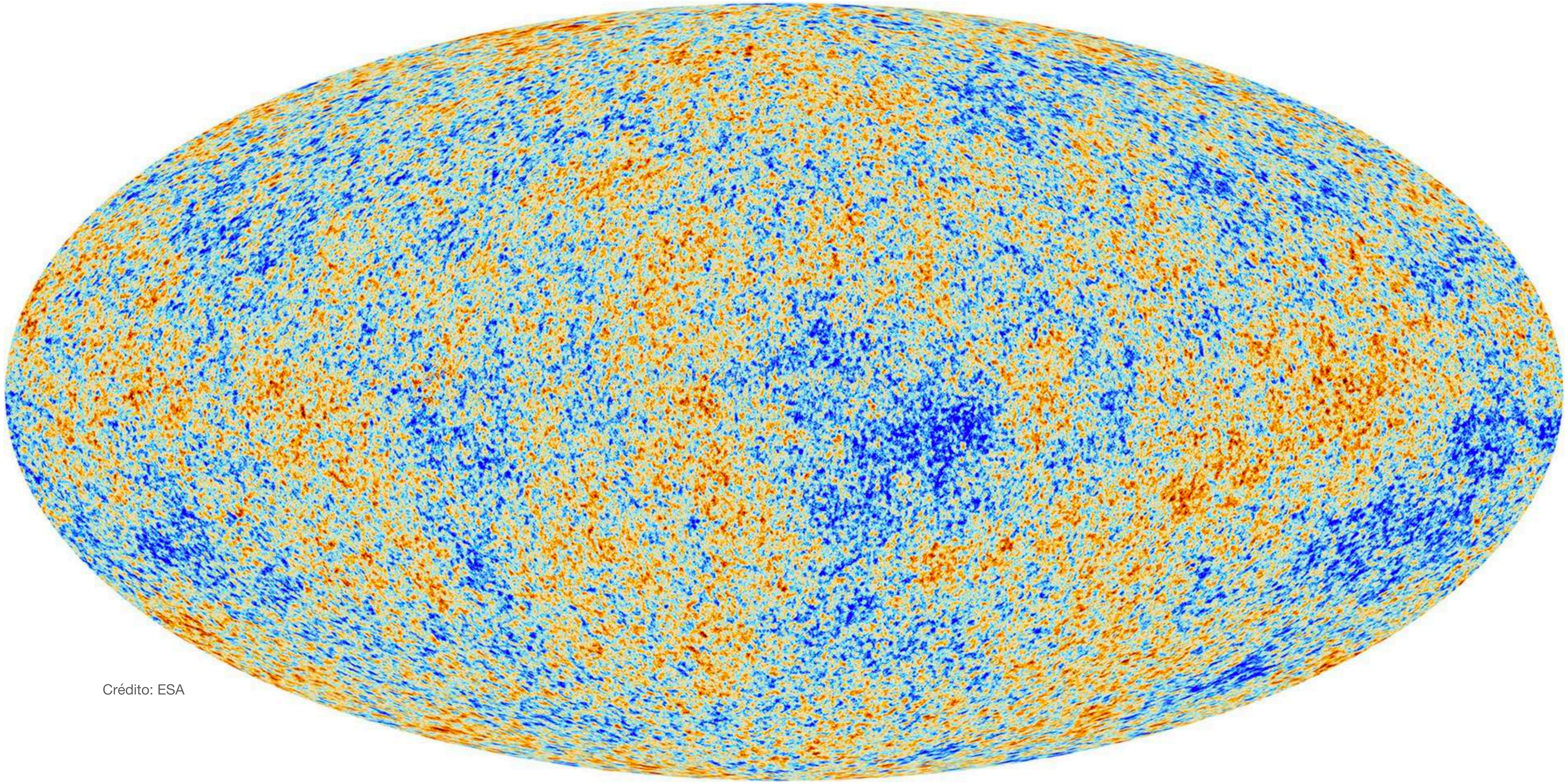
Cosmic **M**icrowave **B**ackground *(CMB)*



Crédito: D. Baumann

Given the expansion of the universe, we observe these photons in microwave.

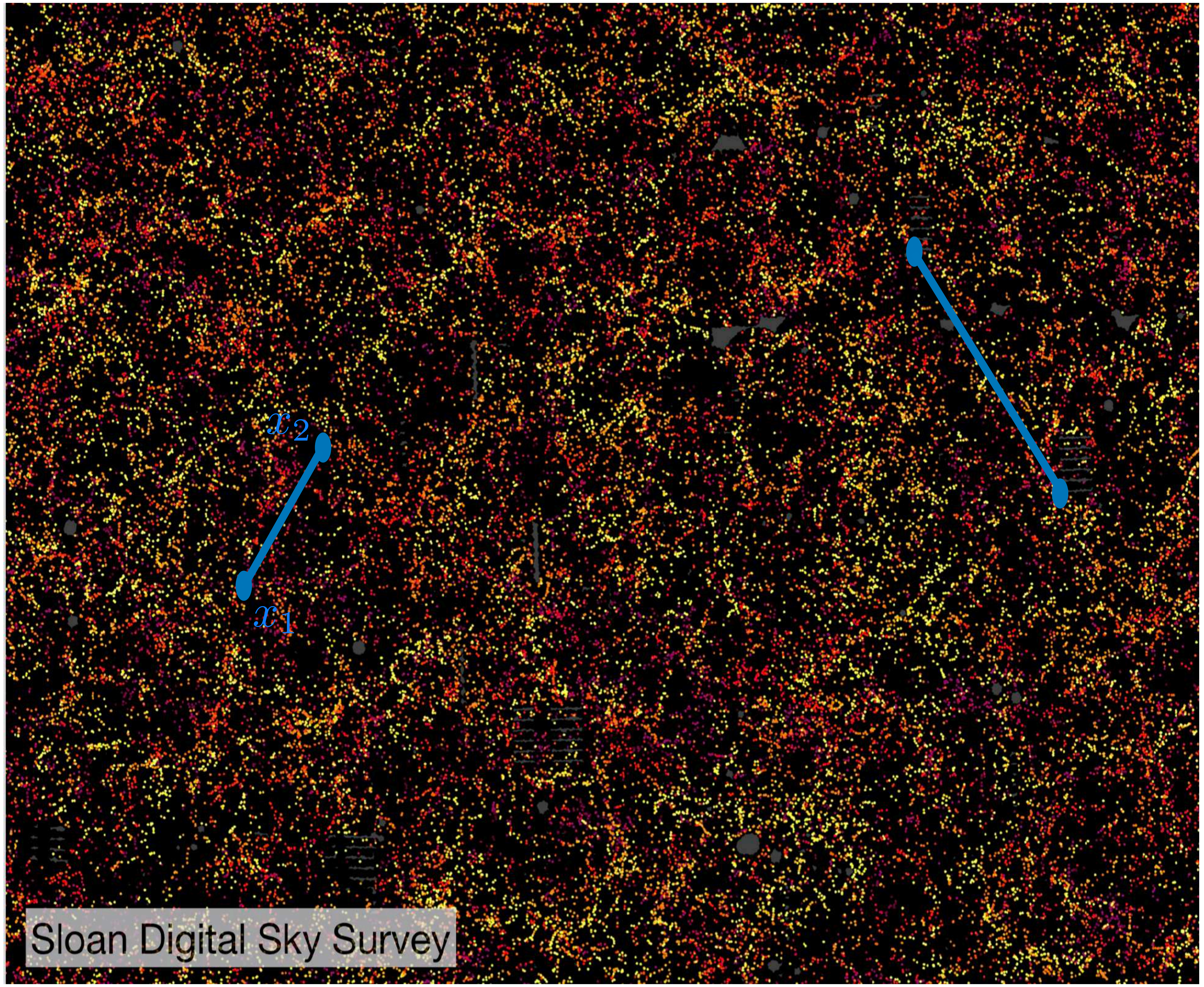
Cosmic **M**icrowave **B**ackground *(CMB)*



Crédito: ESA

Temperature 2.7 K. Small fluctuations - initial condition for the structures of our universe

How to measure structures



2 pontos correlation function:

$$\langle \delta(x_1)\delta(x_2) \rangle$$

If we decompose this into Fourier modes:

$$\delta(x) = \sum_k \delta_k \sin(kx + \phi_k)$$

$$k = 2\pi/\lambda$$

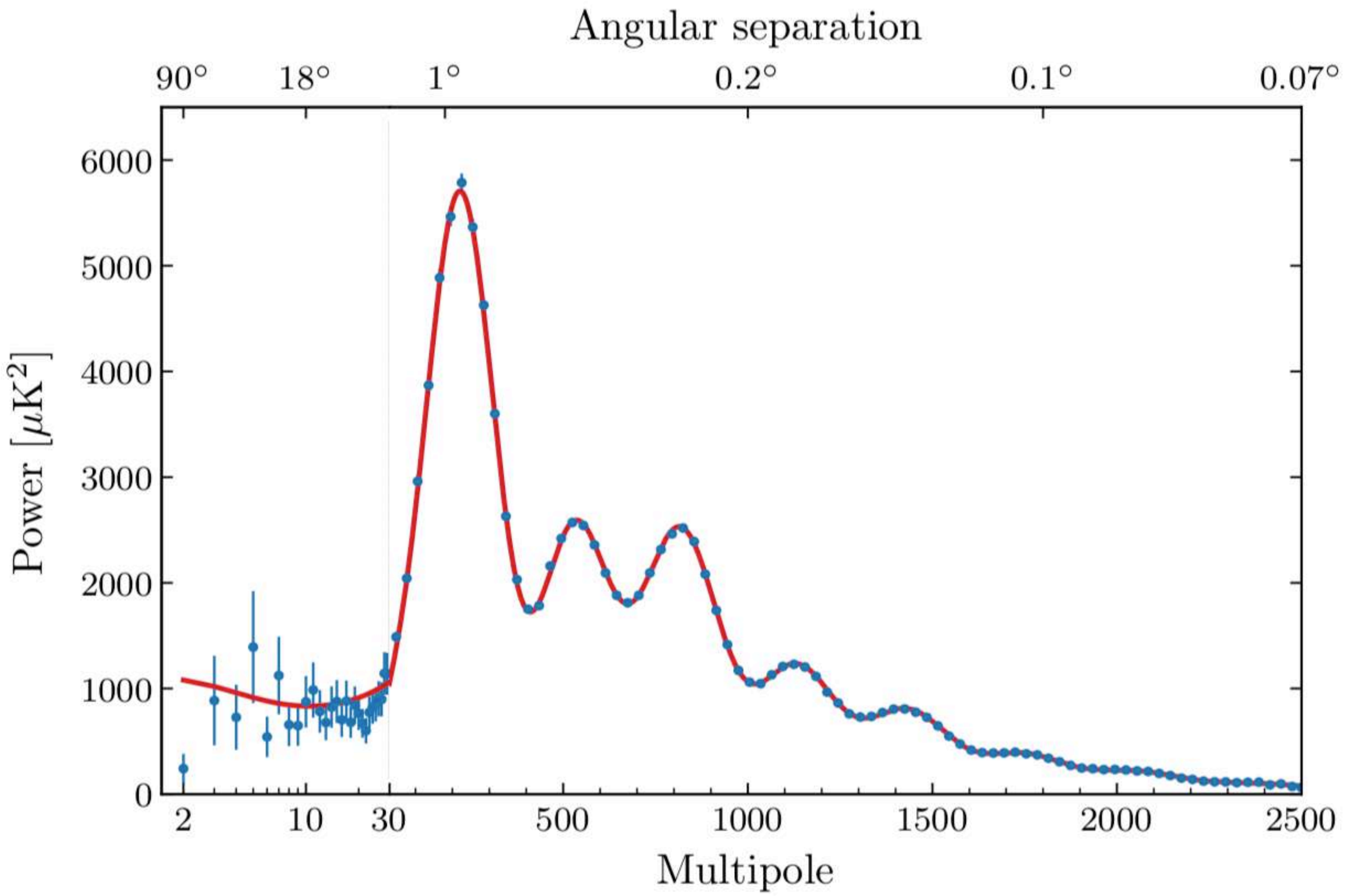
$$\Longrightarrow \quad \boxed{P(k) = |\delta_k|^2}$$

Power spectrum

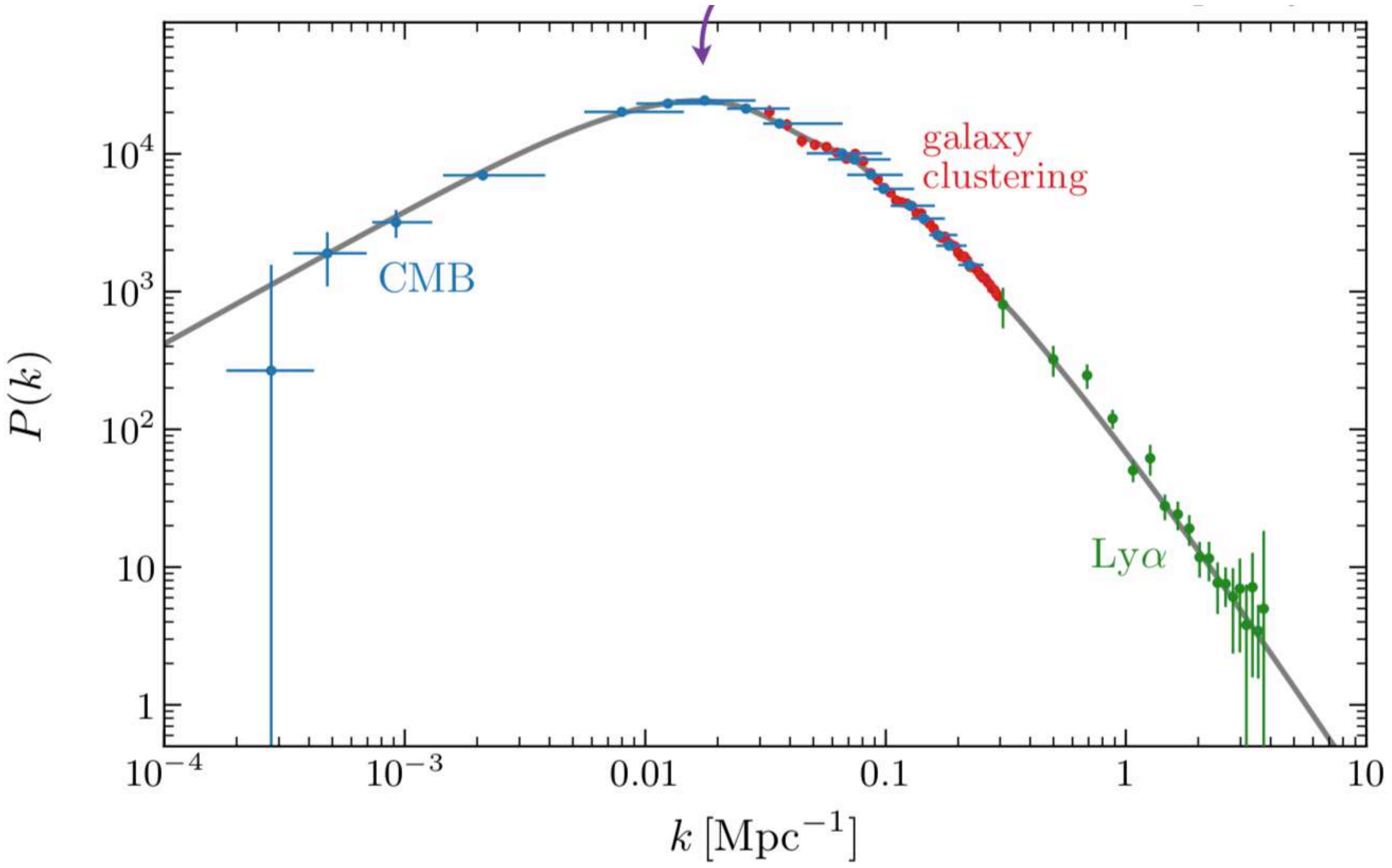
One of the main statistical object in cosmology!

Power spectrum

CMB



Large scale structure



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$$\{\Omega_b, \Omega_m, \Omega_\Lambda, n_s, A_s, \tau\}$$

Using CMB and other LSS probes, can constraint the parameters with incredible precision.

Planck 2018

$\Omega_b = 0.0484 \pm 0.0003 \longrightarrow$ Amount of ordinary/visible matter

$\Omega_m = 0.308 \pm 0.012 \longrightarrow$ Amount of dark matter

$\Omega_\Lambda = 0.692 \pm 0.012 \longrightarrow$ Amount of dark energy

$\mathrm{n}_s = 0.9626 \pm 0.0057 \longrightarrow$ Scale dependence of the initial fluctuations

$10^9 A_s = 2.092 \pm 0.034 \longrightarrow$ Amplitude of the initial fluctuations

$\tau = 0.0522 \pm 0.0080 \longrightarrow$ Optical depth

How opaque the universe is to photons that travel in it

Questions?

Exercise

Cosmological parameters

If you want to learn how to compute the power spectra, you can try this notebook.

$$\{\Omega_b, \Omega_m, \Omega_\Lambda, n_s, A_s, \tau\}$$

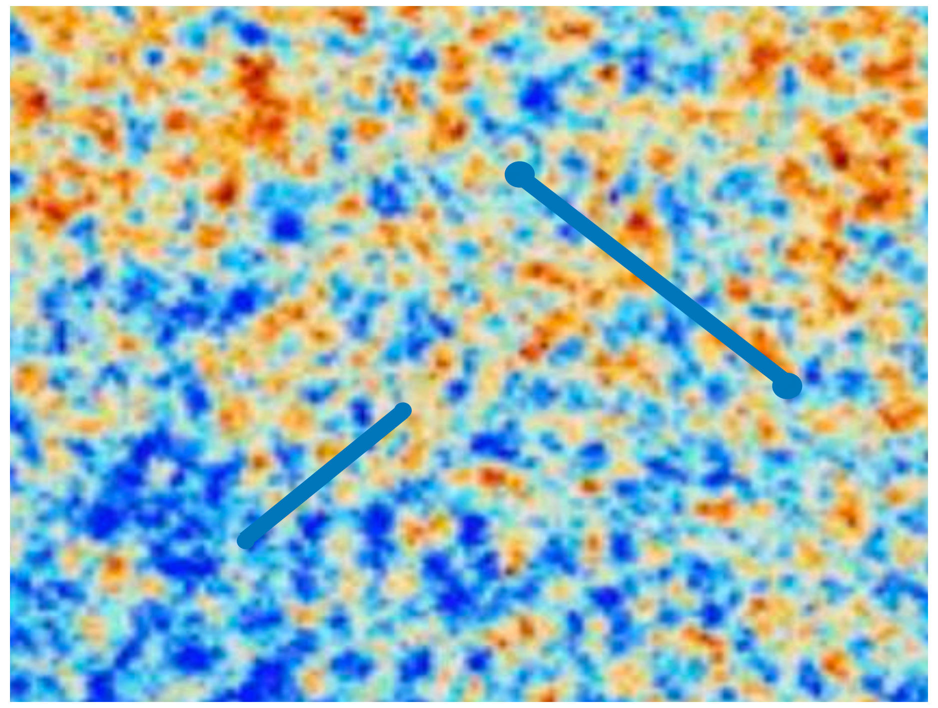
In this notebook, we teach you how to use the codes that calculate the theoretical predictions of the LCDM model. You can see how the cosmological parameters alter these predictions and how this can be compared to data to find the amazing fit we have for those parameters:

Follow the instructions in this notebook:

https://colab.research.google.com/drive/1xVgJ4E4GSn9AqE_IkrkpM7gKrXlRVs31?usp=sharing

Spectrum of the initial perturbations

The initial fluctuations created in inflation, because of the inflationary dynamics, lead to a almost scale invariant spectrum



$$P(k) = A_S \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$n_s \longrightarrow$ Scale-dependency of the initial fluctuations
$A_s \longrightarrow$ Amplitude of the initial fluctuations

Predictions agree with what is measured in the CMB!

$\Omega_b = 0.0484 \pm 0.0003 \longrightarrow$ Amount of visible/standard matter
$\Omega_m = 0.308 \pm 0.012 \longrightarrow$ Amount of dark matter
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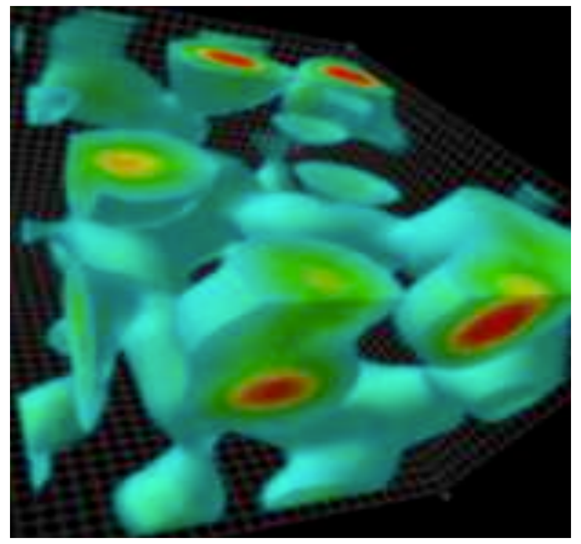
Where everything we see comes from?

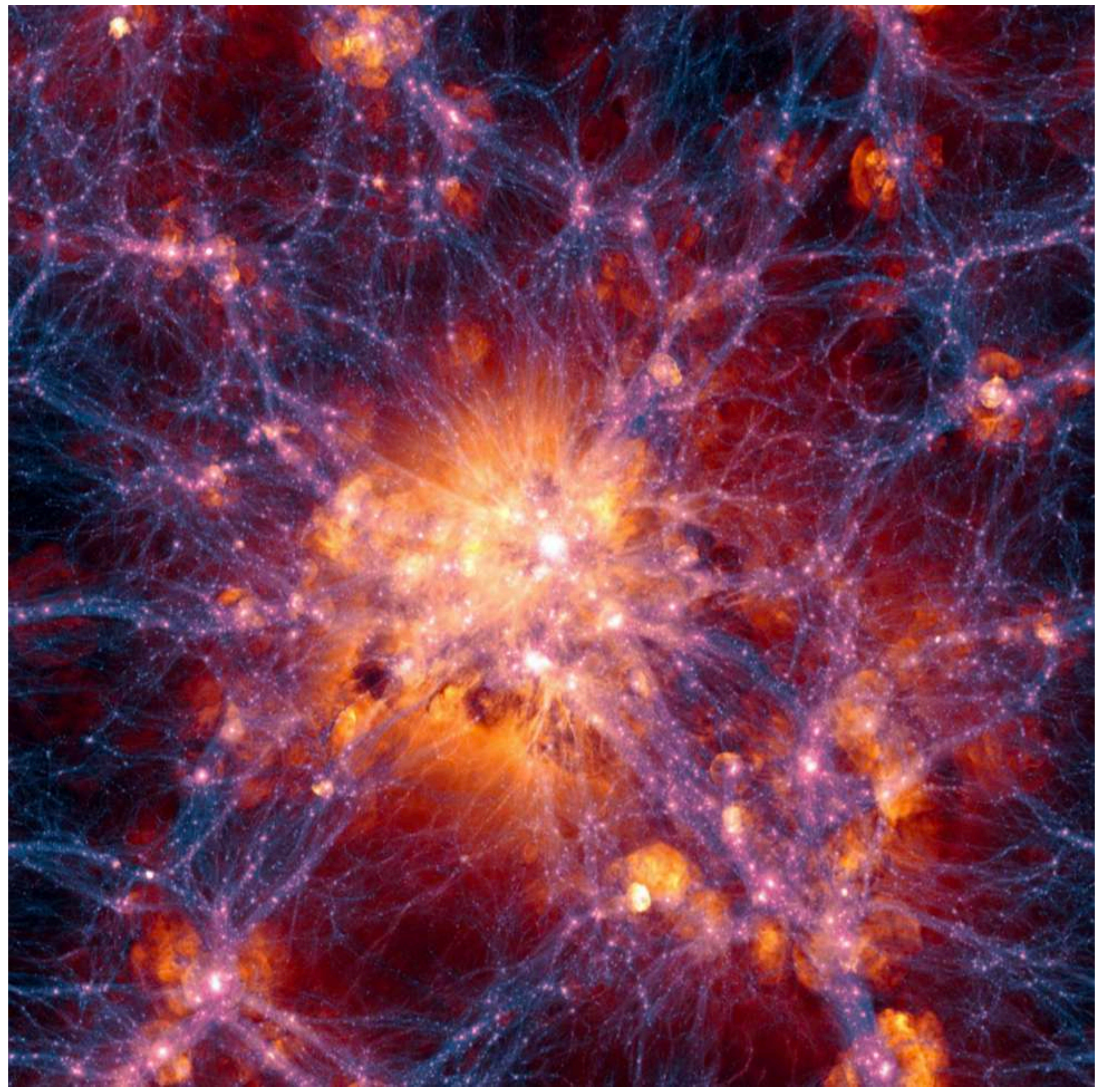
The answer comes from the interesting connection between the really small and really large…

?What is the origin of the initial density fluctuations?



Initial conditions
Initial perturbations

10^{-30} m

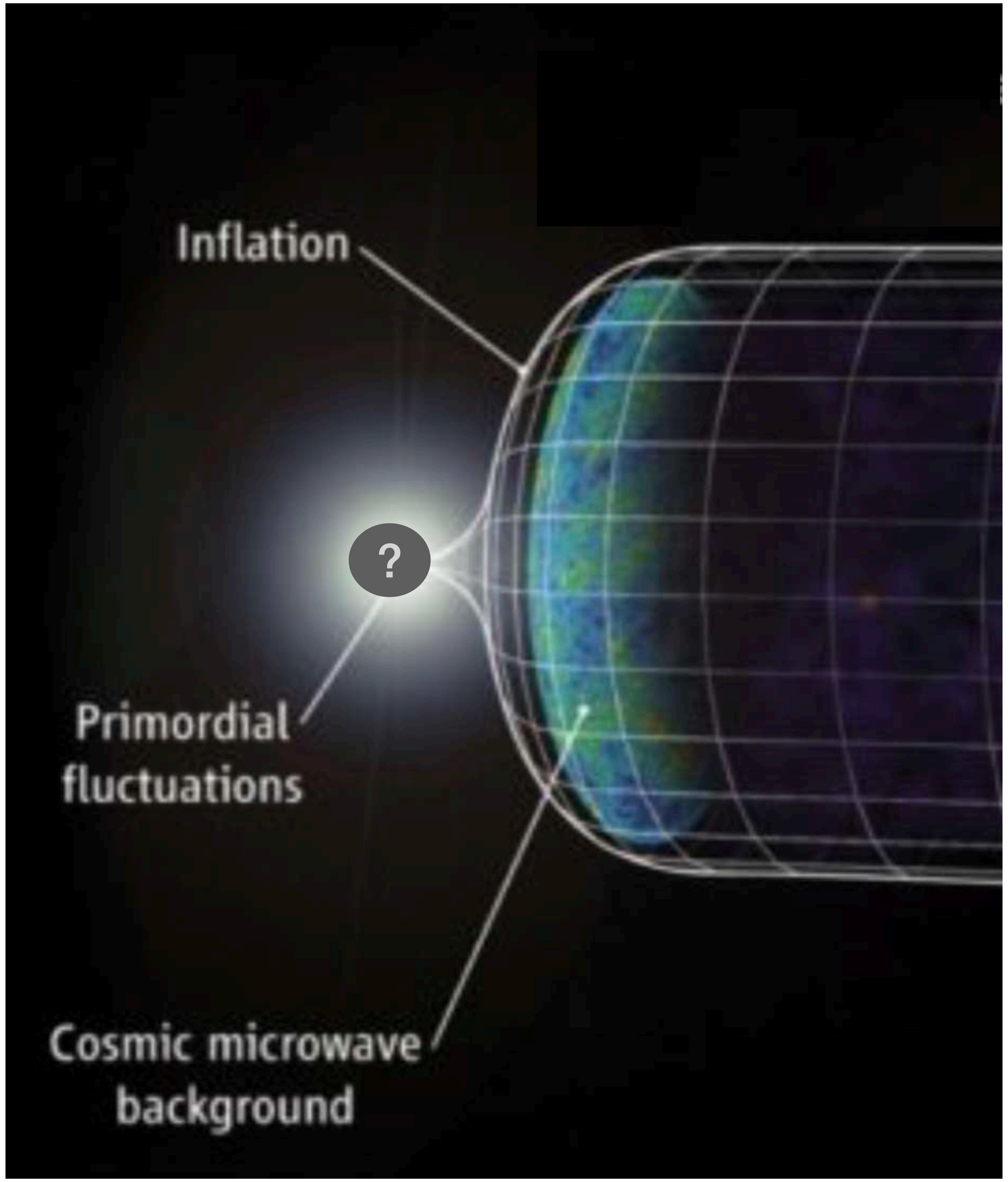


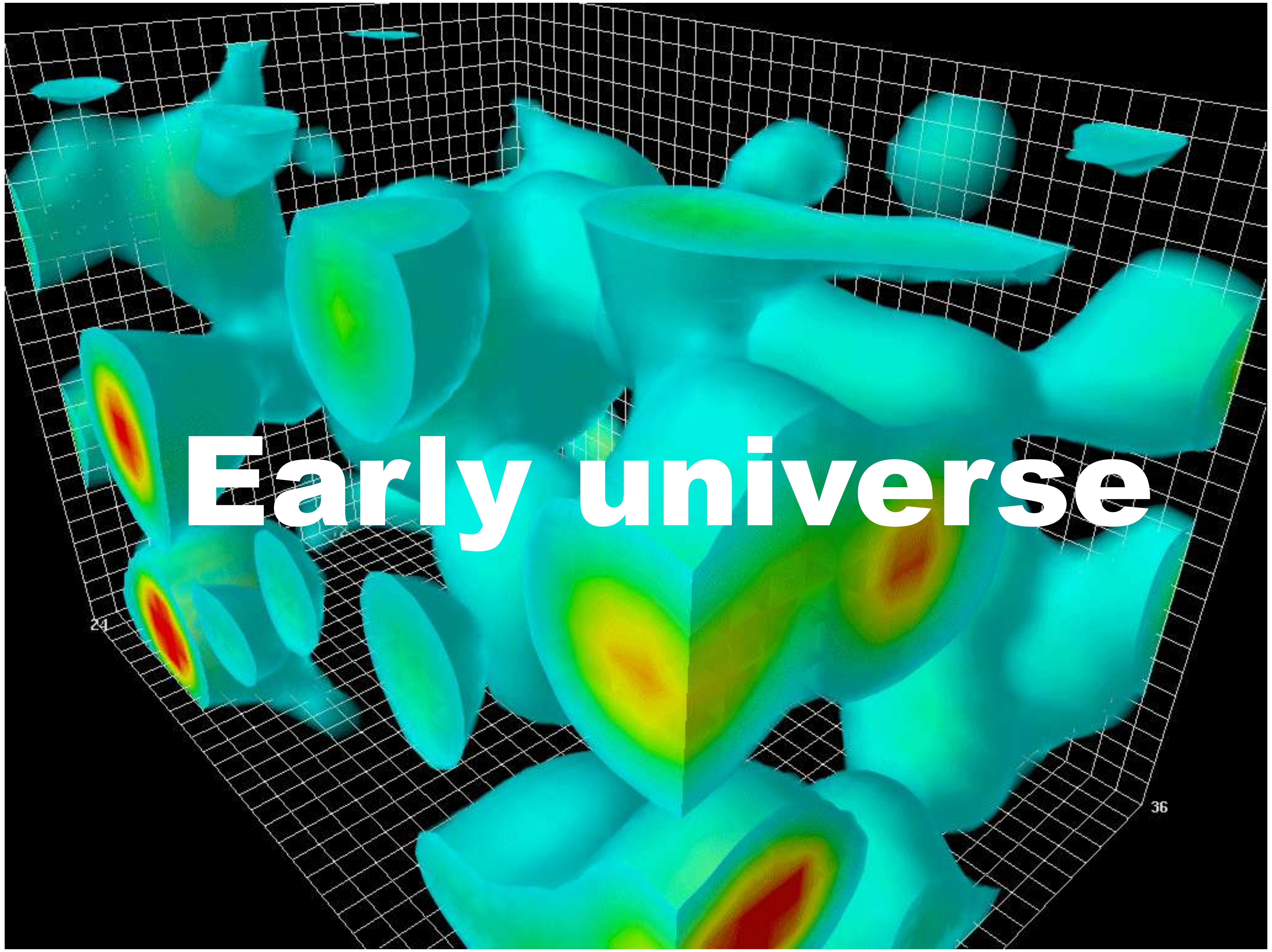
Structures of the universe

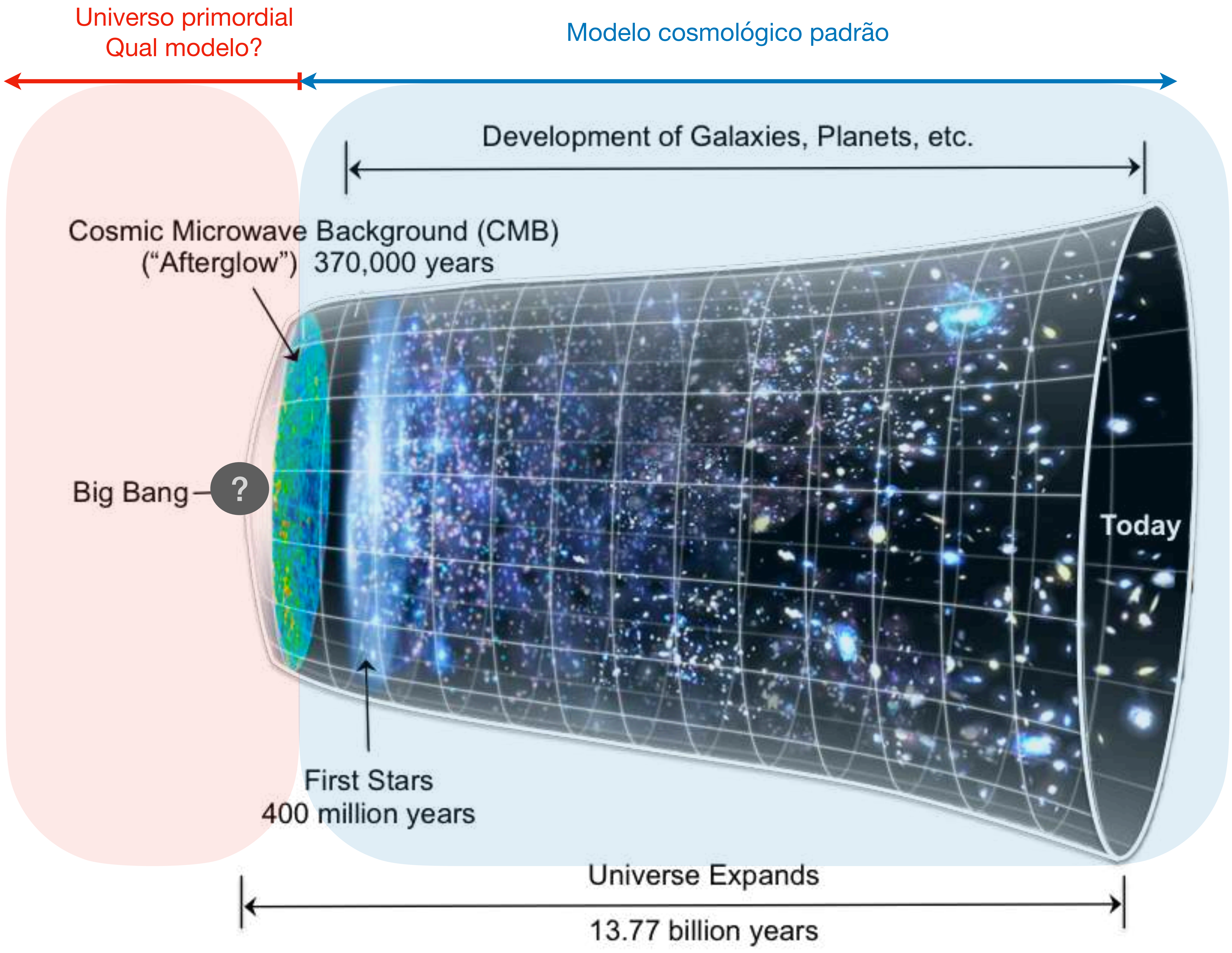
10^{25} m

Crédito: D. Baumann

This is going to depend on how was the evolution of the early universe…




Early universe
24
36



Universo primordial
Qual modelo?
Modelo cosmológico padrão
Development of Galaxies, Planets, etc.
Cosmic Microwave Background (CMB)
("Afterglow") 370,000 years
Big Bang
?
Today
First Stars
400 million years
Universe Expands
13.77 billion years

BUT, this picture (LCDM) has problems…

Problems in the standard model

- Horizon problem
- Problem of the origin of structures da origem das estruturas
- Flatness problem
- Initial singularity
- Dark matter and dark energy
- …

The ΛCDM model is not perfect and presents some problems - needs to be extended (or changed?)

Problems of the standard cosmological model

- Horizon problem
- Problem of the origin of structures
- Flatness problem
- Problem of the magnetic monopoles
- Initial singularity
- DM and DE

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Horizon problem

Crédito: D. Baumann

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

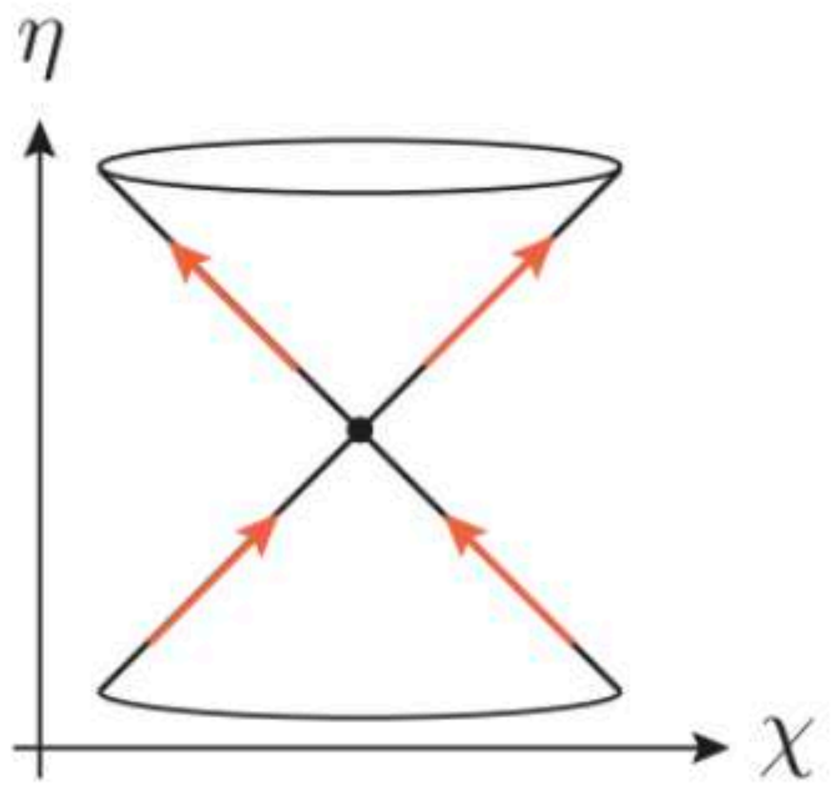
Because of isotropy, we can focus on purely radial geodesics ($d\theta = d\phi = 0$):

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2\right].$$

Photons travel on null geodesics,

$$ds^2 = 0 \quad \Rightarrow \quad \Delta\chi = \pm\Delta\eta$$

(straight lines)



Conformal time $d\eta = dt/a(t)$

outgoing photons and the minus sign to incoming photons

* This shows the main benefit of working with conformal time: light rays correspond to straightlines at 45 degree angles in the-χ, ηcoordinates. If instead we had used physical timet, then the lightcones for curved spacetimes would be curved

Crédito: D. Baumann

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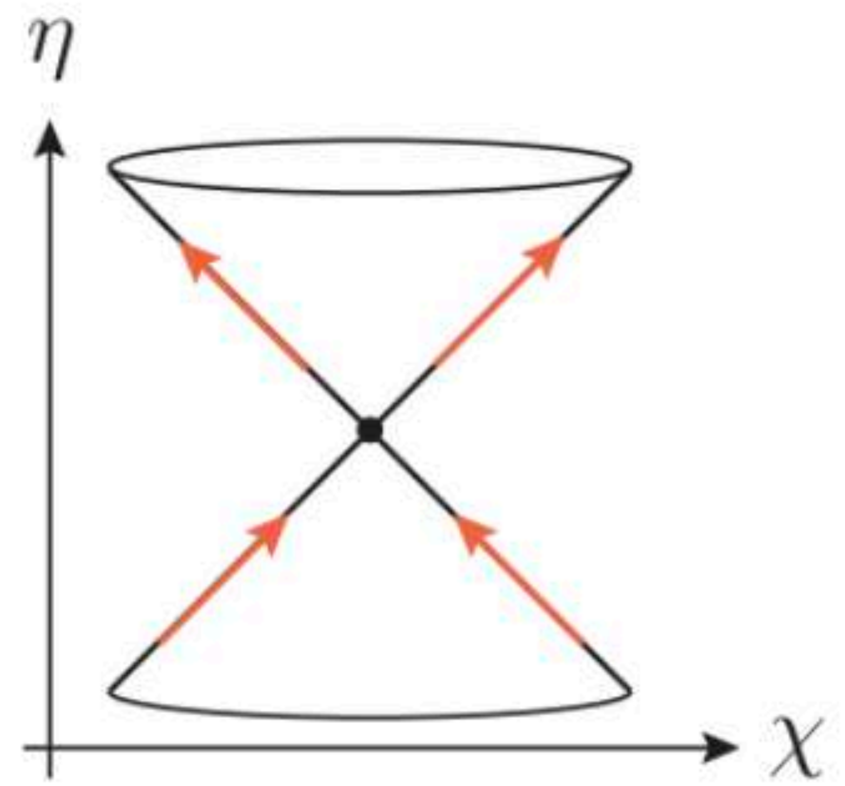


Conformal time $d\eta = dt/a(t)$

outgoing photons and the minus sign to incoming photons

This tells us that: maximum distance light can travel between 2 times is $\Delta\eta = \eta_2 - \eta_1$ $(c = 1)$

Crédito: D. Baumann

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

The maximum distance a photon (and hence any particle) can travel between an initial time t_i and a later time t is

$$\Delta r = \Delta\eta = \eta_2 - \eta_1 = \int_{t_i}^{t} \frac{dt'}{a(t')} \qquad (c = 1)$$

The maximum distance is equal to the amount of conformal time elapsed during the interval Δt

Conformal time $d\eta = dt/a(t)$

The initial time is often taken to be the 'origin of the universe', $t_i = 0$ defined formally by the initial singularity $a_i = a(t_i) = 0$

$$\chi_p = \Delta r_{max} = \int_0^t \frac{dt'}{a(t')} = \eta(t) - \eta(0)$$

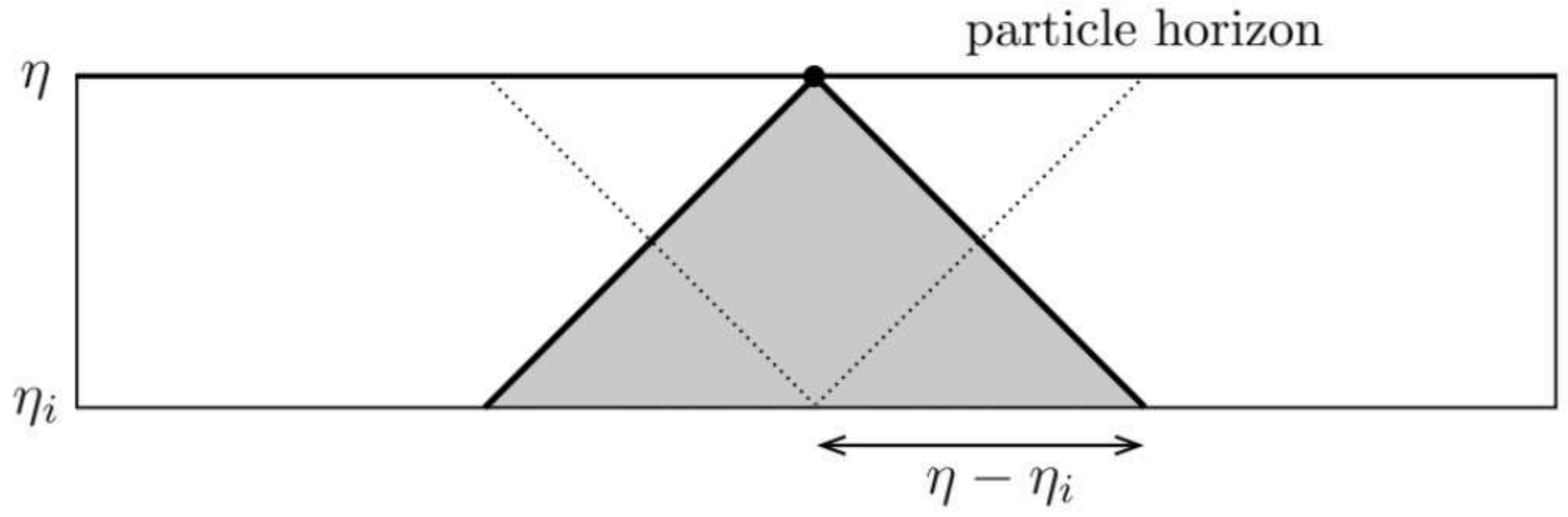
This is the comoving particle horizon

Crédito: D. Baumann

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big Bang



Rewriting in a special way:

(comoving) Hubble radius

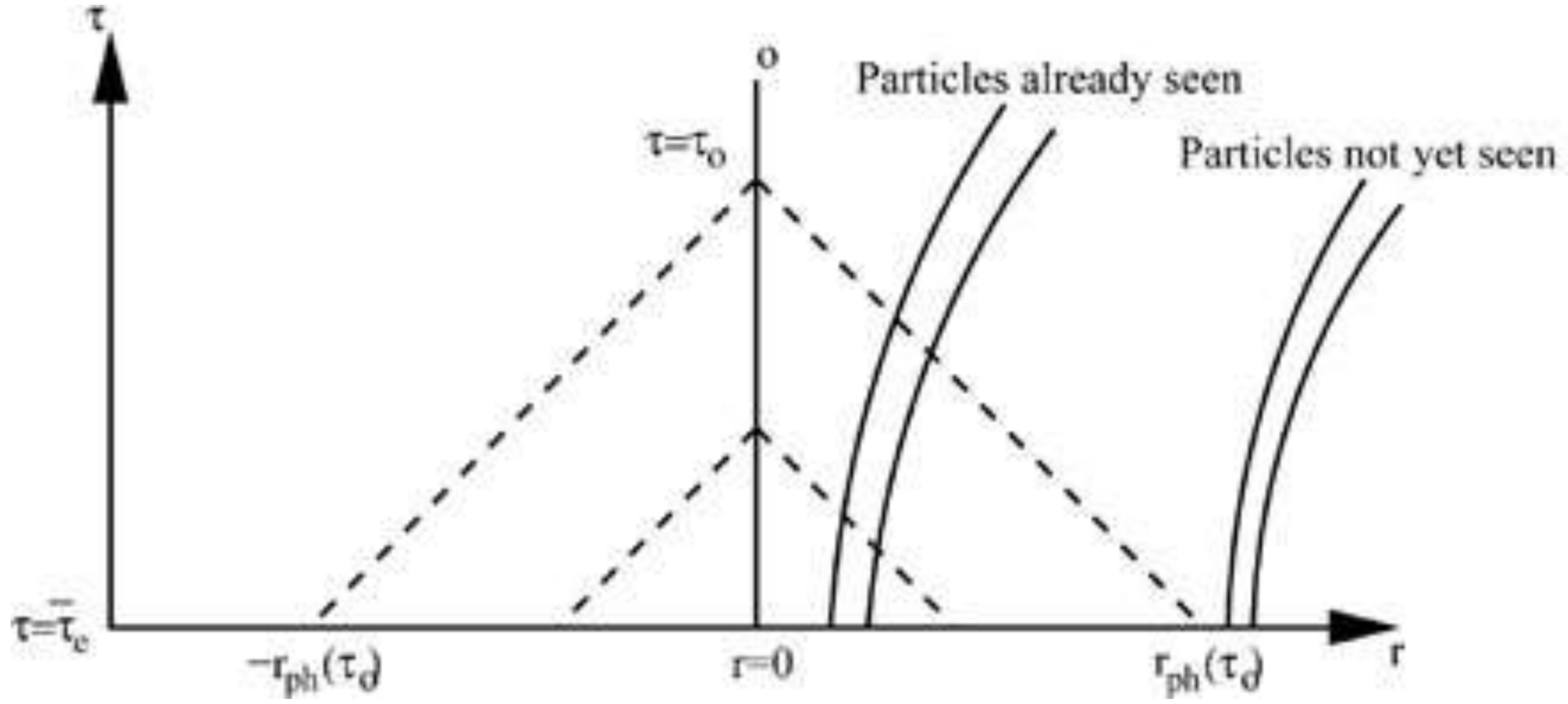
$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Conformal time $d\eta = dt/a(t)$

If the Big Bang started at t=0 the greatest comoving distance from which an observer at time t will be able to receive signals travelling at the speed of light is given by χ_p - comoving particle horizon

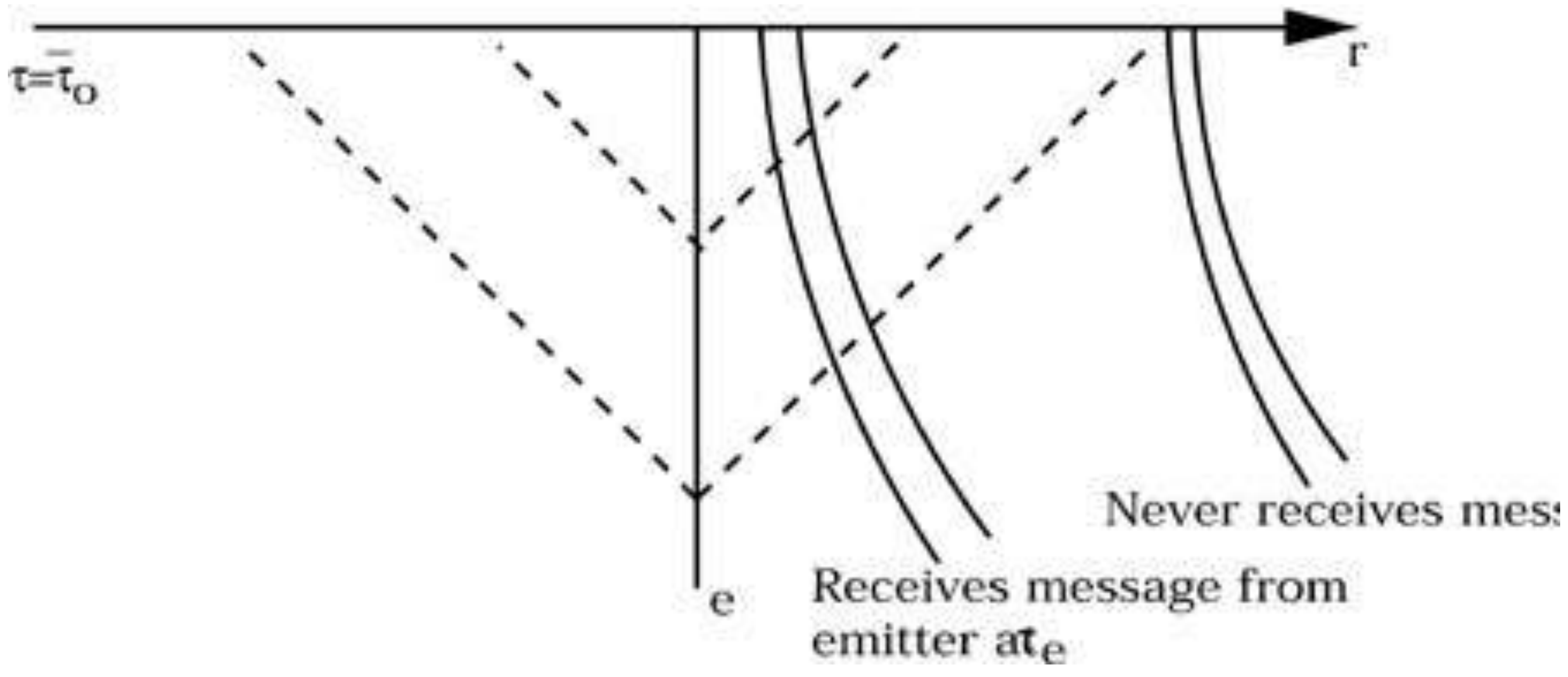
Horizons in cosmology

Particle horizon



Particle horizons arise when the past light cone of an observer O terminates at a finite conformal time. Then there will be worldlines of other particles which do not intersect the past of O, meaning that they were never in causal contact.

Event horizon

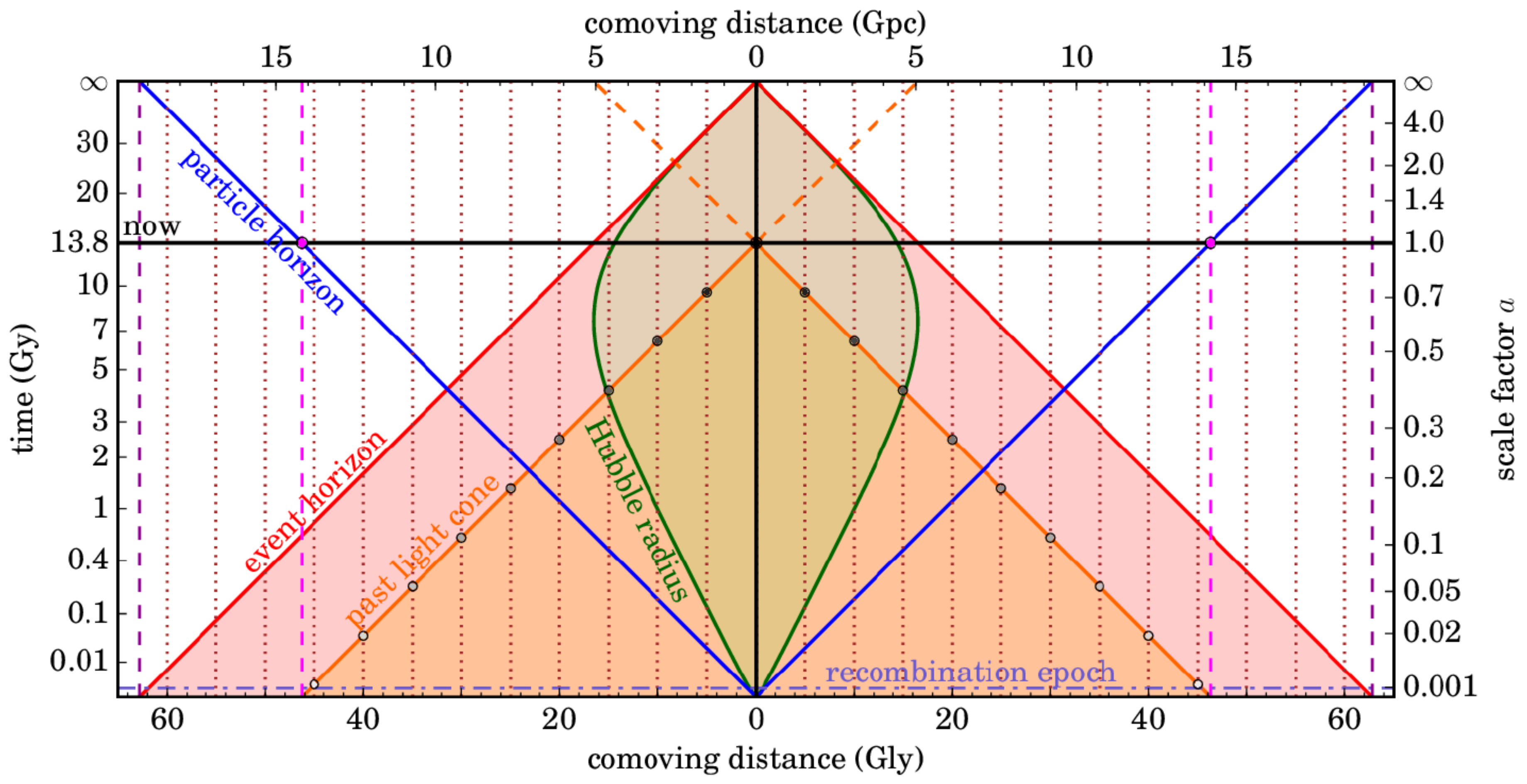


Event horizons arise when the future light cone of an observer o terminates at a finite conformal time. Then there will be worldlines of other particles which do not intersect the future of o, meaning that they cannot possibly influence each other.

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big Bang



(comoving)
Hubble radius

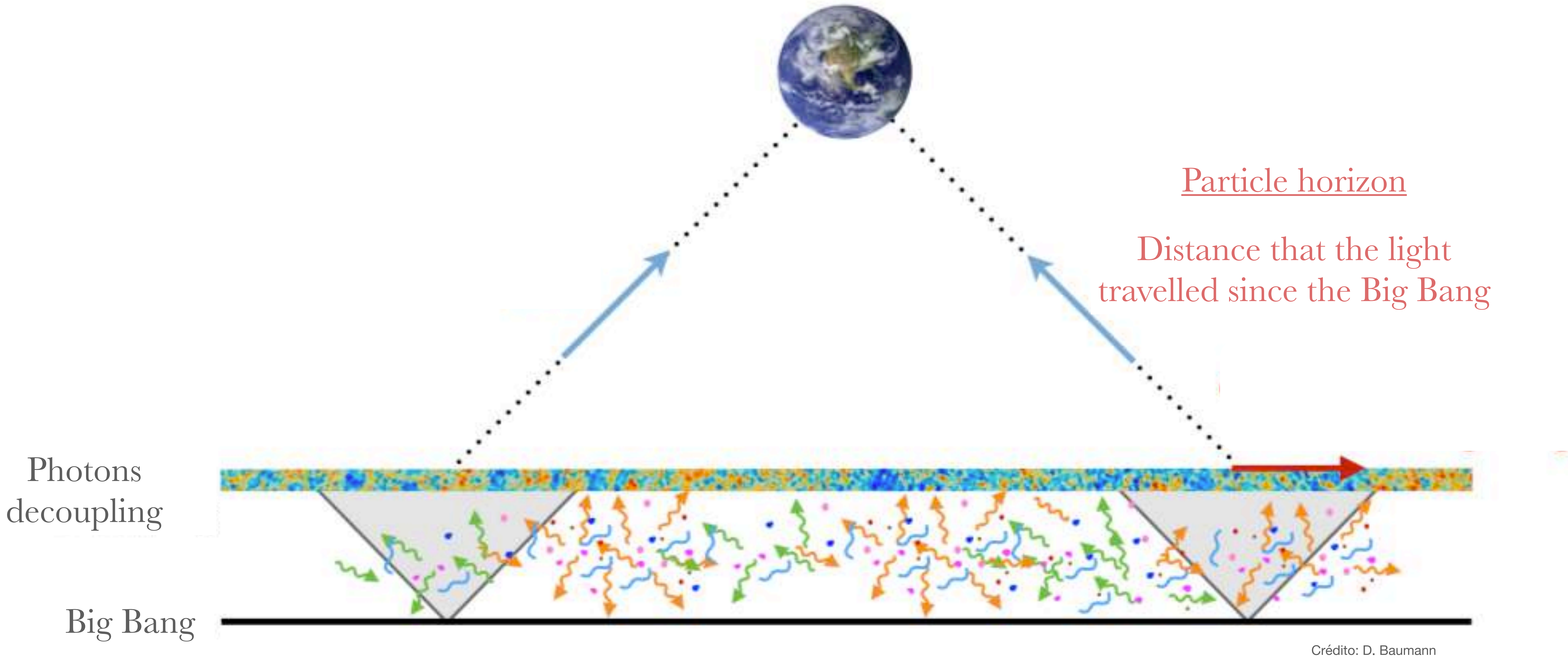
$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Conformal time $d\eta = dt/a(t)$

The size of the particle horizon at η is the intersection of the past light cone of an observer O with the spacelike surface $\eta = \eta_i$

Horizons in cosmology

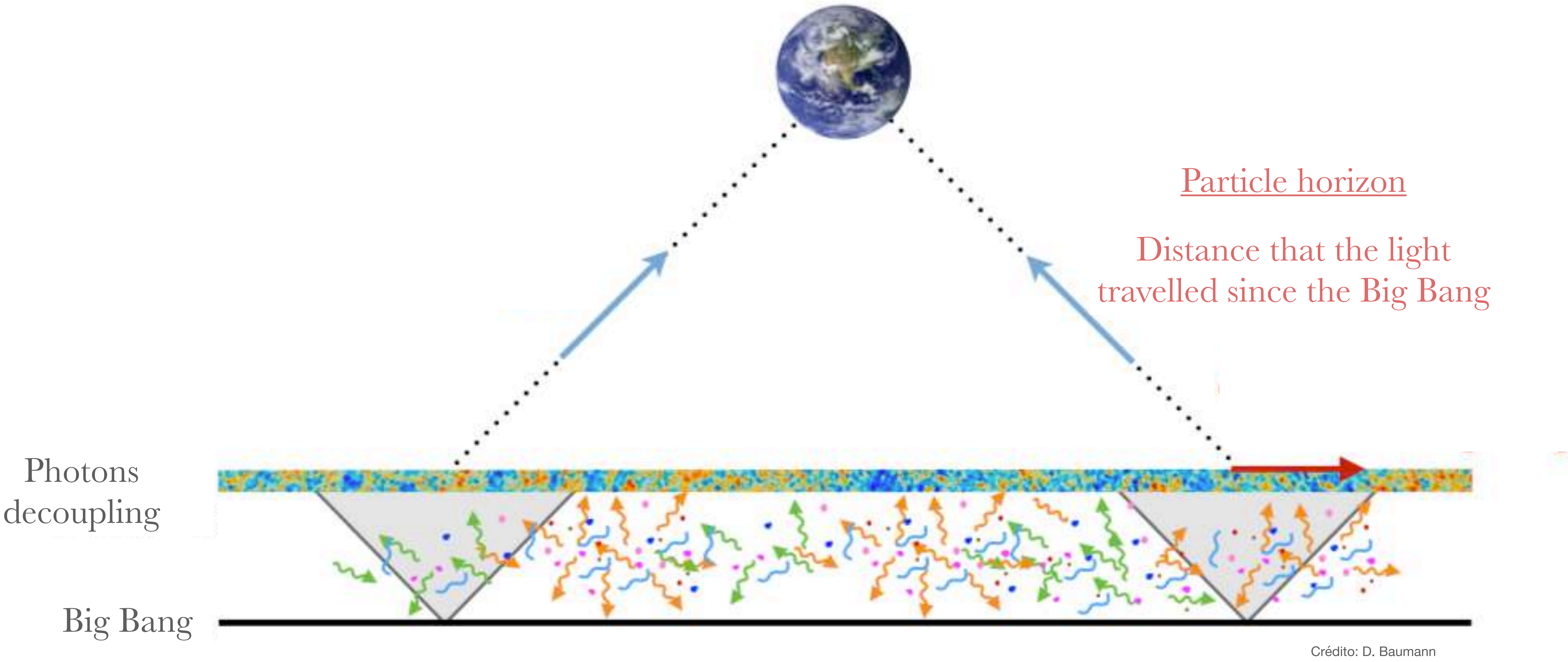
Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.



This limit of what can be observed is known as horizon.

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.



* Notice that the Big Bang singularity is a moment in time (not a point in space).
Figure: singularity described by an extended (possibly infinite) spacelike hypersurface

Horizons in cosmology

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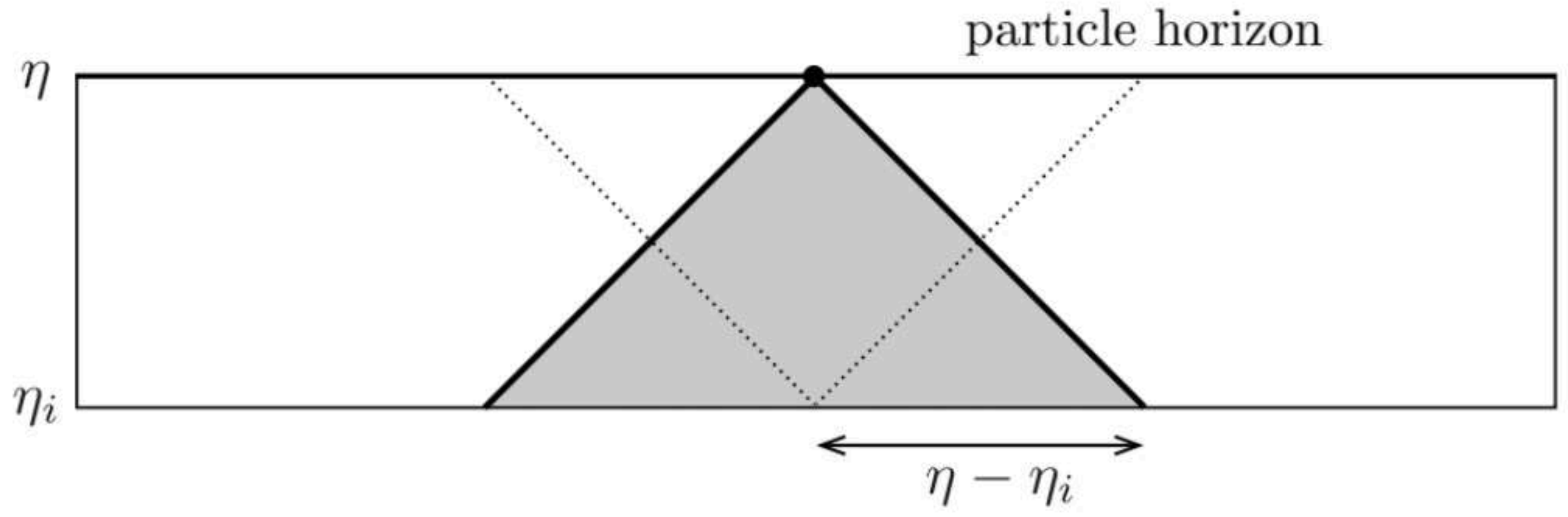
Particle horizon: distance that the light travelled since the Big Bang



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Universe dominated by a fluid with $w = p/\rho$:

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (><)0$$

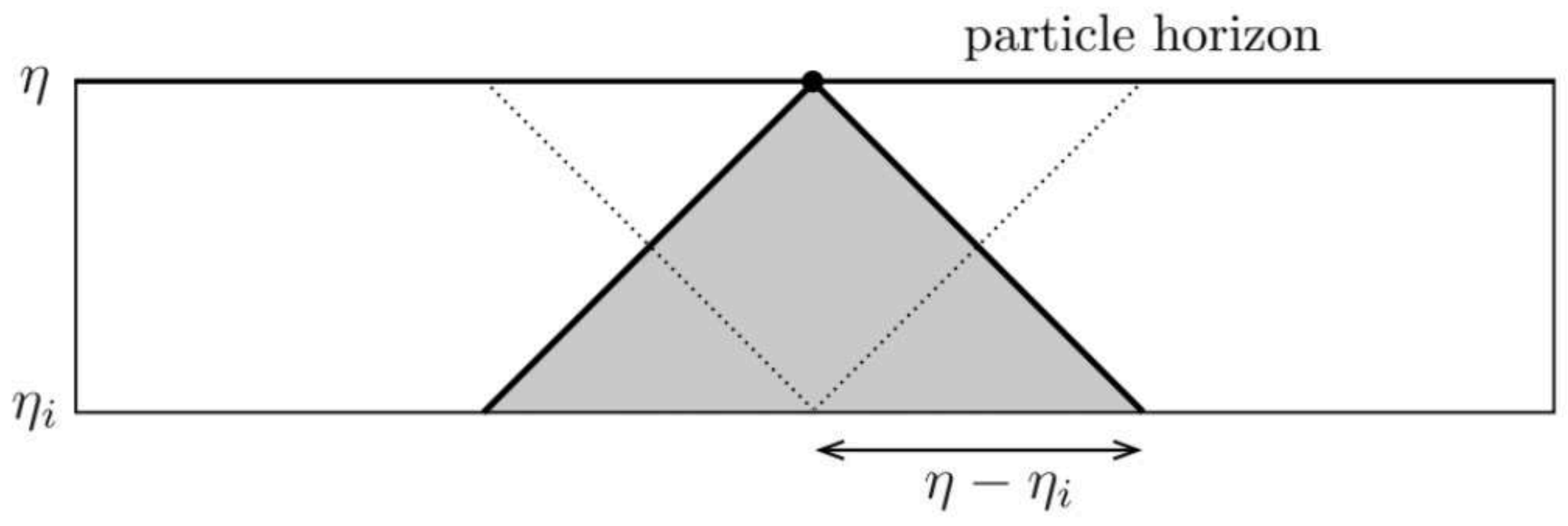
Strong energy condition (SEC) >0

Ordinary matter $(aH)^{-1} \uparrow$

$$\chi_p = \frac{2}{1+3w}(aH)^{-1}$$

Horizons in cosmology

Particle horizon: distance that the light travelled since the Big BangParticle horizon



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Ordinary matter $(aH)^{-1} \uparrow$

$$\chi_p = \frac{2}{1+3w}(aH)^{-1}$$

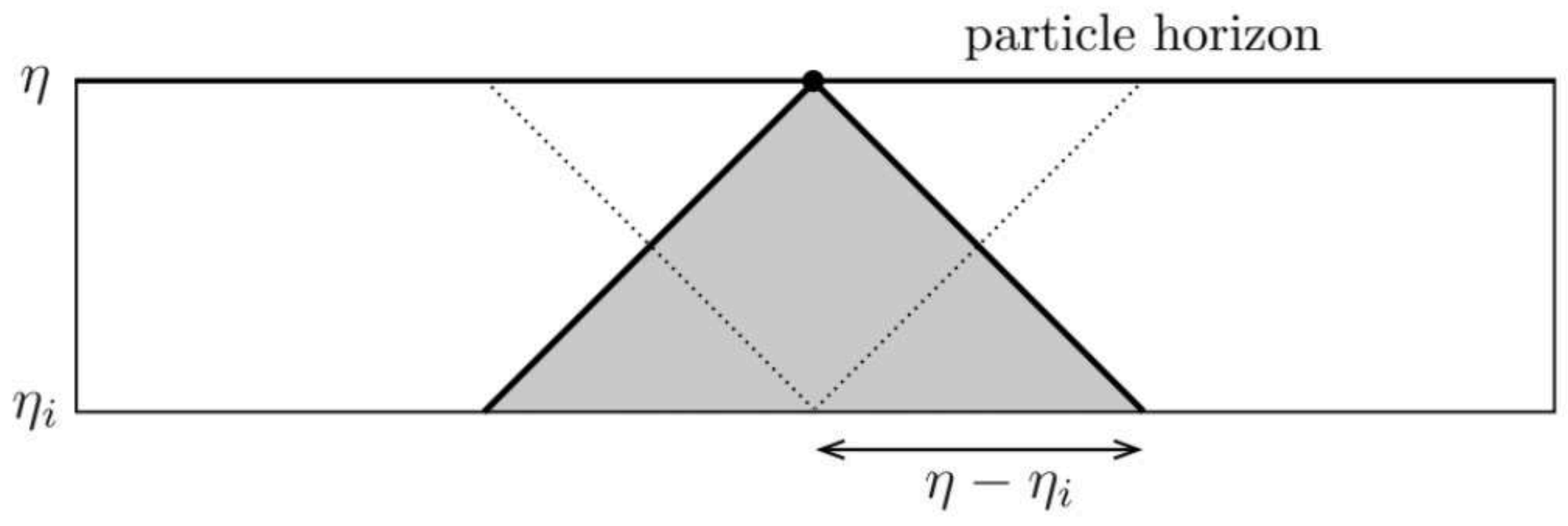
For this reason, $\chi_p \sim (aH)^{-1}$

usually people refer to the particle horizon and Hubble radius as “horizon”.

DON'T do that!

Horizons in cosmology

Particle horizon: distance that the light travelled since the Big BangParticle horizon



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$$\chi_p(\eta) = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Comoving particle horizon is finite!

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\eta_i \propto a_i^{\frac{1}{2}(1+3w)} = 0$$

Ordinary matter

For MD or RD, $w > 0$ or ($w > -1/3$)

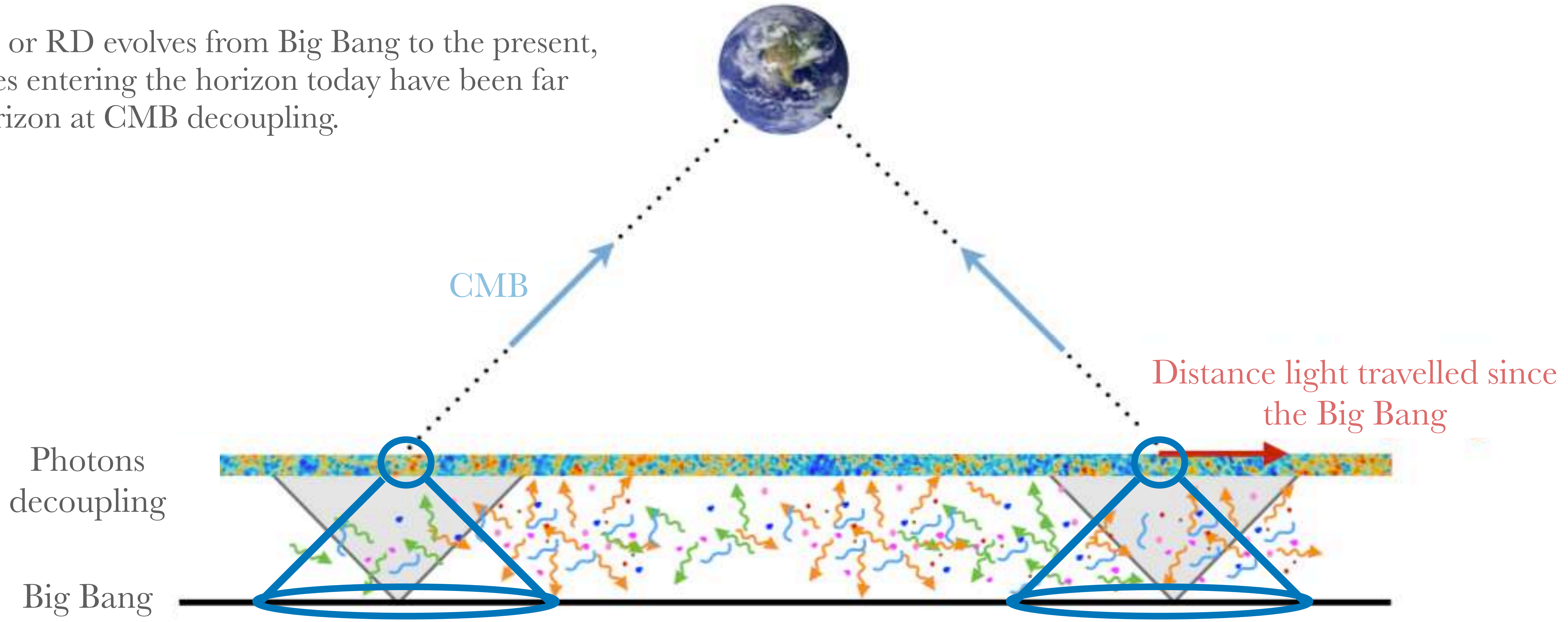
Comoving particle horizon monotonically grows with time

$$(aH)^{-1} \uparrow$$

Horizon problem

As we saw, the CMB presents the same temperature in every point of the observable universe, except from small deviations

Assuming MD or RD evolves from Big Bang to the present, comoving scales entering the horizon today have been far outside the horizon at CMB decoupling.



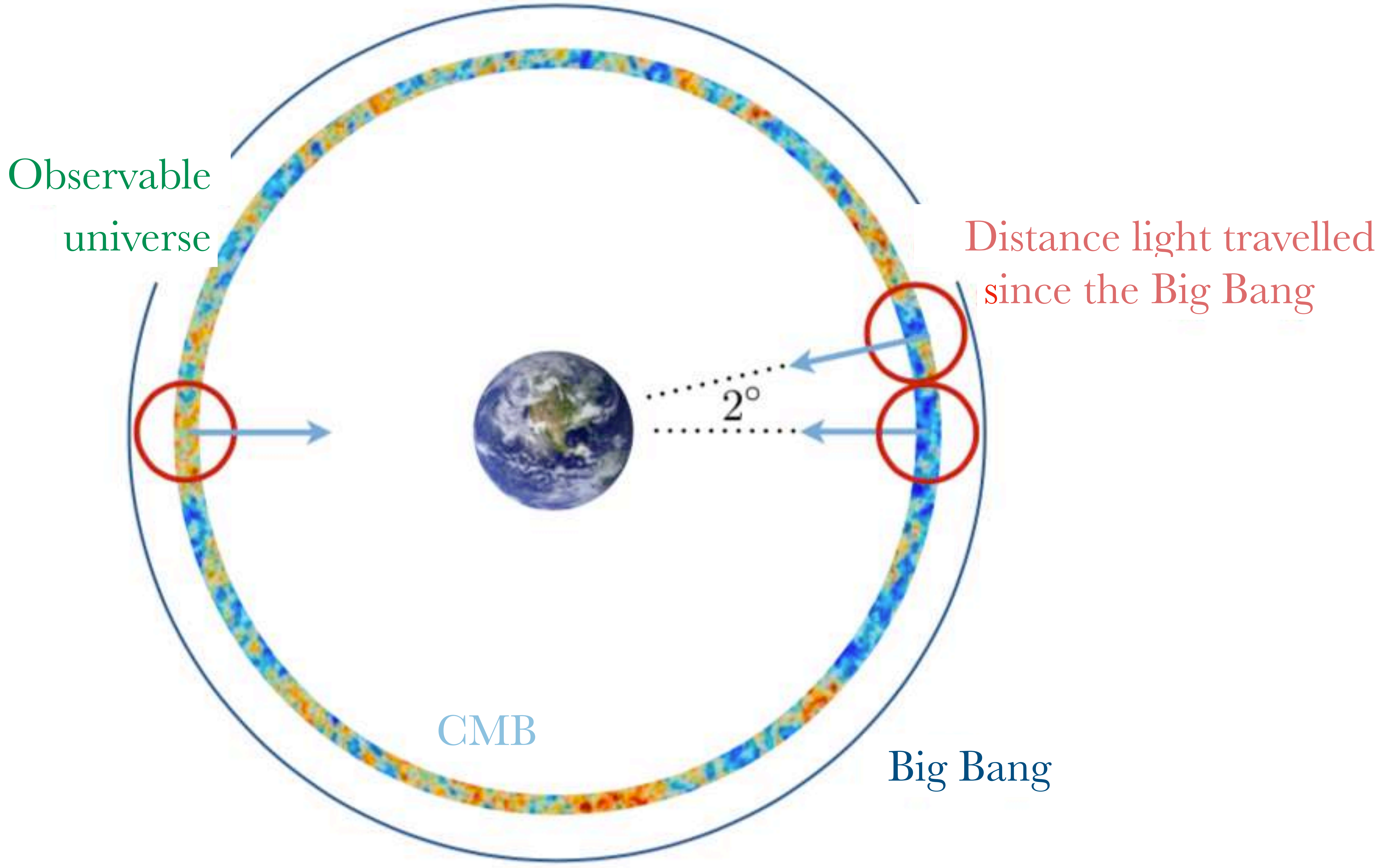
?

However, since there is a particle horizon today, HOW regions that are not in causal contact in the past can present the same characteristics?

Crédito: D. Baumann

Horizon problem

Assuming MD or RD evolves from Big Bang to the present, comoving scales entering the horizon today have been far outside the horizon at CMB decoupling.

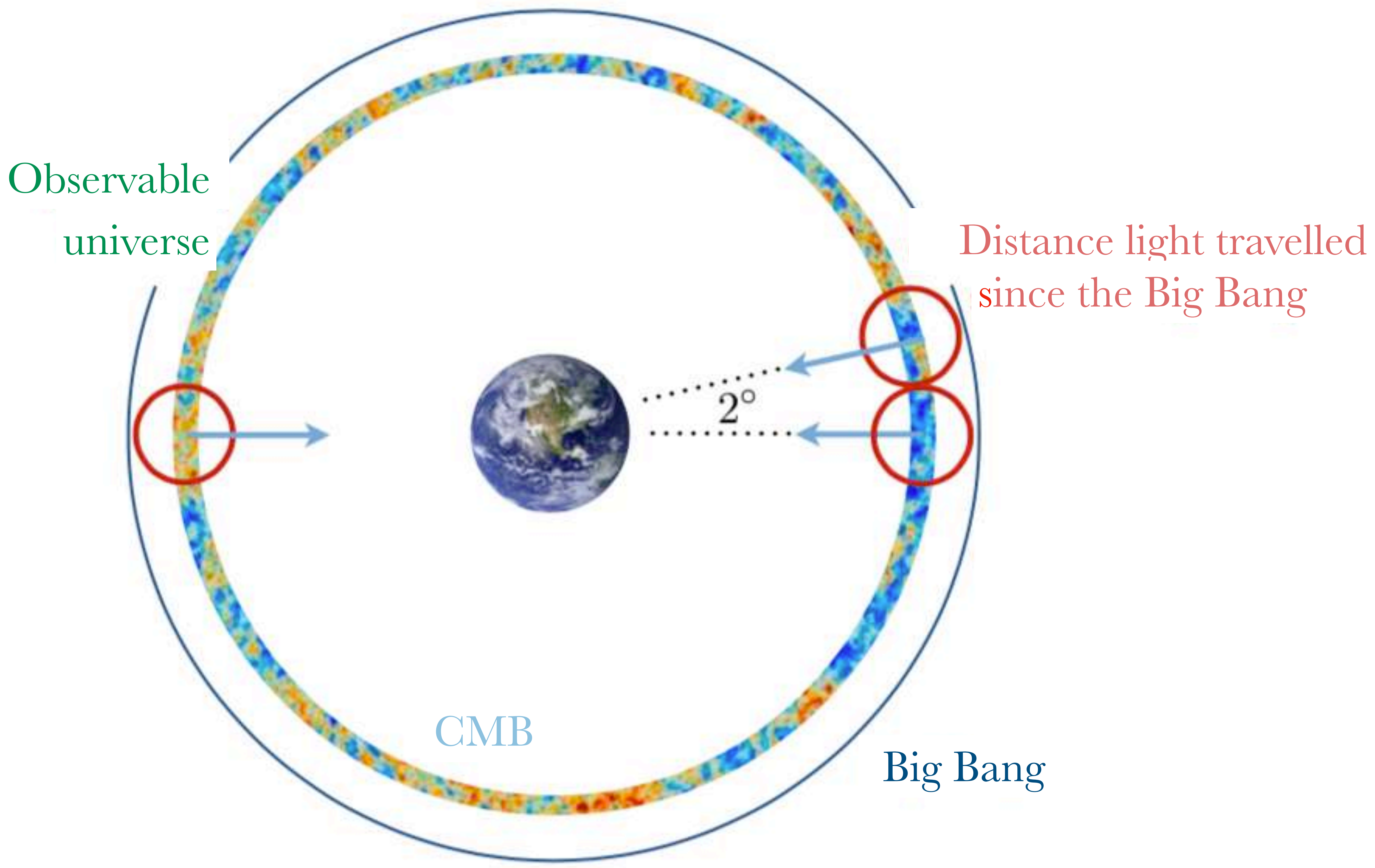


The CMB is made of $10^4 - 10^6$ causally disconnected regions, yet it is observed to be almost perfectly uniform!?

= horizon problem!

Crédito: D. Baumann

Horizon problem



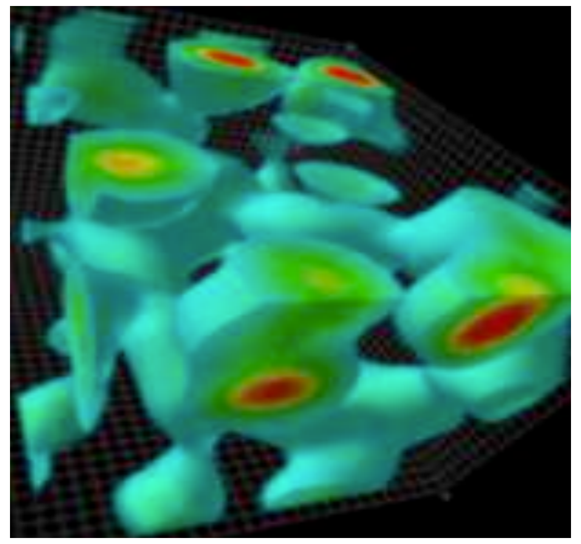
horizon problem

also known as homogeneity and isotropy problem

Crédito: D. Baumann

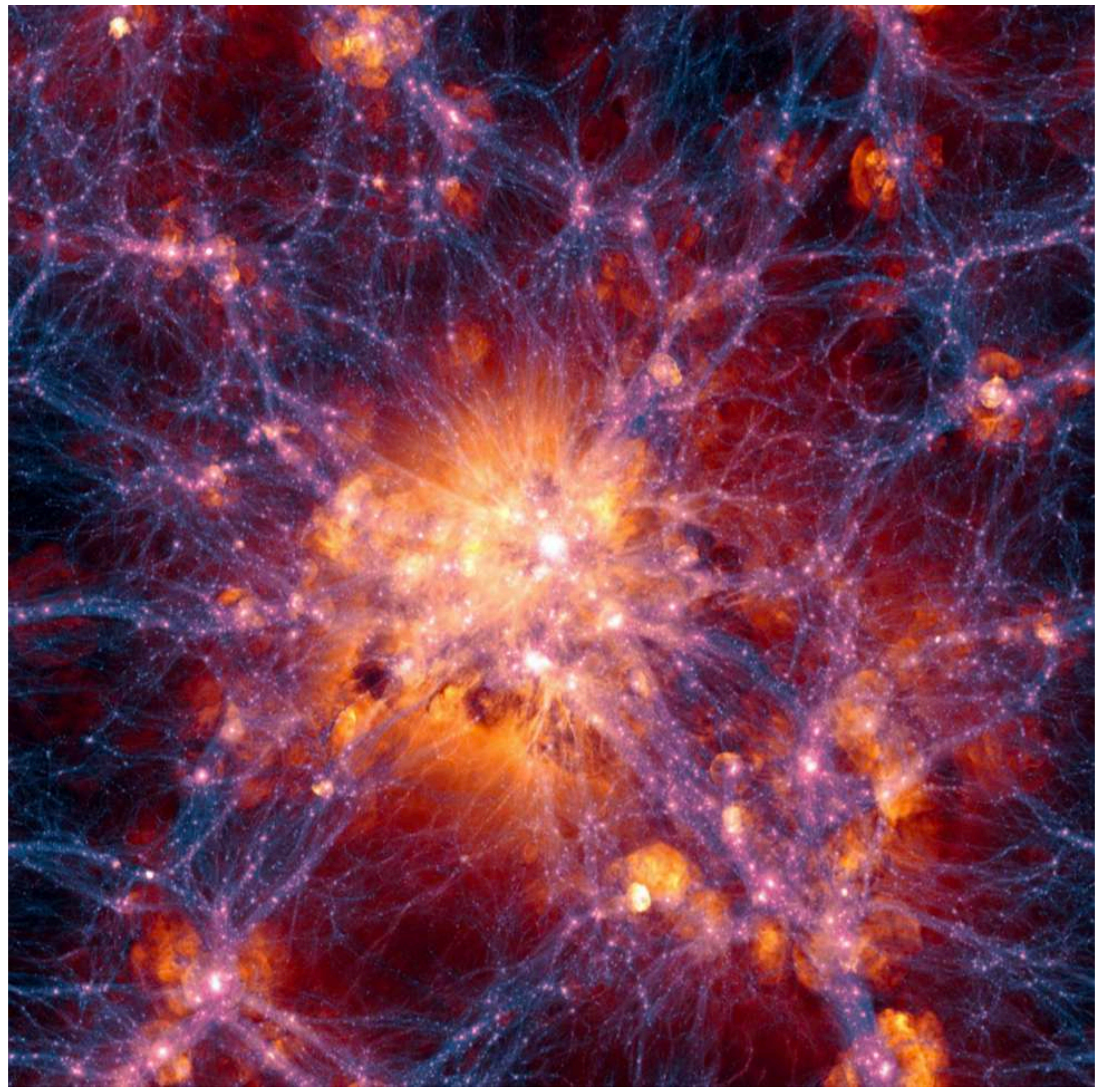
Questions?

Problem of the origin of structures



Initial conditions
Initial perturbations

10^{-30} m



Structures of the universe

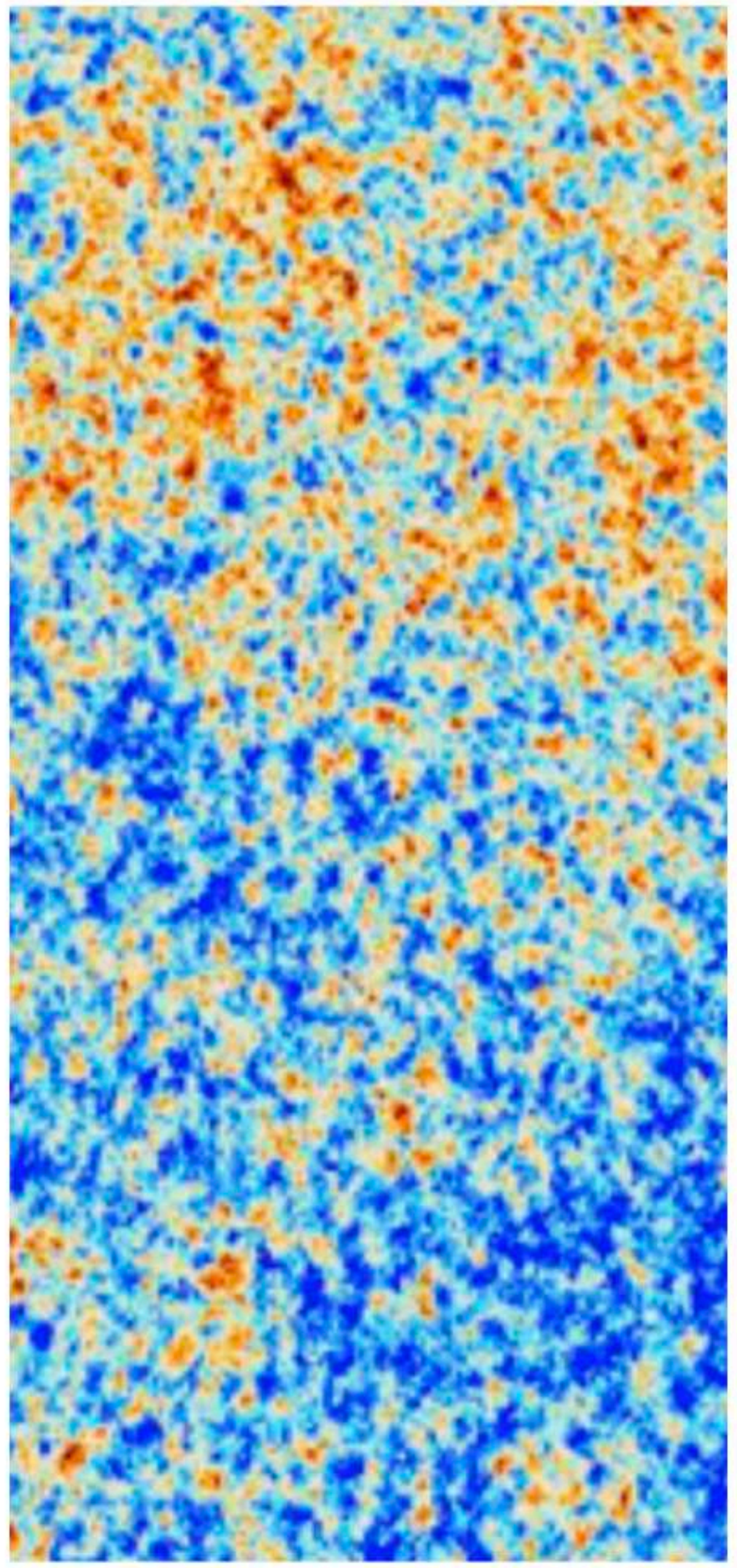
10^{25} m

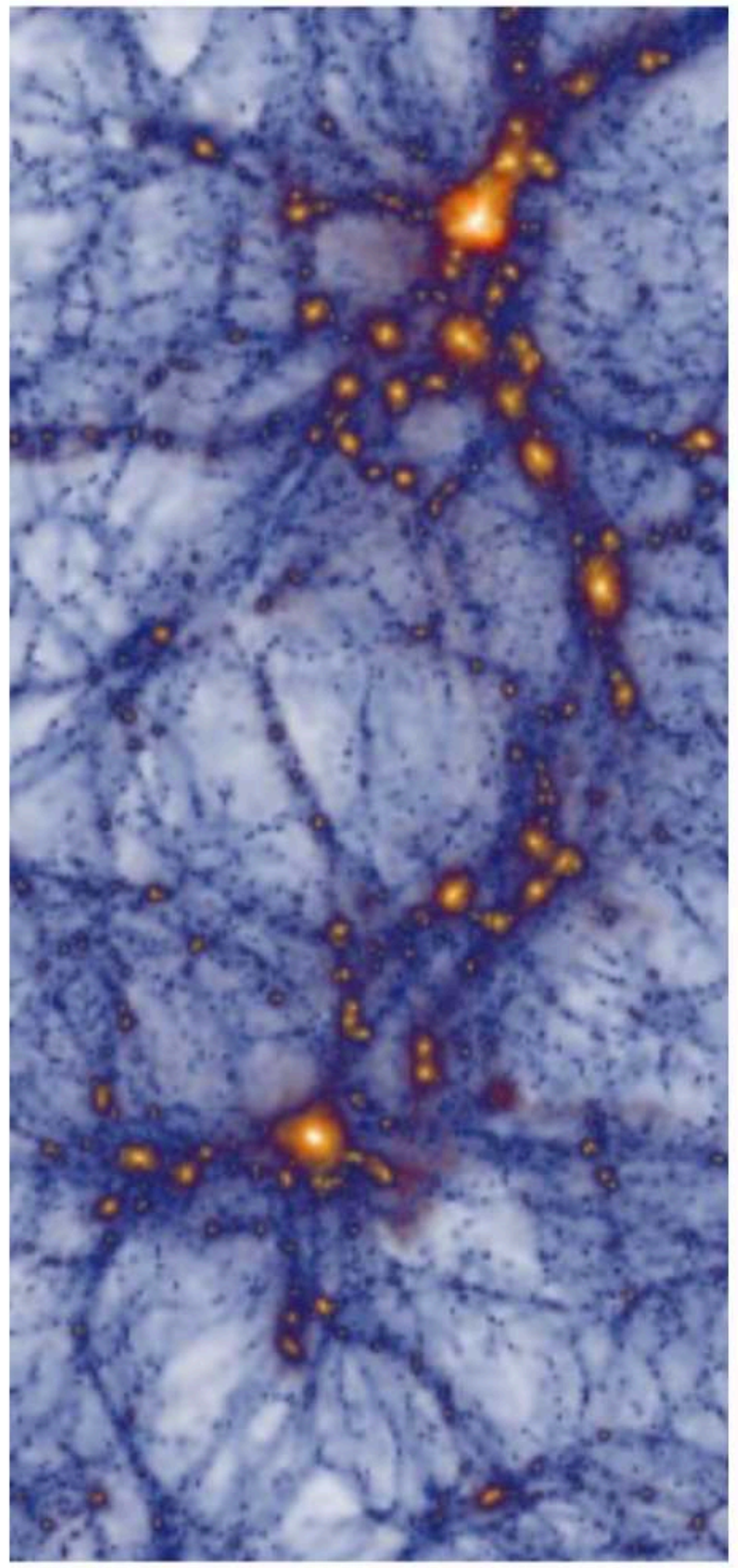
Crédito: D. Baumann

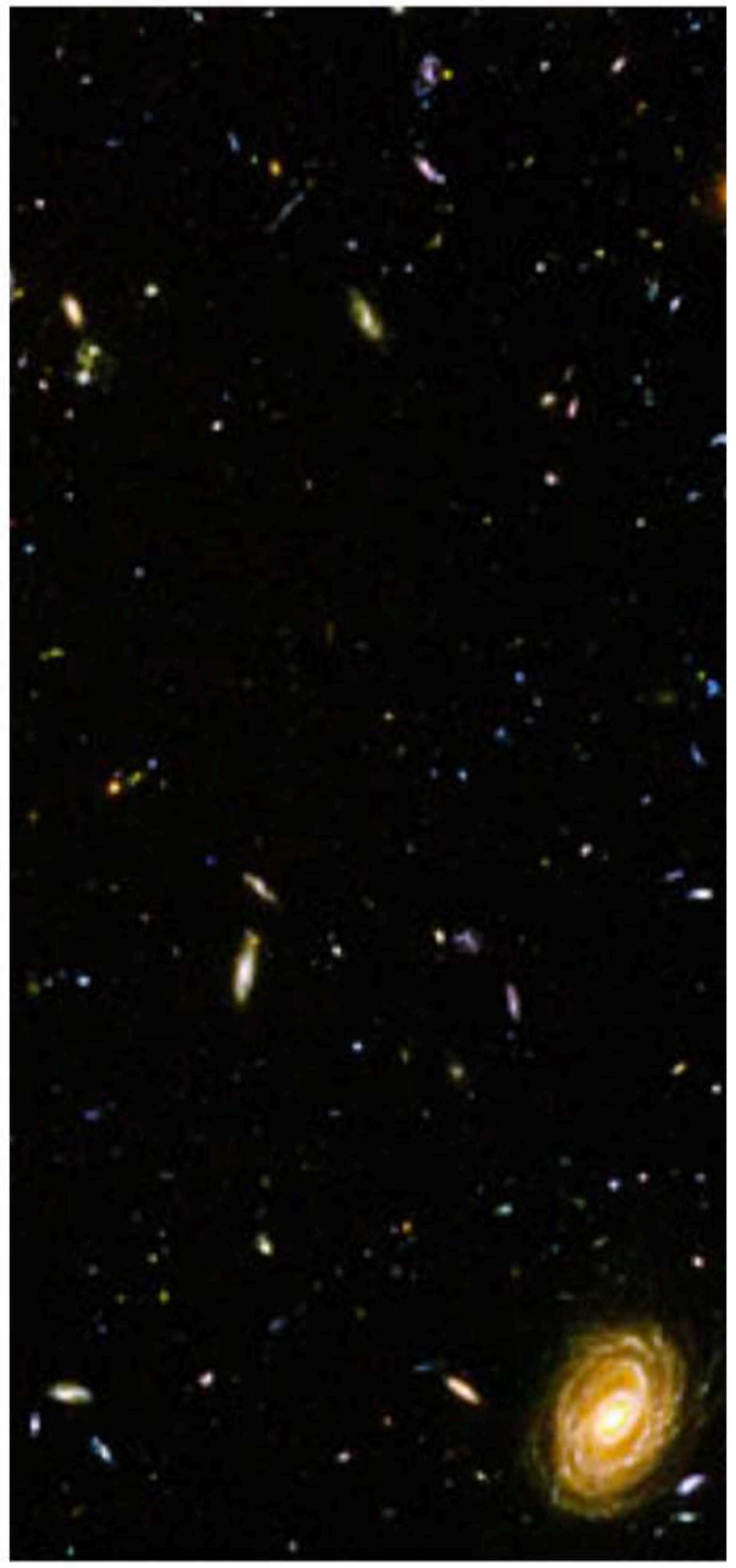
Problem of the origin of structures

Origin of the small perturbations → Small perturbations → Macroscopic structures

$\delta\rho \sim 10^{-5}\,\bar{\rho}$







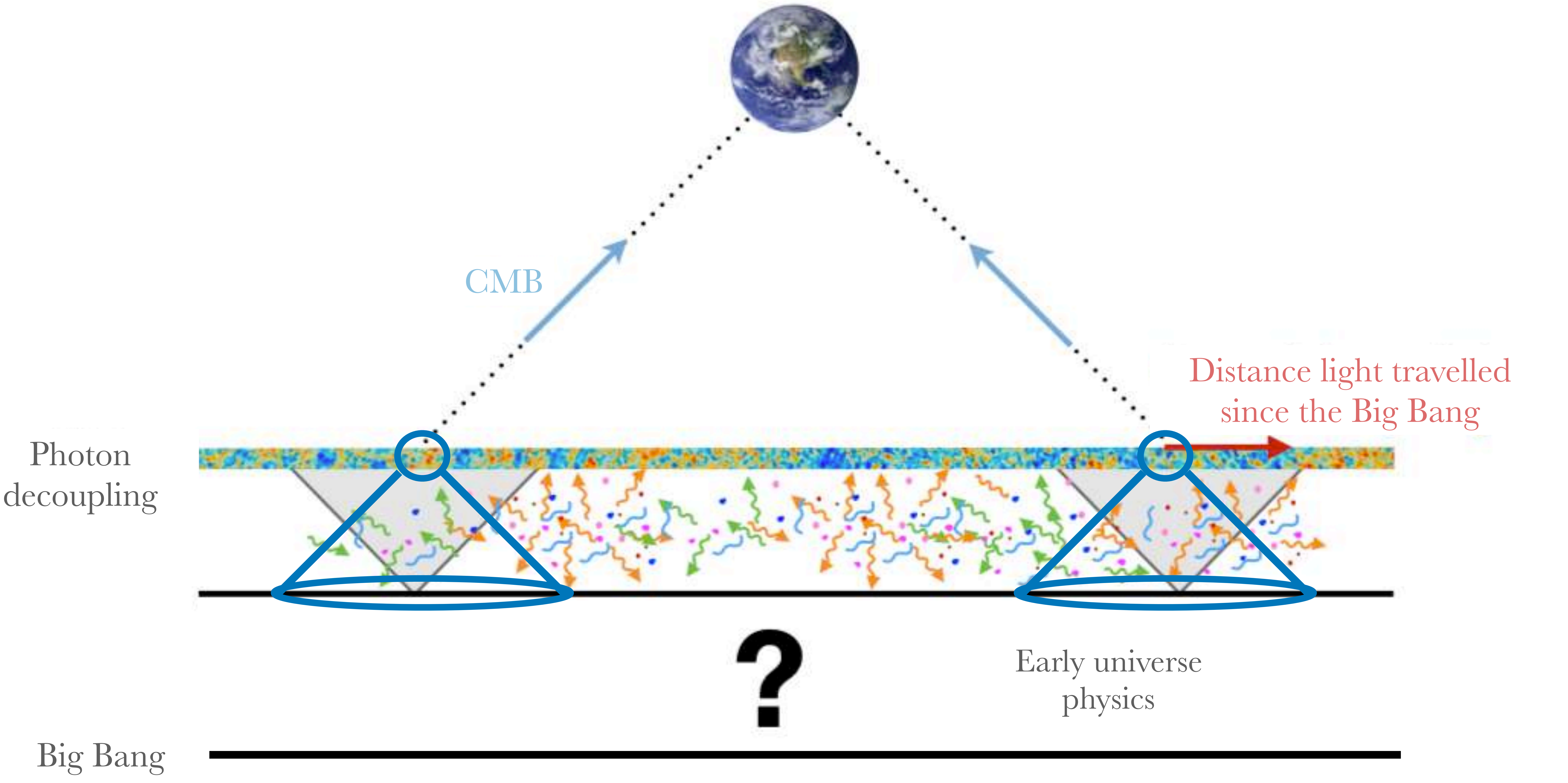
10^{-32} s

380.000 years

13.8 billion of years

Crédito: D. Baumann

We need to understand the primordial universe, explain the origin of the initial fluctuations and make predictions to test these theories of the early universe evolution



Crédito: D. Baumann

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$\tau = 0.0522 \pm 0.0080 \longrightarrow$ Optical depth

$$P_{\mathrm{prim}} = A_s\, k^{n_s - 1}$$

Primordial power spectrum: perturbations were almost scale invariant; red tilted

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\rm crit}$

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

ρ and P here are actually the sum of all the components in the universe $\Longrightarrow$ ρ_{tot}, p_{tot}

We can also rewrite the 1st Friedmann equation as:

$$1 = \Omega_{tot} - \frac{k}{a^2 H^2}$$

Density parameter

$$\Omega_{tot} = \sum_i \Omega_i\,, \qquad \text{onde } \Omega_i = \frac{\rho_i}{\rho_{crit}}$$

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\text{crit}}$ $\Rightarrow$ $(\Omega_0 - 1) \sim \mathcal{O}(1)$

$$(\Omega - 1)a^2 H^2 = k$$

$\propto a^2$ rad

$\propto a$ matter

Extrapolating to earlier times:

$$\frac{|\Omega - 1|_{t=t_{pl}}}{|\Omega_0 - 1|} \approx \frac{a_{pl}^2}{a_0^2} \approx \frac{T_0^2}{T_{pl}^2} \sim \mathcal{O}(10^{-64})$$

for BBN, $\mathcal{O}(10^{-16})$

Given the evolution of the universe, so $(\Omega_0 - 1) \sim \mathcal{O}(1)$, then $(\Omega - 1)$ had to be VERY VERY small!

Fine tuning!

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Can be recast as an "entropy problem" $\longrightarrow$ adiabatic expansion $\Omega - 1 = (k\, m_{pl})/(S^{2/3}\, T^2)$

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\rm crit}$ $\Rightarrow$ $(\Omega_0 - 1) \sim \mathcal{O}(1)$

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Fine tuning!

Amplification of the curvature radius

$$R = \frac{H^{-1}}{|\Omega - 1|^{1/2}} = \left(\frac{a^2}{k}\right)^{1/2}$$

Questions?

What we need

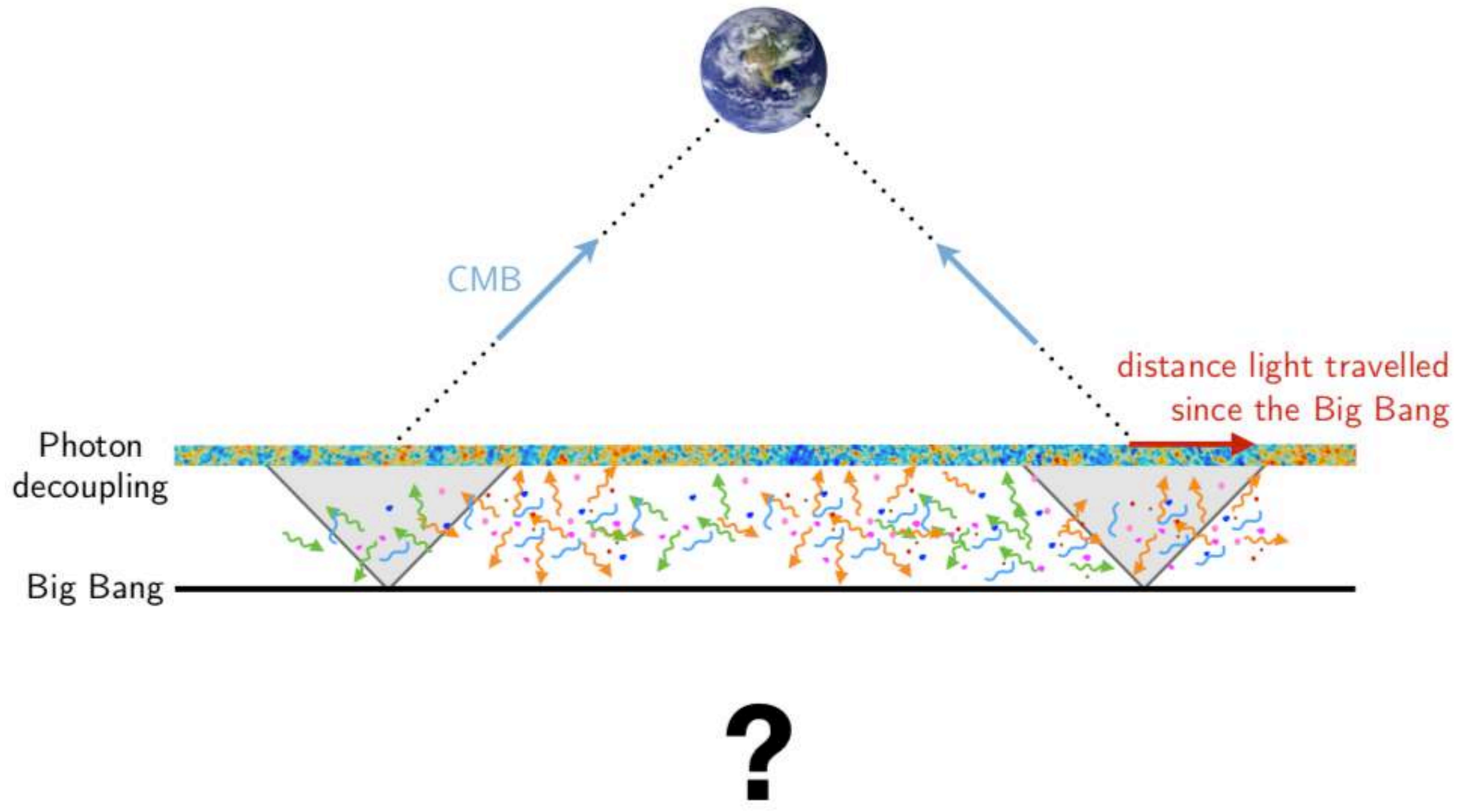
We need a theory of the early universe that solves all of these problems,

- causally connecting the universe and
- generating all the structure we see in the universe today.

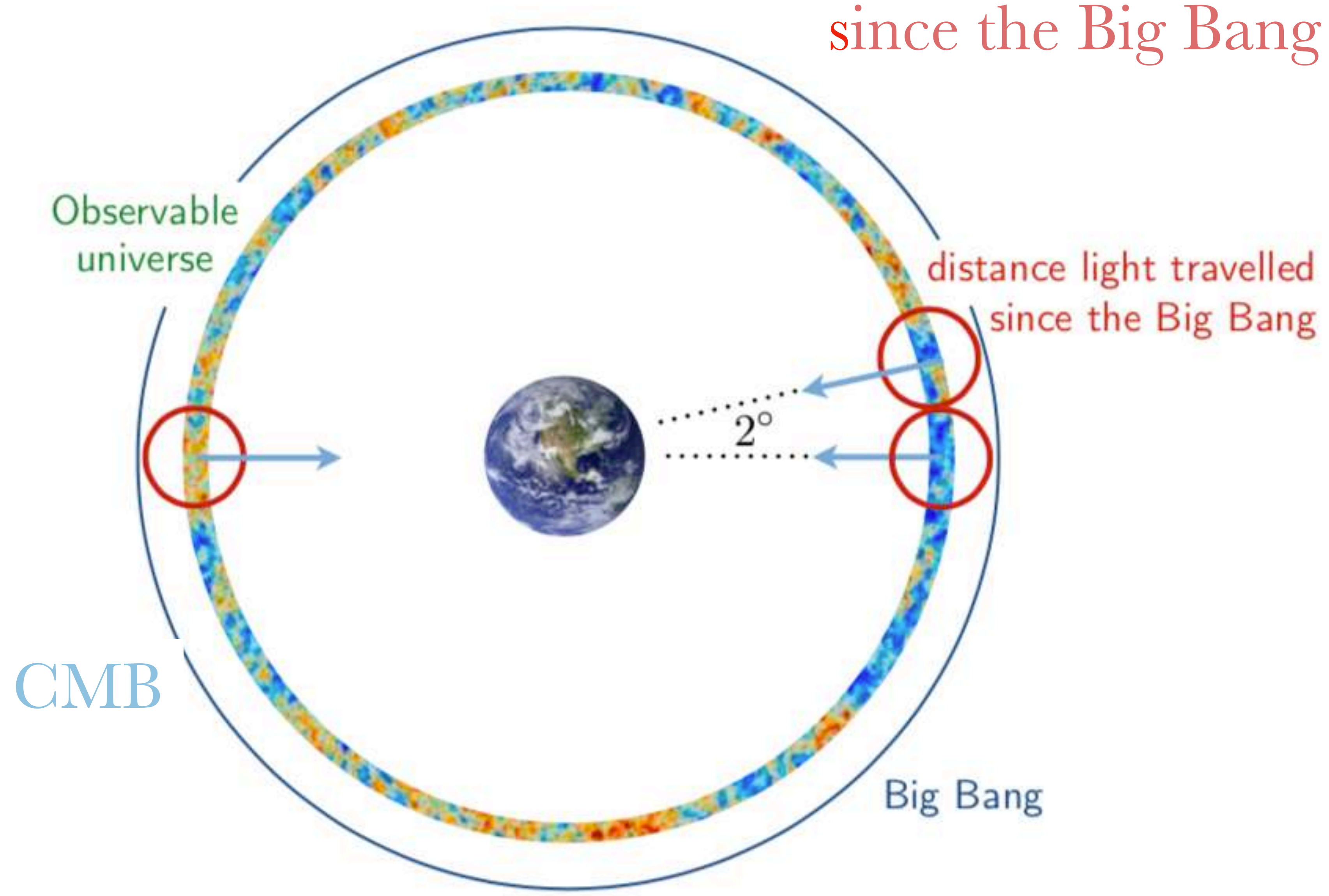
Early universe models

solving the SCM problems

Solving the horizon problem



Distance light travelled
since the Big Bang



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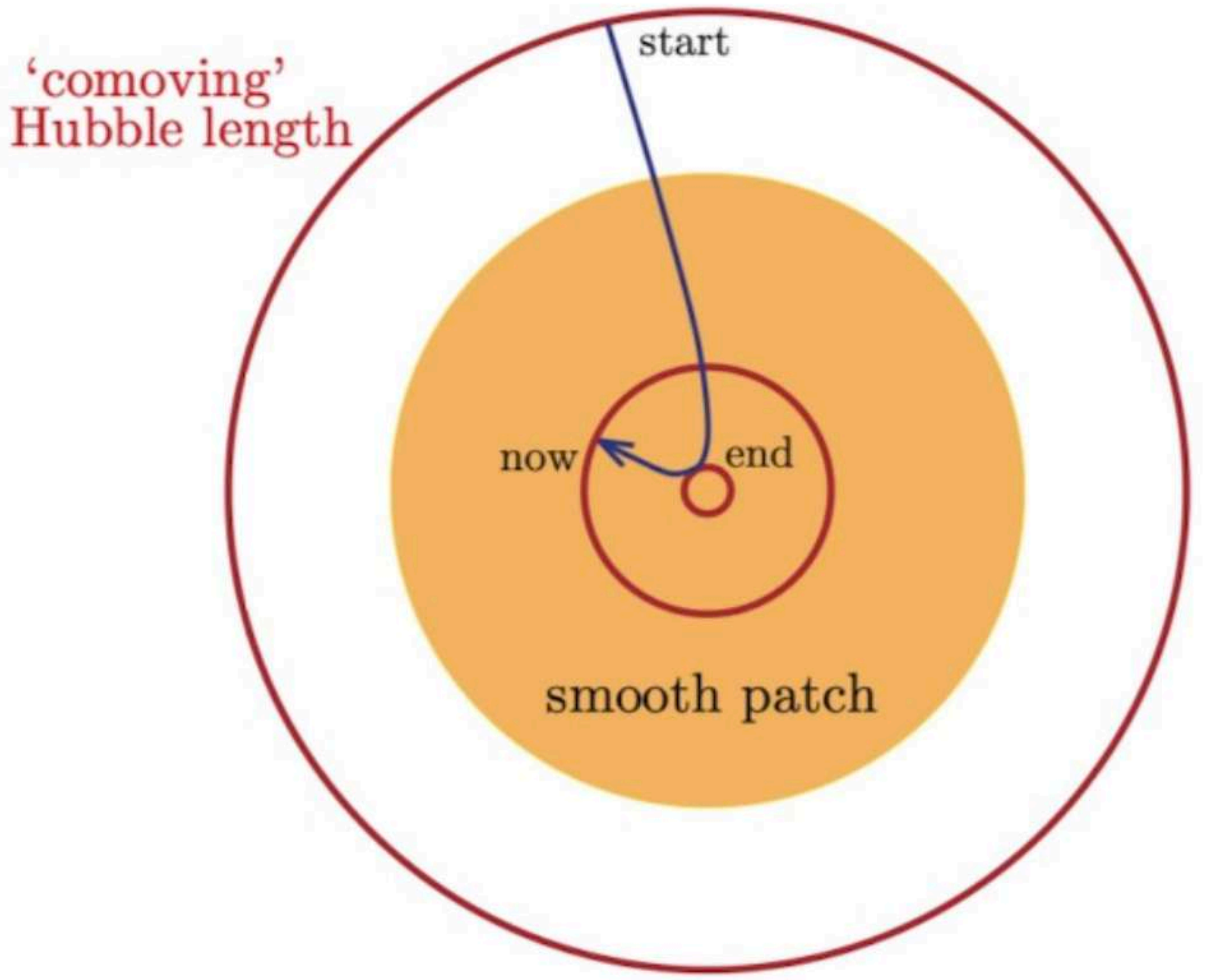
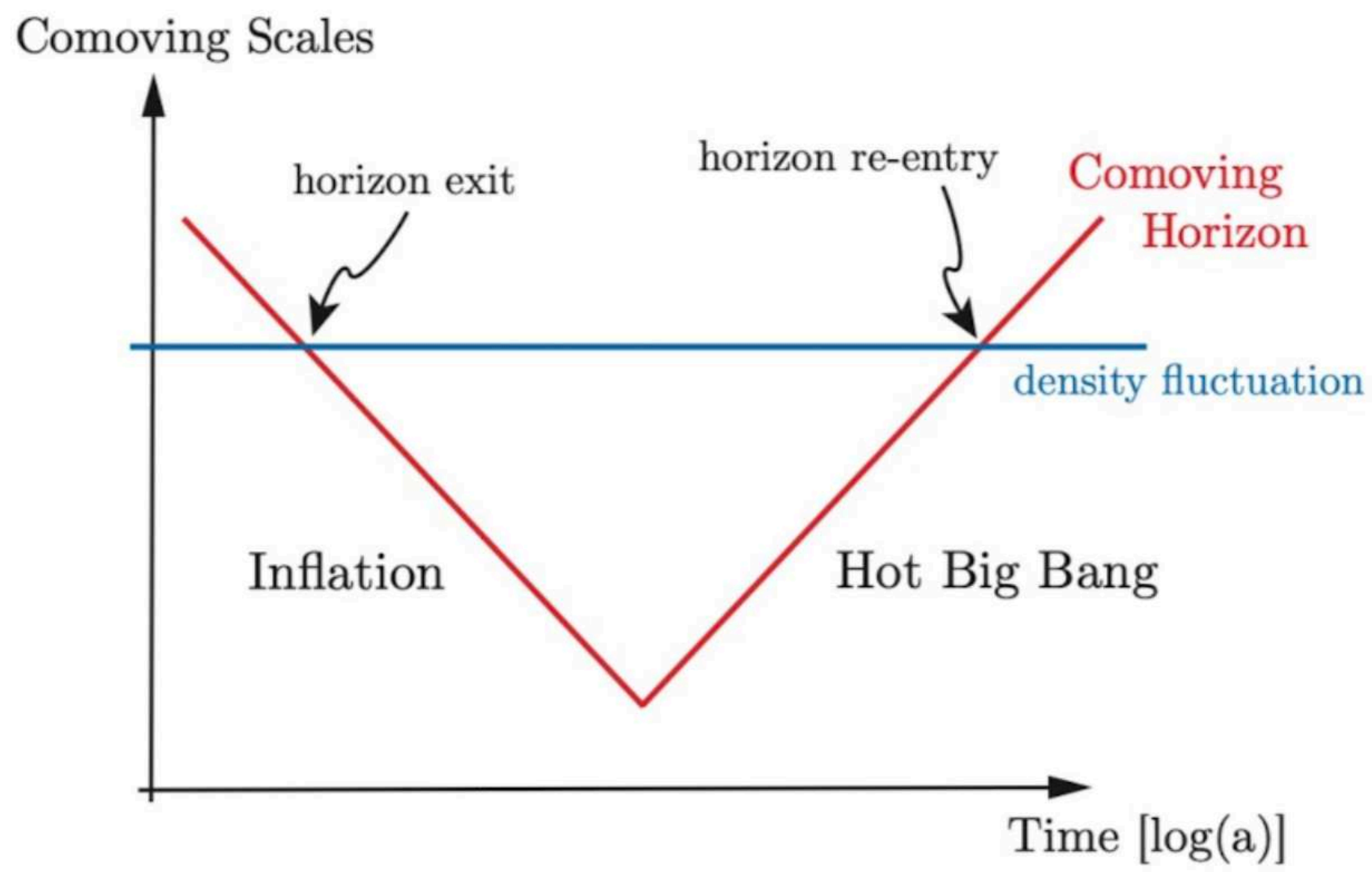
Solving the horizon problem

Idea 1: The shrinking Hubble sphere

A phase of decreasing Hubble radius in the early history of the universe; If this lasts long enough, the horizon problem may be avoided





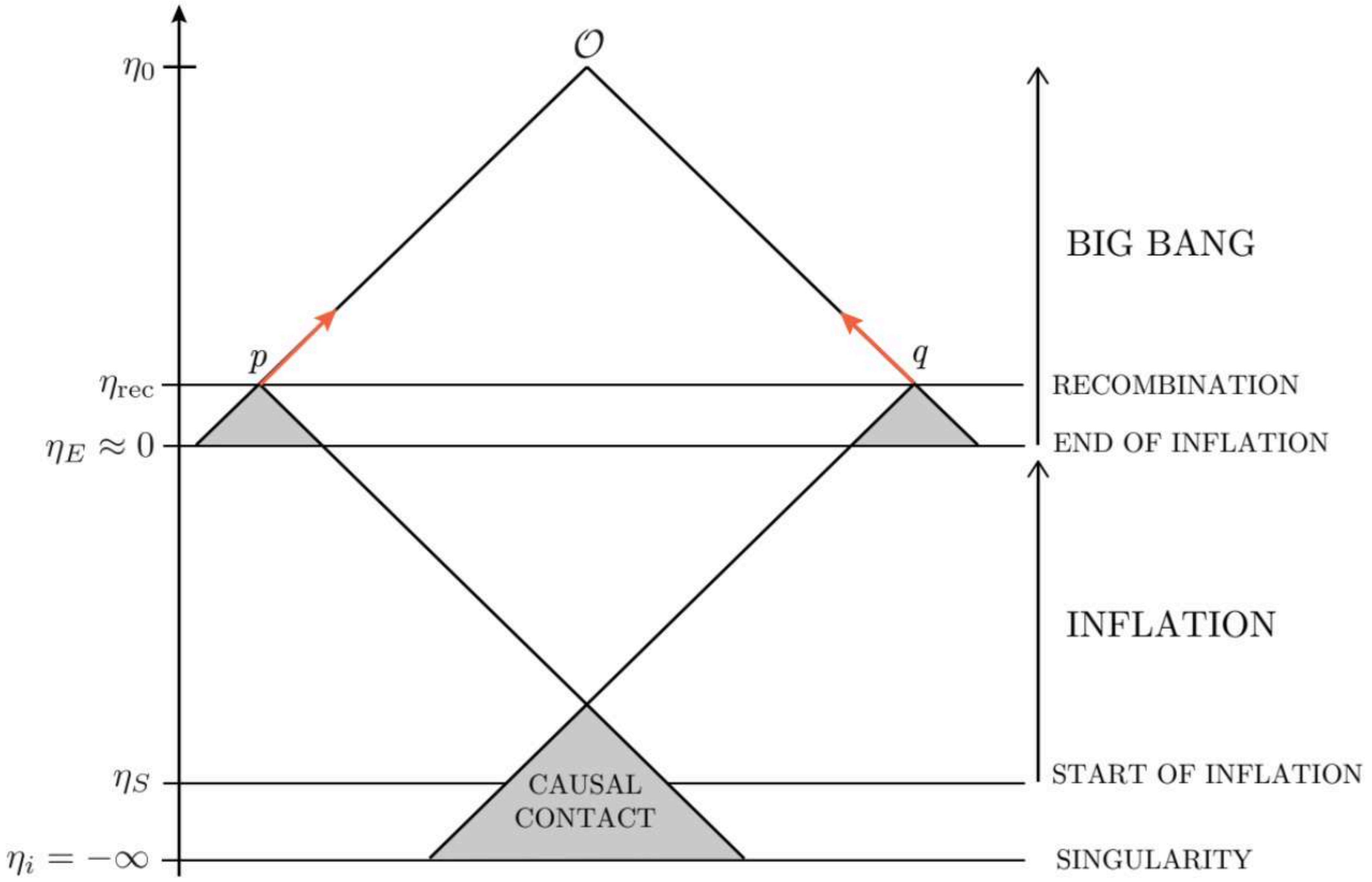
Shrinking Hubble sphere

horizon problem ↔ growing Hubble radius

Solving the horizon problem: **decreasing** Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0$$

If this period lasts long enough, it solves the horizon problem



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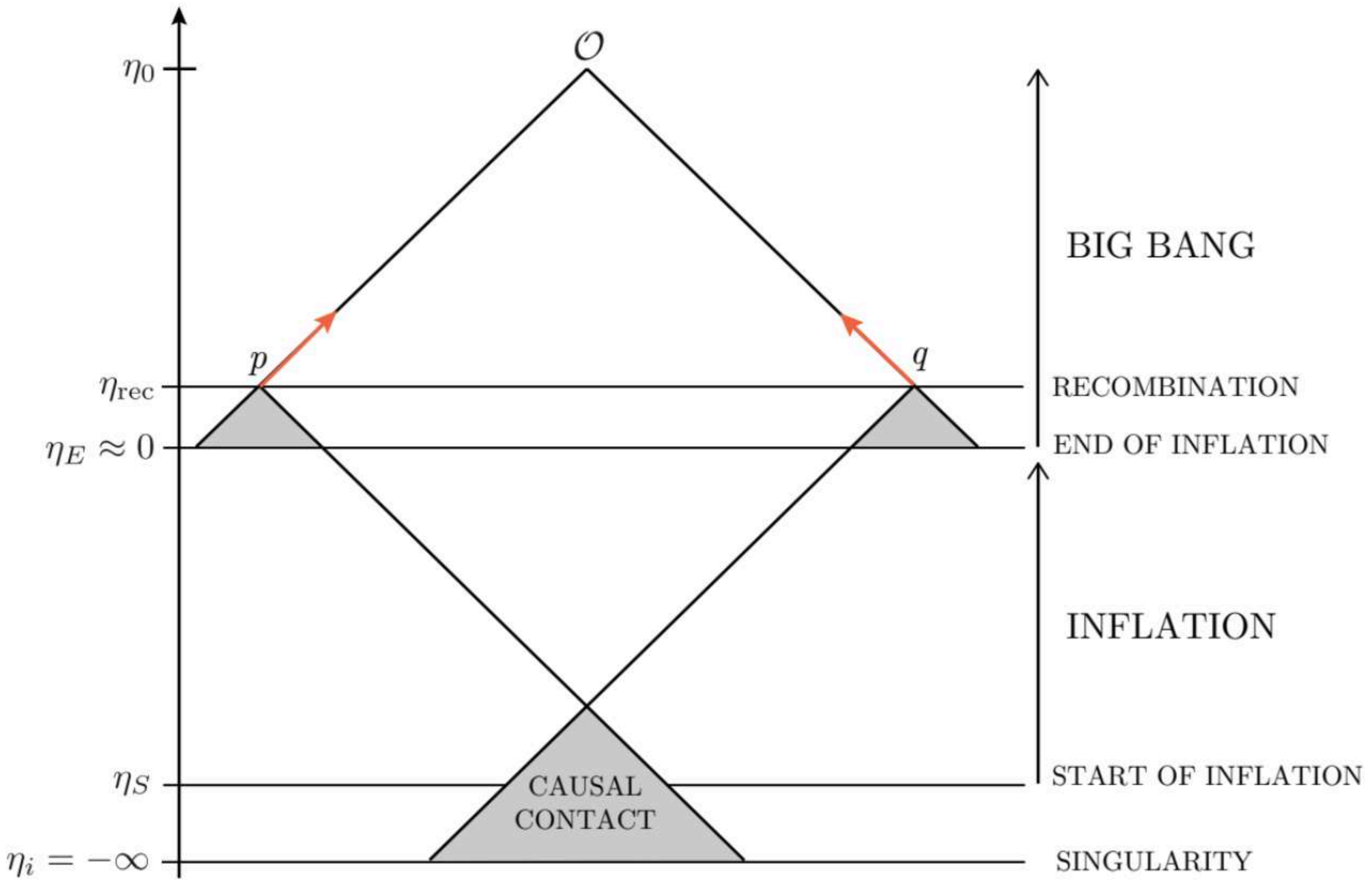
Shrinking Hubble sphere

<u>Solving the horizon problem</u>: **decreasing** Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0$$

- This implies that there was much more conformal time between the singularity and recombination than we had thought!
- The past light cones of widely separated points in the CMB now had time to intersect before the time $\eta = 0$
- For that $\eta = 0$ is NOT the initial singularity; there is time both before and after $\eta = 0$



Crédito: D. Baumann

Shrinking Hubble sphere

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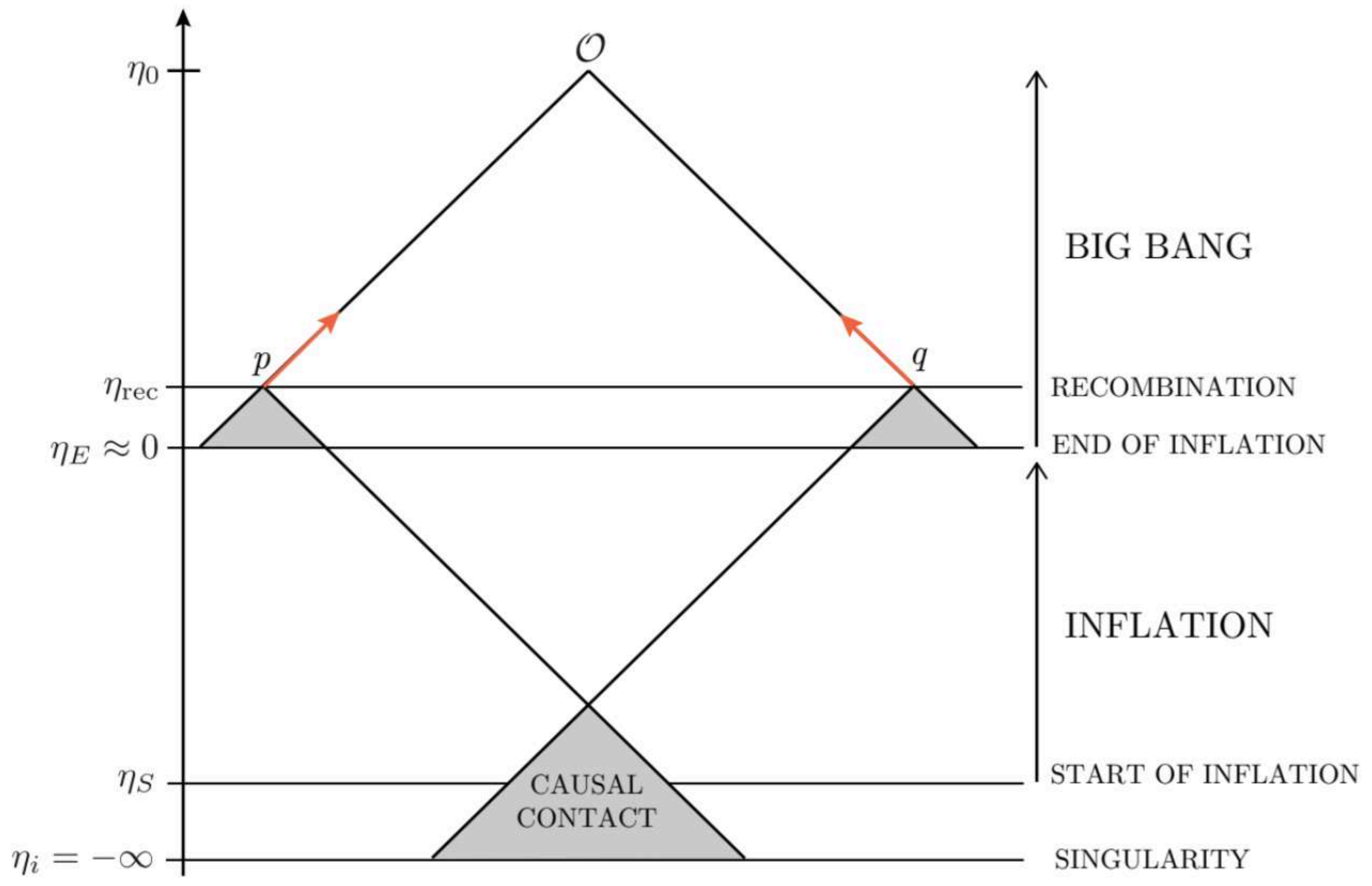
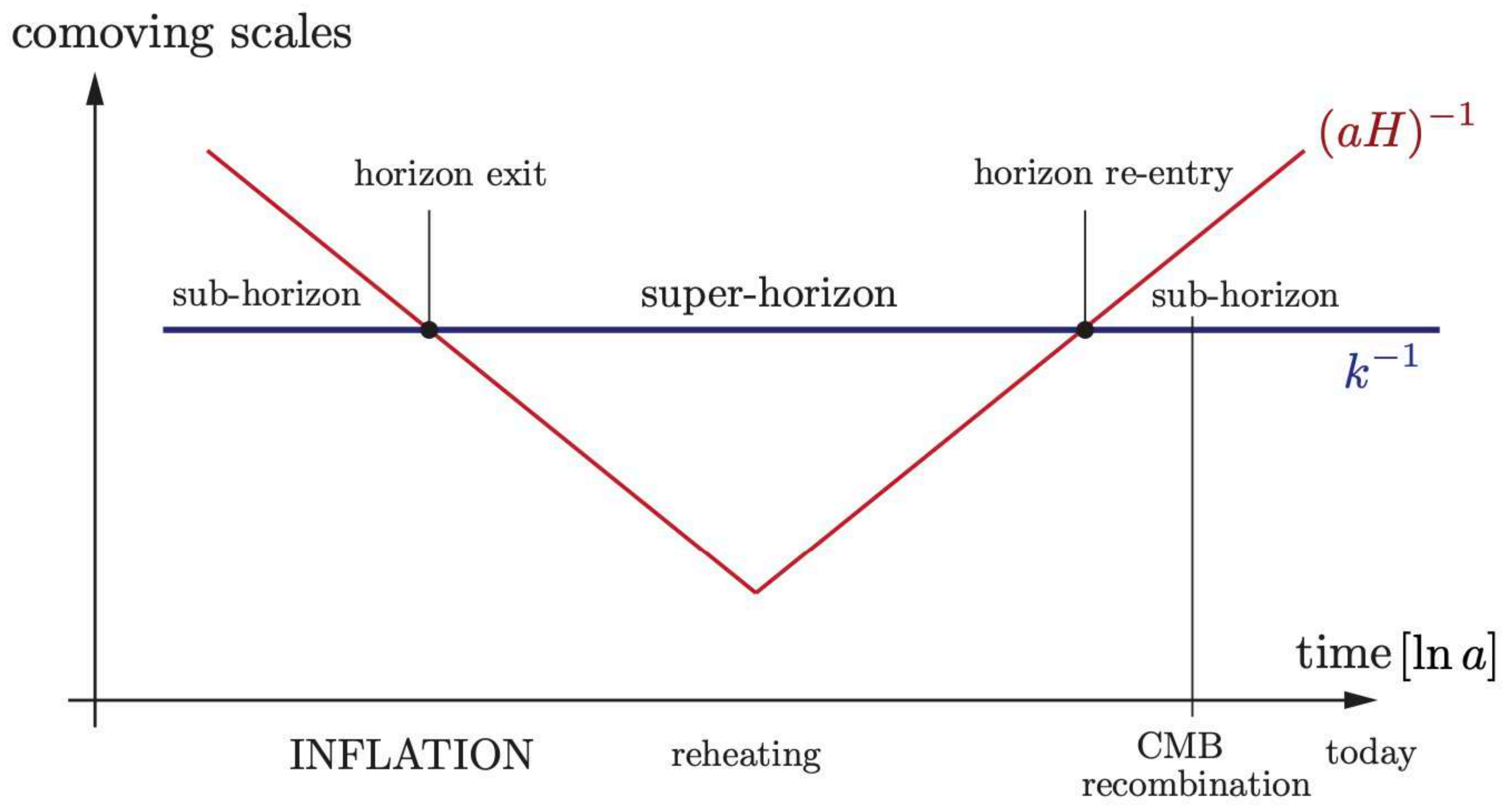
- This implies that there was much more conformal time between the singularity and recombination than we had thought!
- The past light cones of widely separated points in the CMB now had time to intersect before the time $\eta = 0$
- For that $\eta = 0$ is NOT the initial singularity; there is time both before and after $\eta = 0$
- A decreasing comoving horizon means that large scales entering the present universe were inside the horizon before inflation
- Causal physics before inflation therefore had time to establish spatial homogeneity.





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Questions?

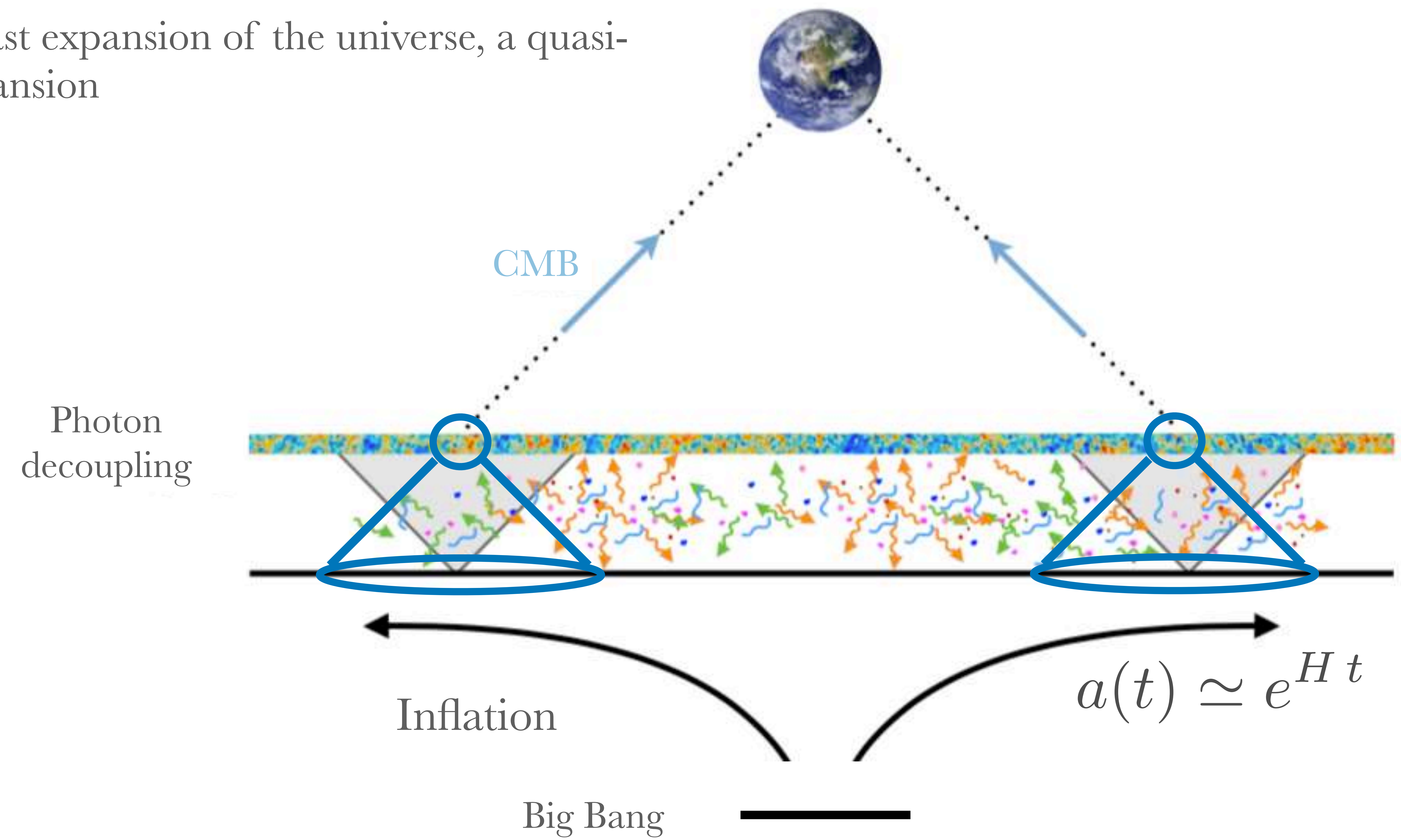
Inflation

Motivation: solve the SCM problems

Inflation

Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

Period of very fast expansion of the universe, a quasi-exponential expansion



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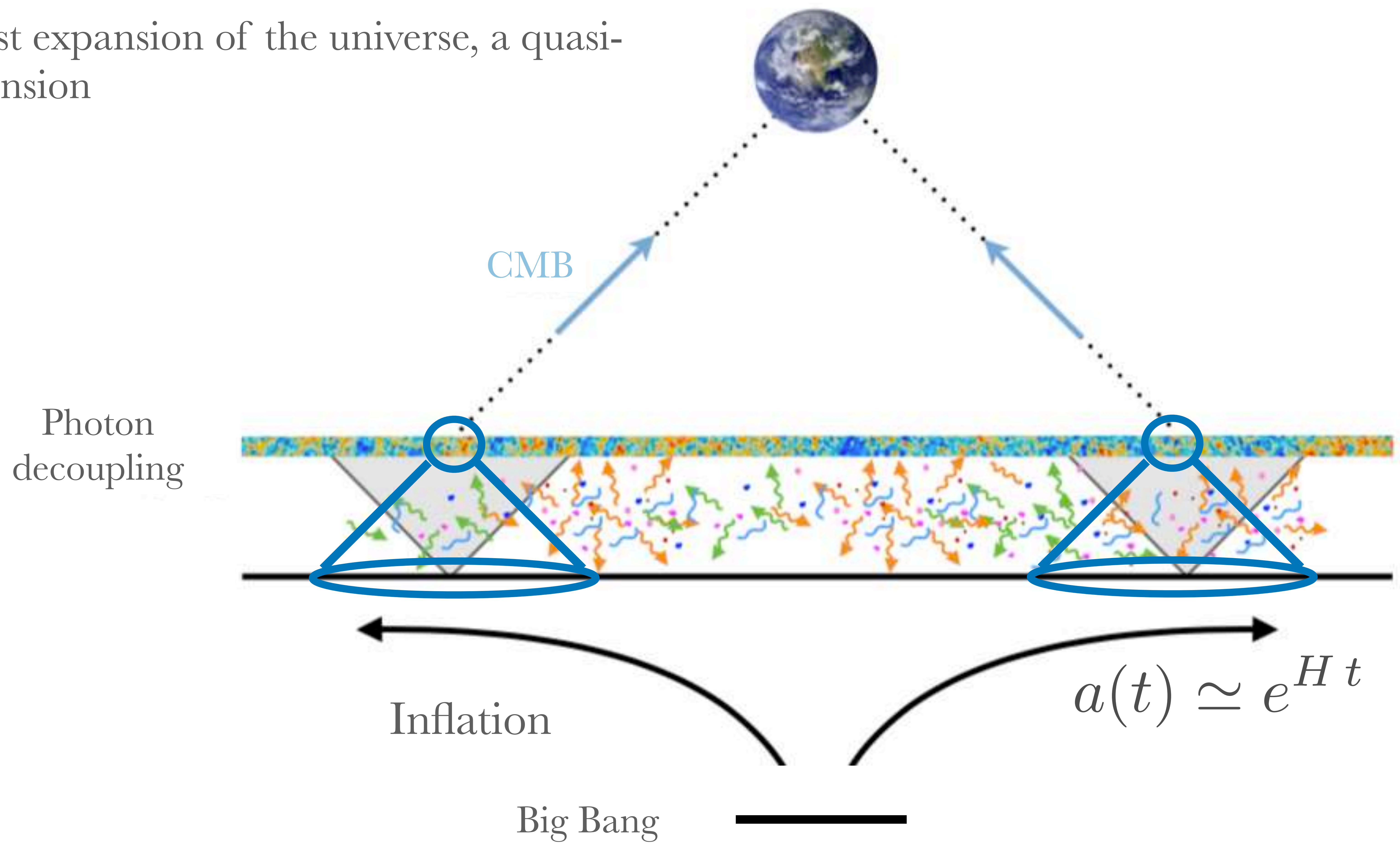

Originally (Guth 1980) - to solve the magnetic monopoles problem

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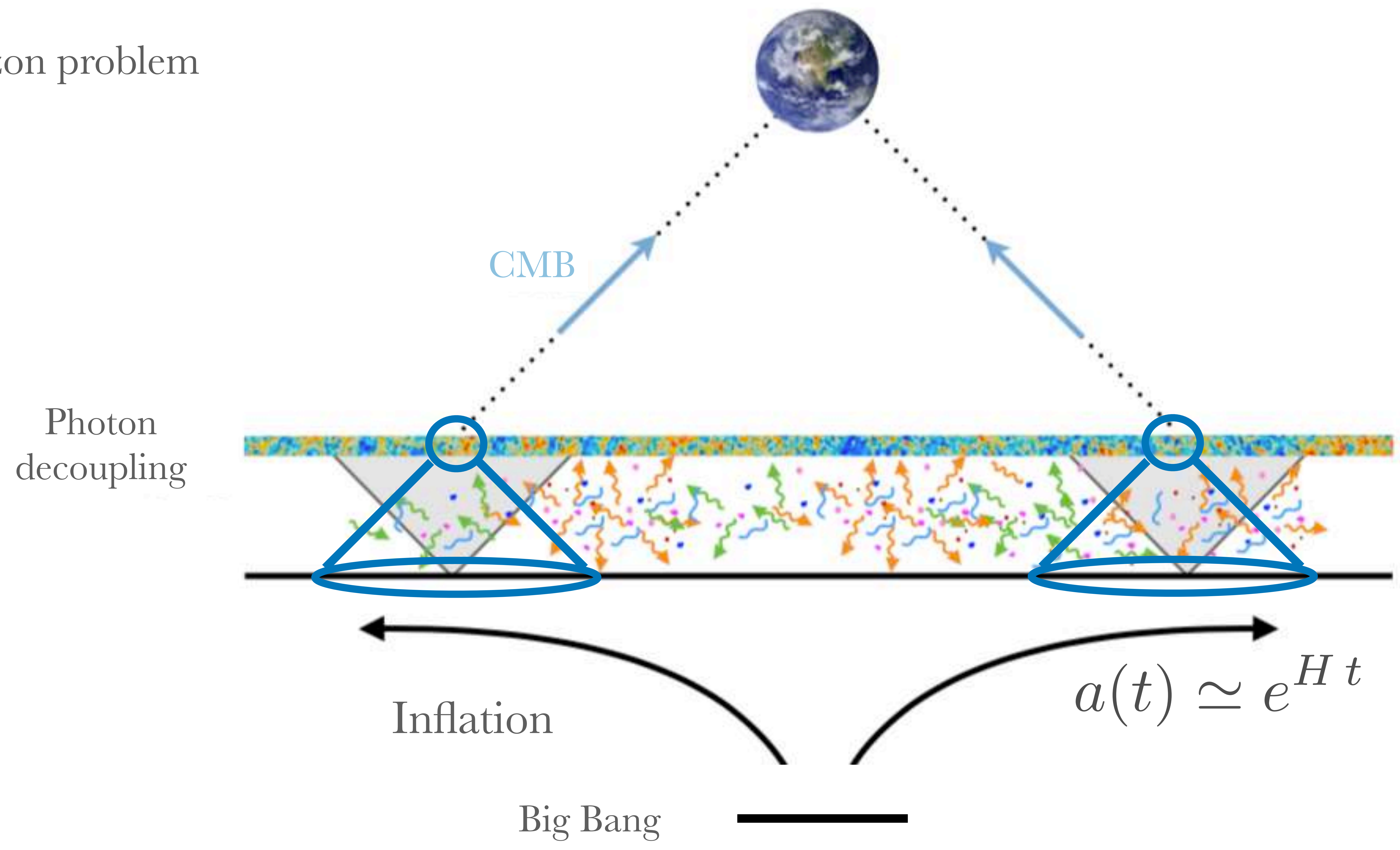
Inflation

Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

Solving the horizon problem



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Inflation

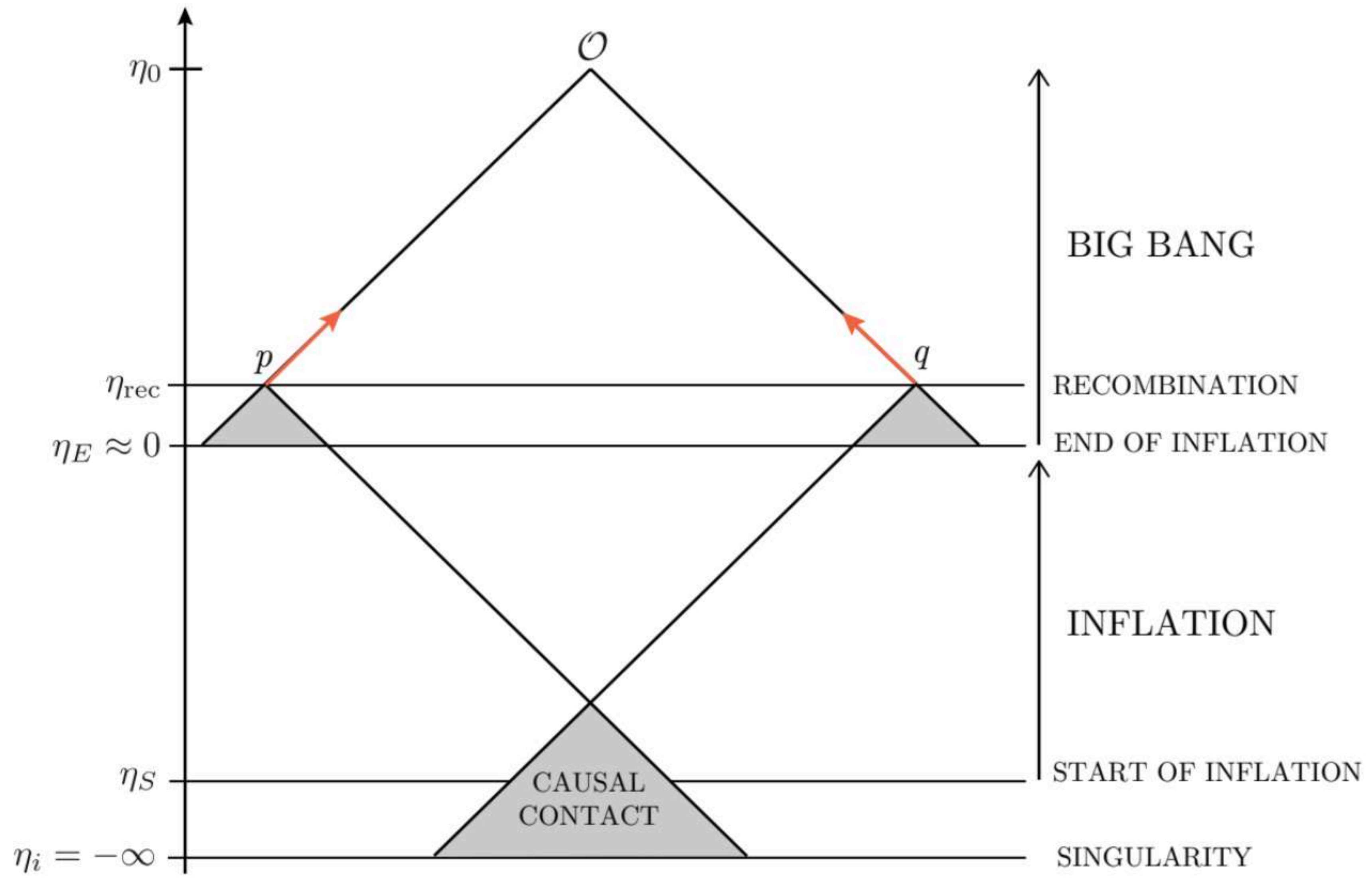
Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

horizon problem $\leftrightarrow$ growing Hubble radius

Solving the horizon problem: **decreasing** Hubble radius

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}(\dot{a})^{-1} = -\frac{\ddot{a}}{(\dot{a})^2} < 0 \ \Rightarrow \ \ddot{a} > 0$$

If this period lasts long enough, it solves the horizon problem



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HOW?

$$\chi_p(\eta) = \int_{\ln a_i}^{\ln a} (aH)^{-1} \, d\ln a$$

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0 \quad \Rightarrow \quad \ddot{a} > 0$$

~~Strong energy condition (SEC) >0~~

$$\eta_i \to -\infty$$

Inflation

Guth (1980)
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Shrinking the Hubble sphere

$$\chi_p(\eta) = \int_{\ln a_i}^{\ln a} (aH)^{-1} \, d\ln a$$

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$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0 \quad \Rightarrow \quad \ddot{a} > 0$$

Strong energy condition (SEC)
>0

$$\eta_i \to -\infty$$

• We notice that the Big Bang singularity is now pushed to negative conformal time

$$\eta_i \propto \frac{2}{(1+3w)} a_i^{\frac{1}{2}(1+3w)} = -\infty$$

(when $w < -1/3$)

- This implies that there was much more conformal time between the singularity and recombination than we had thought!
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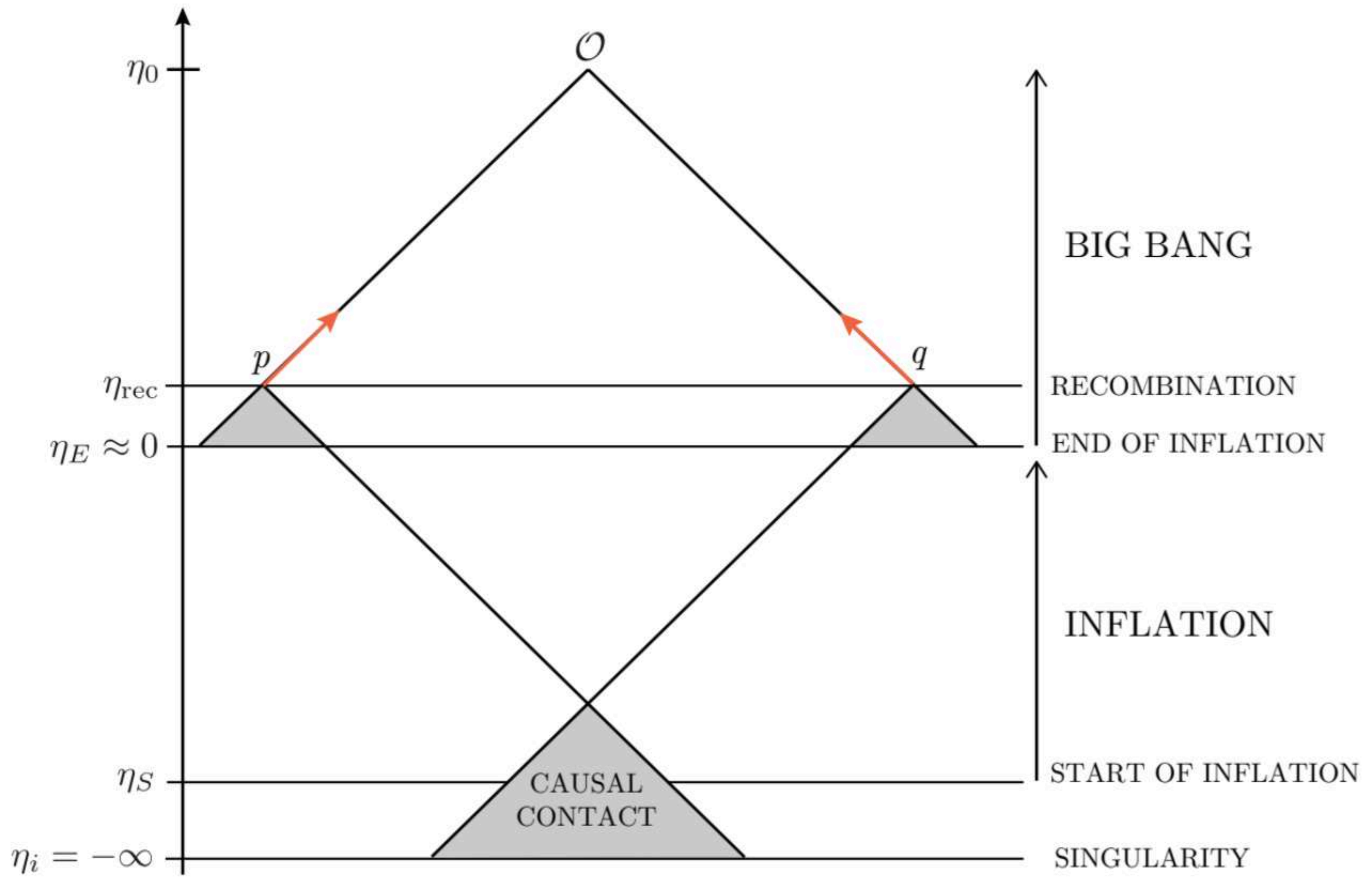
- For that $\eta = 0$ is NOT the initial singularity; there is time both before and after $\eta = 0$



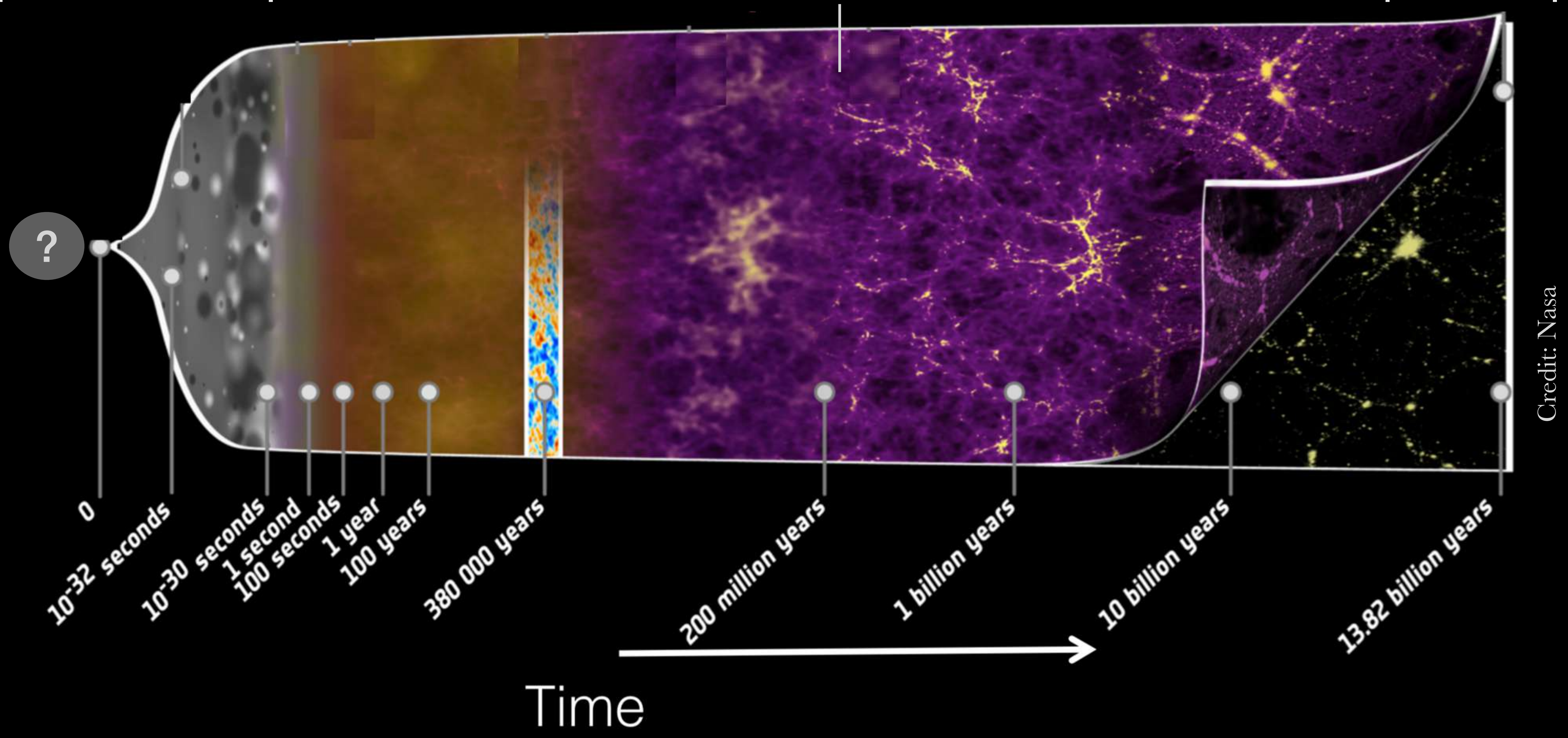
Crédito: D. Baumann

MANY fundamental open questions

Early universe
Big Bang? What is the physics of the early universe?

Dark matter p
What is the dark matter?

Dark energy -
What is the dark energy?



Credit: Nasa

Inflation

- Accelerated expansion

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}(\dot{a})^{-1} = -\frac{\ddot{a}}{(\dot{a})^2} < 0 \;\Rightarrow\; \ddot{a} > 0$$

Shrinking Hubble radius

- Slowly varying Hubble parameter

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = \frac{1}{a}(1-\epsilon) < 0$$

Slow-roll parameter: $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$

- Negative pressure

$$w = \frac{p}{\rho} < -\frac{1}{3}$$

- Constant density

$$\dot{\rho} + 3H\,(\rho + P) = 0$$

$$\Rightarrow \quad |d\ln\rho/d\ln a| = 2\epsilon < 1$$

- Quasi de Sitter expansion

When $\epsilon \to 0 \quad \Rightarrow \quad \mathrm{dS} \quad \Rightarrow \; H = \mathrm{const.}$

Small ϵ, quasi-dS

$$\ddot{a} > 0 \;\leftrightarrow\; p \sim -\rho \;\leftrightarrow\; H = \mathrm{const.} \;\leftrightarrow\; \rho \sim \mathrm{const.} \;\leftrightarrow\; \epsilon < 1 \;\leftrightarrow\; a(t) \simeq \exp(Ht)$$

Constructing inflation

1. Decreasing radius / slowly-varying Hubble parameter

1st slow-roll parameter $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$ Small

2. Inflation persists for long enough (ϵ small for enough time)

2nd slow-roll parameter $\eta \equiv \frac{d \ln \epsilon}{dN} = \frac{\dot{\epsilon}}{H\epsilon}$, $|\eta| < 1$

$N = \ln a$ e-fold

Implementing the inflationary mechanism

How can we implement a microphysical model of the accelerated (exponential

- Adding one (or many) new components that dominate the universe at its begining with $w < -\frac{1}{3} \quad \Longrightarrow \quad \ddot{a} > 0$

$$\rho_{\text{infl}}\,, \;\; p_{\text{infl}}$$

We call this new component the *inflaton*

Acceleration

How can we obtain such an expansion of the universe?

Remember:

- Extra dof: dark energy

acceleration

Dark energy

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[(\rho(t)+p(t))_{R,M} + \rho_{EE}(t)\right] \qquad + \qquad w < -\frac{1}{3}$$

decelerates the expansion

Acceleration

How can we obtain such an expansion of the universe?

Remember:

- Extra component leading to accelerated expansion

acceleration

Inflation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[(\rho(t) + p(t))_{R,M} + \rho_{EE}(t)\right] \qquad + \qquad w < -\frac{1}{3}$$

decelerates the expansion

Implementing the inflationary mechanism

How can we implement a microphysical model of the accelerated (exponential

- Adding one (or many) new components that dominate the universe at its begining with $w < -\frac{1}{3} \implies \ddot{a} > 0$

$$\rho_{\rm infl}\,, \ p_{\rm infl}$$

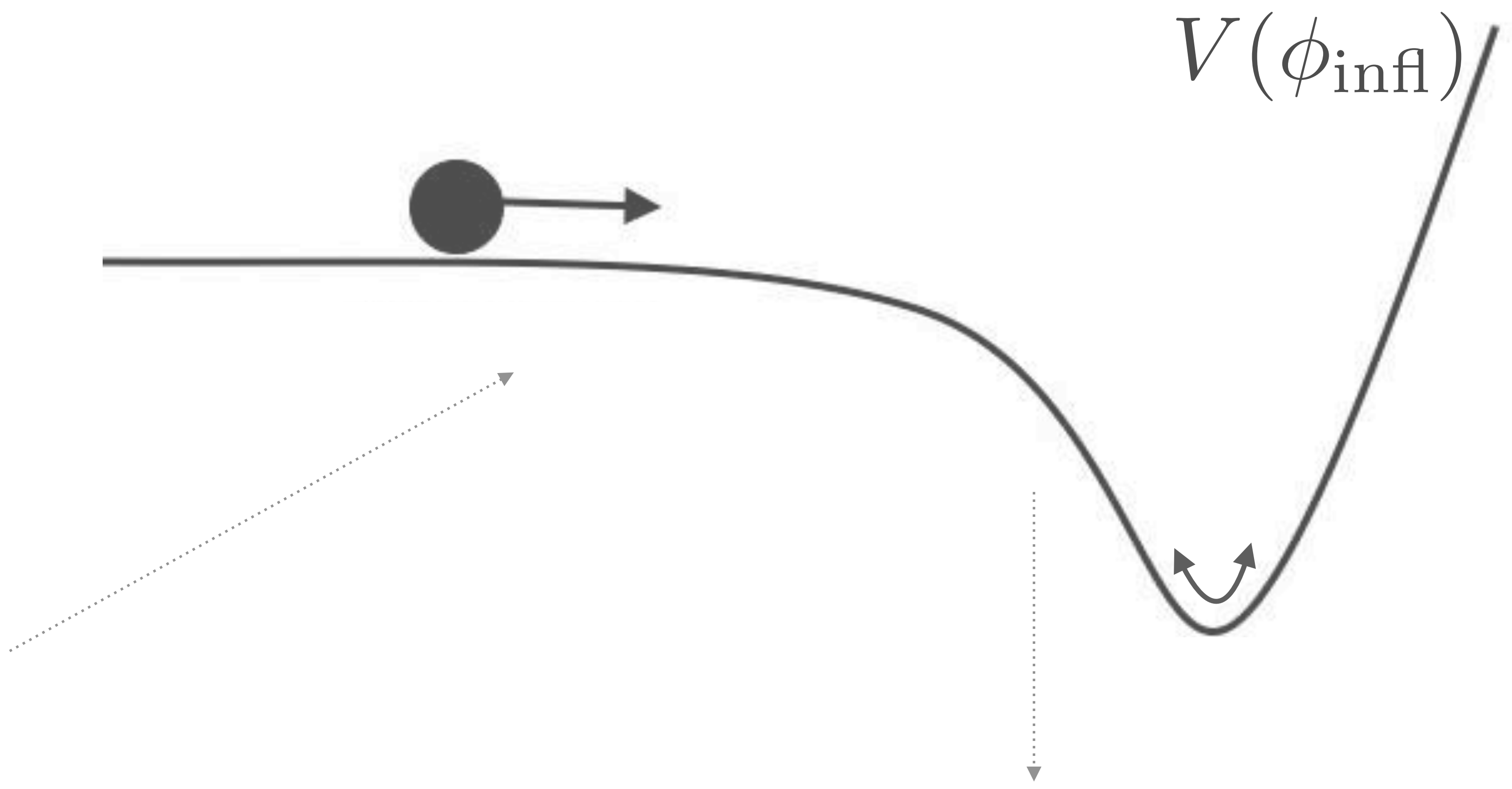
We call this new component the *inflaton*

Inflation

TOY MODEL:

Scalar field (*inflaton*) $\phi(t)$ in a FRW background

To cause the acceleration, the potential has to have the form:



Causes the accelerated expansion - quasi-exponential

However, inflation has to end, so the era of radiation begins - ***graceful exit***

$$\epsilon \sim 1$$

Problems with inflation

- Initial singularity
- Transplanckian problem
- Measure problem
- Hierarchy problem
- …

* still highly debated in the literature

Questions?

Alternatives to inflation

Solving the horizon problem

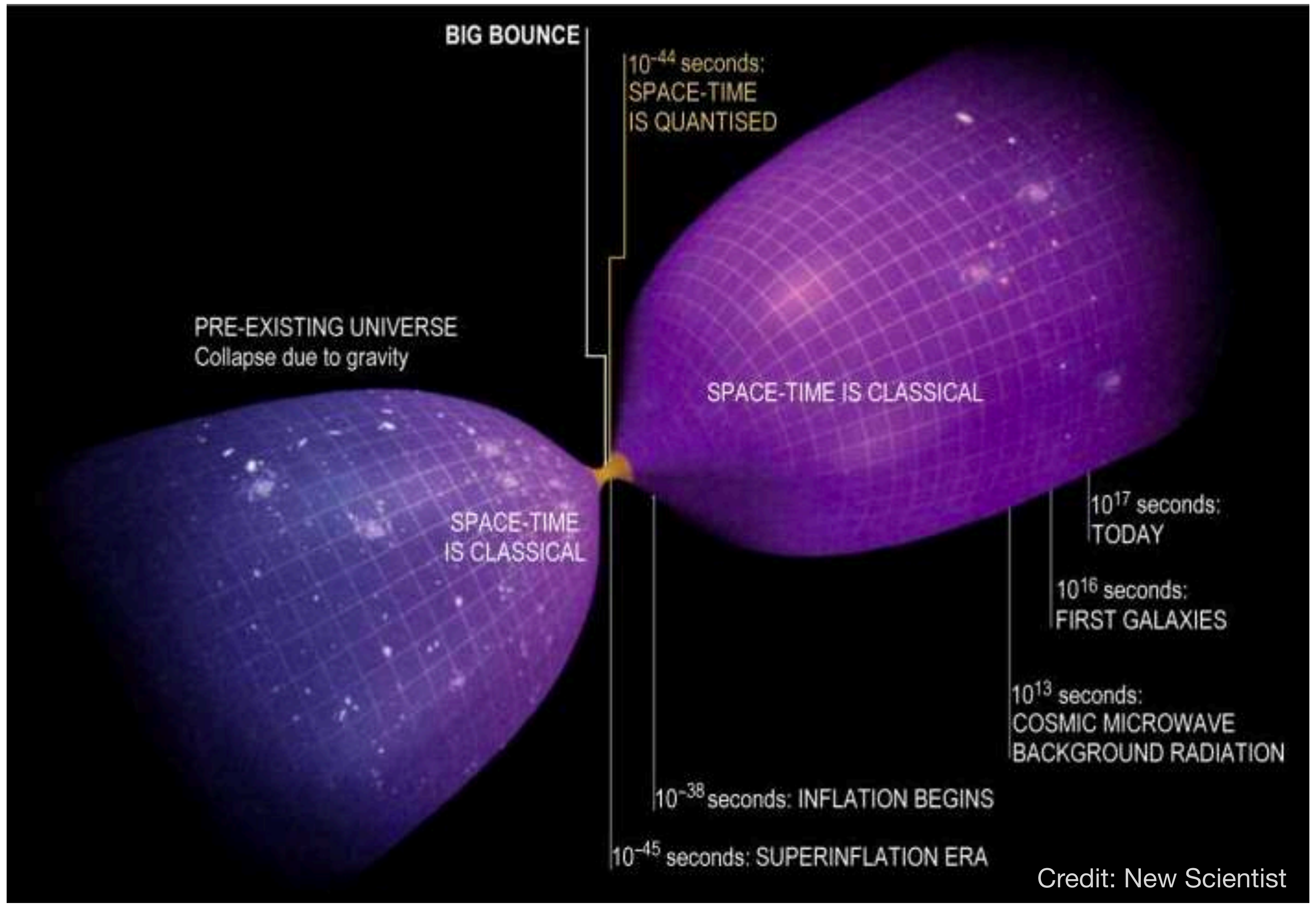
Idea 1: The shrinking Hubble sphere

A phase of decreasing Hubble radius in the early history of the universe; If this lasts long enough, the horizon problem may be avoided

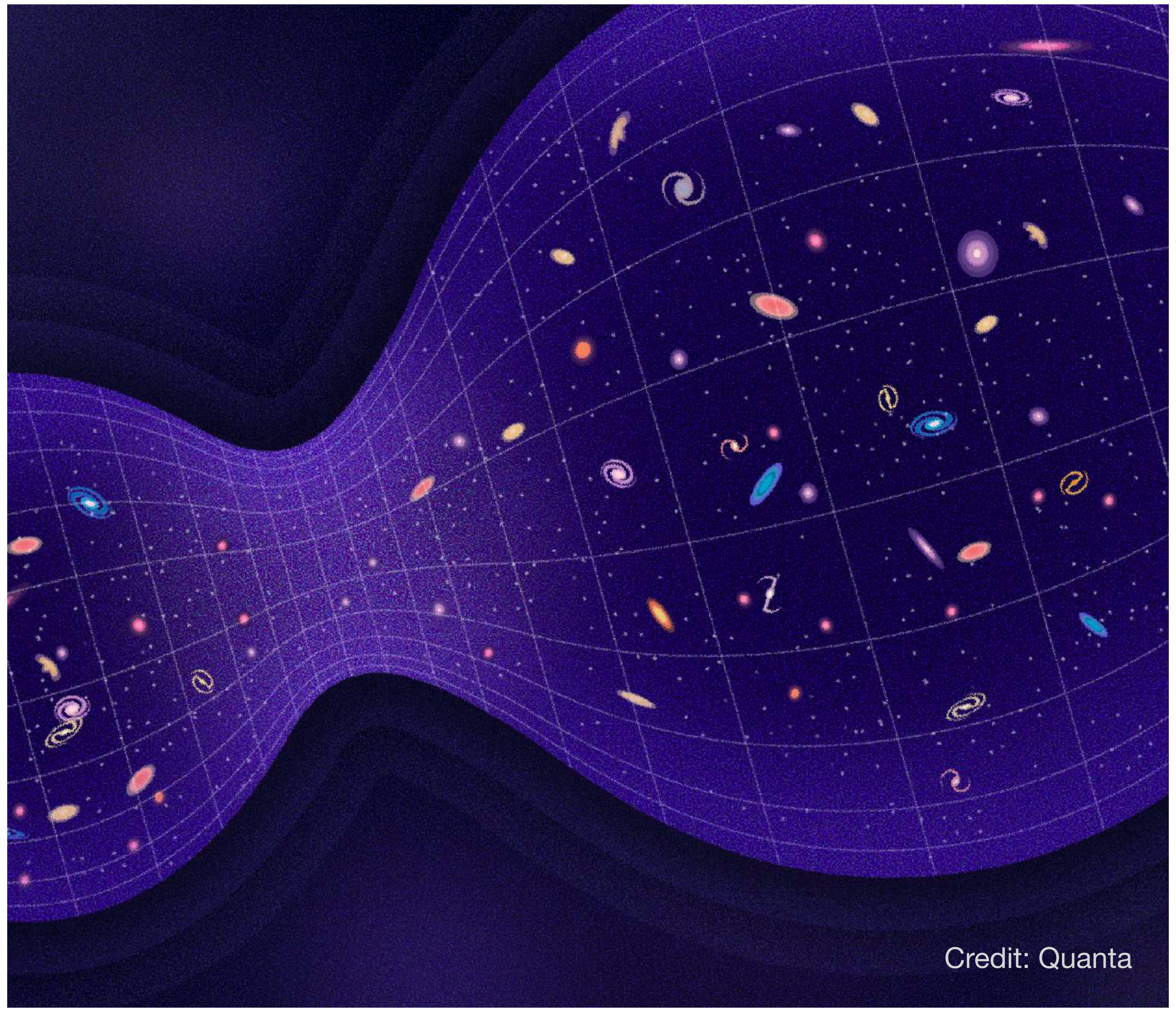
Idea 2:

Bouncing models

What if the universe did not have a begining, but came from a contraction phase, followed by the SCM



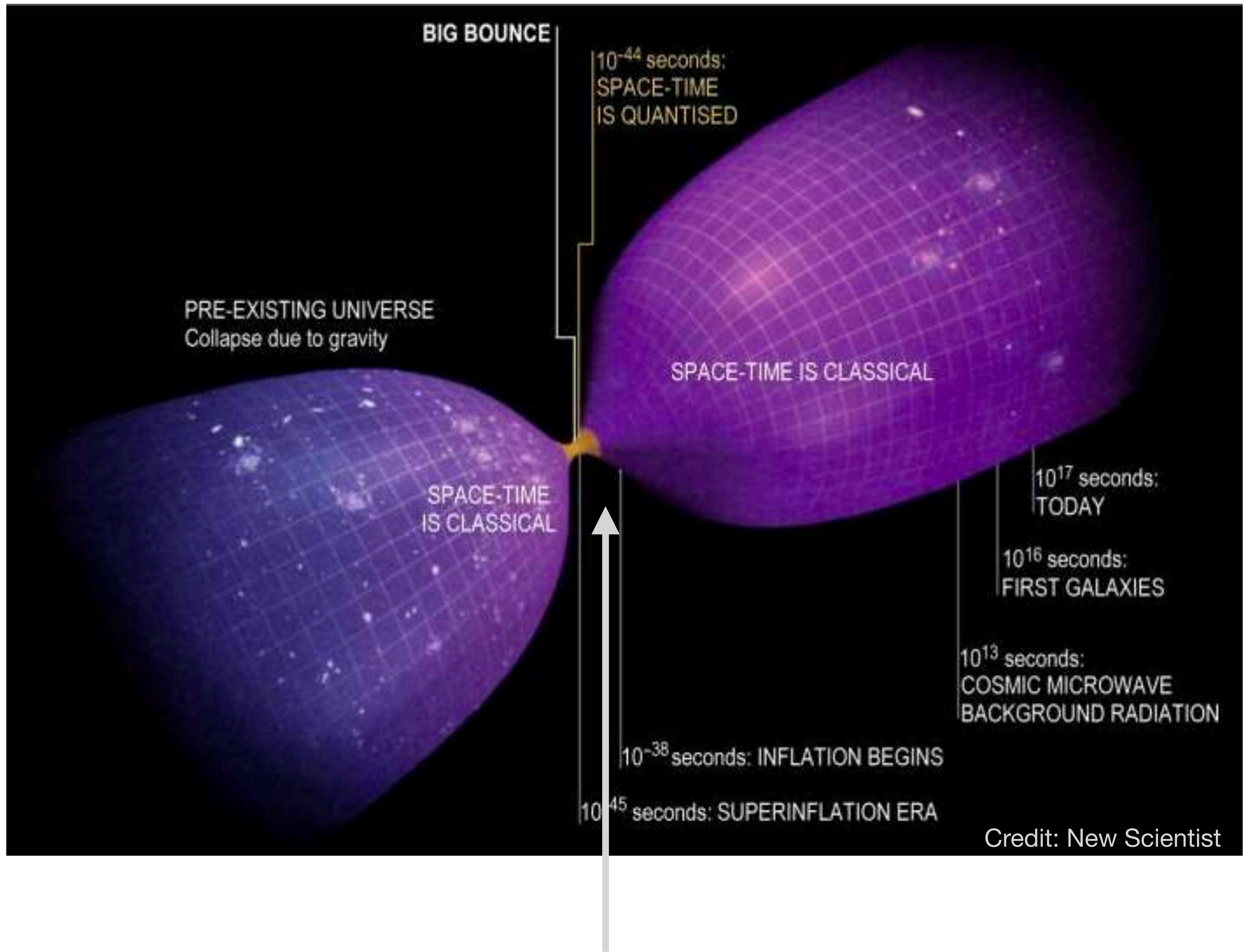
Credit: New Scientist



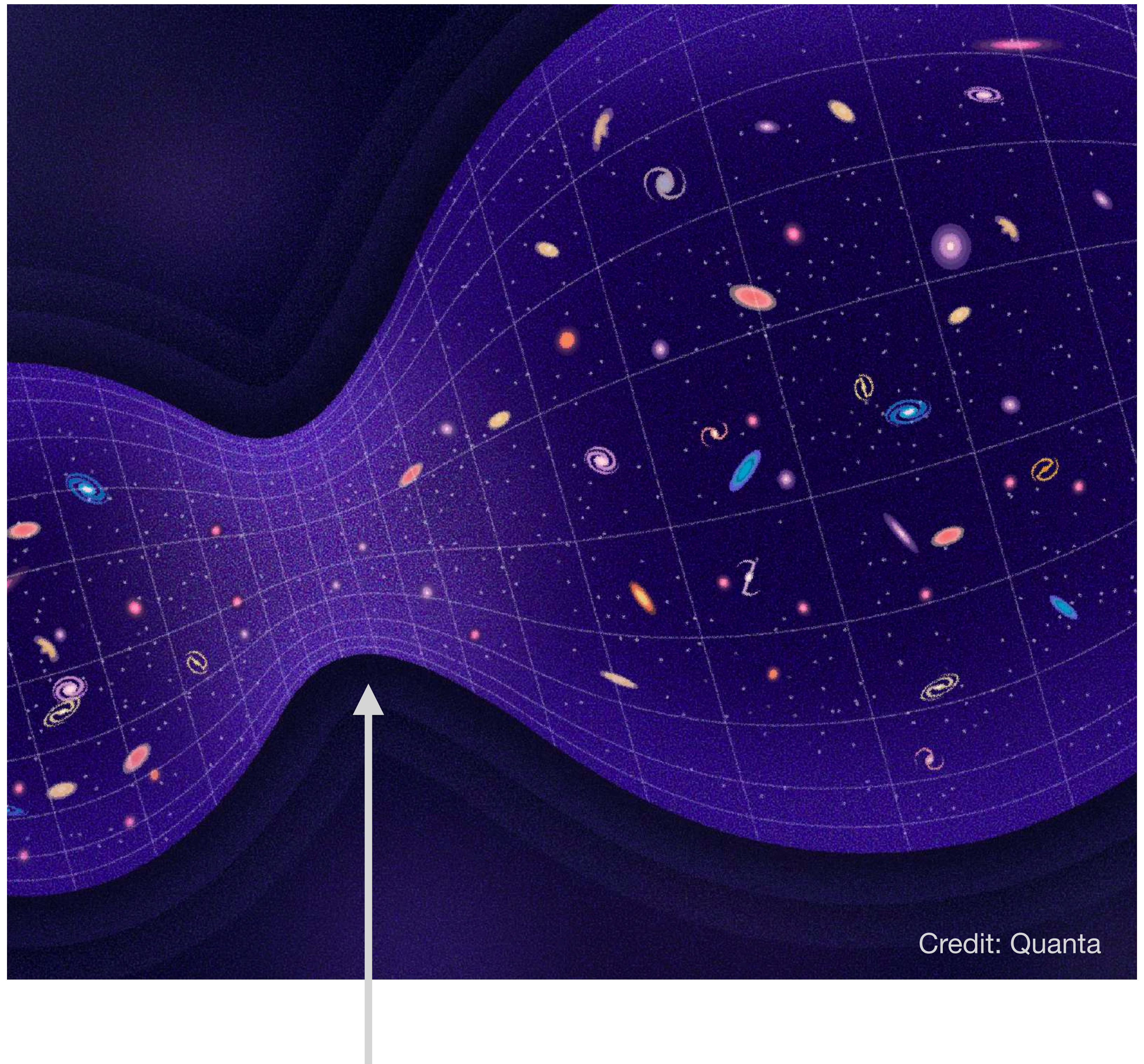
Credit: Quanta

Bouncing models

This *"bounce"* can happen in many ways:



Singular: Big Bang/Big Crunch

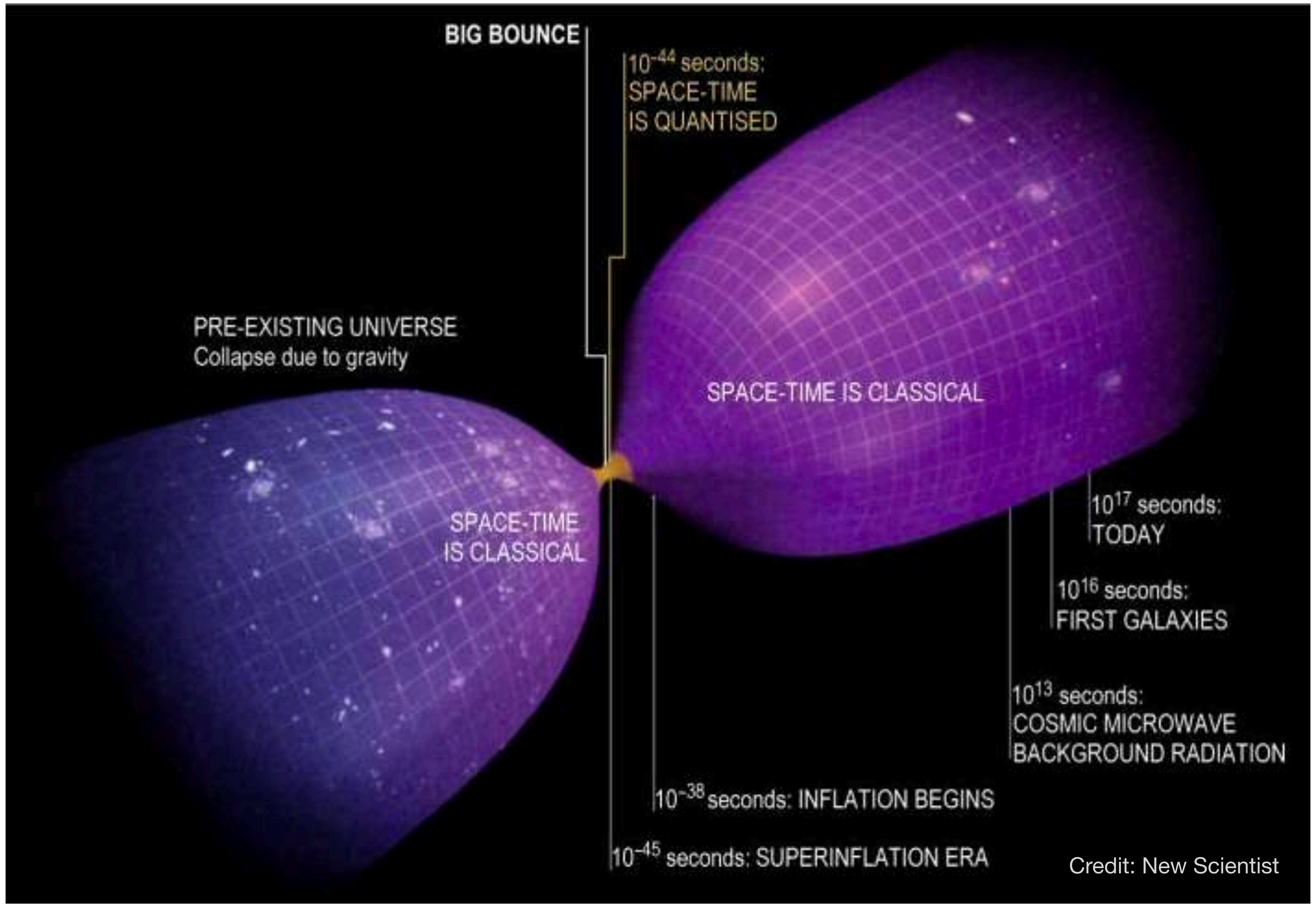


Classical bounce

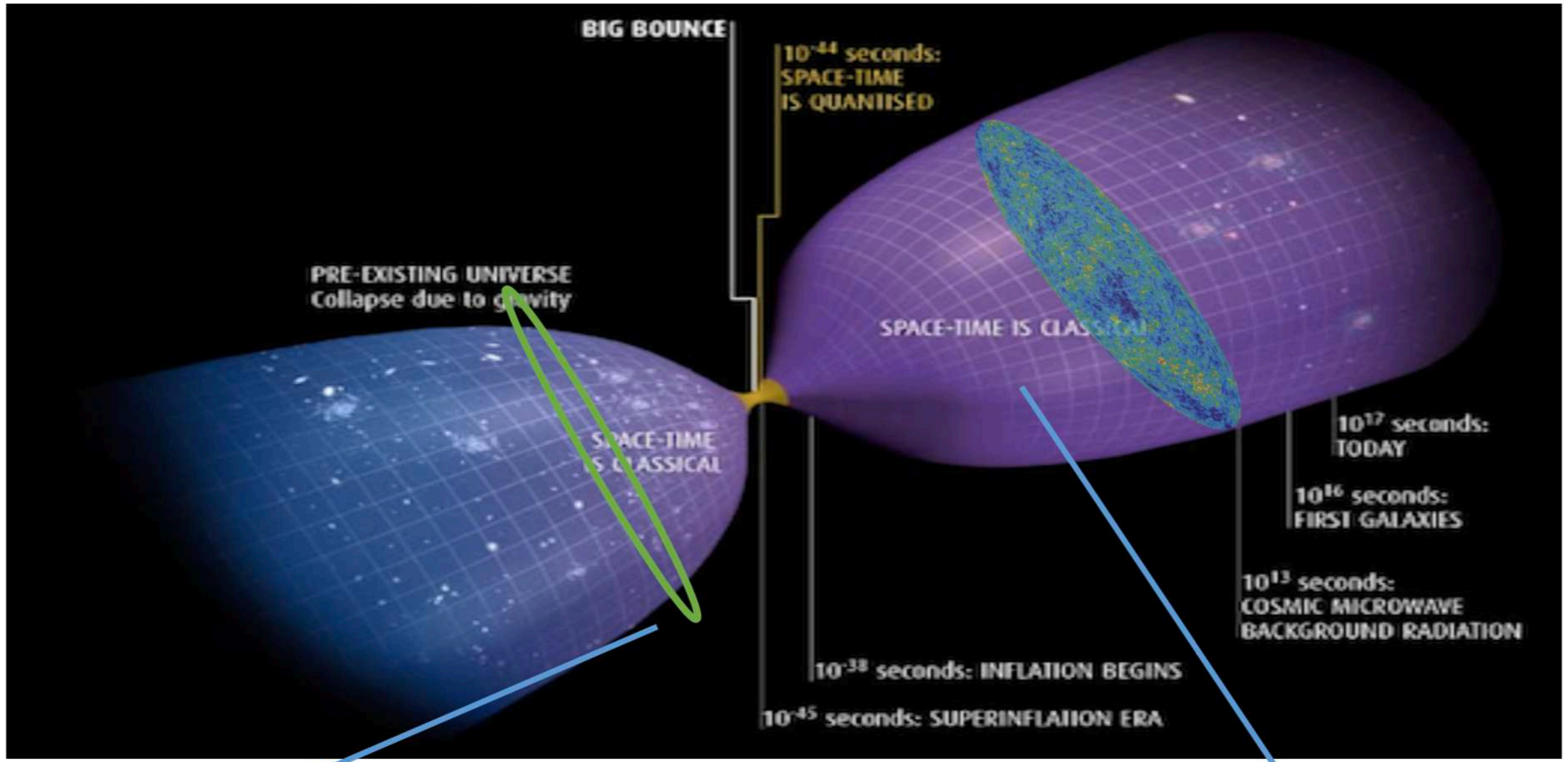
Bouncing models

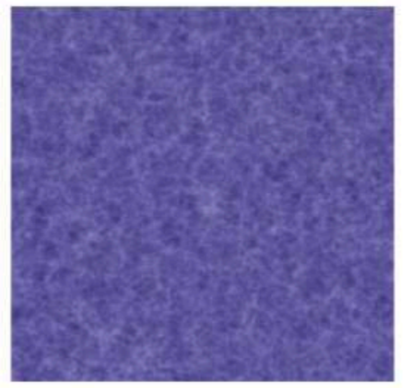
How can we make this?

- Modified gravity
- New components that dominate the universe at contraction
  - Violation of NEC
- …



Bouncing models



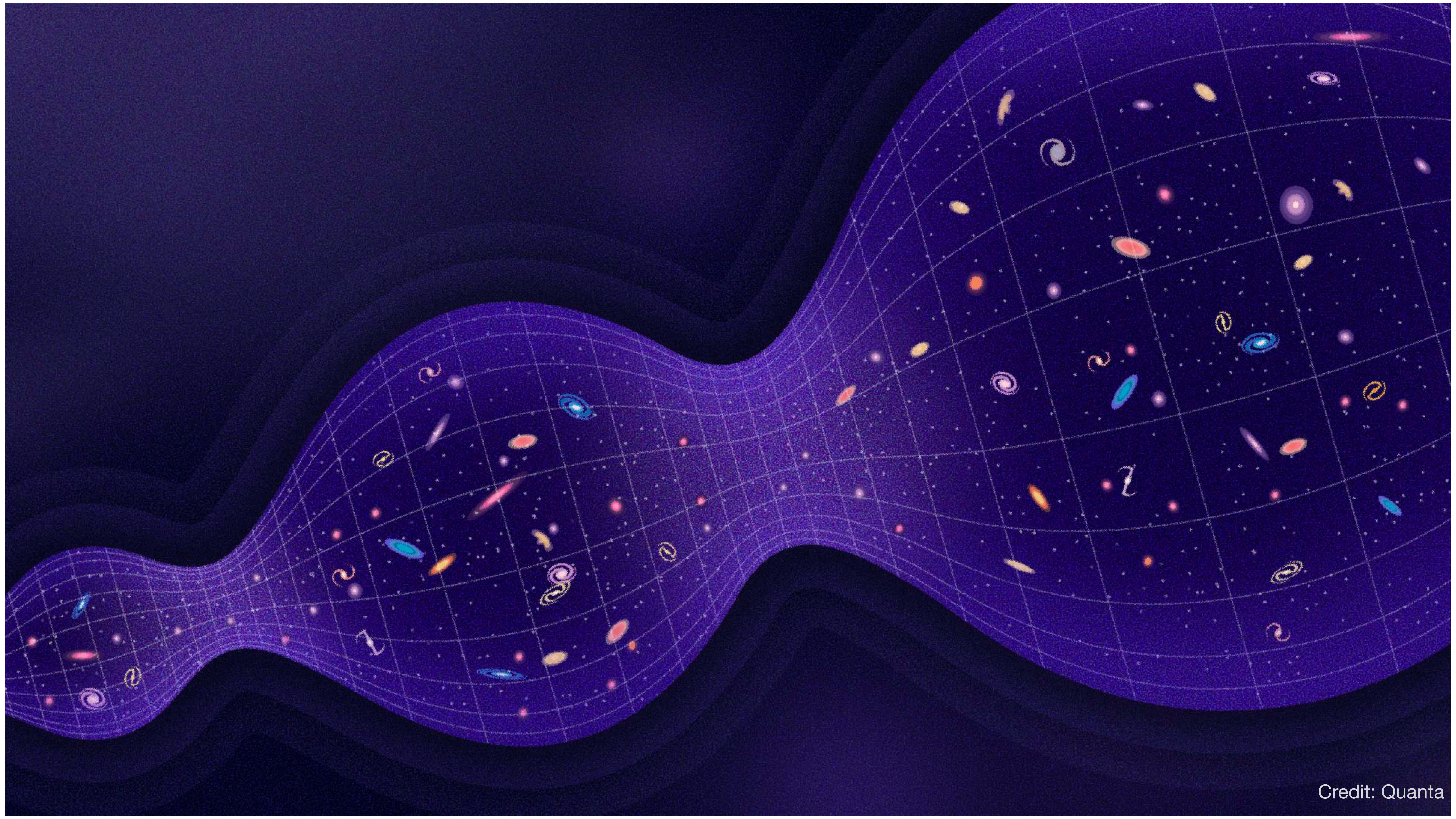


Smoothes and makes the universe flat
Can generate the almost scale invariant spectrum

SCM

Cyclic models

Multiple contraction and expansion periods

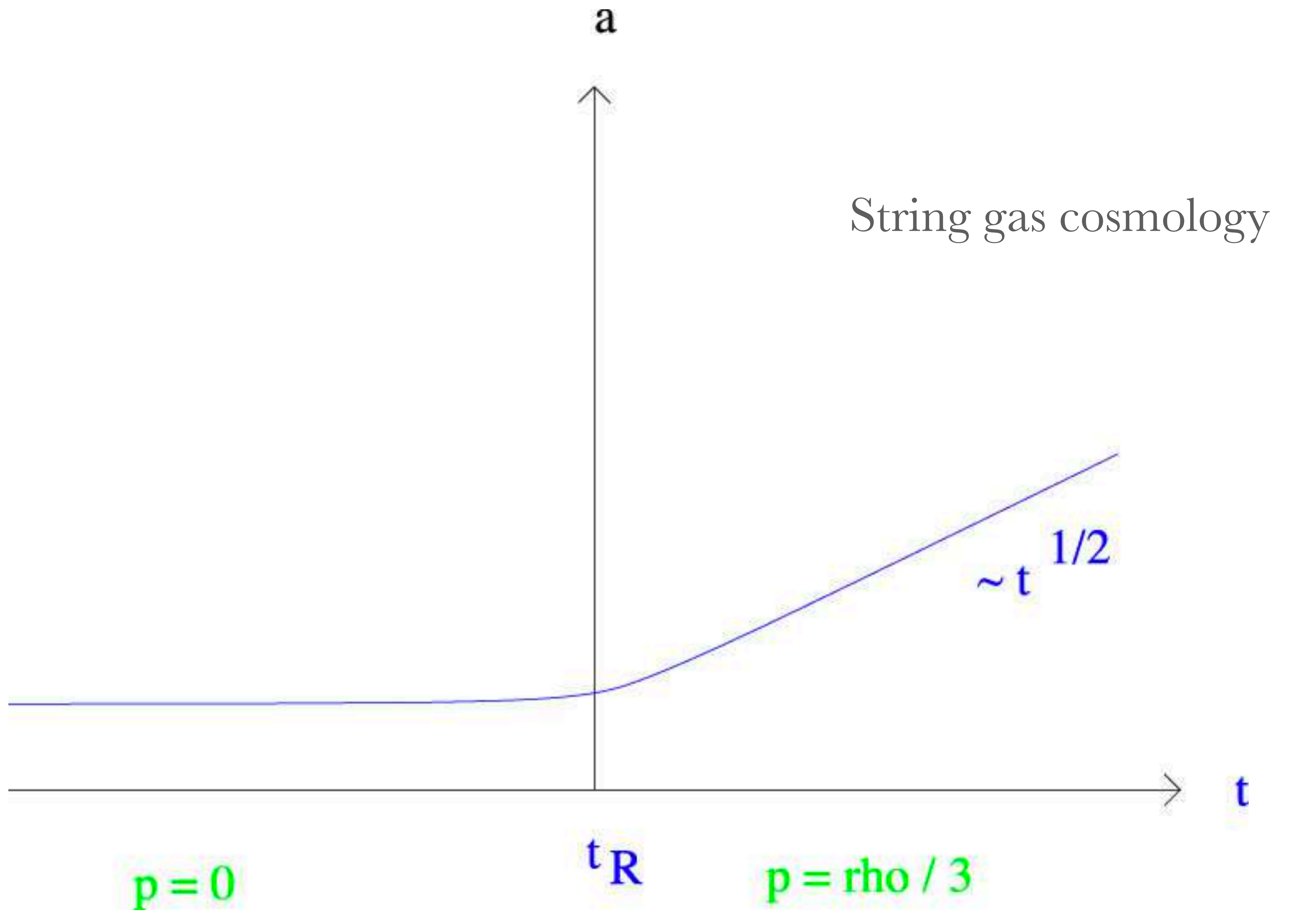


Emergent universe models

No initial singularity. Universe emerges from an initial state

Examples:

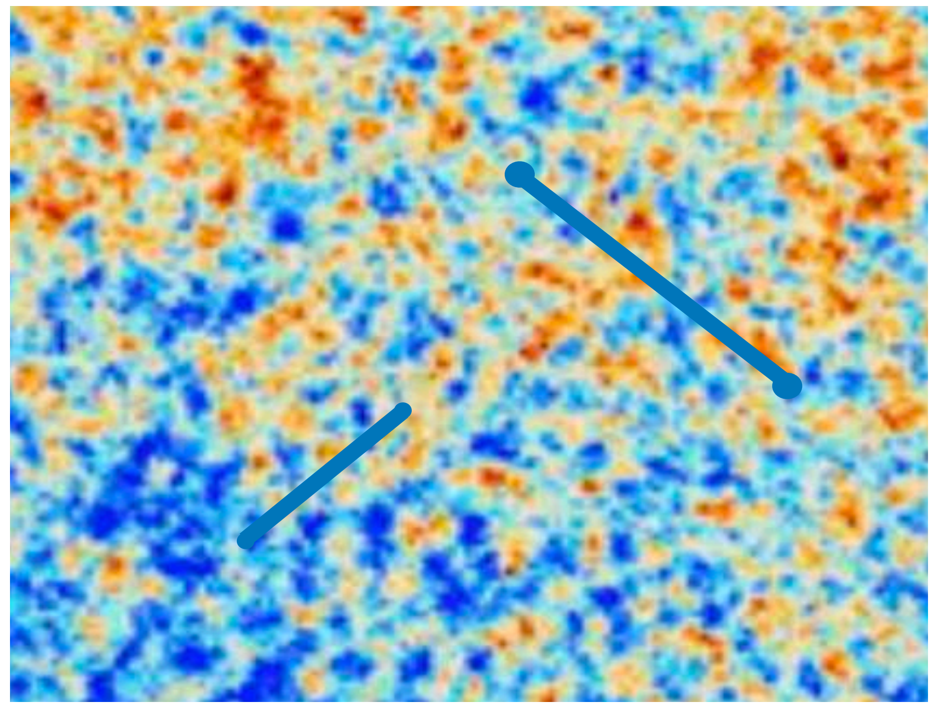
- Static initial stage
- *String gas cosmology*



R. Brandenberger (2015)

Spectrum of the initial perturbations

All of these models have prediction that are in agreement with the measurement from CMB (and LSS)



Predictions agree with what is measured in the CMB!

$$P(k) = A_S \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$\Omega_b = 0.0484 \pm 0.0003$	$\longrightarrow$	Amount of visible/standard matter
$\Omega_m = 0.308 \pm 0.012$	$\longrightarrow$	Amount of dark matter
$\Omega_\Lambda = 0.692 \pm 0.012$	$\longrightarrow$	Amount of dark energy
$\mathrm{n}_s = 0.9626 \pm 0.0057$	$\longrightarrow$	Scale-dependency of the initial fluctuations
$10^9\, A_s = 2.092 \pm 0.034$	$\longrightarrow$	Amplitude of the initial fluctuations
$\tau = 0.0522 \pm 0.0080$	$\longrightarrow$	Optical depth

$\mathrm{n}_s \longrightarrow$ Scale-dependency of the initial fluctuations

$A_s \longrightarrow$ Amplitude of the initial fluctuations

How to distinguish between those models?

All of these models have prediction that are in agreement with the measurement from CMB (and LSS) $\rightarrow$ (n_S, A_S)

So, how can we distinguish these models?

We need to look for predictions that are distinct…

How to distinguish between those models?

Gravitational waves

Besides creating the desnity fluctuations, early universe models also produce gravitacional waves

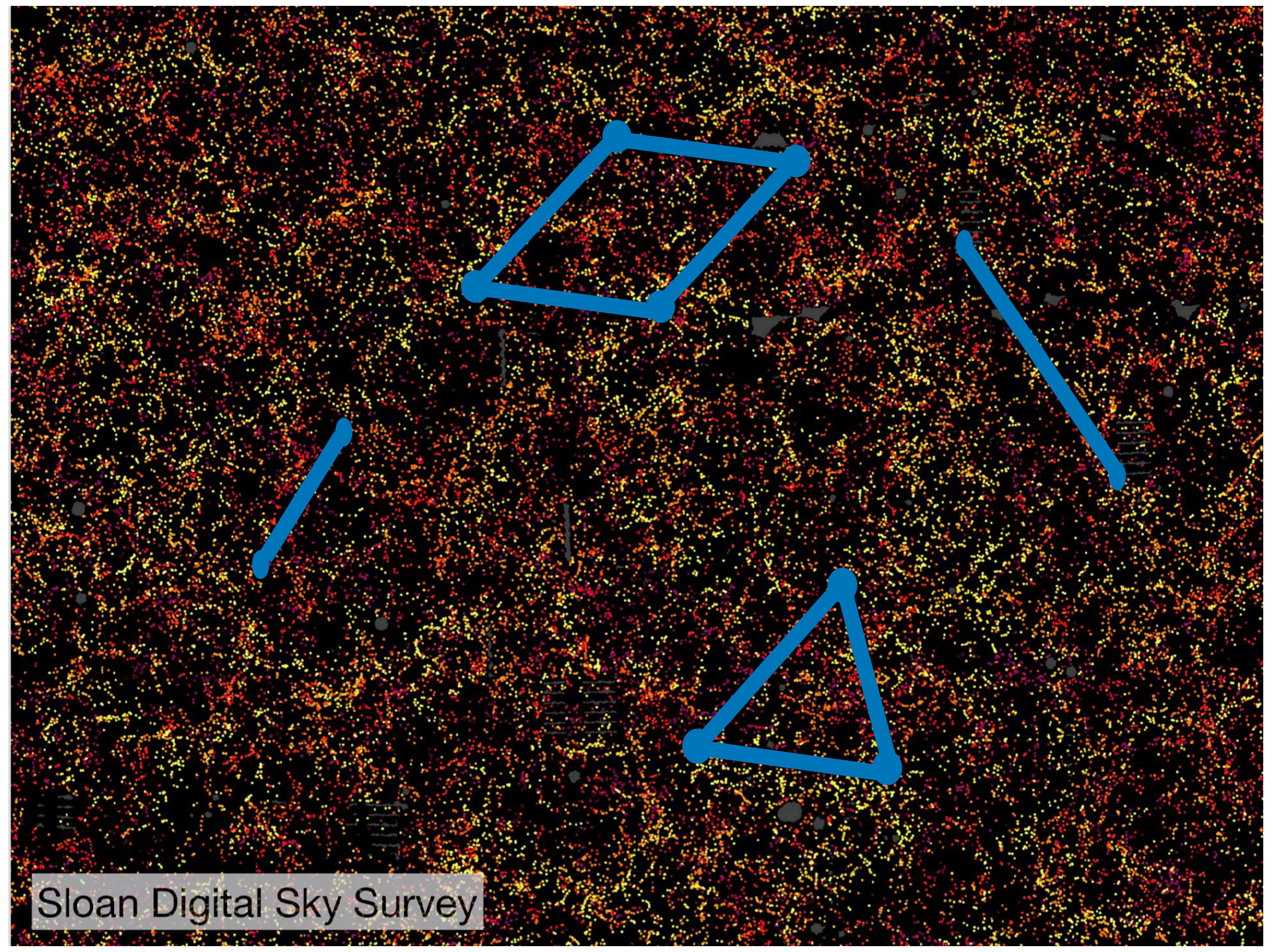
Different models, like inflation and bouncing, have different predictions for them. Even among inflationary models, we have different predictions.

Measuring the primordial GWB would allow us to distinguish between some of these models.tre alguns desses modelos.

+ ? (we have to be creative)

Non-Gaussianities

If the distribution is Gaussian, all of the information is contained in the 2 point correlation function. Otherwise, we have to compute the n-point correlation function:



$$\langle \delta\delta\delta \rangle$$

$$\langle \delta\delta\delta\delta \rangle$$

$$\langle \delta \dots \delta \rangle$$

Very challenging! One of the goals of current cosmology obs

Questions?

Big question

Solve the SCM problems!!

More specifically, find a way to solve the horizon problem or/and flatness problem.

Inflation is a way.
Can you show that this works?

BIG question: is there other way?
Can you show that if we assume something else for the evolution of the particle horizon, this will also solve the horizon problem?

(Can you show that bouncing works?)

Exercise

1. Horizon problem

Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

1.1 Using the Friedmann equations, show that $(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}$

1.2 Show that $\eta \sim (aH)^{-1} \sim a^{\frac{1}{2}(1+3w)}$

which means that the comoving particle horizon is in the same order of magnitude as the comoving Hubble radius

1.3 What is $a(\eta)$ for MD or RD?

Can you see that assuming RD or MD, would imply existence of the Big Bang singularity at $\eta_i = 0$ $(a(\eta_i = 0) = 0)$

PS: Remember, where it is zero or 1 is a definition!

1.4 Show what happens to the particle horizon is $w > -1/3$ and if $w < -1/3$

Exercise

2. Flatness problem

2.1 Show that $\frac{d\Omega}{dlna} = (1 = 3w)(\Omega - 1)$

2.2 Show the fine tuning of the flatness problem. Show that:

$$|\Omega(a_{\rm BBN}) - 1| < \mathcal{O}(10^{-16})$$
$$|\Omega(a_{\rm GUT}) - 1| < \mathcal{O}(10^{-55})$$
$$|\Omega(a_{\rm Pl}) - 1| < \mathcal{O}(10^{-61})$$

(Remember that at RD $T \sim a^{-1}$)

2.3 If $\frac{d|\Omega - 1|}{dlna} > 0$, what this means for the equation of state?

Show that this solves the flatness problem

Big question

Solve the SCM problems!!

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Can you show that if we assume something else for the evolution of the particle horizon, this will also solve the horizon problem?

(Can you show that bouncing works?)

Guided question

Inflation is a way.

Can you show that this works?

For simplicity, let's assume de Sitter, or exponential expansion during inflation

(Bonus question: Can you think why this is not a good model for inflation? Think about what has to come next)

$$\begin{cases} H = const. \\ a(t) \sim \exp(Ht) \end{cases}$$

Calculate:

- conformal time
- $a(\eta)$
- Particle horizon

From that, convince yourself that this solves the horizon problem. Use the same and the Friedmann equation to see how this solves the flatness problem.

Guided question

BIG question: is there other way?

Can you show that if we assume something else for the evolution of the particle horizon, this will also solve the horizon problem?

If we focus on bouncing (we don't have to…your choice):

The key idea is that, unlike in standard Big Bang cosmology, in a bouncing universe the current expanding phase was preceded by a contracting phase, which allows distant regions of the universe to come into causal contact before the bounce.

The main idea is the same:
- shrinking of the Hubble radius H^{-1} during contraction

Guided question

Idea 2: Shrinking of the Hubble radius during contraction

(Toy model)

Let us consider the contracting phase where the universe is dominated by a perfect fluid with constant equation of state w

The contracting phase is described by:

$$a(t) \propto (-t)^{\frac{2}{3(1+w)}}, \quad t < 0$$

with the bounce happening at $t = 0$

Exercise: can you find this from the Friedman equations?

Compute: Hubble parameter, conformal time, comoving particle horizon

Answer next page…but try to work it out first.

[redacted]

Guided question

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with the bounce happening at $t = 0$

Exercise: can you find this from the Friedman equations?

Compute: Hubble parameter, conformal time, comoving particle horizon

Remove the back box to see answer…but try to work it out first.

$$H(t) = \frac{2}{3(1+w)} \cdot \frac{1}{t}, \quad \text{but negative, so:} \quad H(t) = -\frac{2}{3(1+w)} \cdot \frac{1}{|t|}$$

$$\eta(t) \propto (-t)^{\frac{1-w}{1+w}}$$

Guided question

Idea 2: Shrinking of the Hubble radius during contraction

$$a(t) \propto (-t)^{\frac{2}{3(1+w)}}, \quad t < 0$$

(Toy model)

Different bouncing models:

1. Matter-dominated contraction: $w = 0$
2. Radiation-dominated contraction: $w = 1/3$
3. Ekpyrotic contraction: $w \gg 1$

Answer next page…but try to work it out first.

[redacted]

Guided question

Idea 2: Shrinking of the Hubble radius during contraction

$$a(t) \propto (-t)^{\frac{2}{3(1+w)}}, \quad t < 0$$

(Toy model)

Different bouncing models:

1. Matter-dominated contraction: $w = 0$
2. Radiation-dominated contraction: $w = 1/3$
3. Ekpyrotic contraction: $w \gg 1$

- **Matter-dominated contraction:** $w = 0$
  - $a(t) \propto (-t)^{2/3}$
  - $H(t) = -\frac{2}{3|t|}$
  - $\eta(t) \propto (-t)^{1/3}$

- **Radiation-dominated contraction:** $w = 1/3$
  - $a(t) \propto (-t)^{1/2}$
  - $H(t) = -\frac{1}{2|t|}$
  - $\eta(t) \propto (-t)^{1/2}$

- **Ekpyrotic contraction:** $w \gg 1$
  - $a(t) \propto (-t)^{\epsilon}$, with $\epsilon \ll 1$
  - $H(t) \sim -\frac{1}{|t|}$
  - $\eta(t) \propto (-t)^{1-\epsilon} \approx (-t)$

This is nearly Minkowski-like: $\eta \sim t$.

Guided question

Idea 2: Shrinking of the Hubble radius during contraction

$$a(t) \propto (-t)^{\frac{2}{3(1+w)}}, \quad t < 0$$

(Toy model)

Different bouncing models:

With those results:

- Compare the size of the particle horizon with the one from SCM

  Why this solves the problem?

- Plot this!!

<u>BONUS question</u>: how this solves the **flatness problem**? Use the same equations as before…

Guided question

Key Intuition

Idea 1: In inflation, the universe expands rapidly, pushing regions out of causal contact but originating from a small, connected patch.

Idea 2: In bouncing cosmologies, regions start large and contract, so they can easily interact before bouncing and expanding again.


Thank you very much!