

Weak Pion Production off the Nucleon

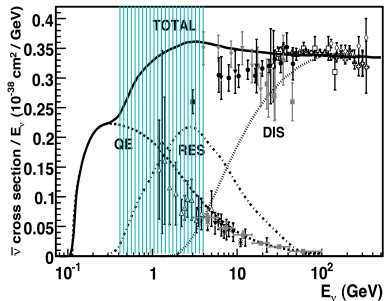
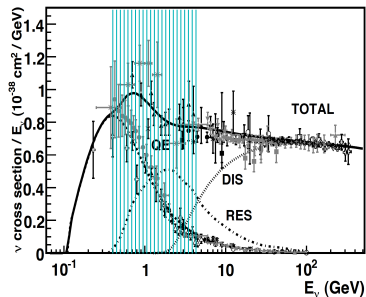
Mohammad Rafi Alam



S. K. Singh, M. Sajjad Athar, S. Chauhan

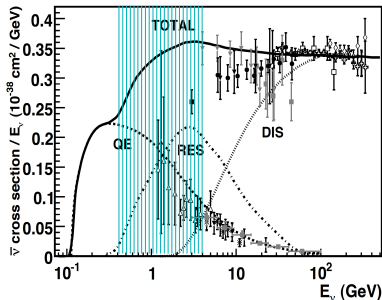
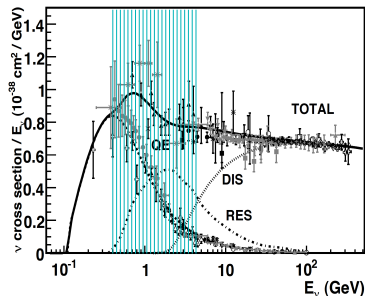
Motivations

Neutrino energy of $\sim 1\text{GeV}$ is quite important for oscillation studies



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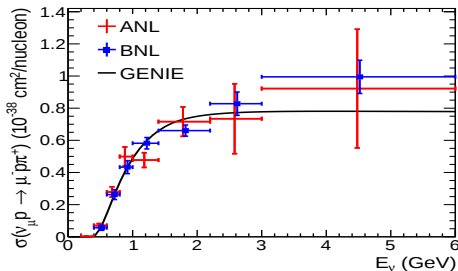
$$\sigma^{Total} = \sigma^{QE} + \sigma^{Inelastic} + \sigma^{DIS}$$

In this energy region the major contribution to the cross section comes from CCQE, CC1 π , NC1 π production processes.

One pion production from nucleons and nuclei has been a topic of great interest because of the measurements by MiniBooNE, K2K, T2K etc. and many experiments like NO ν A, MINER ν A measuring pion production from $\nu/\bar{\nu}$ induced interaction from nuclear targets.

One pion production from nucleons and nuclei has been a topic of great interest because of the measurements by MiniBooNE, K2K, T2K etc. and many experiments like NO ν A, MINER ν A measuring pion production from $\nu/\bar{\nu}$ induced interaction from nuclear targets.

Recently, Wilkinson et al. has reanalyzed ANL and BNL data and it seems the difference between two results have reduced a lot.



Keeping the reanalysis in mind, in this work I am going to present one pion production off the nucleon.

We consider:

- Δ Resonance($J = \frac{3}{2}^+, I = \frac{3}{2}$) contribution
- Background terms due to nucleon and pion pole, contact term using non-linear σ model.
- Higher resonances from second resonance region
 - ▶ $J = \frac{1}{2}^+$ and $I = \frac{1}{2}$: $P_{11}(1440)$
 - ▶ $J = \frac{1}{2}^-$ and $I = \frac{1}{2}$: $S_{11}(1535), S_{11}(1650)$
 - ▶ $J = \frac{3}{2}^+$ and $I = \frac{1}{2}$: $P_{13}(1720)$
 - ▶ $J = \frac{3}{2}^-$ and $I = \frac{1}{2}$: $D_{13}(1520)$
- For the higher resonances we have used Helicity amplitudes obtained from the MAID analysis to determine the vector form factors. For the axial form factors, first to fix the axial coupling strength we have used the decay widths and branching ratio from PDG for $\mathcal{R} \rightarrow N\pi$ pion decay mode.

Charged current(CC)

$$\nu_l p \rightarrow l^- p \pi^+$$

$$\nu_l n \rightarrow l^- n \pi^+$$

$$\nu_l n \rightarrow l^- p \pi^0$$

$$\bar{\nu}_l n \rightarrow l^+ n \pi^-$$

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$$\bar{\nu}_l p \rightarrow l^+ n \pi^0 \quad ; \quad l = e, \mu$$

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Neutral current(NC)

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

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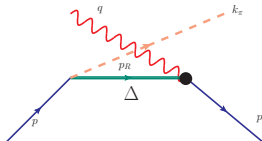
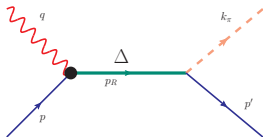
$$\nu_l n \rightarrow l^- p \pi^0$$

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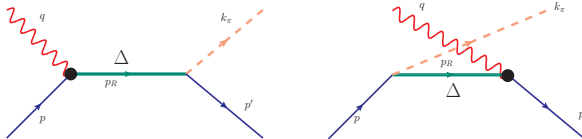
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Δ Resonance



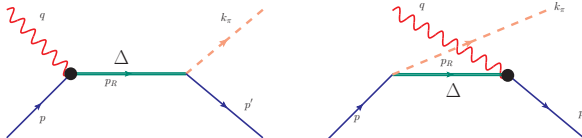
Δ Resonance



$$j^\mu \Big|_R^{\frac{3}{2}} = i V_{ud} C^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + i M_R \Gamma_R} \bar{u}(\vec{p}') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu} u(\vec{p}), \quad p_R = p + q,$$

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Δ Resonance



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$$P_{\alpha\beta}^{3/2}(P) = -(\not{P} + M_R) \left(g_{\alpha\beta} - \frac{2}{3} \frac{P_\alpha P_\beta}{M_R^2} + \frac{1}{3} \frac{P_\alpha \gamma_\beta - P_\beta \gamma_\alpha}{M_R} - \frac{1}{3} \gamma_\alpha \gamma_\beta \right).$$

$$\Gamma_{\nu\mu}^{\frac{3}{2}+} = \left[V_{\nu\mu}^{\frac{3}{2}} - A_{\nu\mu}^{\frac{3}{2}} \right] \gamma_5$$

Δ Resonance

$$\begin{aligned}V_{\nu\mu}^{\frac{3}{2}} &= \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right] \\A_{\nu\mu}^{\frac{3}{2}} &= - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5\end{aligned}$$

Vector Form Factors $\Rightarrow$ $\text{CVC} \longrightarrow C_6^V(Q^2) = 0$
 $\tilde{C}_V^i$, ($i = 3, 4, 5$) are determined from electron scattering

Δ Resonance

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$$A_{\nu\mu}^{\frac{3}{2}} = - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5$$

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$\tilde{C}_V^i$, ($i = 3, 4, 5$) are determined from electron scattering

$$\tilde{C}_3^V(Q^2) = \frac{2.13}{(1 + Q^2/M_V^2)^2} \times \frac{1}{1 + \frac{Q^2}{4M_V^2}},$$

$$\tilde{C}_4^V(Q^2) = \frac{-1.51}{(1 + Q^2/M_V^2)^2} \times \frac{1}{1 + \frac{Q^2}{4M_V^2}},$$

$$\tilde{C}_5^V(Q^2) = \frac{0.48}{(1 + Q^2/M_V^2)^2} \times \frac{1}{1 + \frac{Q^2}{0.776M_V^2}}$$

O. Lalakulich et al, Phys.
Rev. D **74**, 014009 (2006).

Δ Resonance

$$\begin{aligned}
 V_{\nu\mu}^{\frac{3}{2}} &= \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right] \\
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 \end{aligned}$$

Axial Vector Form Factors $\implies$ Dominant Contribution from $\tilde{C}_5^A(Q^2)$

The Q^2 dependence of $\tilde{C}_5^A(Q^2)$ is parameterized by Schreiner and von Hippel in the Adler's model :

$$\tilde{C}_5^A(Q^2) = \frac{C_5^A(0) \left(1 + \frac{a Q^2}{b + Q^2}\right)}{\left(1 + Q^2/M_{A\Delta}^2\right)^2}$$

with coefficients $a = -1.21$ and $b = 2$.

Δ Resonance

$$V_{\nu\mu}^{\frac{3}{2}} = \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right]$$

$$A_{\nu\mu}^{\frac{3}{2}} = - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5$$

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Modified dipole form

$$\tilde{C}_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + Q^2/M_{A\Delta}^2\right)^2} \frac{1}{1 + Q^2/(3M_{A\Delta}^2)}.$$

Δ Resonance

$$\begin{aligned}
 V_{\nu\mu}^{\frac{3}{2}} &= \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right] \\
 A_{\nu\mu}^{\frac{3}{2}} &= - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5
 \end{aligned}$$

Δ Resonance

$$\begin{aligned}V_{\nu\mu}^{\frac{3}{2}} &= \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right] \\A_{\nu\mu}^{\frac{3}{2}} &= - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5\end{aligned}$$

Axial Vector Form Factors $\implies$ PCAC & Goldberger-Treiman relation
Adler's Model

$$\tilde{C}_6^A(Q^2) = \tilde{C}_5^A(Q^2) \frac{M^2}{Q^2 + m_\pi^2}$$

$$\tilde{C}_4^A(Q^2) = -\frac{1}{4} \tilde{C}_5^A(Q^2); \quad \tilde{C}_3^A(Q^2) = 0.$$

Non-resonant background

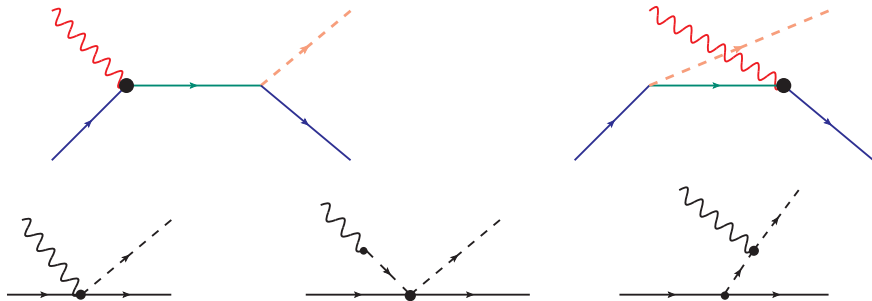
- $SU(2)$ non-linear σ model Lagrangian.
- All the couplings are determined in terms of well known parameters like pion decay constant (f_π) and axial charge (g_A).
- Introduced form factors to incorporate the nucleon structure.

$$\begin{aligned}\mathcal{L}_{\text{int}}^\sigma = & \frac{g_A}{f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} (\partial_\mu \vec{\phi}) \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma_\mu \vec{\tau} (\vec{\phi} \times \partial^\mu \vec{\phi}) \Psi - \frac{1}{6f_\pi^2} (\vec{\phi}^2 \partial_\mu \vec{\phi} \partial^\mu \vec{\phi} - (\vec{\phi} \partial_\mu \vec{\phi})(\vec{\phi} \partial^\mu \vec{\phi})) \\ & + \frac{m_\pi^2}{24f_\pi^2} (\vec{\phi}^2)^2 - \frac{g_A}{6f_\pi^3} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\phi}^2 \frac{\vec{\tau}}{2} \partial_\mu \vec{\phi} - (\vec{\phi} \partial_\mu \vec{\phi}) \frac{\vec{\tau}}{2} \vec{\phi} \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^4}\right)\end{aligned}$$

Vector and axial vector current are obtained using the above Lagrangian.

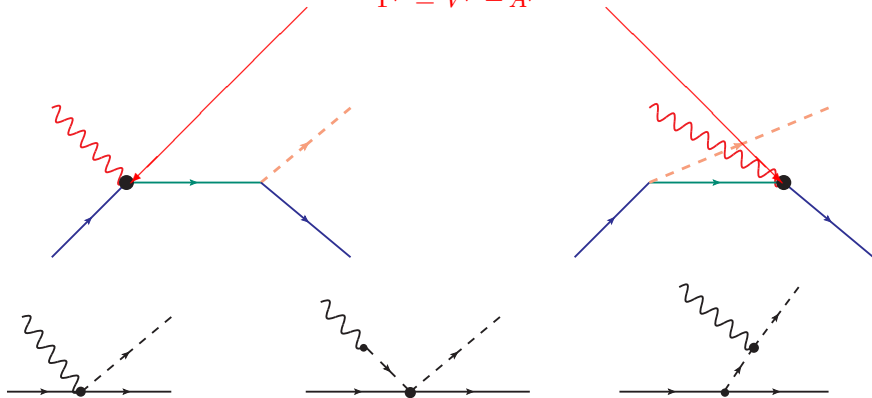
E. Hernandez, J. Nieves and M. Valverde
Phys. Rev. D 76, 033005 (2007).

Non-resonant background



Non-resonant background

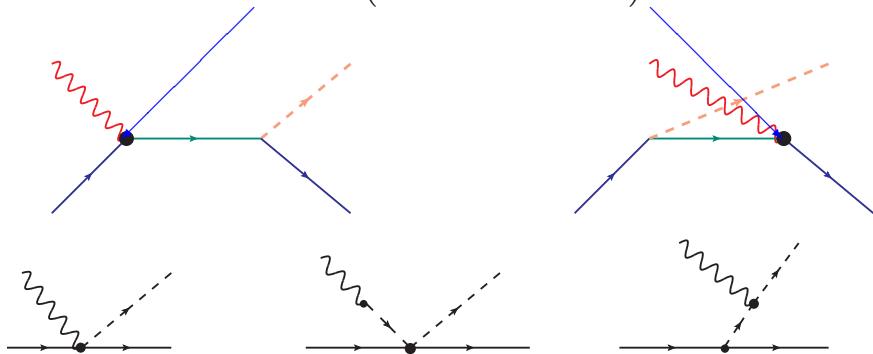
$$\Gamma^\mu = V^\mu - A^\mu$$



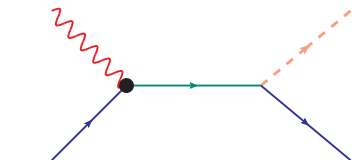
Non-resonant background

$$V^\mu \longrightarrow \tilde{f}_1(Q^2)\gamma^\mu + \tilde{f}_2(Q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M}$$

$$A^\mu \longrightarrow \left(\tilde{f}_A(Q^2)\gamma^\mu + \tilde{f}_P(Q^2)\frac{q^\mu}{M}\right)\gamma^5$$



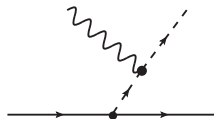
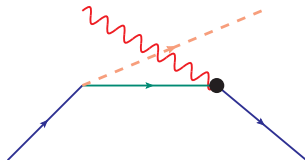
Non-resonant background



$$\Gamma^\mu \sim \gamma^\mu - g_A \gamma^\mu \gamma^5$$



$$\Gamma^\mu \sim \gamma^\mu$$



$$\Gamma^\mu \sim \gamma^\mu \gamma^5$$

Charged Current Process

$$\tilde{f}_{1,2}(Q^2) \longrightarrow f_{1,2}^V(Q^2) = f_{1,2}^p(Q^2) - f_{1,2}^n(Q^2),$$

Axial form factor($\tilde{f}_A(Q^2)$) is generally taken to be of dipole form,

$$\tilde{f}_A(Q^2) = f_A(Q^2) = f_A(0) \left[1 + \frac{Q^2}{M_A^2} \right]^{-2},$$

$$f_P(Q^2) = \frac{2M^2 f_A(Q^2)}{m_\pi^2 + Q^2}.$$

$$\begin{aligned}
j^\mu|_{NP} &= V_{ud} \mathcal{A}^{NP} \bar{u}(\vec{p}') \not{k}_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2 + i\epsilon} [V_N^\mu(q) - A_N^\mu(q)] u(\vec{p}), \\
j^\mu|_{CP} &= V_{ud} \mathcal{A}^{CP} \bar{u}(\vec{p}') [V_N^\mu(q) - A_N^\mu(q)] \frac{\not{p}' - \not{q} + M}{(p'-q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5 u(\vec{p}), \\
j^\mu|_{CT} &= V_{ud} \mathcal{A}^{CT} \bar{u}(\vec{p}') \gamma^\mu \left(g_A f_{CT}^V(Q^2) \gamma_5 - f_\rho ((q - k_\pi)^2) \right) u(\vec{p}), \\
j^\mu|_{PP} &= V_{ud} \mathcal{A}^{PP} f_\rho ((q - k_\pi)^2) \frac{q^\mu}{m_\pi^2 + Q^2} \bar{u}(\vec{p}') \not{q} u(\vec{p}), \\
j^\mu|_{PF} &= V_{ud} \mathcal{A}^{PF} f_{PF}(Q^2) \frac{(2k_\pi - q)^\mu}{(k_\pi - q)^2 - m_\pi^2} 2M \bar{u}(\vec{p}') \gamma_5 u(\vec{p}),
\end{aligned}$$

with $V_{ud} = \cos\theta_C$ for charged current process.

Spin $\frac{1}{2}$ resonance

- 1 $P_{11}(1440) (\frac{1}{2}^+)$
- 2 $S_{11}(1535) (\frac{1}{2}^-)$
- 3 $S_{11}(1650) (\frac{1}{2}^-)$

$$V_{\frac{1}{2}}^{\mu} = \frac{F_1(Q^2)}{(2M)^2} (Q^2 \gamma^{\mu} + \not{q} q^{\mu}) + \frac{F_2(Q^2)}{2M} i \sigma^{\mu\alpha} q_{\alpha}$$

$$A_{\frac{1}{2}}^{\mu} = -F_A(Q^2) \gamma^{\mu} \gamma^5 - \frac{F_P(Q^2)}{M} q^{\mu} \gamma^5,$$

Spin $\frac{1}{2}$ resonance

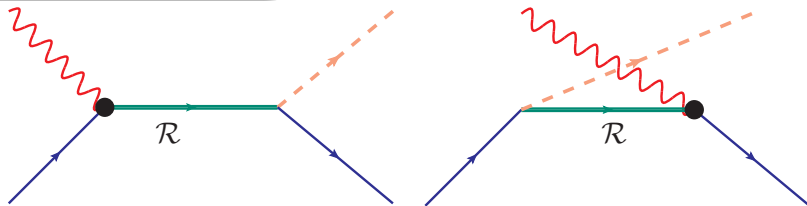
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$$j_{\frac{1}{2}}^{\mu} = \bar{u}(p') \Gamma_{\frac{1}{2}}^{\mu} u(p),$$

Spin $\frac{1}{2}$ resonance

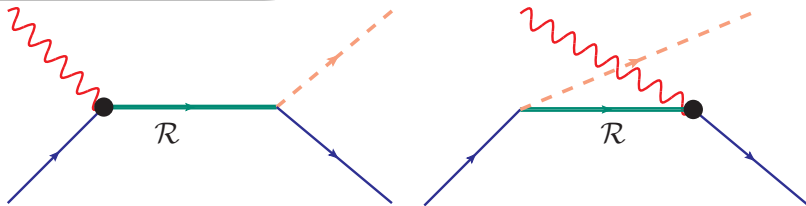
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$$A_{\frac{1}{2}}^{\mu} = -F_A(Q^2) \gamma^{\mu} \gamma^5 - \frac{F_P(Q^2)}{M} q^{\mu} \gamma^5,$$



Positive parity state $\Gamma_{\frac{1}{2}^+}^{\mu} = V_{\frac{1}{2}}^{\mu} - A_{\frac{1}{2}}^{\mu}$

Negative parity state $\Gamma_{\frac{1}{2}^-}^{\mu} = [V_{\frac{1}{2}}^{\mu} - A_{\frac{1}{2}}^{\mu}] \gamma^5$

Spin $\frac{1}{2}$ resonance

$$j^\mu \Big|_R^{\frac{1}{2}} = i V_{ud} \mathcal{C} \mathcal{R} \bar{u}(\vec{p}') \not{k}_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2 + i\epsilon} \Gamma_{\frac{1}{2}}^\mu u(\vec{p}),$$

$$j^\mu \Big|_{CR}^{\frac{1}{2}} = i V_{ud} \mathcal{C} \mathcal{R} \bar{u}(\vec{p}') \Gamma_{\frac{1}{2}}^\mu \frac{\not{p}' - \not{q} + M}{(p'-q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5 u(\vec{p}),$$

Spin $\frac{1}{2}$ resonance

$$j^\mu \Big|_R^{\frac{1}{2}} = i V_{ud} \mathcal{C} \mathcal{R} \bar{u}(\vec{p}') \not{k}_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2 + i\epsilon} \Gamma_{\frac{1}{2}}^\mu u(\vec{p}),$$

$$j^\mu \Big|_{CR}^{\frac{1}{2}} = i V_{ud} \mathcal{C} \mathcal{R} \bar{u}(\vec{p}') \Gamma_{\frac{1}{2}}^\mu \frac{\not{p}' - \not{q} + M}{(p'-q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5 u(\vec{p}),$$

Electromagnetic transition form factors calculated in terms of Helicity amplitudes taken from MAID analysis

$$A_{\frac{1}{2}}^{p,n} = \sqrt{\frac{2\pi\alpha}{M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{Q^2}{4M^2} F_1^{p,n} + \frac{M_R \pm M}{2M} F_2^{p,n} \right]$$

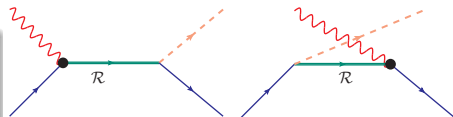
$$S_{\frac{1}{2}}^{p,n} = \mp \sqrt{\frac{\pi\alpha}{M} \frac{(M \pm M_R)^2 + Q^2}{M_R^2 - M^2} \frac{(M_R \mp M)^2 + Q^2}{4M_R M}} \left[\frac{M_R \pm M}{2M} F_1^{p,n} - F_2^{p,n} \right]$$

upper sign for Positive & lower sign for Negative parity state.

Spin $\frac{3}{2}$ & Isospin $\frac{1}{2}$ resonance

④ $D_{13}(1520) (\frac{3}{2}^-)$

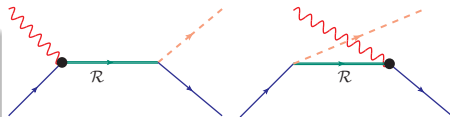
④ $P_{13}(1720) (\frac{3}{2}^+)$



Spin $\frac{3}{2}$ & Isospin $\frac{1}{2}$ resonance

● $D_{13}(1520) \left(\frac{3}{2}^{-}\right)$

● $P_{13}(1720) \left(\frac{3}{2}^{+}\right)$



$$V_{\nu\mu}^{\frac{3}{2}} = \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right]$$

$$A_{\nu\mu}^{\frac{3}{2}} = - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5$$

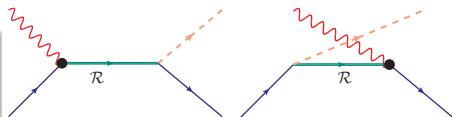
$$j_{\frac{3}{2}}^\mu = \bar{u}_\nu(p') \Gamma_{\frac{3}{2}}^{\nu\mu} u(p),$$

$$\tilde{C}_i^V = C_i^P - C_i^N$$

Spin $\frac{3}{2}$ & Isospin $\frac{1}{2}$ resonance

● $D_{13}(1520) (\frac{3}{2}^-)$

● $P_{13}(1720) (\frac{3}{2}^+)$



$$V_{\nu\mu}^{\frac{3}{2}} = \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right]$$

$$A_{\nu\mu}^{\frac{3}{2}} = - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5$$

Positive parity state $\Gamma_{\nu\mu}^{\frac{3}{2}+} = \left[V_{\nu\mu}^{\frac{3}{2}} - A_{\nu\mu}^{\frac{3}{2}} \right] \gamma_5$

Negative parity state $\Gamma_{\nu\mu}^{\frac{3}{2}-} = V_{\nu\mu}^{\frac{3}{2}} + A_{\nu\mu}^{\frac{3}{2}}$

$$\begin{aligned}
j^\mu \Big|_R^{\frac{3}{2}} &= i V_{ud} C^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + i M_R \Gamma_R} \bar{u}(\vec{p}') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu}(p, q) u(\vec{p}), \quad p_R = p + q, \\
j^\mu \Big|_{CR}^{\frac{3}{2}} &= i a C^{\mathcal{R}} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + i M_R \Gamma_R} \bar{u}(\vec{p}') \hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha}(p', -q) P_{\alpha\beta}^{3/2}(p_R) u(\vec{p}), \quad p_R = p' - q,
\end{aligned}$$

$$\begin{aligned}
j^\mu \Big|_R^{\frac{3}{2}} &= i V_{ud} C^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + i M_R \Gamma_R} \bar{u}(\vec{p}') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu}(p, q) u(\vec{p}), \quad p_R = p + q, \\
j^\mu \Big|_{CR}^{\frac{3}{2}} &= i a C^{\mathcal{R}} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + i M_R \Gamma_R} \bar{u}(\vec{p}') \hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha}(p', -q) P_{\alpha\beta}^{3/2}(p_R) u(\vec{p}), \quad p_R = p' - q,
\end{aligned}$$

Vector form factor are determined from Helicity amplitudes

$$\begin{aligned}
A_{\frac{3}{2}}^{p,n} &= \sqrt{\frac{\pi \alpha}{M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{C_3^{p,n}}{M} (M \pm M_R) \pm \frac{C_4^{p,n}}{M^2} \frac{M_R^2 - M^2 - Q^2}{2} \pm \frac{C_5^{p,n}}{M^2} \frac{M_R^2 - M^2 + Q^2}{2} \right] \\
A_{\frac{1}{2}}^{p,n} &= \sqrt{\frac{\pi \alpha}{3M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{C_3^{p,n}}{M} \frac{M^2 + M M_R + Q^2}{M_R} - \frac{C_4^{p,n}}{M^2} \frac{M_R^2 - M^2 - Q^2}{2} - \frac{C_5^{p,n}}{M^2} \frac{M_R^2 - M^2 + Q^2}{2} \right] \\
S_{\frac{1}{2}}^{p,n} &= \pm \sqrt{\frac{\pi \alpha}{6M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \frac{\sqrt{Q^4 + 2Q^2(M_R^2 + M^2) + (M_R^2 - M^2)^2}}{M_R^2} \\
&\times \left[\frac{C_3^{p,n}}{M} M_R + \frac{C_4^{p,n}}{M^2} M_R^2 + \frac{C_5^{p,n}}{M^2} \frac{M_R^2 + M^2 + Q^2}{2} \right],
\end{aligned}$$

upper sign for Positive & lower sign for Negative parity state.

Axial-Vector form factors

No data are available for higher resonances to fix axial form-factors

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Dipole-form $\Rightarrow$

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} ;$$

$$C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

Axial-Vector form factors

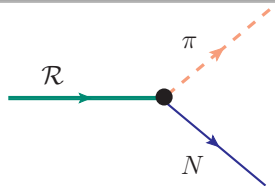
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The couplings $f_{R\frac{3}{2}}$ & $f_{R\frac{1}{2}}$ are determined from the $\mathcal{R} \rightarrow N\pi$ decay rate



Axial-Vector form factors

No data are available for higher resonances to fix axial form-factors

Dipole-form $\Rightarrow$

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} ;$$

$$C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

PCAC & Goldberger-Treiman relation

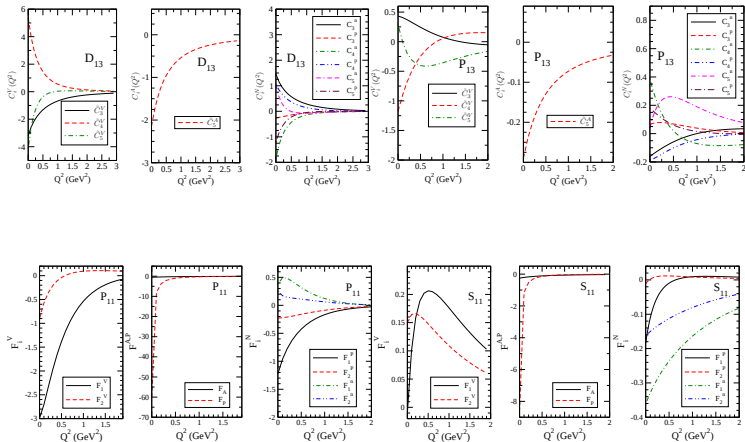
$$F_A(0) = -2f_\pi \frac{f_{R\frac{1}{2}}}{m_\pi},$$

$$C_5^A(0) = -2f_\pi \frac{f_{R\frac{3}{2}}}{m_\pi},$$

$$F_P(Q^2) = \frac{(MM_R \pm M^2)}{m_\pi^2 + Q^2} F_A(Q^2)$$

$$C_6^A(Q^2) = \frac{(MM_R \pm M^2)}{m_\pi^2 + Q^2} C_5^A(Q^2)$$

form-factors



Experiments were performed on Deuteron targets

BNL 7-foot deuterium bubble chamber

Argonne 12-foot bubble chamber

- $\nu_\mu d \rightarrow \mu^- p \pi^+ n_s$
- $\nu_\mu d \rightarrow \mu^- p \pi^0 p_s$
- $\nu_\mu d \rightarrow \mu^- n \pi^+ p_s$

$$\left(\frac{d\sigma}{dQ^2 dW} \right)_{\nu d} = \int d\mathbf{p}_p^d |\Psi_d(\mathbf{p}_p^d)|^2 \frac{M}{E_p^d} \left(\frac{d\sigma}{dQ^2 dW} \right)_{\text{off shell}},$$

Deuteron four momenta $p^\mu = (E_p^d, \mathbf{p}_p^d)$

energy of the off shell proton inside the deuteron

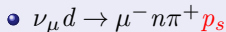
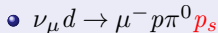
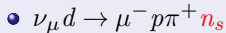
$$E_p^d (= M_{\text{Deuteron}} - \sqrt{M^2 + |\mathbf{p}_p^d|^2})$$

M_{Deuteron} is the deuteron mass

Experiments were performed on Deuteron targets

BNL 7-foot deuterium bubble chamber

Argonne 12-foot bubble chamber



n_s and p_s are spectator nucleon

$$\left(\frac{d\sigma}{dQ^2 dW} \right)_{\nu d} = \int d\mathbf{p}_p^d |\Psi_d(\mathbf{p}_p^d)|^2 \frac{M}{E_p^d} \left(\frac{d\sigma}{dQ^2 dW} \right)_{\text{off shell}},$$

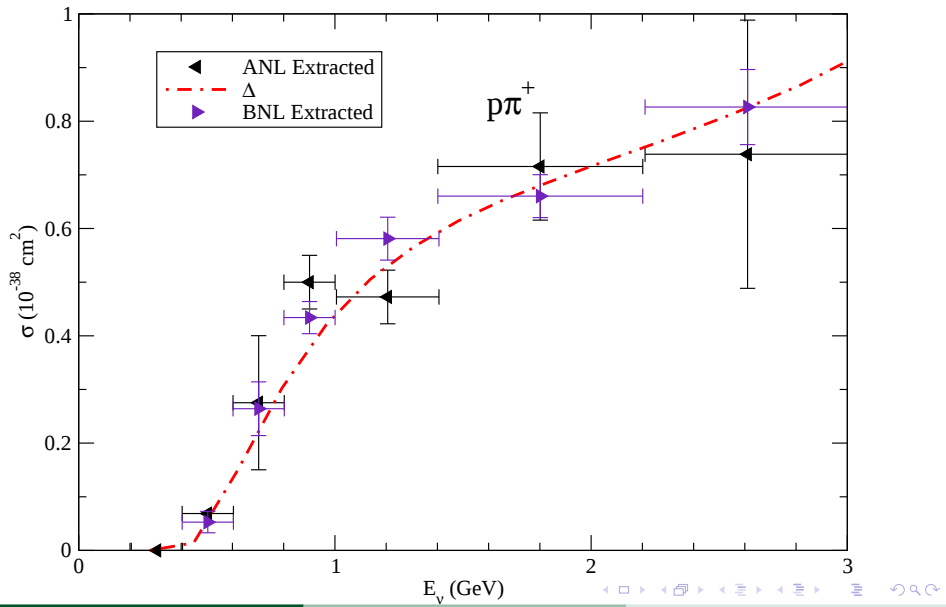
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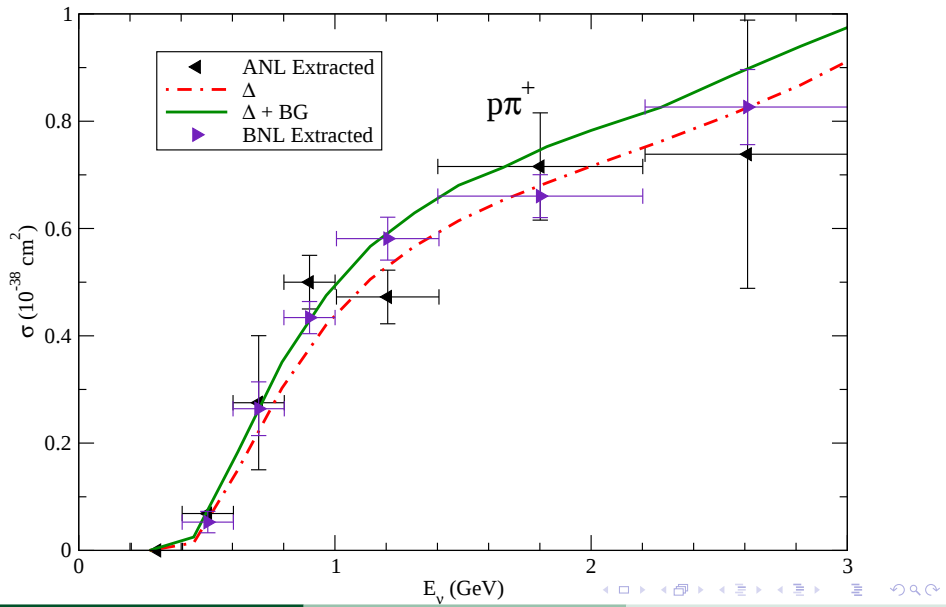
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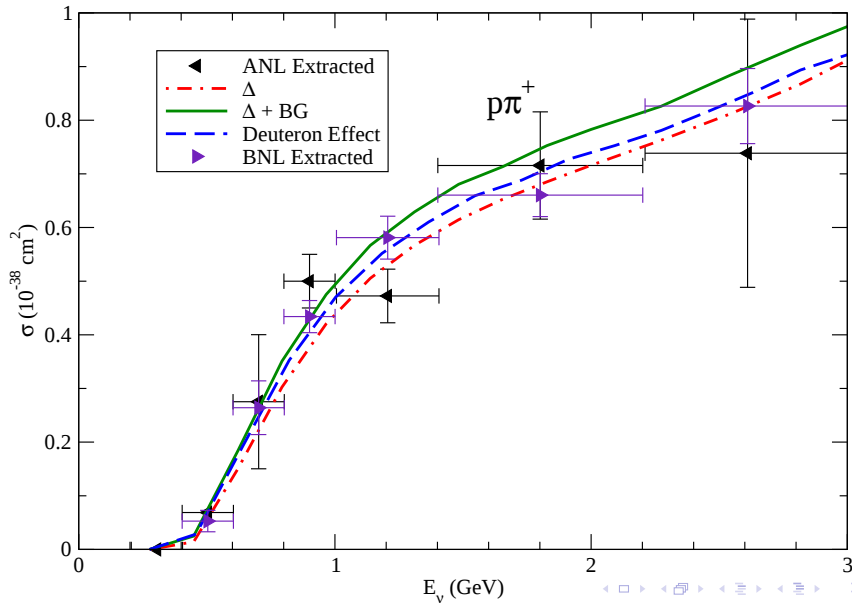
$$\nu_\mu p \rightarrow \mu^- p \pi^+$$



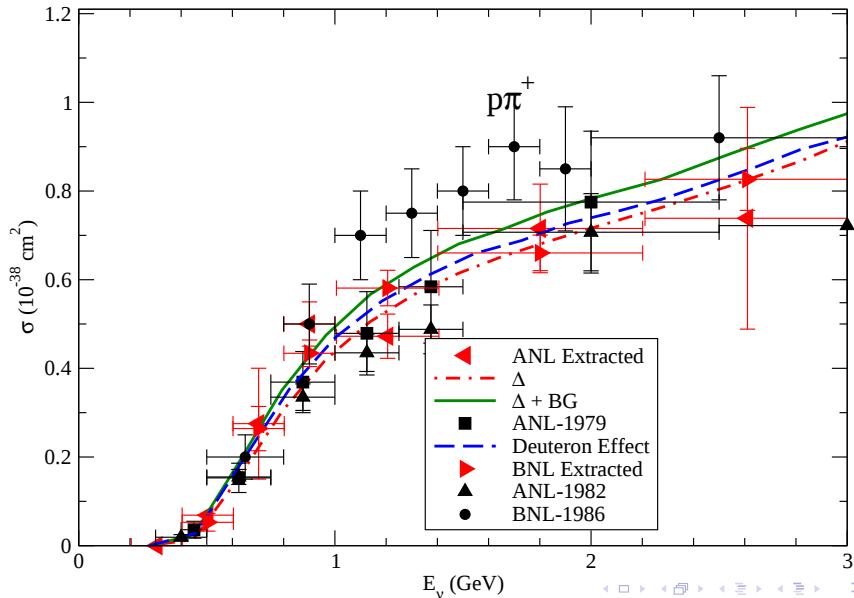
$$\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+}$$



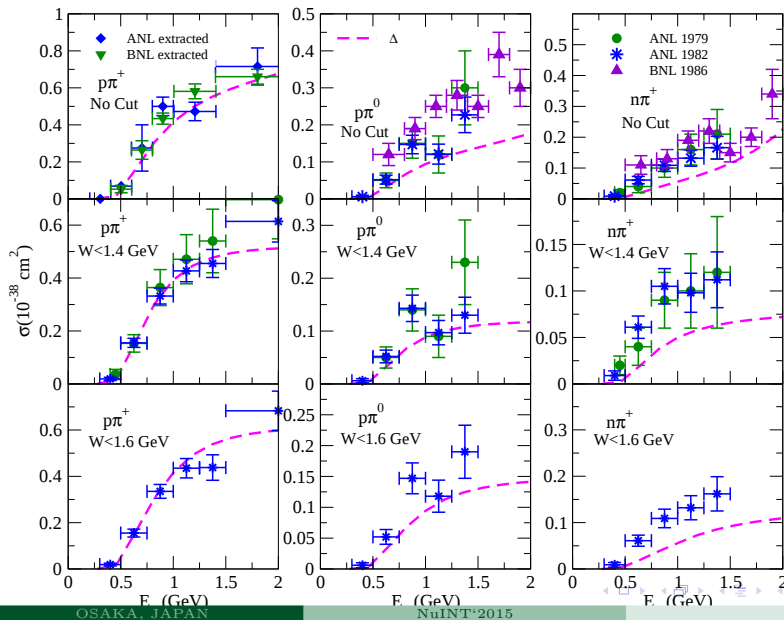
$$\nu_\mu p \rightarrow \mu^- p \pi^+ \quad C_5^A(0)|_{\Delta=1} \text{ and } M_A = 1.026 \text{ GeV}$$



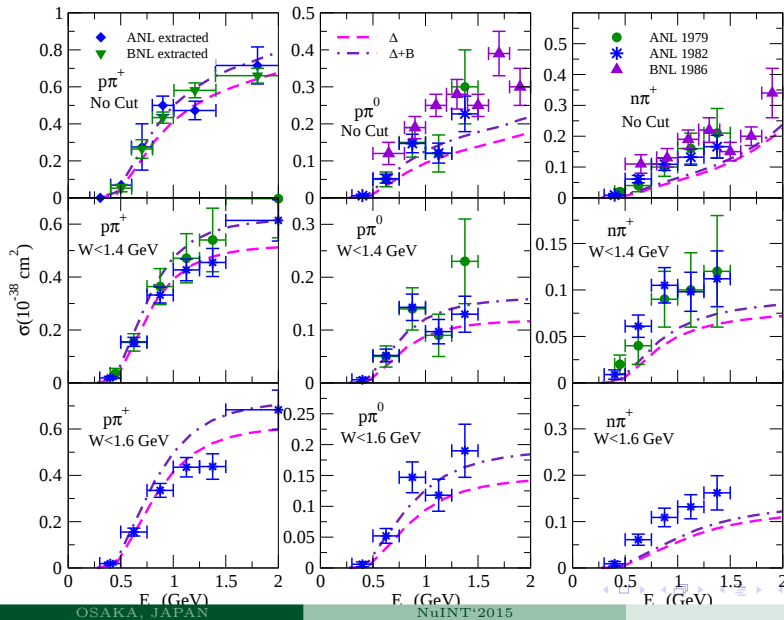
$\nu_\mu p \rightarrow \mu^- p \pi^+$ $C_5^A(0)|_{\Delta=1}$ and $M_A = 1.026 \text{ GeV}$



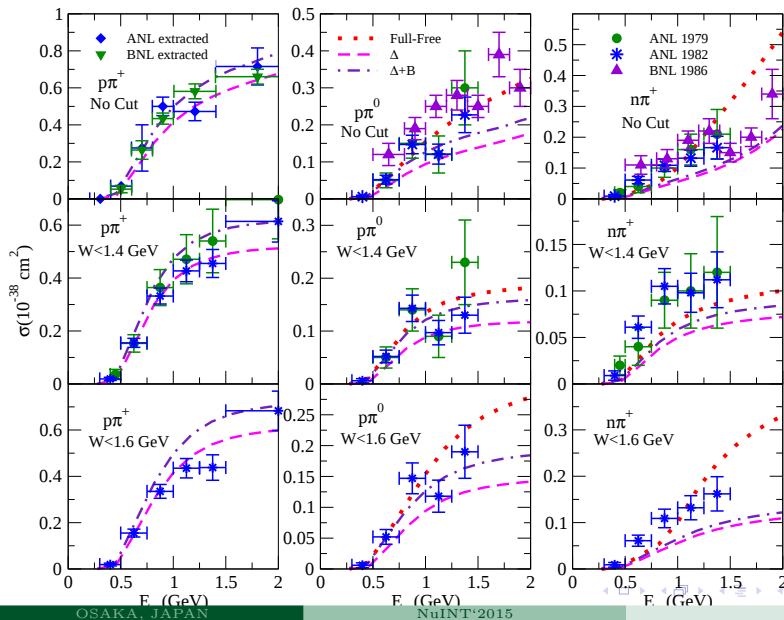
Neutrino induced Charged-current pion production



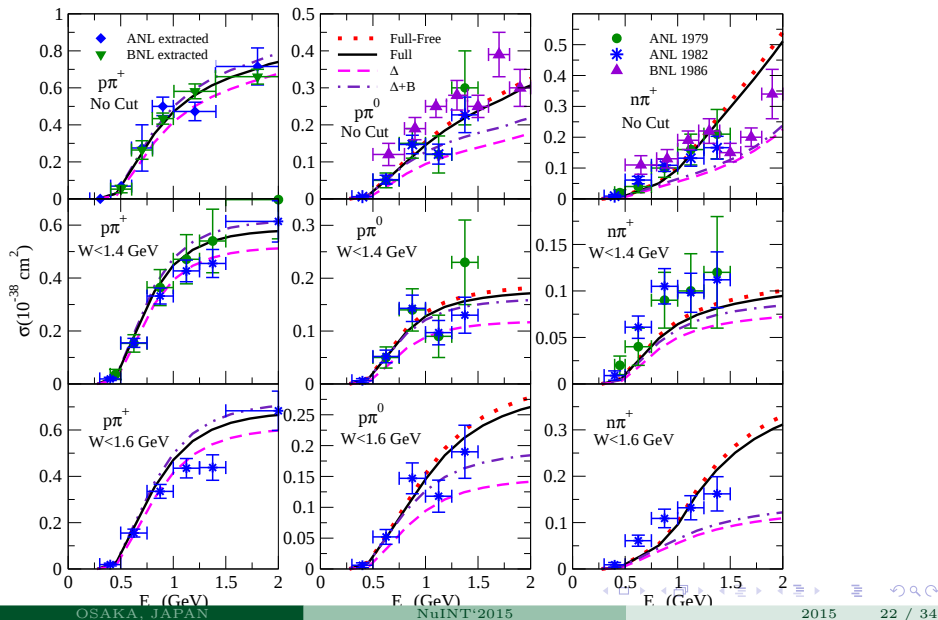
Neutrino induced Charged-current pion production



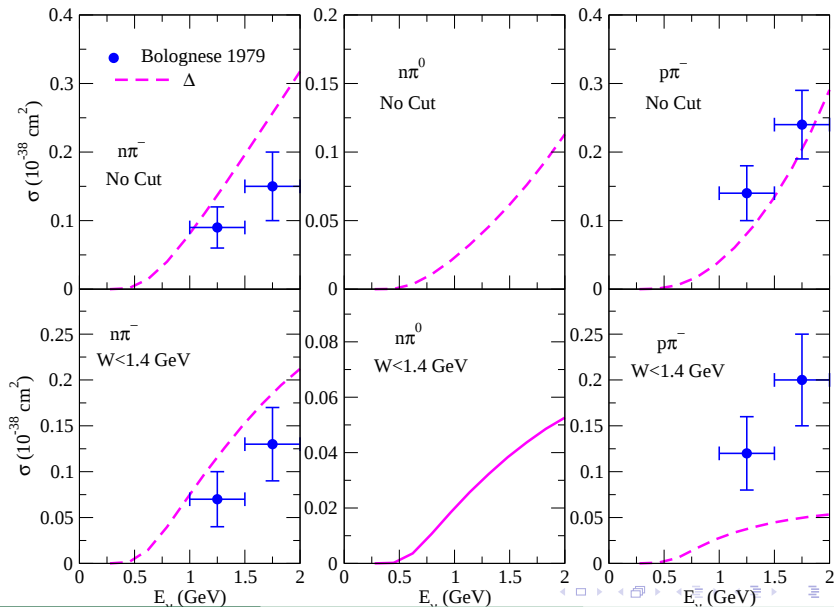
Neutrino induced Charged-current pion production



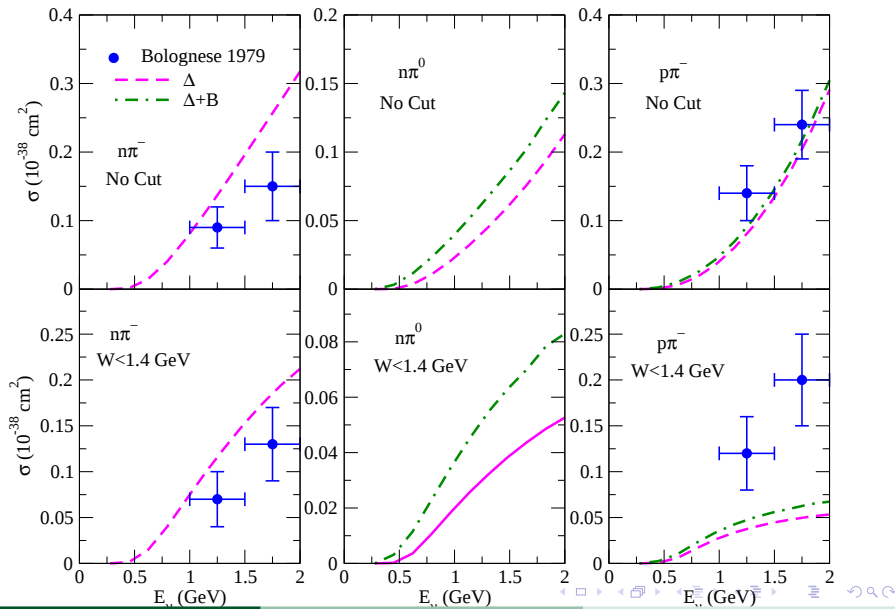
Neutrino induced Charged-current pion production



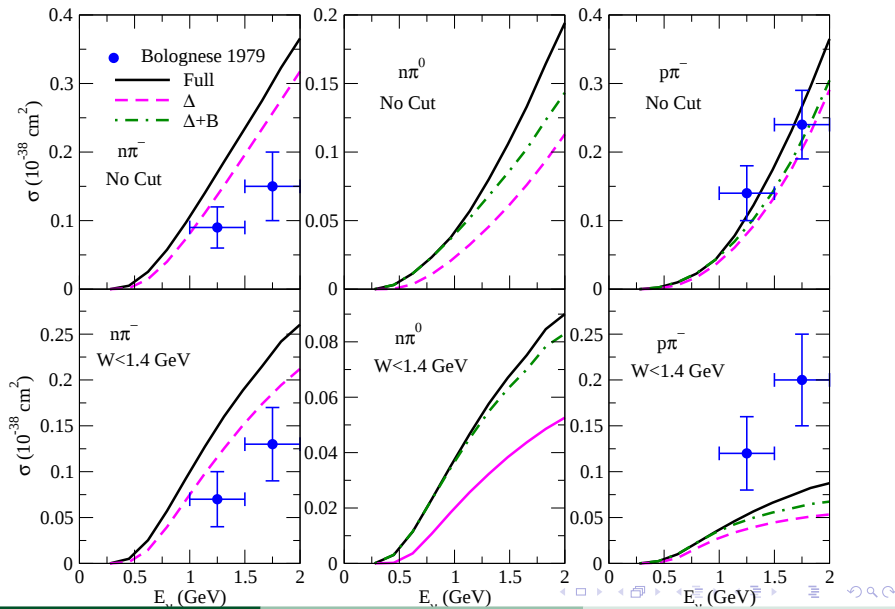
Antineutrino induced Charged-current pion production



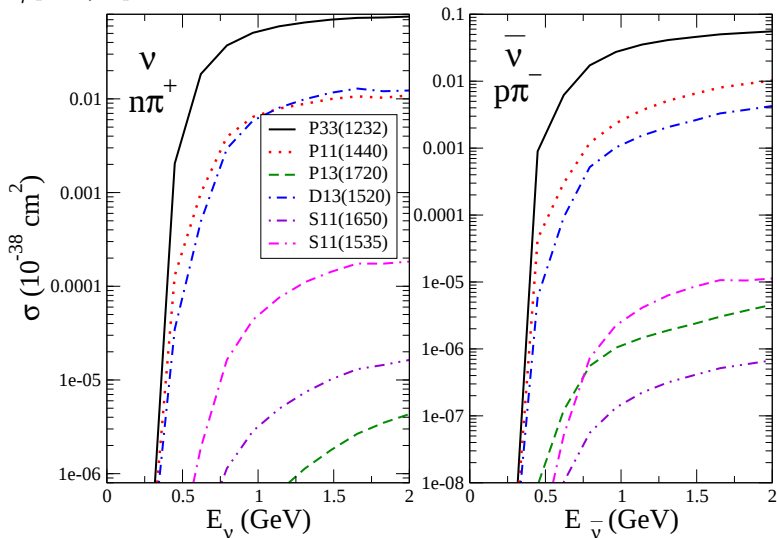
Antineutrino induced Charged-current pion production



Antineutrino induced Charged-current pion production



Individual contribution of various resonances for $\nu_\mu n \rightarrow \mu^- n \pi^+$ and $\bar{\nu}_\mu p \rightarrow \mu^+ p \pi^-$ channel.



Neutral Current induced one pion production

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

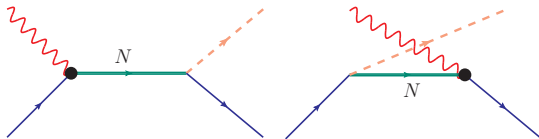
$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

Non-resonant background



$$j^\mu|_{NP} = \mathcal{A}^{NP} \bar{u}(\vec{p}') \not{k}_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2 + i\epsilon} [V_N^\mu(q) - A_N^\mu(q)] u(\vec{p}),$$

$$j^\mu|_{CP} = \mathcal{A}^{CP} \bar{u}(\vec{p}') [V_N^\mu(q) - A_N^\mu(q)] \frac{\not{p}' - \not{q} + M}{(p'-q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5 u(\vec{p}),$$

Only Nucleon-poles will contribute

Non-resonant background

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$j^\mu|_{NP}$$

$$V_N^\mu(q) = \tilde{f}_1(Q^2)\gamma^\mu + \tilde{f}_2(Q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M} \dots q) \Big] u(\vec{p}),$$

$$j^\mu|_{CP}$$

$$A_N^\mu(q) = \left(\tilde{f}_A(Q^2)\gamma^\mu + \tilde{f}_P(Q^2)\frac{q^\mu}{M} \right) \gamma^5, \dots \pi \gamma^5 u(\vec{p}),$$

Only nucleon poles will contribute

Non-resonant background

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

 $j^\mu|_N$

$$\left(\frac{1}{2} - 2\sin^2\theta_W\right) f_{1,2}^p(Q^2) - \frac{1}{2} f_{1,2}^n(Q^2)$$

 $(\vec{p}),$
 $j^\mu|_C$

$$\left(\frac{1}{2} - 2\sin^2\theta_W\right) f_{1,2}^n(Q^2) - \frac{1}{2} f_{1,2}^p(Q^2).$$

 $\mu(\vec{p}),$

where θ_W is the Weinberg angle.

contribute

Non-resonant background

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$(1 - \sin^2 \theta_W) \approx 0.46, \quad 1 - \sin^2 \theta_W \approx 0.54$$

$$\tilde{f}_A(Q^2) = \tilde{f}_A^{p,n}(0) \left[1 + \frac{Q^2}{M_A^2} \right]^{-2}$$

where θ_W is the Weinberg angle.

contribute

Δ -Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$j^\mu \Big|_R^{\frac{3}{2}} = iC\mathcal{R} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu} u(\vec{p}), \quad p_R = p + q,$$

$$j^\mu \Big|_{CR}^{\frac{3}{2}} = iC\mathcal{R} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}') \hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha} P_{\alpha\beta}^{3/2}(p_R) u(\vec{p}), \quad p_R = p' - q,$$

Direct and Cross Δ poles

Δ -Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$\begin{aligned} V_{\nu\mu}^{\frac{3}{2}} &= \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{\epsilon} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right] \\ A_{\nu\mu}^{\frac{3}{2}} &= - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{\epsilon} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p_R - q_\nu p_{R\mu}) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5 \end{aligned}$$

bles

Δ -Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$V_{\nu\mu}^{\frac{3}{2}}$$

$$A_{\nu\mu}^{\frac{3}{2}}$$

$$(\tilde{C}_i^V)^{NC} \longrightarrow (1 - 2 \sin^2 \theta_W) \tilde{C}_i^V$$

$$(\tilde{C}_i^A)^{NC} \longrightarrow -(\tilde{C}_i^A)$$

$$\tilde{C}_6^V \Big]$$

where θ_W is the Weinberg angle.

cross Δ poles

Spin- $\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$j^\mu \Big|_R^{\frac{1}{2}} = i \mathcal{C} \mathcal{R} \bar{u}(\vec{p}') \not{k}_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2 + i\epsilon} \Gamma_{\frac{1}{2}}^\mu u(\vec{p}),$$

$$j^\mu \Big|_{CR}^{\frac{1}{2}} = i \mathcal{C} \mathcal{R} \bar{u}(\vec{p}') \Gamma_{\frac{1}{2}}^\mu \frac{\not{p}' - \not{q} + M}{(p'-q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5 u(\vec{p}),$$

Direct and Cross $\mathcal{R}$ -poles

Spin- $\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$j \quad \begin{aligned} V_{\frac{1}{2}}^{\mu} &= \left[\frac{F_1(Q^2)}{(2M)^2} (Q^2 \gamma^{\mu} + \not{q} q^{\mu}) + \frac{F_2(Q^2)}{2M} i \sigma^{\mu\alpha} q_{\alpha} \right] \gamma_5 \\ A_{\frac{1}{2}}^{\mu} &= -F_A(Q^2) \gamma^{\mu} - \frac{F_P(Q^2)}{M} q^{\mu}, \end{aligned}$$

Direct and Cross $\mathcal{R}$ -poles

$Spin-\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$\left(\frac{1}{2} - 2\sin^2\theta_W\right) F_{1,2}^p(Q^2) - \frac{1}{2} F_{1,2}^n(Q^2)$$
$$\left(\frac{1}{2} - 2\sin^2\theta_W\right) F_{1,2}^n(Q^2) - \frac{1}{2} F_{1,2}^p(Q^2).$$

where θ_W is the Weinberg angle.

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ss $\mathcal{R}$ -poles

Spin- $\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$(1 - \frac{Q^2}{M_A^2})^{-2} \approx 1 + \frac{2Q^2}{M_A^2}$$

$$\tilde{F}_A(Q^2) = \tilde{F}_A^{p,n}(0) \left[1 + \frac{Q^2}{M_A^2} \right]^{-2}$$

where θ_W is the Weinberg angle.

mass $\mathcal{R}$ -poles

Spin- $\frac{3}{2}$ and Isospin- $\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$j^\mu \Big|_R^{\frac{3}{2}} = i\mathcal{C}\mathcal{R} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu}(p, q) u(\vec{p}), \quad p_R = p + q,$$

$$j^\mu \Big|_{CR}^{\frac{3}{2}} = i\mathcal{C}\mathcal{R} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}') \hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha}(p', -q) P_{\alpha\beta}^{3/2}(p_R) u(\vec{p}), \quad p_R = p' - q,$$

Direct and Cross $\mathcal{R}$ poles

Spin- $\frac{3}{2}$ and Isospin- $\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$V_{\nu\mu}^{\frac{3}{2}} = \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu} q \cdot p' - q_\nu p'_\mu) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu} q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right]$$

$$A_{\nu\mu}^{\frac{3}{2}} = - \left[\frac{\tilde{C}_3^A}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^A}{M^2} (g_{\mu\nu} q \cdot p' - q_\nu p'_\mu) + \tilde{C}_5^A g_{\mu\nu} + \frac{\tilde{C}_6^A}{M^2} q_\nu q_\mu \right] \gamma_5$$

bles

Spin- $\frac{3}{2}$ and Isospin- $\frac{1}{2}$ Resonance

$$\nu_l p \rightarrow \nu_l n \pi^+$$

$$\nu_l p \rightarrow \nu_l p \pi^0$$

$$\nu_l n \rightarrow \nu_l n \pi^0$$

$$\nu_l n \rightarrow \nu_l p \pi^-$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l p \pi^0$$

$$\bar{\nu}_l p \rightarrow \bar{\nu}_l n \pi^+$$

$$\bar{\nu}_l n \rightarrow \bar{\nu}_l n \pi^0$$

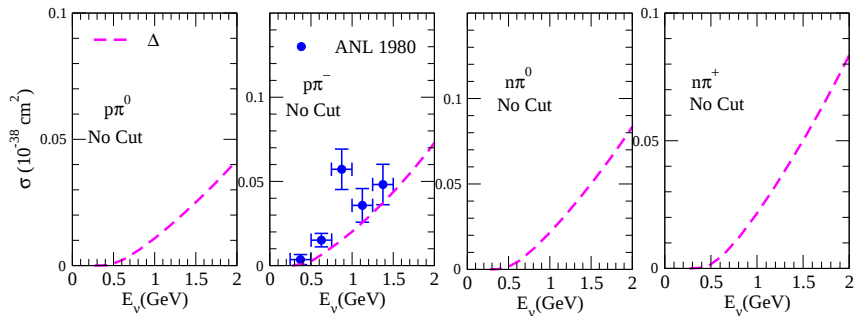
$$\bar{\nu}_l n \rightarrow \bar{\nu}_l p \pi^-.$$

$$\left(\frac{1}{2} - 2 \sin^2 \theta_W \right) \tilde{C}_i^p(Q^2) - \frac{1}{2} \tilde{C}_i^n(Q^2)$$

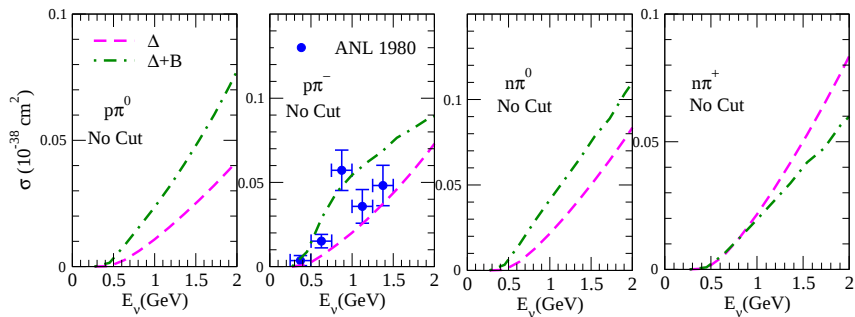
$$\left(\frac{1}{2} - 2 \sin^2 \theta_W \right) \tilde{C}_i^n(Q^2) - \frac{1}{2} \tilde{C}_i^p(Q^2).$$

where θ_W is the Weinberg angle.

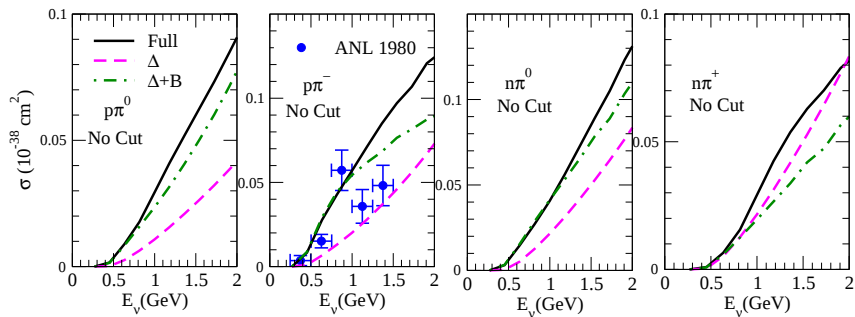
Neutrino induced neutral-current pion production



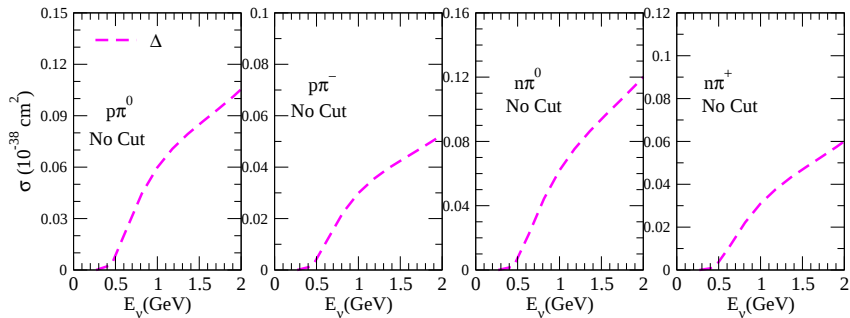
Neutrino induced neutral-current pion production



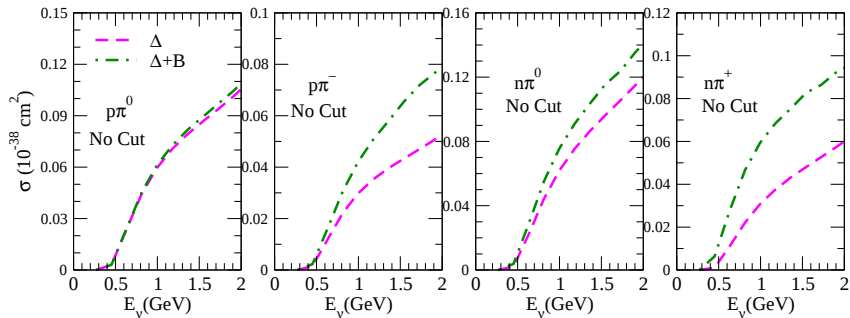
Neutrino induced neutral-current pion production



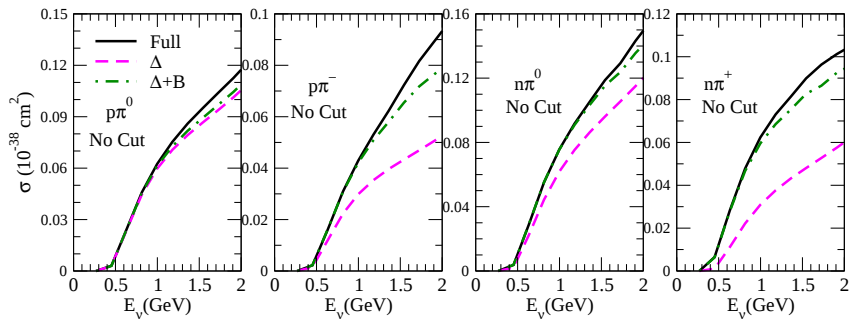
Antineutrino induced neutral-current pion production



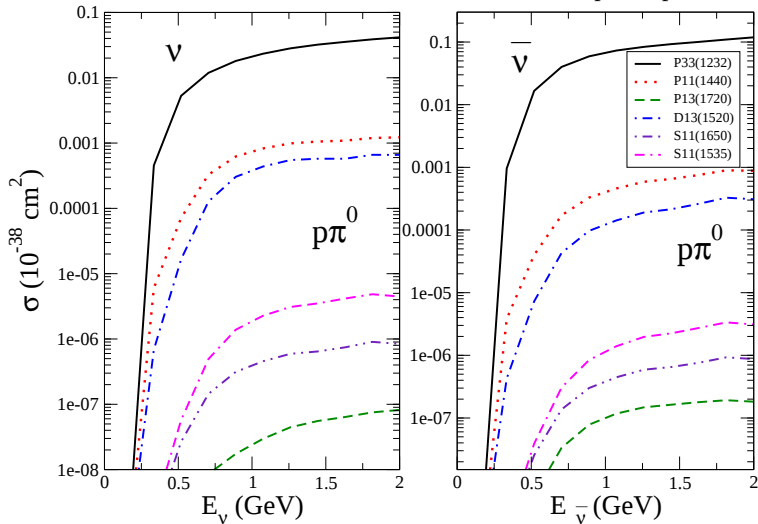
Antineutrino induced neutral-current pion production



Antineutrino induced neutral-current pion production



Individual contribution of various resonances for $\nu p \rightarrow \nu p \pi^0$ and $\bar{\nu} p \rightarrow \bar{\nu} p \pi^0$.



Conclusion

- We have studied single charged current and neutral current one pion production in the energy region of $E_{\nu/\bar{\nu}} \leq 2 \text{ GeV}$ and have considered the following contributions:
 - ▶ Δ resonance
 - ▶ Background nonresonant
 - ▶ Higher resonances
- The axial mass $M_A = 1.026 \text{ GeV}$ and $C_A^5(0) = 1.0$ in the axial form factor $C_A^5(Q^2)$ for Δ resonance are obtained from the reanalyzed data of ANL and BNL when nonresonant background terms and higher resonances are taken into account in a coherent way.
- The enhancement in the cross section due to the presence of non-resonant background terms at $E_{\nu} = 1 \text{ GeV}$ is around 12% for $\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+}$ process, and about 24% for $\bar{\nu}_{\mu} n \rightarrow \mu^{+} n \pi^{-}$ process.

- The non-resonant contributions in the case of $\nu_\mu n \rightarrow \mu^- n \pi^+$ process is around 14% which becomes 42% when higher resonances are also included.
- In the case of $\bar{\nu}_\mu p \rightarrow \mu^+ n \pi^0$ process the enhancement in the cross section due to the presence of non-resonant background terms is $\sim 42\%$ at $E_\nu = 1$ GeV, which in the case of $\bar{\nu}_\mu p \rightarrow \mu^+ p \pi^-$ process is $\sim 16\%$.
- The maximum contribution due to non-resonant terms is for $\nu n \rightarrow \nu p \pi^-$ process and the minimum contribution is for $\bar{\nu} p \rightarrow \bar{\nu} p \pi^0$.
- When contribution of higher resonances are taken into account we find the major contributions from $P_{11}(1440)$ and $D_{13}(1520)$.
- We plan to apply these calculation to obtain $\nu/\bar{\nu}$ nucleus scattering cross section for charged current as well as neutral current pion production processes. We also plan to calculate the Q^2 , pion energy and pion angle distribution.