Neutrino induced two-pion production on nucleons close to threshold

$$\nu_l + N \to l^- + N + \pi + \pi$$

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Outline of the talk

- The model
  - Nonresonant (background) contribution
  - Roper resonance contribution
- Results and conclusions

Two-pion nonresonant production

Interaction Lagrangian

We use a SU(2) nonlinear sigma model. Up to $\mathcal{O}(1/f_\pi^3)$ the interaction Lagrangian reads

$$
\begin{aligned}
\mathcal{L}^\sigma_{\rm int} \quad = \quad & \frac{g_A}{f_\pi}\bar\Psi\gamma^\mu\gamma_5\frac{\vec\tau}{2}(\partial_\mu\vec\phi)\Psi - \frac{1}{4f_\pi^2}\bar\Psi\gamma_\mu\vec\tau\left(\vec\phi\times\partial^\mu\vec\phi\right)\Psi \\
& -\frac{1}{6f_\pi^2}\left(\vec\phi^{\,2}\partial_\mu\vec\phi\partial^\mu\vec\phi - (\vec\phi\partial_\mu\vec\phi)(\vec\phi\partial^\mu\vec\phi)\right) \\
& +\frac{m_\pi^2}{24f_\pi^2}(\vec\phi^{\,2})^2 - \frac{g_A}{6f_\pi^3}\bar\Psi\gamma^\mu\gamma_5\left[\vec\phi^{\,2}\frac{\vec\tau}{2}\partial_\mu\vec\phi - (\vec\phi\partial_\mu\vec\phi)\frac{\vec\tau}{2}\vec\phi\right]\Psi\,,
\end{aligned}
$$

where $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$ is the nucleon field, $\vec\phi$ is the isovector pion field.

To evaluate the different contributions we also need the coupling to the W boson, i.e. we need the vector and axial currents.

Two-pion nonresonant production

Vector and axial currents I

To the corresponding order the vector and axial currents are

$$
\begin{aligned}
\vec{V}^\mu &= \vec{\phi}\times\partial^\mu\vec{\phi} + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi \\
&\quad - \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})
\end{aligned}
$$

$$
\begin{aligned}
\vec{A}^\mu &= f_\pi\partial^\mu\vec{\phi} + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right] \\
&\quad - \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})
\end{aligned}
$$

The normalization is such that $-\sqrt{2}\cos\theta_C(V^\mu_{+1} - A^\mu_{+1})$ provides the W^+ absorption vertex

Two-pion nonresonant production

Vector and axial currents II

The pure nucleonic part of the vector and axial currents are further modified by the inclusion of form factors. For the neutron to proton transition which fixes our normalization we have

$$V_N^\alpha(q) = 2 \times \left(F_1^V(q^2)\gamma^\alpha + \mathrm{i}\mu_V \frac{F_2^V(q^2)}{2M}\sigma^{\alpha\nu}q_\nu \right),$$

$$A_N^\alpha(q) = G_A(q^2) \times \left(\gamma^\alpha\gamma_5 + \frac{\not{q}}{m_\pi^2 - q^2} q^\alpha\gamma_5 \right).$$

The magnetic part in $V_N^\alpha(q)$ is not provided by the sigma model neither the q^2 dependence of the form factors.

Vector current conservation requires that all other terms in the vector current are multiplied by the $F_1^V(q^2)$ form factor.

Two-pion nonresonant production

Vector and axial currents III

For the vector form factors we use the parametrization of S. Galster et al in NPB 32, 221 (1971)

$$F_1^N = \frac{G_E^N + \kappa G_M^N}{1+\kappa}, \qquad \mu_N F_2^N = \frac{G_M^N - G_E^N}{1+\kappa},$$

$$G_E^p = \frac{G_M^p}{\mu_p} = \frac{G_M^n}{\mu_n} = -(1+\lambda_n \kappa)\frac{G_E^n}{\mu_n \kappa} = \left(\frac{1}{1-q^2/M_D^2}\right)^2$$

with $\kappa = -q^2/4M^2$, $M_D = 0.843$ GeV, $\mu_p = 2.792847$, $\mu_n = -1.913043$ and $\lambda_n = 5.6$. Besides,

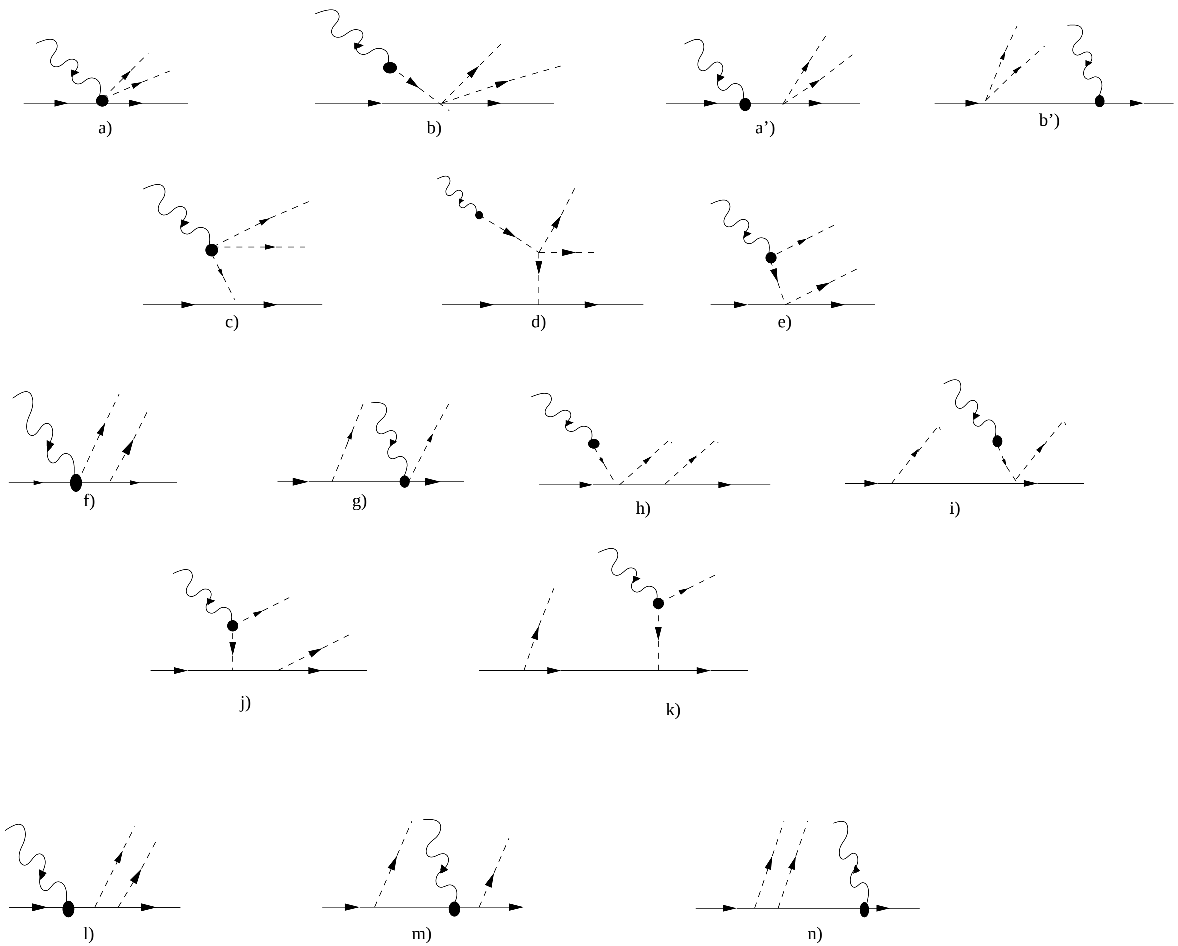
$$F_1^V(q^2) = \frac{1}{2}\left(F_1^p(q^2) - F_1^n(q^2)\right), \qquad \mu_V F_2^V(q^2) = \frac{1}{2}\left(\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)\right).$$

The axial form factor we take from the book of Ericson & Weise (The International Series of Monographs on Physics 74)

$$G_A(q^2) = \frac{g_A}{(1-q^2/M_A^2)^2}, \quad g_A = 1.26, \quad M_A = 1.05\ \mathrm{GeV}$$

Two-pion nonresonant production

We have in total 16 different contributions corresponding to the Feynman diagrams



Two-pion resonant production

Since the Δ does not couple to two pions in s-wave, Δ excitation contributions are expected to be small when only slow pions are produced (close to threshold).

The Roper resonance $N^*(1440)$ on the other hand has a sizeable decay into a scalar pion pair and it is very wide so that its contribution could be large. In fact, the $N^*(1440)$ plays a major role in $\pi N \to \pi\pi N$ and $NN \to \pi\pi NN$ reactions for certain channels and close to threshold.

This work has been the first attempt to include the $N^*(1440)$ in two pion production in weak processes, the study of which could serve to study electroweak nucleon to Roper transition form factors.

Two-pion resonant production

$N^*(1440)$ contribution

In this case we have two contributions coming form direct an crossed Roper excitation

For the s-wave $N^* \to N\pi\pi$ decay we use the Lagrangian

$$\mathcal{L}_{N^*N\pi\pi} = -c_1^* \frac{m_\pi^2}{f_\pi^2} \bar{\psi}_{N^*} \vec{\phi}^2 \Psi + c_2^* \frac{1}{f_\pi^2} \bar{\psi}_{N^*} (\vec{\tau}\partial_0\vec{\phi})(\vec{\tau}\partial_0\vec{\phi})\Psi + h.c. \, .$$

Assuming a branching ratio of 7.5% for the $N(\pi\pi)^{I=0}_{J=0}$ decay mode and a total decay width of $\Gamma_{\rm tot} = 350$ MeV, the best agreement with $NN \to \pi\pi NN$ and the $\pi N \to \pi\pi N$ data was obtained by L. Alvarez-Ruso et al. [NPA633, 519 (1998)] using

$$c_1^* = -7.27 \, \mathrm{GeV}^{-1}, \; c_2^* = 0 \, \mathrm{GeV}^{-1}$$

Since in the above Lagrangian the pions are coupled to isospin 0 the Roper excitation contributes only for final $\pi^+\pi^-$ and $\pi^0\pi^0$ channels

Two-pion resonant production

$N \to N^*(1440)$ weak current

The $W^+ n \to N^{*+}(1440)$ weak transition current is given by

$$\cos\theta_C \, \bar{u}_*(\vec{p}_*) J^\alpha_{cc*} u(\vec{p})\,,$$

with

$$J^\alpha_{cc*} = \frac{F_1^{V*}(q^2)}{\mu^2}(q^\alpha \not{q} - q^2\gamma^\alpha) + i\frac{F_2^{V*}(q^2)}{\mu}\sigma^{\alpha\nu}q_\nu - G_A\gamma^\alpha\gamma_5 - \frac{G_P}{\mu}q^\alpha \not{q}\gamma_5 - \frac{G_T}{\mu}\sigma^{\alpha\nu}q_\nu\gamma_5$$

with $\mu = M + M_*$.

The vector part is the most general compatible with current conservation.

Although the G_T term is in principle allowed by G-parity invariance, we shall assume it is zero.

Two-pion resonant production

$N \to N^*(1440)$ axial form factors

Assuming the pseudoscalar coupling G_P is dominated by the pion pole contribution, and imposing partial conservation of the axial current (PCAC) hypothesis

$$G_P(q^2) = \frac{\mu}{m_\pi^2 - q^2} G_A(q^2)\,.$$

$G_A(0)$ can be related with the $N^* N\pi$ coupling constant at $q^2 = 0$ using the corresponding non-diagonal Goldberger–Treiman relation

$$G_A(0) = 2 f_\pi \frac{\tilde{f}}{m_\pi} = 0.63$$

with $\tilde{f}/m_\pi$ the $N^* N\pi$ strong coupling determined from the $\Gamma_{N^* \to N\pi}$ decay width.

For the q^2 dependence of $G_A(q^2)$ we assumed a dipole form

$$G_A(q^2) = \frac{G_A(0)}{(1 - q^2/M_{A*}^2)^2}\,,$$

with an axial mass $M_{A*} = 1$ GeV

Two-pion resonant production

$N \to N^*(1440)$ vector form factors I

The vector-isovector form factors F_1^{V*}/μ^2 and F_2^{V*}/μ can be related to the isovector part of the electromagnetic form factors, $F_i^{V*} = F_i^{p*} - F_i^{n*}$. The experimental information is given in terms of helicity amplitudes

$$A_{1/2}^N = \sqrt{\frac{2\pi\alpha}{k_R}} \langle N^* \uparrow | \sum_{\rm pol} \epsilon \cdot j_{\rm e.m.}(0) | N \downarrow \rangle \, \xi$$

$$S_{1/2}^N = \sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\vec{q}|}{\sqrt{-q^2}} \langle N^* \uparrow | \sum_{\rm pol} \epsilon \cdot j_{\rm e.m.}(0) | N \uparrow \rangle \, \xi \,,$$

$\alpha = 1/137$, $k_R = (W^2 - M^2)/2W$ with W the energy of the Roper in its center of mass and ϵ stands for the polarization vectors.

ξ is the relative sign between the $NN\pi$ and $N^*N\pi$ couplings which we have taken to be positive (I shall come back to this later).

Two-pion resonant production

$N \to N^*(1440)$ vector form factors II

Taking into account that the electromagnetic current is given by

$$\langle N^*; \vec{p}_* = \vec{p} + \vec{q} | j^\alpha_{\rm e.m.}(0) | N; \vec{p}\rangle = \bar{u}_*(\vec{p}_*) \left[\frac{F_1^{N*}(q^2)}{\mu^2} (q^\alpha \not{q} - q^2 \gamma^\alpha) + i \frac{F_2^{N*}(q^2)}{\mu} \sigma^{\alpha\nu} q_\nu \right] u(\vec{p}) \, .$$

one obtains

$$A^N_{1/2} = |\vec{q}| g(q^2) \left[\frac{F_2^{N*}}{\mu} - \frac{q^2}{W+M} \frac{F_1^{N*}}{\mu^2} \right] \, , \quad S^N_{1/2} = \frac{1}{\sqrt{2}} |\vec{q}|^2 g(q^2) \left[\frac{F_1^{N*}}{\mu^2} - \frac{F_2^{N*}}{\mu} \frac{1}{W+M} \right] ,$$

with

$$g(q^2) = \sqrt{\frac{8\pi\alpha(W+M)W^2}{M(W-M)((W+M)^2 - q^2)}} \, .$$

These relations determine the form factors in terms of the helicity amplitudes

Two-pion resonant production

$N \to N^*(1440)$ vector form factors III

Problem:

There is no information of $\gamma n \to N^{*0}$

Assumptions are needed.

Following quark models predictions [L.A. Copley et al., PLB29, 117 (1969)] we assume

$$A^n_{1/2} = -2/3 A^p_{1/2}, \;\; S^n_{1/2} = 0$$

and then

$$F_1^{V*} = \frac{F_1^{p*}((M+W)^2 - 5q^2/3) + 2/3 F_2^{p*}(M+W)\mu}{(M+W)^2 - q^2}$$

$$F_2^{V*} = \frac{F_2^{p*}(5(M+W)^2 - 3q^2)\mu - 2F_1^{p*} q^2 (M+W)}{3((M+W)^2 - q^2)\mu} \, .$$

Two-pion resonant production

$N \to N^*(1440)$ vector form factors IV

We fit the $\gamma p \to N^*$ electromagnetic form factors to experiment [I.G. Aznauryan et al., PRC71, 015201 (2005); L. Tiator et al., EPJA19, 55 (2004)] using a parametrization from O. Lalakulich et al. [PRD74, 014009 (2006)]

$$F_1^{p*}(q^2) = \frac{g_1^p/D_V}{1 - q^2/X_1 M_V^2}\,,$$
$$F_2^{p*}(q^2) = \frac{g_2^p}{D_V}\left(1 - X_2 \ln\left(1 - \frac{q^2}{1\,\mathrm{GeV}^2}\right)\right)$$

where $D_V = \left(1 - q^2/M_V^2\right)^2$ with $M_V = 0.84$ GeV.
The best fit parameters are:

$$g_1^p = -5.7 \pm 0.9\,, \quad g_2^p = -0.64 \pm 0.04\,, \quad X_1 = 1.4 \pm 0.5\,, \quad X_2 = 2.47 \pm 0.12\,,$$

This defines our FF1 set of vector form factors.

Two-pion resonant production

$N \to N^*(1440)$ vector form factors V

The $\gamma p \to N^*$ data have large uncertainties so that other fits are possible. We shall consider three more sets of vector form factors:

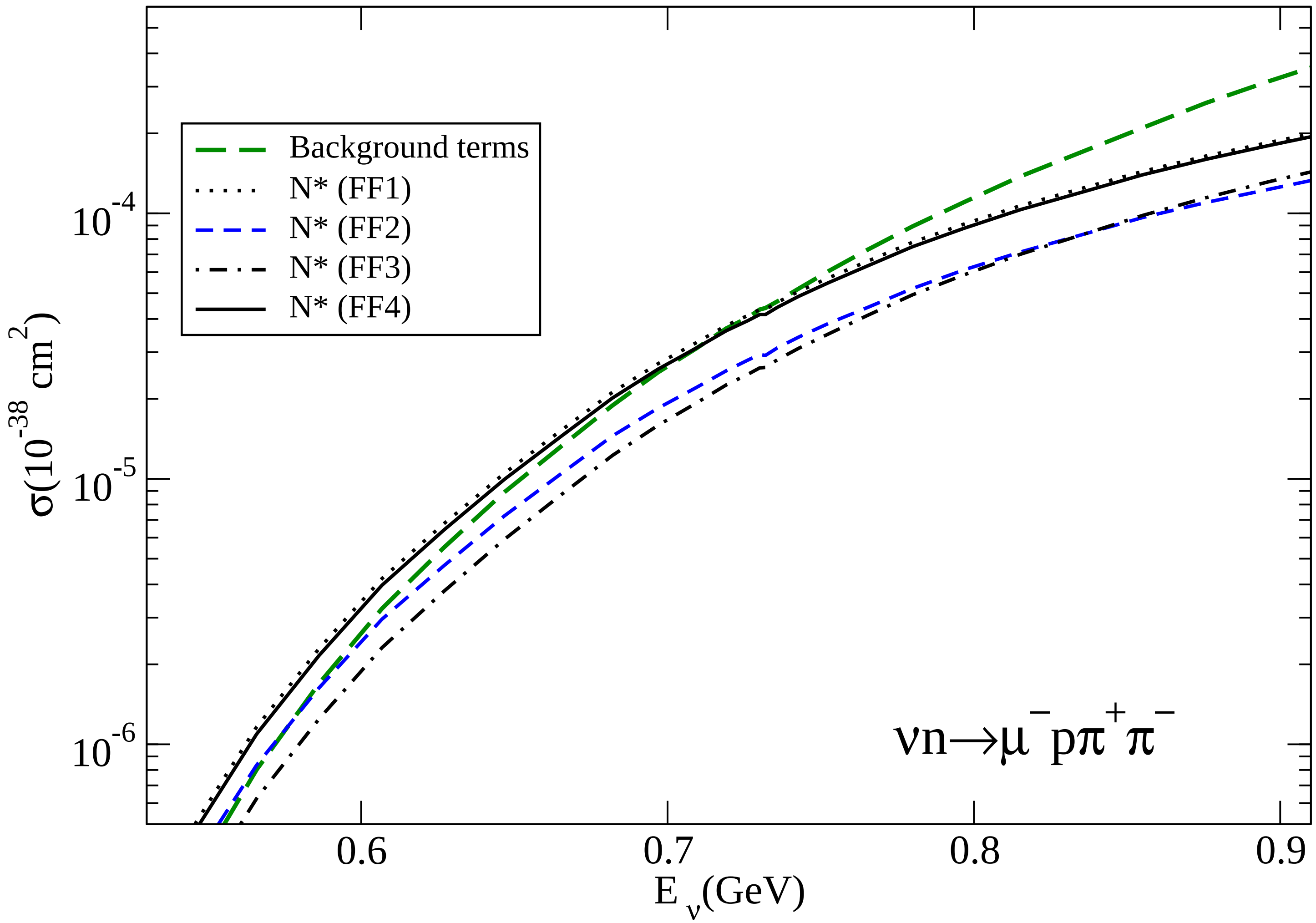
- Set FF2. Quark model calculation of U. Meyer et al., PRC 64, 035203 (2001)
- Set FF3. This we take from O. Lalakulich et al., PRD74, 014009 (2005)
- Set FF4. MAID analysis from D. Drechsel et al., EPJA34, 69 (2007)

$N \to N^*(1440)$ and background relative sign

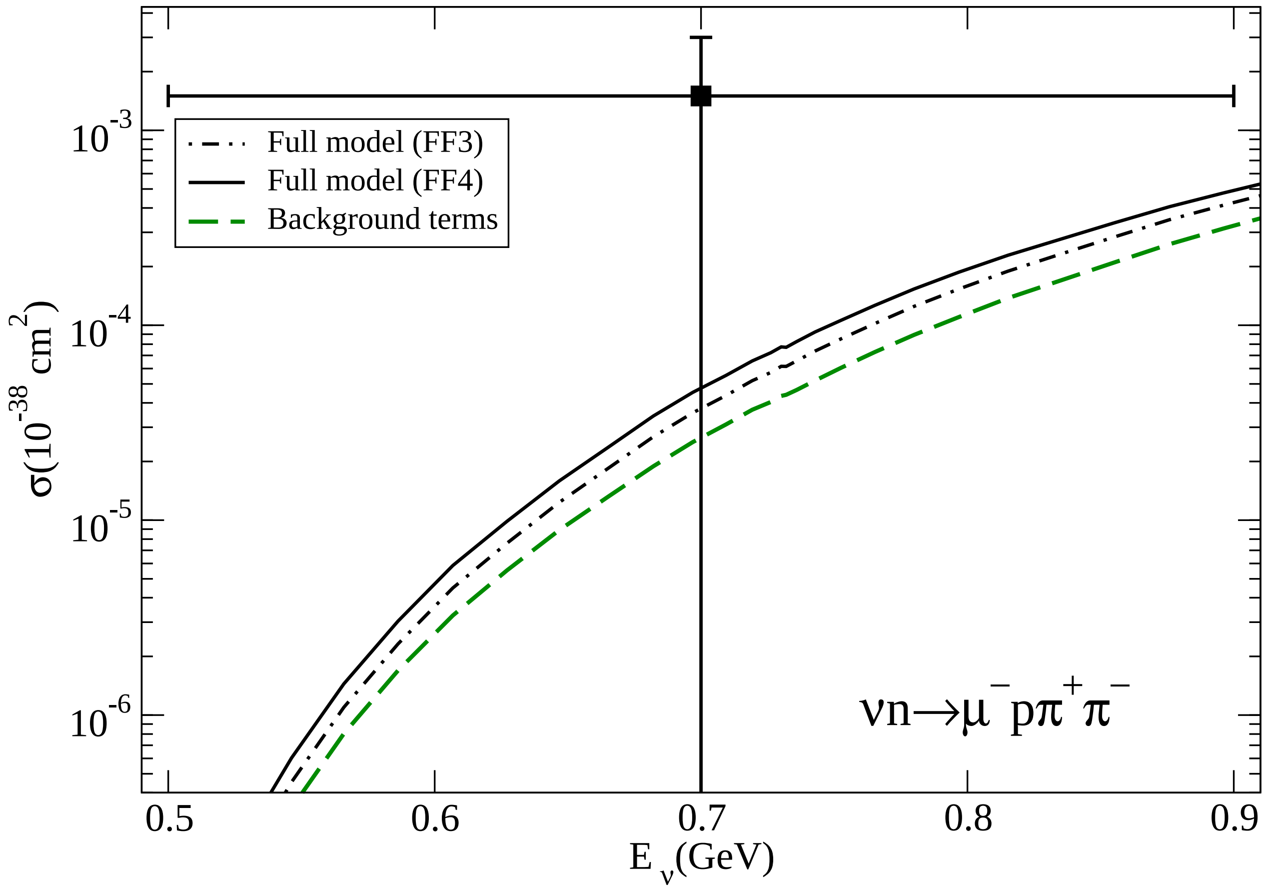
- All relative phases in the background terms are fixed by the nonlinear sigma model Lagrangian.
- We have assumed the sign of the $N^*N\pi$ coupling to be the same as the $NN\pi$ coupling. This fixes the vector part of the $N \to N^*$ weak current. The axial part is also fixed through the nondiagonal Goldberger-Treiman relation. Besides, it also fixes the sign of the c_1^*, c_2^* $N^*N\pi\pi$ coupling constants.

Taking the opposite sign for the $N^*N\pi$ coupling does not affect the results since both the $N \to N^*$ weak current and the c_1^*, c_2^* coupling would change sign leaving the Roper contribution unaltered.

Results I


Background terms
N* (FF1)
N* (FF2)
N* (FF3)
N* (FF4)
$\nu n \to \mu^- p \pi^+ \pi^-$
10^{-4}
10^{-5}
10^{-6}
$\sigma(10^{-38}\ cm^2)$
0.6
0.7
0.8
0.9
E_ν(GeV)

Results II



- BNL data by T. Kitagaki et al., PRD34, 2554 (1986)

Results III

In order to stay close to threshold, S.A. Adjei et al. [PRD23,672 (1981)] suggested to implement the following phase space kinematical cuts

(1) $$q_\pi^2 \leq ((1+\eta/2)m_\pi)^2 \; ,$$

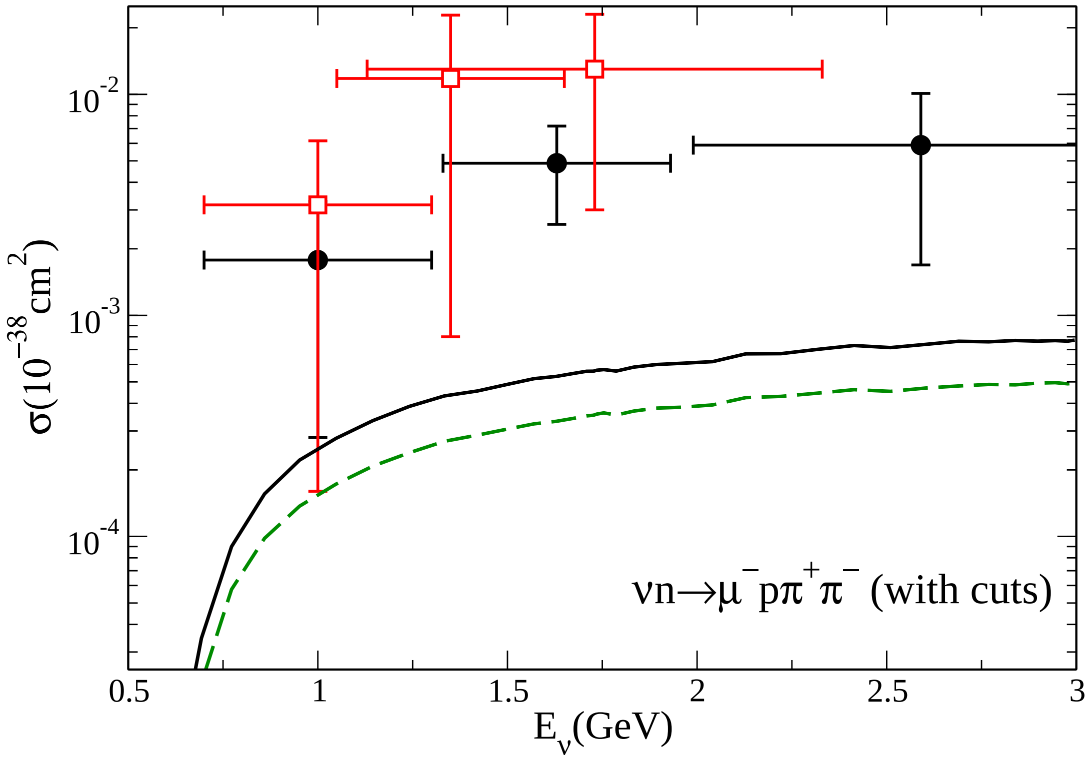
(2) $$2p \cdot q_\pi \leq (M + (1+\eta)m_\pi)^2 - M^2 - m_\pi^2$$

(3) $$2p' \cdot q_\pi \leq (M + (1+\eta)m_\pi)^2 - M^2 - m_\pi^2 \, ,$$

with $q_\pi = (k_{\pi_1} + k_{\pi_2})/2$. $\eta = 1/4$, $2/4$, $3/4$ were proposed.

(1)$\rightarrow$ keeps individual pion momenta close to average pion momentum.
(2) and (3)$\rightarrow$ pole terms should dominate.

Results IV



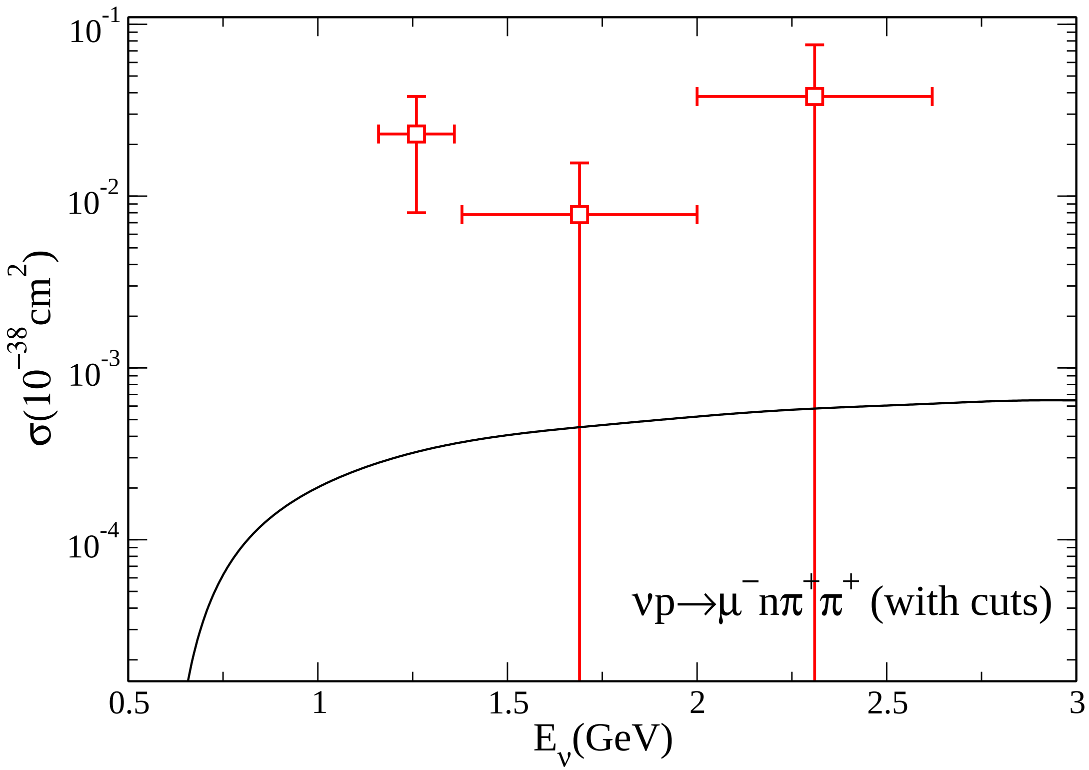
$\eta = 3/4$

Dashed line: Background terms alone

Solid line: Full calculation with FF1 set

- ● BNL data by T. Kitagaki et al., PRD34, 2554 (1986)
- □ ANL data by D. Day et al., PRD28, 2714 (1983)

Results V



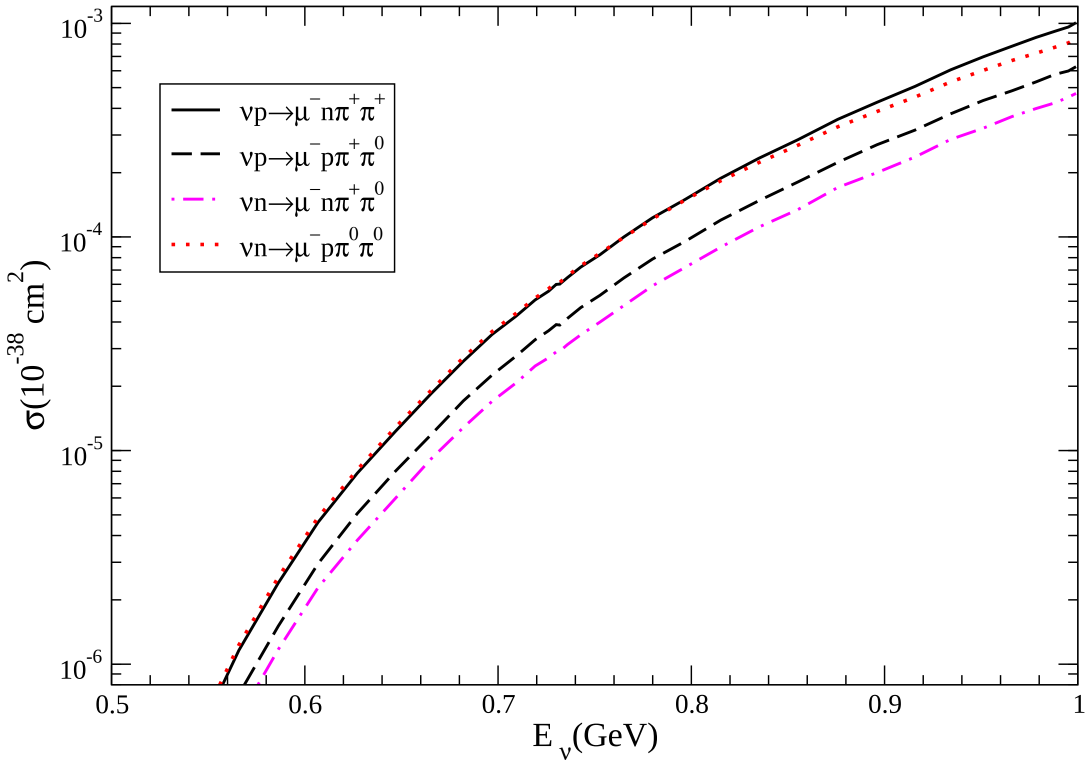
$\eta = 3/4$

Solid line: Background terms alone

□ ANL data by D. Day et al., PRD28, 2714 (1983).

In this channel the Roper excitation does not contribute.

Results VI



$\pi^0\pi^0$: Full model with FF1 set.

Other channels: Background contribution.

Conclusions

- The Roper contribution is important up to neutrino energies $E_\nu \sim 700$ MeV.
- Its contribution is sensitive to the form factors used for the $N \to N^*$ transition.
- Theoretical predictions underestimate the experimental data available so far.
- New data with better statistics would be needed.