Exotic Defect Dynamics in an Axion Cosmology

Yu Komiya | YITP, Kyoto University

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Work Supervised by Assoc. Prof. Fumihiro Takayama

Center for Gravitational Physics and Quantum Information
Yukawa Institute for Theoretical Physics, Kyoto University

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

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Contents

Symmetry Breaking in an Inflationary Universe

Inflation[1] explains observations including flatness, monopole abundances, and the CMB

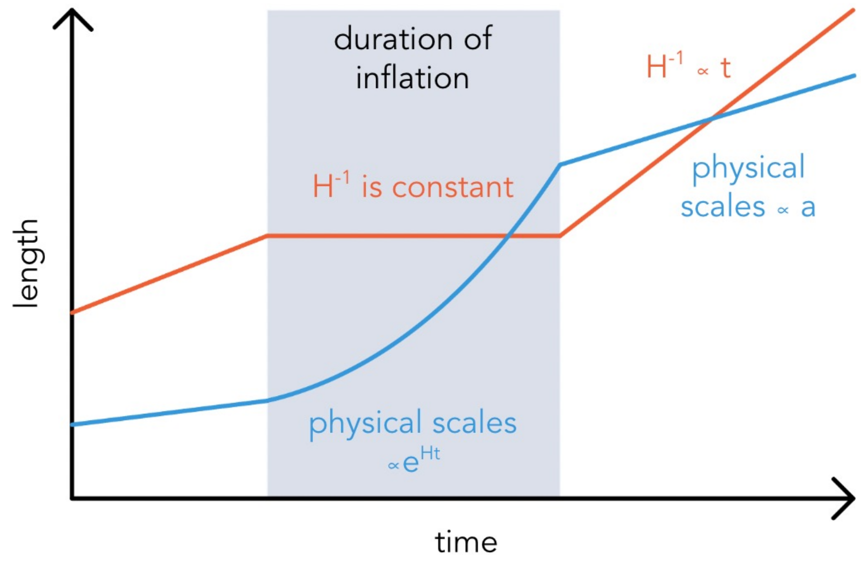
Monopole problem – produced as GUT symmetry broken at early times, overpopulation in absence of effective dilution mechanism

Often predicted by inflationary models:

Symmetry breaking

Degeneracy of vacua

Phase transitions



[1] A. Guth, 1980

The Peccei-Quinn Mechanism and Axion Cosmology

Peccei-Quinn mechanism[1]: an elegant solution to the strong CP problem

- ϑ is dynamical
- Axion acquires mass from QCD anomaly
- QCD axions are pseudo-Nambu-Goldstone bosons
- ϑ stabilised at CP conserving minimum in axion potential
- Non-trivial, physically relevant EM coupling

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial_\mu \varphi - \frac{\lambda}{4}(\varphi^2 - \eta^2)^2$$

$$\varphi = e^{i\vartheta} = \eta e^{ia/\eta}$$

$$\mathcal{L} = \frac{1}{2}\eta^2 \partial_\mu \vartheta \partial_\mu \vartheta - \frac{m_a^2 f_a^2}{N_{DW}}[1 - \cos(N_{DW}\vartheta)]$$

$$f_a \equiv \eta / N_{DW}$$

$$\mathcal{L}_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha N_{DW}\vartheta}{8\pi}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

[1] R. D. Peccei and H. R. Quinn, 1977

The Peccei-Quinn Mechanism and Axion Cosmology

We will proceed with the following choices of parameters

- $N_{DW} = 1$[1]
- PQ symmetry is broken during inflation[2]
- m_a, f_a obey astrophysical bounds[3]:

$$10^{-6} eV < m_a < 0.1 eV$$
$$10^{8} GeV < f_a < 10^{12} GeV$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi - \frac{\lambda}{4}(\varphi^2 - \eta^2)^2$$

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[1] G. Lazarides and Q. Shafi,1982
[2] S. Ge et. al., 2024
[3] M. Redi and A. Tesi, 2023; S. Ge, 2020

Defect Formation

Four classes:
1. Domain walls
2. Cosmic strings
3. Monopoles
4. Textures

Naturally produced in a large number of theories, largely due to

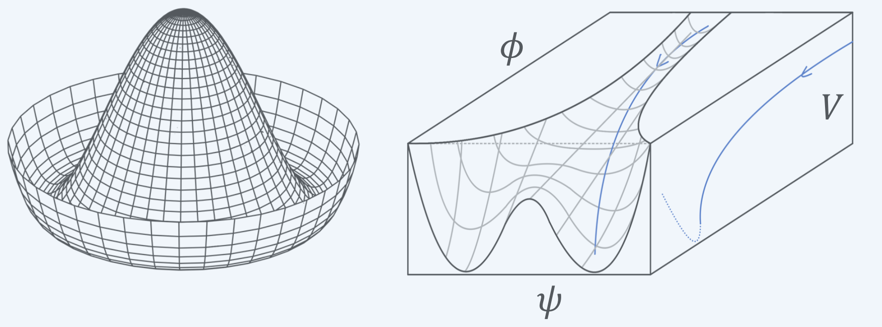
Phase transitions

Symmetry breaking

Degeneracies are common to many theories

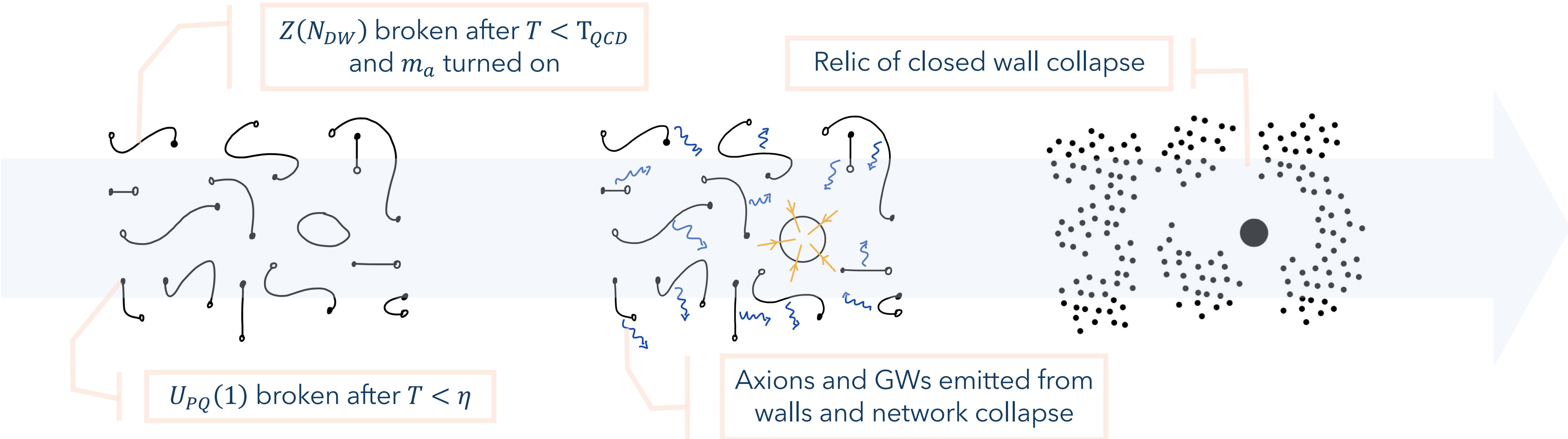
↳ different vacua separated by defects

↳ energies are usually equal



Defect Formation

String-domain wall networks are naturally formed from axion cosmology



Standard Scenario: PBH Formation

Closed domain walls are found to collapse to PBHs (or wormholes[1]) upon horizon re-entry

Assuming for simplicity:

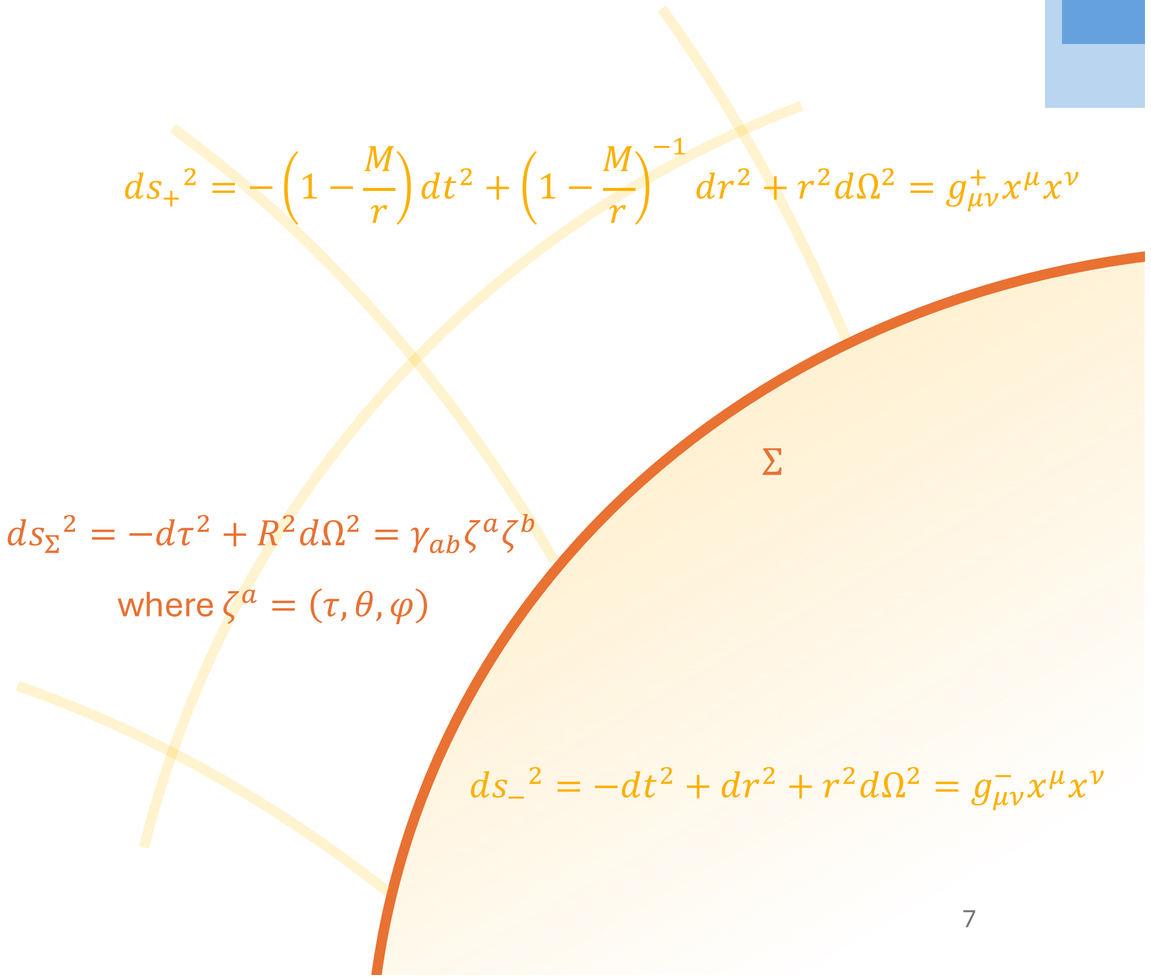
- Spherical symmetry
- Thin wall in comparison to radius

Often assumed to form PBH if[2]

$$2GM \gtrsim r \sim m_a^{-1}$$

Gravitational waves and axion emission upon collapse

(GW emission also due to $\mathcal{O}(\rho) \sim \mathcal{O}(p)$)



[1] Y. Gouttenoire and E. Vitagliano, 2023
[2] S. Ge et. al., 2023

Standard Scenario: PBH Formation

PBHs proposed to be dark matter candidates

Abundance estimated and constrained by

- Axion abundance
- Evaporation
- Observational data

↳ other possibilities allowing for larger/difference viable parameter space?

↳ other final states other than black holes?

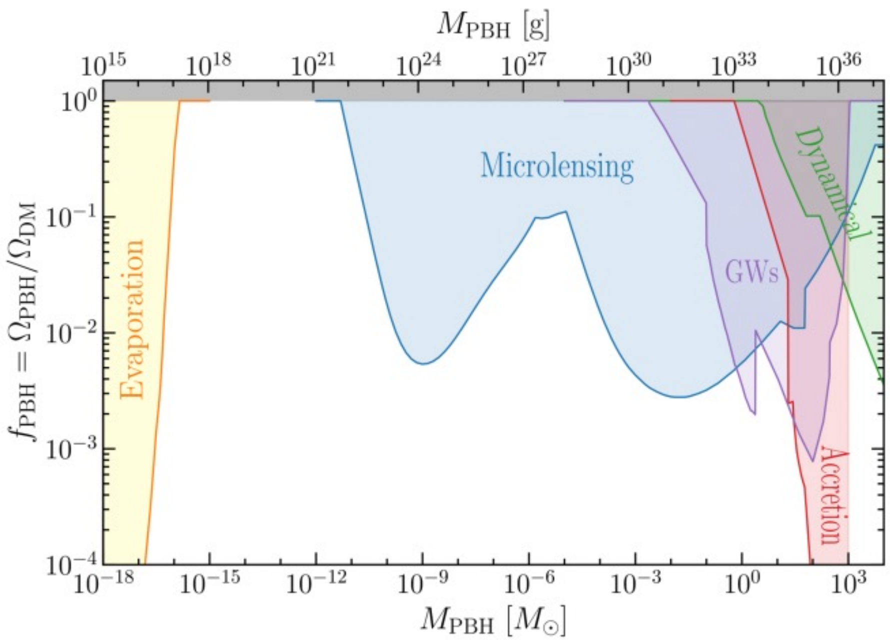


Figure from P. C. W. Davies et. al., 2025

The Monopole-Domain Wall System

Monopoles and axion fields interactions unique due to anomalous $\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}$ term in the theory's Lagrangian

- Formed in gauge symmetry breaking, contains fixed mass and stable core region ('t Hooft-Polyakov monopoles)
- Monopoles have dyonic (electrically charged) excitations
- Charge is dependent on value of ϑ (Witten effect[1])
- Charge conservation ⇒ induced charge on domain wall

Dirac quantisation condition dictating electric and magnetic charge on a particle[2]:

$$qg = 2\pi N, \qquad N \in \mathbb{Z}$$

$$q = ne - e\frac{\vartheta}{2\pi}$$

[1] E. Witten, 1979
[2] J. Schwinger, 1966

The Monopole-Domain Wall System

*This hybrid defect is proposed to have a stable final state–**the monopole bag**[1]*

⇒ domain wall must collapse without forming a PBH

Suppose $\vartheta(r \to 0) = 2\pi, \vartheta(r \to \infty) = 0$:

$$B_r = \frac{1}{er^2}$$

$$E_r = -\frac{e^2}{8\pi^2}\vartheta(r)B_r(r) + \frac{e}{r^2} = \frac{e}{4\pi r^2}\left(1 - \frac{\vartheta(r)}{2\pi}\right)$$

$\boldsymbol{E} \cdot \boldsymbol{B}$ term implies an effective potential:

$$V(\vartheta) = m_a^2 f_a^2 (1 - \cos\vartheta) + \frac{e^2}{32\pi^2 r^4}\left(\frac{\vartheta}{2\pi} - 1\right)^2$$

When $m_a^2 f_a^2 \ll \frac{e^2}{32\pi^2 r^4}$: $\quad \vartheta(r) = 2\pi\left[1 - \exp\left(\frac{R_b}{r}\right)\right]$

$$R_b = \frac{e^2}{8\pi^2 f_a}$$

[1] I. Kogan, 1992

The Monopole-Domain Wall System

Notable features of the monopole bag

- For physical minimal case with $N_{DW} = 1$, complete screening of dyon charge is possible
- Potential dark matter component
- Constraints include
  - Astrophysical bounds (Parker bound[1])
  - (Non-)Detection
  - Primordial fields
  - Clustering behaviour

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[1] M. S. Turner et. al., 1982

Regular Magnetic Black Holes

Black holes with non-singular centres as an alternative final state of collapse

Monopole naturally provides a regular core region with de Sitter geometry[1]

At critical mass $M_{crit} = q_m$, exterior collapses to a black hole geometry with

- one degenerate Killing horizon if $M = M_{crit}$
- two Killing horizons at $r_\pm$ if $M > M_{crit}$

Domain wall and exterior:

$$ds_{RN}^2 = -\left(1 - \frac{r_s}{r} + \frac{q_m^2}{r^2}\right) dt^2 + \left(1 - \frac{r_s}{r} + \frac{q_m^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Monopole core:

$$ds_{dS}^2 = -(1 - H^2 r^2) dt^2 + (1 - H^2 r^2)^{-1} dr^2 + r^2 d\Omega^2$$

$$H = \frac{8\pi G}{3} \frac{\lambda \eta^4}{4}$$

[1] D. Harari and C. Loustó, 1990

Regular Magnetic Black Holes

Dynamics are non-standard with observationally relevant consequences

Evaporation described by Vaidya-like spacetime[1]:

$$ds_V^2 = -\left(1 - \frac{r_s}{r} + \frac{q_m^2}{r^2}\right)du^2 - 2dudr + r^2 d\Omega^2$$

where u is retarded time

Extremal magnetic black holes can be stable relics

Non-singular relics as dark matter

[1] P. C. Vaidya, 1951

Domain wall and exterior:

$$ds_{RN}^2 = -\left(1 - \frac{r_s}{r} + \frac{q_m^2}{r^2}\right)dt^2 + \left(1 - \frac{r_s}{r} + \frac{q_m^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Monopole core:

$$ds_{ds}^2 = -(1 - H^2r^2)dt^2 + (1 - H^2r^2)^{-1}dr^2 + r^2 d\Omega^2$$

$$H = \frac{8\pi G}{3}\frac{\lambda\eta^4}{4}$$

Conclusions and Outlook

We considered a specific physical scenario in which axion domain walls are produced in an early-time inflationary cosmology

Monopole problem continues to plague a host early universe models[1], with existing solutions including dissociation on domain walls[2] and additional symmetry breaking phases[3]

We propose alternative final states to a well-studied class of axion models:
monopole bags and regular magnetic black holes

Opens additional parameters space for considering dark matter from primordial processes

Close ties to other studies of regular solutions such as gravastars, and to dynamics of other early universe objects such as vacuum bubbles formed in phase transitions

Updates to bounds and constraints for application to cases such as ours necessary

[1] S. Kasuya and M. Kawasaki, 1998; A. Rajantie, 2004 | [2] L. Pogosian and T. Vachaspati, 2000
[3] P. Langacker and S.-Y. Pi, 1980; K.-I. Hikasa,1982

Appendices

1. Standard Domain Wall Collapse
2. More Details on Monopole Bags
3. Regular Black Hole Formation and Conditions

Standard Domain Wall Collapse ①

Condition $2GM \gtrsim r \sim m_a^{-1}$ derived from point when radiation from bubble/defect becoming significant[1]

There are caveat and assumptions:

- Sphericity maintained
- No radiation backreaction
- No angular momentum or other unexpected degrees of freedom

Bounds for when BH very unlikely to be formed, but not for when BHs definitely formed

Repulsive gravitational potential property shared by domain walls and monopoles
$\Downarrow$
Formation of regular BHs from standard domain wall collapse?

[1] S. Ge et. al., 2019; T. Vachaspati, 2018; J. Fort and T. Vachaspati, 1993

More Details on Monopole Bags

2

Given that $B_r = \frac{1}{er^2}$, $E_r = -\frac{e^2}{8\pi^2}\vartheta(r)B_r(r) + \frac{e}{r^2} = \frac{e}{4\pi r^2}(1 - \frac{\vartheta(r)}{2\pi})$

The equation of motion of ϑ is

$$0 = f_a^2\ddot{\vartheta} - f_a^2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\vartheta}{\partial r}\right) + m_a^2 f_a^2 \sin\vartheta - \frac{e^2}{8\pi^2}\boldsymbol{E}\cdot\boldsymbol{B} = f_a^2\ddot{\vartheta} - f_a^2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\vartheta}{\partial r}\right) + m_a^2 f_a^2 \sin\vartheta - \frac{e^2}{32\pi^3 r^4}\left(1 - \frac{\vartheta}{2\pi}\right)$$

The energy for the stationary axion configuration can be derived from the classical effective action and found to be

$$E = \int dr r^2\left[\frac{1}{2}(\boldsymbol{\nabla}\vartheta)^2 + \underbrace{m_a^2 f_a^2(1-\cos\vartheta) + \frac{e^2}{32\pi^2 r^4}\left(1 - \frac{\vartheta}{2\pi}\right)^2}_{V(\vartheta)}\right]$$

The effective potential $V(\vartheta)$ implies a critical radius a which the term $m_a^2 f_a^2(1-\cos\vartheta)$ is negligible.
Solving the equation of motion gives the field profile

$$\vartheta(r) = 2\pi\left[1 - \exp\left(\frac{R_b}{r}\right)\right]$$

with stable monopole bag radius $R_b = \frac{e^2}{8\pi^2 f_a}$

Calculation from "Axions, Monopoles and Cosmic Strings", I. Kogan, 1993

Regular BH Formation & Conditions

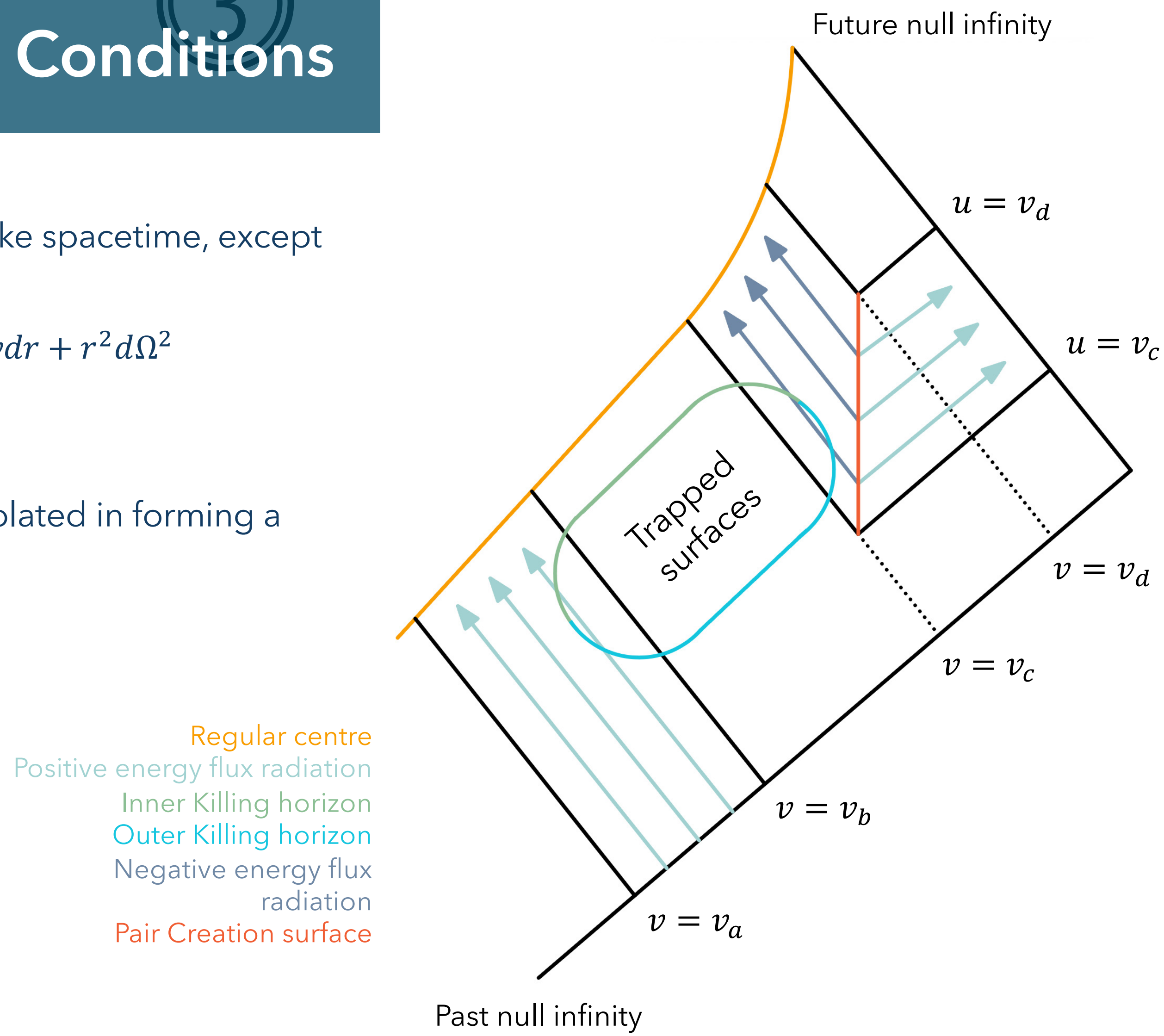
Ingoing radiation also modelled by a Vaidya-like spacetime, except with[1]

$$ds_V^2 = -\left(1 - \frac{r_s}{r} + \frac{q_m^2}{r^2}\right)dv^2 - 2dvdr + r^2 d\Omega^2$$

in which v is the advanced time $v = t + \int \frac{dr}{F(r)}$

Some traditional energy conditions may be violated in forming a regular space time:

WEC	$\rho \geq 0, \rho + p_\perp \geq 0$
SEC	$\rho + \mathrm{p}_\parallel + 2p_\perp \geq 0$
NEC	$\rho + p_\perp \geq 0, \rho + p_\parallel \geq 0$
DEC	$\rho \geq 0, p_{\parallel,\perp} \in [-\rho, \rho]$



[1] S. A. Hayward, 2005