# **Enumerative Geometry in East Asia 2025**

### **Report of Contributions**

Contribution ID: 1 Type: **not specified** 

### Geometric and Algebraic Constructions of Quantum Integrable Hierarchies

Tuesday 7 October 2025 15:30 (1 hour)

The theory of integrable hierarchies is deeply related to the enumerative geometry, and in particular, quantum integrable hierarchies provide efficient tools to compute Gromov-Witten invariants via the symplectic field theory. In this talk, I will compare two different ways for constructing quantum integrable hierarchies. One construction is geometric and given by A. Buryak and P. Rossi by considering the Hodge integrals over double ramification cycles, and the other one is algebraic by applying Weyl quantization to vertex algebras. I will explain these two constructions through the example of the quantum dispersionless KdV hierarchy which corresponds to the symplectic field theory of a disk.

**Presenter:** WANG, Zhe (RIKEN)

Contribution ID: 2 Type: not specified

#### Quantum K-theory at roots of unity

Wednesday 8 October 2025 11:30 (1 hour)

I will describe how the quasimap approach to equivariant quantum K-theory is modified when the curve-counting parameter is sent not to unity, but to a primitive root of unity instead. In particular, this leads to the appearance of the Frobenius action on the moduli space. Upon reducing the quantum difference equation modulo primes, we arrive at the Grothendieck–Katz p-curvature and prove that it is isospectral to a standard curvature operator precomposed with Frobenius.

**Presenter:** KOROTEEV, Peter (UC Berkeley)

Contribution ID: 3 Type: not specified

#### **Refined Gromov-Witten invariants**

Tuesday 7 October 2025 11:30 (1 hour)

I will discuss a definition of refined curve counting invariants of Calabi-Yau threefolds with a C\*-action in terms of stable maps on Calabi-Yau fivefolds. The corresponding disconnected generating function should conjecturally equate the Nekrasov-Okounkov K-theoretic membrane index under a refined version of the Gromov-Witten/Pandharipande-Thomas correspondence. I'll present several acid tests validating the conjecture, both in the A and the B-model. This is based on joint work with Yannik Schuler (ETH Zurich), arXiv:2410.00118.

Presenter: BRINI, Andrea (University of Scheffield)

Contribution ID: 4 Type: not specified

#### Mirror theorem and shift operators

Tuesday 7 October 2025 14:00 (1 hour)

The genus-zero Gromov-Witten invariants of a smooth projective variety can be encoded in an infinite-dimensional Lagrangian submanifold, known as the Givental cone, within the loop space of the cohomology group. A Givental-style mirror theorem states that a certain explicit cohomology-valued hypergeometric series, called the I-function, lies in this Givental cone. In joint work with Coates, Corti and Tseng, we established a Givental-style mirror theorem for toric Deligne-Mumford stacks. In this talk, I will explore the connection between Givental-style mirror theorems and shift operators for equivariant parameters, and I will reframe our results using this perspective. This is partly based on joint work with Fumihiko Sanda.

**Presenter:** IRITANI, Hiroshi (Kyoto University)

Contribution ID: 5 Type: not specified

### Remodeling Conjecture with inner branes, descendants and primaries

Monday 6 October 2025 10:00 (1 hour)

The Remodeling Conjecture proposed by Bouchard-Klemm-Mariño-Pasquetti relates all-genus open-closed Gromov-Witten invariants in a toric Calabi-Yau 3-manifold/3-orbifold to the Chekhov-Eynard-Orantin Topological Recursion (TR) invariants of its mirror curve. In this talk, I will describe the Remodeling Conjecture when 1) there are multiple Aganagic-Vafa Lagrangian branes, including inner branes; 2) Gromov-Witten invariants have descendant and primary insertions. Descendant insertions correspond to oscillatory integrals and Gamma classes, while primary insertions correspond to period integral of TR forms by special geometry. Furthermore, in the non-equivariant limit, we prove a conjecture of Hosono which equates quantum cohomology central charges of compactly supported coherent sheaves with period integrals of a holomorphic 3-form along integral 3-cycles on the Hori-Vafa mirror. This talk is based on joint work with Chiu-Chu Melissa Liu, Song Yu, and Zhengyu Zong.

Presenter: FANG, Bohan (Peking University)

Contribution ID: 6 Type: **not specified** 

### Chiral correlations on elliptic curves and enumerative geometry

Monday 6 October 2025 11:30 (1 hour)

We explain an elliptic trace formula for correlations of chiral QFT on elliptic curves. As an application, we show how this leads to a corresponding holomorphic anomaly equation and enumerative geometry on elliptic curves.

Presenter: LI, Si (Tsinghua University)

Contribution ID: 7 Type: not specified

### Satake's good basic invariants for finite complex reflection groups

Monday 6 October 2025 14:00 (1 hour)

In 1980, Saito, Sekiguchi and Yano found "flat generator system" on the orbit spaces of irreducible finite Coxeter Groups. Their construction can be understood in the framework of the almost duality of Frobenius manifolds proposed by Dubrovin.

In late 2010', the story was extended to well-generated finite complex reflection groups by Arsie-Lorenzoni, Kato-Mano-Sekiguchi, and Konishi-Minabe-Shiraishi.

In 2019, in a conference held at Kyoto University, Satake proposed the notion of good basic invariants for finite complex reflection groups and showed that good basic invariants are flat for finite Coxeter groups. I think that Satake's definition, given in terms of regular elements, opened a new perspective. In the joint work with S. Minabe, we showed that good basic invariants are flat and found that taking a reflection subquotient which preserves the largest degree induces good basic invariants on the reflection subquotient. In this talk, I will explain these results.

**Presenter:** KONISHI, Yukiko (Tsuda University)

Contribution ID: 8 Type: not specified

### Quantum K-Rings of Partial Flag Varieties, the Coulomb Branch, and the Bethe Ansatz

Monday 6 October 2025 16:00 (1 hour)

We prove two conjectured presentations of the quantum K-ring of type A partial flag varieties, one coming from quantum field theory, the other coming from quantum integrable systems. Our main tool to do this is using abelianization, which also gives a purely geometric interpretation for the Bethe Ansatz equations, which appear in both conjectures.

**Presenter:** HUQ-KURUVILLA, Irit (Academia Sinica)

Contribution ID: 9 Type: not specified

#### Symplectic duality and the Tutte polynomial

Thursday 9 October 2025 10:00 (1 hour)

The Tutte polynomial was introduced in the 1940s as a two-variable generalisation of the chromatic polynomial of a graph. It is the universal matroid invariant satisfying a deletion-contraction relation, and is the subject of much recent work.

I will describe a geometric realisation of the Tutte polynomial via the cohomology of a symplectic dual pair of hypertoric varieties. The same construction associates an interesting two-variable polynomial to any pair of symplectically dual spaces, whose one-variable specialisations recover the respective Poincare polynomials. Joint work with Ben Davison.

Presenter: MCBREEN, Michael (The Chinese University of Hong Kong)

Contribution ID: 10 Type: not specified

### Descendent transformations in Donaldson-Thomas wall-crossings

Tuesday 7 October 2025 10:00 (1 hour)

In recent work with Nick Kuhn and Felix Thimm, we proved a Joyce-style "universal" wall-crossing formula for certain equivariant moduli problems of 3-Calabi-Yau type, including Donaldson-Thomas theory. An immediate and productive question is how tautological classes like descendents transform under such wall-crossings. I will present an explicit descendent transformation formula for the Donaldson-Thomas/Pandharipande-Thomas wall-crossing of equivariant vertices, explain how the computation works, and speculate on how it may be generalized. This serves as a fairly down-to-earth example of how such wall-crossing formulas may be applied.

**Presenter:** LIU, Henry (Kavli IPMU)

Contribution ID: 11 Type: not specified

#### Calabi-Yau four wall-crossing

Wednesday 8 October 2025 14:00 (1 hour)

The Gross-Joyce-Tanaka conjecture proposes a universal wall-crossing formula for Calabi-Yau 4 theories. I will present recent progress towards proving an equivariant refinement of this framework, generalized to include insertions. This refinement can be formulated in terms of particular deformations of vertex algebras produced by adapting Joyce's construction. The utility of the theory is demonstrated through applications to Nekrasov's Magnificent Four conjecture and to proving tautological stable pair correspondences. This project includes joint work with Kuhn, Liu, and Thimm.

Presenter: BOJKO, Arkadij (SIMIS)

Contribution ID: 12 Type: not specified

#### Stable envelopes for critical loci

Wednesday 8 October 2025 10:00 (1 hour)

In a joint work with Andrei Okounkov, Yehao Zhou and Zijun Zhou. We introduce stable envelopes in critical cohomology and K-theory for symmetric quiver varieties with potentials and related geometries.

Critical stable envelopes are compatible with dimensional reductions, specializations, Hall products, and other geometric constructions. In particular, for tripled quivers with canonical cubic potentials, critical stable envelopes

reproduce those on Nakajima quiver varieties, constructed by Maulik and Okounkov.

If time permits, we also mention their applications to enumerative geometric problems. Applications to geometric representation theory will be mentioned in Yehao's talk.

Presenter: CAO, Yalong (Chinese Academy of Sciences)

Contribution ID: 13 Type: not specified

#### Critical stable envelopes and Yangians

Thursday 9 October 2025 11:30 (1 hour)

Using the critical stable envelopes (see Yalong's talk), we can construct solutions to Yang-Baxter equations with R-matrixes acting on the critical cohomologies of symmetric quiver varieties with potentials. Then, the FRT formalism gives natural (shifted) (super) Yangian action on these critical cohomologies. For example, in the case of tripled quiver with the canonical cubic potential, the zero shifted Yangian reproduces the Maulik-Okounkov Yangian. In general, given a symmetric quiver Q with potential W, we get a family of shifted Yangians with shift homomorphisms and coproducts, where the zero shifted one is a deformation of enveloping algebra of g[u] for a certain Lie algebra g determined by (Q,W). The positive part of g is expected to be related to the BPS Lie algebra of (Q,W). This talk is based on joint work with Yalong Cao, Andrei Okounkov, and Zijun Zhou.

Presenter: ZHOU, Yehao (SIMIS)

Contribution ID: 14 Type: not specified

### **KP Integrability in Topological Recursion**

Thursday 9 October 2025 14:00 (1 hour)

Topological recursion is a powerful tool in mathematical physics, applicable to various problems in enumerative geometry, such as intersections on moduli spaces and Hurwitz numbers. In my talk, I will discuss the KP integrability of topological recursion, which arises naturally in the context of the x-y swap relation. This integrability can be described through certain integral transforms, leading to Kontsevich-like matrix models.

This talk is based on a joint work with Boris Bychkov, Petr Dunin-Barkowski, Maxim Kazarian, and Sergey Shadrin.

Presenter: ALEXANDROV, Alexander (IBS center for Geometry and Physics)

Contribution ID: 15 Type: not specified

## Br\'ezin-Gross-Witten numbers and Witten's intersection numbers: their combinatorics and uniform large genus asymptotics

Thursday 9 October 2025 16:00 (1 hour)

Br\'ezin-Gross-Witten (BGW) numbers and Witten's intersection numbers are two families of rational numbers, that both have physical origins, topological meanings and backgrounds from matrix models. Their partition functions are known to be particular tau-functions of the KdV hierarchy, satisfying Virasoro constraints. In view of integrable systems, the origin of the BGW numbers and Witten's intersection numbers goes back to bispectrality after Duistermaat and Gr\"unbaum. In this talk, we study combinatorics of these numbers, and discover and prove their uniform large genus asymptotics. Applications to the Painlev\'e II hierarchy and to the Painlev\'e I equation are given. The talk is based on joint works with Jindong Guo, Paul Norbury and Don Zagier.

**Presenter:** YANG, Di (University of Science and Tehnology)

TBA

Contribution ID: 16 Type: not specified

#### **TBA**

Friday 10 October 2025 10:00 (1 hour)

**Presenter:** ZHANG, Yingchun (Shanghai Jiao Tong University)

Contribution ID: 17 Type: not specified

### Quantum Cohomology, Shift operators, and Coulomb branches

Friday 10 October 2025 11:30 (1 hour)

Given a complex reductive group G and a G-representation N, there is an associated Coulomb branch algebra defined by Braverman–Finkelberg–Nakajima. In joint work with Chan and Lam, we show that these Coulomb branch algebras can be described as the largest subcomodules of the equivariant BM-homology of the affine Grassmannian on which certain shift operators admit non-equivariant limits. I will outline the main ideas of the proof, and explain how the defining equations of Coulomb branches reflect the properness of the moduli spaces needed to construct shift operators.

As an application, I will also present work in preparation with Chan, Lam, and Chow that gives a geometric construction of a (degenerate) DAHA action on the equivariant quantum cohomology of  $T^*(G/P)$  via stable envelopes.

**Presenter:** CHAN, Kifung (The Chinese University of Hong Kong)

Contribution ID: 18 Type: not specified

### Topological Langlands duality for 3-manifolds via DT cohomology

Friday 10 October 2025 14:00 (1 hour)

This talk concerns the Langlands program for 3-manifolds, initiated by Ben-Zvi–Gunningham–Jordan–Safronov. From the physical perspective, it arises as part of S-duality in 4d N=4 supersymmetric quantum field theory. One formulation of the Langlands duality conjecture for 3-manifolds involves infinite-dimensional DT cohomology, defined as the cohomology of certain perverse sheaves on the stack of representations of the fundamental group.

I will introduce a finite-dimensional subspace of DT cohomology, called the BPS cohomology, and explain that DT cohomology decomposes under parabolic induction (a.k.a. the Eisenstein series map) into the BPS cohomology. This reduces the Langlands duality conjecture to an isomorphism of finite-dimensional vector spaces. In addition, we provide a formula for the dimension of BPS cohomology for the 3-manifold T^3, and show that Langlands duality for types B/C follows from the Jacobi identity of the theta function.

This talk involves joint works with Hyeonjun Park, Pavel Safronov, Chenjing Bu, Ben Davison, Anders Ibáñez Núñez, Tudor Pădurariu, and Lucien Hennecart.

**Presenter:** KINJO, Tasuki (Kyoto University)

Contribution ID: 19 Type: not specified

#### Microlocalization of virtual fundamental classes

Friday 10 October 2025 16:00 (1 hour)

I will report on joint work in progress with Tasuki Kinjo, Hyeonjun Park, and Pavel Safronov, regarding Joyce's conjecture on a (-1)-shifted microlocalization of the virtual fundamental class. This construction subsumes not only the usual virtual fundamental class for quasi-smooth moduli spaces (read: perfect obstruction theories), but also other virtual class type constructions appearing e.g. in DT4 and GLSM invariants.

Presenter: KHAN, Adeel (Academia Sinica)