Nuclear effects in deep inelastic scattering and transition region

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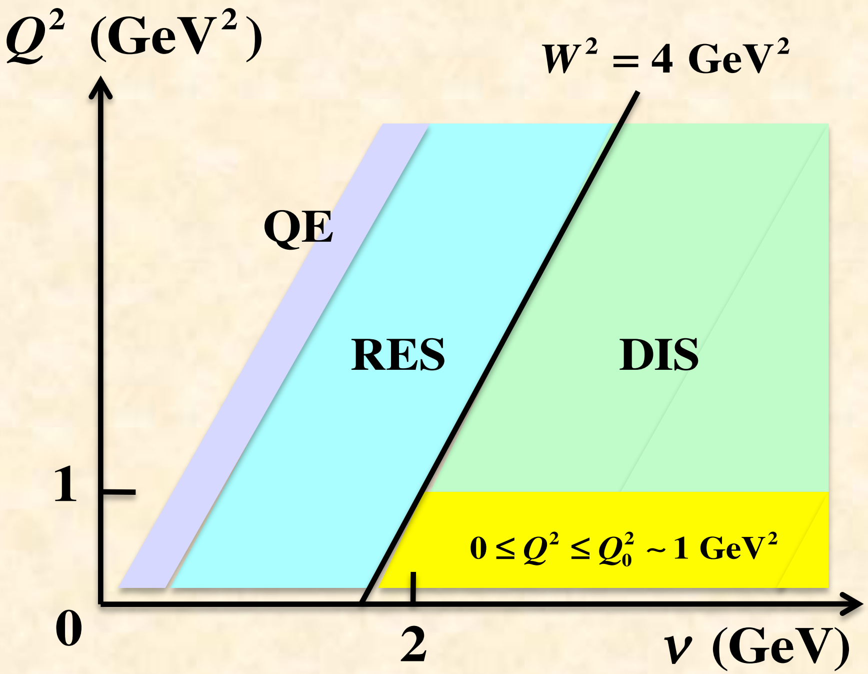
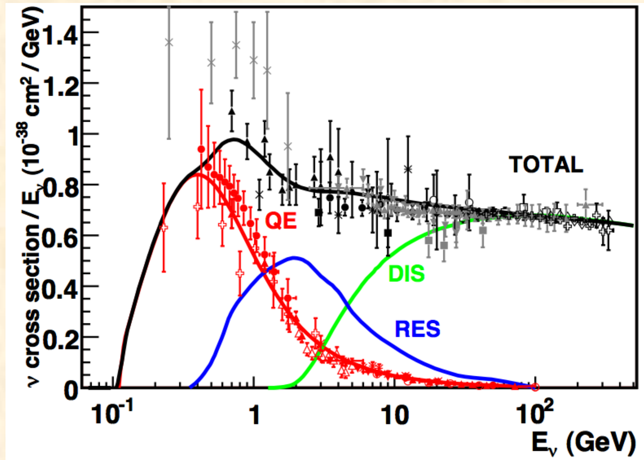
http://indico.ipmu.jp/indico/conferenceDisplay.py?ovw=True&confId=46

November 16, 2015

Kinematical regions of neutrino-nucleus scattering





J.L. Hewett *et al.*, arXiv:1205.2671,
Proceedings of the 2011 workshop
on Fundamental Physics at the Intensity Frontier

Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic)

T2K systematic errors

ν flux and cross section	w/ ND measurement	2.7%
ν cross section due to difference of nuclear target btw. near and far		5.0%
Final or Secondary Hadronic Interaction		3.0%
Super-K detector		4.0%
total	w/ ND measurement	7.7%

ν interactions

A.K.Ichikawa@KEK workshop 2015

Contents

1. Small Q^2 region
2. Deep inelastic scattering (DIS)
3. Parton distributions functions (PDFs)
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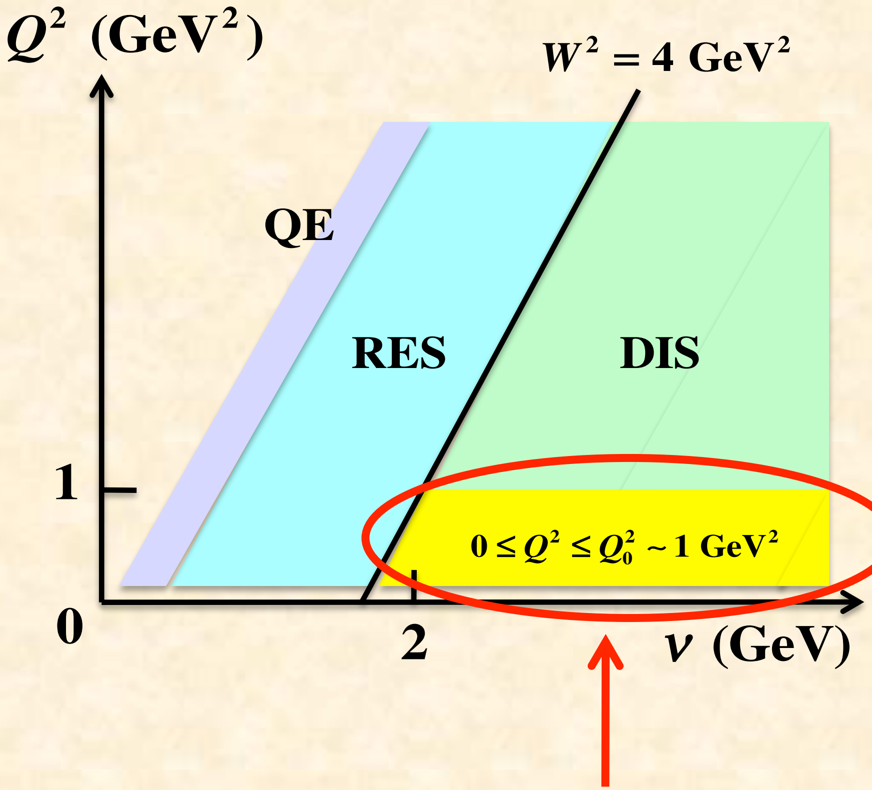
5. Nuclear PDFs
6. Summary

Small Q^2 region

$Q^2 \to 0$ region: Theoretical background



$$F_{T,L} = \frac{\gamma}{\pi} Q^2 \sigma_{T,L}, \quad \gamma = \frac{|\vec{q}\,|}{q_0} = \sqrt{1 + \frac{Q^2}{\nu^2}}$$

$\sigma_{T,L}$ = Total ν cross section

$$\sim \sum_f (2\pi)^4 \delta(p + q - p_f) \left| \langle f \,|\, \varepsilon_{T,L} \cdot J(0) \,|\, p \rangle \right|^2$$

$F_{T,L}$ = transverse, longitudinal cross section

Vector current conservation: $q_\mu W^{\mu\nu} = 0$

$$\Rightarrow F_L^V \sim Q^2 F_T^V \text{ as } Q^2 \to 0$$

PCAC (Partially Conserved Axial-vector Current):

$\partial_\mu A^\mu(x) = f_\pi m_\pi^2 \pi(x)$, A^μ = Axial-vector current,

f_π = Pion-decay constant, π = Pion field

$$\Rightarrow F_L^A \sim \frac{f_\pi^2}{\pi} \sigma_\pi \text{ as } Q^2 \to 0,$$

Pion-scattering cross section: σ_π

References:
A. Donnachie and P. V. Landshoff, Z. Phys. C 61 (1994) 139
B. Z. Kopeliovich, Nucl. Phys. B 139 (2005) 219;
S. A. Kulagin, Phys. Rev. D 76 (2007) 094023

$Q^2 \to 0$ region: Practical description

$F_{1,2,3}^{\nu A}(x, Q^2 \to 0)$

(1) FLUKA, G. Battistoni *et al.*,
Acta Phys. Pol. B 40 (2009) 2431

$$F_{2,3}(x,Q^2) = \frac{2Q^2}{Q_0^2 + Q^2} F_{2,3}(x,Q_0^2)$$

(2) A. Bodek and U.-K. Yang, arXiv:1011.6592

charged-lepton:

$$F_2^{e/\mu}(x, Q^2 < 0.8\ \text{GeV}^2) = K_{valence}^{vector}(Q^2) F_{2,LO}^{valance}(\xi_w, Q^2 = 0.8\ \text{GeV}^2) + K_{sea}^{vector}(Q^2) F_{2,LO}^{sea}(\xi_w, Q^2 = 0.8\ \text{GeV}^2)$$

$$K_{valence}^{vector}(Q^2) = \frac{Q^2}{Q^2 + C_s}, \quad K_{sea}^{vector}(Q^2) = \left[1 - G_D^2(Q^2)\right] \frac{Q^2 + C_{v2}}{Q^2 + C_{v1}}$$

$$G_D(Q^2) = \frac{1}{(1 + Q^2/0.71)^2}, \quad \xi_w = \frac{2x(Q^2 + M_f^2 + B)}{Q^2\left[1 + \sqrt{1 + 4M^2x^2/Q^2}\right] + 2Ax}$$

neutrino:

Separate $F_i^\nu(x,Q^2)$ into vector and axial-vector parts.

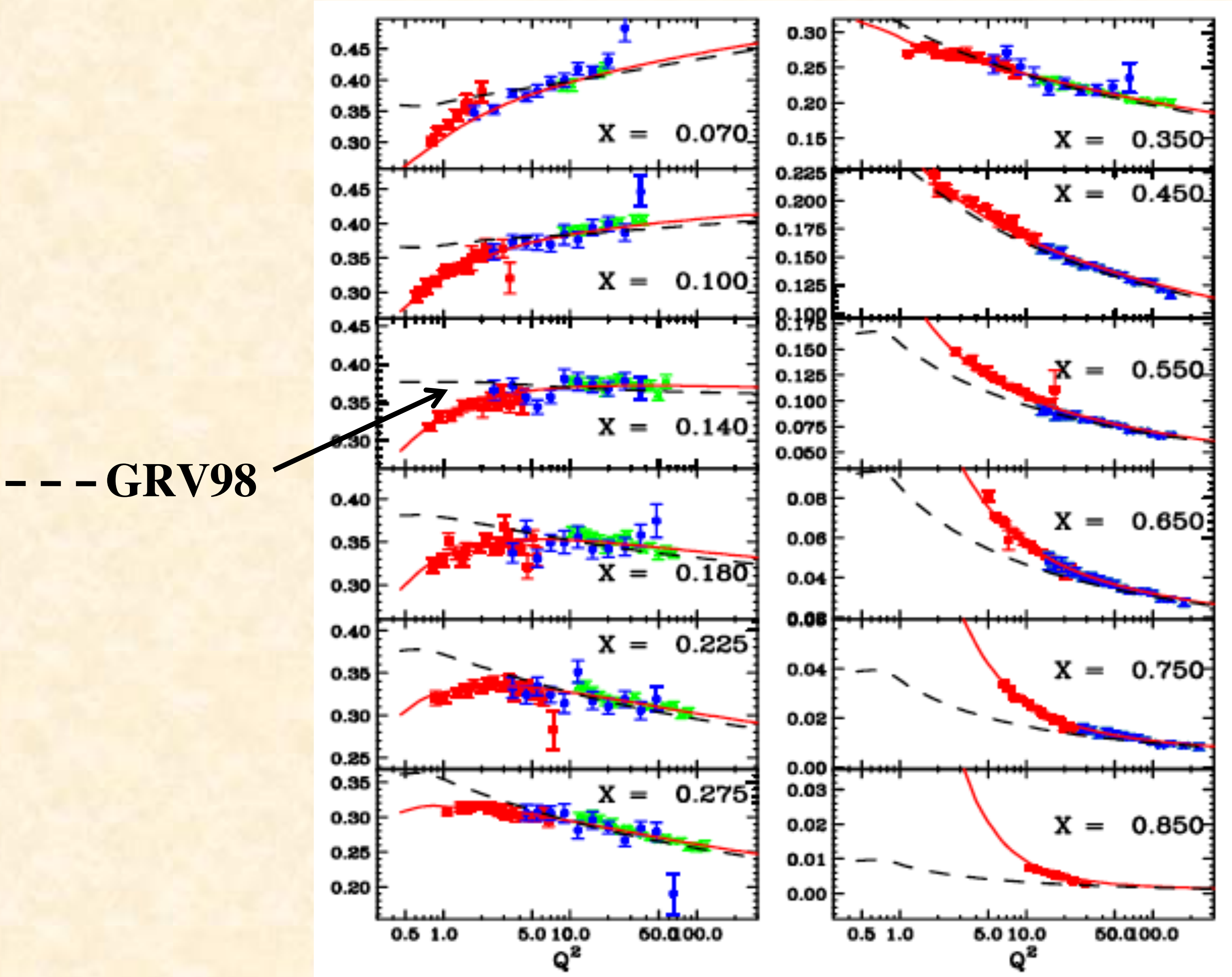
$F_i^\nu(x,Q^2)_{\text{vector}} \to Q^2 \to 0 \quad (Q^2 \to 0)$ as the charged-lepton case.

$F_i^\nu(x,Q^2)_{\text{axial-vector}} \neq 0 \quad (Q^2 \to 0)$ due to PCAC.

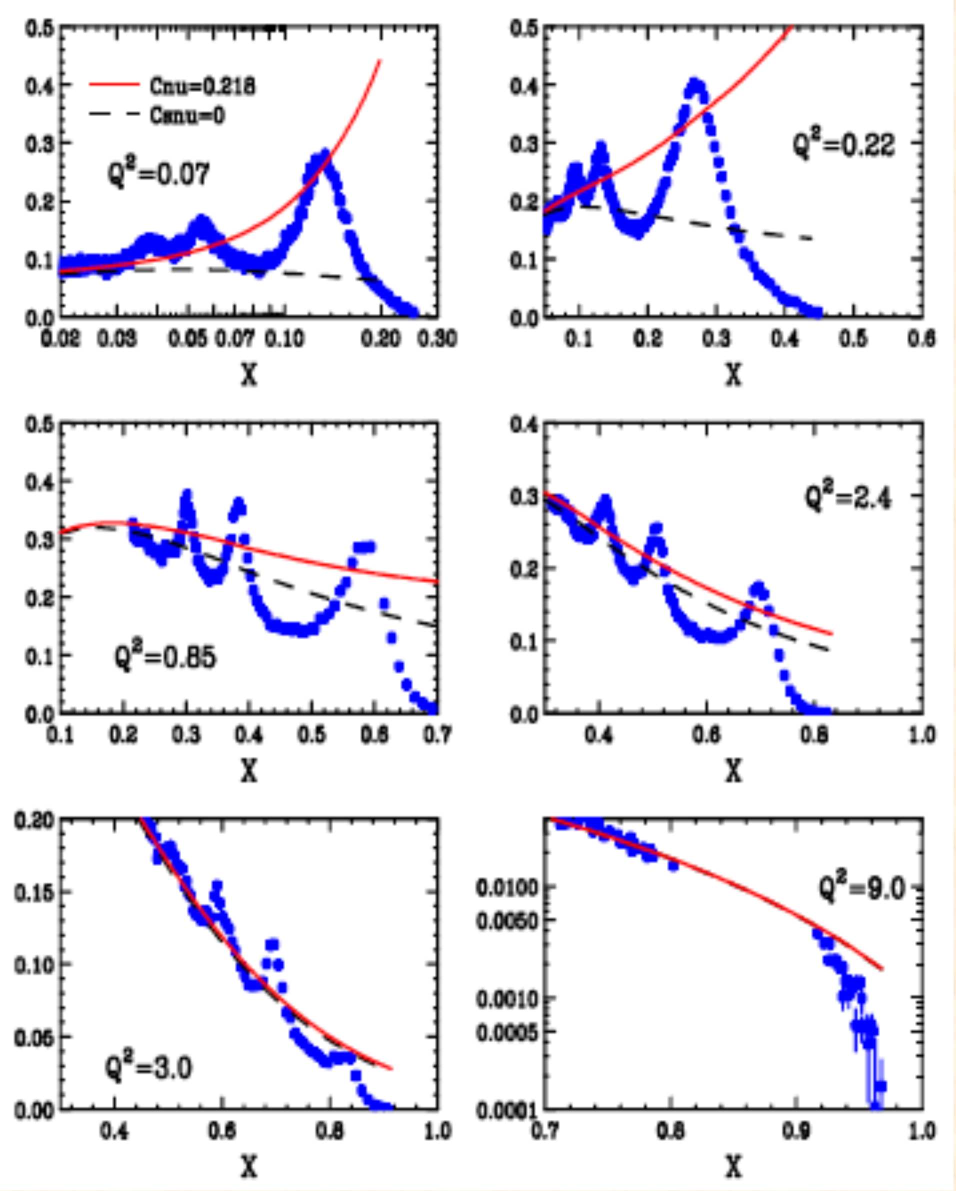
Actual expressions are slightly complicated (see the original paper).

Comparison with charged-lepton data

A. Bodek and U.-K. Yang, arXiv:1011.6592



Comparison with charged-lepton data



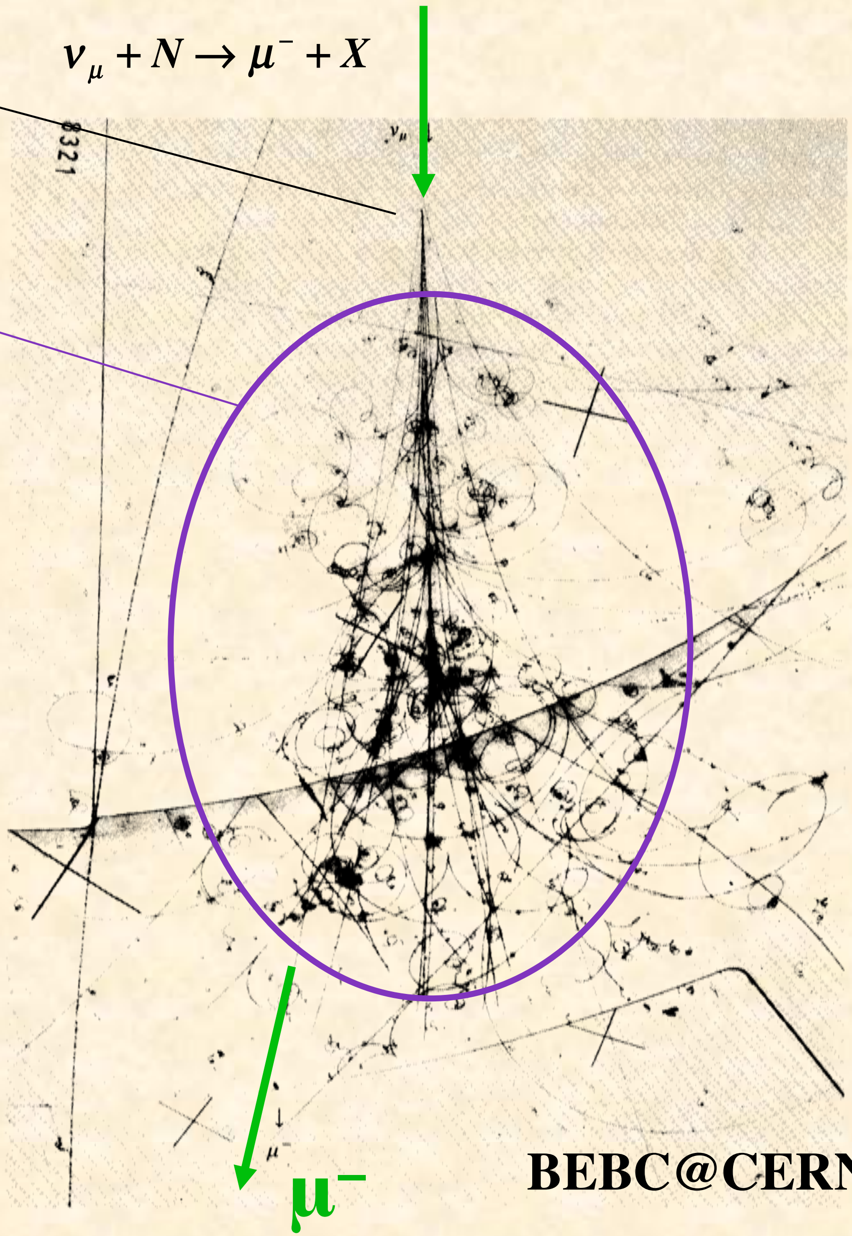
Introduction to neutrino deep inelastic scattering (DIS)

Deep inelastic scattering

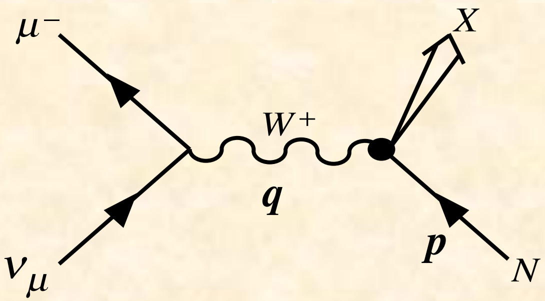
ν_μ ~ 200 GeV

A nucleon is broken up by a high-energy neutrino.

Hadrons are produced; however, these are not usually measured. (inclusive reaction)



BEBC@CERN

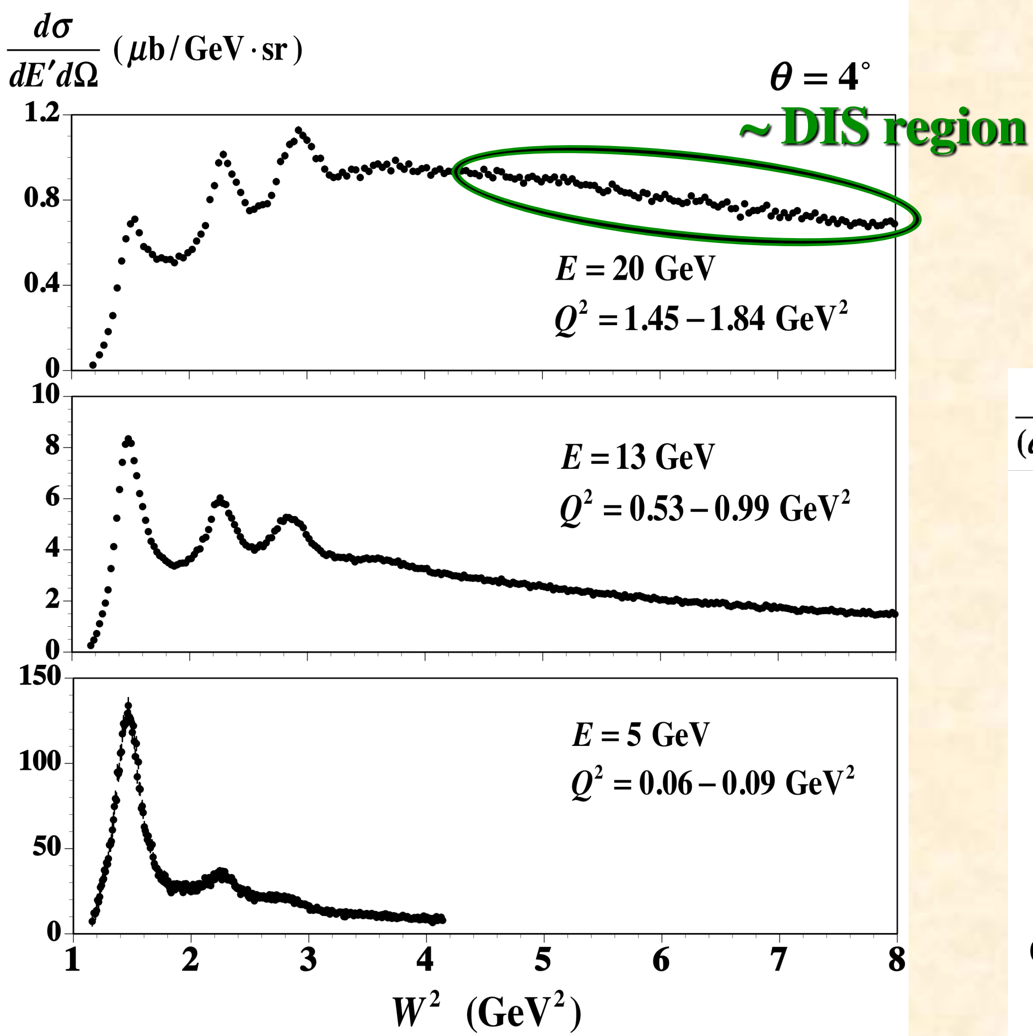


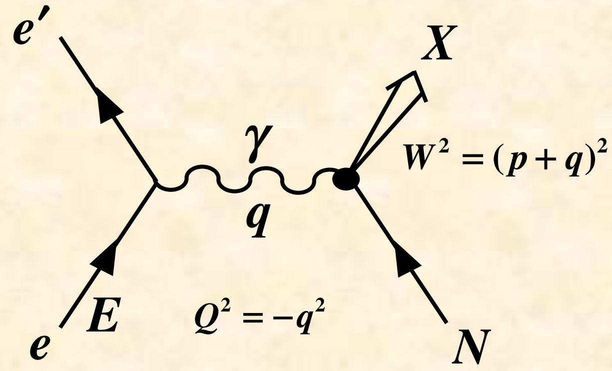
Momentum transfer: $q^2 = (k - k')^2 = -Q^2$

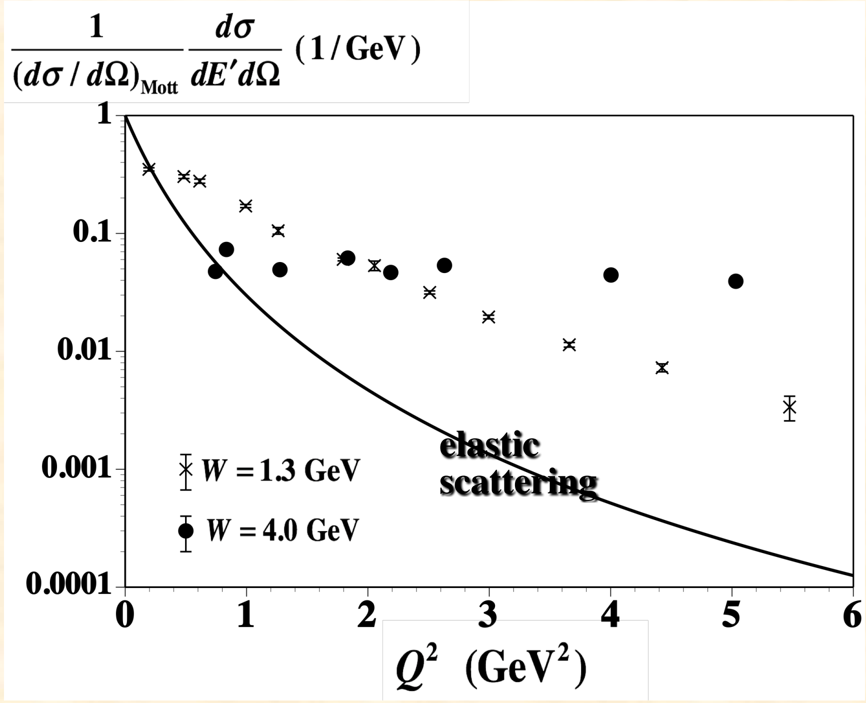
Bjorken scaling variable: $x = \dfrac{Q^2}{2p \cdot q}$

Invariant mass: $W^2 = p_X^2 = (p+q)^2$

Lepton scattering

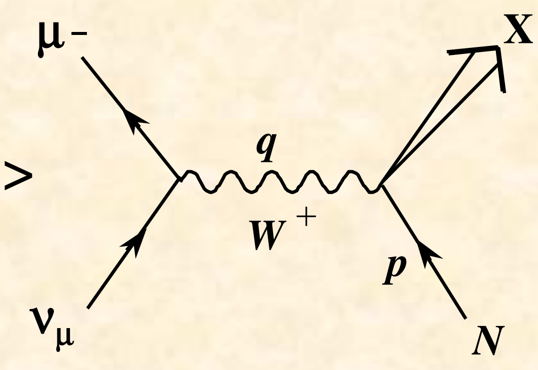






Neutrino deep inelastic scattering (CC: Charged Current)

$$d\sigma = \frac{1}{4k\cdot p}\,\frac{1}{2}\sum_{spins}\sum_X (2\pi)^4\,\delta^4(k+p-k'-p_X)\,|M|^2\,\frac{d^3k'}{(2\pi)^3\,2E'}$$



$$M = \frac{1}{1+Q^2/M_W^2}\,\frac{G_F}{\sqrt{2}}\,\bar{u}(k',\lambda')\,\gamma^\mu\,(1-\gamma_5)\,u(k,\lambda)\,<X|J_\mu^{CC}|p,\lambda_p>$$

$$\frac{d\sigma}{dE'd\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2}\,\frac{k'}{32\pi^2 E}\,L^{\mu\nu}\,W_{\mu\nu}$$

$$L^{\mu\nu} = 8\left[k^\mu k'^\nu + k'^\mu k^\nu - k\cdot k' g^{\mu\nu} + \underline{i\varepsilon^{\mu\nu\rho\sigma}k_\rho k'_\sigma}\right], \quad \varepsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2\frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) + \frac{i}{2M^2}\underline{W_3\varepsilon_{\mu\nu\rho\sigma}p^\rho q^\sigma}$$

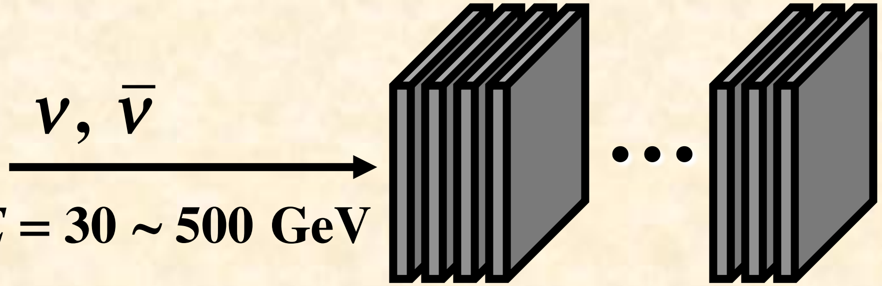
$$MW_1 = F_1\ ,\ \ \nu W_2 = F_2\ ,\ \ \nu W_3 = F_3\ ,\ \ x = \frac{Q^2}{2p\cdot q}\ ,\ \ y = \frac{p\cdot q}{p\cdot k}$$

$$\frac{d\sigma^{CC}_{\nu,\bar{\nu}}}{dx\,dy} = \frac{G_F^2\,(s-M^2)}{2\pi\,(1+Q^2/M_W^2)^2}\left[x\,y^2F_1^{CC} + \left(1-y-\frac{M\,x\,y}{2E}\right)F_2^{CC} \underline{\pm x\,y\left(1-\frac{y}{2}\right)F_3^{CC}}\right]$$

Neutrino DIS experiments

• CDHS,	H. Abramowicz *et al*.,	Z. Phys. C 25 (1984) 29
• WA25,	D. Allasia *et al*.,	Z. Phys. C 28 (1985) 321
• WA59,	K. Varvell *et al*.,	Z. Phys. C 36 (1987) 1
• CDHSW,	P. Berge *et al*.,	Z. Phys. C 49 (1991) 187
• Serpukhov,	A. V. Sidorov *et al*.,	Eur. Phys. J. C 10 (1999) 405
• CCFR,	U.-K. Yang *et al*.,	PRL 86 (2001) 2742
• NuTeV/CCFR $\mu^+\mu^-$,	M. Goncharov *et al*.,	PRD 64 (2001) 112006
• CHORUS,	G. Onengut *et al*.,	PLB 632 (2006) 65
• NuTeV,	M. Tzanov *et al*.,	PRD 74 (2006) 012008
• Minverνa,	in progress	

Neutrino DIS experiments



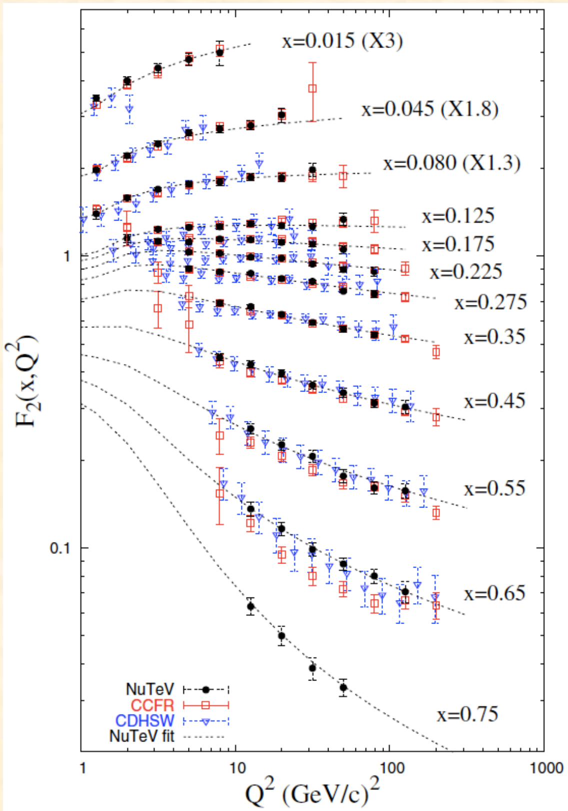
E = 30 ~ 500 GeV

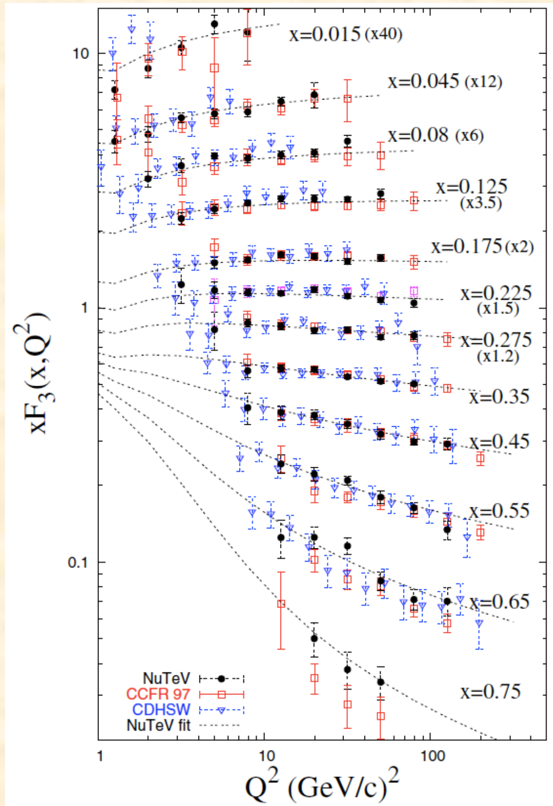
Huge Fe target (690 ton)

Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

MINERνA (He, C, Fe, Pb), …

M. Tzanov *et al.* (NuTeV),
PRD74 (2006) 012008.

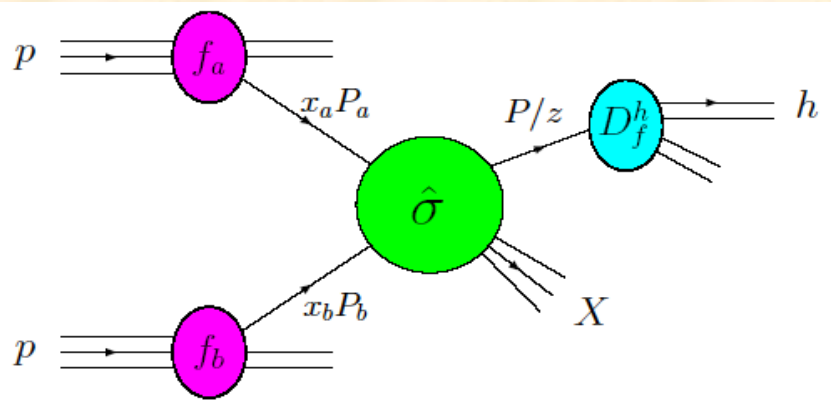




Parton Distribution Functions in the Nucleon

High-energy nuclear reactions

Nuclear PDFs are needed for describing high-energy nuclear reactions in order to find any new phenomena.



$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \to cX) \otimes D_c^h(z, Q^2)$$

$f_a(x_a, Q^2)$: **parton distribution functions**

$\hat{\sigma}(ab \to cX)$: **partonic cross sections**

$D_c^h(z, Q^2)$: **fragmentation functions**

Recent works on unpolarized PDFs

ABKM (Alekhin, Blümlein, Klein, Moch)

ABKM-2010, 2011, S. Alekhin *et al*., Phys. Rev. D 81 (2010) 014032; Phys. Rev. D86（2012）054009;

ABM-2014, S. Alekhin *et al*., Phys. Rev. D89（2014）054028.

CTEQ (Coordinated Theoretical-Experimental Project on QCD)

CTEQ6.6, P. M. Nadolsky *et al*., Phys. Rev. D 78 (2008) 013004.

CT10, H.-L. Lau *et al*., Phys. Rev. D 82 (2010) 074024.

CT12, J. F. Owens *et al*., Phys. Rev. D 87 (2013) 094012. CT14, S. Dulat *et al*., arXiv:1506.07443.

GJR (Glück, Jimenez-Delgado, Reya)

GJR-2008, M. Gluck *et al*., Eur. Phys. J. C 53 (2008) 355; PRD79 (2009) 074023;

JR-2014, Phys.Rev. D89（2014）074049.

HERA (H1 and ZEUS collaborations)

HERAPDF, F. D. Aaron *et al*., JHEP 01 (2010) 109: Eur. Phys. J. C73（2013）2311.

MSTW (Martin, Stirling, Thorne, Watt, L. A. Harland-Lang, P. Motylinski)

MSTW2008, A. D. Martin *et al*., Eur. Phys. J. C 63 (2009) 189;

MMHT2014, A. Harland-Lang *et al*., Eur. Phys. J. C (2015) 75.

Neural Network (Ball, Bertone, Carrazza, Del Debbio, Forte, Guffanti, Hartland, Latorre, Rojo, Ubiali, ...)

NNPDF2.0, R. D. Ball *et al*., Nucl. Phys. B 838 (2010) 136; B855（2012）153;

B867（2013）244; B874（2013）36; B877（2013）290; JHEP 04 (2015) 040.

Parton distribution functions are determined by fitting various experimental data.

- electron/muon: $\mu + p \to \mu + X$
- neutrino: $\nu_\mu + p \to \mu + X$
- Drell-Yan: $p + p \to \mu^+\mu^- + X$
- $\cdots$

(1) assume functional form of PDFs at fixed $Q^2 (\equiv Q_0^2)$:

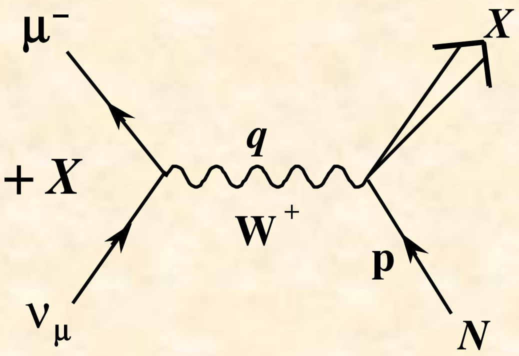
e.g. $f_i(x, Q_0^2) = A_i x^{\alpha_i} (1-x)^{\beta_i} (1+\gamma_i x)$,

where $i = u_v,\ d_v,\ \bar{u},\ \bar{d},\ \bar{s},\ g$

(2) calculate observables at their experimental Q^2 points.

(3) then, the parameters A_i, α_i, β_i, γ_i are determined so as to minimize χ^2 in comparison with data.

Determination of each distribution



Valence quark

$$q_v(x) \equiv q(x) - \bar{q}(x) \qquad \nu_\mu + p \to \mu^- + X$$

$$M = \frac{1}{1+Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k',\lambda') \gamma^\mu (1-\gamma_5) u(k,\lambda) < X | J_\mu^{CC} | p, \lambda_p >$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = 8\left[k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k' + i\varepsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \right] \qquad \text{where } \varepsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_N^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) + \frac{i}{2M_N^2} W_3 \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$$

$$MW_1 = F_1 , \ \nu W_2 = F_2 , \ \nu W_3 = F_3 , \ x = \frac{Q^2}{2p \cdot q} , \ y = \frac{p \cdot q}{p \cdot k}$$

$$\frac{d\sigma_{\nu,\bar{\nu}}^{CC}}{dx\, dy} = \frac{G_F^2 (s - M^2)}{2\pi (1+Q^2/M_W^2)^2} \left[x y^2 F_1^{CC} + \left(1 - y - \frac{M x y}{2E} \right) F_2^{CC} \pm x y \left(1 - \frac{y}{2} \right) F_3^{CC} \right]$$

$$\frac{1}{2} [F_3^{\nu p} + F_3^{\bar{\nu} p}]_{CC} = u_v + d_v + s - \bar{s} + c - \bar{c}$$

Note: Nuclear corrections in CCFR/NuTeV (ν+Fe).

Valence: also F_2 at large x

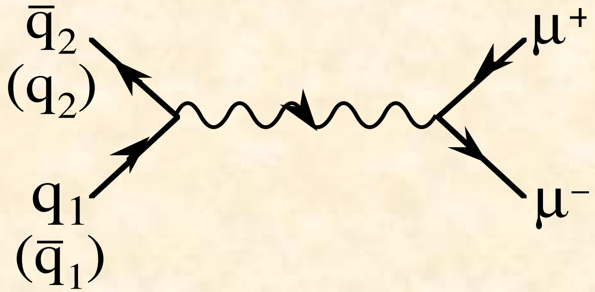
Sea quark

e/μ scattering

$$F_2^N = \frac{F_2^p + F_2^n}{2} = \frac{x}{2}\left[\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}+s+\bar{s}) + \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}+s+\bar{s})\right] = \frac{x}{2}\left[\frac{5}{9}(u+\bar{u}+d+\bar{d}) + \frac{2}{9}(s+\bar{s})\right]$$

$$= \frac{x}{2}\left[\frac{5}{9}(u_v+d_v) + \frac{10}{9}(\bar{u}+\bar{d}) + \frac{2}{9}(s+\bar{s})\right] = \frac{5}{18}x(u_v+d_v) + \frac{2}{18}x\left[5(\bar{u}+\bar{d}) + (s+\bar{s})\right]$$

Drell-Yan (lepton-pair production)



$$p_1 + p_2 \to \mu^+\mu^- + X$$

$$d\sigma \propto q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)$$

at large $x_F = x_1 - x_2$

projectile target

$$d\sigma \propto q_V(x_1)\bar{q}(x_2)$$

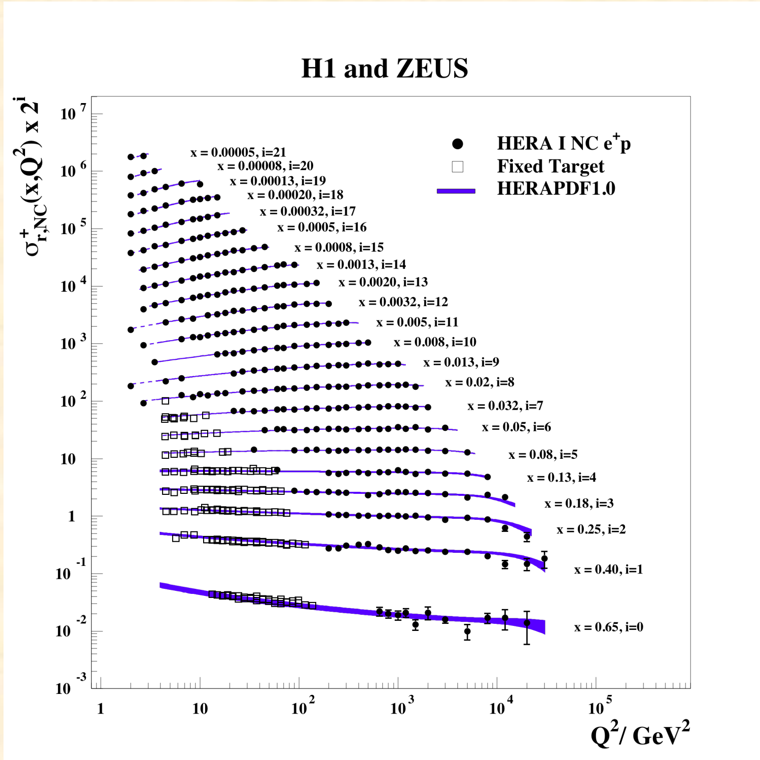
$\bar{q}(x_2)$ can be obtained if $q_V(x_1)$ is known.

Gluon

scaling violation of F_2

$$\frac{\partial}{\partial(\ln Q^2)}\begin{pmatrix} q_s(x,t) \\ g(x,t) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}(x/y) & P_{qg}(x/y) \\ P_{gq}(x/y) & P_{gg}(x/y) \end{pmatrix}\begin{pmatrix} q_s(y,t) \\ g(y,t) \end{pmatrix}$$

H1 and ZEUS
JHEP01(2010)109

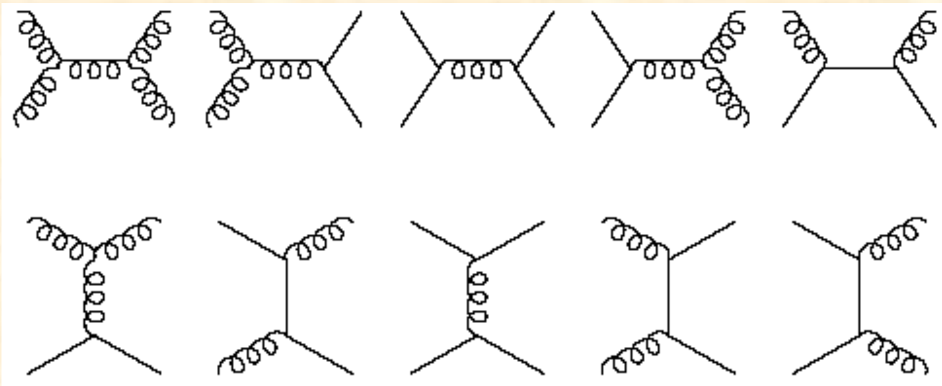


at small x

$$\frac{\partial F_2}{\partial(\ln Q^2)} \approx \frac{10\,\alpha_s}{27\pi}g$$

K. Prytz, Phys. Lett. B311 (1993) 286.

jet production



Situation of data for PDFs of the nucleon **(CTEQ14)**

ID#	Experimental data set	N_{pt}	χ_e^2	χ_e^2/N_{pt}	S_n
101	BCDMS F_2^p [24]	337	384	1.14	1.74
102	BCDMS F_2^d [25]	250	294	1.18	1.89
104	NMC F_2^d/F_2^p [26]	123	133	1.08	0.68
106	NMC σ_{red}^p [26]	201	372	1.85	6.89
108	CDHSW F_2^p [27]	85	72	0.85	-0.99
109	CDHSW F_3^p [27]	96	80	0.83	-1.18
110	CCFR F_2^p [28]	69	70	1.02	0.15
111	CCFR xF_3^p [29]	86	31	0.36	-5.73
124	NuTeV $\nu\mu\mu$ SIDIS [30]	38	24	0.62	-1.83
125	NuTeV $\bar{\nu}\mu\mu$ SIDIS [30]	33	39	1.18	0.78
126	CCFR $\nu\mu\mu$ SIDIS [31]	40	29	0.72	-1.32
127	CCFR $\bar{\nu}\mu\mu$ SIDIS [31]	38	20	0.53	-2.46
145	H1 σ_r^b [32]	10	6.8	0.68	-0.67
147	Combined HERA charm production [33]	47	59	1.26	1.22
159	HERA1 Combined NC and CC DIS [34]	579	591	1.02	0.37
169	H1 F_L [35]	9	17	1.92	1.7

ID#	Experimental data set	N_{pt}	χ_e^2	χ_e^2/N_{pt}	S_n
201	E605 Drell-Yan process [37]	119	116	0.98	-0.15
203	E866 Drell-Yan process, $\sigma_{pd}/(2\sigma_{pp})$ [38]	15	13	0.87	-0.25
204	E866 Drell-Yan process, $Q^3 d^2\sigma_{pp}/(dQdx_F)$ [39]	184	252	1.37	3.19
225	CDF Run-1 electron A_{ch}, $p_{T\ell} > 25$ GeV [40]	11	8.9	0.81	-0.32
227	CDF Run-2 electron A_{ch}, $p_{T\ell} > 25$ GeV [41]	11	14	1.24	0.67
234	DØ Run-2 muon A_{ch}, $p_{T\ell} > 20$ GeV [42]	9	8.3	0.92	-0.02
240	LHCb 7 TeV 35 pb^{-1} W/Z $d\sigma/dy_\ell$ [43]	14	9.9	0.71	-0.73
241	LHCb 7 TeV 35 pb^{-1} A_{ch}, $p_{T\ell} > 20$ GeV [43]	5	5.3	1.06	0.30
260	DØ Run-2 Z rapidity [44]	28	17	0.59	-1.71
261	CDF Run-2 Z rapidity [45]	29	48	1.64	2.13
266	CMS 7 TeV 4.7 fb^{-1}, muon A_{ch}, $p_{T\ell} > 35$ GeV [46]	11	12.1	1.10	0.37
267	CMS 7 TeV 840 pb^{-1}, electron A_{ch}, $p_{T\ell} > 35$ GeV [47]	11	10.1	0.92	-0.06
268	ATLAS 7 TeV 35 pb^{-1} W/Z cross sec., A_{ch} [48]	41	51	1.25	1.11
281	DØ Run-2 9.7 fb^{-1} electron A_{ch}, $p_{T\ell} > 25$ GeV [15]	13	35	2.67	3.11
504	CDF Run-2 inclusive jet production [49]	72	105	1.45	2.45
514	DØ Run-2 inclusive jet production [50]	110	120	1.09	0.67
535	ATLAS 7 TeV 35 pb^{-1} incl. jet production [51]	90	50	0.55	-3.59
538	CMS 7 TeV 5 fb^{-1} incl. jet production [52]	133	177	1.33	2.51

CERN-BCDMS, NMC

CDHSW, CCFR, NuTeV
(ν deep inelastic)

HERA (NC, CC, charm)

Fermilab-E866 (Drell-Yan)

Tevatron (jets, W, Z)

LHC (jets, W, Z, Drell-Yan)

Recent progress

Structure functions in parton model for neutrino-nucleon scattering

$F_2 = 2\,x\,F_1$

$$F_2^{\nu p} = 2\,x\,(d + s + \bar{u} + \bar{c})$$
$$F_2^{\bar{\nu} p} = 2\,x\,(u + c + \bar{d} + \bar{s})$$

$$F_2^{\nu n} = 2\,x\,(u + s + \bar{d} + \bar{c})$$
$$F_2^{\bar{\nu} n} = 2\,x\,(d + c + \bar{u} + \bar{s})$$

$$xF_3^{\nu p} = 2\,x\,(d + s - \bar{u} - \bar{c})$$
$$xF_3^{\bar{\nu} p} = 2\,x\,(u + c - \bar{d} - \bar{s})$$

$$xF_3^{\nu n} = 2\,x\,(u + s - \bar{d} - \bar{c})$$
$$xF_3^{\bar{\nu} n} = 2\,x\,(d + c - \bar{u} - \bar{s})$$

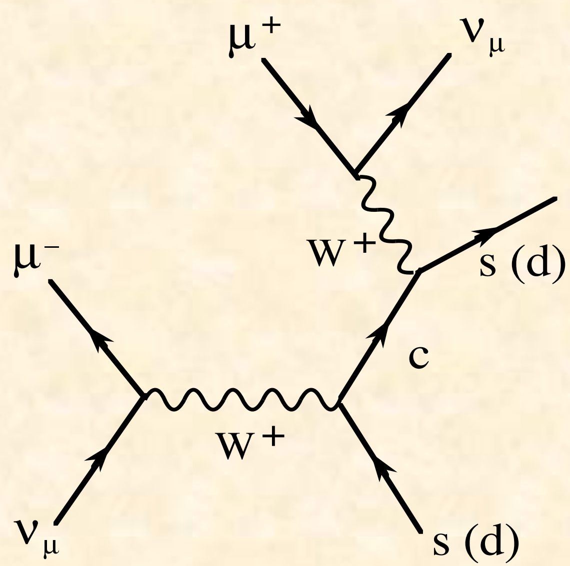
→

$$F_3^{\nu p} + F_3^{\bar{\nu} p} = 2\,\underline{(u_v + d_v)} + 2\,(s - \bar{s}) + 2\,(c - \bar{c})$$

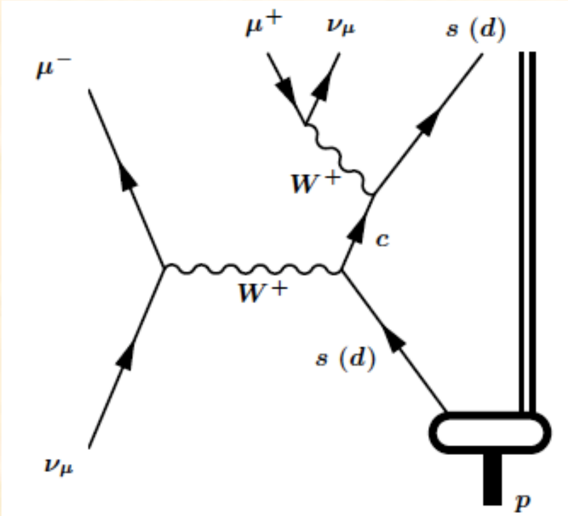
valence-quark distributions

$$F_3^{\nu(p+n)/2} - F_3^{\bar{\nu}(p+n)/2} = 2\,(s + \bar{s}) - 2\,(c + \bar{c})$$

also $\nu\,p \to \mu^- \mu^+ X$ for finding $2\,\bar{s}\,/\,(\bar{u} + \bar{d})$



$s(x)$ from neutrino-induced opposite-sign dimuon events



A. Kayis-Topaksu *et al.*, NPB7 98 (2008) 1.
U. Dore, arXiv: 1103.4572 [hep-ex].

$$\kappa = \frac{\int dx\, x\,[s(x,Q^2)+\bar{s}(x,Q^2)]}{\int dx\, x\,[\bar{u}(x,Q^2)+\bar{d}(x,Q^2)]}$$

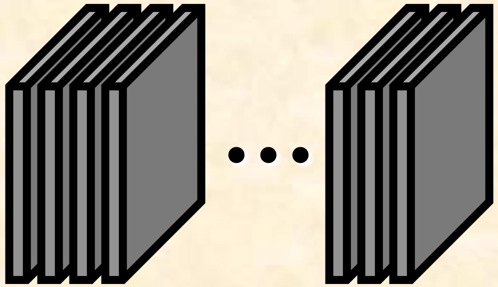
$Q^2 = 20\ \mathrm{GeV}^2$

CCFR, NuTeV

Experiment	κ
This analysis	0.33 ± 0.07
CDHS [1]	0.47 ± 0.09
CCFR [2]	0.44 ± 0.09
CHARM II [3]	0.39 ± 0.09
NOMAD [4]	0.48 ± 0.17
NuTeV [5]	0.38 ± 0.08

$\nu,\ \bar{\nu}$

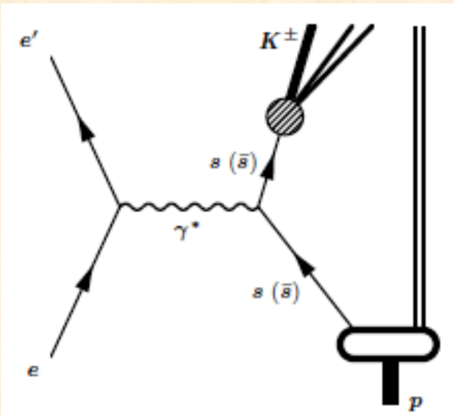
$E = 30 \sim 500$ GeV

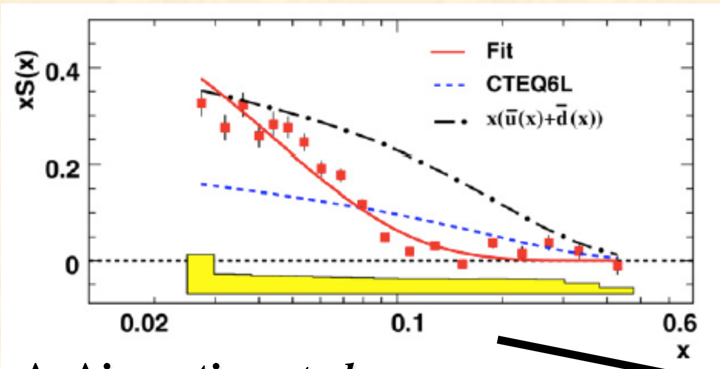


Huge Fe target (690 ton)

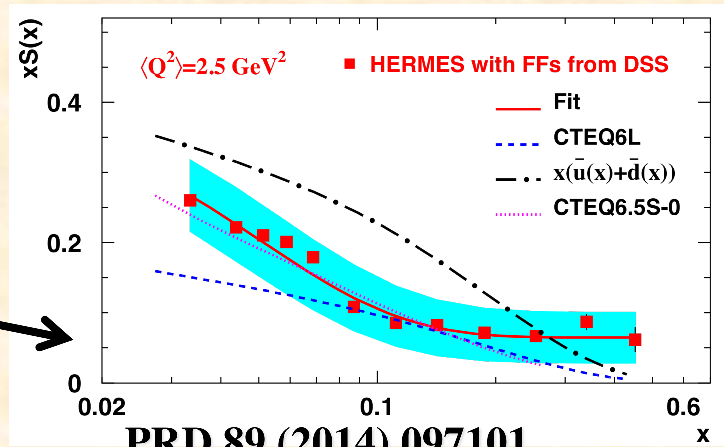
Issue: nuclear corrections

HERMES semi-inclusive measurement





A. Airapetian *et al.*,
PLB 666 (2008) 446.



PRD 89 (2014) 097101.

MMLT-2014

L. A. Harland-Lang, A. D. Martin, P. Motylinkski, and R. S. Thorne,
Eur. Phys. J. C 75 (2015) 204.

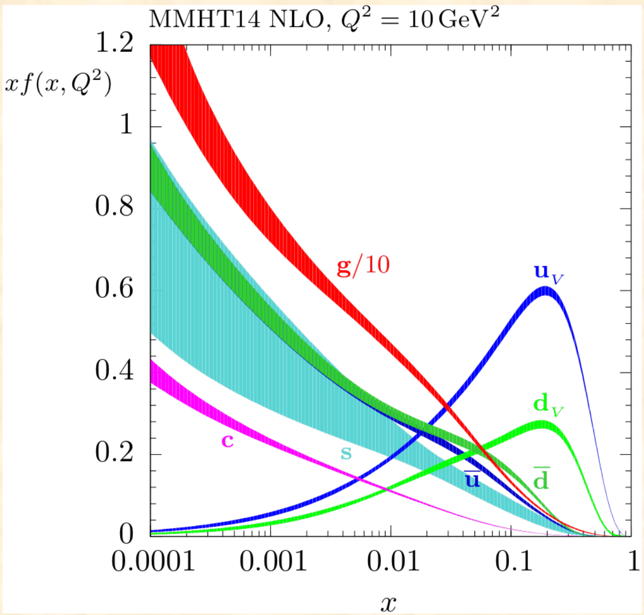
Functional form: $Q_0^2 = 1\ \text{GeV}^2$

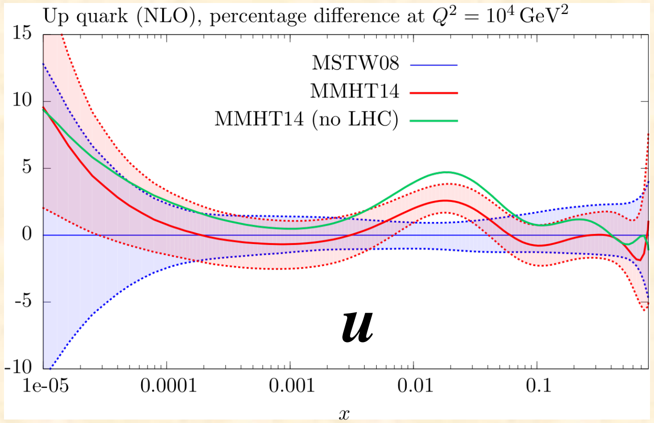
$$xf(x,Q_0^2) = Ax^{\delta}(1-x)^{\eta}\left[1+\sum_{i=1}^{n} a_i T_i(y(x))\right], \quad y = 1-2\sqrt{x}, \; n=4, \; f = u_v, \; d_v, \; S, \; s_+ = s+\bar{s}, \; \delta_S = \delta_{s_+}$$

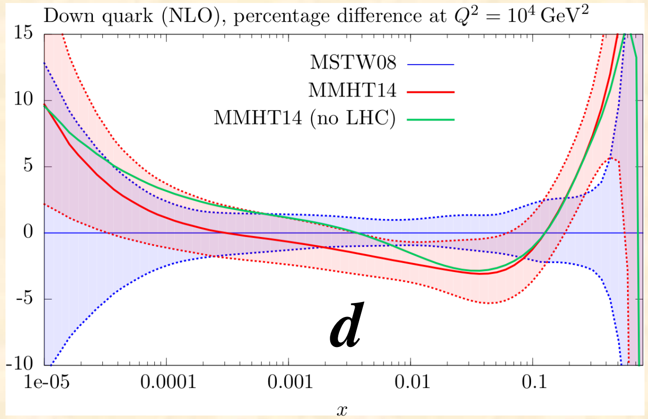
$$xg(x) = A_g x^{\delta_g}(1-x)^{\eta_g}\left[1+\sum_{i=1}^{2} a_{gi} T_i(y(x))\right] + A_g' x^{\delta_g'}(1-x)^{\eta_g'}$$

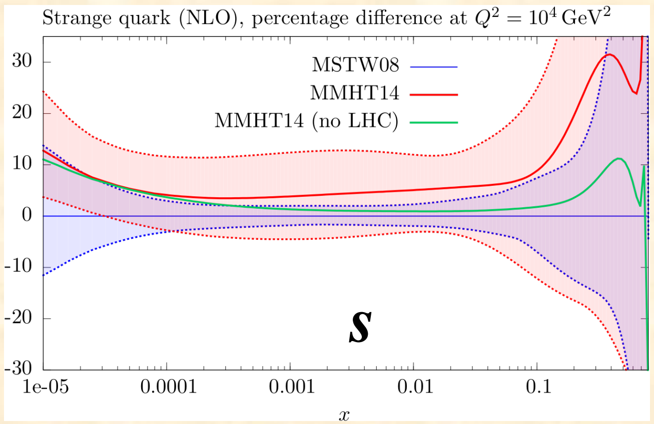
$$x\Delta(x) \equiv x(\bar{d}-\bar{u}) = Ax^{\delta}(1-x)^{\eta}(1+\gamma x+\varepsilon x^2), \qquad xs_- = x\left[s(x)-\bar{s}(x)\right] = A_- x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0)$$

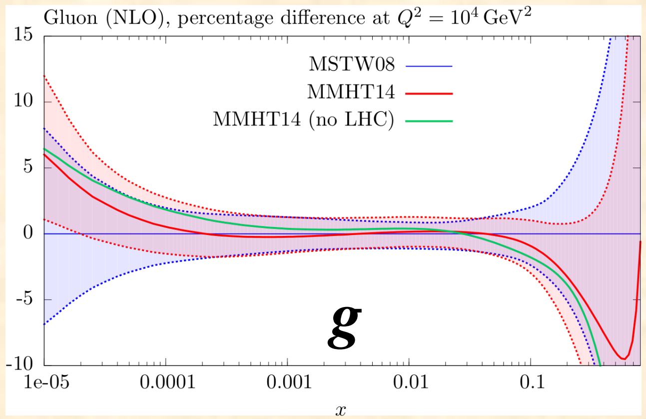
- **NLO, NNLO**



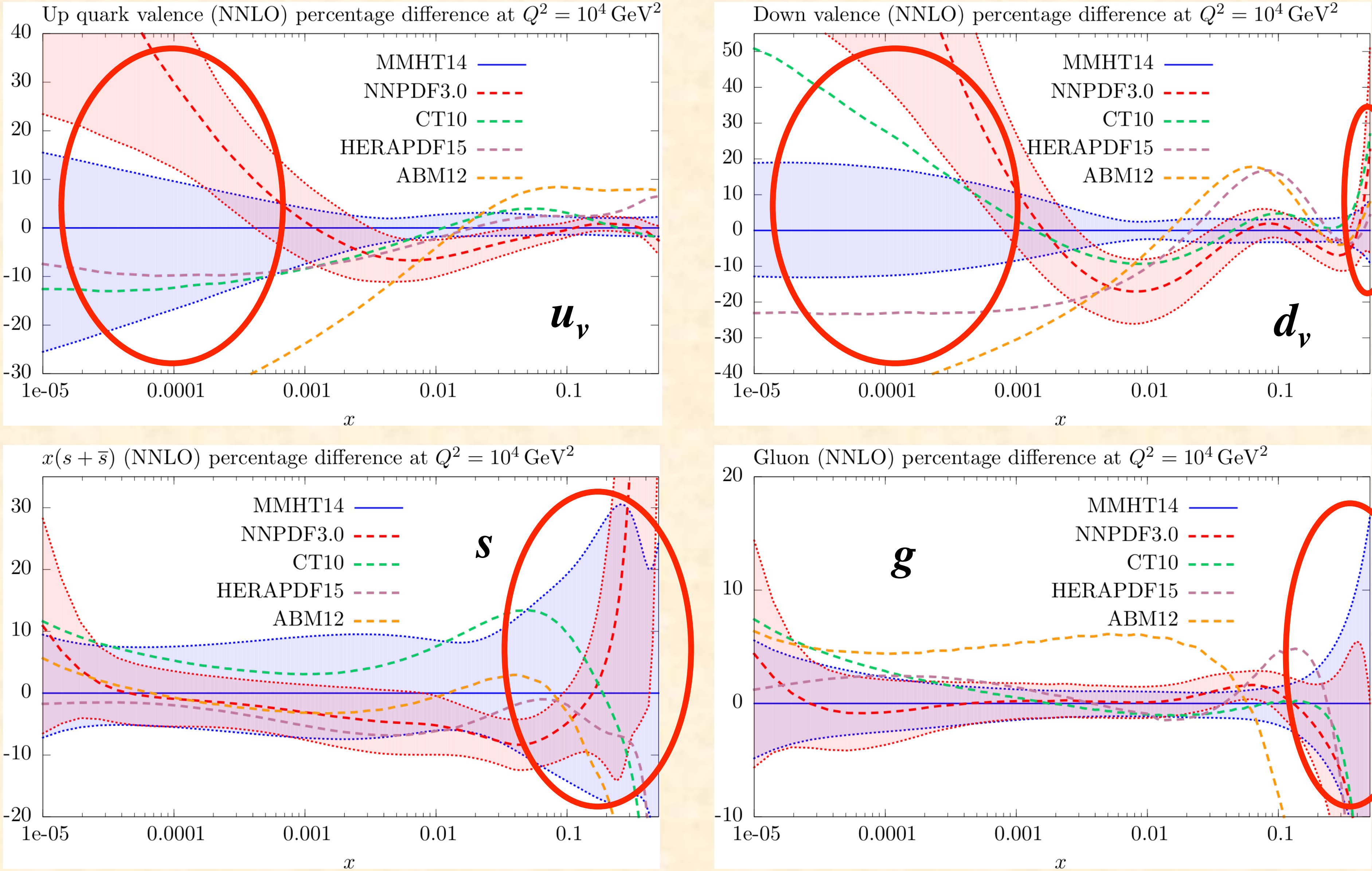








Comparisons with various PDFs



CTEQ14

CT14, S. Dulat *et al*., arXiv:1506.07443

Functional form: $Q_0^2 = (1.3)^2$ GeV^2

$$xf(x,Q_0^2) = x^{a_1}(1-x)^{a_2}P(x)$$

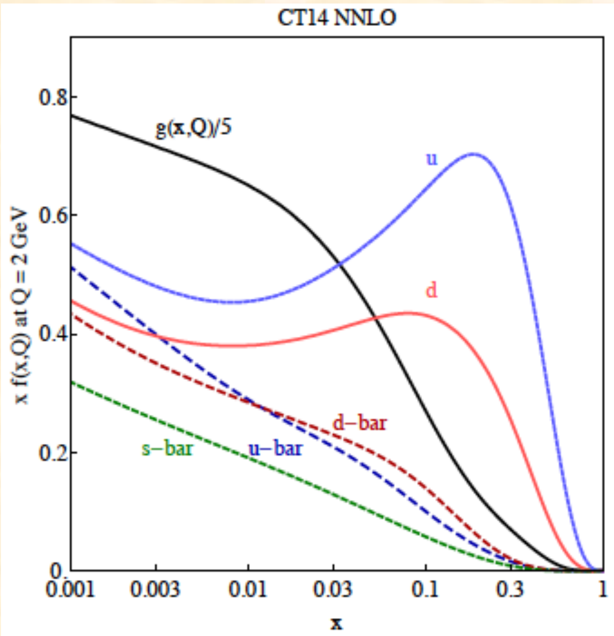
used to be $P(x) = \exp\left(a_0 + a_3\sqrt{x} + a_4 x + a_5 x^2\right)$

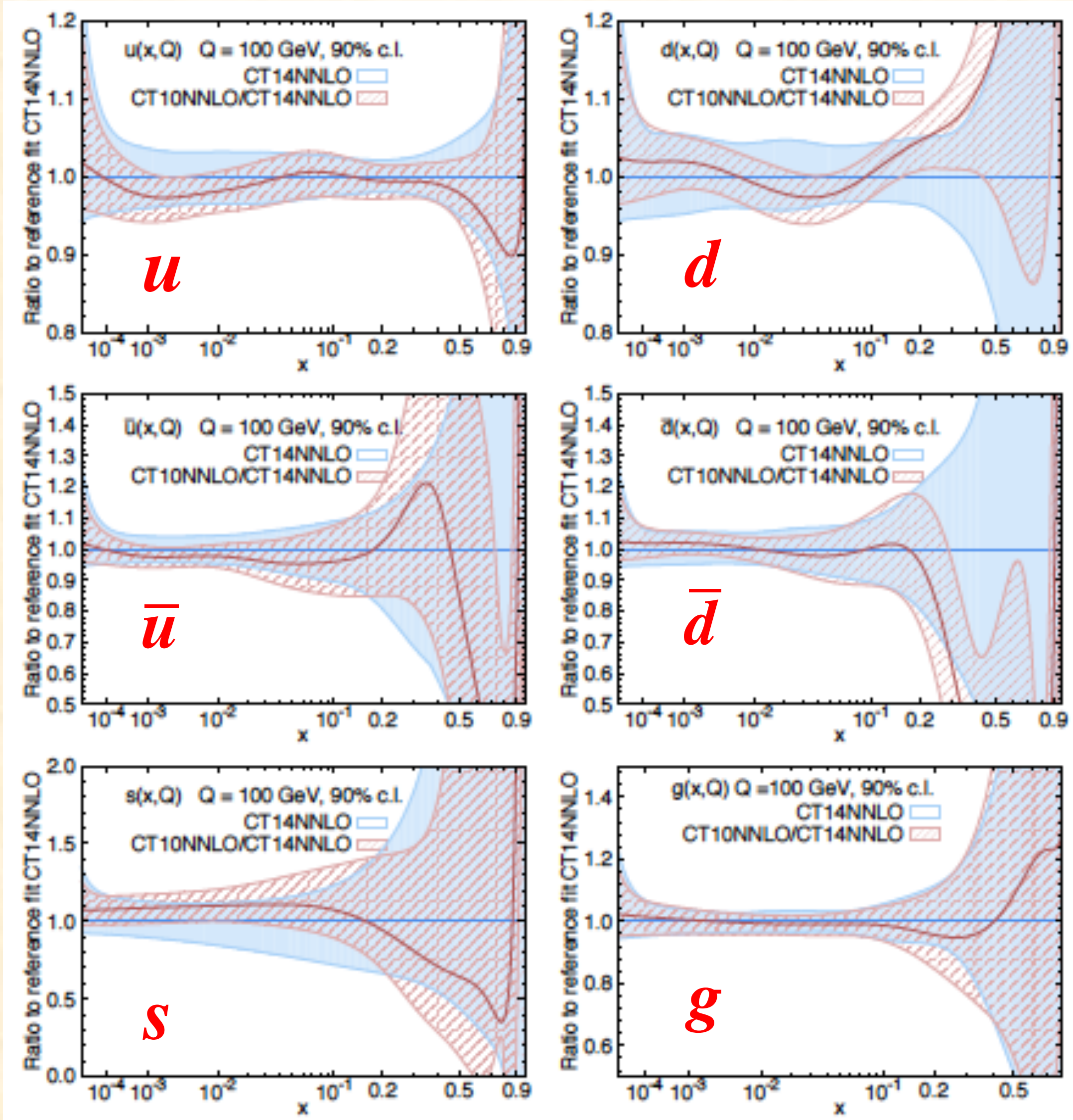
Expansion by Bernstein polynomials:

$$P(x) = \sum_{i=0}^{n} d_i p_i(y=\sqrt{x}),\ \ p_0(y) = (1-y)^4,$$

$$p_1(y) = 4y(1-y)^3,\ \ p_2(y) = 6y^2(1-y)^2,$$

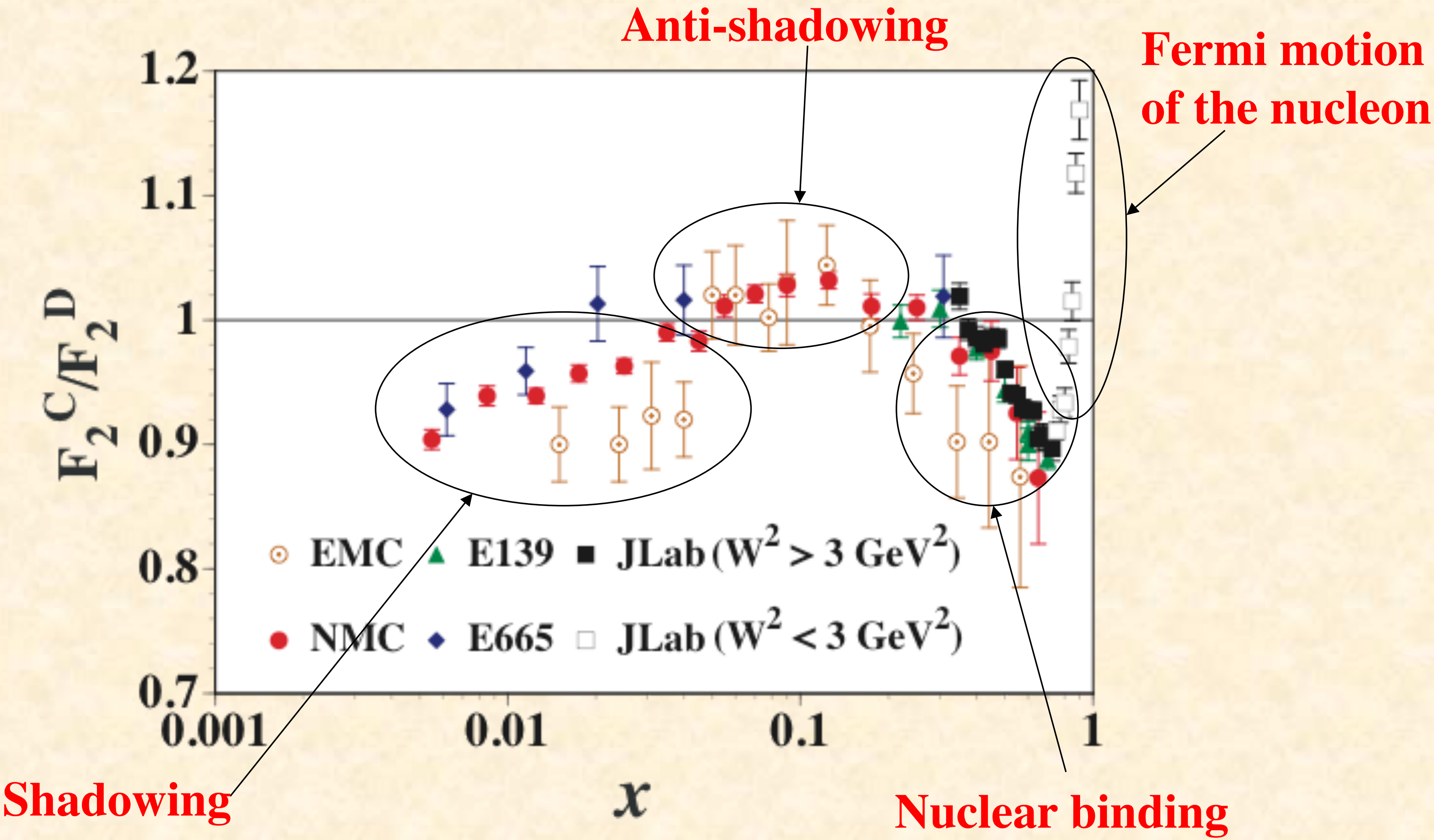
$$p_3(y) = 4y^3(1-y),\ \ p_4(y) = y^4$$





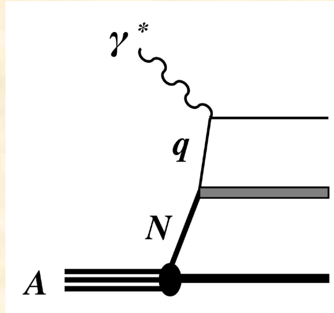
Nuclear modifications of parton distribution functions: Physics mechanisms

Nuclear modifications of structure function F_2



D. F. Geesaman, K. Saito, A. W. Thomas,
Ann. Rev. Nucl. Part. Sci. 45 (1995) 337

Binding and Fermi motion



Convolution: $W^A_{\mu\nu}(p_A, q) = \int d^4p\, S(p)\, W^N_{\mu\nu}(p_N, q)$

$S(p)$ = Spectral function = nucleon momentum distribution in a nucleus

In a simple shell model: $S(p) = \sum_i |\phi_i(\vec{p})|^2 \delta(p_0 - M_N - \varepsilon_i)$

Separation energy: ε_i

$$\hat{P}_2^{\mu\nu} = -\frac{M_N^2 \nu}{2\tilde{p}^2}\left(g^{\mu\nu} - \frac{3\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2}\right)$$

$$\hat{P}_2^{\mu\nu} W_{\mu\nu} = F_2$$

Projecting out F_2: $F_2^A(x, Q^2) = \sum_i \int dz\, f_i(z)\, F_2^N(x/z, Q^2)$

$$z = \frac{p \cdot q}{M_N \nu} \simeq \frac{p \cdot q}{p_A \cdot q / A} \simeq \frac{p^+}{p_A^+ / A}$$ lightcone momentum fraction

$$a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}}$$

$$q = (\nu,\ 0,\ 0,\ -\sqrt{\nu^2 + Q^2})$$

$$q^+ = -\frac{M x}{\sqrt{2}},\quad q^- = \frac{2\nu + M x}{\sqrt{2}} = \sqrt{2}\nu \gg M$$

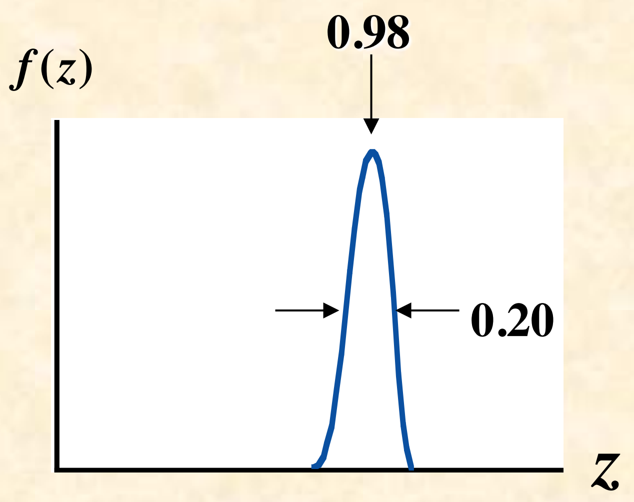
$$p \cdot q = p^+ q^- + p^- q^+ - \vec{p}_T \cdot \vec{q}_T \simeq p^+ q^-$$

$$f_i(z) = \int d^3p\, z\, \delta\left(z - \frac{p \cdot q}{M_N \nu}\right) |\phi_i(\vec{p})|^2$$ lightcone momentum distribution for a nucleon i

$$F_2^A(x,Q^2) = \sum_i \int dz\, f_i(z)\, F_2^N(x/z,Q^2) \qquad f_i(z) = \int d^3p\, z\, \delta\left(z - \frac{p\cdot q}{M_N \nu}\right) |\phi_i(\vec{p})|^2$$

$$z = \frac{p\cdot q}{M_N \nu} = \frac{p^0 \nu - \vec{p}\cdot\vec{q}}{M_N \nu} = 1 - \frac{|\varepsilon_i|}{M_N} - \frac{\vec{p}\cdot\vec{q}}{M_N \nu} \approx 1.00 - 0.02 \pm 0.20 \text{ for a medium-size nucleus}$$

If $f_i(z)$ were $f_i(z) = \delta(z-1)$, there is no nuclear modification: $F_2^A(x,Q^2) = F_2^N(x,Q^2)$.

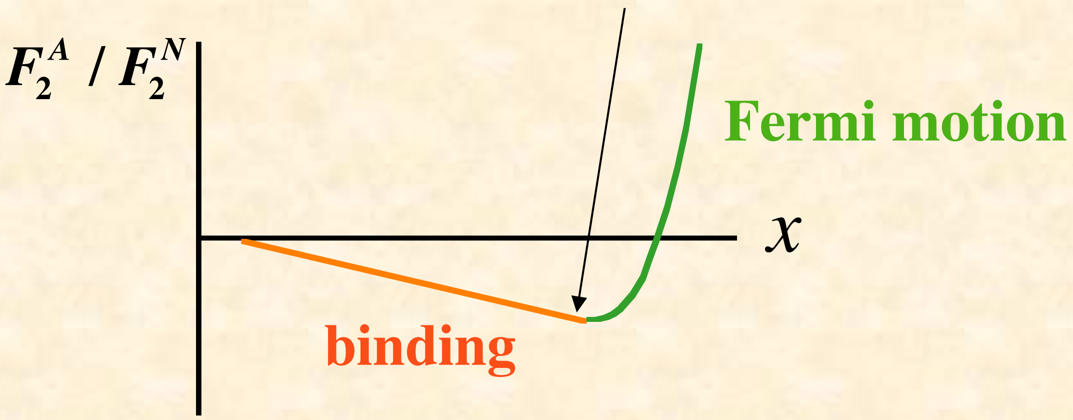


Because the peak shifts slightly (1→ 0.98), nuclear modification of F_2 is created.

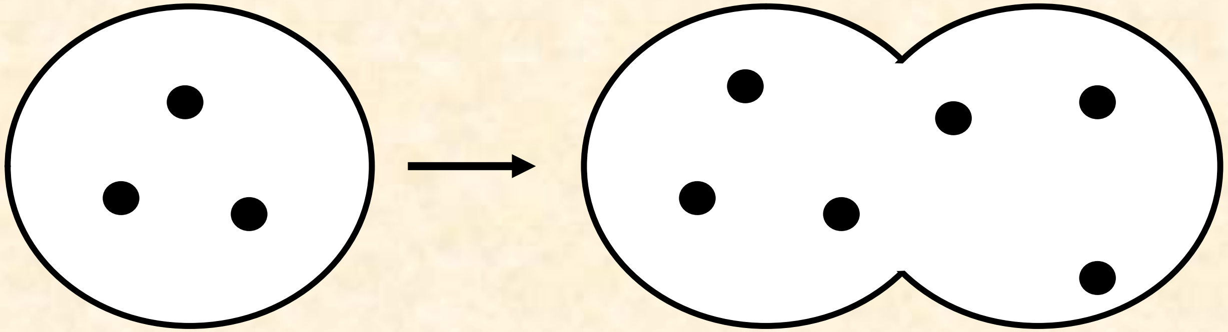
$$F_2^A(x,Q^2) \simeq F_2^N(x/0.98,Q^2)$$

For $x = 0.60$, $x/0.98 = 0.61$

$$\frac{F_2^N(x=0.61)}{F_2^N(x=0.60)} = \frac{0.021}{0.024} = 0.88$$



Theoretical Ideas at Medium x: Q^2 Rescaling Model

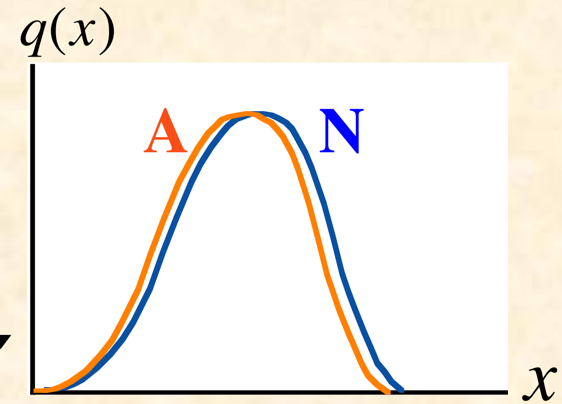


Average nucleon separation (2 fm) ≈ Nucleon diameter

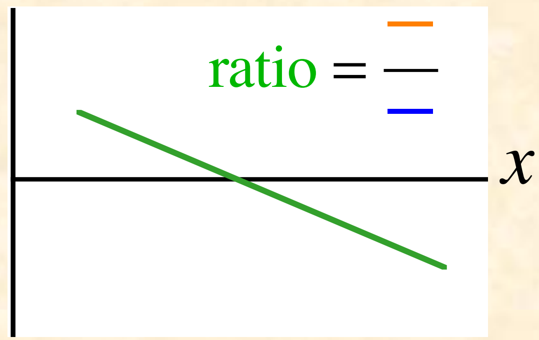
Free nucleon **Nucleon may overlap in a nucleus.**

Confinement radius changes: $\lambda_A > \lambda_N$

Quark momentum distribution changes.

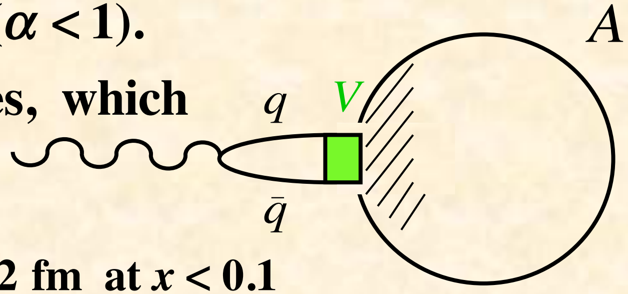


Ratio $q_A(x)/q_N(x)$ is similar to the observed EMC data in 1983.



Shadowing

- Shadowing means that internal constituents are shadowed due to the existence of nuclear surface ones, so that the cross section is smaller than the each nucleon contribution: $\sigma_A = A^{\alpha}\sigma_N \ (\alpha < 1)$.
- A virtual photon transforms into vector meson (or $q\bar{q}$) states, which then interact with a target nucleus.

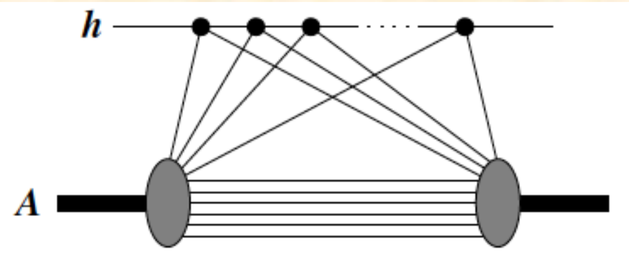


Propagation length of V ($q\bar{q}$): $\lambda = \frac{1}{|E_V - E_\gamma|} = \frac{2\nu}{M_V^2 + Q^2} = \frac{0.2 \text{ fm}}{x} > 2 \text{ fm}$ at $x < 0.1$

At small x, the virtual photon interacts with the target nucleus as if it were a vector meson (or $q\bar{q}$).

- Shadowing takes place due to multiple scattering.

For example, the vector meson interacts elastically with a surface nucleon and then interacts inelastically with a central nucleon.



Because this amplitude is opposite in phase to the one-step amplitude for an inelastic interaction with the central nucleon, the nucleon sees a reduced hadronic flux (namely the shadowing).

Nuclear
Parton Distribution Functions

Experimental data

(1) F_2^A / F_2^D

NMC: p, He, Li, C, Ca
SLAC: He, Be, C, Al, Ca, Fe, Ag, Au
EMC: C, Ca, Cu, Sn
E665: C, Ca, Xe, Pb
BCDMS: N, Fe
HERMES: N, Kr

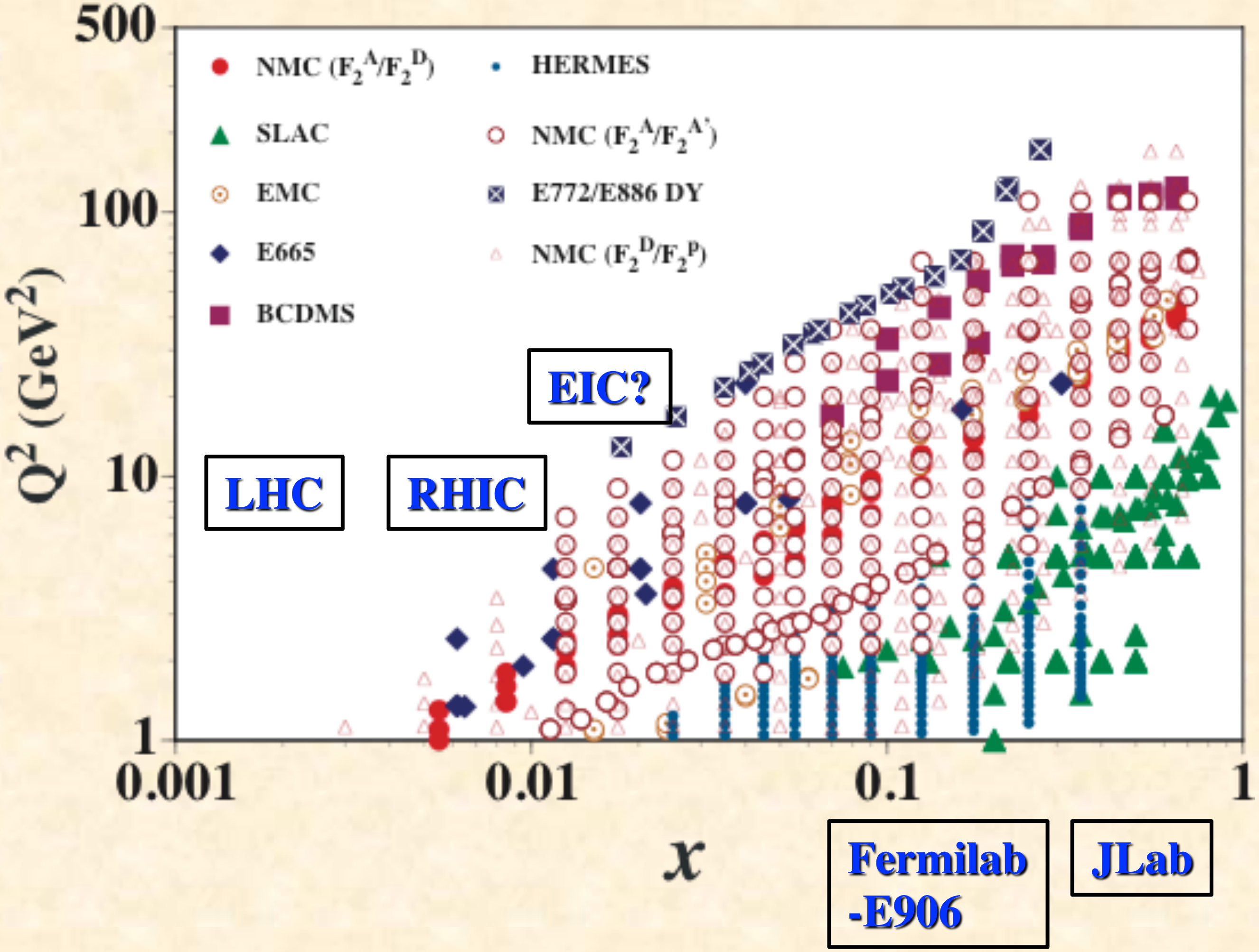
+ JLab data

(2) $F_2^A / F_2^{A'}$

NMC: Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$

E772: C / D, Ca / D, Fe / D, W / D
E866: Fe / Be, W / Be



Current nuclear data are kinematically limited.

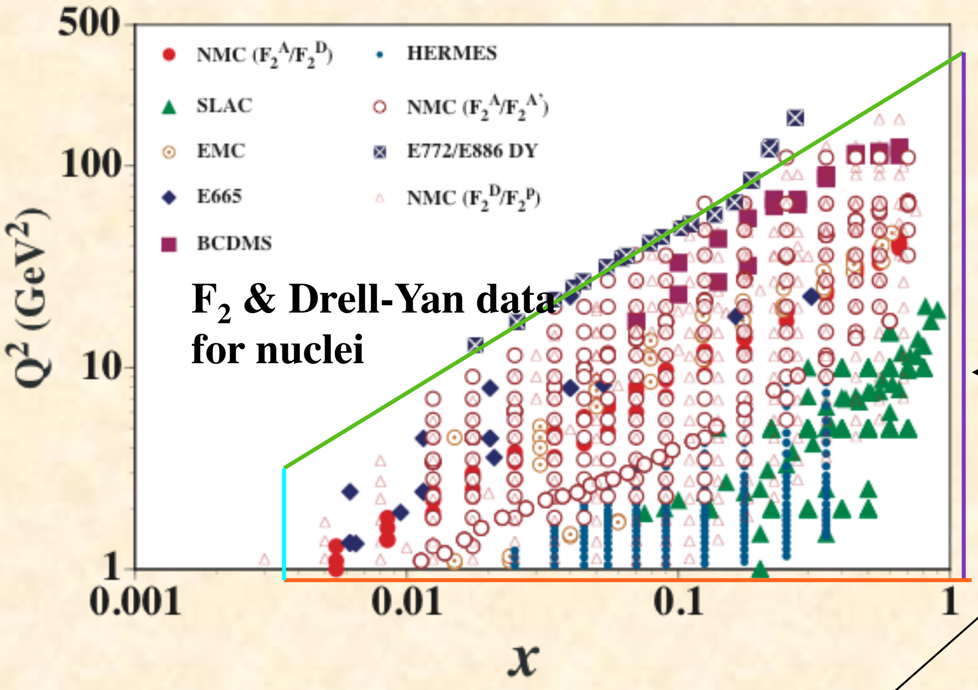
$$x = \frac{Q^2}{2p\cdot q} \simeq \frac{Q^2}{ys}$$

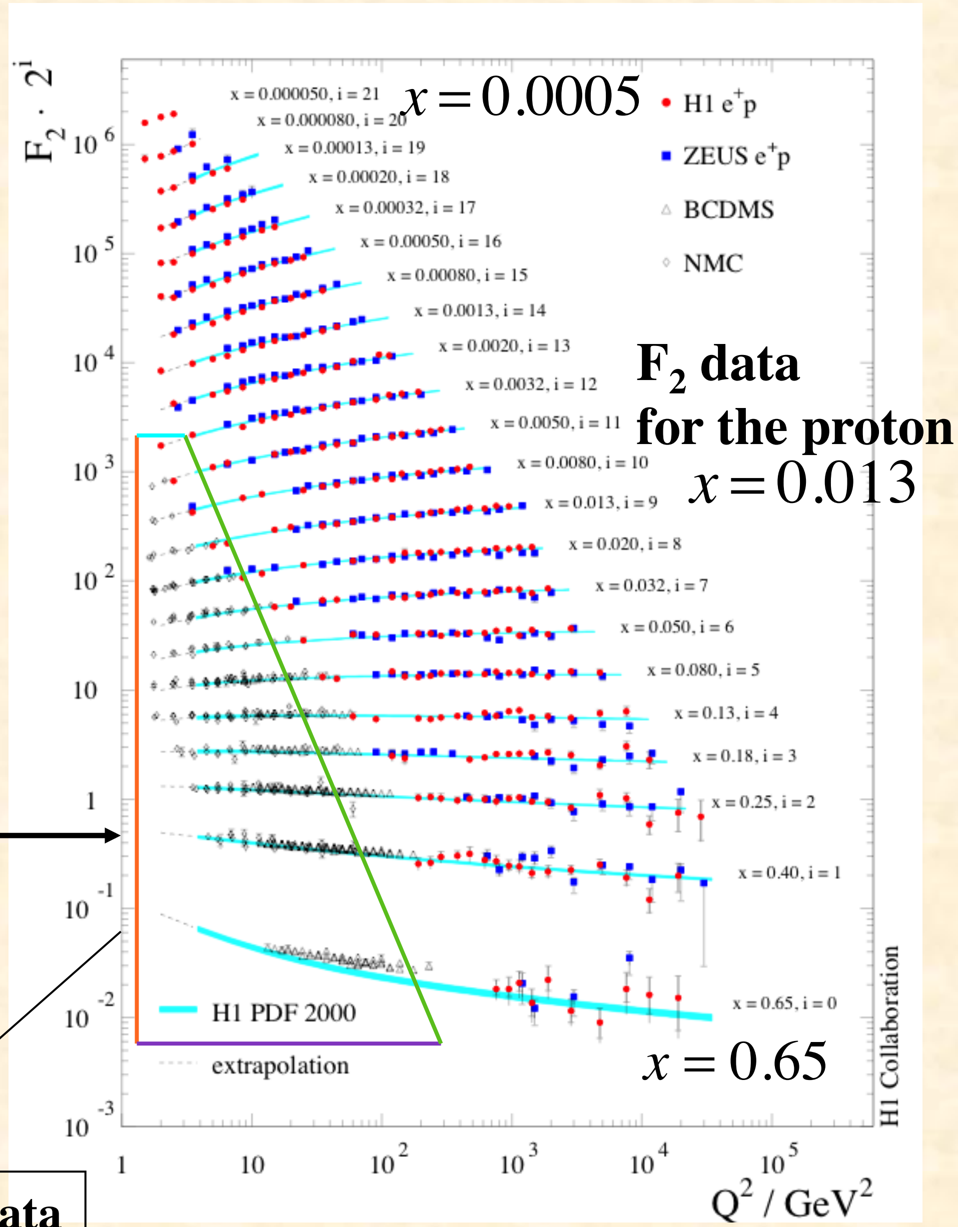
(from H1 and ZEUS, hep-ex/0502008)

$$\text{fixed target:}\ \ \min(x) = \frac{Q^2}{2M_N E_{lepton}} \le \frac{1}{2E_{lepton}(\text{GeV})}$$

$$\text{if}\ \ Q^2 \ge 1\ \text{GeV}^2$$

$$\text{for } E_{lepton}(\text{NMC}) = 200\ \text{GeV},\ \ \min(x) = \frac{1}{2\cdot 200} = 0.003$$





region of nuclear data

Functional form

Nuclear PDFs "per nucleon"

If there were no nuclear modification

$$Au^A(x) = Zu^p(x) + Nu^n(x), \quad Ad^A(x) = Zd^p(x) + Nd^n(x) \qquad p = \text{proton}, \quad n = \text{neutron}$$

Isospin symmetry : $u^n = d^p \equiv d, \qquad d^n = u^p \equiv u$

$$\rightarrow \quad u^A(x) = \frac{Zu(x) + Nd(x)}{A}, \qquad d^A(x) = \frac{Zd(x) + Nu(x)}{A}$$

Take account of nuclear effects by $w_i(x, A)$

$$u_v^A(x) = w_{u_v}(x, A)\frac{Zu_v(x) + Nd_v(x)}{A}, \quad d_v^A(x) = w_{d_v}(x, A)\frac{Zd_v(x) + Nu_v(x)}{A}$$

$$\bar{u}^A(x) = w_{\bar{q}}(x, A)\frac{Z\bar{u}(x) + N\bar{d}(x)}{A}, \qquad \bar{d}^A(x) = w_{\bar{q}}(x, A)\frac{Z\bar{d}(x) + N\bar{u}(x)}{A}$$

$$\bar{s}^A(x) = w_{\bar{q}}(x, A)\bar{s}(x)$$

$$g^A(x) = w_g(x, A)\,g(x)$$

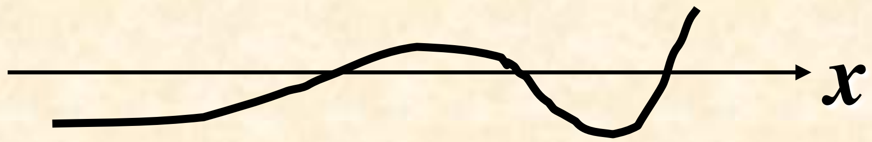
at Q^2=1 GeV^2 ($\equiv Q_0^2$)

Nuclear modifications and constraints

$$f_i^A(x,Q_0^2) = w_i(x,A) f_i(x,Q_0^2) \qquad i = u_v,\, d_v,\, \bar{u},\, \bar{d},\, \bar{s},\, g$$

$$w_i(x,A) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^\beta}$$

Note: The region $x > 1$ cannot be described by this parametrization.



A simple function = cubic polynomial

Three constraints

Nuclear charge: $Z = A\int dx\left[\frac{2}{3}\left(u^A - \bar{u}^A\right) - \frac{1}{3}\left(d^A - \bar{d}^A\right) - \frac{1}{3}\left(s^A - \bar{s}^A\right)\right] = A\int dx\left[\frac{2}{3}u_v^A - \frac{1}{3}d_v^A\right]$

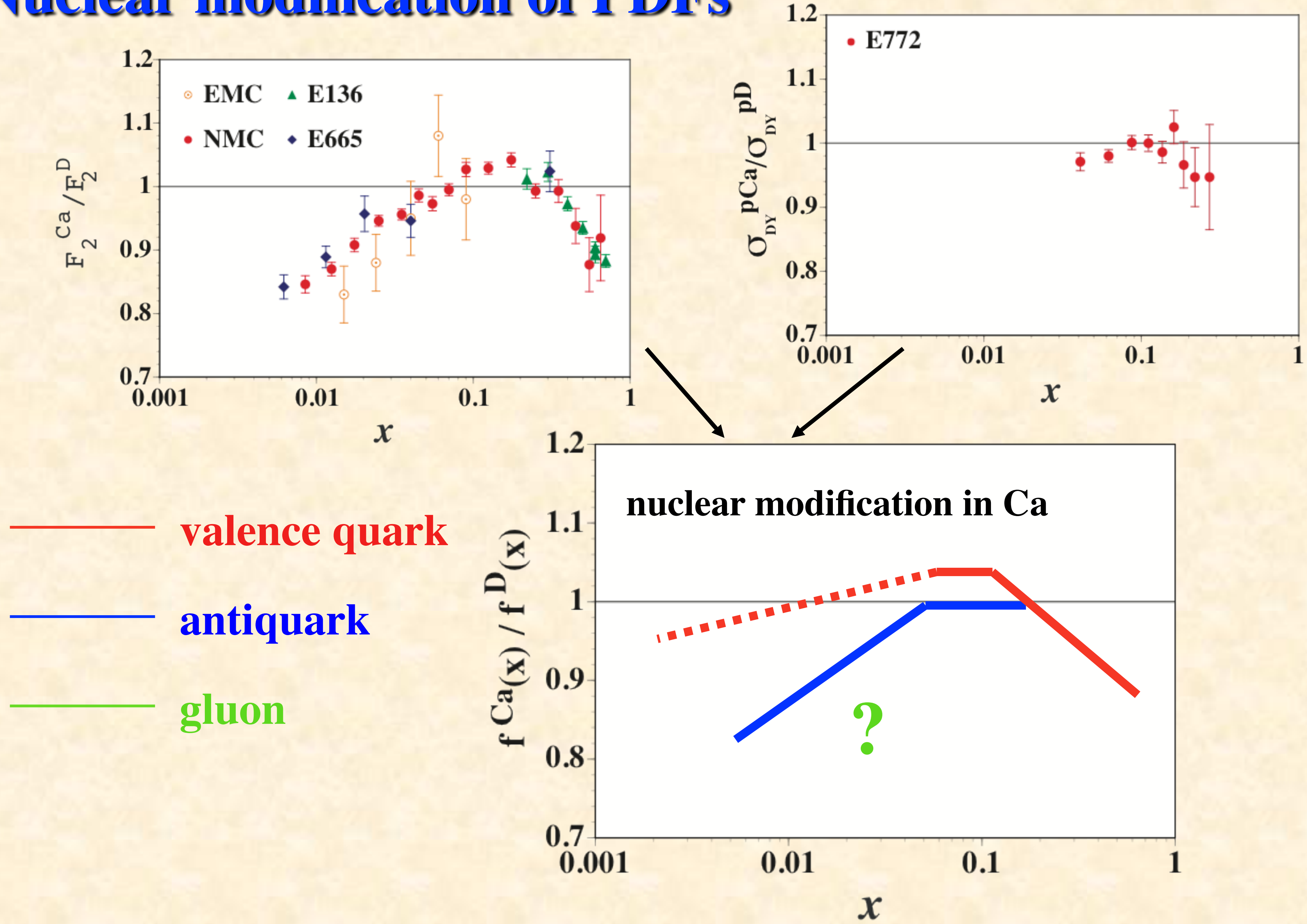
Baryon number: $A = A\int dx\left[\frac{1}{3}\left(u^A - \bar{u}^A\right) + \frac{1}{3}\left(d^A - \bar{d}^A\right) + \frac{1}{3}\left(s^A - \bar{s}^A\right)\right] = A\int dx\left[\frac{1}{3}u_v^A + \frac{1}{3}d_v^A\right]$

Momentum:

$$A = A\int dx\left[u^A + \bar{u}^A + d^A + \bar{d}^A + s^A + \bar{s}^A + g\right]$$

$$= A\int dx\left[u_v^A + d_v^A + 2\left(\bar{u}^A + \bar{d}^A + \bar{s}^A\right) + g\right]$$

Nuclear modification of PDFs



Recent global analyses on nuclear PDFs

I may miss some papers.

HKN07

- M. Hirai, S. Kumano, and T. -H. Nagai, Phys. Rev. C 76 (2007) 065207.
- Charged-lepton DIS, DY.

EPS09

- K. J. Eskola, H. Paukkunen, and C. A. Salgado, JHEP 04 (2009) 065.
- Charged-lepton DIS, DY, π^0 production in *dAu*.

CTEQ

- I. Schienbein, J. Y. Yu, C. Keppel, J. G. Morfin, F. I. Olness, J. F. Owens,
  Phys. Rev. D 77 (2008) 054013; D80 (2009) 094004;
  K. Kovarik *et al.*, PRL 106 (2011) 122301; PoS DIS2013 (2013) 274;
  PoS DIS2014 (2014) 047; arXiv:1509.00792.
- Neutrino DIS, Charged-lepton DIS, DY.

DSZS12

- D. de Florian, R. Sassot, P. Zurita, M. Stratmann, Phys. Rev. D85 (2012) 074028.
- Charged-lepton DIS, DY, RHIC-π

See also L. Frankfurt, V. Guzey, and M. Strikman, Phys. Rev. D 71 (2005) 054001;
Phys. Lett. B687 (2010) 167; Phys. Rept. 512 (2012) 255; arXiv:1310.5879.
S. A. Kulagin and R. Petti, Phys. Rev. D 76 (2007) 094023; C 82 (2010) 054614;
C 90 (2014) 045204.
A. Bodek and U.-K. Yang, arXiv:1011.6592.

Functional form of initial distributions at Q_0^2

Initial nuclear PDFs at

$$f_i^A(x) = \frac{1}{A}\left[Z f_i^{p/A}(x) + (A-Z) f_i^{n/A}(x)\right]$$

$f_i^{N/A}(x)$: **PDF of bound nucleon in the nucleus**

Isospin symmetry is assumed: $u \equiv d^n = u^p,\ d \equiv u^n = d^p$

Functional forms

- **HKN07** ($Q_0^2 = 1$ GeV^2)

$$f_i^A(x) = w_i(x,A,Z)\frac{1}{A}\left[Z f_a^p(x) + (A-Z) f_a^n(x)\right], \qquad w_i(x,A,Z) = 1 + \left(1 - \frac{1}{A^{1/3}}\right)\frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{0.1}}$$

- **EPS09** ($Q_0^2 = 1.69$ GeV^2)

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\mathrm{CTEQ6.1M}}(x,Q_0^2),\ R_i^A(x) = \begin{cases} a_0 + (a_1 + a_2 x)[\exp(-x) - \exp(-x_a)] & (x \le x_a : \text{shadowing}) \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & (x_a \le x \le x_e : \text{antishadowing}) \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & (x_e \le x \le 1 : \text{EMC\&Fermi}) \end{cases}$$

- **CTEQ-08** ($Q_0^2 = 1.69$ GeV^2)

$$x f_i^{N/A}(x) = \begin{cases} A_0 x^{A_1}(1-x)^{A_2} e^{A_3 x}(1 + e^{A_4} x)^{A_5} & : i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s} \\ A_0 x^{A_1}(1-x)^{A_2} + (1 + A_3 x)(1-x)^{A_4} & : i = \bar{d}/\bar{u} \end{cases}$$

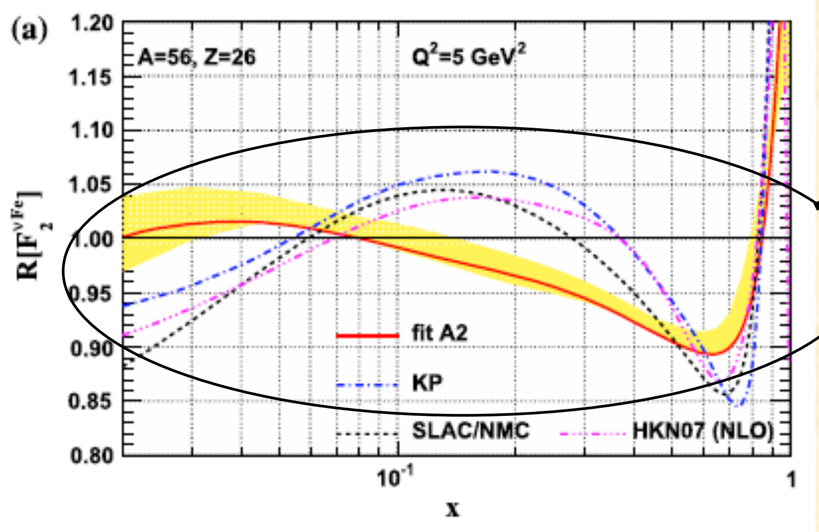
- **DSZS12** ($Q_0^2 = 1.0$ GeV^2)

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{MSTW\,2009}(x,Q_0^2),\ R_v^A(x) = \varepsilon_1 x^{\alpha_v}(1-x)^{\beta_1}[1 + \varepsilon_2(1-x)^{\beta_2}][1 + a_v(1-x)^{\beta_3}]$$

$$R_s^A(x) = R_v^A(x)\frac{\varepsilon_s}{\varepsilon_1}\frac{1 + a_s x^{\alpha_s}}{1 + a_s}, \quad R_g^A(x) = R_g^A(x)\frac{\varepsilon_g}{\varepsilon_1}\frac{1 + a_g x^{\alpha_g}}{1 + a_g}$$

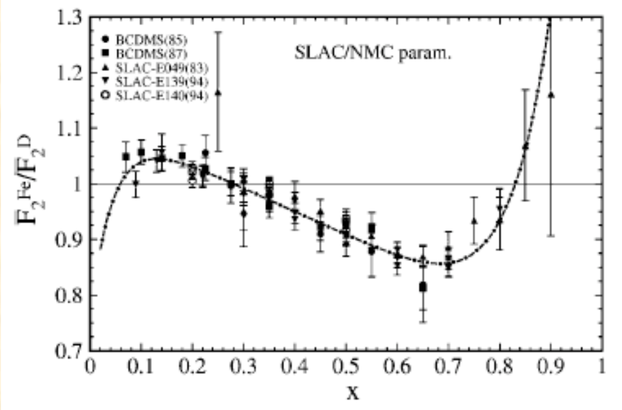
Analysis of CTEQ-2008 (Schienbein *et al.*)

I. Schienbein *et al.*,
PRD 77 (2008) 054013

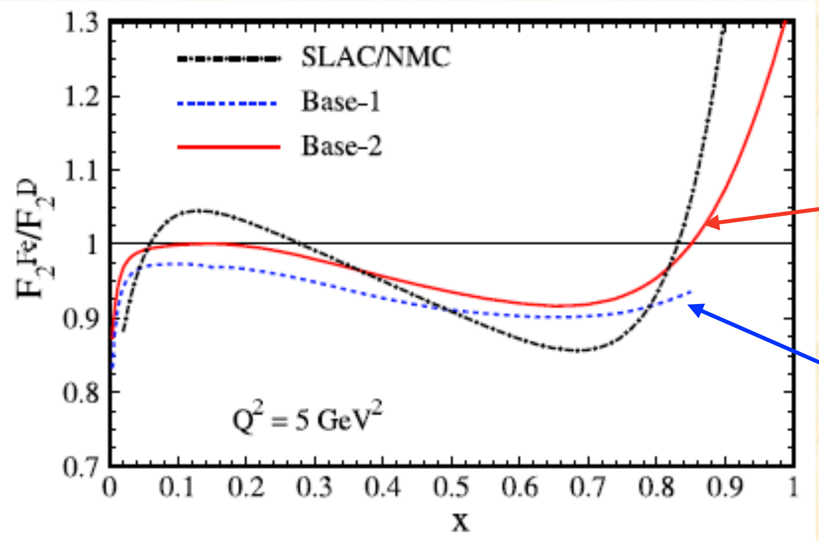


Differences
from typical NPDFs.

Charged-lepton scattering



Neutrino scattering



Base-1
- remove CCFR data
- incorporate deuteron corrections

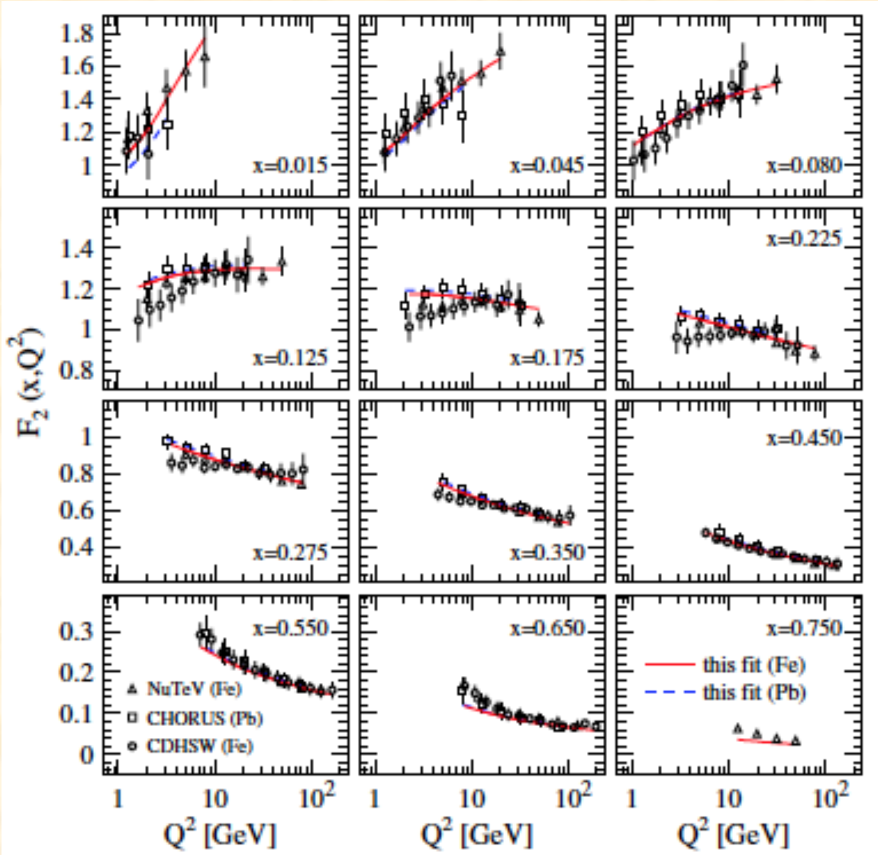
Base-2 corresponds to CTEQ6.1M with s≠sbar
- include CCFR data
  Charged-lepton correction factors are applied.
- s≠sbar

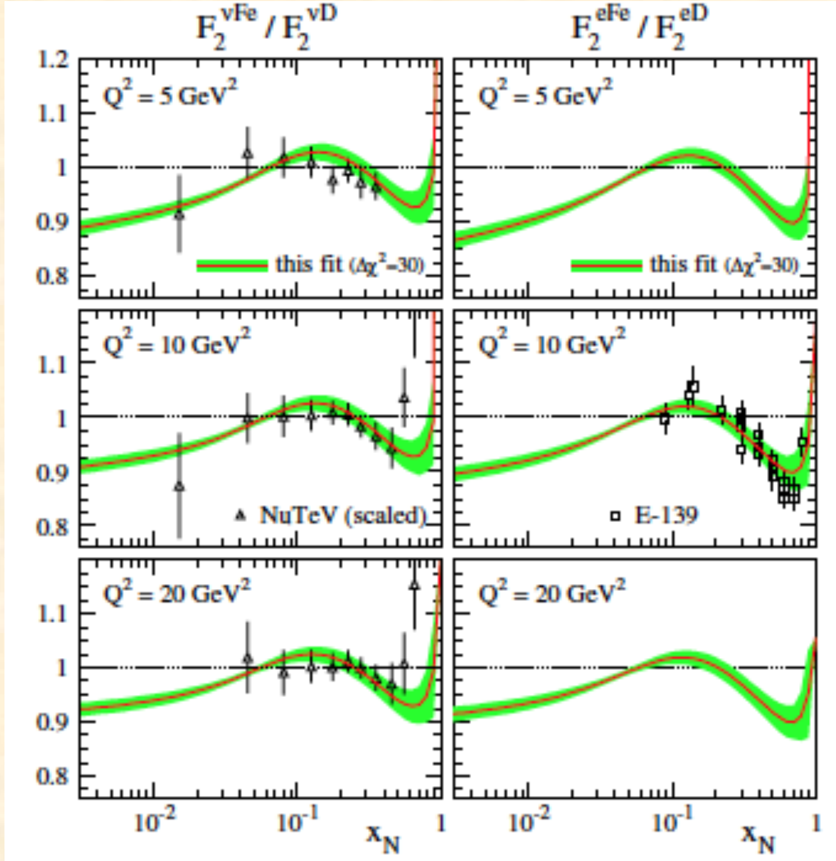
Base-2: Using current nucleonic PDFs, they (and MRST) obtained very different corrections from charged-lepton data.

Base-1: However, it depends on the analysis method for determining "nucleonic" PDFs.

Neutrino DIS ⇔ Charged DIS issue

D. de Florian, R. Sassot, P. Zurita, and M. Stratmann,
Phys. Rev. D 85 (2012) 074028.

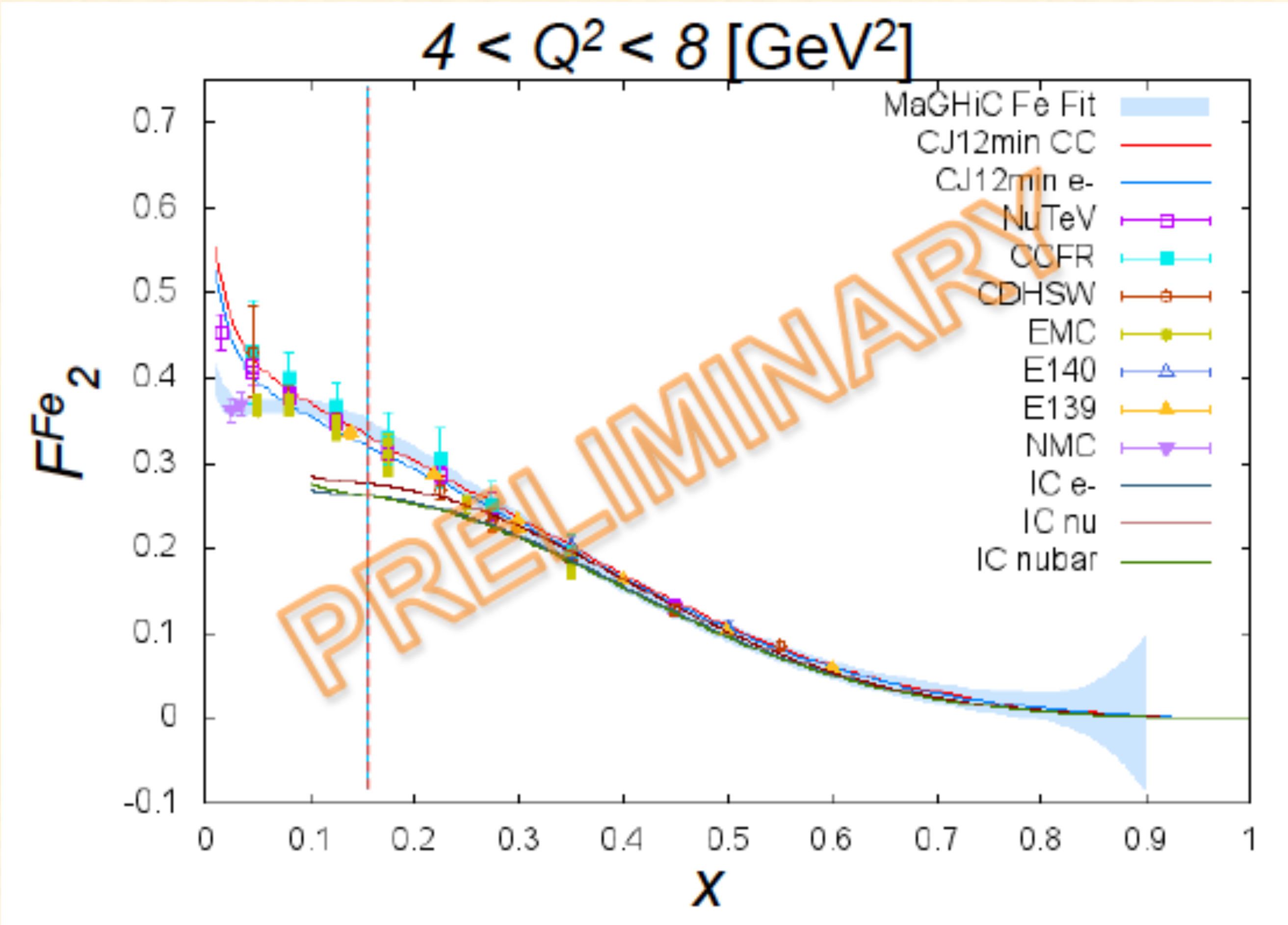




According to their analysis, the issue does not exist!?

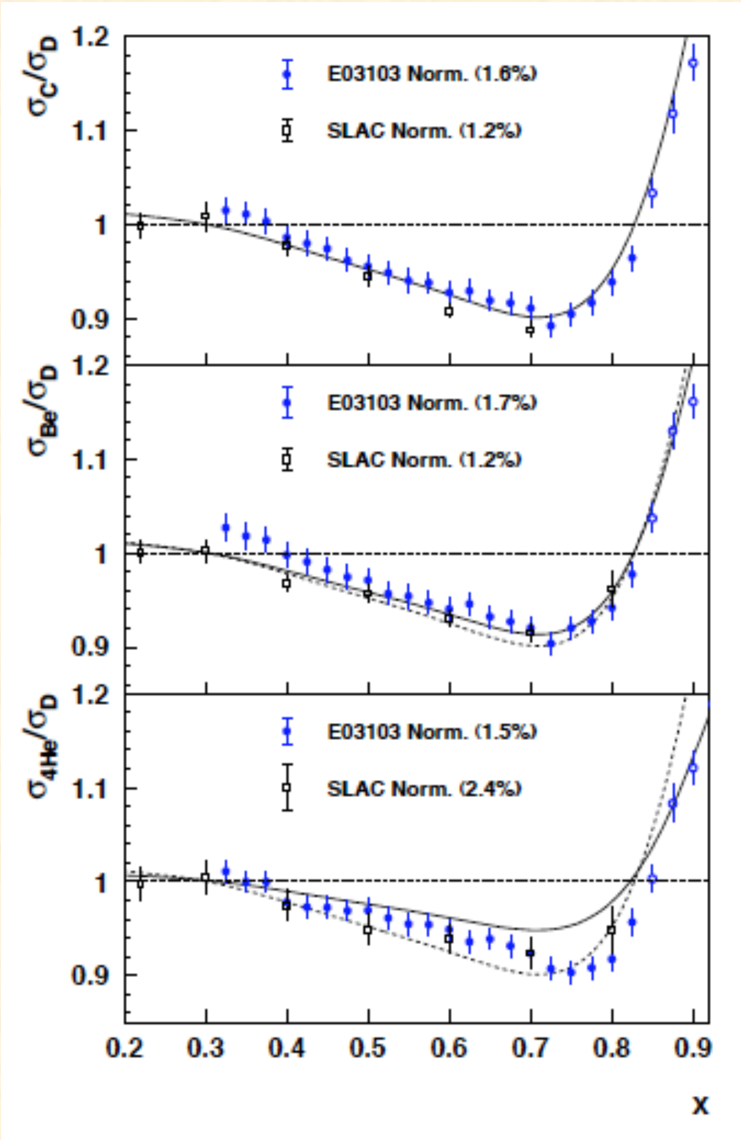
N. Kalantarians: Neutrino DIS ⇔ Charged DIS

N. Kalantarians on Nov.19 at this workshop



According to this analysis, both structure functions are same except for the small-x region (x<0.05) !?

Measurements at JLab

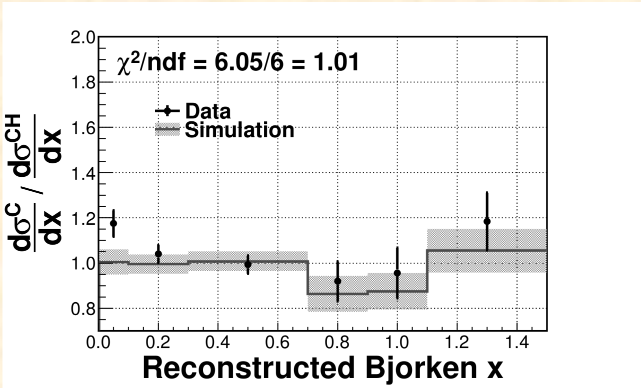


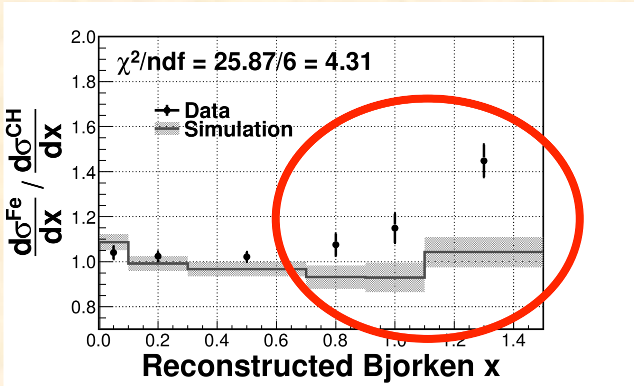
J. Seely *et al.*,
Phys. Rev. Lett. 103 (2009) 202301.

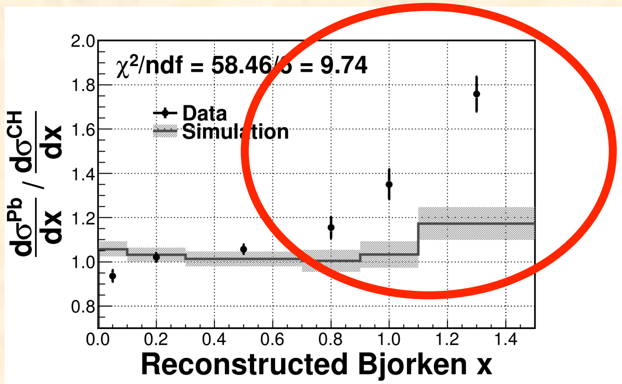
Results indicate that nuclear modifications may not be described by usual *A* (and density) dependence for light nuclei.

Measurements by Minerva

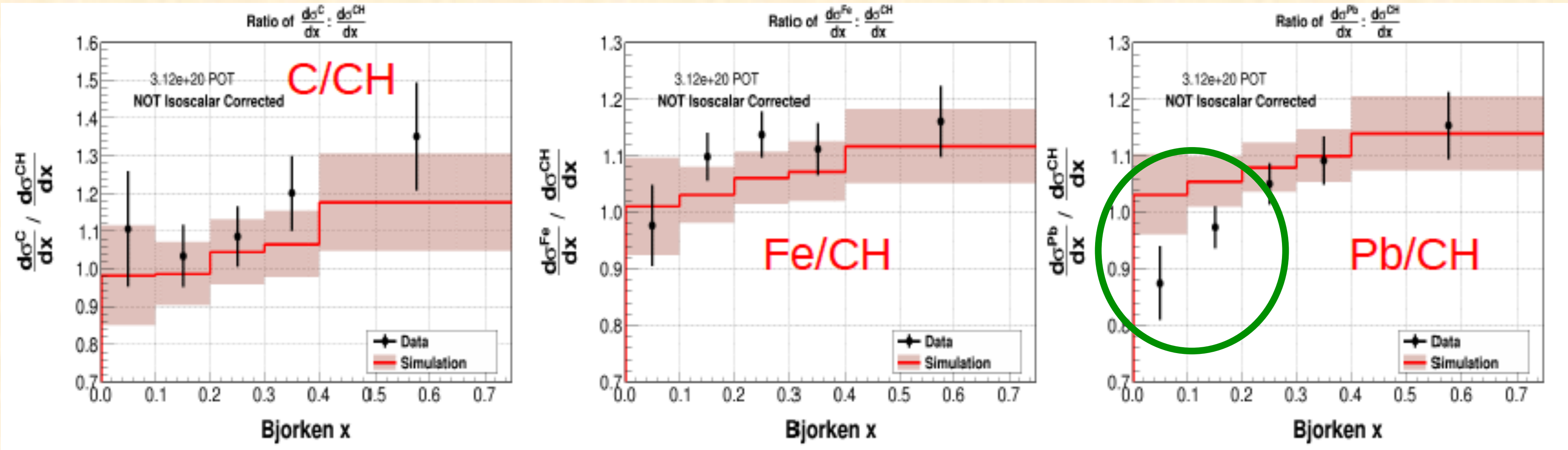
B. G. Tice et al., PRL 112 (2014) 231801







Observed nuclear modifications are different from simulation (GENIE 2.6.2) results.



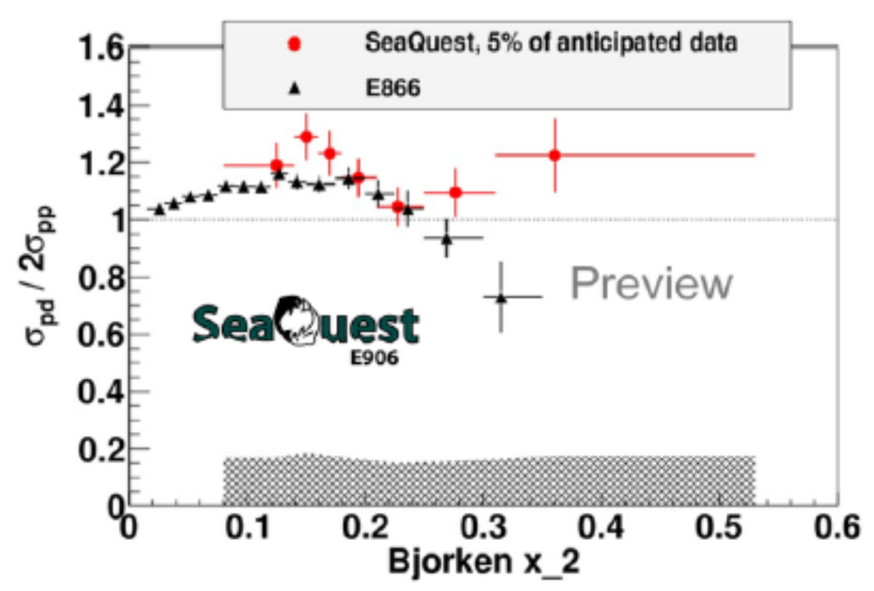
Mousseau@this workshop, Nov.19

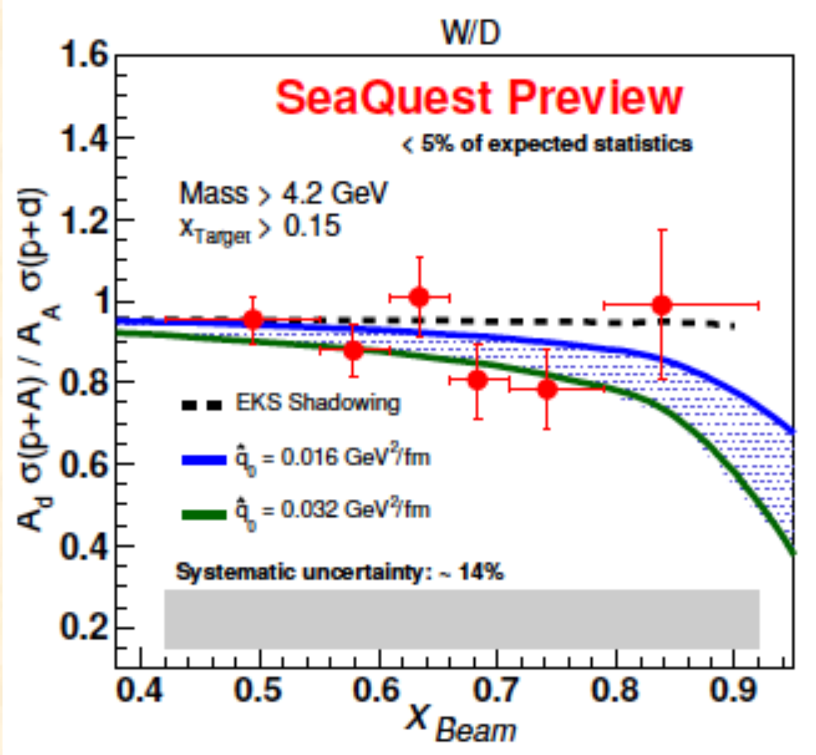
Different shadowing from charged-lepton case?!

Fermilab-E906 Drell-Yan

Fermilab-E906 in progress!

K. Nakano
@Pacific-Spin2015





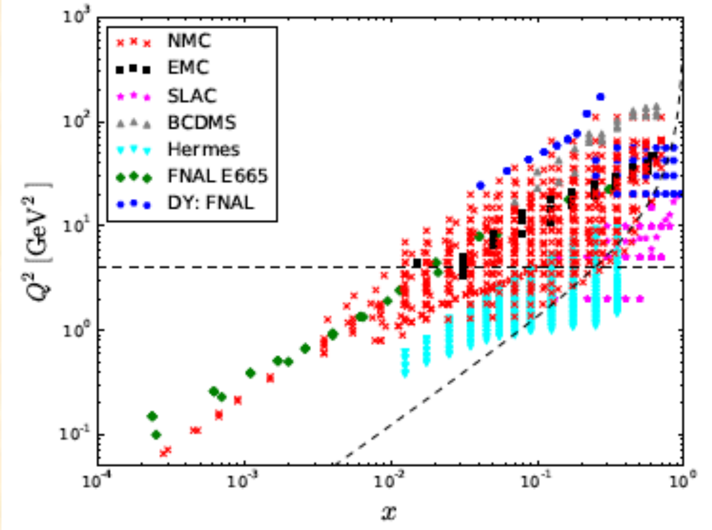
Recent analysis by nCTEQ (2015): data set

K. Kovarik *et al.*, arrXiv:1509.00792

Charged-lepton DIS

F_2^A/F_2^D : Observable	Experiment	ID	Ref.	# data	# data after cuts	χ^2
D	NMC-97	5160	[47]	292	201	247.73
He/D	Hermes	5156	[48]	182	17	18.02
	NMC-95,re	5124	[49]	18	12	10.64
	SLAC-E139	5141	[50]	18	3	1.04
Li/D	NMC-95	5115	[51]	24	11	3.94
Be/D	SLAC-E139	5138	[50]	17	3	0.44
C/D	FNAL-E665-95	5125	[52]	11	3	3.53
	SLAC-E139	5139	[50]	7	2	1.15
	EMC-88	5107	[53]	9	9	7.06
	EMC-90	5110	[54]	9	0	0.00
	NMC-95	5113	[51]	24	12	7.39
	NMC-95,re	5114	[49]	18	12	13.36
N/D	Hermes	5157	[48]	175	19	10.46
	BCDMS-85	5103	[55]	9	9	4.66
Al/D	SLAC-E049	5134	[56]	18	0	0.00
	SLAC-E139	5136	[50]	17	3	0.66
Ca/D	NMC-95,re	5121	[49]	18	12	12.24
	FNAL-E665-95	5126	[52]	11	3	4.87
	SLAC-E139	5140	[50]	7	2	1.43
	EMC-90	5109	[54]	9	0	0.00
Fe/D	SLAC-E049	5131	[57]	14	2	0.67
	SLAC-E139	5132	[50]	23	6	8.20
	SLAC-E140	5133	[58]	10	0	0.00
	BCDMS-87	5101	[59]	10	10	6.47
	BCDMS-85	5102	[55]	6	6	2.83
Cu/D	EMC-93	5104	[60]	10	9	4.31
	EMC-93(chariot)	5105	[60]	9	9	5.72
	EMC-88	5106	[53]	9	9	3.97
Kr/D	Hermes	5158	[48]	167	12	9.68
Ag/D	SLAC-E139	5135	[50]	7	2	1.36
Sn/D	EMC-88	5108	[53]	8	8	17.88
Xe/D	FNAL-E665-92	5127	[61]	10	2	0.74
Au/D	SLAC-E139	5137	[50]	18	3	1.55
Pb/D	FNAL-E665-95	5129	[52]	11	3	5.91
Total:				1205	414	417.92

$F_2^A/F_2^{A'}$: Observable	Experiment	ID	Ref.	# data	# data after cuts	χ^2
C/Li	NMC-95,re	5123	[49]	25	7	5.22
Ca/Li	NMC-95,re	5122	[49]	25	7	1.49
Be/C	NMC-96	5112	[62]	15	14	7.25
Al/C	NMC-96	5111	[62]	15	14	4.98
Ca/C	NMC-95,re	5120	[49]	25	7	3.31
	NMC-96	5119	[62]	15	14	5.18
Fe/C	NMC-96	5143	[62]	15	14	10.38
Sn/C	NMC-96	5159	[63]	146	111	62.95
Pb/C	NMC-96	5116	[62]	15	14	9.09
Total:				296	202	109.85

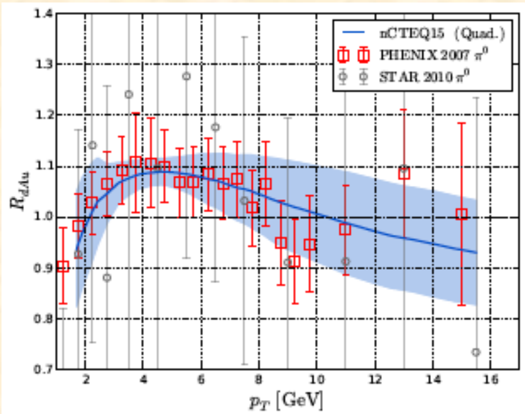


Drell-Yan

$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$: Observable	Experiment	ID	Ref.	# data	# data after cuts	χ^2
C/H2	FNAL-E772-90	5203	[64]	9	9	11.10
Ca/H2	FNAL-E772-90	5204	[64]	9	9	3.11
Fe/H2	FNAL-E772-90	5205	[64]	9	9	3.33
W/H2	FNAL-E772-90	5206	[64]	9	9	7.30
Fe/Be	FNAL-E886-99	5201	[65]	28	28	26.09
W/Be	FNAL-E886-99	5202	[65]	28	28	25.61
Total:				92	92	76.54

Pion-production in dA

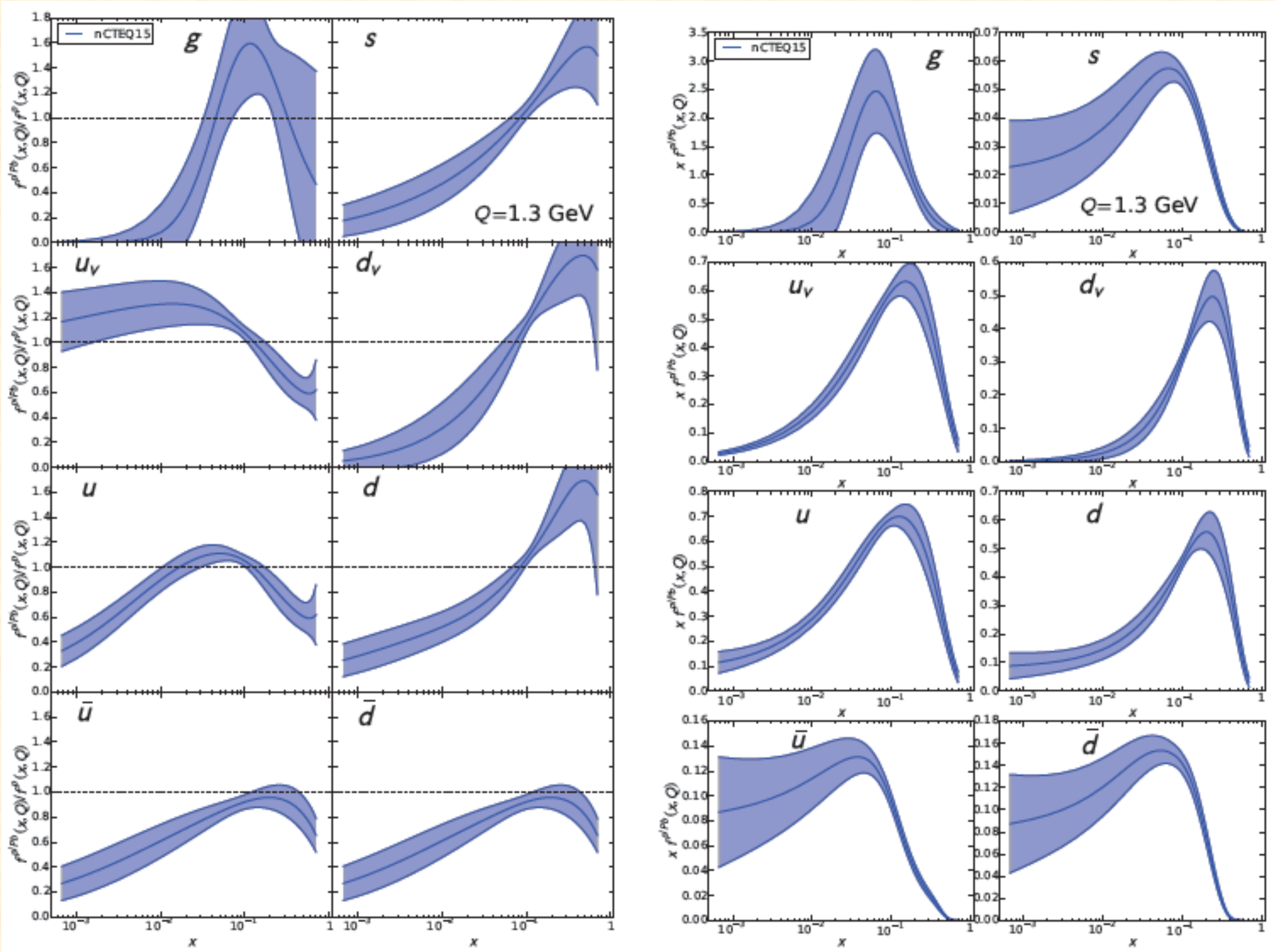
$R_{dAu}^{\pi}/R_{pp}^{\pi}$: Observable	Experiment	ID	Ref.	# data	# data after cuts	χ^2
dAu/pp	PHENIX	PHENIX	[66]	21	20	5.07
	STAR-2010	STAR	[67]	13	12	1.30
Total:				34	32	6.37



- DIS: $Q > 2$ GeV and $W > 3.5$ GeV
- DY: $2 < M < 300$ GeV
- π^0 production: $p_T > 1.7$ GeV

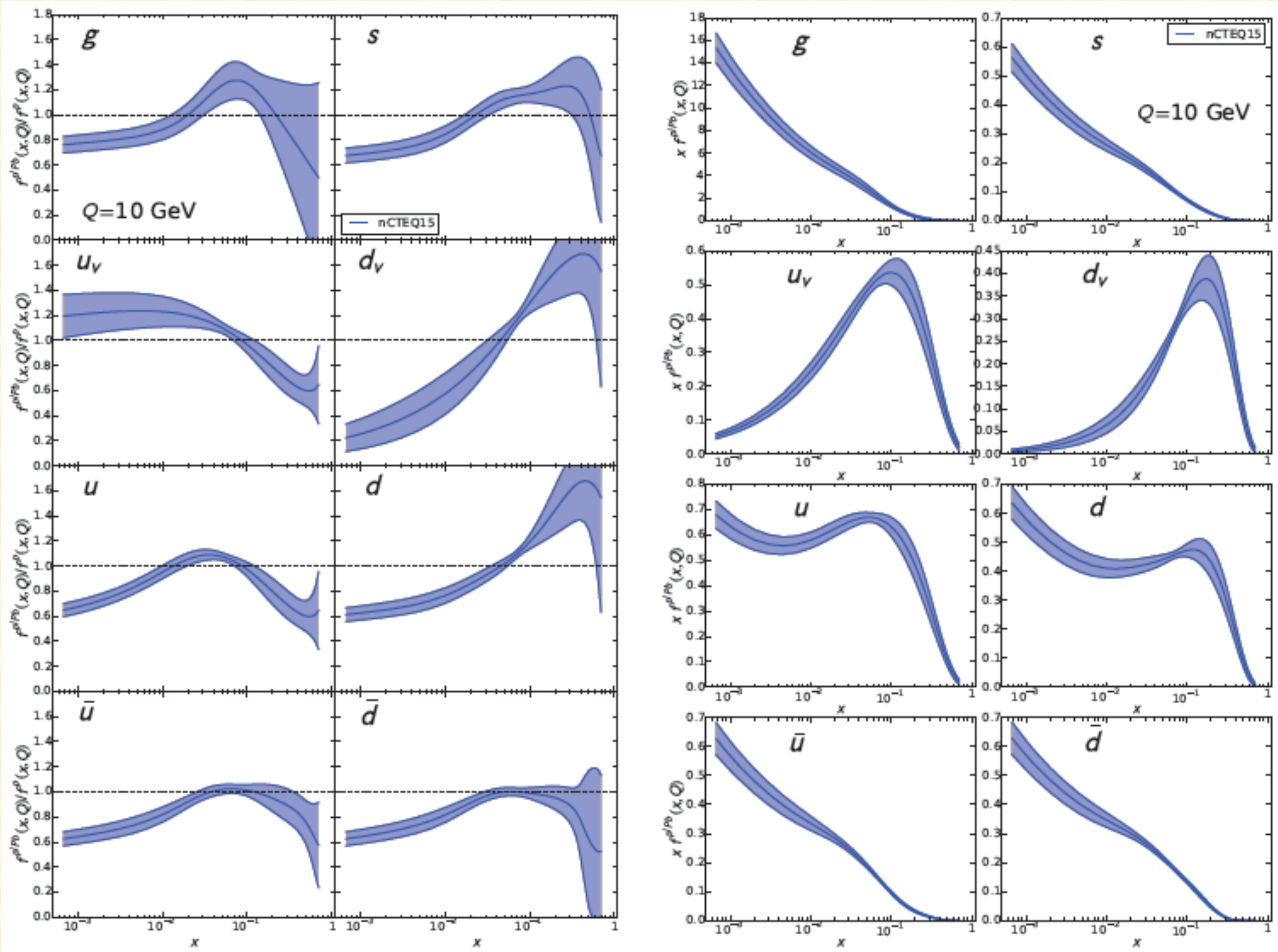
nCTEQ (2015)

$Q^2 = (1.3)^2\ \text{GeV}^2$



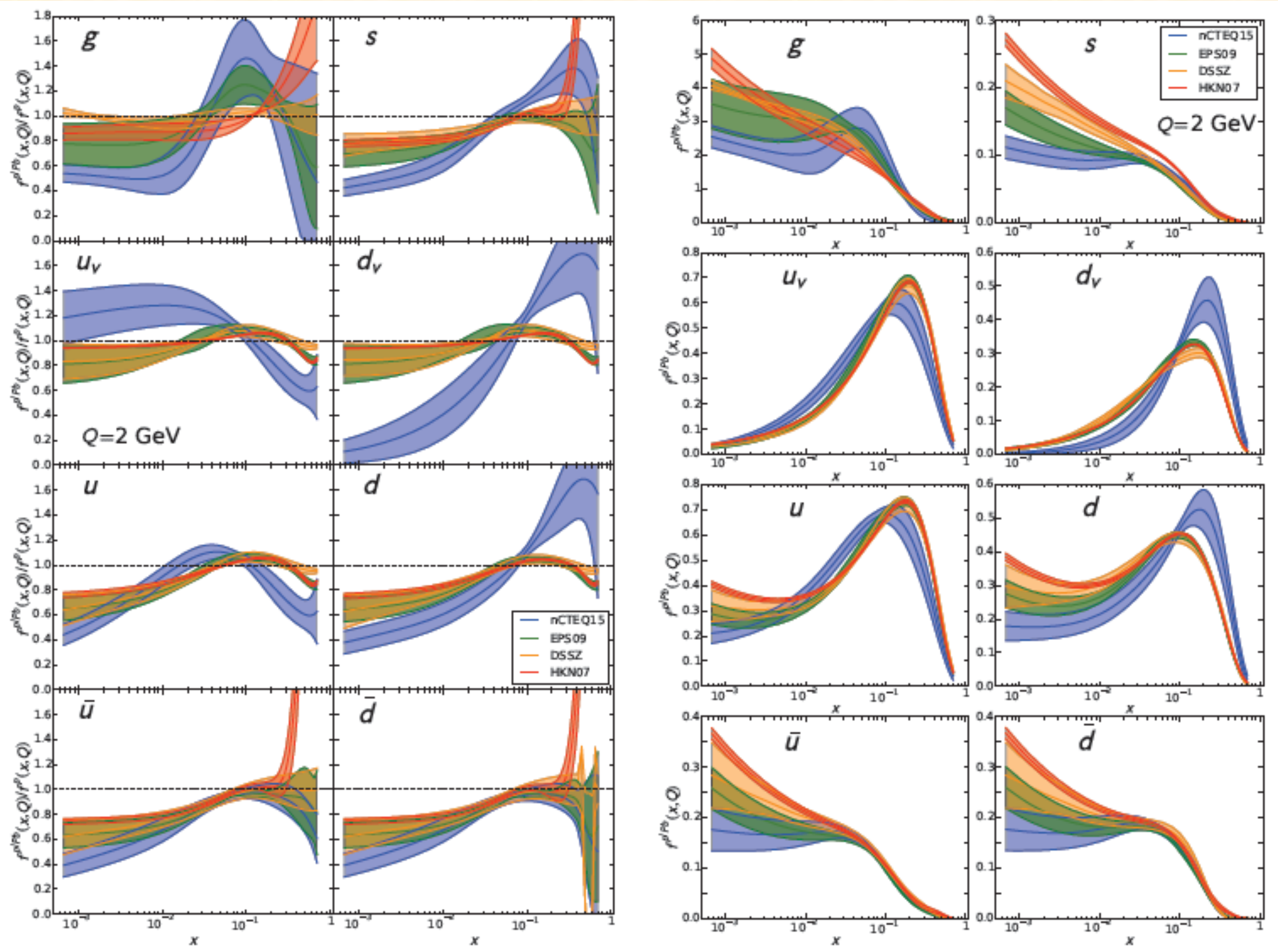
nCTEQ (2015)

$Q^2 = (10)^2\ \mathrm{GeV}^2$



nCTEQ (2015): Comparison with others

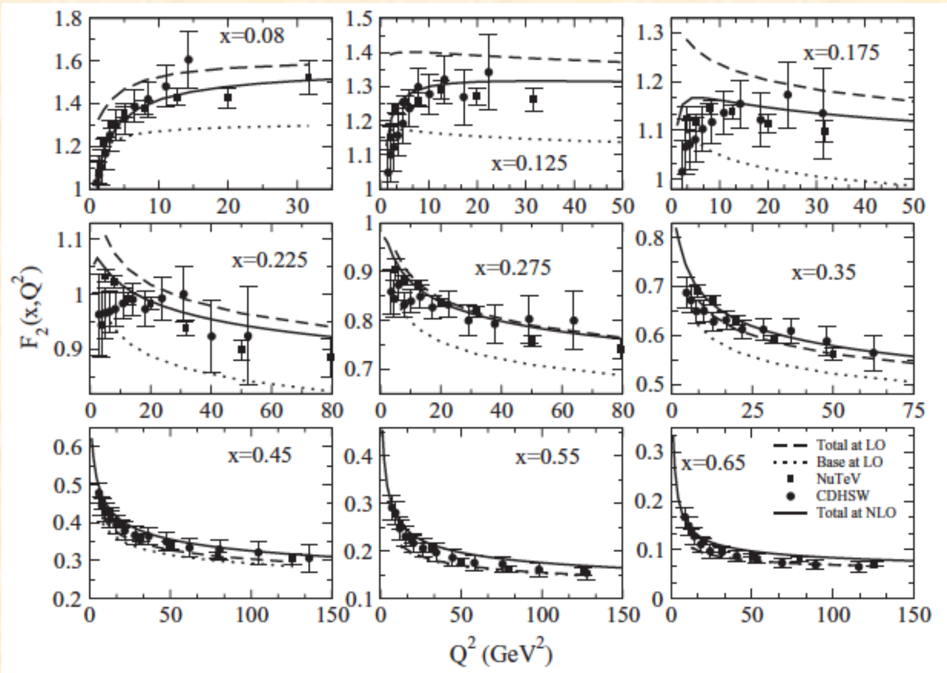
$Q^2 = (2)^2 \text{ GeV}^2$

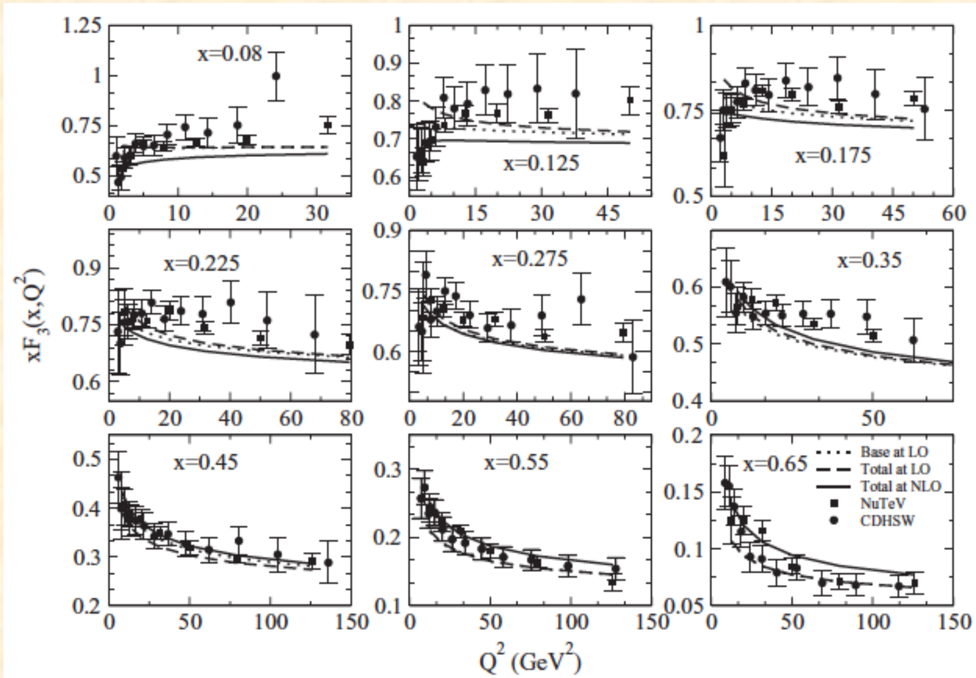


Comparison of theoretical models with neutrino data

For example, see H. Haider *et al*., PRC 84 (2011) 054610; 85 (2012) 055201.

Theoretical model: Spectral function (binding, Fermi motion, correlation) + π, ϱ





Recent works at this workshop

Nov.19, 2015

Update of HKN nuclear PDFs, M. Hirai

MINERvA DIS results, J. Mousseau

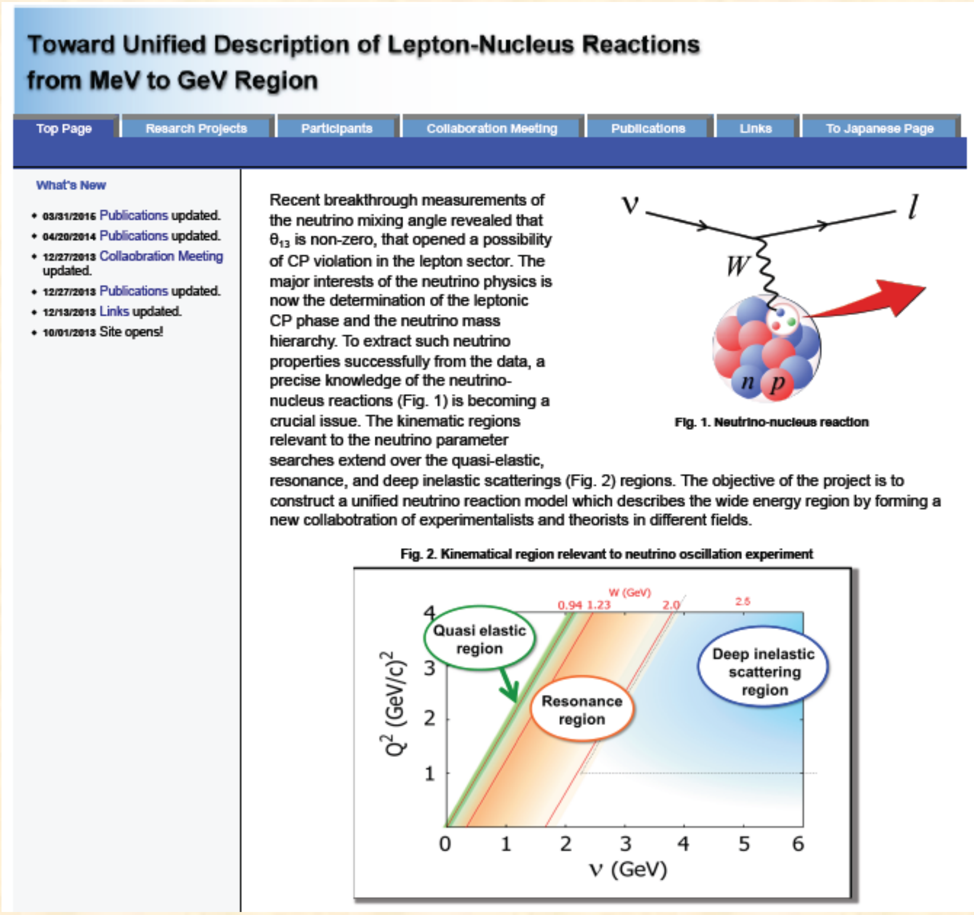
Nuclear PDFs by nCTEQ, J. G. Morfin

Generators for SIS and DIS, C. Bronner

Fragmentation studies in NOMAD, A. Chukanov

Comparison of the F2 Structure Function in Iron as Measured by Charged Lepton and Neutrino Probes, N. Kalantarians

ν-interaction collaboration at J-PARC


Toward Unified Description of Lepton-Nucleus Reactions from MeV to GeV Region

Top Page | Resarch Projects | Participants | Collaboration Meeting | Publications | Links | To Japanese Page

What's New

- 03/31/2016 Publications updated.
- 04/20/2014 Publications updated.
- 12/27/2013 Collaobration Meeting updated.
- 12/27/2013 Publications updated.
- 12/13/2013 Links updated.
- 10/01/2013 Site opens!

Recent breakthrough measurements of the neutrino mixing angle revealed that θ_{13} is non-zero, that opened a possibility of CP violation in the lepton sector. The major interests of the neutrino physics is now the determination of the leptonic CP phase and the neutrino mass hierarchy. To extract such neutrino properties successfully from the data, a precise knowledge of the neutrino-nucleus reactions (Fig. 1) is becoming a crucial issue. The kinematic regions relevant to the neutrino parameter searches extend over the quasi-elastic, resonance, and deep inelastic scatterings (Fig. 2) regions. The objective of the project is to construct a unified neutrino reaction model which describes the wide energy region by forming a new collabotration of experimentalists and theorists in different fields.

Fig. 1. Neutrino-nucleus reaction

Fig. 2. Kinematical region relevant to neutrino oscillation experiment

Y. Hayato, M. Hirai, W. Horiuchi, H. Kamano, S. Kumano, T. Murata, S. Nakamura, K. Saito, M. Sakuda, T. Sato

http://nuint.kek.jp/index_e.html, ν interactions

http://j-parc-th.kek.jp/ J-PARC branch, KEK Theory

Summary

Global analyses for the nuclear PDFs
by using data of charged-lepton, neutrino DIS, pA, AA collisions

Valence quark: reasonably good, in progress at JLab, Minerva for large x

Antiquark: good only at $x = 0.1$, in progress at Fermilab (E906) $x = 0.1 \sim 0.4$.

Gluon: large uncertainties in the whole-x region, LHC, RHIC

Issues

- Charged-lepton DIS ⇔ Neutrino DIS
- Matching with resonance model, $Q^2 \rightarrow 0$
- Gluon distributions

New experimental information

- JLab, Fermilab-DY, Minerva, LHC, …

The End

The End