# Direct extraction of nuclear effects on $^{12}$C

Kevin McFarland [1] **Callum Wilkinson** [2]

[2]University of Bern

[1]University of Rochester

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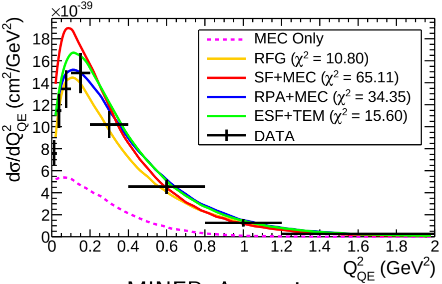
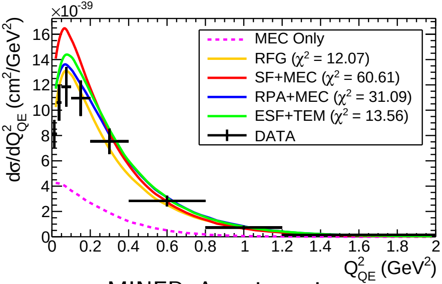
# Motivation

- CCQE models are becoming increasingly sophisticated in order to model nuclear effects.
- No available models seem to fit all of the available data on heavy targets in the few-GeV energy region.
- Currently the only way to test nuclear models against data is with large multi-parameter fits, which can have some issues...
- A measurement of suppression/enhancement of the cross section due to nuclear effects would be a powerful way to discriminate between models.

# MINER$\nu$A published results



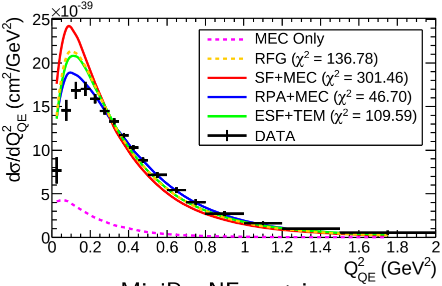

MINER$\nu$A neutrino



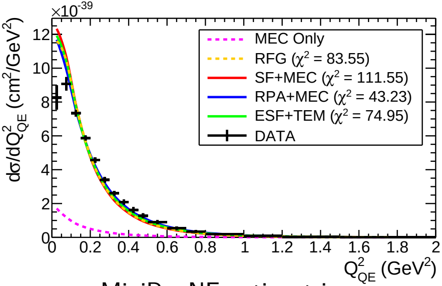

MINER$\nu$A antineutrino

- MINER$\nu$A CCQE results are given for a CH target (plastic scintillator).
- Release full covariance between their results, including cross correlations between neutrino and antineutrino datasets.
- The updated MINER$\nu$A CCQE results are used consistently on all slides **except this one!**

# MiniBooNE published results




MiniBooNE neutrino




MiniBooNE antineutrino

- MiniBooNE CCQE results are given for a $CH_2$ target (mineral oil).
- No covariance matrix is provided. Only released shape-only bin variances and separate total normalization uncertainties.
- Both experiments release their flux predictions.
- The cross sections for both experiments are flux-integrated and given per neutron or per proton.

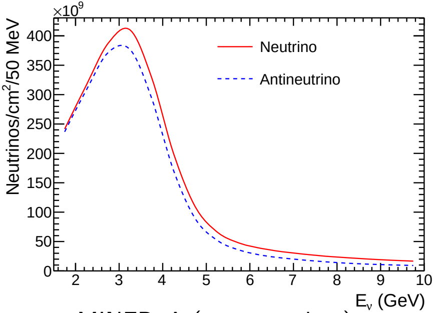
# Basic idea

- There are three differences between the neutrino and antineutrino CCQE results from an experiment with a hydrocarbon target:
  - Neutrino and antineutrino cross sections are different
  - Neutrino and antineutrino fluxes from the same beamline are different
  - Antineutrino results include additional interactions on hydrogen
- If we assume for a moment that we understand the antineutrino/neutrino cross section and flux differences perfectly, then it is easy to see that the following relation is true (for a CH target):

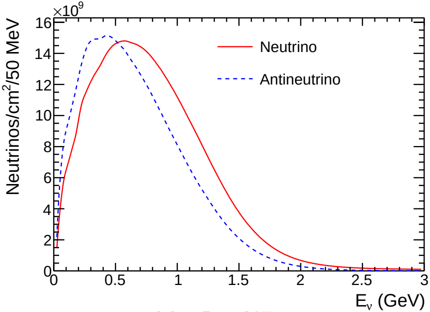
$$\frac{6\sigma^{\bar{\nu}}_{\mathrm{H}}}{\sigma^{\bar{\nu}}_{\mathrm{C}}} = \frac{[7\widetilde{\sigma}^{\bar{\nu}}_{\mathrm{CH}} - 6\lambda\widetilde{\sigma}^{\nu}_{\mathrm{CH}}]}{\lambda\widetilde{\sigma}^{\nu}_{\mathrm{CH}}},$$

  where $\sigma$ is the flux-integrated cross section for a given target (subscript), and $\widetilde{\sigma}$ is a cross section per nucleon (the numerical factors convert between the two). $\lambda$ corrects for the antineutrino/neutrino flux and cross section differences.
- **This would be a direct measurement of the nuclear effects in carbon.**

# Neutrino/antineutrino difference - 1




MINERνA (recent update)




MiniBooNE

- The fluxes are considerably different for neutrino and antineutrino running for both experiments.
- In principle the uncertainty on these fluxes enters into $\lambda$. However, this information is not available, and the uncertainties are likely to largely cancel in the ratio (for normalization at least).

# Neutrino/antineutrino CCQE difference - 2

- In the Llewellyn-Smith formalism, the neutrino/antineutrino difference (due to chirality) is in the axial-vector interference term ($B(Q^2)$):

$$\frac{d\sigma}{dQ^2}\begin{pmatrix}\nu_l + n \rightarrow l^- + p \\ \bar{\nu}_l + p \rightarrow l^+ + n\end{pmatrix} = \frac{M^2 G_{\mathrm{F}}^2 \cos^2\vartheta_C}{8\pi E_\nu^2}$$
$$\times \left[A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right],$$

$M$ is the mass of the nucleon; $G_{\mathrm{F}}$ is Fermi's constant; $\vartheta_C$ is the Cabibbo angle; $E_\nu$ is the incoming neutrino energy; and $s$ and $u$ are the Mandelstam variables.

- The functions $A(Q^2)$, $B(Q^2)$ and $C(Q^2)$ are given:

$$A(Q^2) = \frac{(m_l^2 + Q^2)}{M^2}\left[(1+\tau)F_A^2 - (1-\tau)(F_V^1)^2 + \tau(1-\tau)(\xi F_V^1)^2 + 4\tau(F_V^1 \xi F_V^2)\right.$$
$$\left. - \frac{m_l^2}{4M^2}\left((F_V^1 + \xi F_V^2)^2 + (F_A + 2F_P)^2 - 4(1+\tau)F_P^2\right)\right],$$
$$B(Q^2) = \frac{Q^2}{M^2}F_A(F_V^1 + \xi F_V^2),\; C(Q^2) = \frac{1}{4}\left(F_A^2 + (F_V^1)^2 + \tau(\xi F_V^2)^2\right),$$

where $\tau = \frac{Q^2}{4M^2}$, $\xi = (\mu_p - \mu_n) - 1$, $m_l$ is the outgoing lepton mass and $\mu_p$, $\mu_n$ are the proton and neutron magnetic moments.

## Neutrino/antineutrino difference - 3

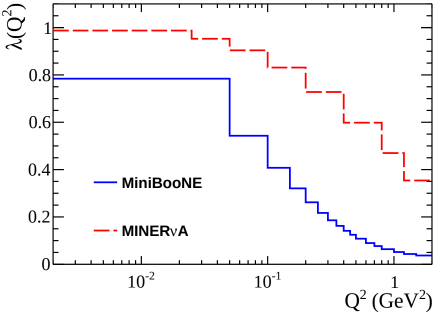
- The $\bar{\nu}/\nu$ difference is not known perfectly, but we can calculate a $\lambda(Q^2)$ correction for the L-S model, and test whether this result is "good enough" to trust the relationship:

$$\frac{6\sigma^{\bar{\nu}}_{\mathrm{H}}}{\sigma^{\bar{\nu}}_{\mathrm{C}}} \approx \frac{\left[7\widetilde{\sigma}^{\bar{\nu}}_{\mathrm{CH}} - 6\lambda(Q^2)\widetilde{\sigma}^{\nu}_{\mathrm{CH}}\right]}{\lambda(Q^2)\widetilde{\sigma}^{\nu}_{\mathrm{CH}}},$$

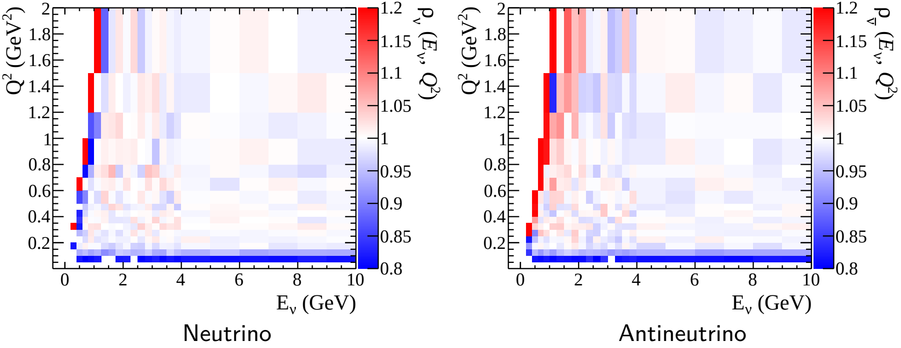
- The result will be model dependent:
  - The result will be biased by additional contributions to the axial-vector interference term.
  - The size of any bias is easy to calculate for any given nuclear model.
  - In the limit $Q^2 = 0$, the only difference comes from the flux, so model dependence is minimal in low-$Q^2$ bins.

- This result is a slightly model dependent measurement of the suppression of the antineutrino cross section in carbon, relative to a free proton. **Direct measurement of the nuclear effects in carbon.**

# Calculating $\lambda(Q^2)$

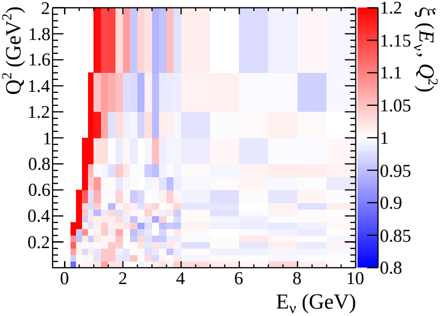



- Generate CCQE interactions on **free** nucleons in GENIE (v2.8.6) using the relevant experimental fluxes.
- Calculate $\lambda(Q^2) = \sigma_p^{\bar{\nu}}/\sigma_n^{\nu}$, where $\sigma$ is the flux-integrated cross section.
- The central values and errors for the test statistic can be extracted using this ratio, and the central values and covariance of the data (publicly available).




- Investigate the ratio of RFG/L-S predictions for neutrino ($\rho_\nu = \sigma_\nu^{\text{RFG}}/\sigma_\nu^{\text{L-S}}$) and antineutrino ($\rho_{\bar{\nu}} = \sigma_{\bar{\nu}}^{\text{RFG}}/\sigma_{\bar{\nu}}^{\text{L-S}}$) CCQE cross sections.
- If this ratio is different for neutrino and antineutrino cross sections there will be a bias in $\lambda(Q^2)$...

# Limitations of the approach



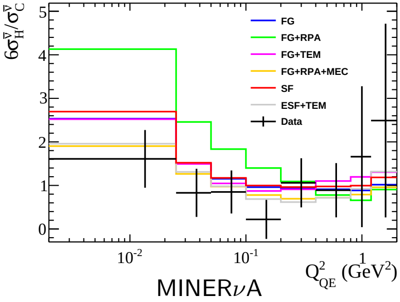

- The double ratio $\xi = \frac{\sigma_{\bar{\nu}}^{\text{RFG}}/\sigma_{\nu}^{\text{RFG}}}{\sigma_{\bar{\nu}}^{\text{L-S}}/\sigma_{\nu}^{\text{L-S}}}$ gives the size of that bias (for the RFG model):
  - Clear bias around the kinematic boundary
  - Fermi motion adds additional smearing which makes neutrino and antineutrino differ (away from a boundary this effect averages out).
- Expect the method to work well for MINER$\nu$A ($1.5 \leq E_\nu \leq 10$ GeV), but there will be a bias for MiniBooNE ($0 \leq E_\nu \leq 3$ GeV), although it's not clear how big this bias will be.

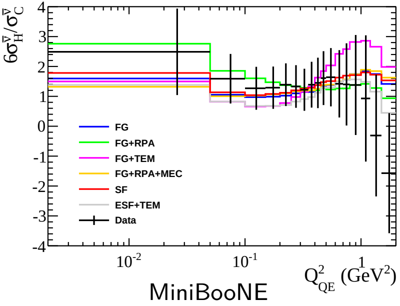
## Pre-results recap

- There are three main differences between the neutrino and antineutrino CCQE results from an experiment with a hydrocarbon target:
  - Neutrino and antineutrino cross sections are different – difficult to correct for
  - Neutrino and antineutrino fluxes from the same beamline are different – easy to correct for
  - Antineutrino results include additional interactions on hydrogen
- Take existing results, which cannot differentiate between models very well. Add a bit of model dependence, and produce a "new" measurement of $6\sigma_{\mathrm{H}}/\sigma_{\mathrm{C}}$, which is a direct measurement of nuclear effects.
- Test statistic (MINER$\nu$A):

$$\frac{6\sigma^{\bar{\nu}}_{\mathrm{H}}}{\sigma^{\bar{\nu}}_{\mathrm{C}}} \approx \frac{\left[7\widetilde{\sigma}^{\bar{\nu}}_{\mathrm{CH}} - 6\lambda(Q^2)\widetilde{\sigma}^{\nu}_{\mathrm{CH}}\right]}{\lambda(Q^2)\widetilde{\sigma}^{\nu}_{\mathrm{CH}}}$$

# Test statistic for various models



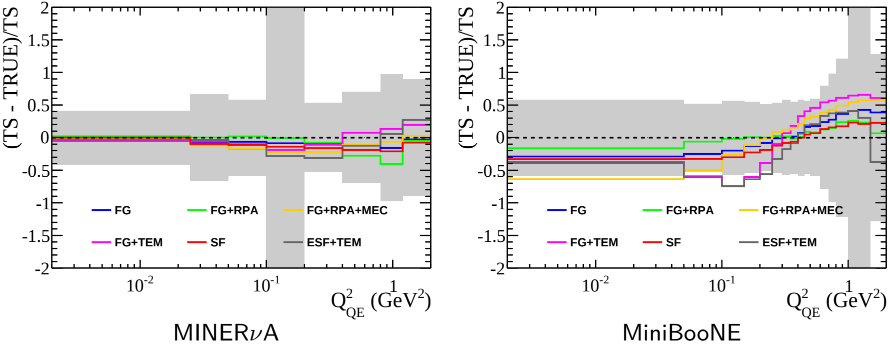

- For each model, calculate $\chi^2 = \left(\nu_i^{\text{DATA}} - \nu_i^{\text{MC}}\right) M_{ij}^{-1} \left(\nu_j^{\text{DATA}} - \nu_j^{\text{MC}}\right)$.

| Model | $\chi^2$/DOF MINER$\nu$A | $\chi^2$/DOF MiniBooNE |
|---|---|---|
| FG | 14.8/8 | 6.0/17 |
| FG+RPA | 44.3/8 | 6.0/17 |
| FG+RPA+MEC | 13.6/8 | 6.8/17 |
| FG+TEM | 13.4/8 | 23.4/17 |
| SF | 15.9/8 | 6.1/17 |
| ESF+TEM | 12.8/8 | 6.2/17 |

## How model dependent are the results?

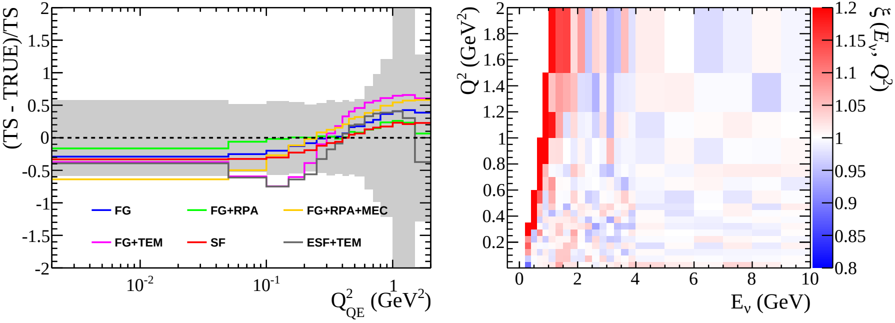
- Important to check whether these results are biased by the $\lambda(Q^2)$ correction.
- Easy to calculate the bias for any given nuclear model:
  - For each model, calculate $6\sigma^{\bar{\nu}}_{\mathrm{H}}/\sigma^{\bar{\nu}}_{\mathrm{C}}$ directly. This is the "true" model for the nuclear suppression.
  - Calculate the test statistic (TS).
  - The fractional error on the test statistic (TS - TRUE)/TS as a function of $Q^2_{\mathrm{QE}}$ gives the size of the bias **for that nuclear model**.
- The "model spread" gives an idea of the size of any bias in the technique itself, which can be compared to the size of the errors on the extracted data.

# How model dependent are the results?




- The model dependence is shown as a fractional error on the test statistic, and is compared with the $1\sigma$ bin errors on the measurement from data.
- Model bias is relatively small for MINER$\nu$A, and is very small in the lowest $Q^2_{\mathrm{QE}}$ bins.
- There is clearly a serious issue for MiniBooNE which was anticipated earlier. We retain MiniBooNE for completeness, but it is not a particularly useful result.

# How model dependent are the results?

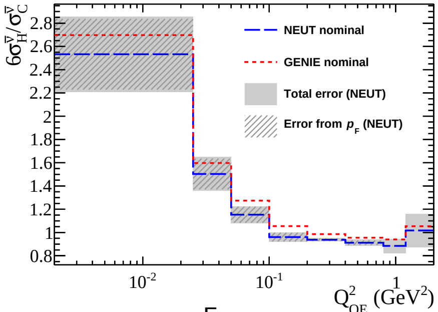



- The double ratio plot shows the bias for low neutrino energies: $\xi = \frac{\sigma_{\bar{\nu}}^{\text{RFG}}/\sigma_{\nu}^{\text{RFG}}}{\sigma_{\bar{\nu}}^{\text{L-S}}/\sigma_{\nu}^{\text{L-S}}}$
- The bias for MiniBooNE increases with $Q^2_{\text{QE}}$, this is due to the increasing proportion of the flux that can fill each $Q^2$ bin being affected by the bias around the kinematic boundary.
- We retained MiniBooNE for completeness up to this point, but it is not a particularly useful result.

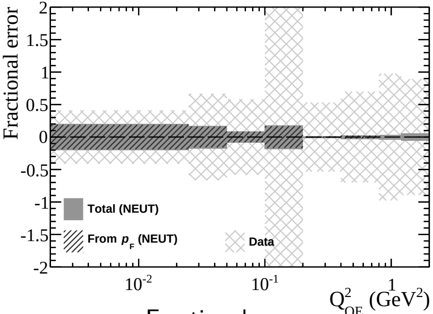
## Systematic uncertainties

- Some models will have theoretical uncertainties which should be treated as systematic errors when calculating the $\chi^2$.
- Consider only the RFG model (because it has well known uncertainties). Four sources of error are considered:
  - $M_{\mathrm{A}} = 1.00 \pm 0.03$ GeV (BBBA05 fits),
  - $E_{\mathrm{b}} = 25 \pm 3$ MeV (Moniz 1972),
  - $p_{\mathrm{F}} = 217 \pm 5$ MeV (Moniz 1972),
  - Variations in $E_{\mathrm{b}}$ for neutrino or antineutrino. Consider a 3 MeV shift to only one mode.
- These errors are combined in quadrature, and shown in the plots on the next slide.
- Additionally, the default GENIE prediction is shown as a cross-check $\rightarrow$ reasonable agreement with NEUT.

# Systematic uncertainties




Error



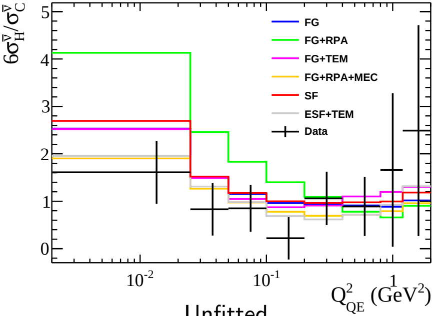

Fractional error

- The error on $p_{\mathrm{F}}$ is a dominant at low $Q^2$, $M_{\mathrm{A}}$ is dominant at high $Q^2$.
- Try fitting to $p_{\mathrm{F}}$ for all of the RFG-derived models to give a more conservative comparison between all of the models.

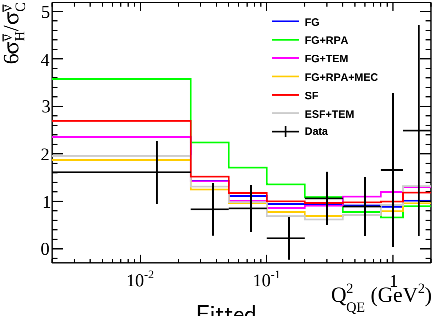
$$\chi^2(\Delta_{p_{\mathrm{F}}}) = \left(\nu_i^{\mathrm{DATA}} - \nu_i^{\mathrm{MC}}\right) M_{ij}^{-1} \left(\nu_j^{\mathrm{DATA}} - \nu_j^{\mathrm{MC}}\right) + \left(\frac{\Delta_{p_{\mathrm{F}}} - N_{p_{\mathrm{F}}}}{\sigma_{p_{\mathrm{F}}}}\right)^2,$$

where $\sigma_{p_{\mathrm{F}}}$ is 5 MeV, as defined on the previous slide.

# Systematic uncertainties




Unfitted




Fitted

- The fitted $\chi^2$ and $p_F$ values for MINER$\nu$A are:

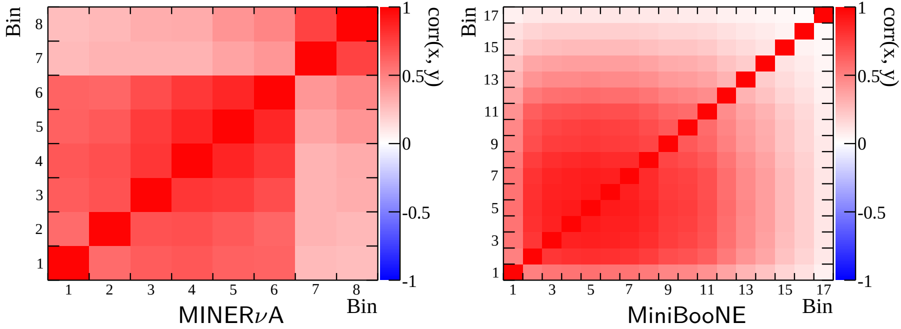
| Model | $\chi^2$/DOF | | $p_F$ (GeV$^2$) |
|---|---|---|---|
| | Nominal | Fit | |
| FG | 14.8 | 14.1 | 213.8 $\pm$ 4.0 |
| FG+RPA | 44.3 | 38.2 | 207.6 $\pm$ 4.0 |
| FG+RPA+MEC | 13.6 | 13.5 | 214.1 $\pm$ 3.9 |
| FG+TEM | 13.4 | 12.8 | 215.8 $\pm$ 4.5 |

# Summary

- Presented a direct measurement of nuclear effects in carbon extracted from existing MINER$\nu$A data:
  - Weakly model dependent, but biases are small compared with the size of the errors.
  - The model dependence is minimal at low $Q^2$.
  - The measurement can be compared to any model with the test statistic by calculating the ratio $6\sigma_{\mathrm{H}}^{\bar{\nu}}/\sigma_{\mathrm{C}}^{\bar{\nu}}$ for the MINER$\nu$A flux.
  - Currently a weak constraint, but with higher statistics MINER$\nu$A datasets, this might be quite a powerful technique.
- Also performed the analysis for MiniBooNE, but the model dependence of the method is a problem.
- This technique could also be used with measurements on water targets to extract a measurement of nuclear suppression in oxygen. Similar techniques could also be used with different interaction modes.

# Backup

# Correlation matrics




- The correlation matrix for the test statistic is shown for both experiments.
- Note that MiniBooNE picks up off diagonal correlations because of the fully correlated normalization errors which are put into the matrix.

# Added complication: $Q^2 \to Q^2_{QE}$

- The results from MINER$\nu$A and MiniBooNE are given as a function of reconstructed $Q^2_{\mathrm{QE}}$, assuming it is a CCQE interaction:

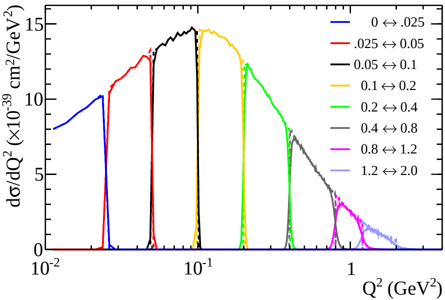
$$E_\nu^{\text{QE, RFG}} = \frac{2M_i' E_\mu - (M_i'^2 + m_\mu^2 - M_f^2)}{2(M_i' - E_\mu + \sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu)},$$

$$Q^2_{\mathrm{QE}} = -m_\mu^2 + 2E_\nu^{\text{QE, RFG}}(E_\mu - \sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu),$$

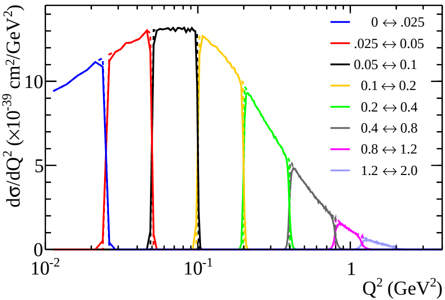
  where $E_\mu$ is the muon energy, $m_\mu$ is the muon mass, $M_i$ and $M_f$ are the initial and final nucleon masses respectively, and $M_i' = M_i - V$ where $V$ is the binding energy of carbon assumed in the analysis. For all datasets (except MINER$\nu$A antineutrino), $V = 34\,\mathrm{MeV}$ ($V = 30\,\mathrm{MeV}$).

- $Q^2_{\mathrm{QE}}$ is nuclear model dependent, so is part of the measurement.
- However, if the ratio of $\sigma(p_\mu, \theta_\mu)$ for bound and free nucleons was different for neutrinos and antineutrinos, this would produce a bias in the result.

# $Q^2 \rightarrow Q^2_{QE}$ MINER$\nu$A



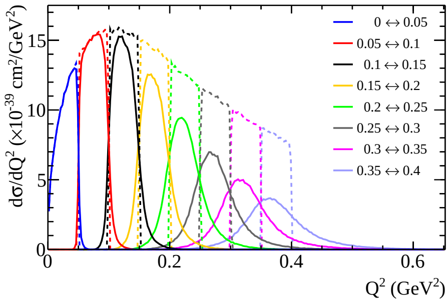

Neutrino



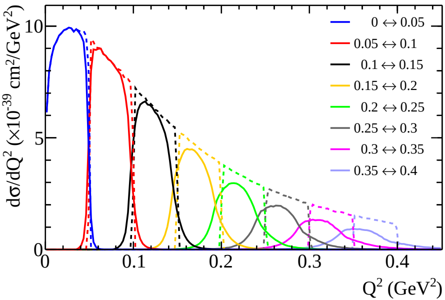

Antineutrino

- The legend gives the $Q^2_{\mathrm{QE}}$ bin edges used by MINER$\nu$A. The dashed lines give the flux-integrated cross section prediction for the RFG model (NEUT) as a function of $Q^2_{\mathrm{QE}}$ (broken down into the MINER$\nu$A binning). The solid lines show the true $Q^2$ distribution of events in each $Q^2_{\mathrm{QE}}$ bin.
- For MINER$\nu$A, the mapping between $Q^2$ and $Q^2_{\mathrm{QE}}$ is pretty good.

# $Q^2 \to Q^2_{QE}$ MiniBooNE



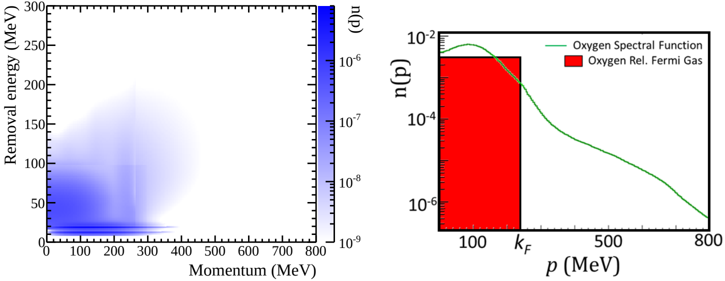

Neutrino




Antineutrino

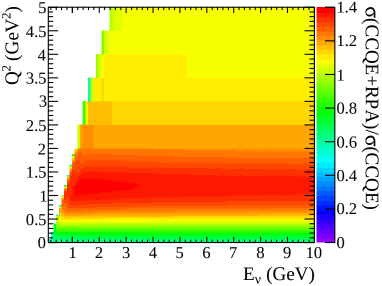
- The legend gives the $Q^2_{\mathrm{QE}}$ bin edges used by MiniBooNE. The dashed lines give the flux-integrated cross section prediction for the RFG model (NEUT) as a function of $Q^2_{\mathrm{QE}}$ (broken down into the MINER$\nu$A binning). The solid lines show the true $Q^2$ distribution of events in each $Q^2_{\mathrm{QE}}$ bin.
- For MiniBooNE, the mapping between $Q^2$ and $Q^2_{\mathrm{QE}}$ is pretty bad.

# Base nuclear model

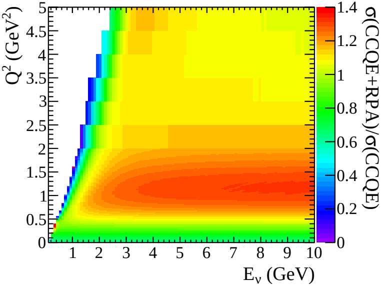



- Relativistic Fermi Gas (RFG), used for a long time in generators due to its simplicity (NEUT <v5.3.1).
- Omar Benhar's 2D Spectral Function in momentum and removal energy has been implemented in NEUT (v5.3.1).

# Random phase approximation (2D)



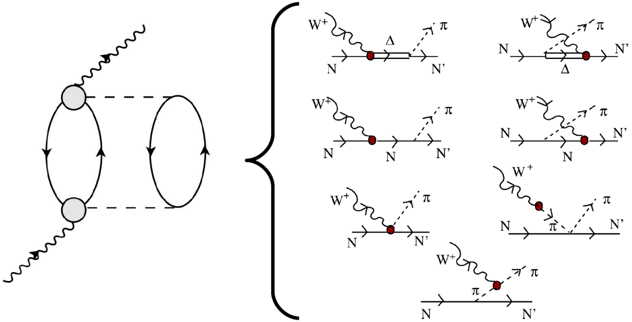

Neutrino




Antineutrino

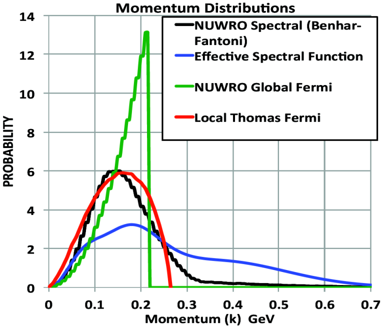
- Random Phase Approximation (RPA), nuclear screening effect due to long range nucleon-nucleon correlations.
- NEUT implementation is dependent on $Q^2$ and $E_\nu$.

# Nieves multi-nucleon interaction model



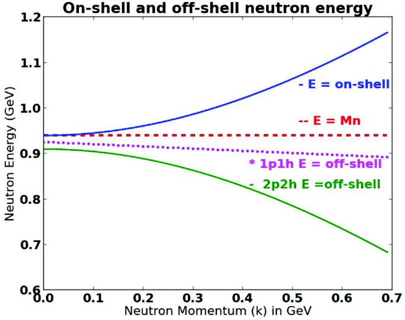

- Multi-nucleon interactions (MEC) from Nieves *et al.*, see Peter Sinclair's NuInt2014 talk for full implementation details (NEUT v5.3.2).
- Includes the high $E_\nu$ extension. The low q3 part of the cross-section is accurate up to high energies.

# Effective spectral function



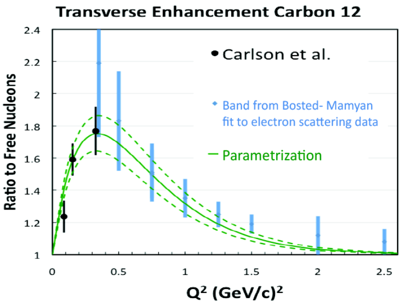

- **Basic idea:** model outgoing lepton kinematic distribution by changing initial state nucleon model. This effective modification is designed to cover a range of sins (additional nuclear effects), but in a way which is easy to implement in generators.
- Effective SF based on a parameterisation of the momentum distribution from Benhar's SF (from NOMAD collaboration), but parameters modified to fit superscaling function.
- Note that a significant high momentum component is required to fit electron scattering.

# Effective spectral function




- Constant probability of being in a correlated state with another nucleon (2p2h), which affects how off-shell the interacting nucleon is.
- Difference is whether momentum and energy are being balanced by on-shell proton (2p2h), or on-shell A-1 nuclear remnant.
- On-shell proton in 2p2h events is also simulated (with equal and opposite momentum).

# Transverse enhancement model




- $Q^2$ dependent excess in the transverse response compared with longitudinal response observed in electron scattering data.
- This excess is parameterised as a modification to the magnetic form factors for free nucleons:

$$G_{M_n}^{nuclear} = G_{M_n} \times \sqrt{1 + AQ^2 \exp(-Q^2/B)}$$
$$G_{M_p}^{nuclear} = G_{M_p} \times \sqrt{1 + AQ^2 \exp(-Q^2/B)}$$