

# Stringent Constraints on Self-Interacting Dark Matter Using Milky-Way Satellite Galaxies Kinematics

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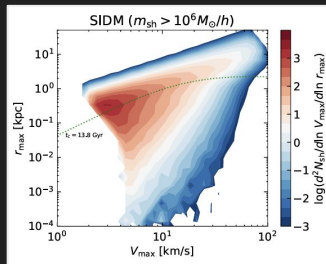
# Stringent Constraints on Self-Interacting Dark Matter Using Milky-Way Satellite Galaxies Kinematics

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Kinematics of dSphs

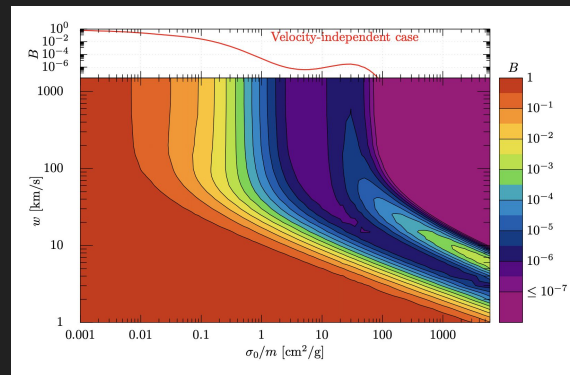
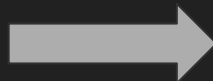
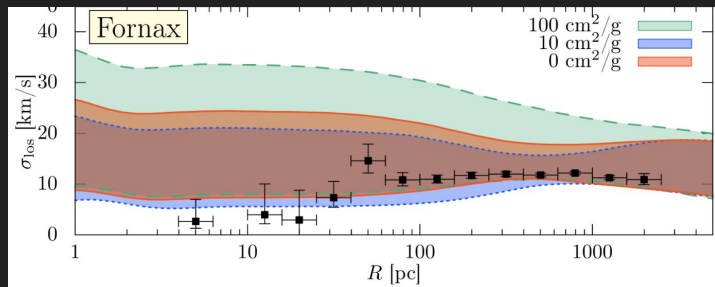


SIDM cosmology



- We verify SIDM models by comparing
  - theory (SIDM cosmology)
  - observation (DM halos in dSphs)
- We obtain constraints on SIDM parameters
  - e.g. In velocity independent case:

$$\sigma/m \lesssim 0.2 \text{ cm}^2/\text{g}$$



# Introduction

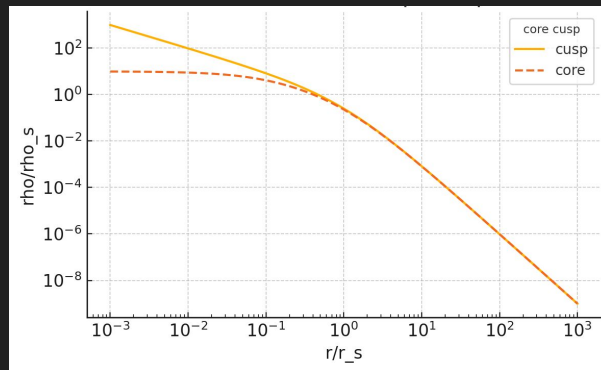
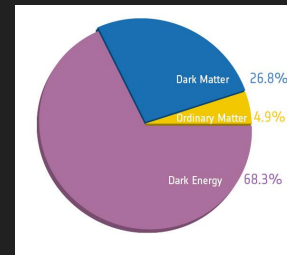
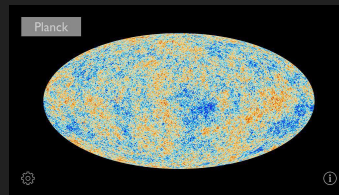
# Dark matter in our universe

$\Lambda$ CDM explains the **LSS** of the universe

How about the **small** scale?

- Core vs. cusp
- Missing satellite
- Too-big-to-fail
- ...

→ Any other solutions than CDM?

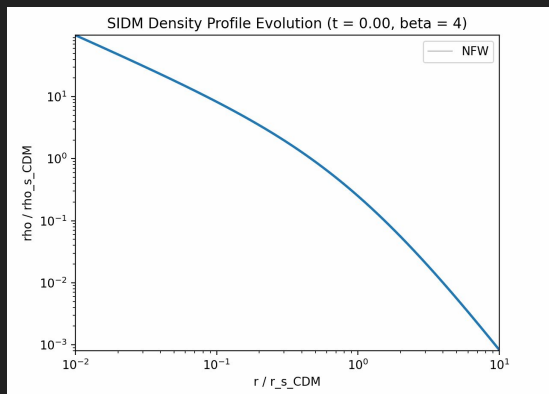


# Self-Interacting Dark Matter (SIDM)

Large self-scattering cross section

$$\sigma/m \simeq 1 \text{ cm}^2/\text{g}$$

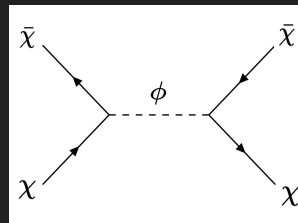
→ Gravothermal evolution of halos



c.f. CDM (WIMP)

$$\sigma/m \lesssim 10^{-14} \text{ cm}^2/\text{g}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\sigma_0}{2[1 + (v/w)^2 \sin^2(\theta/2)]^2}$$



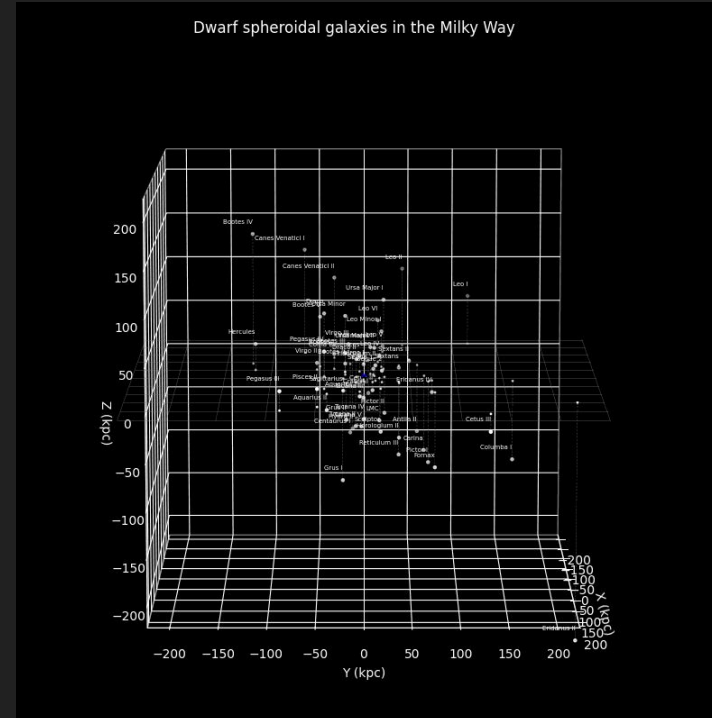
# Dwarf spheroidal galaxies (dSphs)

Large M/L ratio:  $Y=10\text{-}100$

→ dark matter dominant objects

Important targets to investigate small scale issues of dark matter (core/cusp)

Q. Are SIDM predictions consistent with the observed dSph profiles?



# Method

# Bayesian analysis

$$\mathcal{L}_{\text{eff}}(\overbrace{r_s, \rho_s, r_c}^{\text{DM halo parameters}}) = \int d[\overbrace{r_\beta, \eta, \beta_{0,\infty}, r_{1/2}}^{\text{nuisance parameters}}] \mathcal{L}$$

$$\underbrace{Z(\sigma_0/m, w)}_{\text{SIDM paramters}} = \int d\rho_s dr_s dr_c \mathcal{L}_{\text{eff}}(r_s, \rho_s, r_c) P(\rho_s, r_s, r_c | \sigma_0/m, w)$$

- $L$ : Likelihood from dSph observation
- $P$ : Prior from SIDM Cosmology
- $Z(\sigma/m, w)$ : marginal likelihood (model *evidence*)
- $B(\sigma/m, w) = Z(\sigma/m, w)/Z(0, 0)$ : Bayes factor



# Likelihood: Jeans analysis

Jeans equation:

$$\frac{\partial \nu(r) \sigma_r^2(r)}{\partial r} + \frac{2\beta_{\text{ani}}(r) \sigma_r^2(r)}{r} = -\nu(r) \frac{\partial \Phi}{\partial r}$$

$\nu(r)$ : Number density (Plummer model)

$\beta(r)$ : Anisotropy (general model by Baes & van Hese (2007))

$\sigma(r)$ : Velocity dispersion

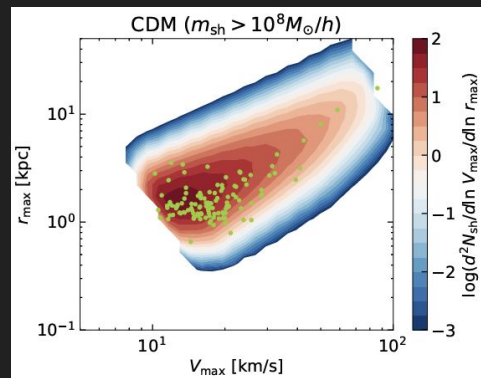
$\Delta\Phi(r) = 4\pi G \rho(r)$ :

$$\rho(r) = \frac{\rho_s r_s^3}{(r^4 + r_c^4)^{1/4} (r + r_s)^2} \begin{cases} \xrightarrow{r \gg r_c} \frac{\rho_s r_s^3}{r(r + r_s)^2} & \text{NFW} \\ \xrightarrow{r \ll r_c} \rho_s (r_s/r_c)^3 & \text{const. (cored)} \end{cases}$$

# Prior: Cosmological constraint of SASHIMI-SIDM

**SASHIMI (Semi-Analytical SubHalo Inference Modelling)** [arXiv:1803.07691], [arXiv:1903.11427]

- calculate various subhalo properties efficiently using semi-analytical models
  - Extended Press-Schechter formalism
  - Tidal stripping effect
- Outputs: catalogue of subhalos
  - accretion redshift
  - mass
  - $V_{\max}$ ,  $r_{\max}$
  - ...

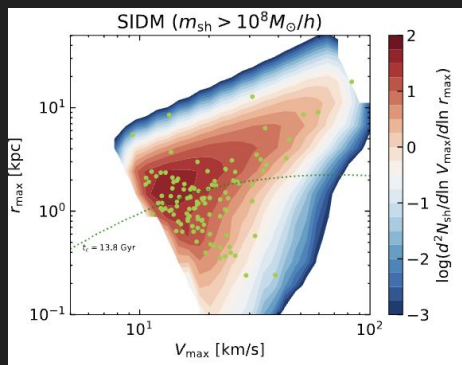


For CDM: [arXiv:2403.16633]

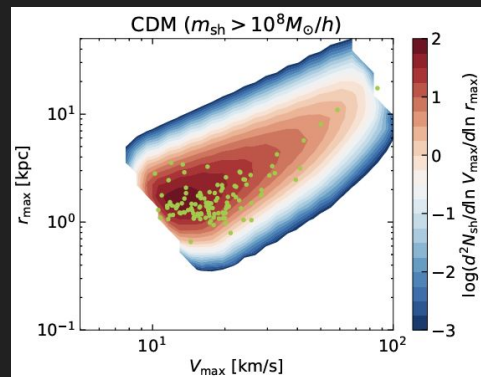
# Prior: Cosmological constraint of SASHIMI-SIDM

**SASHIMI (Semi-Analytical SubHalo Inference Modelling) for SIDM** [arXiv:2403.16633]

- Parametric model of SIDM halos [Yang+(2023)] to reinterpret CDM  $\rightarrow$  SIDM



For **SIDM**: [arXiv:2403.16633]



For **CDM**: [arXiv:2403.16633]

# Prior: Cosmological constraint of SASHIMI-SIDM

## SASHIMI (Semi-Analytical SubHalo Inference Modeling) for SIDM [arXiv:2403.16633]

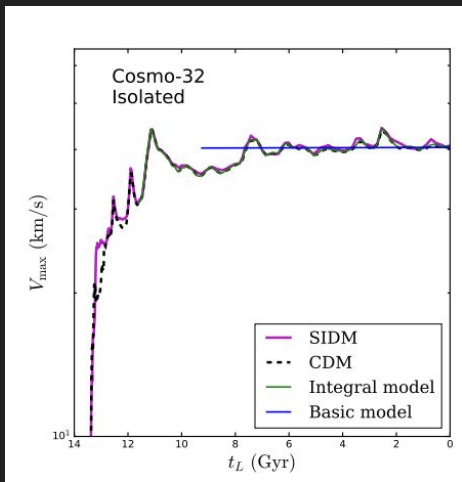
- Parametric model of SIDM halos [Yang+(2023)] to reinterpret CDM  $\rightarrow$  SIDM
  - Empirical function of  $\tau = t/t_c$ 
    - $t_c$ : collapse time scale:

$$t_c = \frac{150}{C} \frac{1}{(\sigma_{\text{eff}}/m)\rho_{\text{eff}}r_{\text{eff}}} \frac{1}{\sqrt{4\pi G\rho_{\text{eff}}}},$$

SIDM

$\frac{V_{\text{max}}}{V_{\text{max},0}} = 1 + 0.1777\tau - 4.399\tau^3 + 16.66\tau^4 - 18.87\tau^5 + 9.077\tau^7 - 2.436\tau^9$

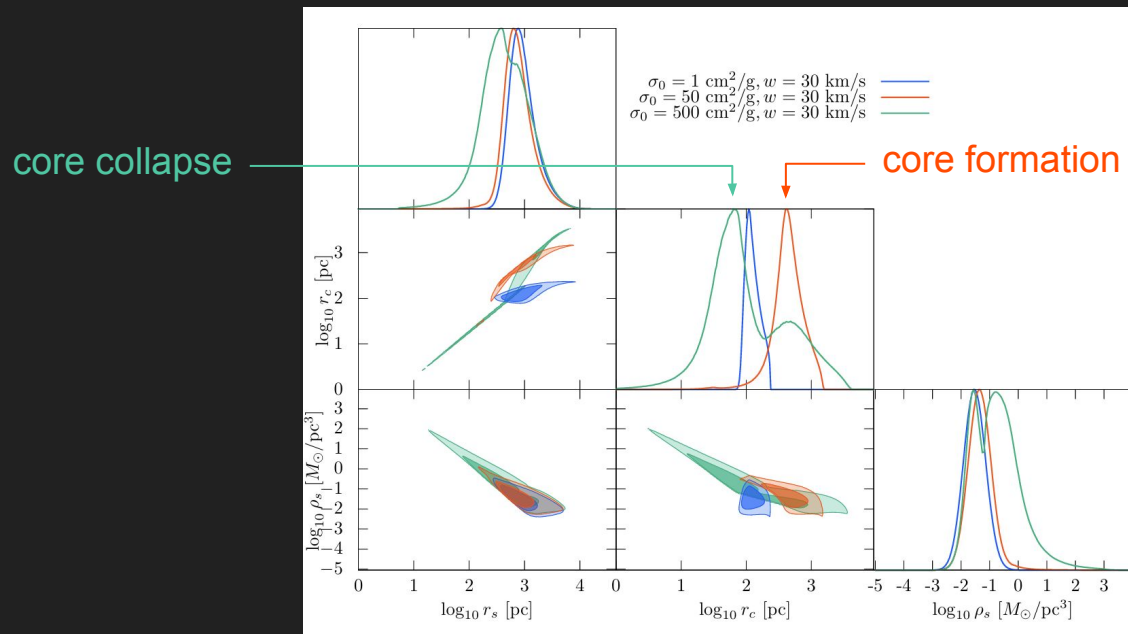
CDM



Yang+(2023)

# Prior: Cosmological constraint of SASHIMI-SIDM

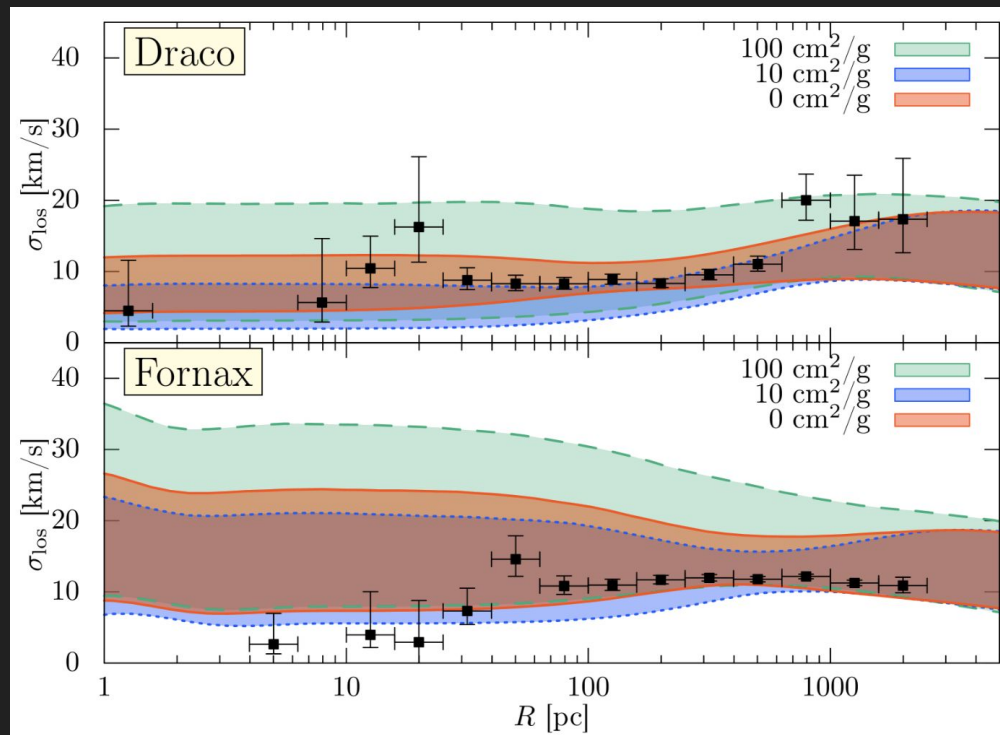
Example: Prior distributions for different SIDM parameters

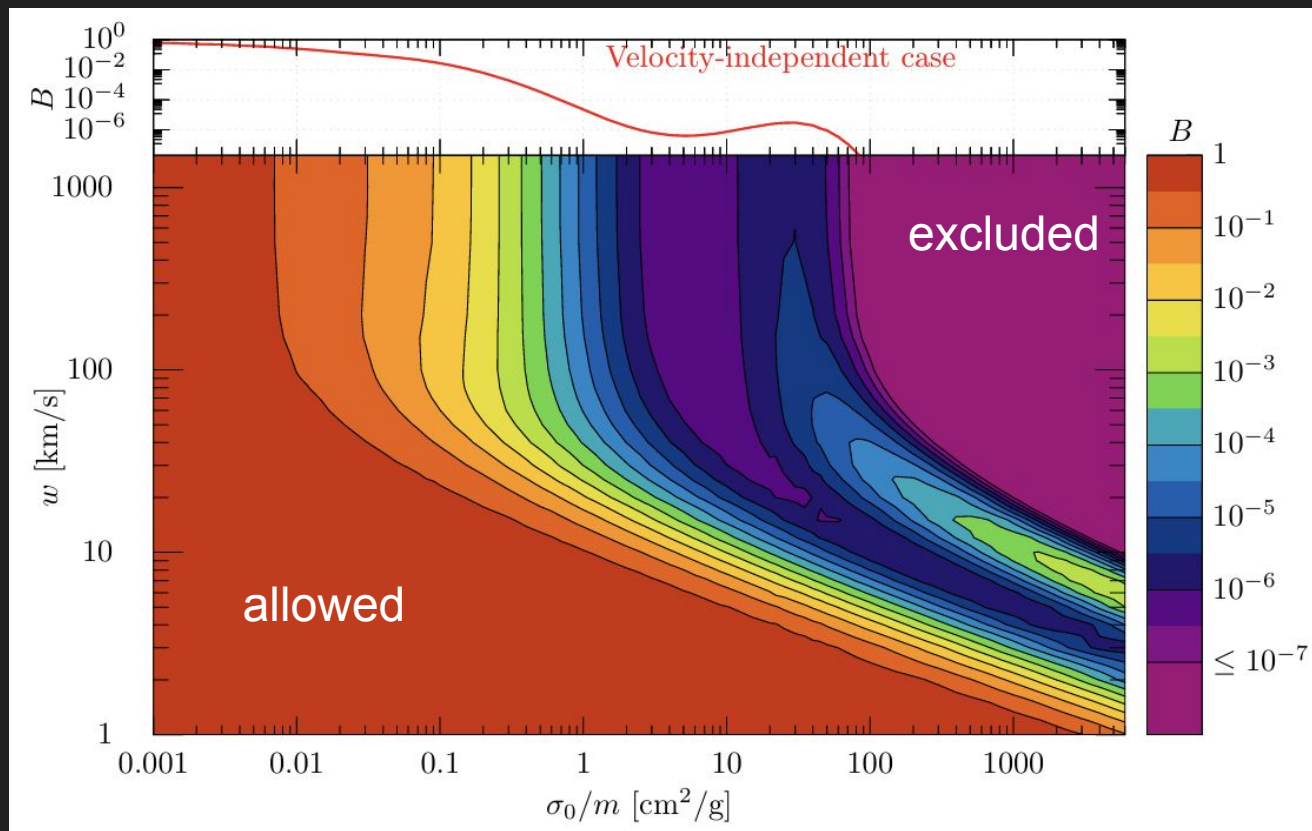


# Results

# Results

- Too large  $\sigma/m$  cannot fit observed velocity dispersion of stars
- To marginalize the diversity of dSphs, we compute total Bayes factor  $B = \prod_i B_i$

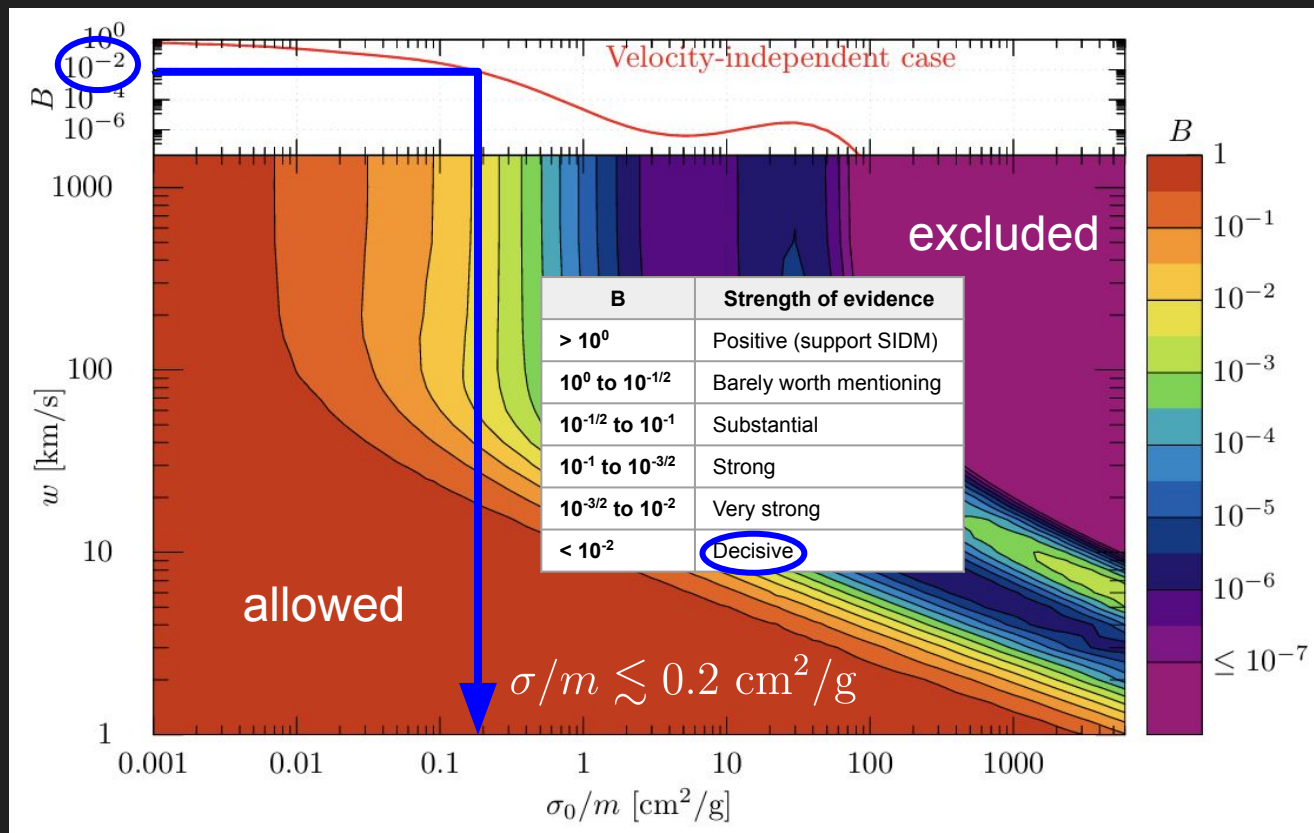




$$B = \prod_i B_i$$

$$B_i = \frac{Z_{\text{SIDM},i}}{Z_{\text{CDM},i}}$$





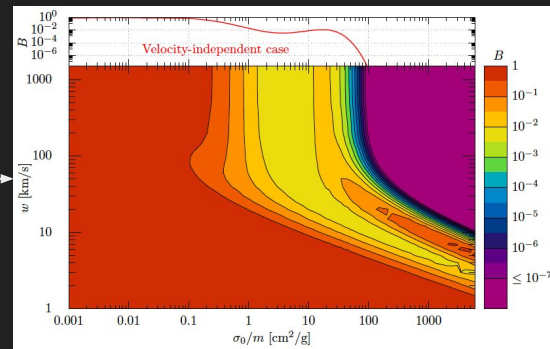
$$B = \prod_i B_i$$

$$B_i = \frac{Z_{\text{SIDM},i}}{Z_{\text{CDM},i}}$$

# Discussion & Future direction

- Baryonic feedback
  - Classical dSphs might be affected
- Galaxy formation model
  - Uncertainty: DM halo  $\rightarrow$  Galaxy
- Spatial SASHIMI
  - Simulate precise mass-loss history of dSphs by tracing their orbital information
- Axisymmetric SASHIMI
  - Considering axisymmetry & tidal disruption

(e.g.  $V_{50} = 18$  km/s)



# Summary

- DSphs are promising targets to verify the small scale issues of DM
- We investigate if SIDM can explain DM profiles of dSphs by considering
  - **Kinematics of dSph:** (spherical) Jeans equation
  - **Cosmology:** SASHIMI-SIDM (semianalytical SIDM cosmology)
    - including **gravothermal core-collapse effect**
- We found that
  - SIDM with large self-interaction cannot explain the observed dSph profiles
    - Velocity independent case:  $\sigma/m \lesssim 0.2 \text{ g/cm}^2$
- Future works
  - Better understanding of subhalo & satellite galaxy

# Backup

# Likelihood: Jeans analysis

$$-2 \log(\mathcal{L}) = \sum_i \left[ \frac{(v_i - V)^2}{\sigma_i^2} + \log(2\pi\sigma_i^2) \right]$$

$\nu(r)$ : number density  
 $\sigma(r)$ : velocity dispersion  
 $\beta(r)$ : anisotropy

Jeans equation:

$$\frac{\partial \nu(r) \sigma_r^2(r)}{\partial r} + \frac{2\beta_{\text{ani}}(r) \sigma_r^2(r)}{r} = -\nu(r) \frac{\partial \Phi}{\partial r}$$

Projection:

$$\sigma_{\text{los}}^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty dr \left( 1 - \beta_{\text{ani}}(r) \frac{R^2}{r^2} \right) \frac{\nu(r) \sigma_r^2(r)}{\sqrt{1 - R^2/r^2}}$$

# Likelihood: Jeans analysis

## Models

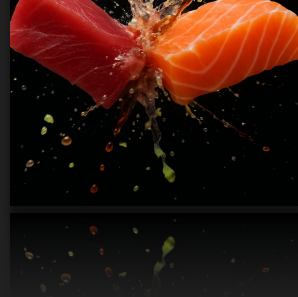
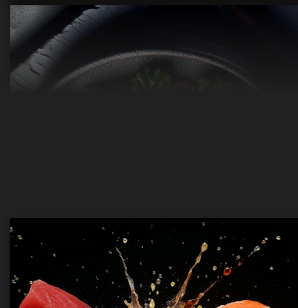
- $v(r)$ : Plummer profile
- $\beta(r)$ : Baes & van Hese (2007)
- SIDM profile:

$$\rho(r) = \frac{\rho_s r_s^3}{(r^4 + r_c^4)^{1/4} (r + r_s)^2}$$

Asymptotic behavior of the SIDM profile:

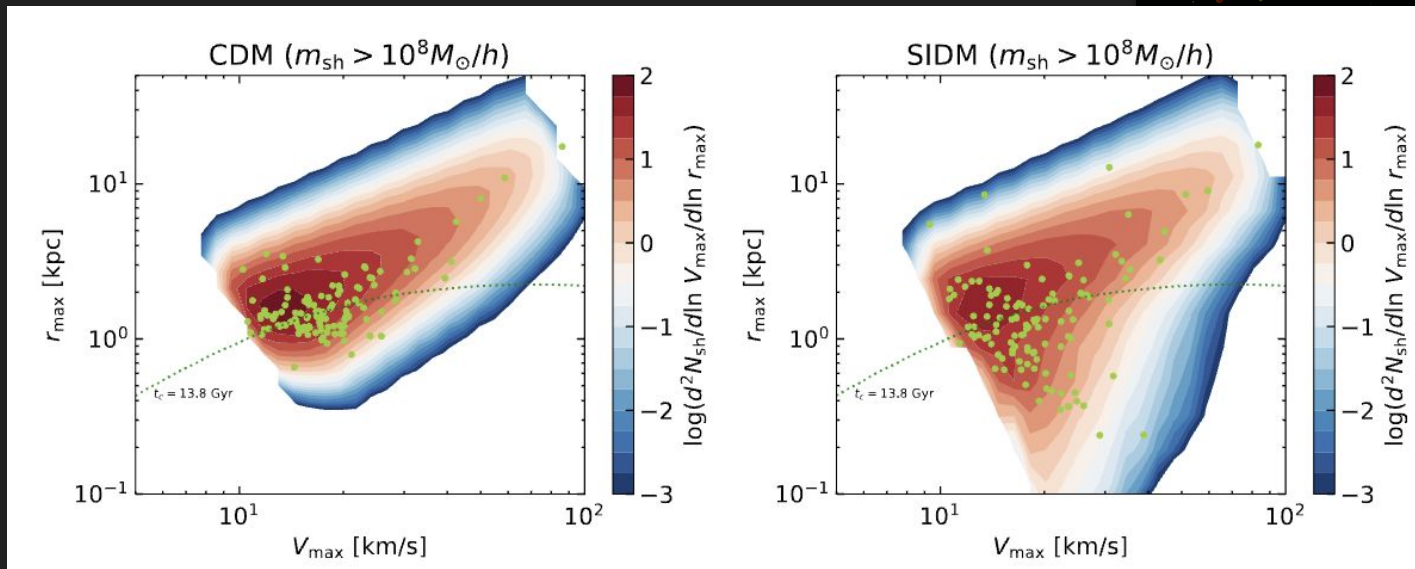
- For  $r \gg r_c$ :  $\rho(r) \rightarrow \frac{\rho_s r_s^3}{r(r + r_s)^2}$  (NFW)
- For  $r \ll r_c$ :  $\rho(r) \rightarrow \rho_s (r_s/r_c)^3$  (const. (cored))

# Various SASHIMI



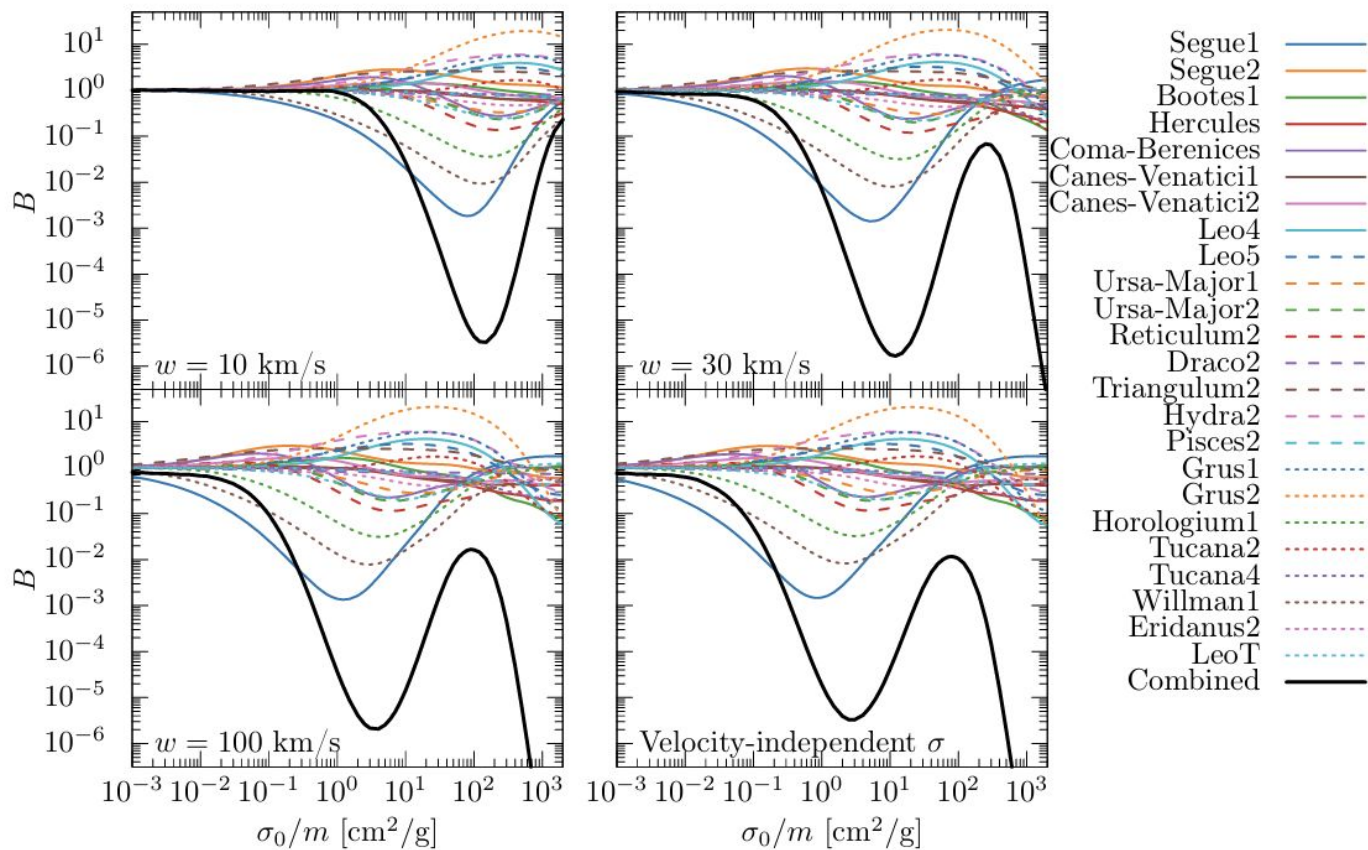
# Testing and Tasting SASHIMI-SIDM

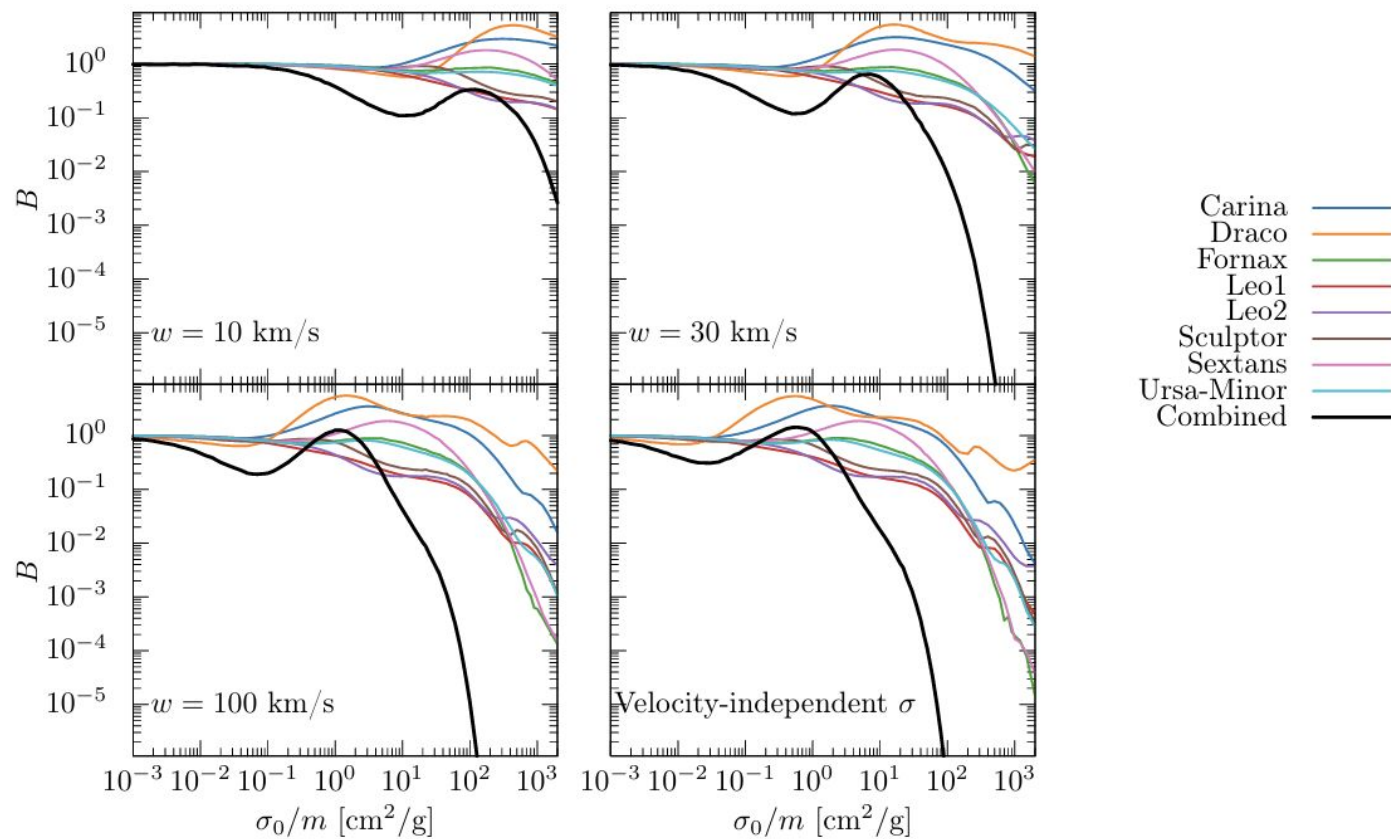
SASHIMI-SIDM can reproduce N-body results



SASHIMI-SIDM allows us to play with SIDM cosmology easily in a few minutes







# Future works

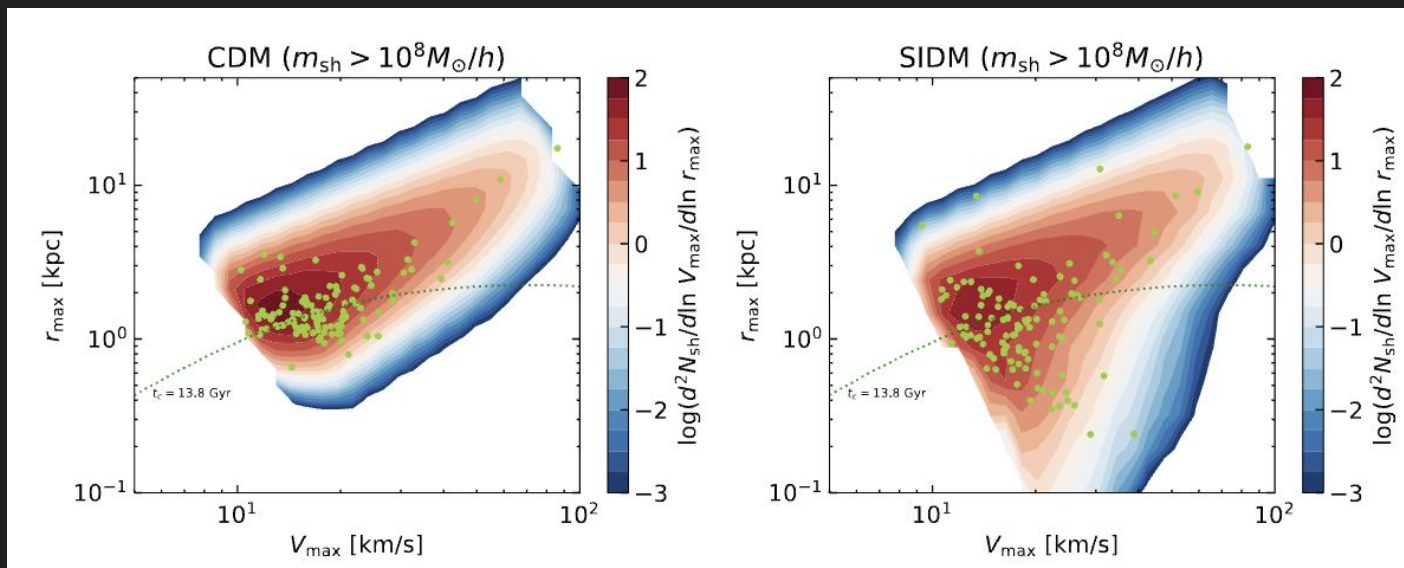
- Baryonic feedback
  - Classical dSphs might be affected
- Formation history
  -
- Spatial SASHIMI
  - Simulate precise mass-loss history of dSphs by tracing their orbital information
- Axisymmetric SASHIMI
  -

# SIDM halo structure and evolution: Semi-analytical and effective models

S. Ando, S. Horigome, E. O. Nadler,  
D. Yang, H.-B. Yu [in prep.]

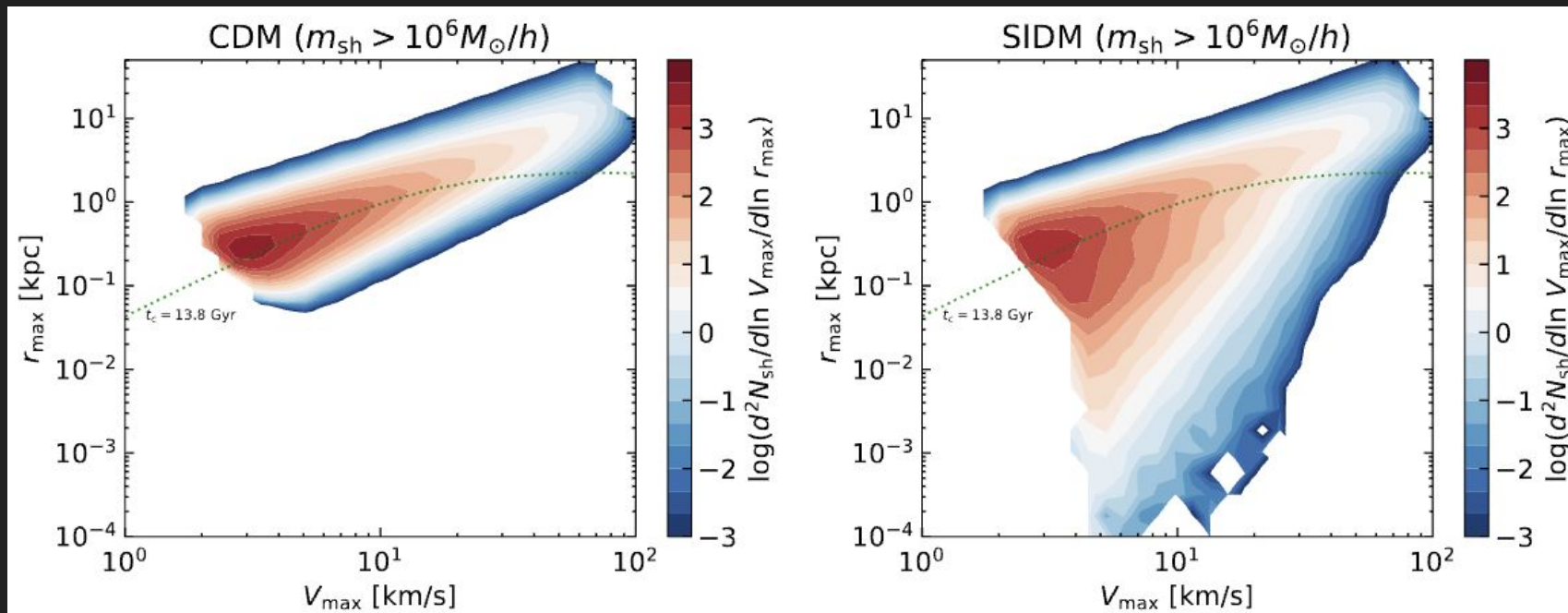
SASHIMI-SIDM: Semi-analytical approach to simulate SIDM models

- Quick calculation of SIDM halo properties and subhalo mass functions



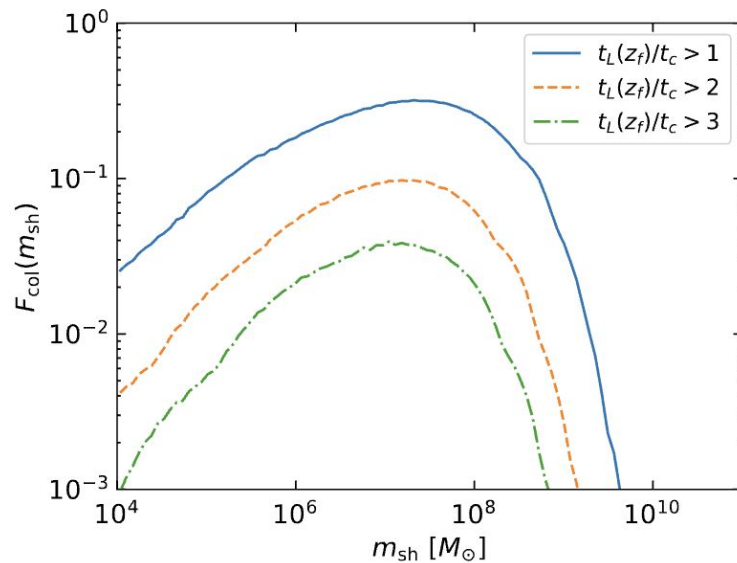
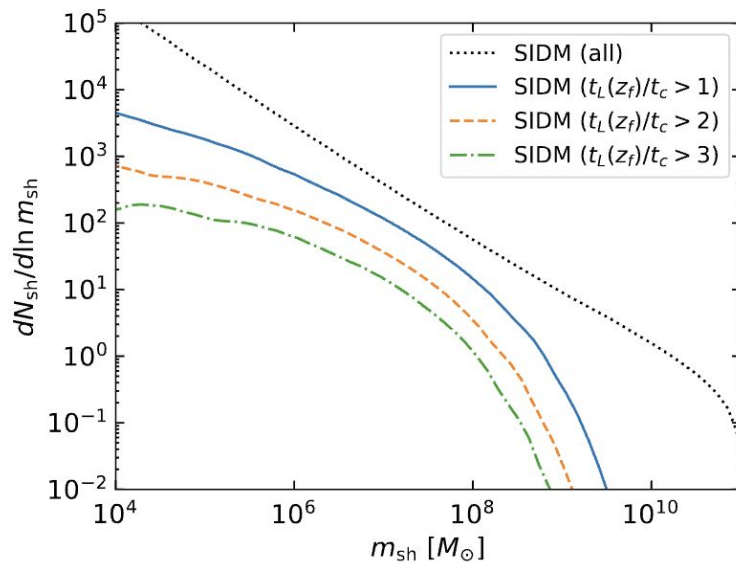
# SASHIMI-SIDM

Use Case: Property for lighter subhalos



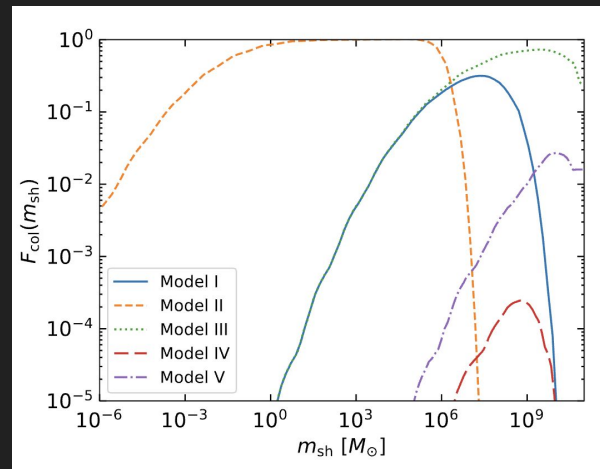
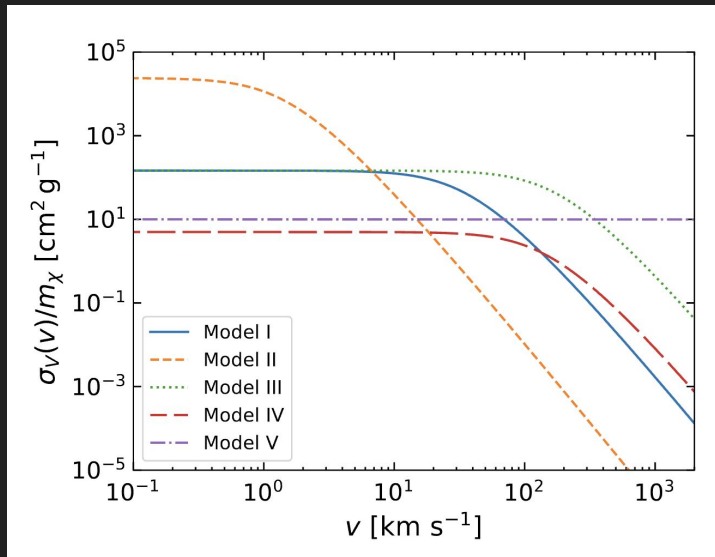
# SASHIMI-SIDM

Use case: Subhalo mass function



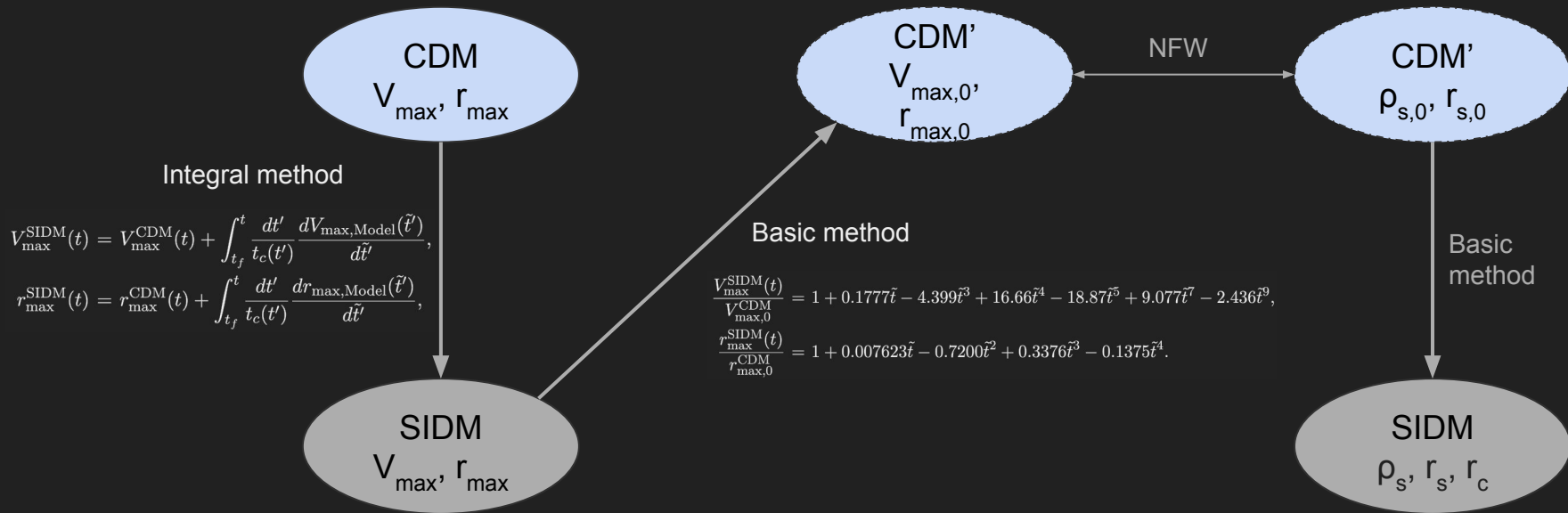
# SASHIMI-SIDM

## Subhalo mass function for various SIDM models



It works for lighter subhalo masses

# Parametric Model



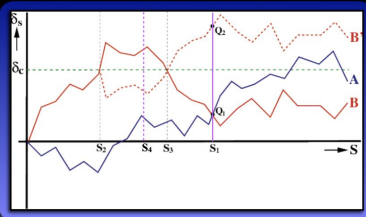


# EPS formalism

[Press & Schechter(1974)]

Ref: [http://www.astro.yale.edu/vdbosch/astro610\\_lecture9.pdf](http://www.astro.yale.edu/vdbosch/astro610_lecture9.pdf)

### The Excursion Set Formalism



Three trajectories corresponding to three different mass elements in a Gaussian random field. Note that  $B'$  is obtained mirroring trajectory  $B$  in the line  $\delta_S = \delta_c$  for  $S \geq S_2$ . Since the trajectories are Markovian  $B$  and  $B'$  are equally likely!

The problem with the PS ansatz is that it fails to account for trajectories such as  $B$  when counting mass elements in haloes with mass  $M > M_1$ .

Correcting for this is easy though, by realizing that each trajectory  $B$  has a mirror version,  $B'$ , that is equally likely (as a result of the Markovian nature of the trajectories).

Double-counting trajectories with  $\delta_S > \delta_c$  at  $S_1$  corrects for 'missed trajectories'....

➡ **A natural explanation for the fudge-factor two in PS formalism!**

ASTR 610: Theory of Galaxy Formation © Frank van den Bosch, Yale University

### The Excursion Set Formalism

In the excursion set formalism, also called the Extended Press-Schechter (EPS) formalism, one uses the (statistics of) Markovian random walks (the trajectories of mass elements in  $(S, \delta_S)$ -space) to infer the halo mass function (and more).

**PS ansatz:** fraction of mass elements with  $\delta_S > \delta_c(t)$  is equal to the mass fraction that at time  $t$  resides in haloes with masses  $> M$ , where  $S$  and  $M$  are related according to  $S = \sigma^2(M)$

↓

**EPS ansatz:** fraction of trajectories with a first upcrossing (FU) of the barrier  $\delta_S = \delta_c(t)$  at  $S > S_1 = \sigma^2(M_1)$  is equal to the mass fraction that at time  $t$  resides in haloes with masses  $M < M_1$

Since, each trajectory is guaranteed to upcross the barrier  $\delta_S = \delta_c(t)$  at some (arbitrarily large)  $S$ , the EPS ansatz predicts that every mass element is in a halo of some (arbitrarily low) mass

➡  $F(< M_1) = 1 - F(> M_1)$

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### The EPS Mass Function

Based on the EPS ansatz, we can write the EPS mass function as:

$$n(M, t) dM = \frac{\bar{\rho}}{M} \frac{\partial F(> M)}{\partial M} dM = - \frac{\bar{\rho}}{M} \frac{\partial F(< M)}{\partial M} dM$$
$$= - \frac{\bar{\rho}}{M} \frac{\partial F_{FU}(> S)}{\partial S} \frac{dS}{dM} dM = \frac{\bar{\rho}}{M} f_{FU}(S, \delta_c) \left| \frac{dS}{dM} \right| dM$$

Here  $f_{FU}(S, \delta_c) dS$  is the fraction of trajectories that have their first upcrossing of barrier  $\delta_c(t)$  between  $S$  and  $S + dS$ .

Without proof:  $f_{FU}(\nu) = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{S^{3/2}} \exp\left[-\frac{\delta_c^2}{2S}\right] = \frac{1}{2S} f_{PS}(\nu)$  (see MBW §7.2.2 for derivation)

where, as before, we defined  $\nu = \delta_c(t)/\sigma(M) = \delta_c/\sqrt{S}$  and we expressed the result in terms of the PS multiplicity function  $f_{PS}(\nu) = \sqrt{2/\pi} \nu \exp(-\nu^2/2)$

It is straightforward to show that this yields exactly the same **halo mass function** as before, but this time there has been no need for a fudge factor....

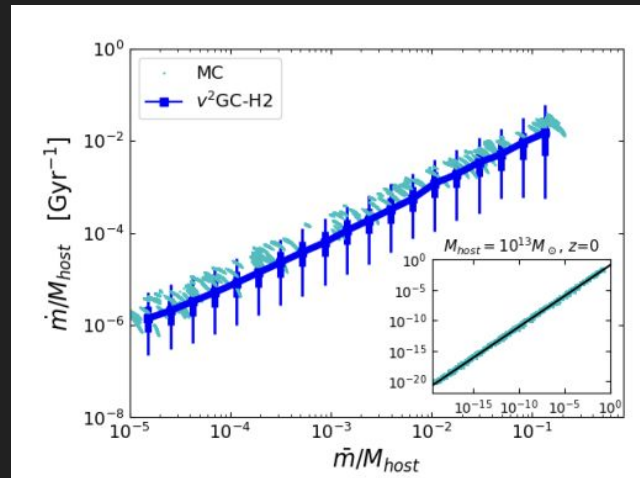
ASTR 610: Theory of Galaxy Formation © Frank van den Bosch, Yale University

# SASHIMI

Tidal stripping: [[Hiroshima+\(2018\)](#)]

$$\dot{m}(z) = -A \frac{m(z)}{\tau_{\text{dyn}}(z)} \left[ \frac{m(z)}{M(z)} \right]^\zeta,$$

$$\begin{aligned} \log A &= \left[ -0.0003 \log \left( \frac{M_{\text{host}}}{M_\odot} \right) + 0.02 \right] z \\ &\quad + 0.011 \log \left( \frac{M_{\text{host}}}{M_\odot} \right) - 0.354, \\ \zeta &= \left[ 0.00012 \log \left( \frac{M_{\text{host}}}{M_\odot} \right) - 0.0033 \right] z \\ &\quad - 0.0011 \log \left( \frac{M_{\text{host}}}{M_\odot} \right) + 0.026. \end{aligned}$$



# SIDM review

[Sean Tulin, Hai-Bo Yu, "Dark Matter Self-interactions and Small Scale Structure"](#)