Domain Walls in multi-axion models

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FY2024 What is dark matter?
Comprehensive study of the huge discovery space in dark matter
April 25th 2025

TOHOKU UNIVERSITY

Talks and papers

FY2022 B06

"Isotropic cosmic birefringence from early dark energy"

by KM, Fumihiro Naokawa, Toshiya Namikawa, Eiichiro Komatsu

FY2023 A01 and B01

"Misalignment production of vector boson dark matter from axion-SU(2) inflation"

by Tomohiro Fujita, KM, Kazunori Nakayama, Wen Yin

Today A01

"Induced domain walls of QCD axion, and gravitational waves"

"More is different: multi-axion dynamics changes topological defect evolution"

by Junseok Lee, KM, Fuminobu Takahashi, Wen Yin

Introduction

- **QCD axion** [Peccei & Quinn (1977), Weinberg (1978), Wilczek (1978)]

  {
  a solution to the strong CP problem
  a pseudo-NG boson arising at SSB of the Peccei-Quinn symmetry

  The QCD axion acquires a potential. → Dark matter candidate

  - Pre-inflationary PQ breaking

    → The axion field becomes (nearly) uniform.

    Typically $f_a = \mathcal{O}(10^{12})\,\mathrm{GeV}$ for all DM.

    "misalignment mechanism"

    [Preskill, Wise, Wilczek (1983), Abbott & Sikivie (1983), Dine & Fischler (1983)]

  - Post-inflationary PQ breaking [Davis (1986)]

    → Topological defects are formed.

    DM can be produced from the collapse of defects.

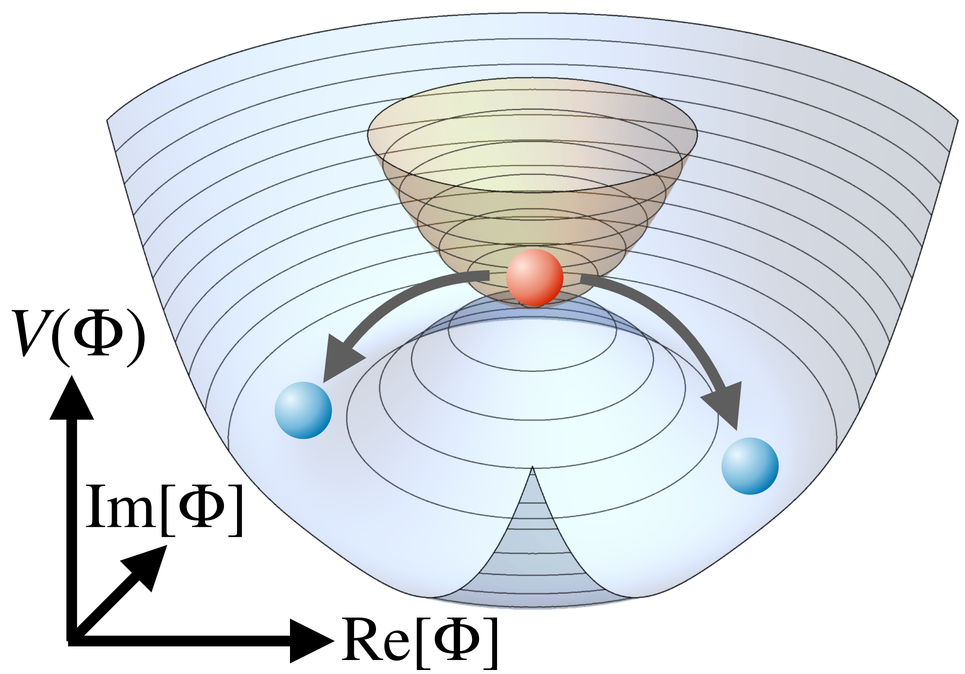
- Cosmic string

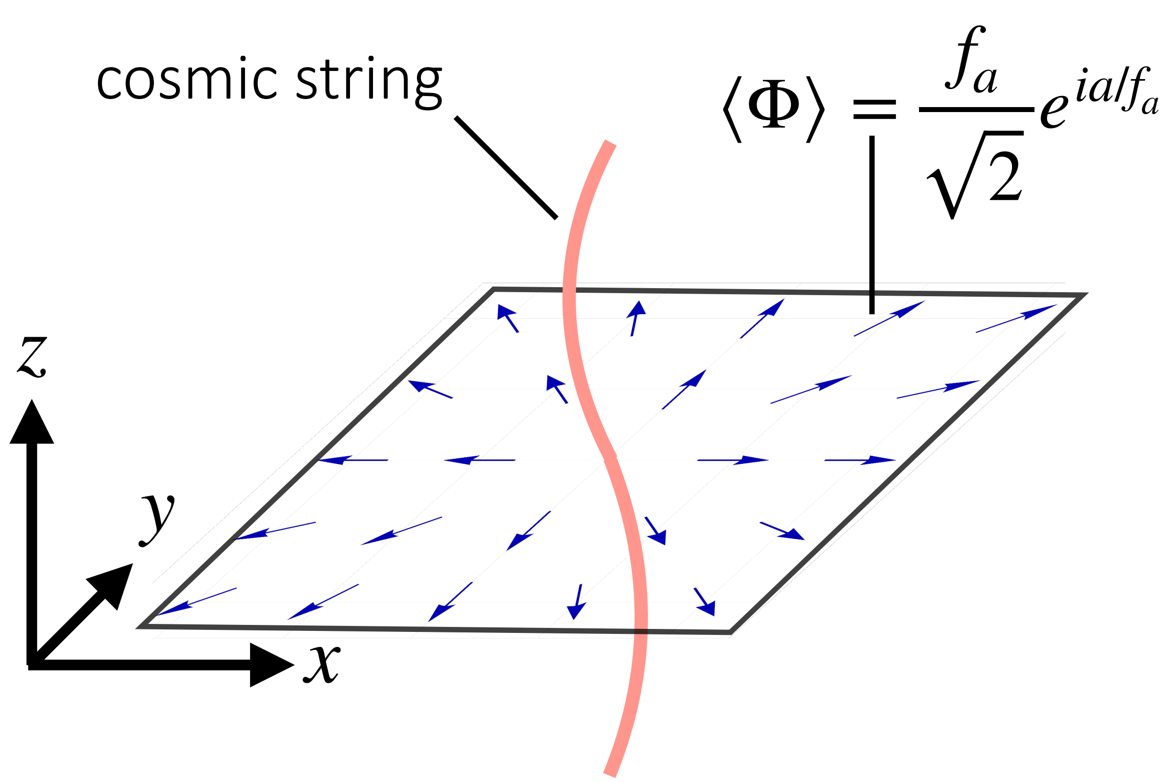
A complex scalar Φ with a wine-bottle $V(\Phi)$

At high temperatures,
Φ is stabilized at the origin.

At low temperatures,
the VEV, $\langle\Phi\rangle$, becomes nonzero.
An axion arises as a NG boson.

→ Formation of cosmic string





Domain wall

Explicit breaking of U(1) symmetry

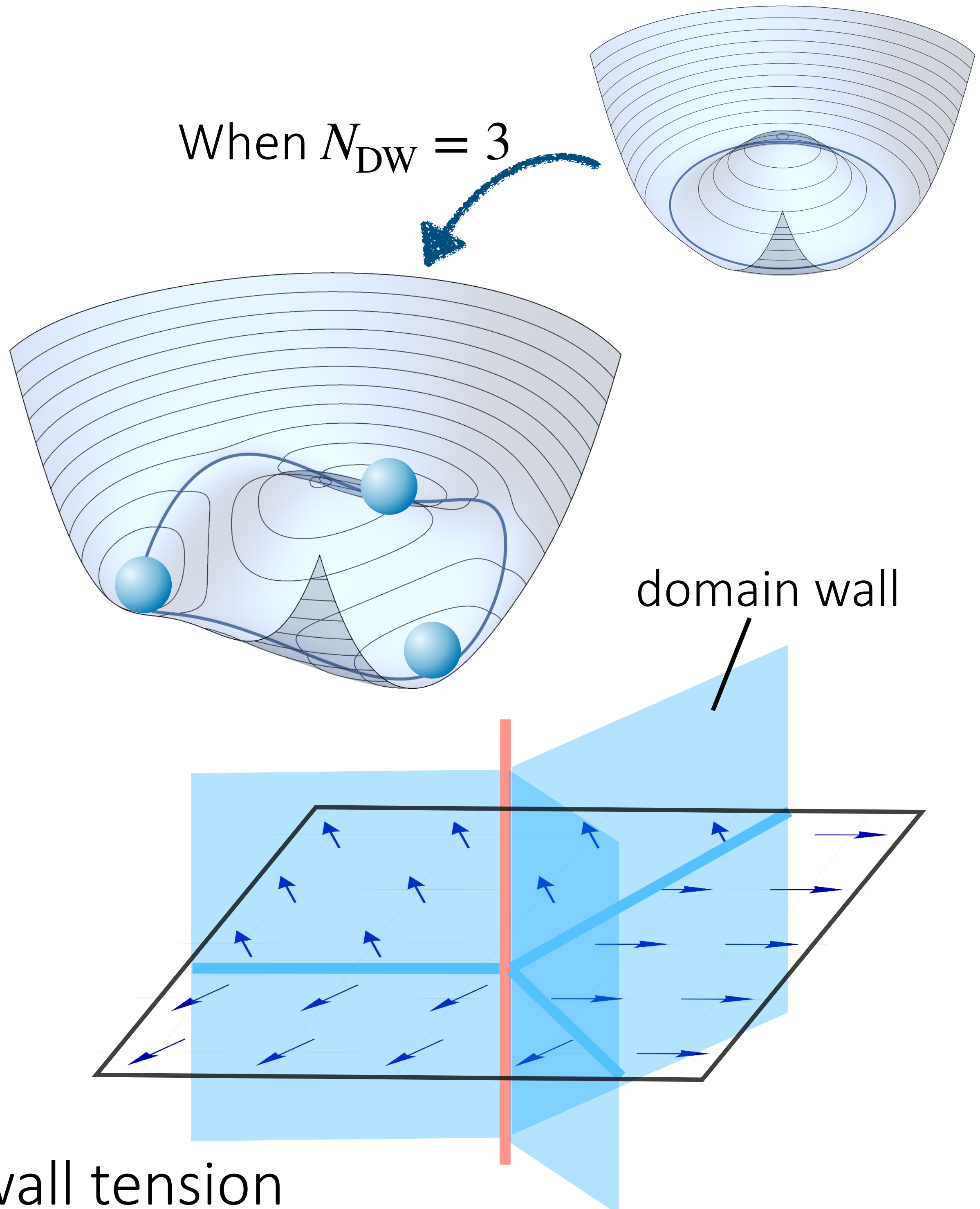
$$\mathrm{U}(1) \to \mathbb{Z}_{N_{\mathrm{DW}}}$$

$$V_{\mathrm{QCD}} = \chi \left[1 - \cos\left(N_{\mathrm{DW}} \frac{a}{f_a} \right) \right]$$

When $m_a > H$,
the axion rolls down the potential.

→ Formation of domain walls

$$\begin{cases} N_{\mathrm{DW}} = 1 & \text{: rapidly decay due to the wall tension} \\ N_{\mathrm{DW}} > 1 & \text{: long-lived network of strings and wall} \end{cases}$$



Introduction

- Cosmological implications

If $N_{\mathrm{DW}} > 1$ and the string-wall network is long-lived,
it approaches the "scaling regime": $\mathcal{O}(1)$ DWs in each Hubble volume.

→ Overclosure of the universe, "domain wall problem"

With a bias potential, the degeneracy of the minima can be broken and the network decays.

→ Emission of { axions / gravitational waves }

cf.) Kitajima, Lee, KM, Takahashi, Yin (2023)
Kitajima, Lee, Takahashi, Yin (2023)

Topological defects have been well studied in the literature.

Q. How about multiple axions?

In the string axiverse, many axion-like particles are expected.
We find new phenomena of defects in multi-axion models.

Induced domain walls

- **Setting**

Let us consider the QCD axion a + a heavy ALP ϕ with

$$V(a,\phi) = \Lambda^4 \left[1 - \cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right] + \chi(T)\left[1 - \cos\left(N_a\frac{a}{f_a} + N_\phi\frac{\phi}{f_\phi}\right)\right]$$

$$\Lambda^4 \equiv \frac{m_\phi^2 f_\phi^2}{N_{\rm DW}^2} \qquad \chi(T) \equiv \frac{m_a^2(T) f_a^2}{N_a^2}$$

We assume

order of DW formation: $m_\phi \gg m_{a0} \equiv m_a(T=0)$

hierarchy of DW tension: $\dfrac{m_\phi f_\phi^2}{N_{\rm DW}^2} \gg \dfrac{m_{a0} f_a^2}{N_a^2}$

Induced domain walls

$$V \simeq \Lambda^4 \left[1 - \cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]$$

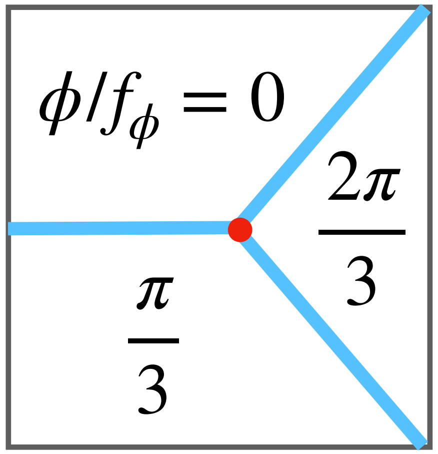
- Initial condition

As the initial condition, we consider

- the pre-inflationary for a → Initially homogeneous, no winding of a
- the post-inflationary for ϕ → Cosmic strings of ϕ

When $m_\phi \gtrsim 3H$, DWs of ϕ are formed.

In each domain, $\phi_k = 2\pi k \dfrac{f_\phi}{N_{\rm DW}}$.



$(N_{\rm DW} = 3)$

Ordinary DWs:

Potential height: $\Lambda^4 = \dfrac{m_\phi^2 f_\phi^2}{N_{\rm DW}^2}$

Tension: $\sigma_\phi = \dfrac{8 m_\phi f_\phi^2}{N_{\rm DW}^2}$

DW width: $\sim m_\phi^{-1}$

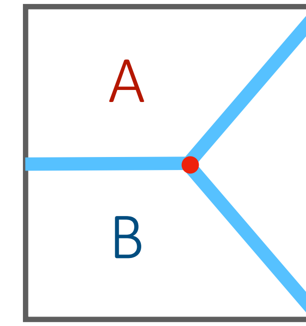
Induced domain walls

$$\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right], \quad \phi_k = 2\pi k\frac{f_\phi}{N_{\rm DW}}$$

- Domain walls of ϕ

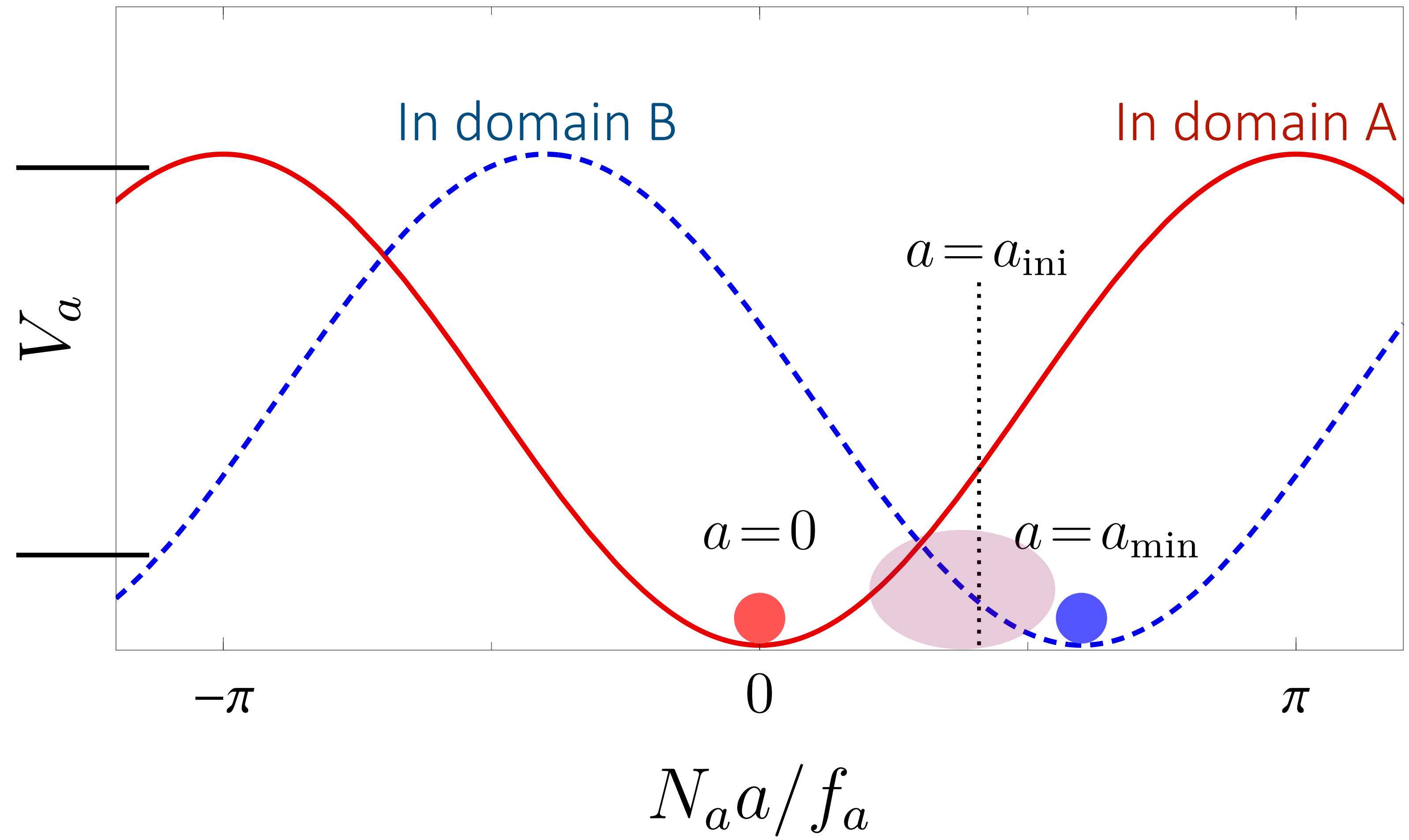
  When $m_a \gtrsim 3H$, a starts to oscillate in the potential.

  Due to the difference in ϕ, a has different potential minima.



$$\frac{m_a^2 f_a^2}{N_a^2}\left[1-\cos\left(N_a\frac{a}{f_a}\right)\right]$$

$$\frac{m_a^2 f_a^2}{N_a^2}\left[1-\cos\left(N_a\frac{a}{f_a}+\frac{2\pi N_\phi}{N_{\rm DW}}\right)\right]$$



a has a potential barrier between the domains. → "Induced domain walls"

Induced domain walls

- Induced domain walls

The DW of a is induced around that of ϕ.

The width is much larger for the indued DW.

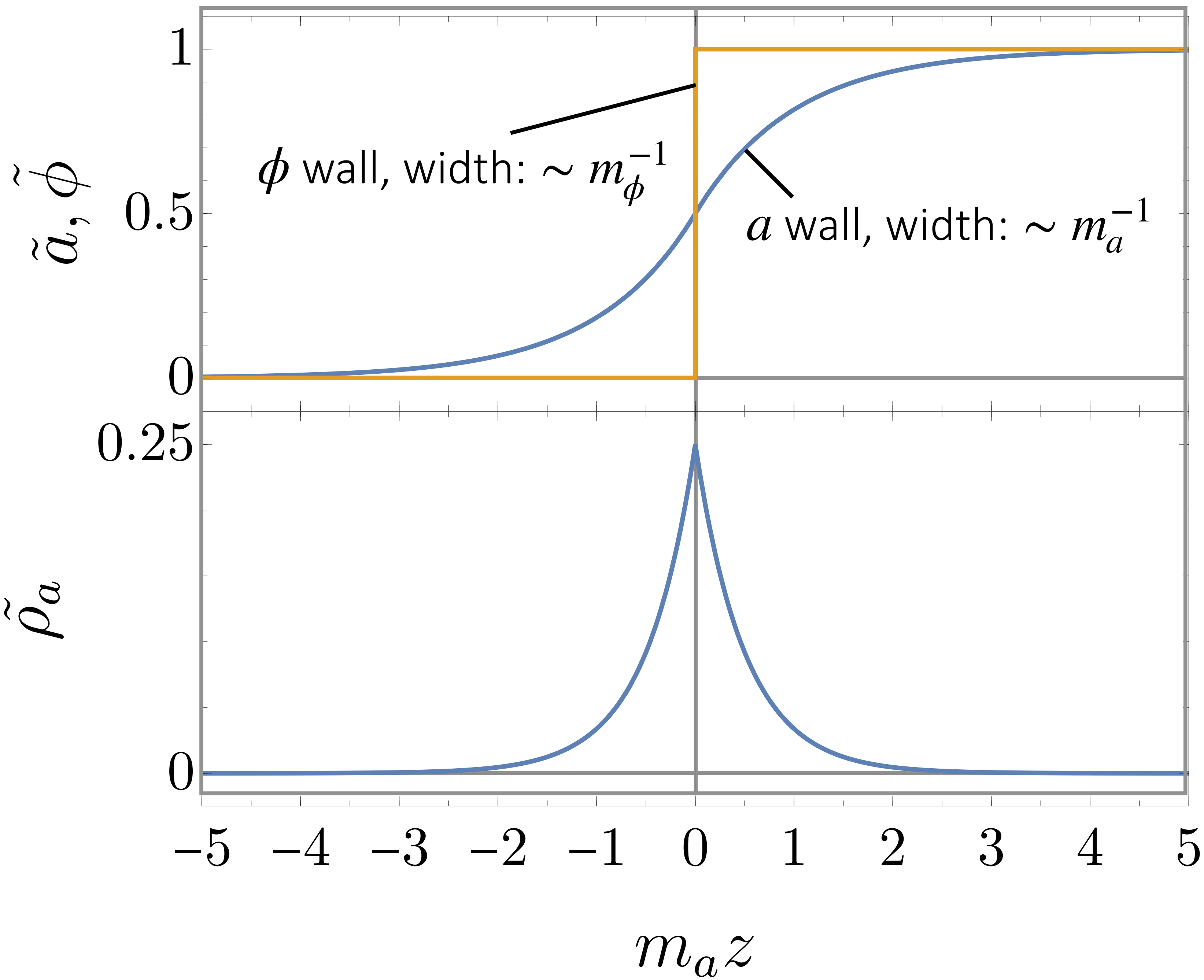
The energy is localized around the induced DW.

The DW tension is

$$\sigma_a \propto m_a a_{\min}^2$$

$\Uparrow$

cf.) $\sigma_\phi \propto m_\phi f_\phi^2$



Induced domain walls

- Production of dark matter and gravitational waves

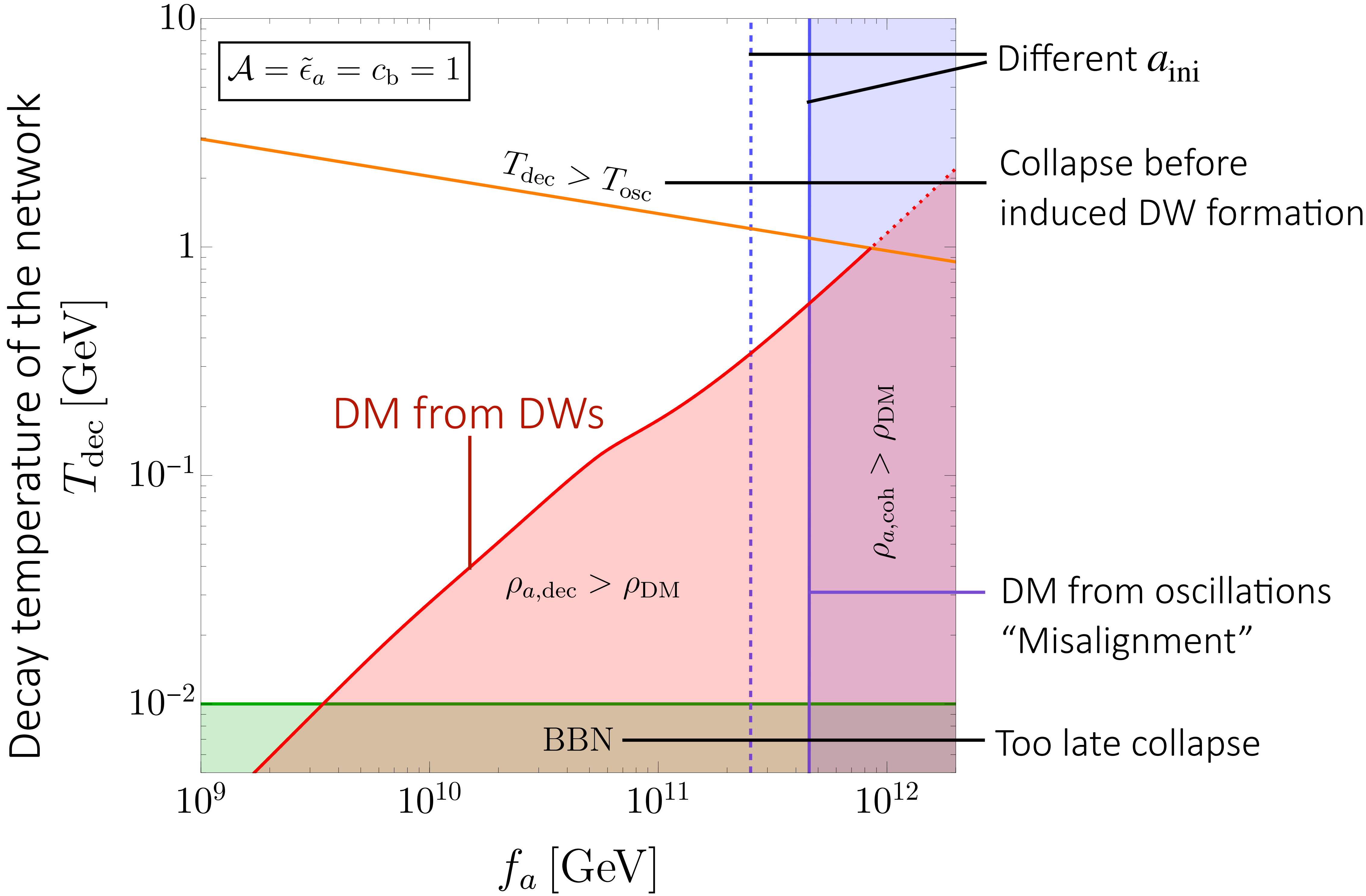
With a potential bias, the network finally decays.
The induced DWs decay at the same time as the heavy DWs.

Heavy DWs → Observable gravitational waves (with decaying ϕ particles)

Induced DWs → a particles as dark matter (with negligible GWs)

Due to the hierarchy of the tension,
DM and observable GW can be produced at the same time.

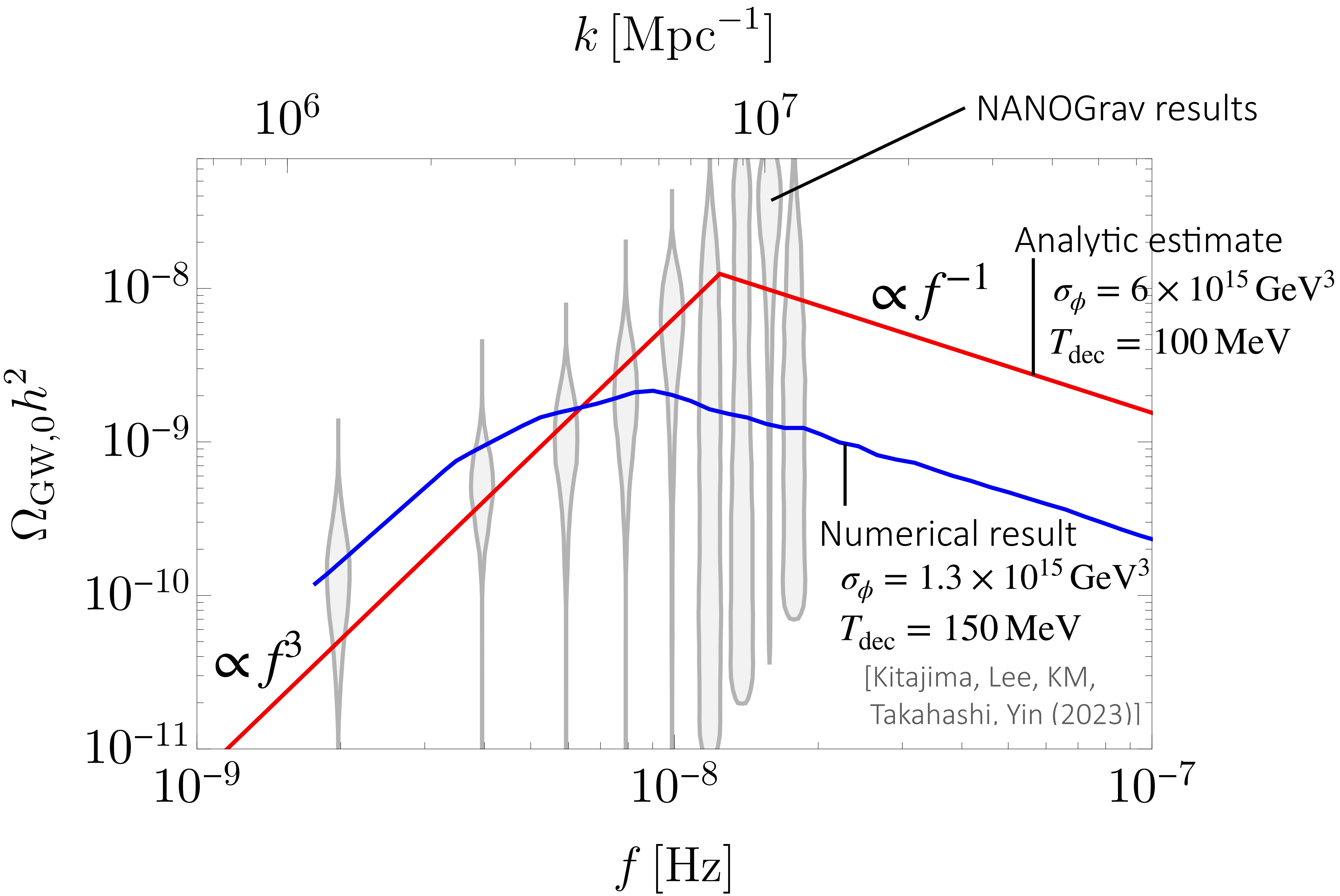
Dark matter production



Dark matter can be explained with $f_a < 10^{12}\,\mathrm{GeV}$.

($f_a \sim 10^{12}\,\mathrm{GeV}$ in the conventional scenario)

Gravitational wave production



NANOGrav results can also be explained.

Classification of general cases

General setup

We consider more general cases:

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2} \right) \right]$$

$$V_2(\phi_1, \phi_2) = \tilde{\Lambda}^4 \left[1 - \cos\left(n_1' \frac{\phi_1}{f_1} + n_2' \frac{\phi_2}{f_2} + \alpha \right) \right]$$

Each axions follows either of pre- or post-inflationary scenarios.

pre- → homogeneous; post-inflationary → defects

We can consider pre-pre, pre-post, post-post initial conditions.

→ We classify the evolution of defects by n's and the initial conditions.

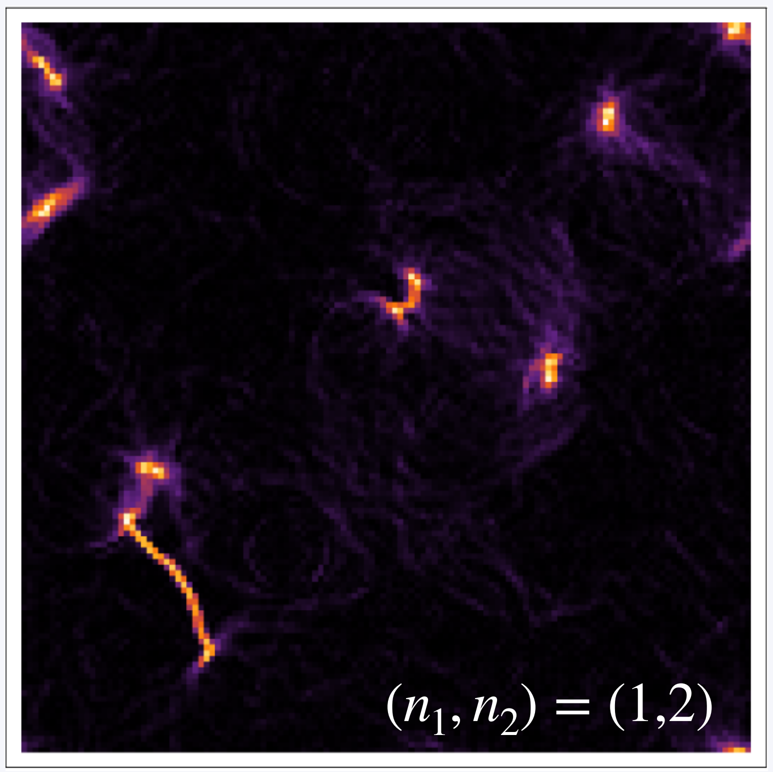
n_1	n_2	$N_{\rm DW}$	V_1	V_2
1	0	-	decay of ϕ_1 strings	ϕ_2 strings with $\lvert n_2' \rvert$ DWs
≥ 2	0	-	ϕ_1 network & ϕ_2 strings	stable network
≥ 1	1	0	θ_L string bundles	θ_L string bundles
		1		decay of network
		≥ 2		stable network
≥ 2	≥ 2	-	stable network	stable network with induced DWs or collapse by transient bias

(Details in the paper)

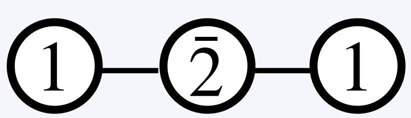
Post-post scenario

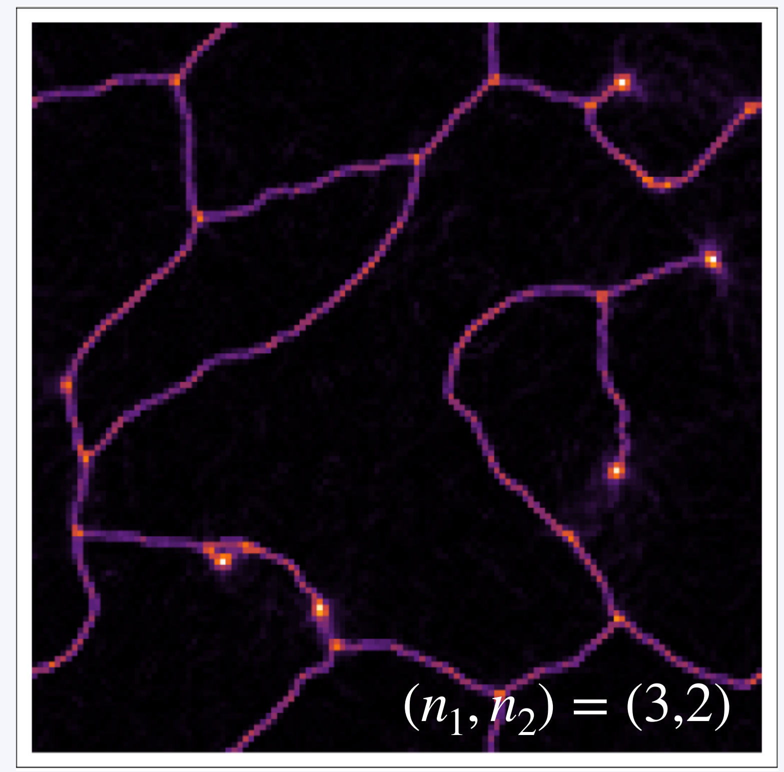
- Formation of string bundles

Let us consider the post-post scenario with $V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2} \right) \right]$



“String bundles” are formed.



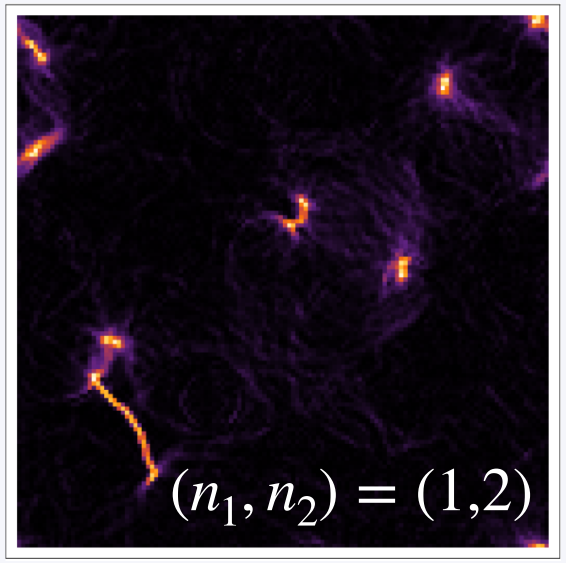


Complex network remains.

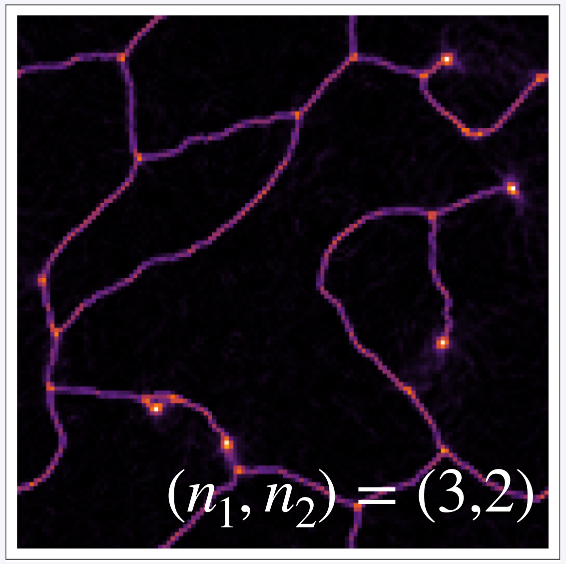
Post-post scenario

- Fate of defects

With the second potential: $V_2(\phi_1, \phi_2) = \tilde{\Lambda}^4 \left[1 - \cos\left(n'_1 \frac{\phi_1}{f_1} + n'_2 \frac{\phi_2}{f_2} + \alpha \right) \right]$



Each bundle has $N_{\rm DW}$ DWs with $N_{\rm DW} = \frac{|n_1 n'_2 - n_2 n'_1|}{\gcd(|n_1|, |n_2|)}$.



Induced domain walls are formed.
The network is long-lived.

Summary

- Heavy axion DWs can induce QCD axion DWs via mixing even with a homogeneous initial condition for the QCD axion.
- QCD axions from induced DWs can account for all dark matter with f_a as small as $\mathcal{O}(10^9)\,\mathrm{GeV}$.
- GWs from induced DWs can account for the NANOGrav result.
- We classified the evolution of string-wall network in multi-axion models.