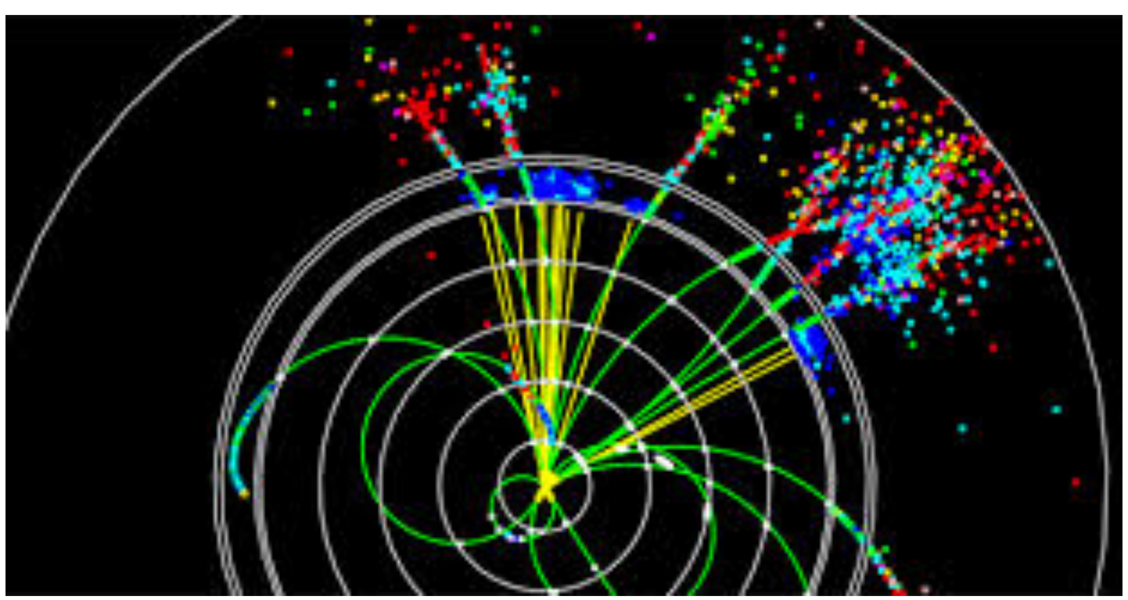
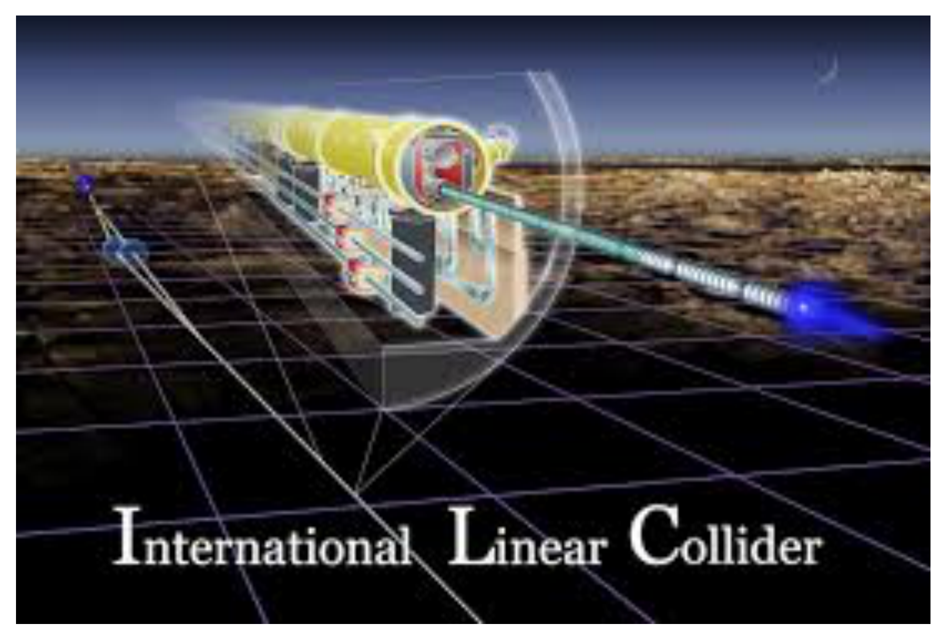
Supersymmetric naturalness: the best reason to build the ILC

Howard Baer
University of Oklahoma
Kaoru-fest, 2015





It's a real honor to be speaking at ``Kaoru-fest" since Kaoru is one of my best friends and our collaboration dates to 1983 when I was a grad student at UW and Kaoru was a post-doc

some of our early papers:

The Background to $t-$Quarks Signals from Higher-Order QCD Contributions (with V. Barger, K. Hagiwara, A.D. Martin and R.J.N. Phillips), Phys. Rev. **D29**,1923 (1984).

Fourth Generation Quarks and Leptons (with V. Barger, K. Hagiwara and R.J.N. Phillips), Phys. Rev. **D30**, 947 (1984).

Testing Models of Anomalous Radiative Decays of the Z Boson (with V. Barger and K. Hagiwara), Phys. Rev. **D30**, 1513 (1984).

Single Production of Very Heavy Particles at $\overline{p}p$ Colliders (with V. Barger and K. Hagiwara), Phys. Lett. **146B**, 57 (1984).

Testing Spinless Boson Parent Models for Anomalous $l^+l^-\gamma$ Events (with K. Hagiwara and J. Ohnemus), Phys. Rev. **D32**, 82 (1985).

Consequences of Models for Monojet Events from Z^0 Boson Decay (with K. Hagiwara and S. Komamiya), Phys. Lett. **156B**, 117 (1985).

Can the CERN $p\overline{p}$ Collider Data Limit Gaugino Masses? (with K. Hagiwara and X. Tata), Phys. Rev. Letter **57**, 294 (1986).

Gauginos as a Signal for Supersymmetry at $p\overline{p}$ Colliders (with K. Hagiwara and X. Tata), Phys. Rev. **D35**, 1598 (1987).

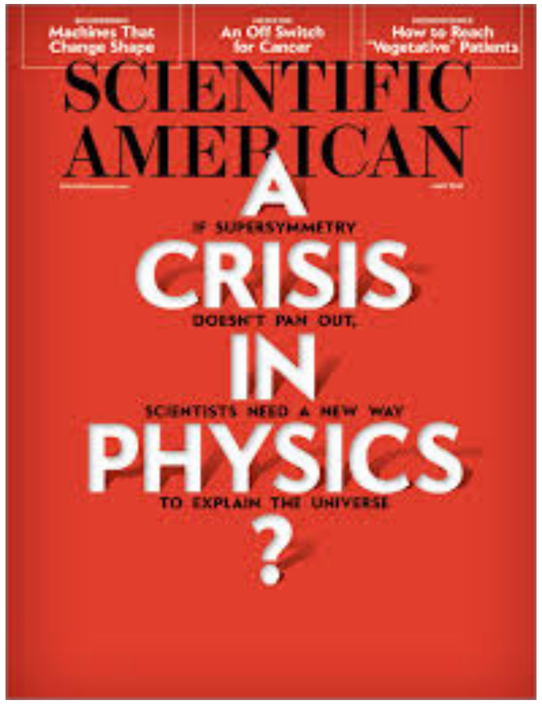
On the Model Dependence of the Chargino Mass Bound from the CERN Collider Data (with K. Hagiwara and X. Tata). Phys. Rev. **D38**, 1485 (1988).

Kaoru: thank you for your patience and wisdom!

Biggest result from LHC7/LHC8:
discovery of Higgs boson m(h)~125 GeV

- fantastic discovery!
- but puzzling: how can fundamental scalar particles exist? (Wilson/Susskind)
- either composite or need protective ``supersymmetry"
- certainly looks fundamental
- likely SUSY

Is SUSY dead or dying?



- many authors think so: crisis for SUSY: SUSY not as we understood it,...???
- LHC8: m(gluino)>1.3 TeV   [m(gl)<<m(sq)]
- m(gluino)>1.8 TeV   [m(sq)~m(gl)]
- m(h)~125 GeV: need TeV scale highly mixed stops
- But we have been hearing for years: naturalness means sparticles at or near weak scale as typified by m(W,Z,h)~100 GeV
- typical claim: SUSY fine-tuned to 0.1%
- This is serious: fine-tuning indicative of some pathology in theory

We believe these claims stem from overestimates of EW fine-tuning which arise from violations of the

Prime directive on fine-tuning:

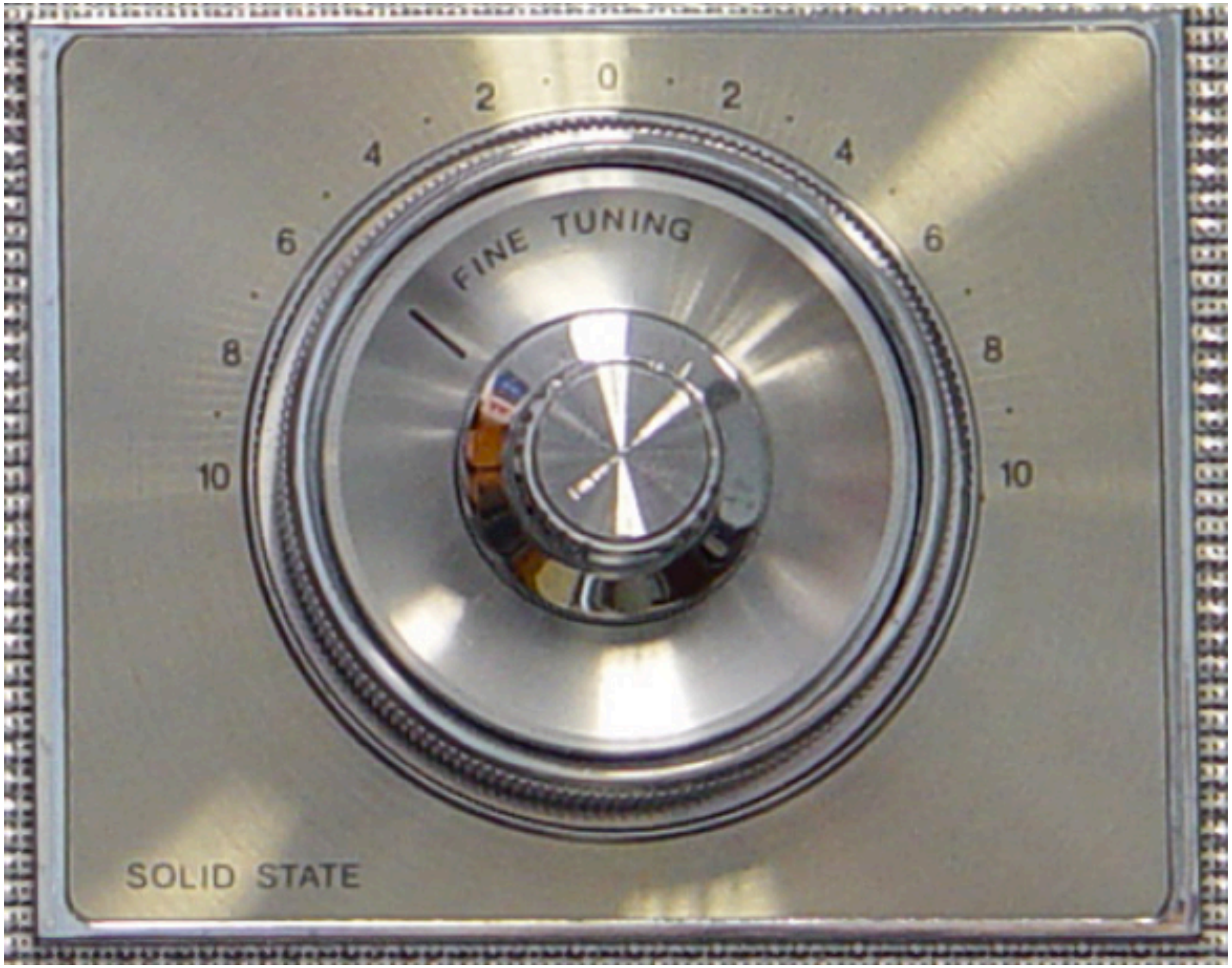
"Thou shalt not claim fine-tuning of dependent quantities one against another!"

Is observable $\mathcal{O}$ fine-tuned?

$\mathcal{O} = \mathcal{O} + b - b$

HB, Barger, Mickelson, Padeffke-Kirkland, PRD89 (2014) 115019

Three measures of fine-tuning:

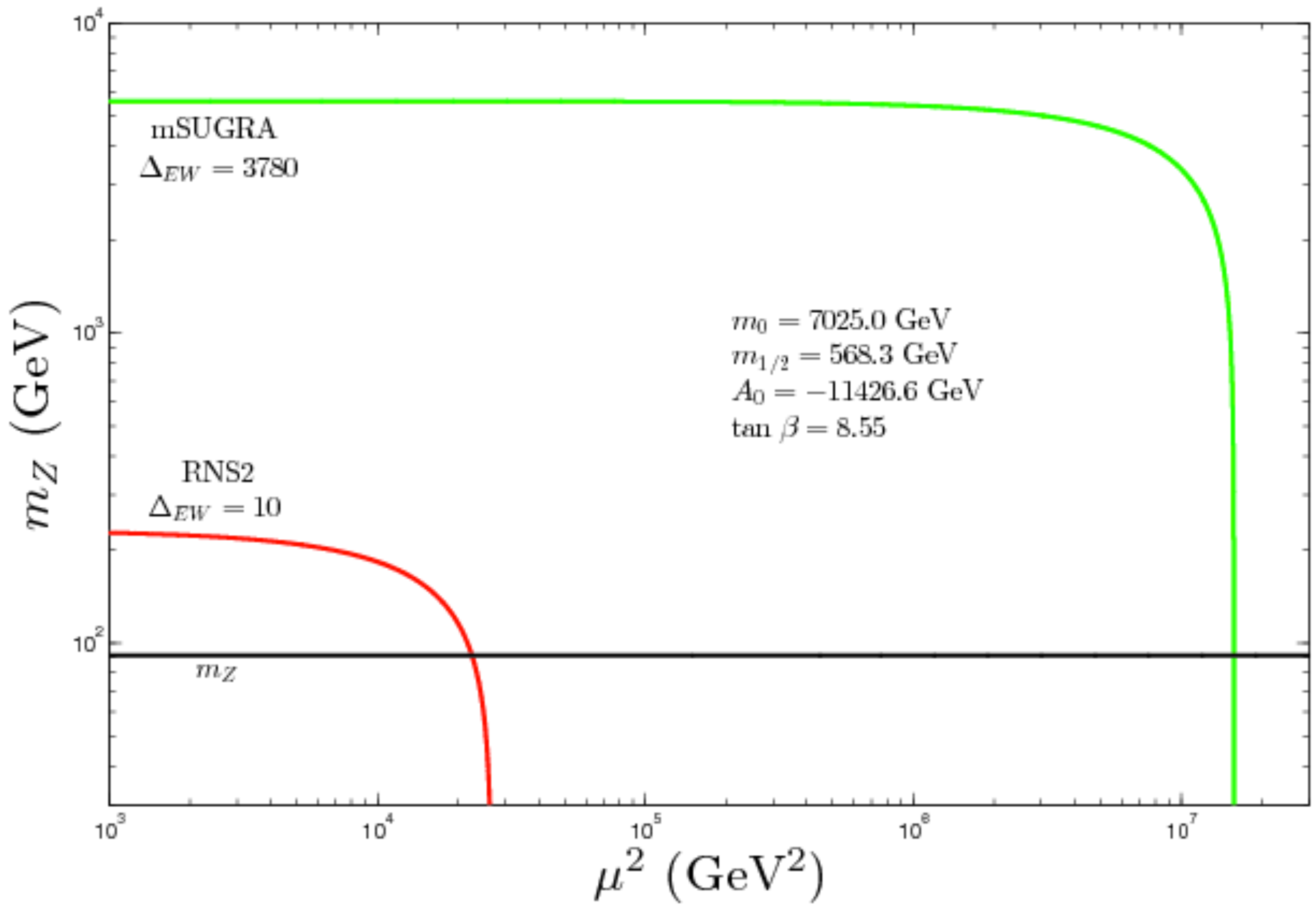


First: simple electroweak fine-tuning:

dial the value of mu so that Z mass comes out right:

everybody does it, but it is hidden inside spectra codes (Isajet, SuSpect, SoftSUSY, Spheno, SSARD)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2$$



e.g. in CMSSM/ mSUGRA: one then concludes nature gives this:

#1: Simplest SUSY measure: Δ_{EW}

Working only at the weak scale, minimize scalar potential: calculate m(Z) or m(h)

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \quad \sim -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

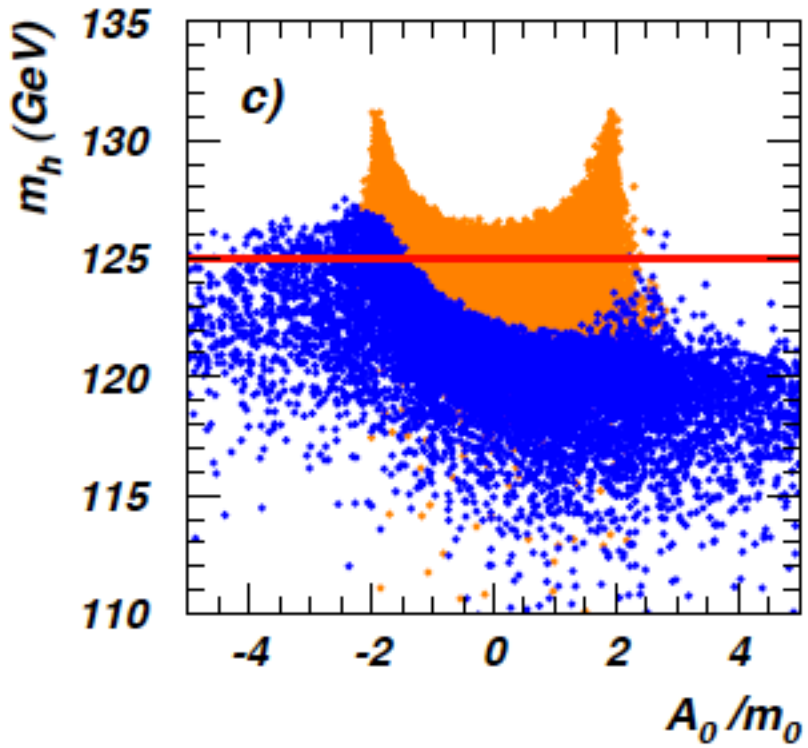
$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$ with $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ etc.

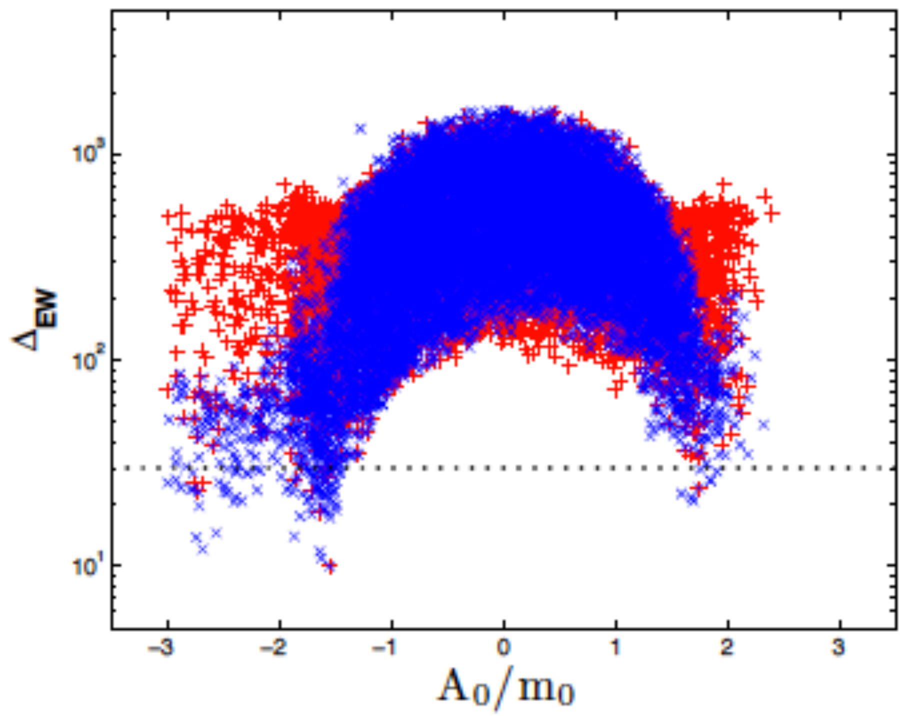
simple, direct, unambiguous interpretation:

- $|\mu| \sim m_Z \sim 100 - 200$ GeV
- $m_{H_u}^2$ should be driven to small negative values such that $-m_{H_u}^2 \sim 100 - 200$ GeV at the weak scale and
- that the radiative corrections are not too large: $\Sigma_u^u \lesssim 100 - 200$ GeV

Radiative natural SUSY with a 125 GeV Higgs boson (with V. Barger, P. Huang, A. Mustafayev and X. Tata), Phys. Rev. Letters **109** 161802 (2012).

Large value of A_t reduces $\Sigma_u^u(\tilde{t}_{1,2})$ contributions to Δ_{EW} while uplifting m_h to ~ 125 GeV





$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2}F(m^2_{\tilde{t}_{1,2}})\left[f_t^2 - g_Z^2 \mp \frac{f_t^2A_t^2 - 8g_Z^2(\frac{1}{4}-\frac{2}{3}x_W)\Delta_t}{m^2_{\tilde{t}_2}-m^2_{\tilde{t}_1}}\right]$$

$$\Delta_t = (m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R})/2 + M_Z^2\cos 2\beta(\tfrac{1}{4}-\tfrac{2}{3}x_W)$$

$$F(m^2) = m^2\left(\log\frac{m^2}{Q^2} - 1\right) \qquad Q^2 = m_{\tilde{t}_1}m_{\tilde{t}_2}$$

#2: Higgs mass or large-log fine-tuning Δ_{HS}

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{rad}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from m(SUSY) to Lambda

$$\delta m_{H_u}^2|_{rad} \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2)\ln\left(\Lambda^2/m_{SUSY}^2\right)$$

$\Delta_{HS} \sim \delta m_h^2/(m_h^2/2) < 10$ then

$m_{\tilde{t}_{1,2},\tilde{b}_1} < 500$ GeV

$m_{\tilde{g}} < 1.5$ TeV

A_t can't be too big

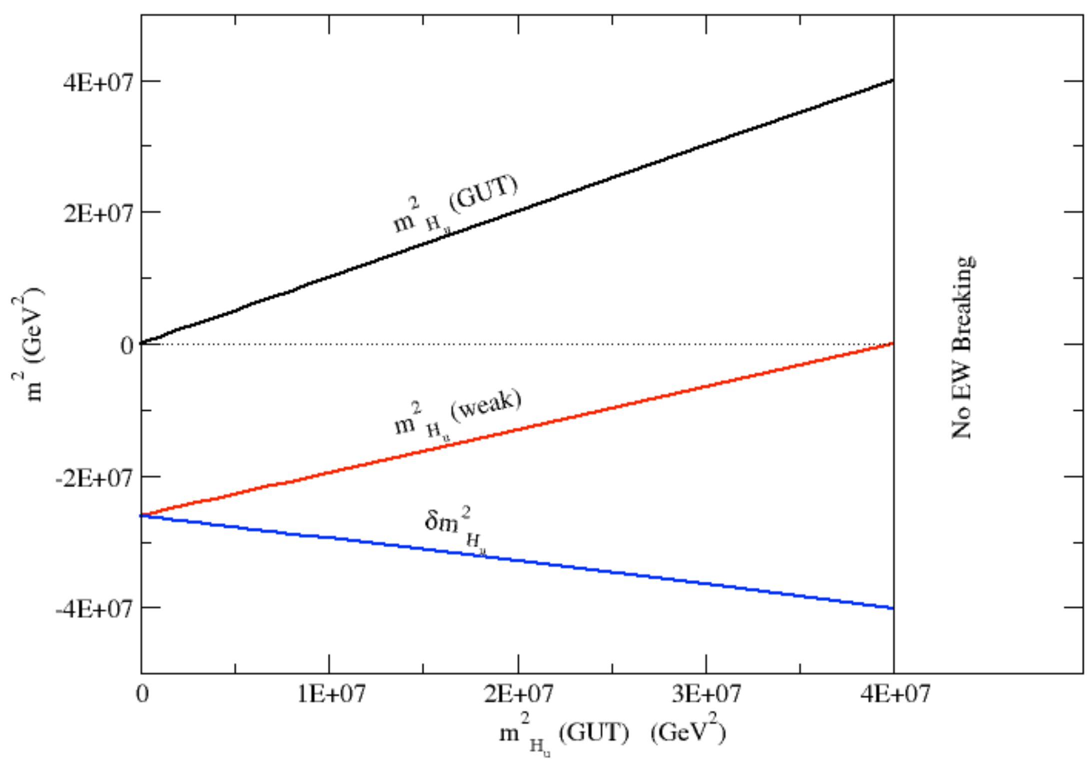
old natural SUSY

What's wrong with this argument?

In zeal for simplicity, have made several simplifications: most egregious is that one sets m(Hu)^2=0 at beginning to simplify

$$m^2_{H_u}(\Lambda) \text{ and } \delta m^2_{H_u} \text{ are } \textit{not} \text{ independent!}$$

violates prime directive!



The larger $m^2_{H_u}(\Lambda)$ becomes, then the larger becomes the cancelling correction!

To fix: combine dependent terms:

$m_h^2 \simeq \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ where now both μ^2 and $\left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ are $\sim m_Z^2$

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

Instead of: the radiative correction $\delta m_{H_u}^2 \sim m_Z^2$
we now have: the radiatively-corrected $m_{H_u}^2 \sim m_Z^2$

#3: EENZ/BG traditional measure Δ_{BG}

Such a re-grouping is properly used in the EENZ/BG measure:

$$\Delta_{BG} \equiv max_i\,[c_i]\,, \quad \text{where} \quad c_i = \left|\frac{\partial \ln m_Z^2}{\partial \ln p_i}\right| = \left|\frac{p_i}{m_Z^2}\frac{\partial m_Z^2}{\partial p_i}\right|$$

the p_i constitute the fundamental parameters of the model.

for pMSSM, obviously $\Delta_{BG} \simeq \Delta_{EW}$

What about models defined at high scale?

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \mu^2$$

express weak scale value in terms of high scale parameters

Express m(Z) in terms of GUT scale parameters:

$$m_Z^2 \simeq -2m_{H_u}^2 - 2\mu^2$$ (weak scale relation)

$$
\begin{aligned}
-2\mu^2(m_{SUSY}) &= -2.18\mu^2 \\
-2m_{H_u}^2(m_{SUSY}) &= 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\
&+0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\
&-0.025M_1A_t + 0.22A_t^2 + 0.004m_3A_b \\
&-1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\
&+0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\
&+0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\
&+0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2,
\end{aligned}
$$

all GUT scale parameters

Ibanez, Lopez, Munoz; Lleyda, Munoz

Kane, King

Abe, Kobayashi, Omura; S. P. Martin

For generic parameter choices, Δ_{BG} *is large*

But if: $m_{Q_{1,2}} = m_{U_{1,2}} = m_{D_{1,2}} = m_{L_{1,2}} = m_{E_{1,2}} \equiv m_{16}(1,2)$ then $\sim 0.007m_{16}^2(1,2)$

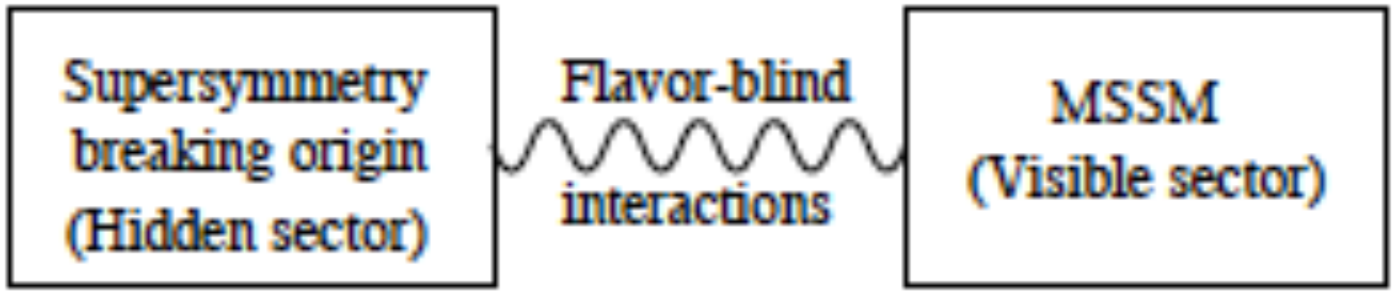
Even better: $m_{H_u}^2 = m_{H_d}^2 = m_{16}^2(3) \equiv m_0^2$ => $-0.017m_0^2$

For correlated parameters, EWFT collapses in 3rd gen. sector!

- Usually Δ_{BG} is applied to *multi-parameter effective theories* where multiple soft terms are adopted as parameter set.
- For these theories, the multiple soft terms parametrize our ignorance of details of the hidden sector SUSY breaking.
- But in supergravity, for any given hidden sector, soft terms are all *dependent* and can be computed as multiples of $m_{3/2}$.

Thus, the usual evaluation of Δ_{BG} also violates the prime directive!

To properly apply BG measure, need to identify independent soft breaking terms



examine gravity mediation

For any particular SUSY breaking hidden sector, each soft term is some multiple of gravitino mass m(3/2)

$$
\begin{aligned}
m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\
m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\
A_t &= a_{A_t} \cdot m_{3/2}, \\
M_i &= a_i \cdot m_{3/2}, \\
& \cdots .
\end{aligned}
$$

Soni, Weldon (1983);
Kaplunovsky, Louis (1992);
Brignole, Ibanez, Munoz (1993)

Since we don't know hidden sector, we impose parameters which parameterize our ignorance:

but this doesn't mean each parameter is independent

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3} m_{3/2}$

Writing each soft term as a multiple of m(3/2) then we allow for correlations/cancellations:

GUT scale param's

$$m_Z^2 = -2.18\mu^2 + a \cdot m_{3/2}^2$$

numerical co-efficient which depends on hidden sector

for naturalness, then

$\mu^2 \sim m_Z^2$ and $a \cdot m_{3/2}^2 \sim m_Z^2$

either $m_{3/2} \sim m_Z$ *or* a is small

$$m_Z^2 \simeq -2\mu^2(weak) - 2m_{H_u}^2(weak) \simeq -2.18\mu^2(GUT) + a \cdot m_{3/2}^2$$

then $-m_{H_u}^2(weak) \sim a \cdot m_{3/2}^2 \sim m_Z^2$

$$\lim_{n_{SSB}\to 1} \Delta_{BG} \to \Delta_{EW}$$

Thus, correctly applying these measures by first collecting dependent quantities, we find that- at tree level- all agree:

$$\Delta_{HS} \simeq \Delta_{BG} \simeq \Delta_{EW}$$

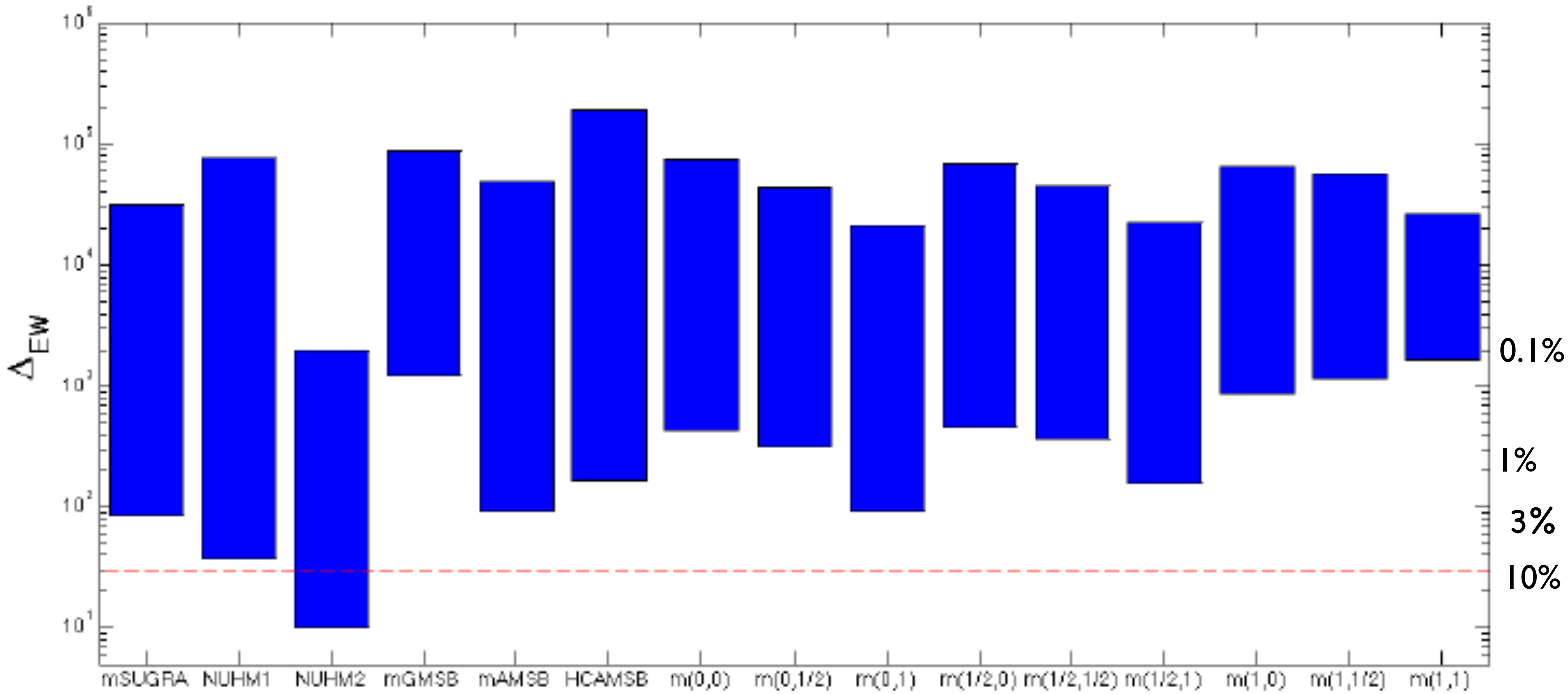
Due to ease of use and including radiative corrections, and due to its explicit model independence, we will use

$$\Delta_{EW}$$

for remainder of talk

Often claimed that Δ_{EW} doesn't include high scale effects: not true: it selects out high scale models which can naturally produce m(W,Z,h)~100 GeV

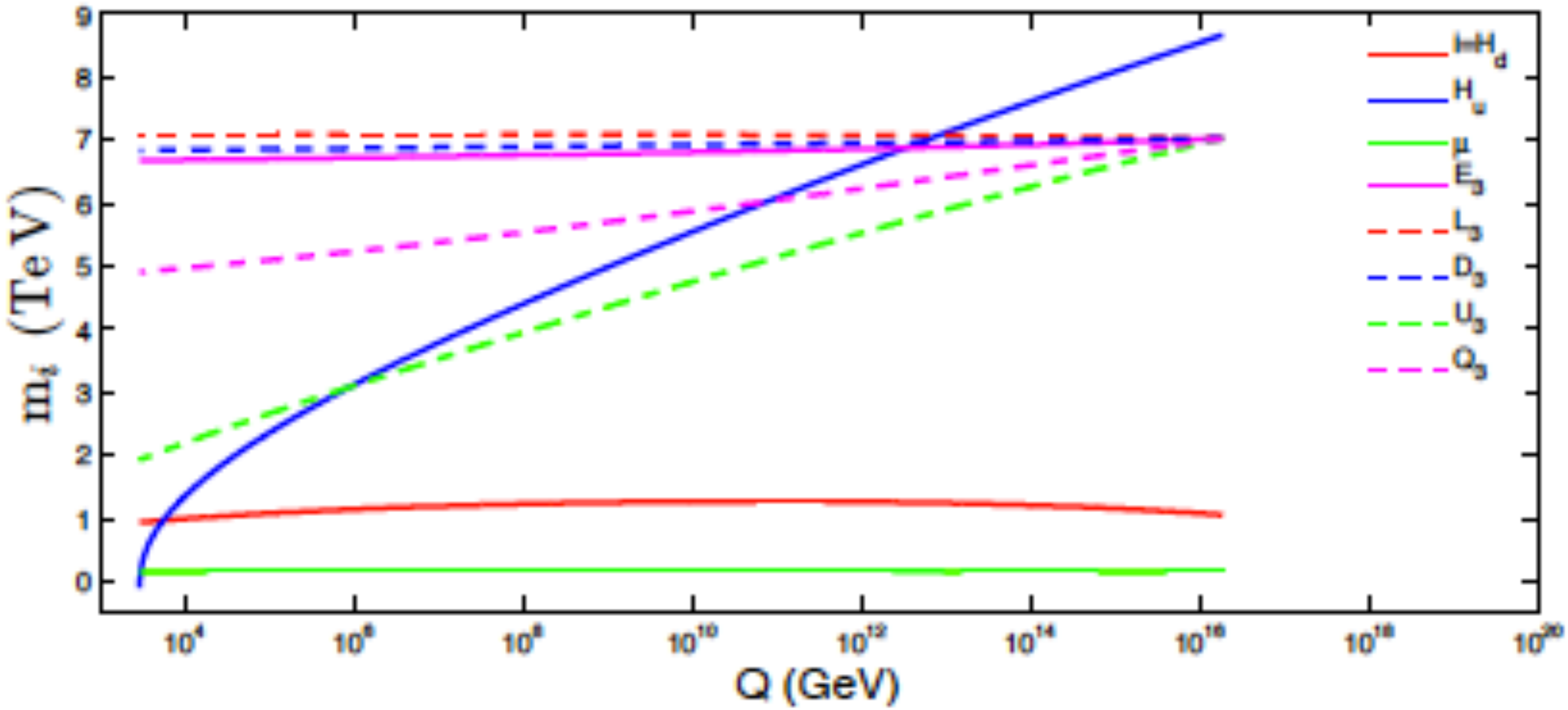
scan over p-space with m(h)=125.5+-2.5 GeV:



need large A_t and some non-universality e.g. NUHM2 model

HB, Barger, Mickelson,Padeffke-Kirkland, PRD89 (2014) 115019

Applied properly, all three measures agree:
naturalness is unambiguous and highly predictive!

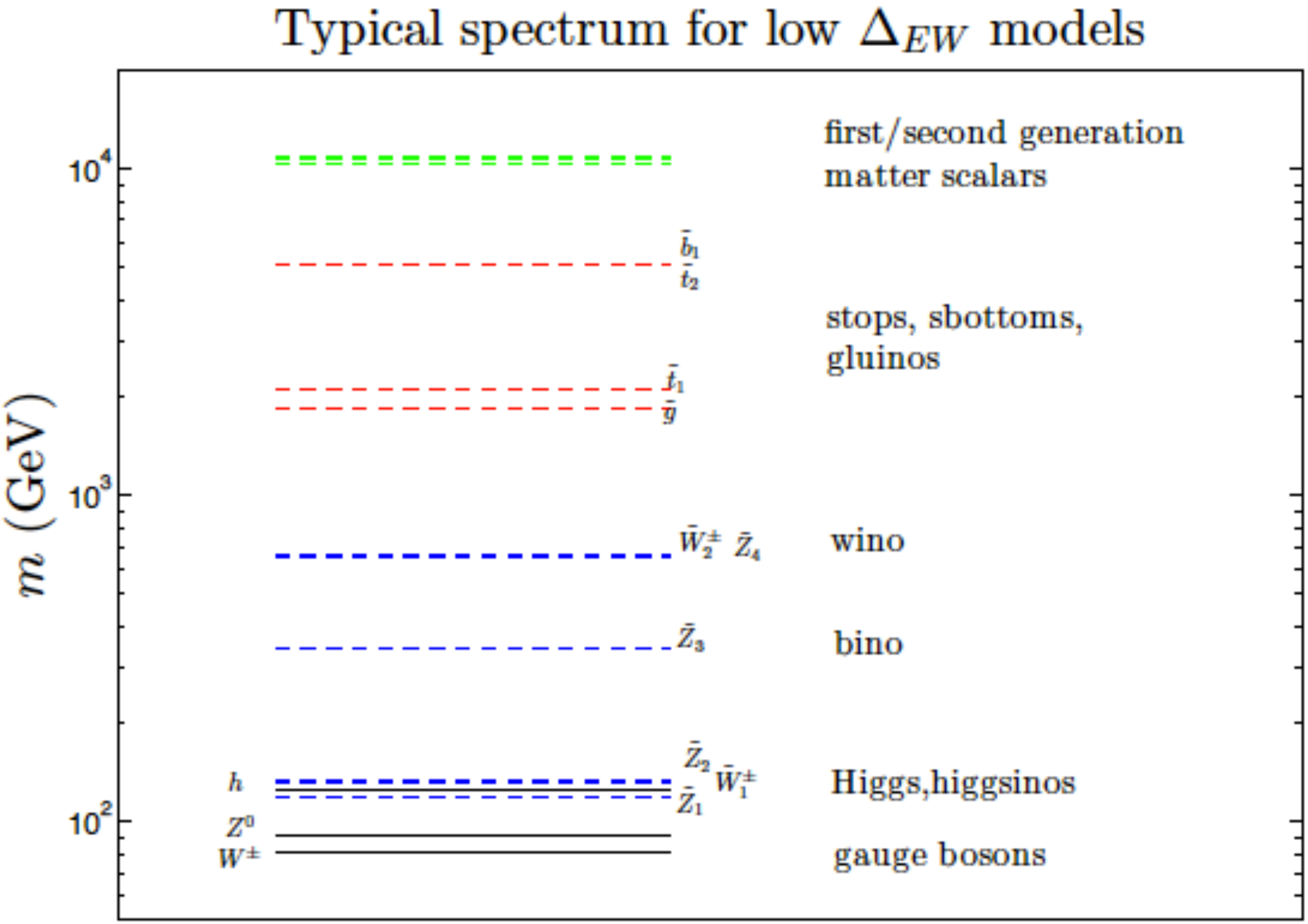


Radiatively-driven natural SUSY, or RNS:

(typically need mHu~25-50% higher than m0)

H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Phys. Rev. Lett.* **109** (2012) 161802.

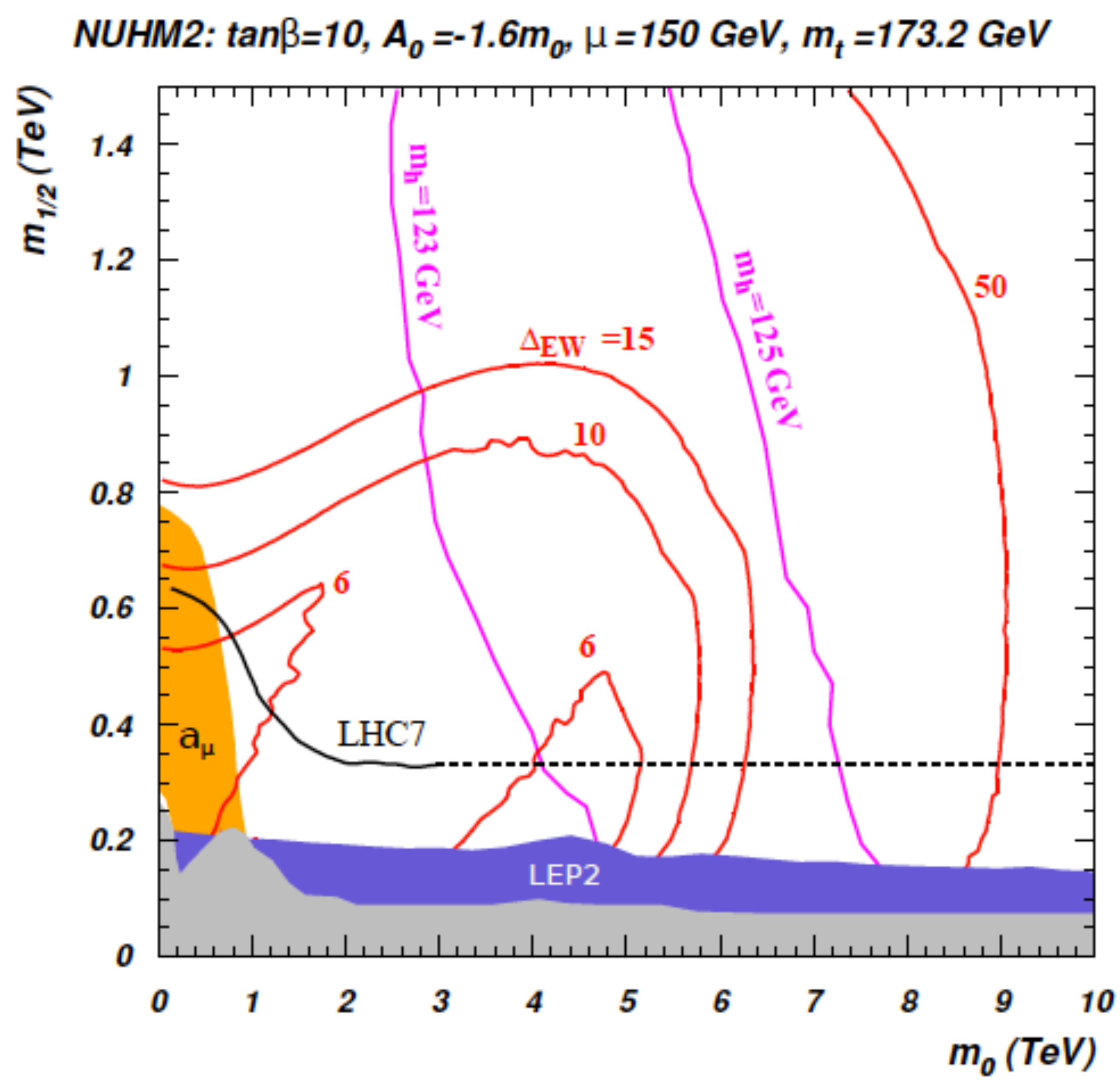
H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Phys. Rev.* **D 87** (2013) 115028 [arXiv:1212.2655 [hep-ph]].



There is a Little Hierarchy, but it is no problem

$m^2_{H_u}$ is radiatively driven to natural values and $\mu \ll m_{3/2}$

Good old m0 vs. mhf plane still viable, but require low mu (NUHM2)



$\mu = 150$ GeV throughout which is allowed for NUHM2

SUSY mu problem: mu term is SUSY, not SUSY breaking: expect mu~M(Pl) but phenomenology requires mu~m(Z)

- NMSSM: mu~m(3/2); beware singlets!
- Giudice-Masiero: mu forbidden by some symmetry: generate via Higgs coupling to hidden sector
- Kim-Nilles: invoke SUSY version of DFSZ axion solution to strong CP:

KN: PQ symmetry forbids mu term, but then it is generated via PQ breaking

$$W_{DFSZ} \ni \lambda S^2 H_u H_d / M_P$$

$$\mu \sim \lambda f_a^2 / M_P$$

Little Hierarchy due to mismatch between PQ breaking and SUSY breaking scales?

$$m_{3/2} \sim m_{hid}^2 / M_P$$

$$f_a \ll m_{hid}$$

Higgs mass tells us where to look for axion!

$$m_a \sim 6.2 \mu\mathrm{eV} \left(\frac{10^{12}\ \mathrm{GeV}}{f_a} \right)$$

Little Hierarchy from radiative PQ breaking? explore within context of MSY model

Murayama, Suzuki, Yanagida (1992);
Gherghetta, Kane (1995)

Choi, Chun, Kim (1996)

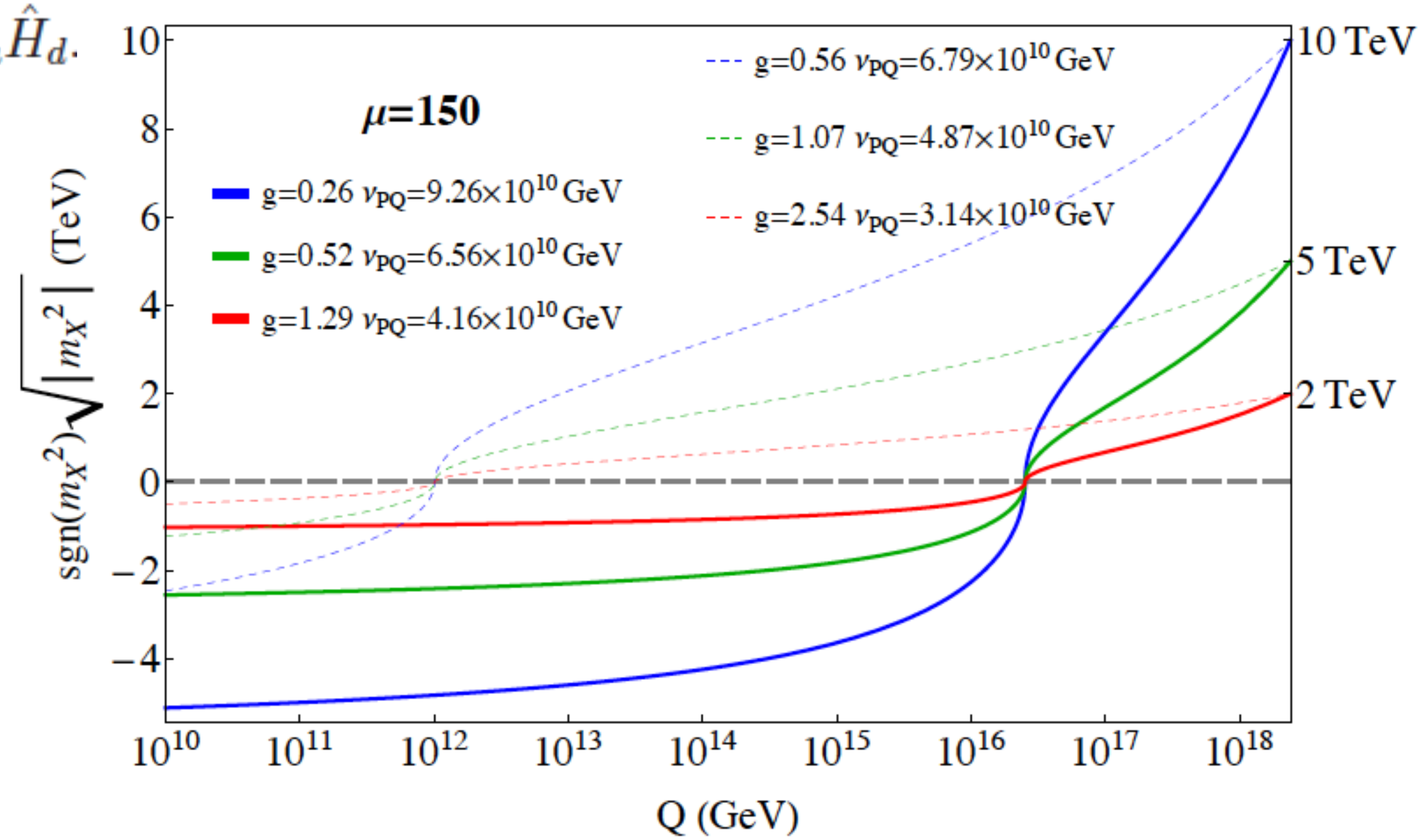
Bae, HB, Serce, PRD91 (2015) 015003

augment MSSM with PQ charges/fields:

$$\hat{f}' = \frac{1}{2} h_{ij} \hat{X} \hat{N}_i^c \hat{N}_j^c + \frac{f}{M_P} \hat{X}^3 \hat{Y} + \frac{g}{M_P} \hat{X} \hat{Y} \hat{H}_u \hat{H}_d.$$

$$M_{N_i^c} = v_X \; h_i|_{Q=v_X}$$

$$\mu = g \frac{v_X v_Y}{M_P} \; .$$

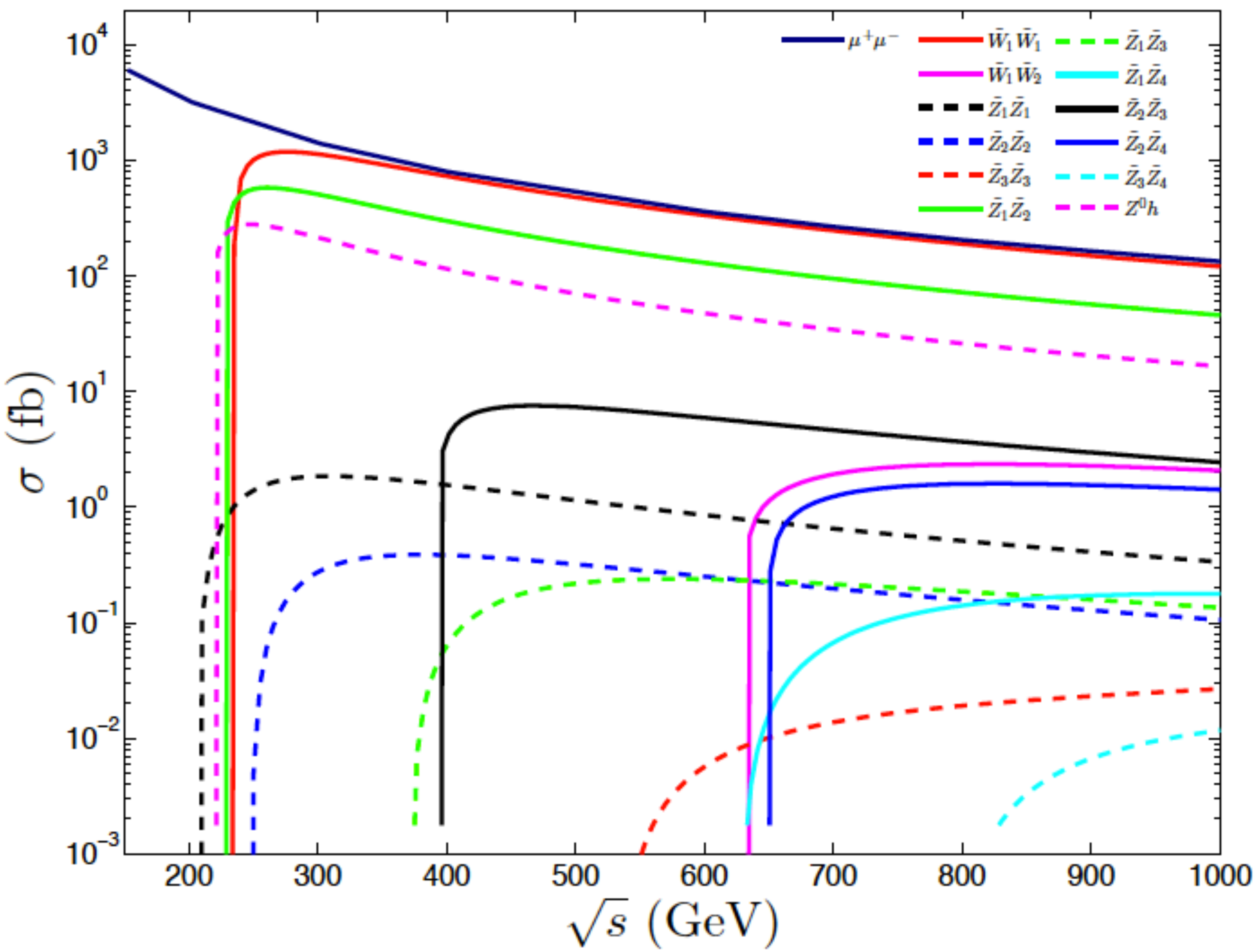


Large $m_{3/2}$ generates small $\mu \sim 100 - 200$ GeV!

Smoking gun signature: light higgsinos at ILC:

ILC is Higgs/higgsino factory!

ILC1: $m_0 = 7025$ GeV, $m_{1/2} = 568.3$ GeV, $A_0 = -11426.6$ GeV, $\tan\beta = 10$, $\mu = 115$ GeV, $m_A = 1000$ GeV



compressed higgsino spectrum very hard to see at LHC

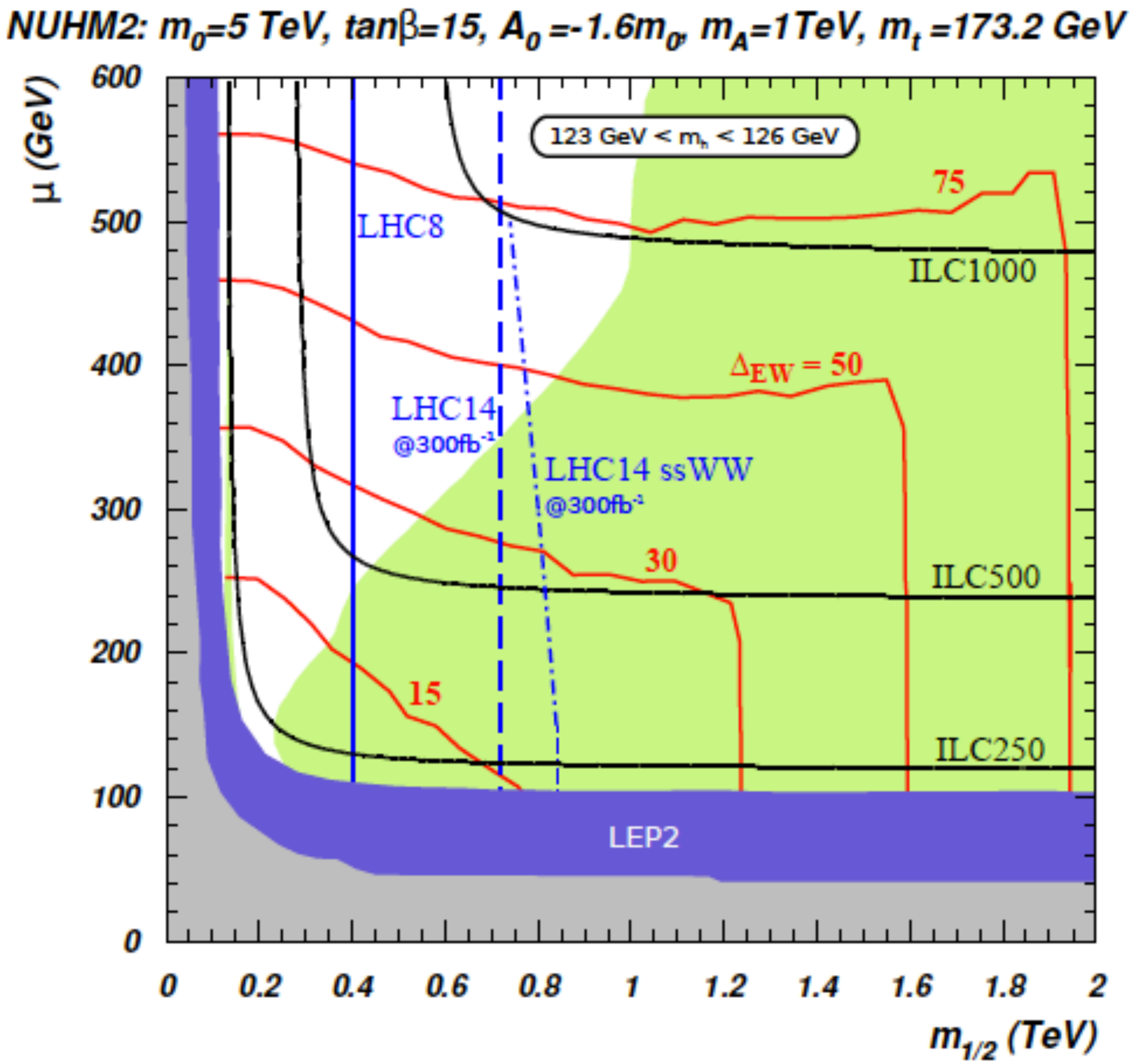
10-20 GeV higgsino mass gaps are no problem in clean ILC environment

$$\sigma(higgsino) \gg \sigma(Zh)$$

HB, Barger, Mickelson, Mustafayev, Tata

ILC either sees light higgsinos or natural SUSY dead

LHC/ILC complementarity

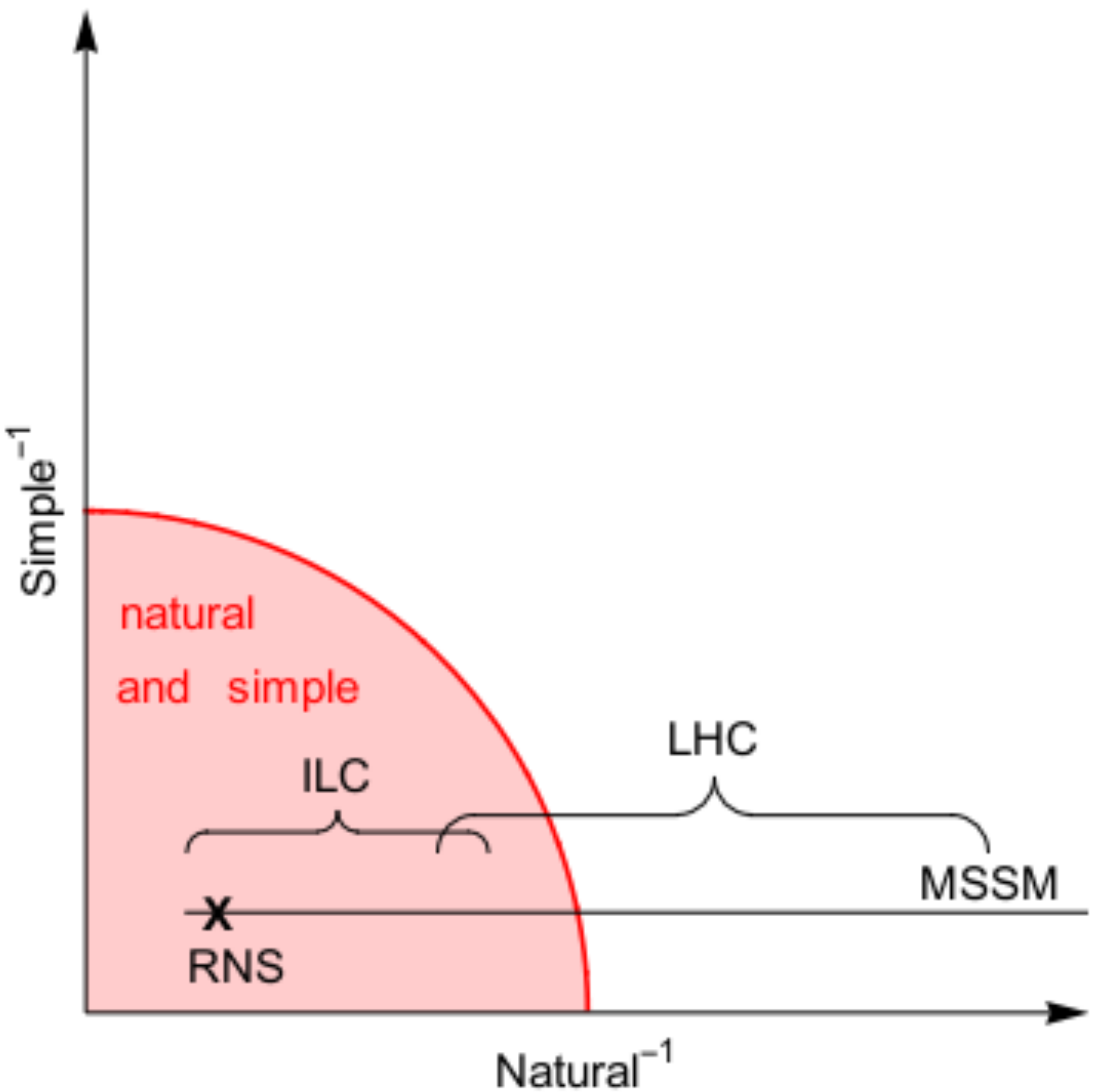


When to give up on naturalness in SUSY?
If ILC(500-600 GeV) sees no light higgsinos

The best reason to build ILC:

discovery of SUSY

in its most natural, parsimonius form!



But so far we have addressed only Part 1
of fine-tuning problem:

In QCD sector, the term $\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu}\tilde{F}_A^{\mu\nu}$ must occur

But neutron EDM says it is not there: strong CP problem

(frequently ignored by SUSY types)

Best solution after 35 years:
PQWW/KSVZ/DFSZ invisible axion

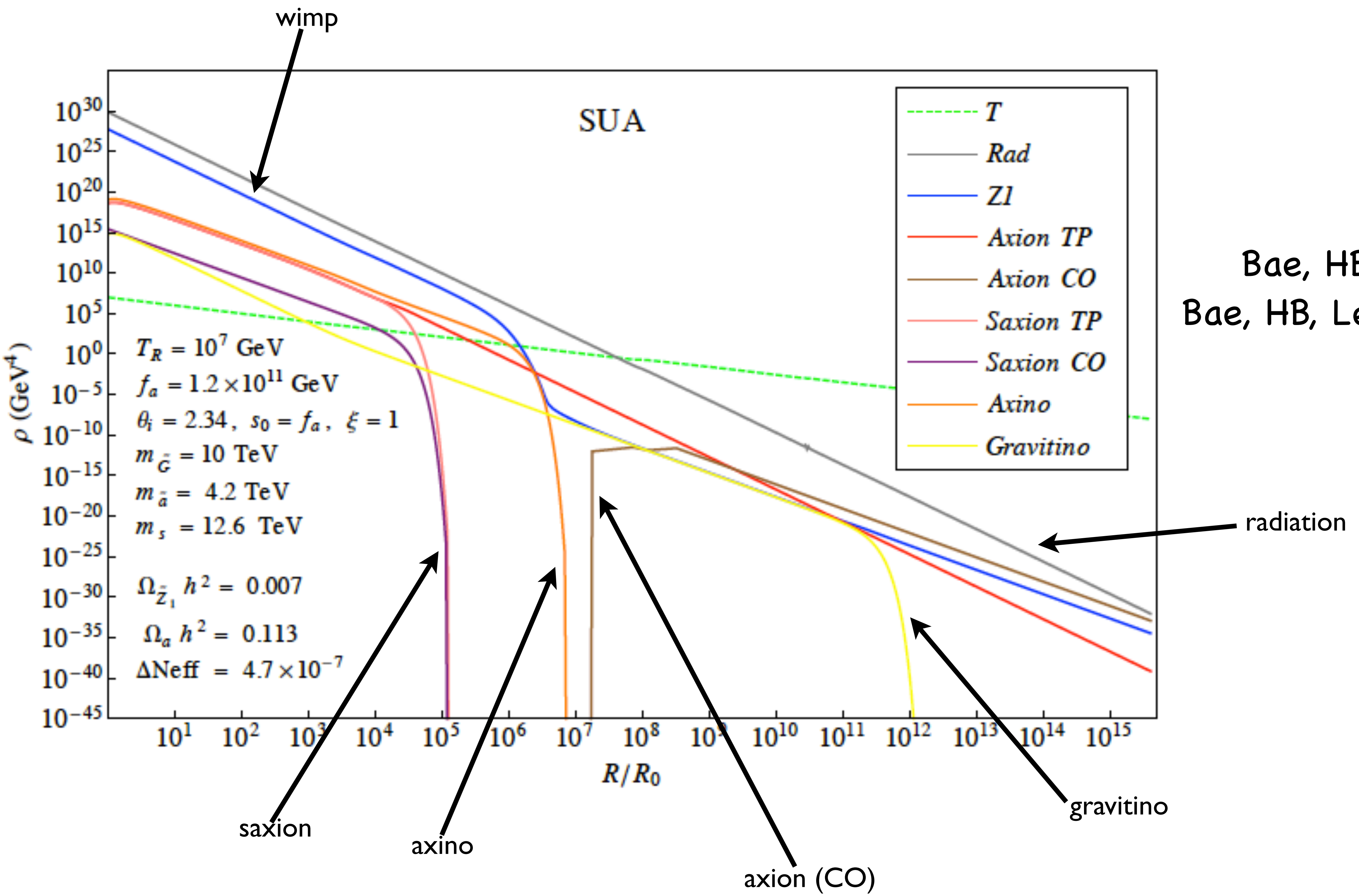
In SUSY, axion accompanied by axino and saxion

Changes DM calculus:
expect mixed WIMP/axion DM (2 particles)

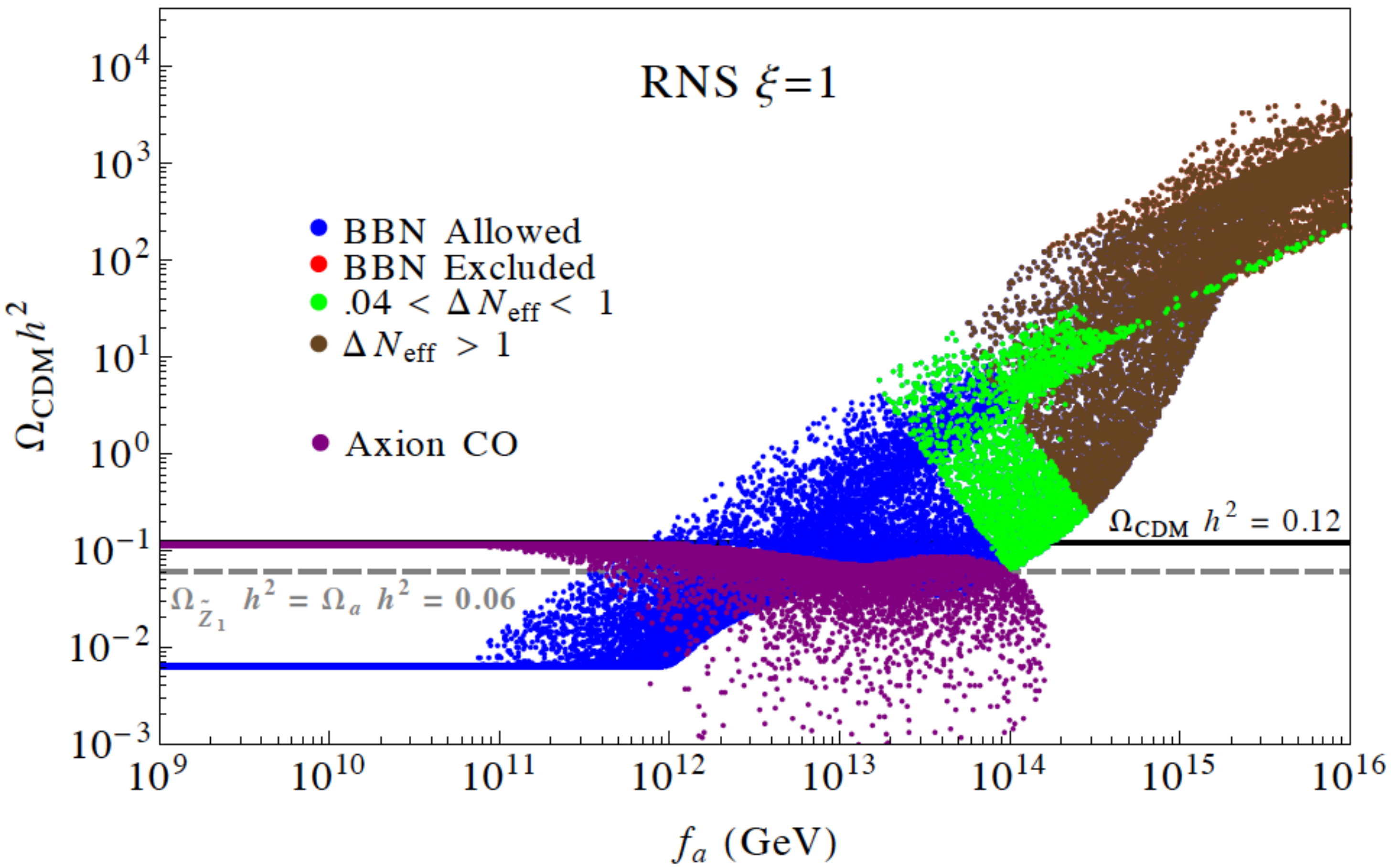
mixed axion-neutralino production in early universe

- neutralinos: thermally produced (TP) or NTP via $\tilde{a}$, s or $\tilde{G}$ decays
  - re-annihilation at $T_D^{s,\tilde{a}}$
- axions: TP, NTP via $s \to aa$, bose coherent motion (BCM)
- saxions: TP or via BCM
  - $s \to gg$: entropy dilution
  - $s \to SUSY$: augment neutralinos
  - $s \to aa$: dark radiation ($\Delta N_{eff} < 1.6$)
- axinos: TP
  - $\tilde{a} \to SUSY$ augments neutralinos
- gravitinos: TP, decay to SUSY

DM production in SUSY DFSZ: solve eight coupled Boltzmann equations

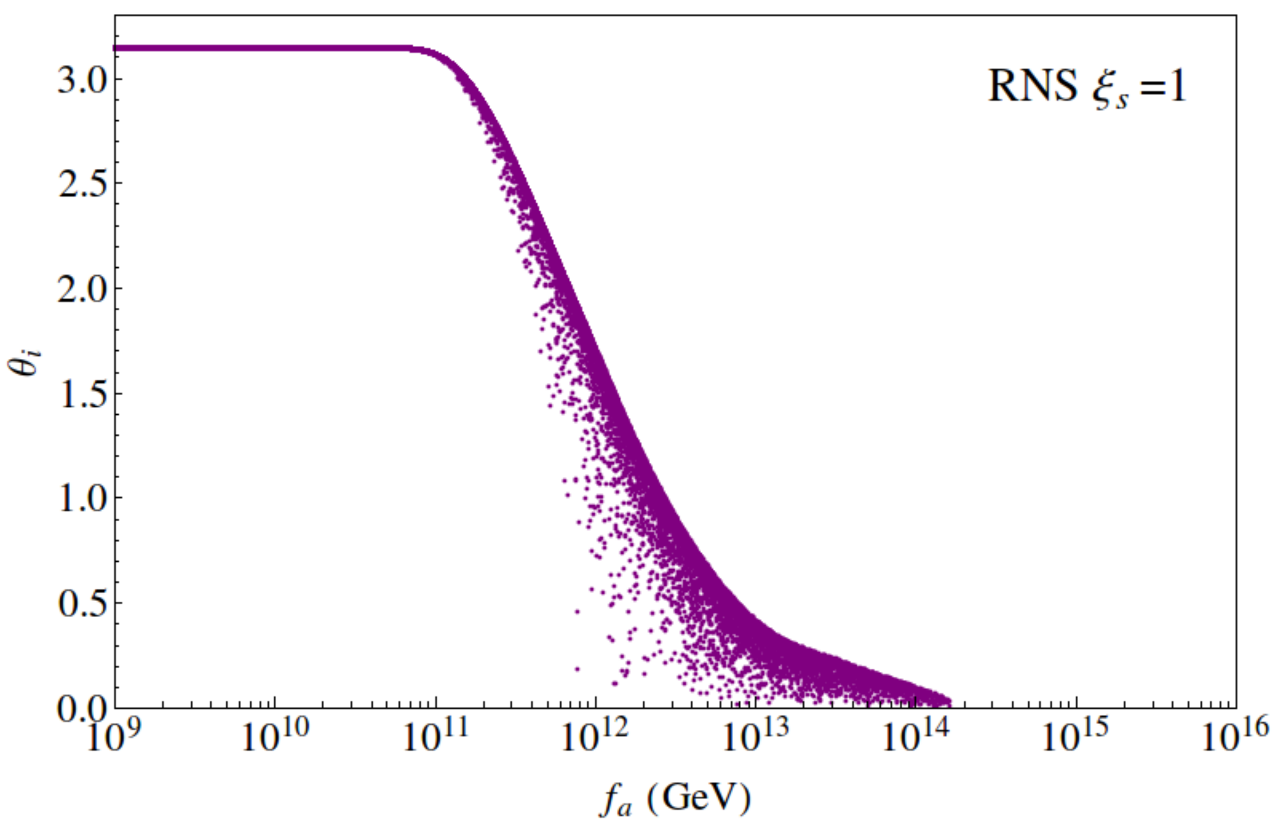


Bae, HB, Chun;
Bae, HB, Lessa, Serce



higgsino abundance

axion abundance



mainly axion CDM

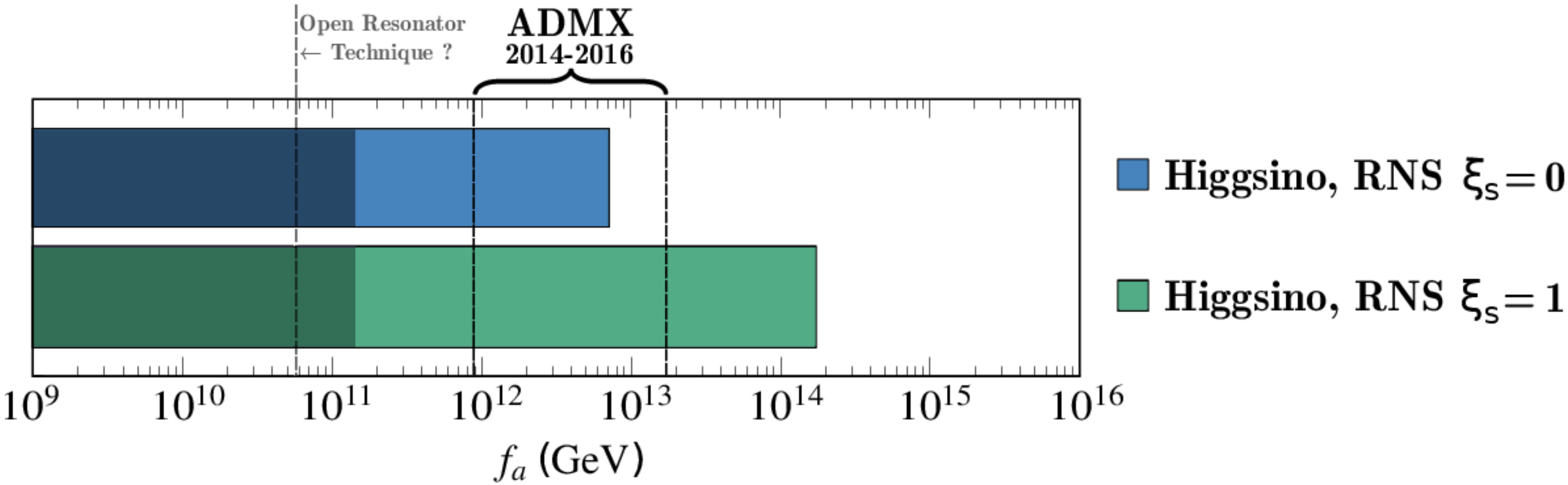
for fa<~10^12 GeV;

for higher fa, then

get increasing wimp

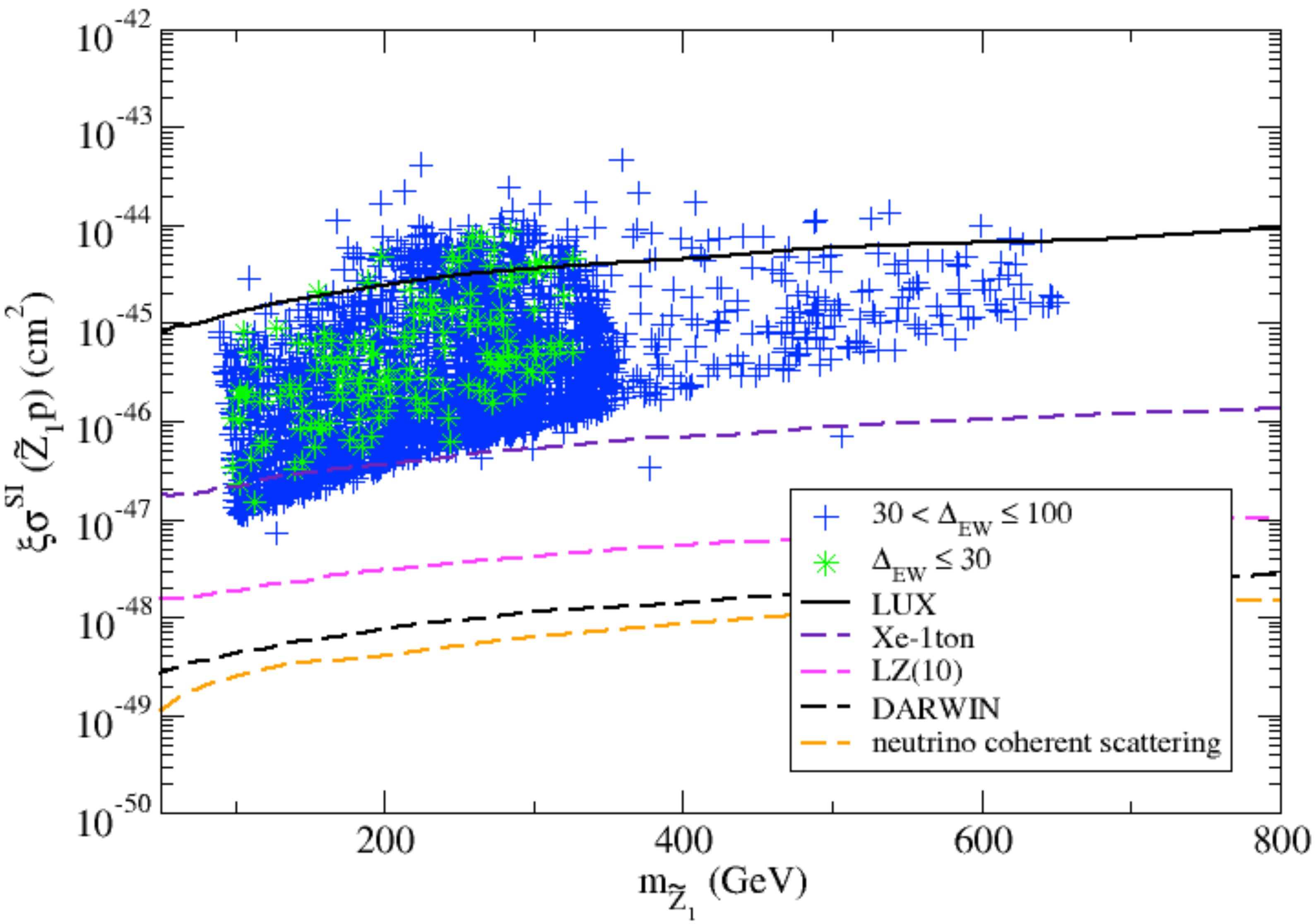
abundance

Bae, HB,Lessa,Serce



range of f_a expected from SUSY
with radiatively-driven naturalness
compared to ADMX axion reach

Direct higgsino detection rescaled for minimal local abundance



Bae, HB, Barger,Savoy,Serce

$$\mathcal{L} \ni -X_{11}^h \overline{\widetilde{Z}}_1 \widetilde{Z}_1 h$$

$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)} \sin\alpha - v_1^{(1)} \cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

Deployment of Xe-1ton, LZ, SuperCDMS coming soon!

Can test completely with ton scale detector or equivalent (subject to minor caveats)

Conclusions: status of SUSY post LHC8

- SUSY EWFT non-crisis: EWFT allowed at 10% level in radiatively-driven natural SUSY: SUGRA GUT paradigm is just fine in NUHM2 but CMSSM/others fine-tuned
- naturalness maintained for mu~100-200 GeV; t1~1-2 TeV, t2~2-4 TeV, highly mixed; m(glno)~1-5 TeV
- LHC14 w/ 300 fb^-1 can see about half of RNS parameter space
- e+e- collider with sqrt(s)~500-600 GeV needed to find predicted light higgsino states
- Discovery of and precision measurements of light higgsinos at ILC!
- RNS spectra characterized by mainly higgsino-like WIMP: standard relic underabundance
- SUSY DFSZ/MSY invisible axion model:
  solves strong CP and mu problems while allowing for mu~m(Z)
- Expect mainly axion CDM with 5-10% higgsino-like WIMPs over much of p-space
- Ultimately detect both axion and higgsino-like WIMP

Naturalness in the Standard Model

SM case: invoke a single Higgs doublet

$$V = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

$$m_h^2 = m_h^2|_{tree} + \delta m_h^2|_{rad}$$

$$m_h^2|_{tree} = 2\mu^2 \qquad \delta m_h^2|_{rad} \simeq \frac{3}{4\pi^2}\left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda\right)\Lambda^2$$

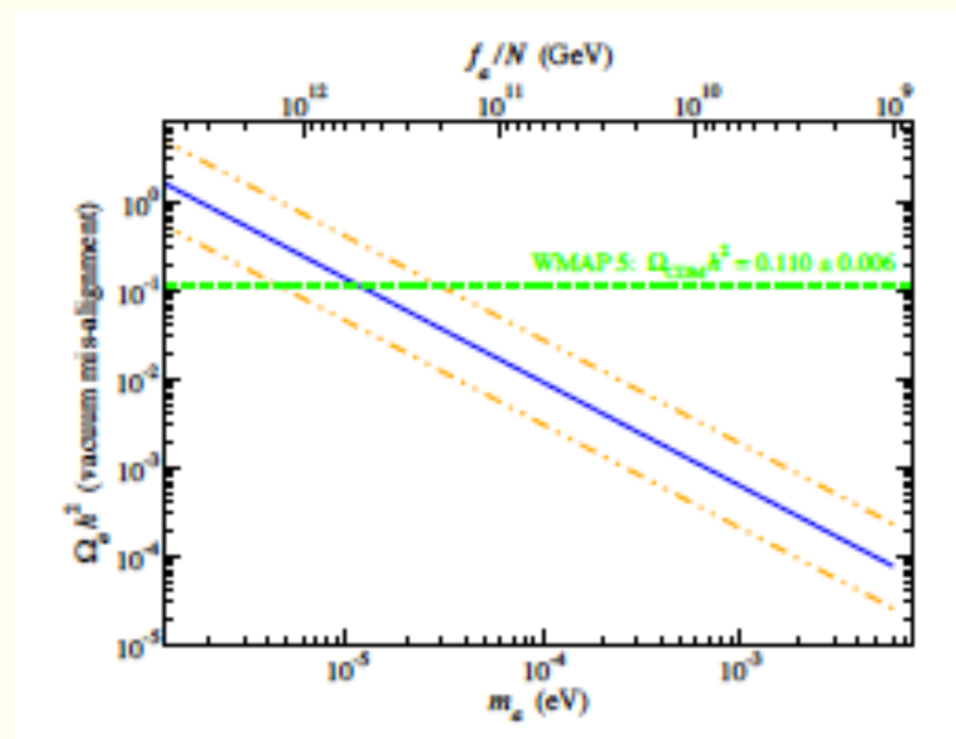
$m_h^2|_{tree}$ and $\delta m_h^2|_{rad}$ are *independent*,

If δm_h^2 blows up, can freely adjust (tune) $2\mu^2$ to maintain $m_h = 125.5$ GeV

$$\Delta_{SM} \equiv \delta m_h^2|_{rad}/(m_h^2/2) \qquad \Delta_{SM} < 1 \Rightarrow \ \Lambda \sim 1\ TeV$$

Axion cosmology

- ★ Axion field eq'n of motion: $\theta = a(x)/f_a$
  - $\ddot{\theta} + 3H(T)\dot{\theta} + \frac{1}{f_a^2}\frac{\partial V(\theta)}{\partial \theta} = 0$
  - $V(\theta) = m_a^2(T) f_a^2 (1 - \cos\theta)$
  - Solution for T large, $m_a(T) \sim 0$:
    $\theta = const.$
  - $m_a(T)$ turn-on ~ 1 GeV
- ★ a(x) oscillates,
  creates axions with $\vec{p} \sim 0$:
  production via vacuum mis-alignment



- ★ $\Omega_a h^2 \sim \frac{1}{2}\left[\frac{6\times10^{-6} eV}{m_a}\right]^{7/6} \theta_i^2 h^2$
- ★ astro bound: stellar cooling $\Rightarrow f_a \gtrsim 10^9 GeV$