Low energy excitations in CRPA calculations

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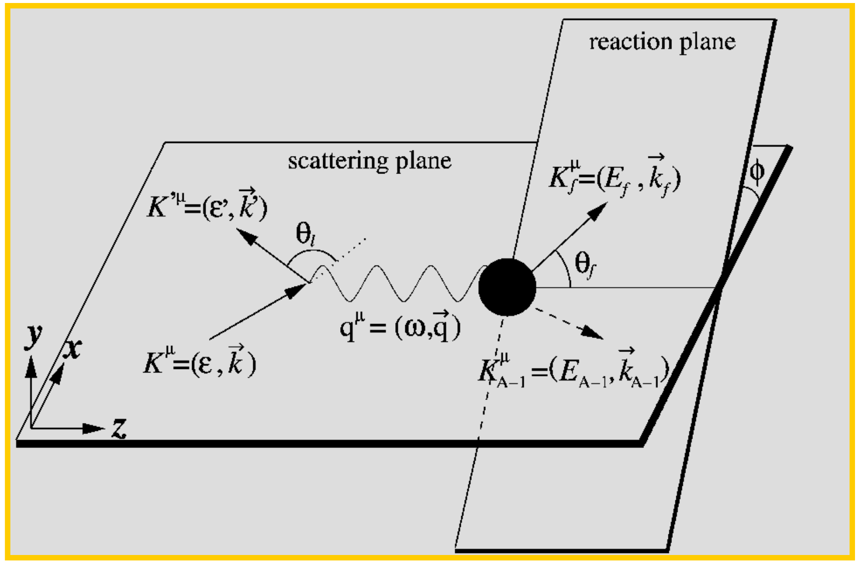
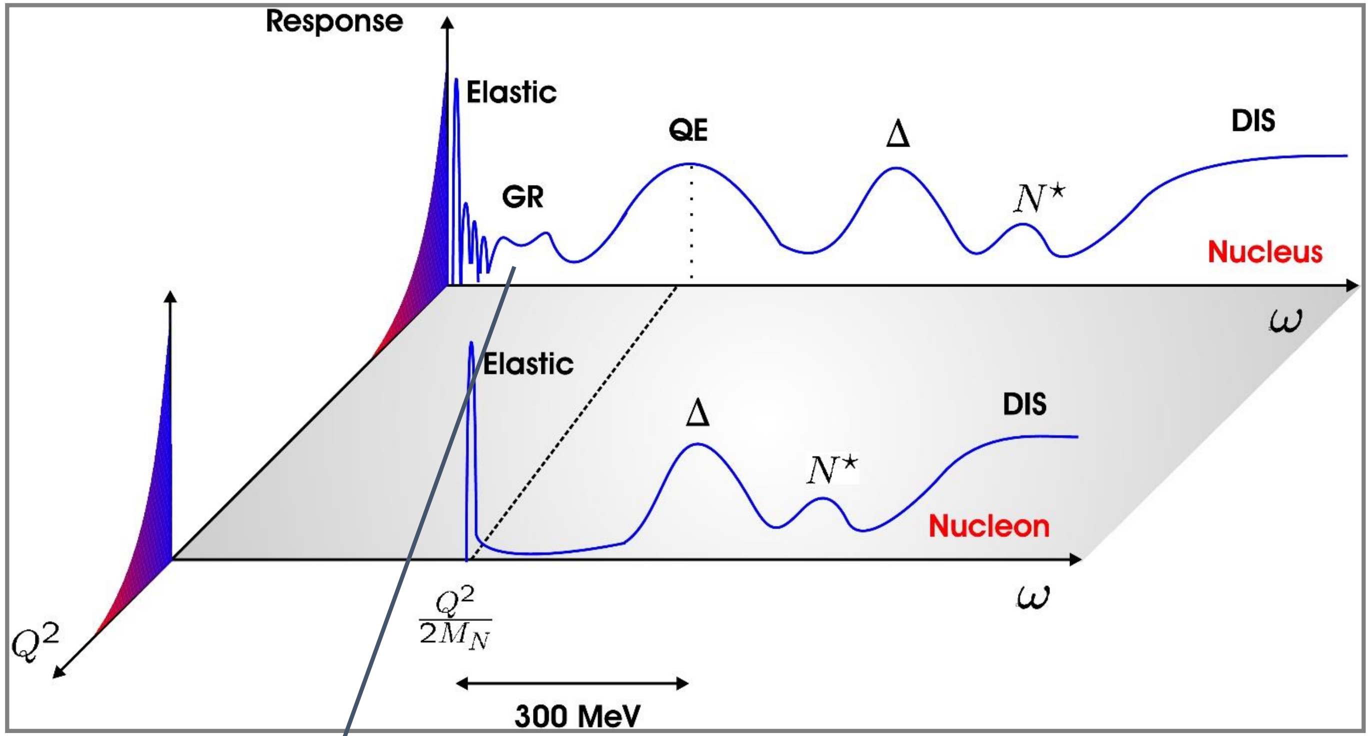
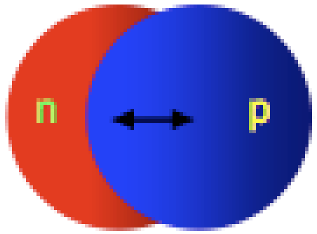
Neutrino-hadron scattering



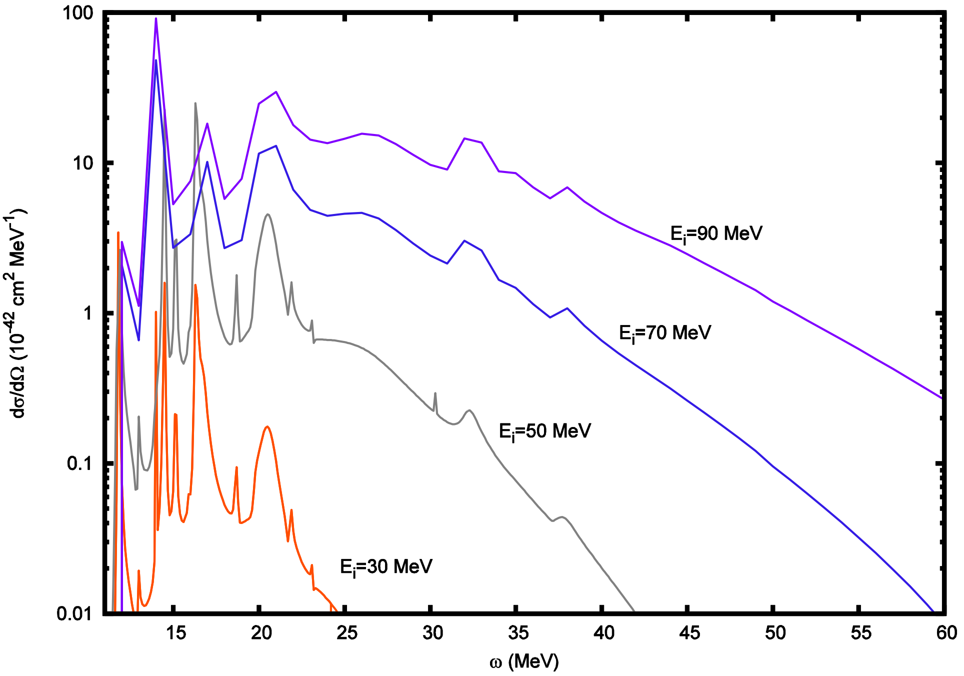


e.g.

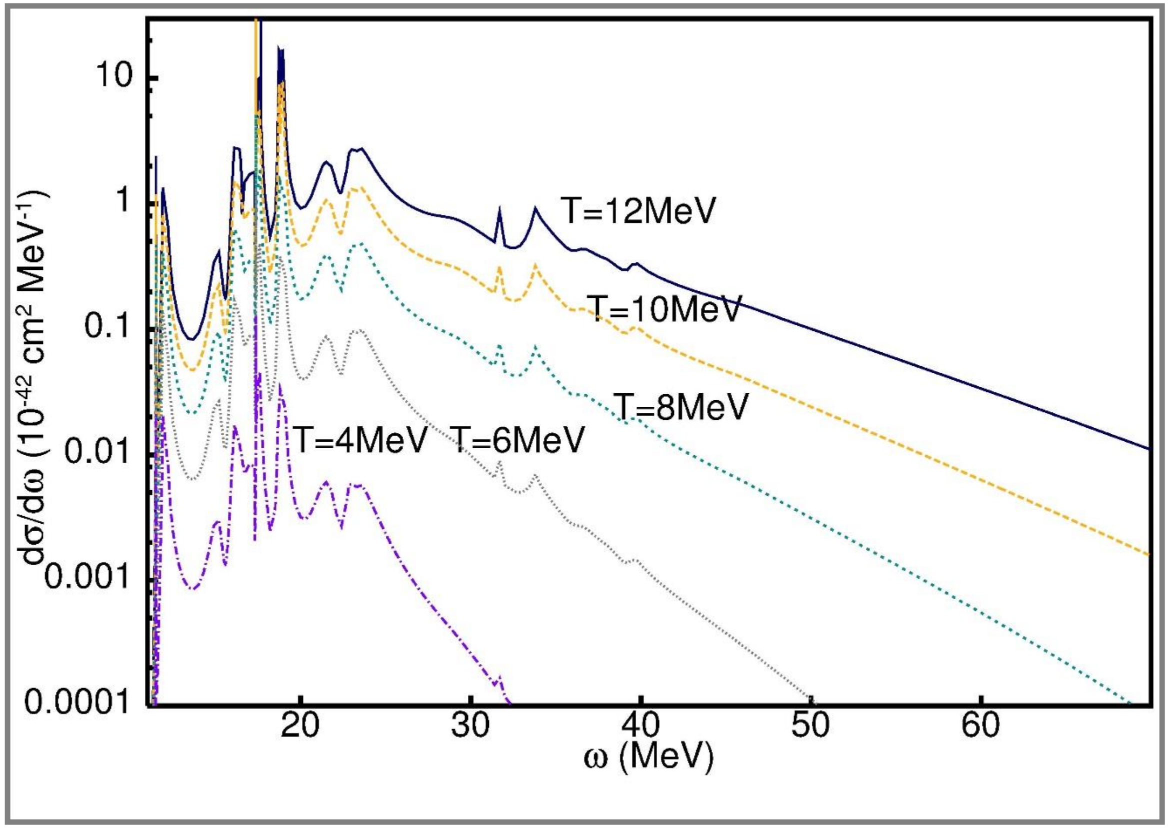


Neutrino scattering results at low energies :

$^{16}O(\nu, \nu')\,^{16}O^*$

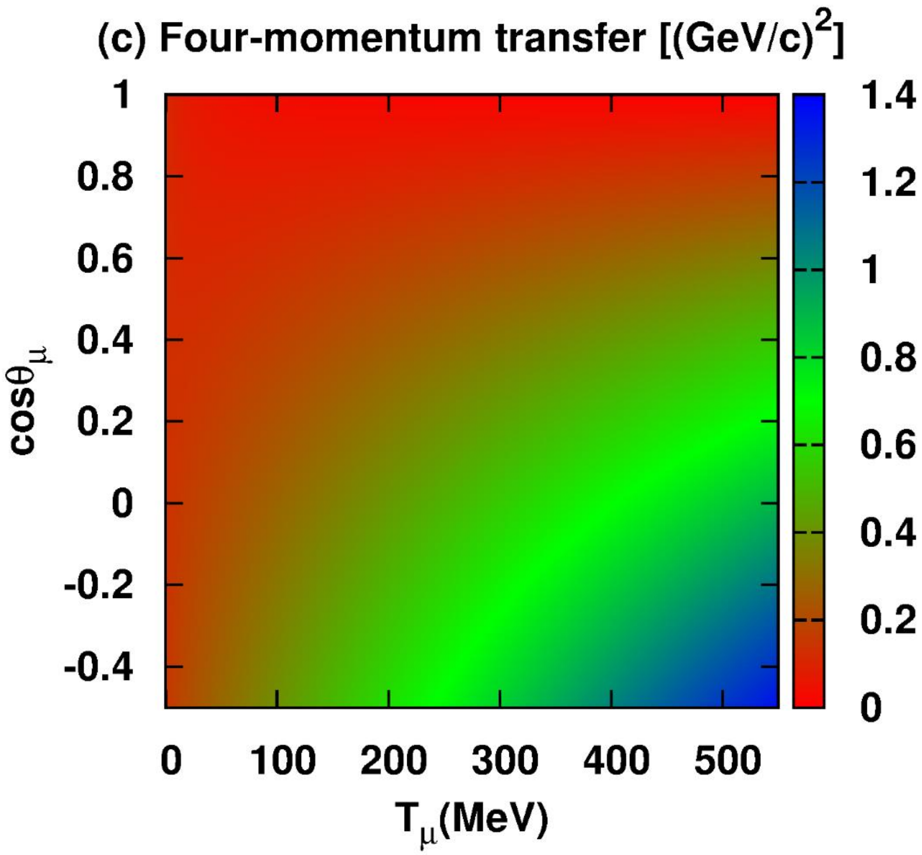


Folded cross sections supernova neutrino spectra :



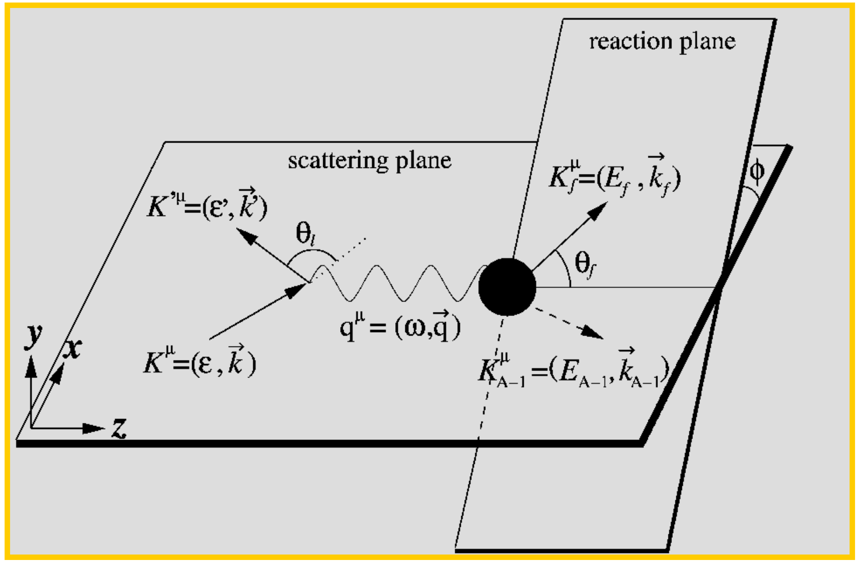
What is ‘low energy’ ?

E_ν=700MeV



Neutrino-nucleus interactions

$$\widehat{H}_W = \frac{G}{\sqrt{2}} \int d\vec{x}\, \hat{j}_{\mu,lepton}(\vec{x})\, \hat{J}^{\mu,hadron}(\vec{x})$$



Hadron current

$$J^{\mu} = F_1(Q^2)\gamma^{\mu} + i\frac{\kappa}{2M_N}F_2(Q^2)\sigma^{\mu\nu}q_{\nu} + G_A(Q^2)\gamma^{\mu}\gamma_5 + \frac{1}{2M_N}G_P(Q^2)q^{\mu}\gamma_5$$

Lepton tensor

$$l_{\alpha\beta} \equiv \overline{\sum_{s,s'}}[\overline{u}_l\gamma_{\alpha}(1-\gamma_5)u_l]^{\dagger}[\overline{u}_{\nu}\gamma_{\beta}(1-\gamma_5)u_{\nu}]$$

$$\vec{J}_V^{\alpha}(\vec{x}) = \vec{J}^{\alpha}_{convection}(\vec{x}) + \vec{J}^{\alpha}_{magnetization}(\vec{x})$$

with

$$\vec{J}_c^{\alpha}(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_E^{i,\alpha}\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\,\delta(\vec{x}-\vec{x}_i)\right],$$

$$\vec{J}_m^{\alpha}(\vec{x}) = \frac{1}{2M}\sum_{i=1}^{A} G_M^{i,\alpha}\,\overrightarrow{\nabla}\times\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$\vec{J}_A^{\alpha}(\vec{x}) = \sum_{i=1}^{A} G_A^{i,\alpha}\,\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$J_V^{0,\alpha}(\vec{x}) = \rho_V^{\alpha}(\vec{x}) = \sum_{i=1}^{A} G_E^{i,\alpha}\,\delta(\vec{x}-\vec{x}_i),$$

$$J_A^{0,\alpha}(\vec{x}) = \rho_A^{\alpha}(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_A^{i,\alpha}\,\vec{\sigma}_i\cdot\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\,\delta(\vec{x}-\vec{x}_i)\right]$$

$$J_P^{0,\alpha}(\vec{x}) = \rho_P^{\alpha}(\vec{x}) = \frac{m_\mu}{2M}\sum_{i=1}^{A} G_P^{i,\alpha}\,\vec{\nabla}\cdot\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i)$$

for NC reactions

$$G_E^{V,o} = \left(\frac{1}{2}-\sin^2\theta_W\right)\tau_3 - \sin^2\theta_W,$$

$$G_M^{V,o} = \left(\frac{1}{2}-\sin^2\theta_W\right)(\mu_p-\mu_n)\,\tau_3 - \sin^2\theta_W\,(\mu_p+\mu_n)$$

$$G^{A,0} = g_a\,\frac{\tau_3}{2} = -\,\frac{1.262}{2}\,\tau_3$$

for CC reactions

$$G_E^{V,\pm} = \tau_\pm$$

$$G_M^{V,\pm} = (\mu_p-\mu_n)\,\tau_\pm$$

$$G^{A,\pm} = g_a\,\tau_\pm = -1.262\,\tau_\pm$$

$G = (1+Q^2/M^2)^{-2}$ Q^2 dependence : dipole parametrization

Cross section

$$\frac{d^2\sigma}{d\Omega\, d\omega} = (2\pi)^4\, k_f \varepsilon_f \sum_{s_f, s_i} \frac{1}{2J_i+1} \sum_{M_f, M_i} \left| \left\langle f \left| \widehat{H}_W \right| i \right\rangle \right|^2$$

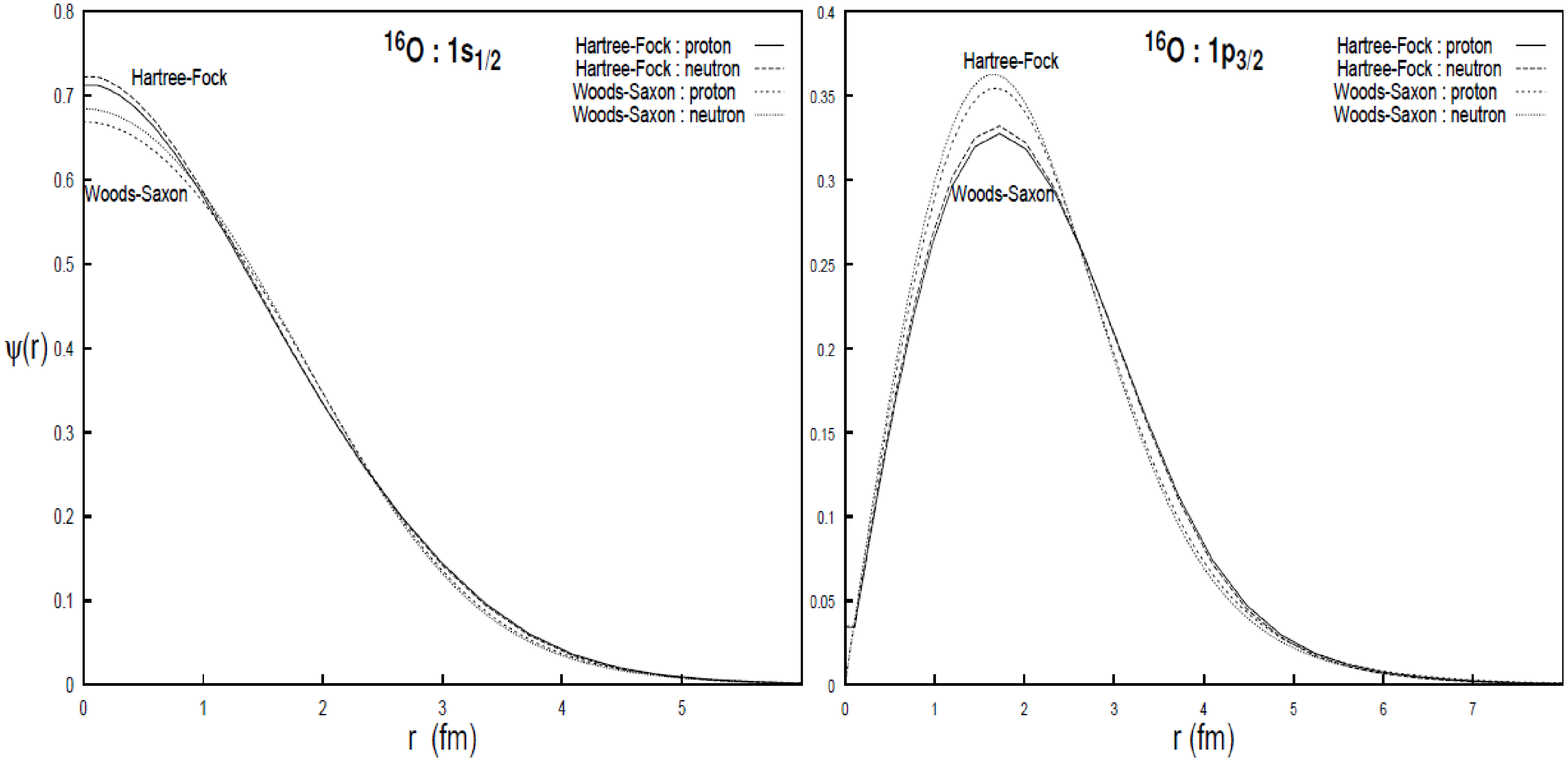
$$\left(\frac{d^2\sigma_{i\to f}}{d\Omega d\omega} \right)_{\frac{\nu}{\bar{\nu}}} = \frac{G^2 \varepsilon_f^2}{\pi} \frac{2\cos^2\left(\frac{\theta}{2}\right)}{2J_i+1} \left[\sum_{J=0}^{\infty} \sigma_{CL}^J + \sum_{J=1}^{\infty} \sigma_T^J \right]$$

$$\sigma_{CL}^J = \left| \left\langle J_f \left\| \widehat{\mathcal{M}}_J(\kappa) + \frac{\omega}{|\vec{q}|} \widehat{\mathcal{L}}_J(\kappa) \right\| J_i \right\rangle \right|^2$$

$$\sigma_T^J = \left(-\frac{q_\mu^2}{2|\vec{q}|^2} + \tan^2\left(\frac{\theta}{2}\right) \right) \left[\left| \left\langle J_f \left\| \widehat{\mathcal{J}}_J^{mag}(\kappa) \right\| J_i \right\rangle \right|^2 + \left| \left\langle J_f \left\| \widehat{\mathcal{J}}_J^{el}(\kappa) \right\| J_i \right\rangle \right|^2 \right]$$

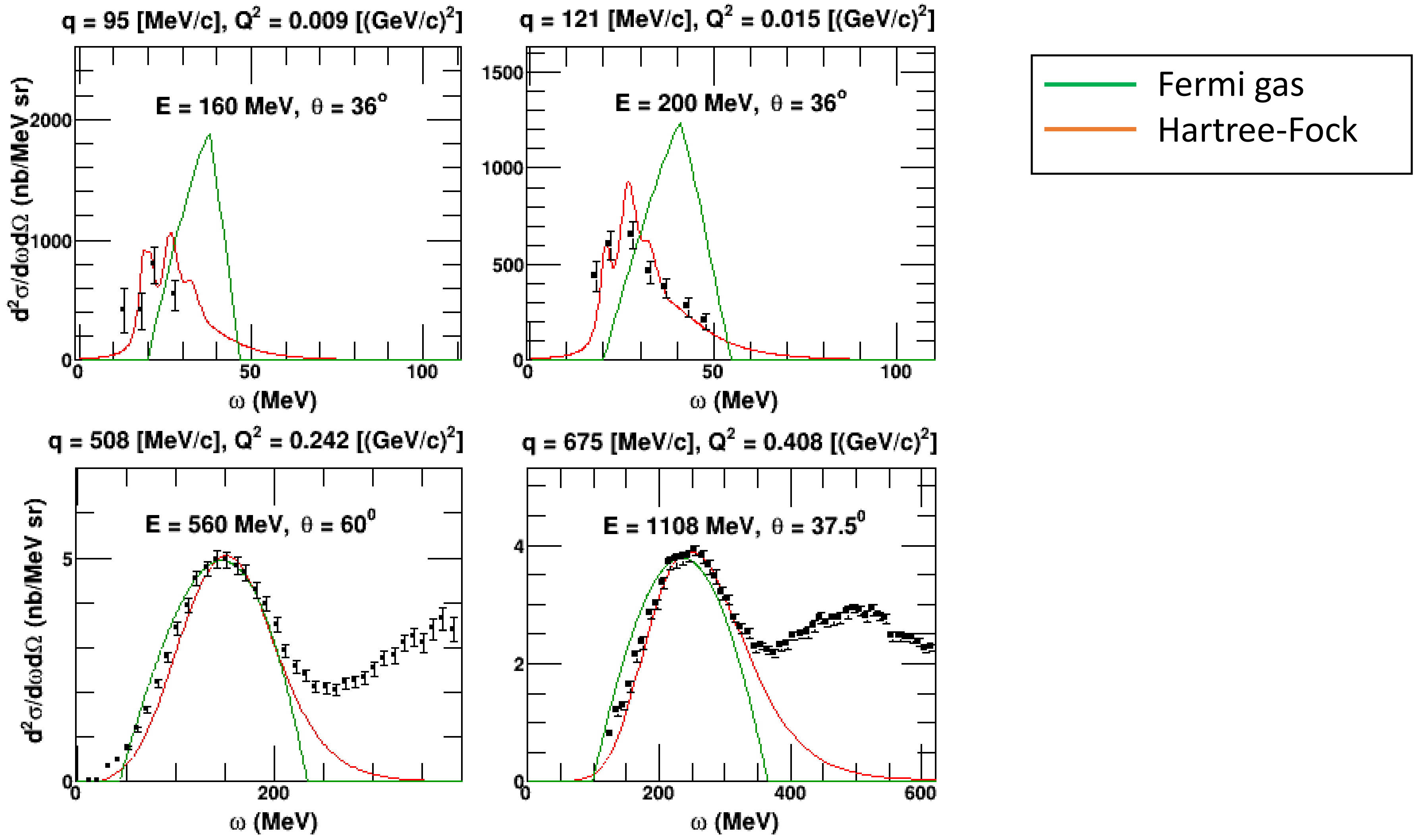
$$\mp \tan\left(\frac{\theta}{2}\right) \sqrt{-\frac{q_\mu^2}{|\vec{q}|^2} + \tan^2\left(\frac{\theta}{2}\right)} \left[2\Re\left(\left\langle J_f \left\| \widehat{\mathcal{J}}_J^{mag}(\kappa) \right\| J_i \right\rangle \left\langle J_f \left\| \widehat{\mathcal{J}}_J^{el}(\kappa) \right\| J_i \right\rangle^* \right) \right]$$

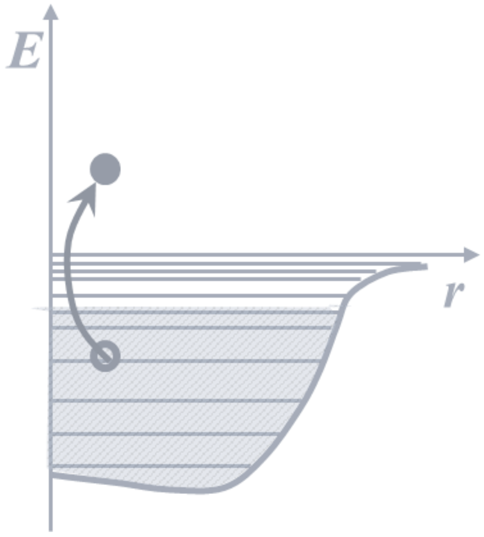
Bound state wave functions



Hartree-Fock single-particle wave functions (Skyrme)

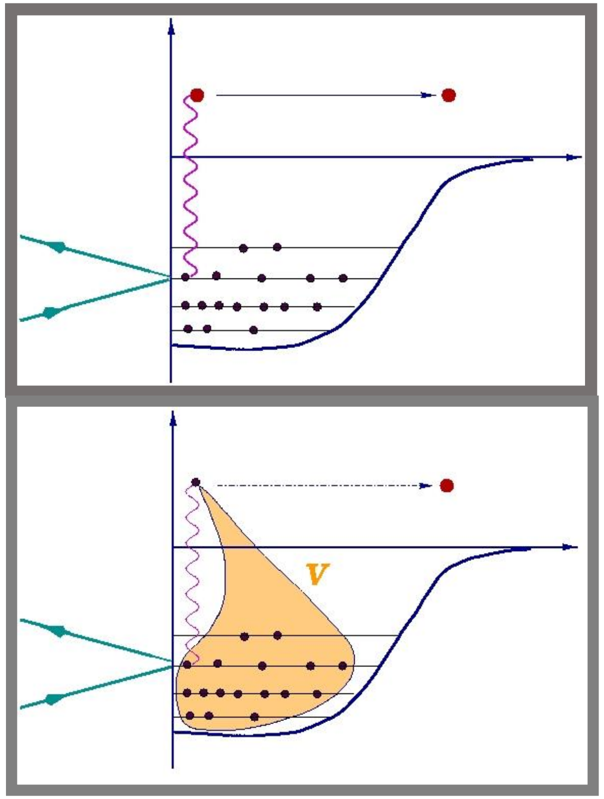
- Pauli blocking
- binding

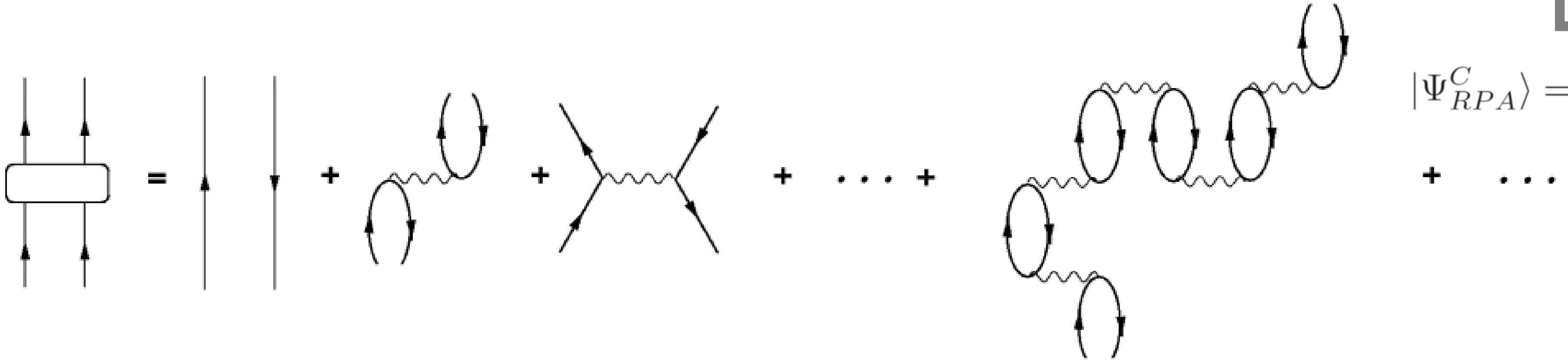

q = 95 [MeV/c], Q^2 = 0.009 [(GeV/c)^2]
E = 160 MeV, θ = 36°
q = 121 [MeV/c], Q^2 = 0.015 [(GeV/c)^2]
E = 200 MeV, θ = 36°
q = 508 [MeV/c], Q^2 = 0.242 [(GeV/c)^2]
E = 560 MeV, θ = 60°
q = 675 [MeV/c], Q^2 = 0.408 [(GeV/c)^2]
E = 1108 MeV, θ = 37.5°
$d^2\sigma/d\omega d\Omega$ (nb/MeV sr)
ω (MeV)
Fermi gas
Hartree-Fock



Continuum RPA

- Accounts for long-range correlations
- Green's function approach
- Skyrme SkE2 residual interaction
- ground state : Hartree-Fock single-particle wave functions (Skyrme)
- self-consistent calculations



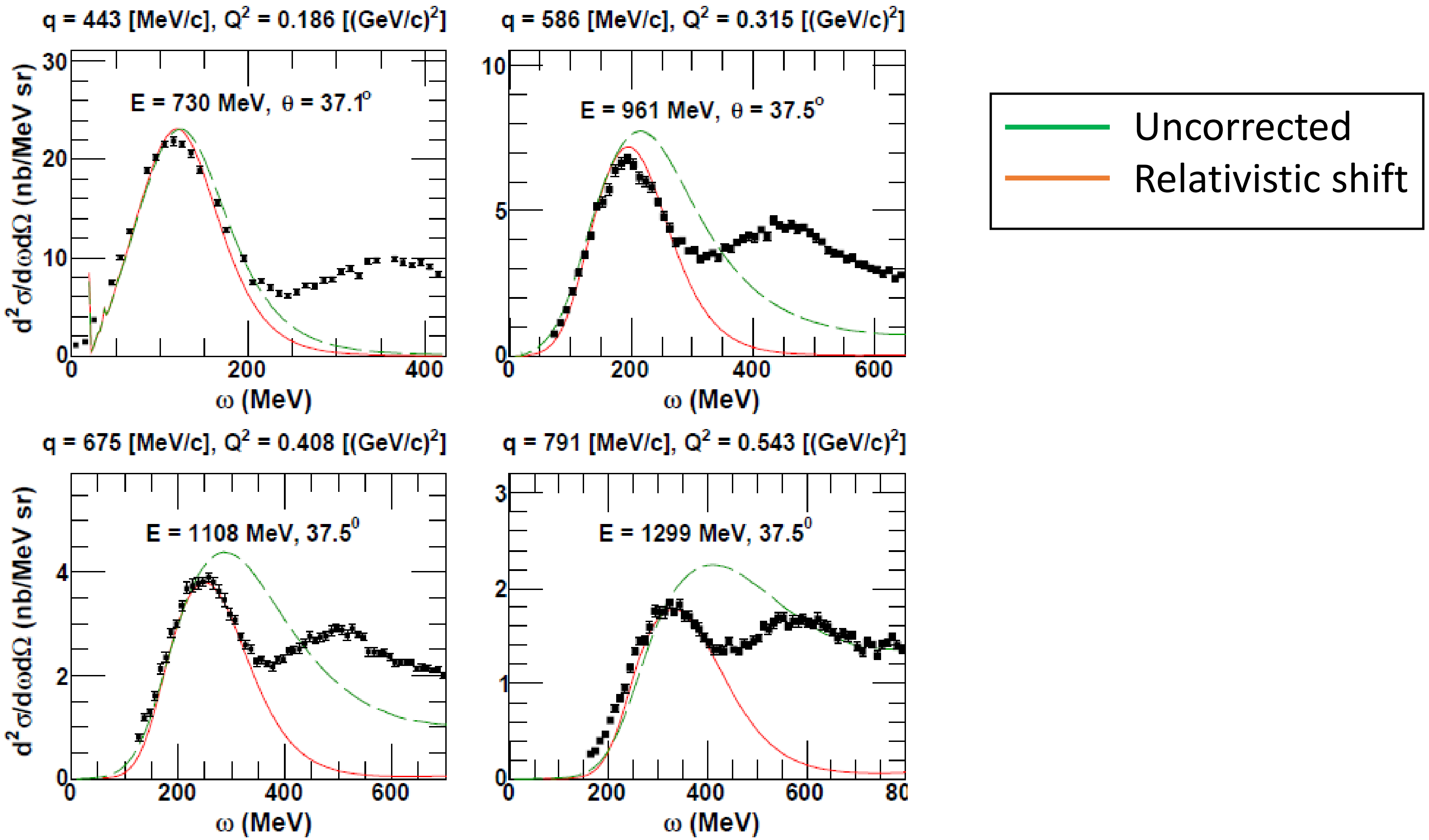


$$|\Psi^{C}_{RPA}\rangle = \sum_{C'} \left[X_{C,C'} \; |p'h'^{-1}\rangle - \; Y_{C,C'} \; |h'p'^{-1}\rangle \right]$$

$$\Pi^{(RPA)}(x_1, x_2; E_x) = \Pi^{(0)}(x_1, x_2; E_x) + \frac{1}{\hbar} \int dx dx' \Pi^{0}(x_1, x; E_x) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; E_x)$$

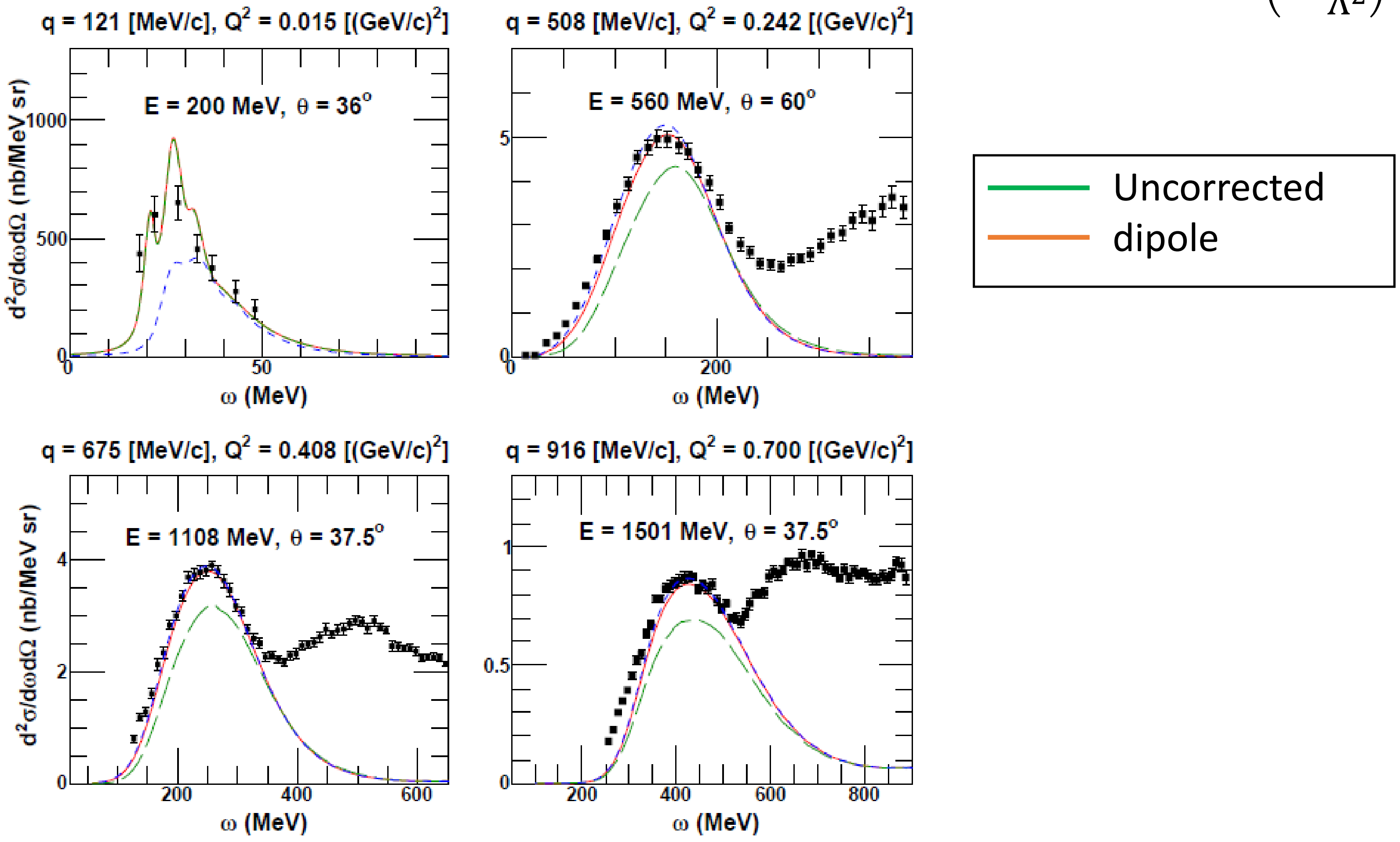
•Relativistic corrections at higher energies (J. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):

$$\lambda \to \lambda(\lambda + 1) \qquad \lambda = \omega / 2M_N$$



•Regularization of the residual interaction :

$$V(Q^2) = V(Q^2 = 0)\frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}$$

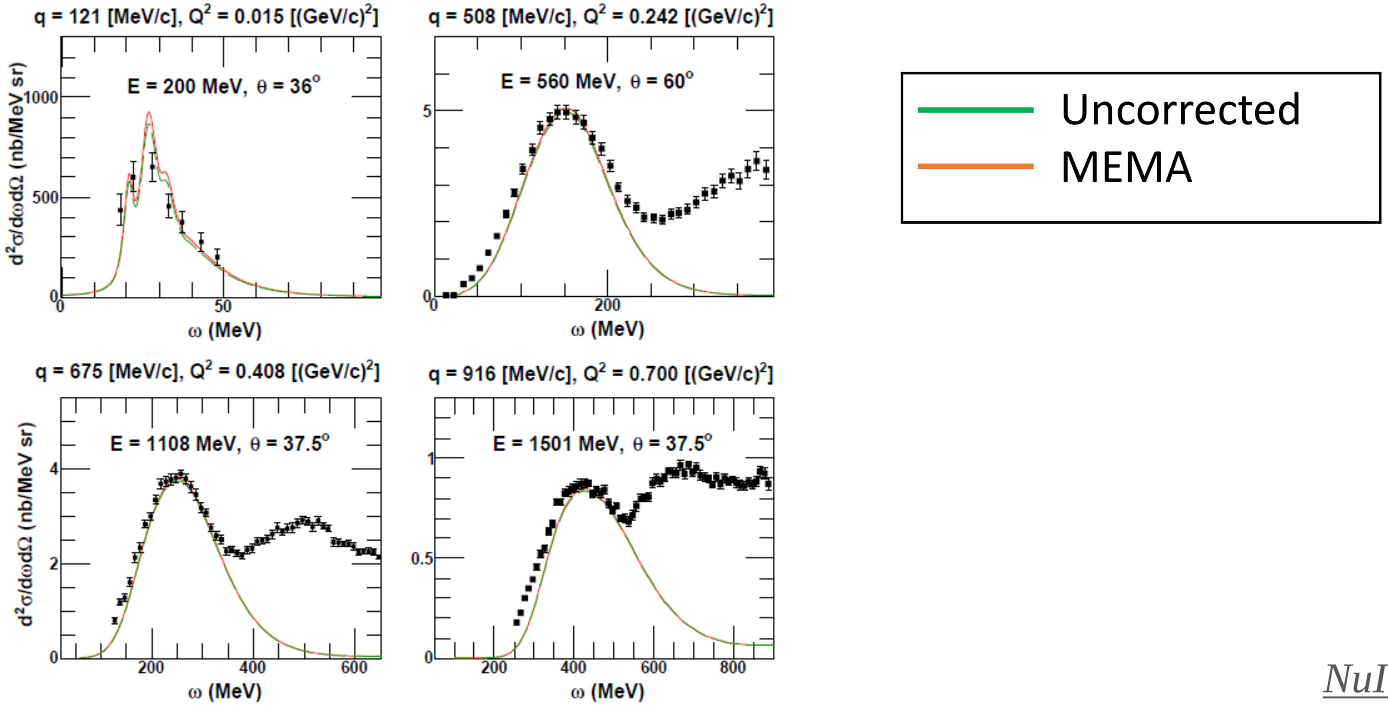


- Coulomb correction for the outgoing lepton in charged-current interactions :
  - ✓ Low energies : Fermi function

$$F(Z', E) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \qquad \eta \sim \mp Z'\alpha$$

  - ✓ High energies : modified effective momentum approximation (J. Engel, PRC57,2004 (1998))

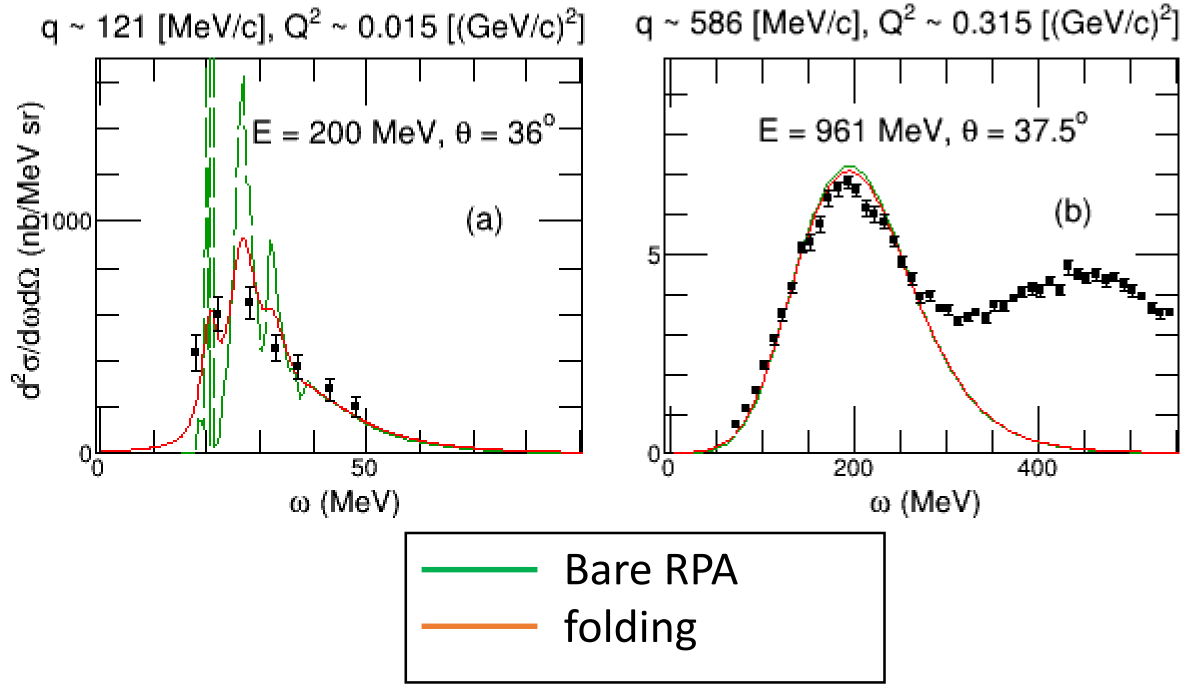
$$q_{eff} = q + 1.5 \left(\frac{Z'\alpha\hbar c}{R}\right) \qquad \Psi_l^{eff} = \zeta(Z', E, q)\ \Psi_l \qquad \zeta(Z', E, q) = \sqrt{\frac{q_{eff} E_{eff}}{qE}}$$



- Final state interactions :

-taken into account through the calculations of the wave function of the outgoing nucleon in the (real) nuclear potential generated using the Skyrme force

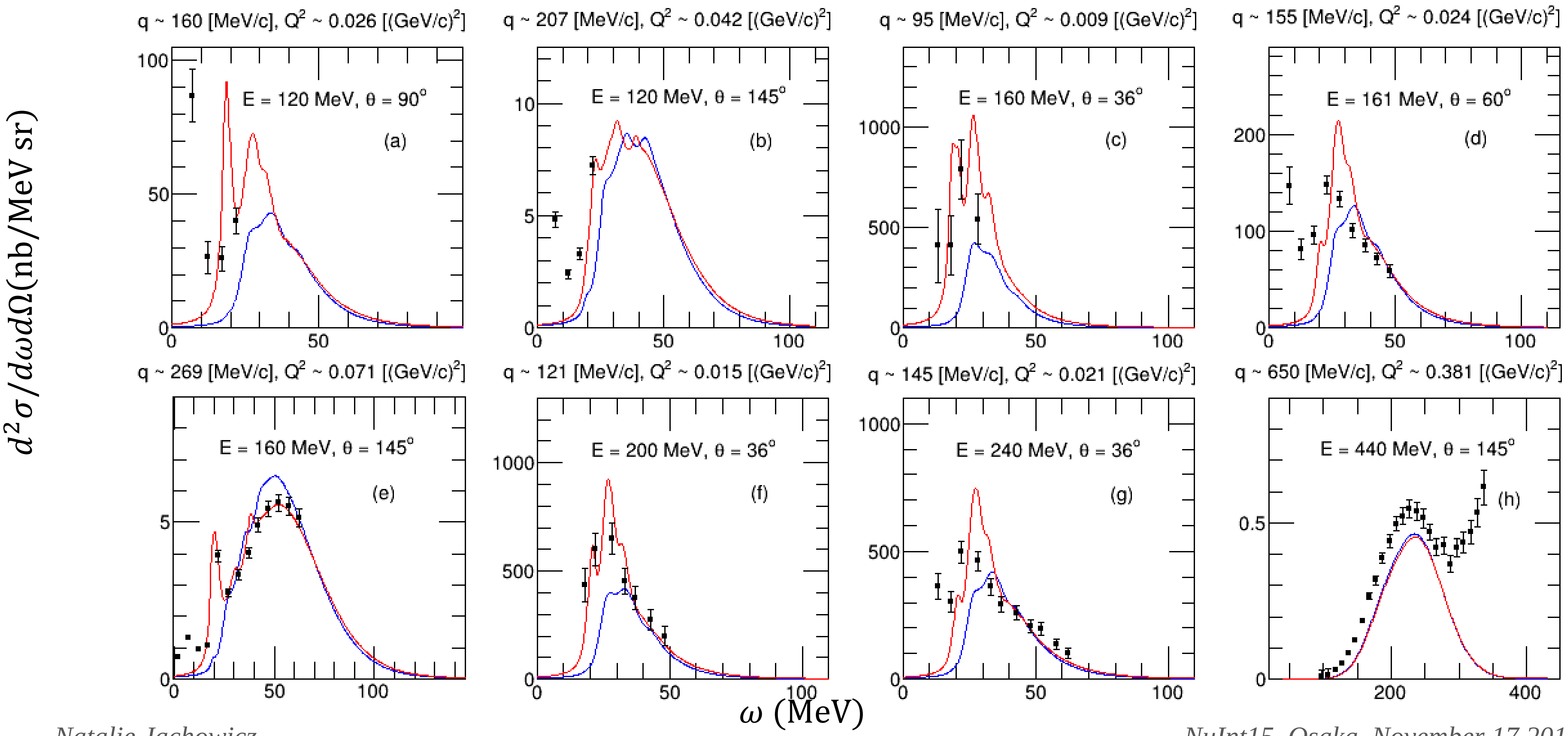
-influence of the spreading width of the particle states is implemented through a folding procedure

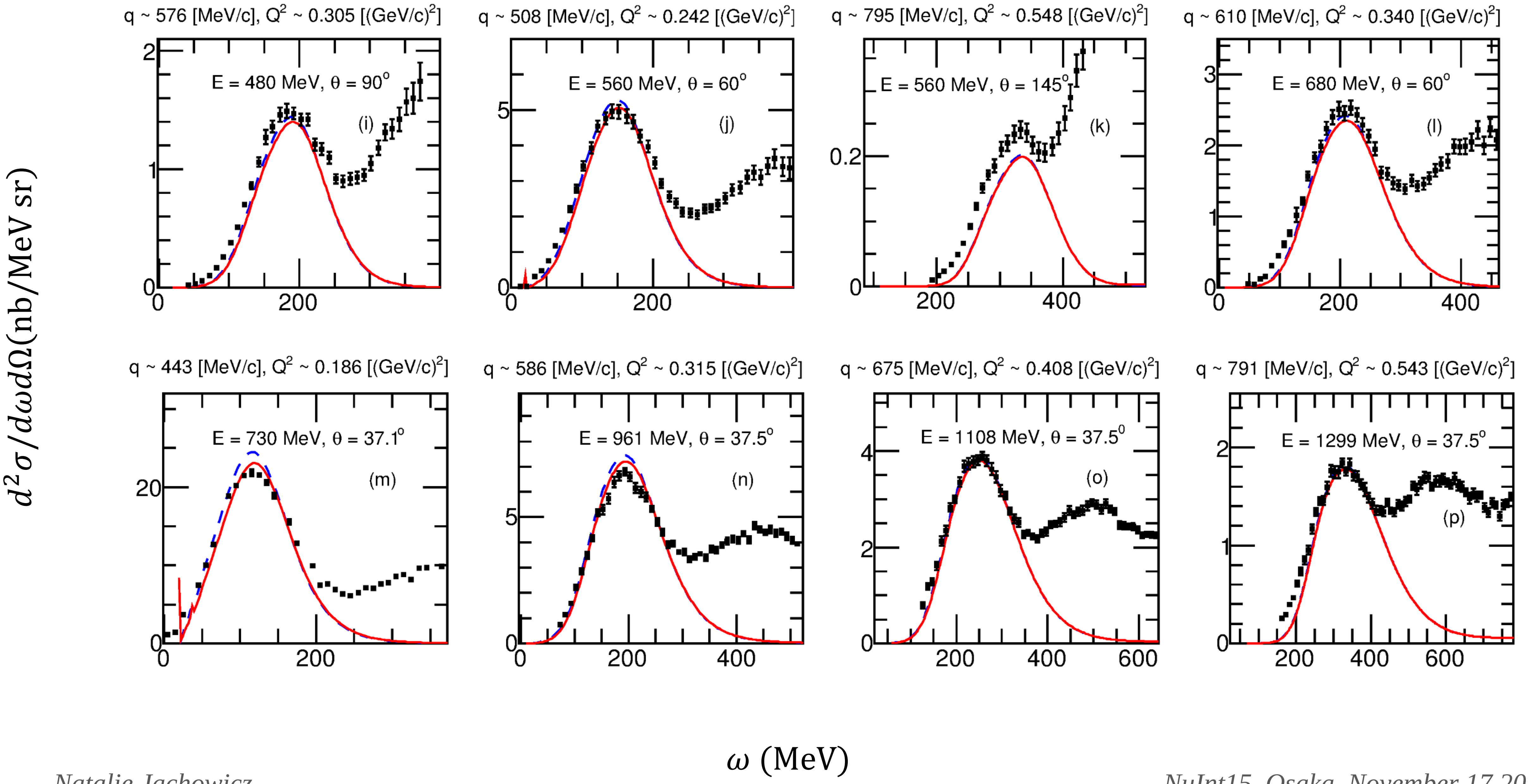


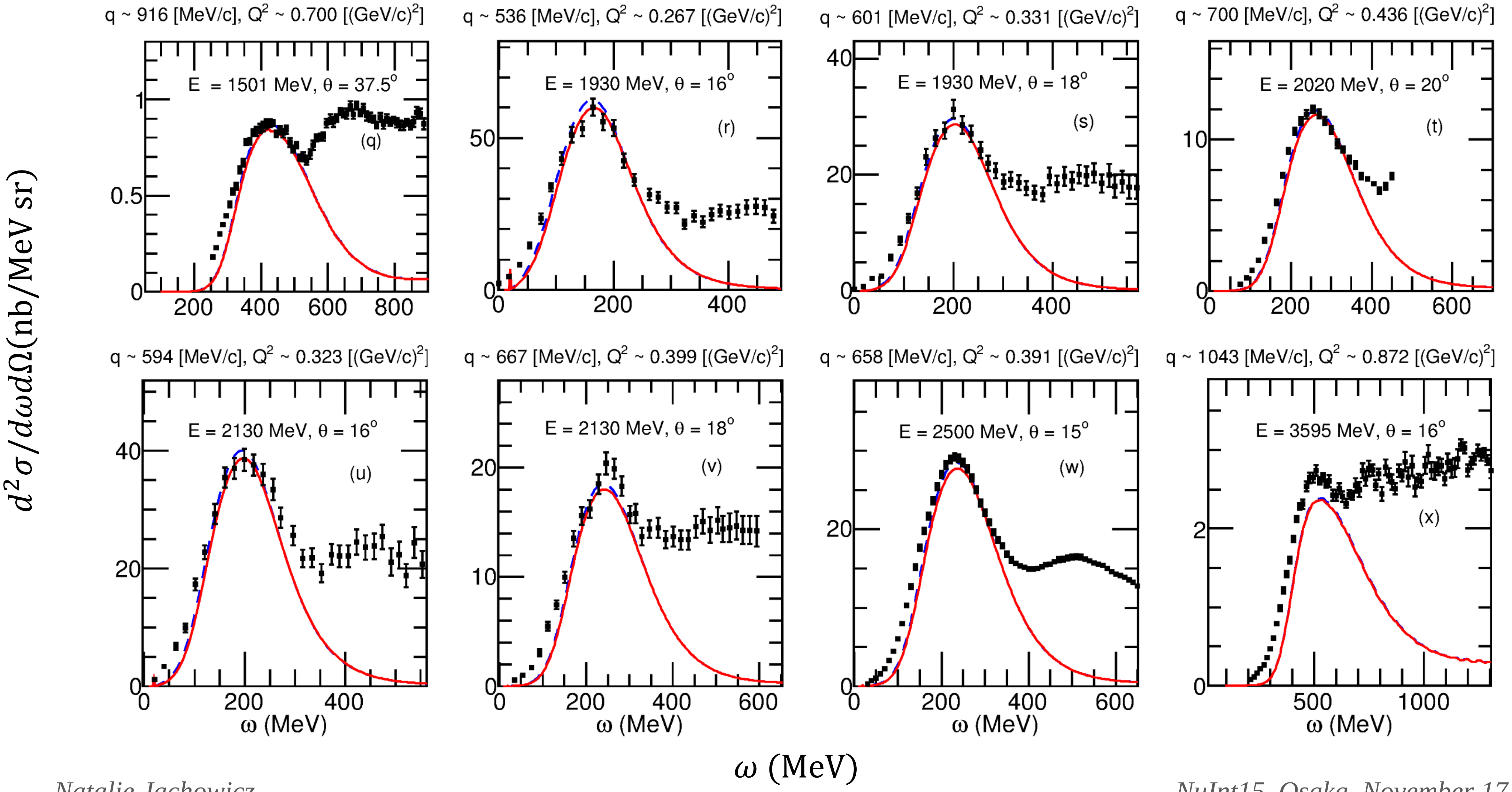
$$R'(q,\omega') = \int_{-\infty}^{\infty} d\omega \; R(q,\omega) \; L(\omega,\omega')$$

$$L(\omega,\omega') = \frac{1}{2\pi}\left[\frac{\Gamma}{(\omega-\omega')^2 + (\Gamma/2)^2}\right]$$

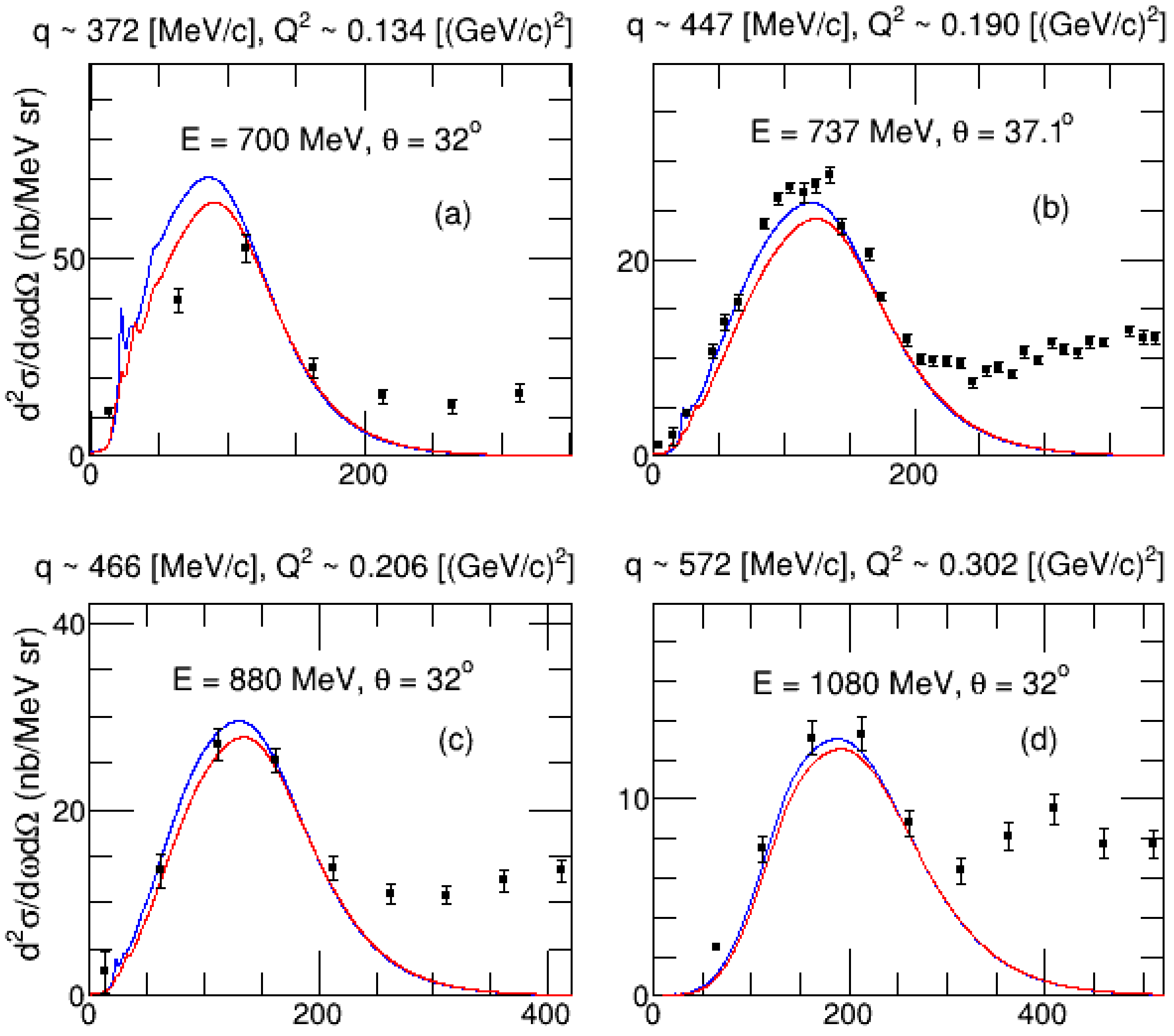
Comparison with electron scattering data $^{12}C(e, e')$




q ~ 576 [MeV/c], Q² ~ 0.305 [(GeV/c)²]
E = 480 MeV, θ = 90°
(i)
q ~ 508 [MeV/c], Q² ~ 0.242 [(GeV/c)²]
E = 560 MeV, θ = 60°
(j)
q ~ 795 [MeV/c], Q² ~ 0.548 [(GeV/c)²]
E = 560 MeV, θ = 145°
(k)
q ~ 610 [MeV/c], Q² ~ 0.340 [(GeV/c)²]
E = 680 MeV, θ = 60°
(l)
q ~ 443 [MeV/c], Q² ~ 0.186 [(GeV/c)²]
E = 730 MeV, θ = 37.1°
(m)
q ~ 586 [MeV/c], Q² ~ 0.315 [(GeV/c)²]
E = 961 MeV, θ = 37.5°
(n)
q ~ 675 [MeV/c], Q² ~ 0.408 [(GeV/c)²]
E = 1108 MeV, θ = 37.5°
(o)
q ~ 791 [MeV/c], Q² ~ 0.543 [(GeV/c)²]
E = 1299 MeV, θ = 37.5°
(p)
d²σ/dωdΩ(nb/MeV sr)
ω (MeV)


q ~ 916 [MeV/c], Q² ~ 0.700 [(GeV/c)²]
E = 1501 MeV, θ = 37.5°
(q)
q ~ 536 [MeV/c], Q² ~ 0.267 [(GeV/c)²]
E = 1930 MeV, θ = 16°
(r)
q ~ 601 [MeV/c], Q² ~ 0.331 [(GeV/c)²]
E = 1930 MeV, θ = 18°
(s)
q ~ 700 [MeV/c], Q² ~ 0.436 [(GeV/c)²]
E = 2020 MeV, θ = 20°
(t)
q ~ 594 [MeV/c], Q² ~ 0.323 [(GeV/c)²]
E = 2130 MeV, θ = 16°
(u)
q ~ 667 [MeV/c], Q² ~ 0.399 [(GeV/c)²]
E = 2130 MeV, θ = 18°
(v)
q ~ 658 [MeV/c], Q² ~ 0.391 [(GeV/c)²]
E = 2500 MeV, θ = 15°
(w)
q ~ 1043 [MeV/c], Q² ~ 0.872 [(GeV/c)²]
E = 3595 MeV, θ = 16°
(x)
ω (MeV)
d²σ/dωdΩ(nb/MeV sr)
ω (MeV)

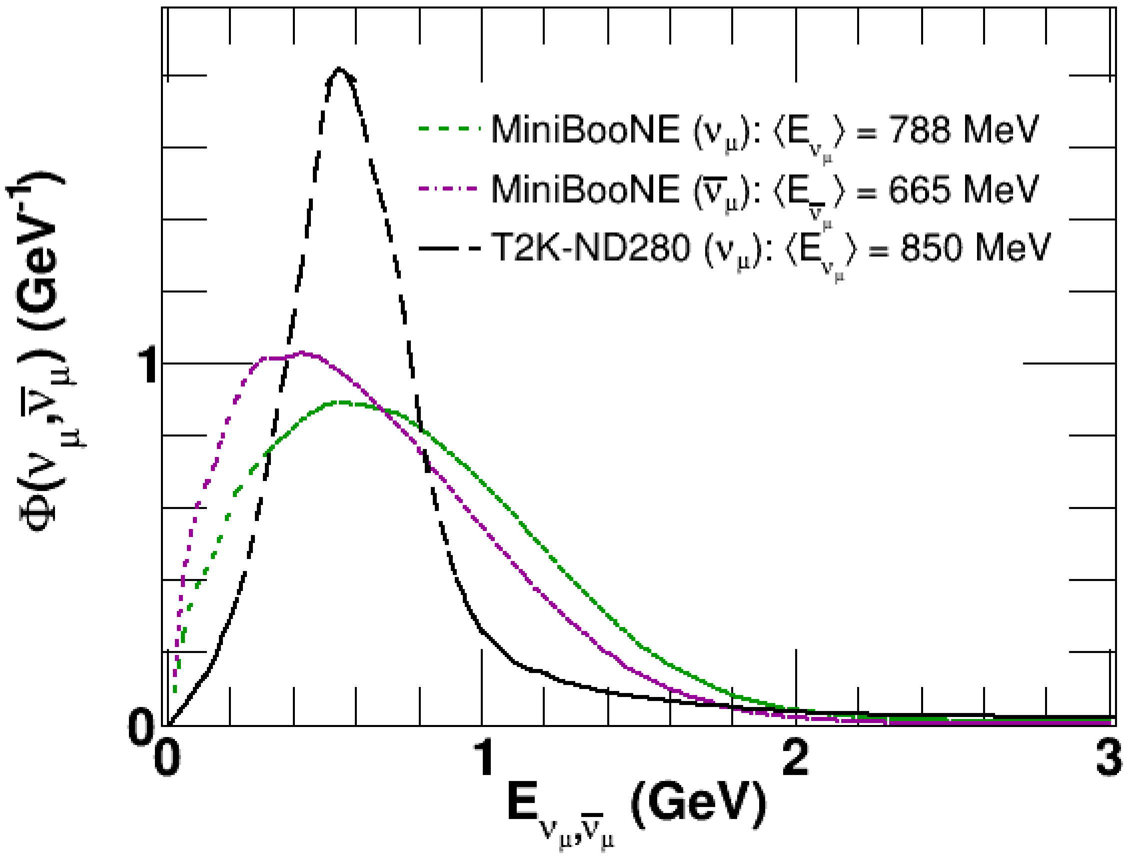
$^{16}O(e, e')$

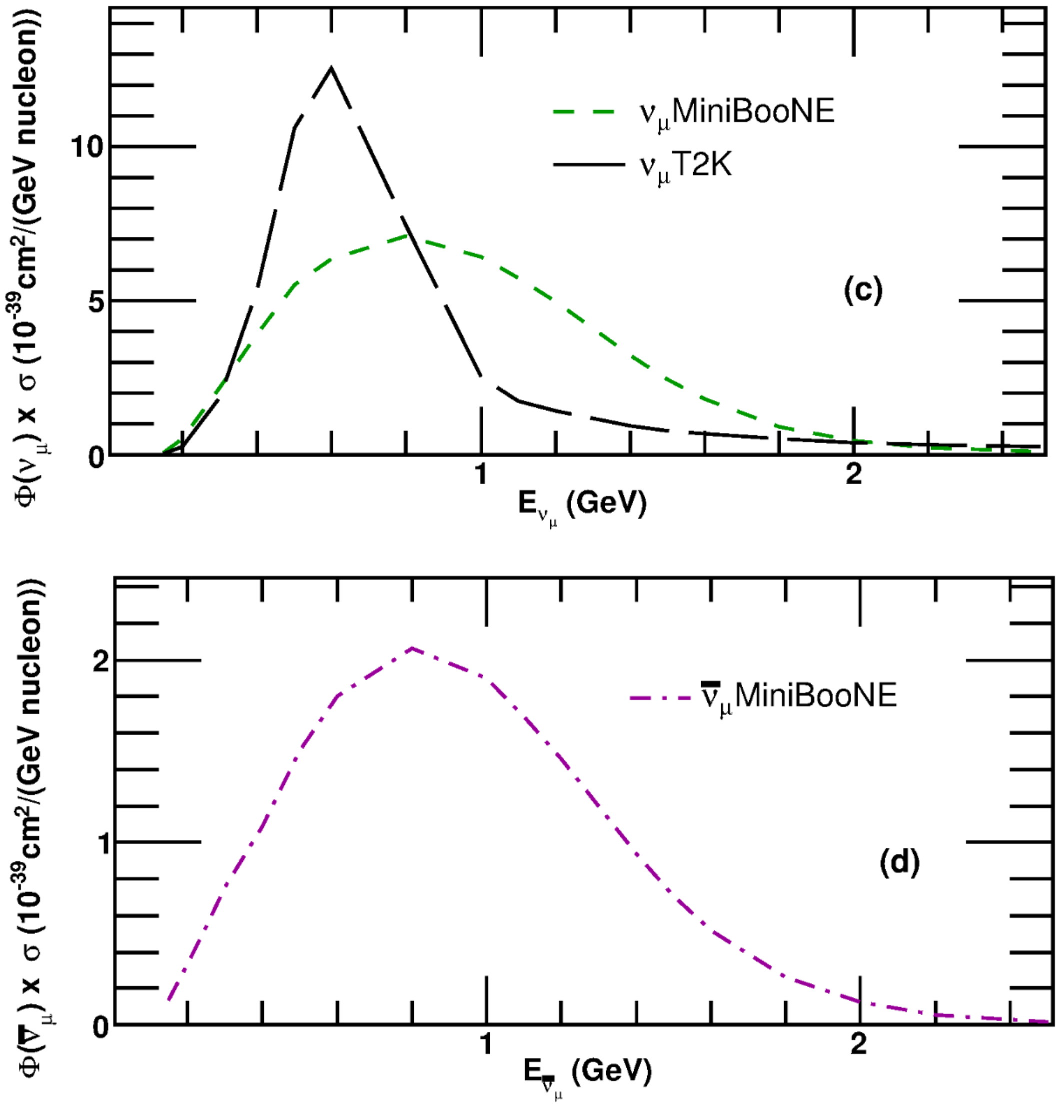


- Good overall agreement with e-scattering data

P. Barreau et al., Nucl. Phys. A402, 515 (1983), J. S. O'Connell et al., Phys. Rev. C35, 1063 (1987), R. M. Sealock et al., Phys. Rev. Lett.62, 1350 (1989).,D. S. Bagdasaryan et al., YERPHI-1077-40-88 (1988),D. B. Day et al., Phys. Rev. C 48, 1849 (1993).,D. Zeller, DESY-F23-73-2 (1973).

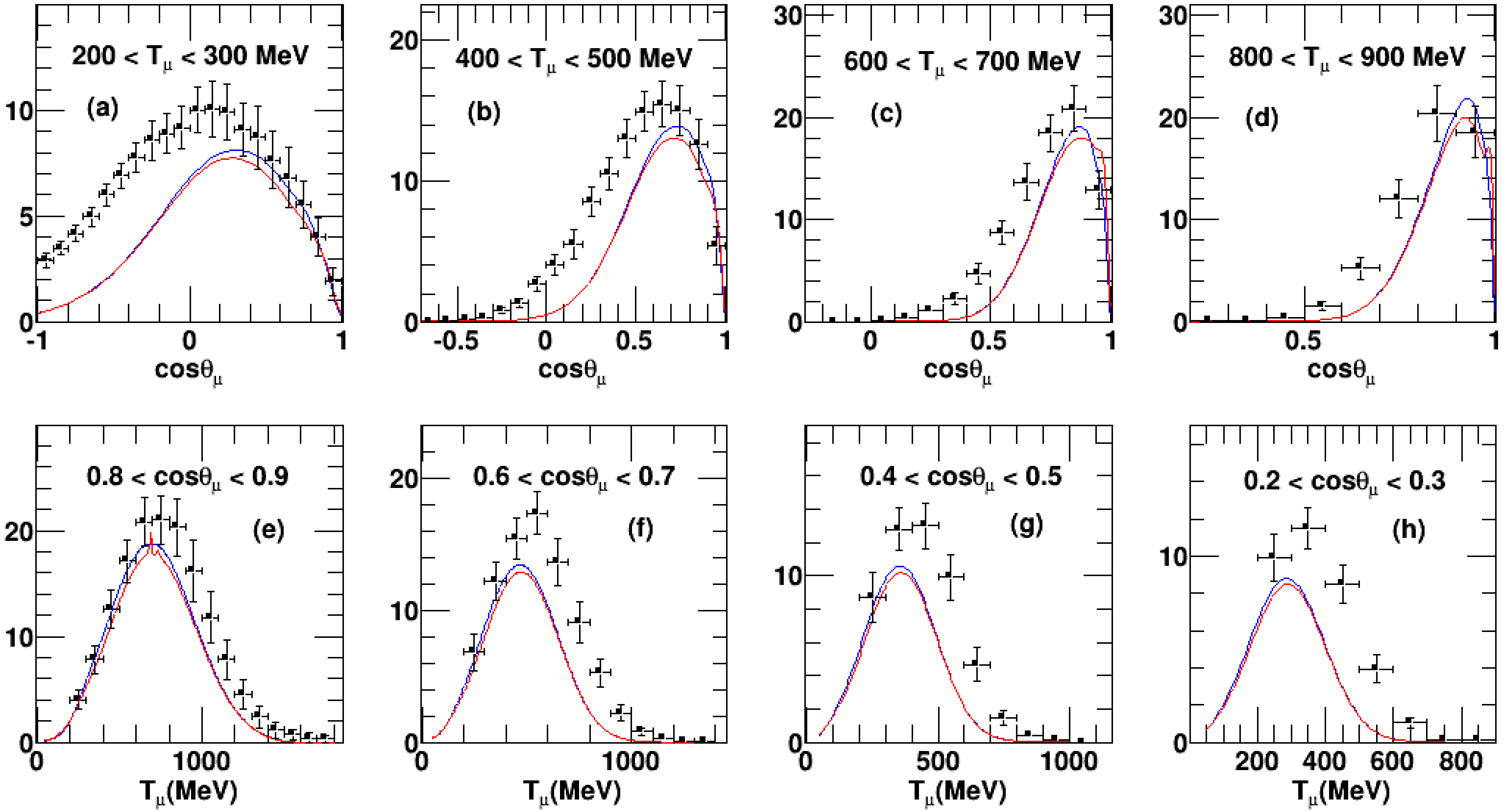
Comparison with neutrino data





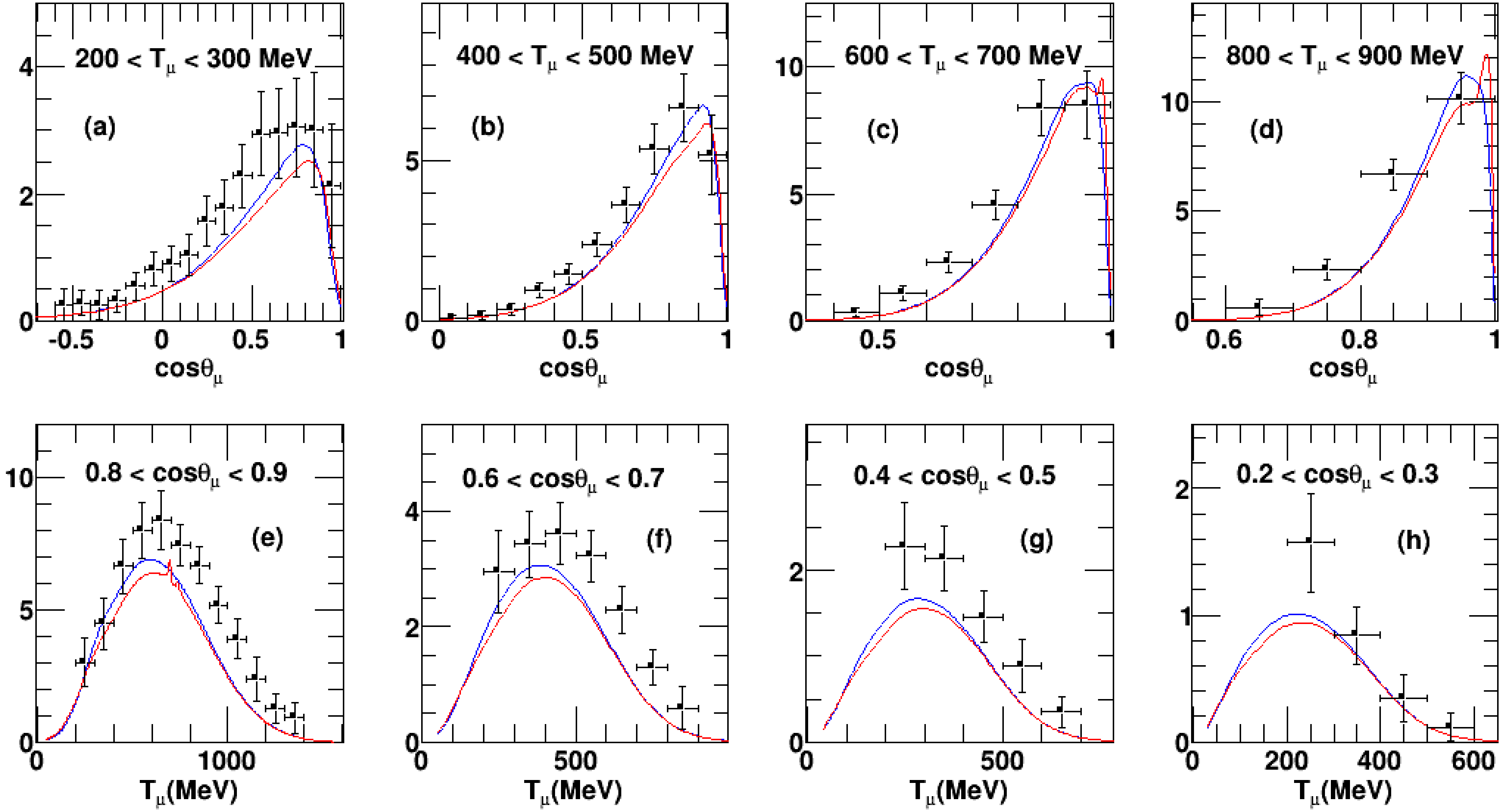
MiniBooNe ν_μ

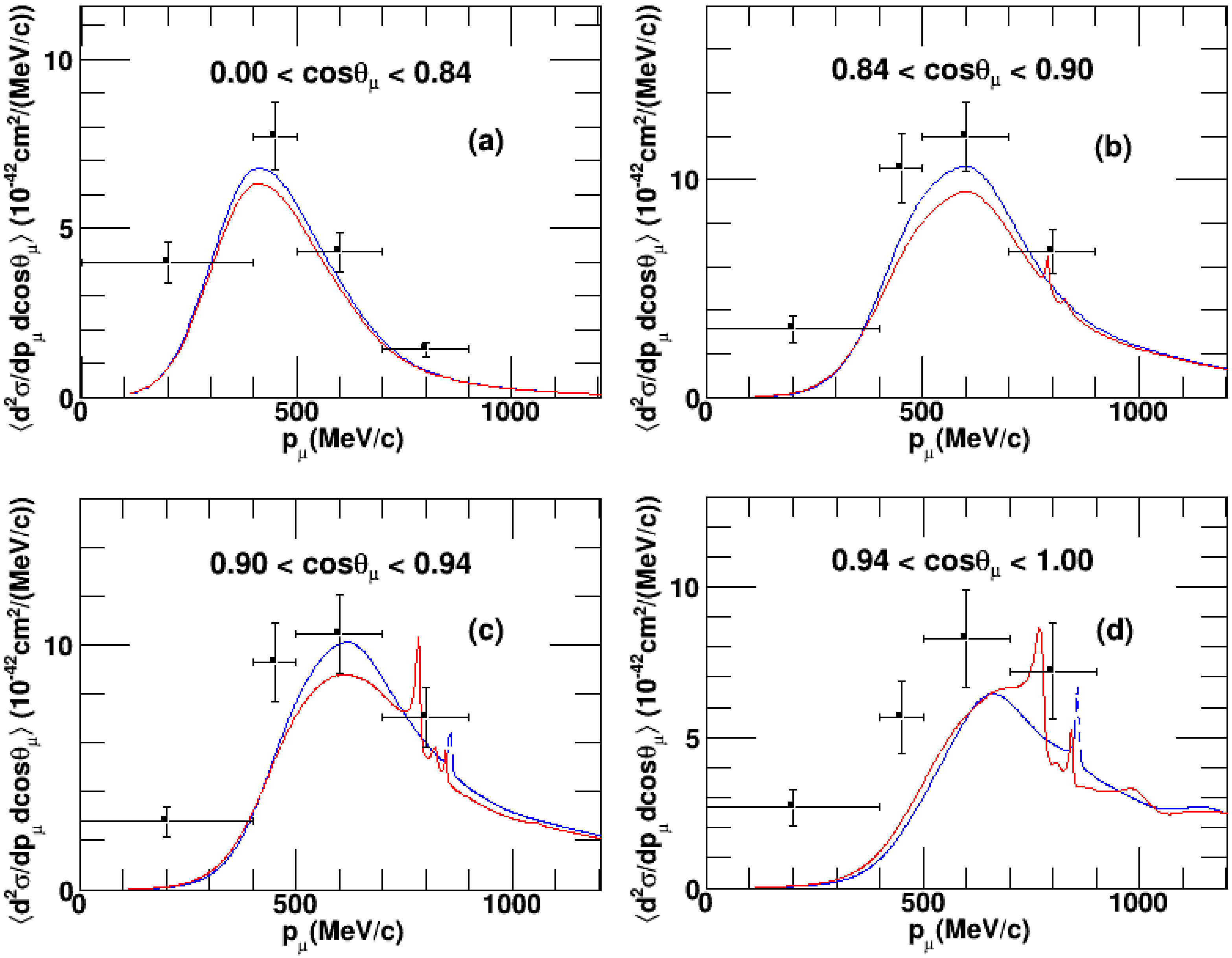
- Satisfactory general agreement
- Good agreement for forward scattering
- Missing strength for low T_μ, backward scattering



MiniBooNe $\bar{\nu}_\mu$

- Good general agreement
- Good agreement for forward scattering
- Missing strength for high T_μ, backward scattering
- Better agreement with data than neutrino cross sections



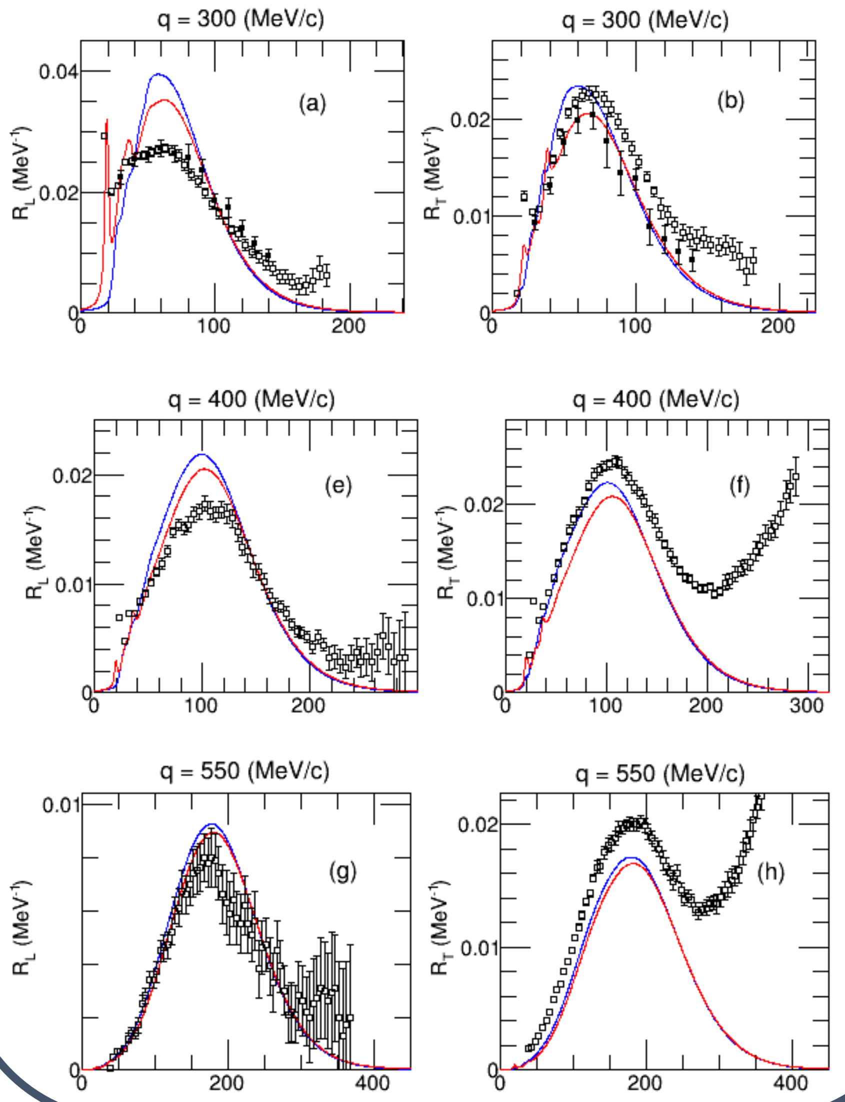


T2K ν_μ

- General agreement quite good
- Missing strength for low p_μ

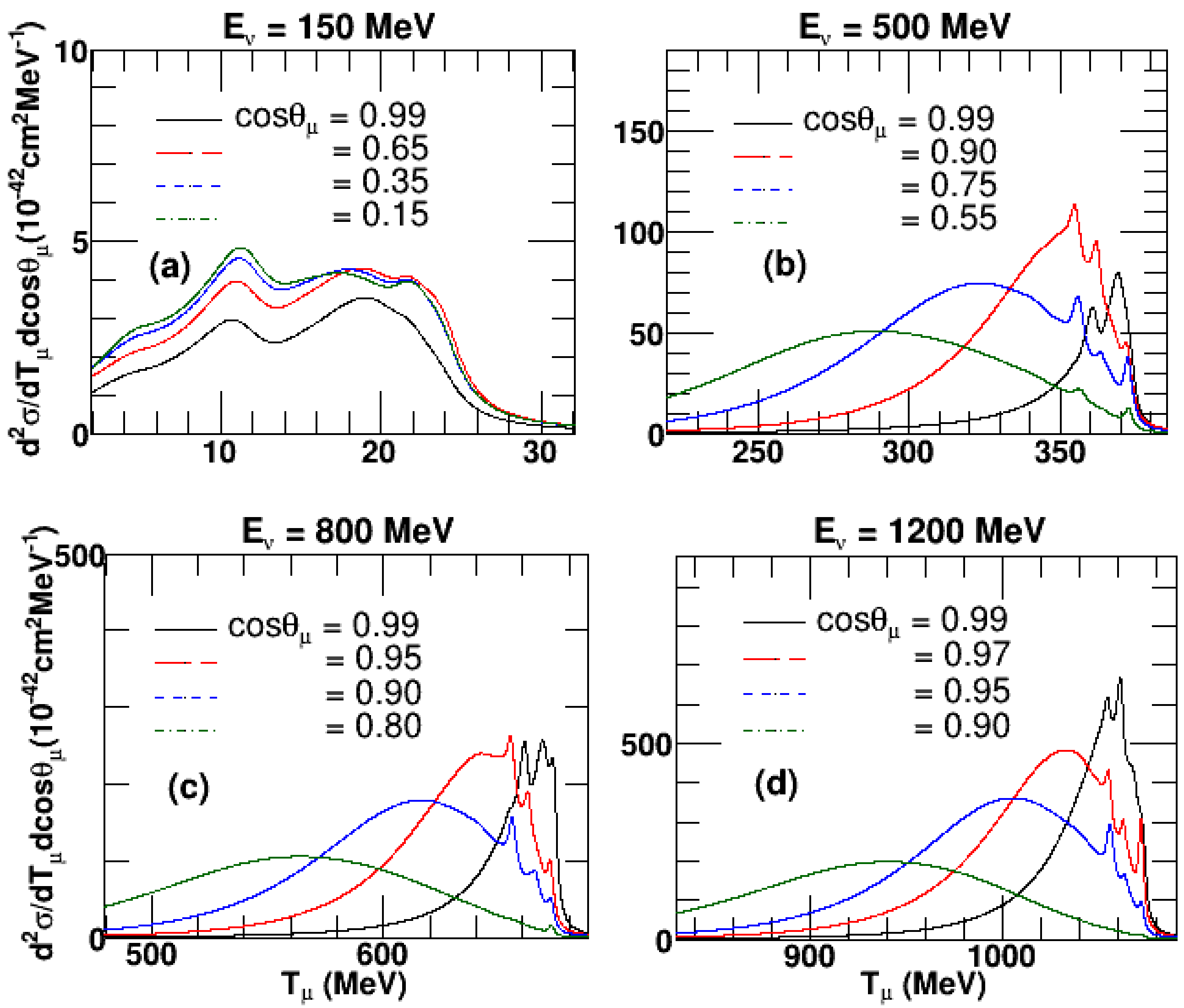
More detailed cross section contributions

Transverse/longitudinal structure

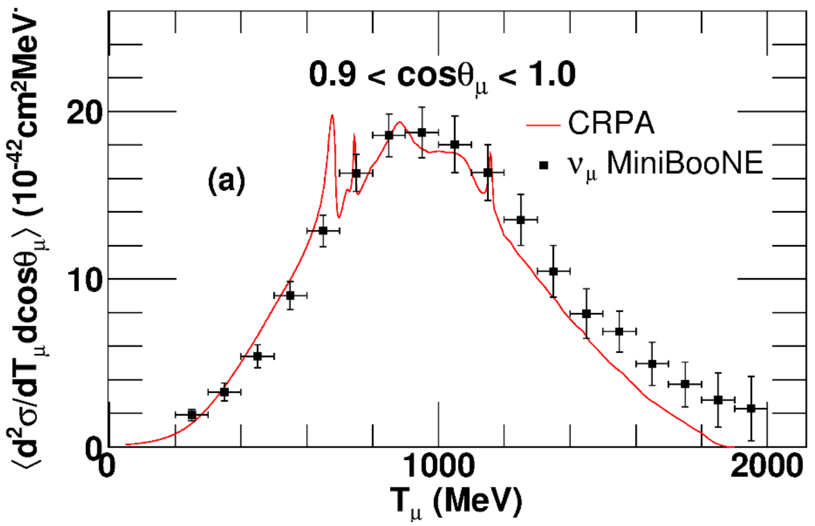


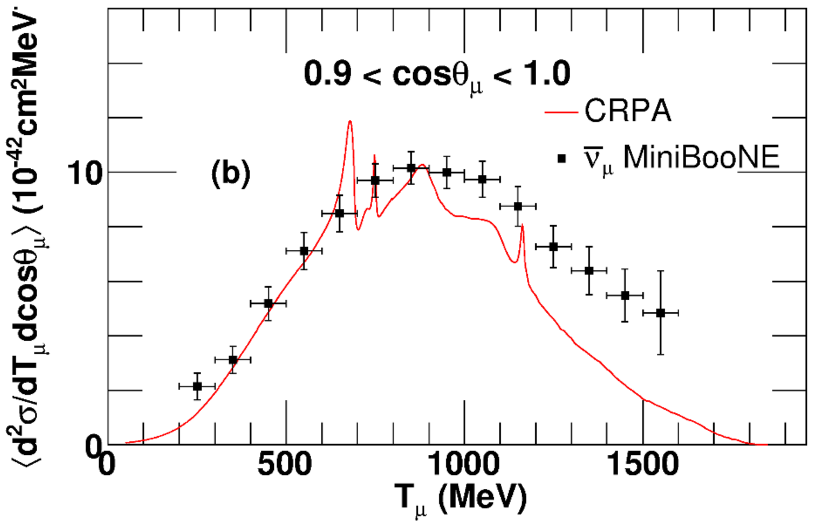
- ➢ Missing strength mainly attributed to transverse responses

Forward scattering

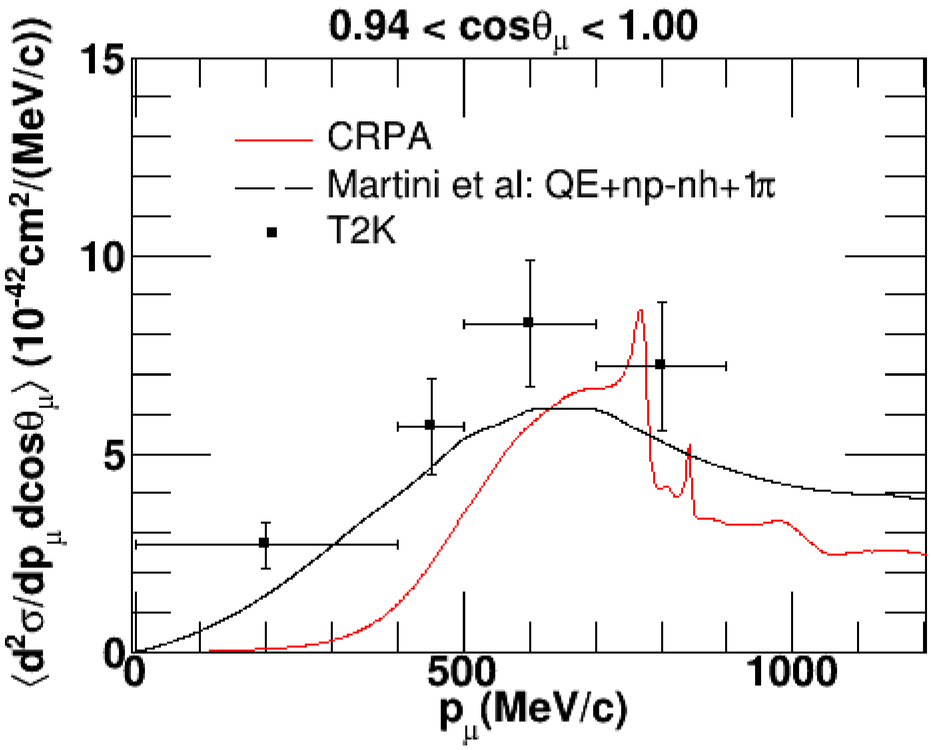


MiniBooNe

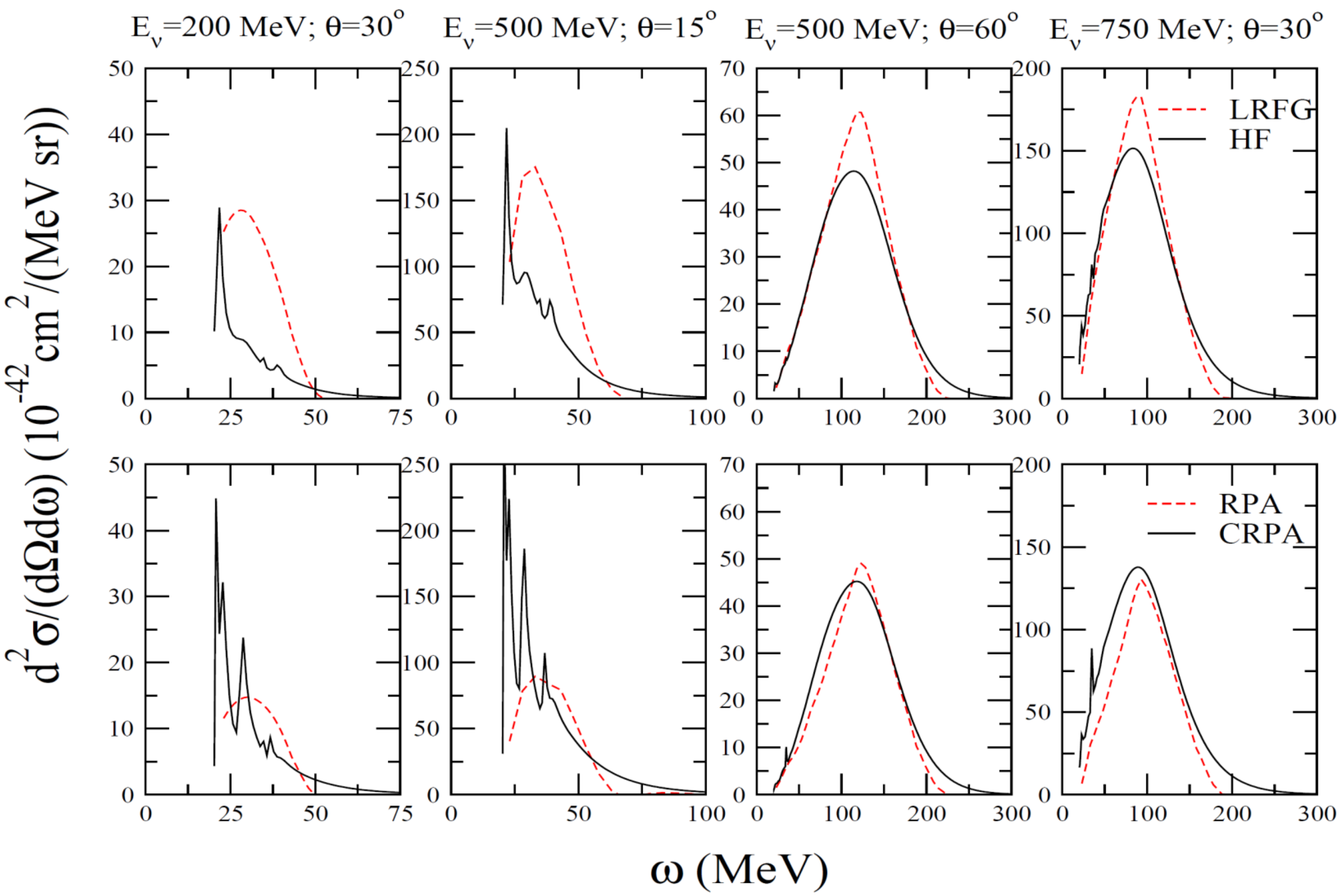


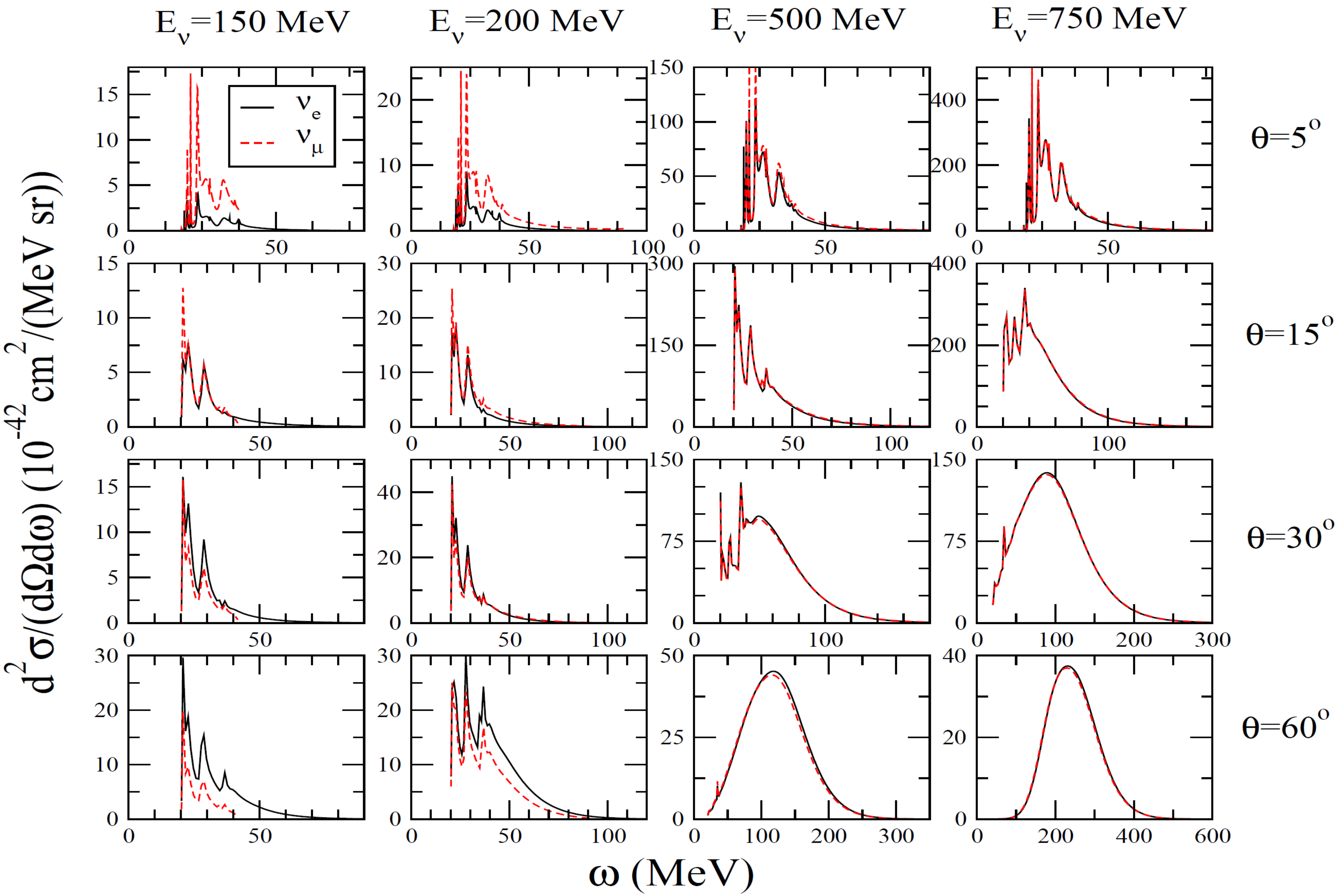


T2K



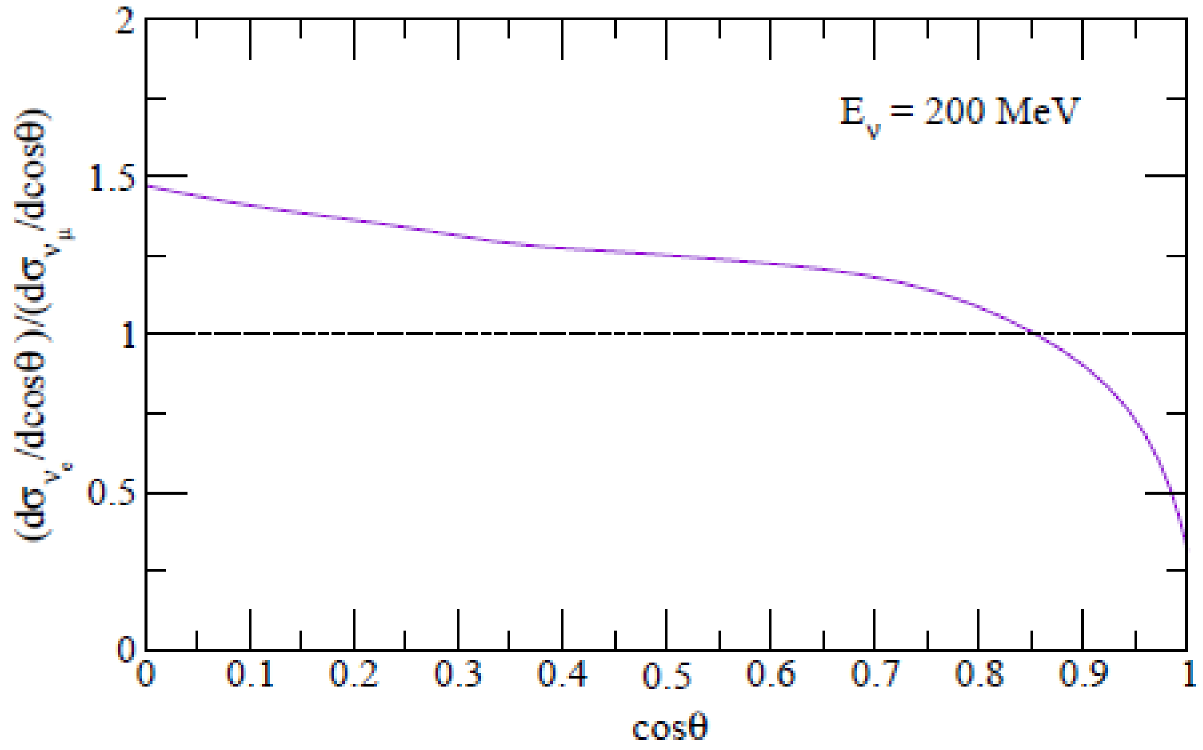
Electron neutrino scattering





E_ν=150 MeV
E_ν=200 MeV
E_ν=500 MeV
E_ν=750 MeV
ν_e
ν_μ
θ=5°
θ=15°
θ=30°
θ=60°
$d^2\sigma/(d\Omega d\omega)$ (10^{-42} cm^2/(MeV sr))
ω (MeV)

Electronneutrino vs muonneutrino ratio :



Summary

- CRPA calculations provide extra strength for forward scattering arising from low-energy excitations
- This might affect CCQE neutrino cross sections as measured by MiniBooNe and T2K

- Refs. : V. Pandey, N. Jachowicz et al : PRC89,024601, PRC92,024606.