UNIVERSITY OF LIVERPOOL

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- Introduction and overview
- SM prediction of g-2
- a_μ^{HVP}: status/puzzles/outlook

Introduction & motivation: EDMs and MDMs

SM `too' successful, but incomplete:

- ν masses (small) and mixing point towards some high-scale (GUT) physics, so LFV in neutral sector established, but no Charged LFV & EDMs seen so far
- Need to explain dark matter
- Not enough CP violation in the SM for matter-antimatter asymmetry
- And: $a_\mu^{EXP} - a_\mu^{SM}$ at 3.x σ

Is there a common New Physics (NP) explanation for all these puzzles?

- Uncoloured leptons are particularly `clean' probes to establish and constrain/ distinguish NP, complementary to high energy searches at the LHC

- No direct signals for NP from LHC so far:
  - some models like CMSSM are in trouble already when trying to accommodate LHC exclusion limits and to solve muon g-2
  - is there any TeV scale NP out there? Or new low scale physics?

Low energy observables may provide the key

Introduction: Lepton Dipole Moments

- Dirac equation (1928) combines non-relativistic Schroedinger Eq. with rel. Klein-Gordon Eq. and describes spin-1/2 particles and interaction with EM field $A_\mu(x)$:

$$(i\partial_\mu + eA_\mu(x))\,\gamma^\mu\psi(x) \;=\; m\,\psi(x)$$

with gamma matrices $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}I$ and 4-spinors ψ(x).

- Great success: Prediction of anti-particles and magnetic moment $\vec{\mu} = g\frac{Qe}{2m}\vec{s}$ with g = 2 (and not 1) in agreement with experiment.

- Dirac already discussed electric dipole moment together with MDM: $\vec{\mu}\cdot\vec{H} + i\rho_1\vec{\mu}\cdot\vec{E}$ but discarded it because imaginary.

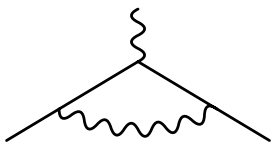
- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with g_s= 2.00229 ± 0.00008.

Introduction: Lepton Dipole Moments

- 1948: Schwinger calculates the famous radiative correction:

  that g = 2 (1+a), with

  a = (g-2)/2 = α/(2π) = 0.001161



This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

`` If you can't join 'em, beat 'em "

- The anomaly a (Anomalous Magnetic Moment) is from the Pauli term:

$$\delta\mathcal{L}_{\rm eff}^{\rm AMM} = -\frac{Qe}{4m} a\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)F_{\mu\nu}(x)$$

This is a dimension 5 operator, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But it occurs through radiative corrections and is calculable in perturbation theory.

- Similarly, an EDM can come from a term $\delta\mathcal{L}_{\rm eff}^{\rm EDM} = -\frac{d}{2}\bar{\psi}(x)\, i\, \sigma^{\mu\nu}\gamma_5\psi(x)F_{\mu\nu}(x)$

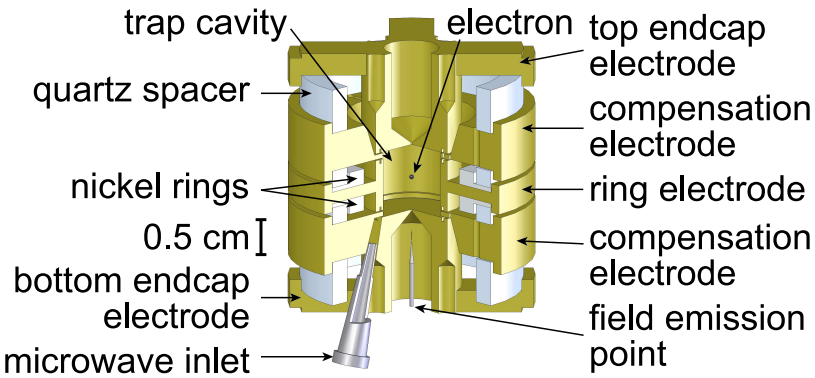
Magnetic Moments: a_e vs. a_μ

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]

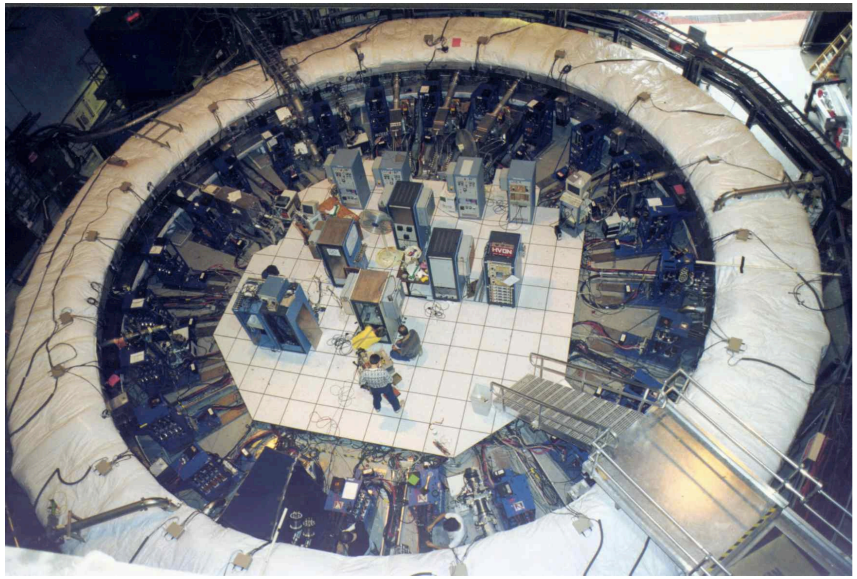
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]

Bennet et al., PRD 73(2006)072003



one electron quantum cyclotron



- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales

- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{\rm NP} \sim \mathcal{C}\, m_\ell^2/\Lambda_{\rm NP}^2$

→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, but a_e provides best determination of α

Magnetic Moments: a_e^{SM}

- General structure: $a_e^{\rm SM} = a_e^{\rm QED} + a_e^{\rm hadronic} + a_e^{\rm weak}$

- Weak and hadronic contributions suppressed as induced by particles heavy compared to electron, hence a_e^{SM} dominated by QED → Kinoshita-san's work

$a_e^{SM} = 1\,159\,652\,181.78(77) \times 10^{-12}$ [Aoyama+Hayakawa+Kinoshita+Nio, PRL 109(2012)111807]

including 5-loop QED and using α measured with Rubidium atoms [α to 0.66 ppb]
[Bouchendira et al., PRL106(2011)080801; Mohr et al., CODATA, Rev Mod Phys 84(2012)1527]

Of this only

$a_e^{\text{had, LO VP}} = 1.875(18) \times 10^{-12}$ [or our newer $1.866(11) \times 10^{-12}$]

$a_e^{\text{had, NLO VP}} = -0.225(5) \times 10^{-12}$ [or our newer $-0.223(1) \times 10^{-12}$]

$a_e^{\text{had, L-by-L}} = 0.035(10) \times 10^{-12}$

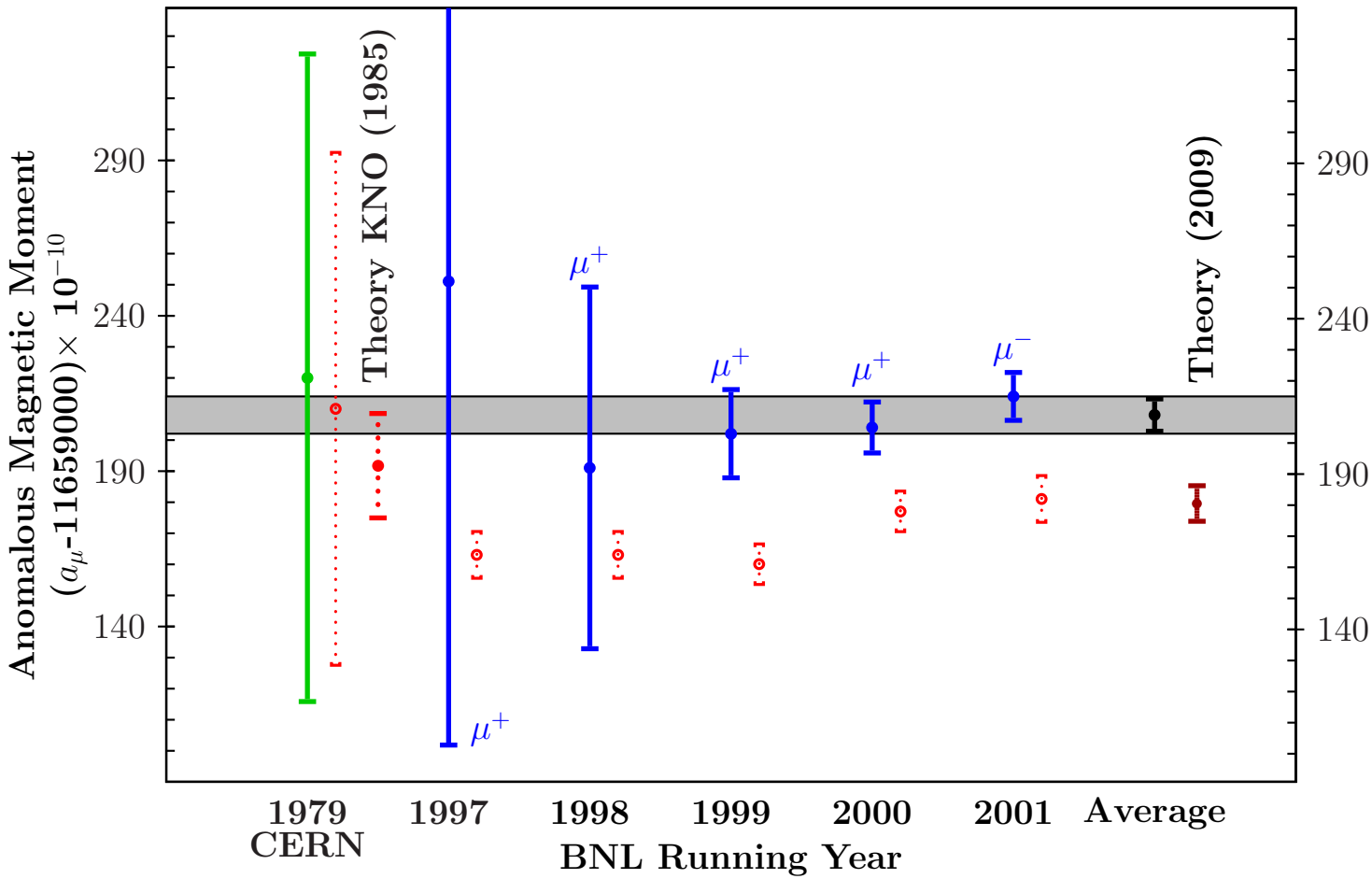
$a_e^{\text{weak}} = 0.0297(5) \times 10^{-12}$,

whose calculations are a byproduct of the μ case which I will discuss in more detail.

- In turn a_e^{EXP} and a_e^{SM} can be used to get the most precise determination of α, to 0.25 ppb, consistent with Rubidium experiment and other determinations.

Magnetic Moments: a_μ

g-2 history plot and
motto from Fred Jegerlehner's book: `The closer you look the more there is to see'



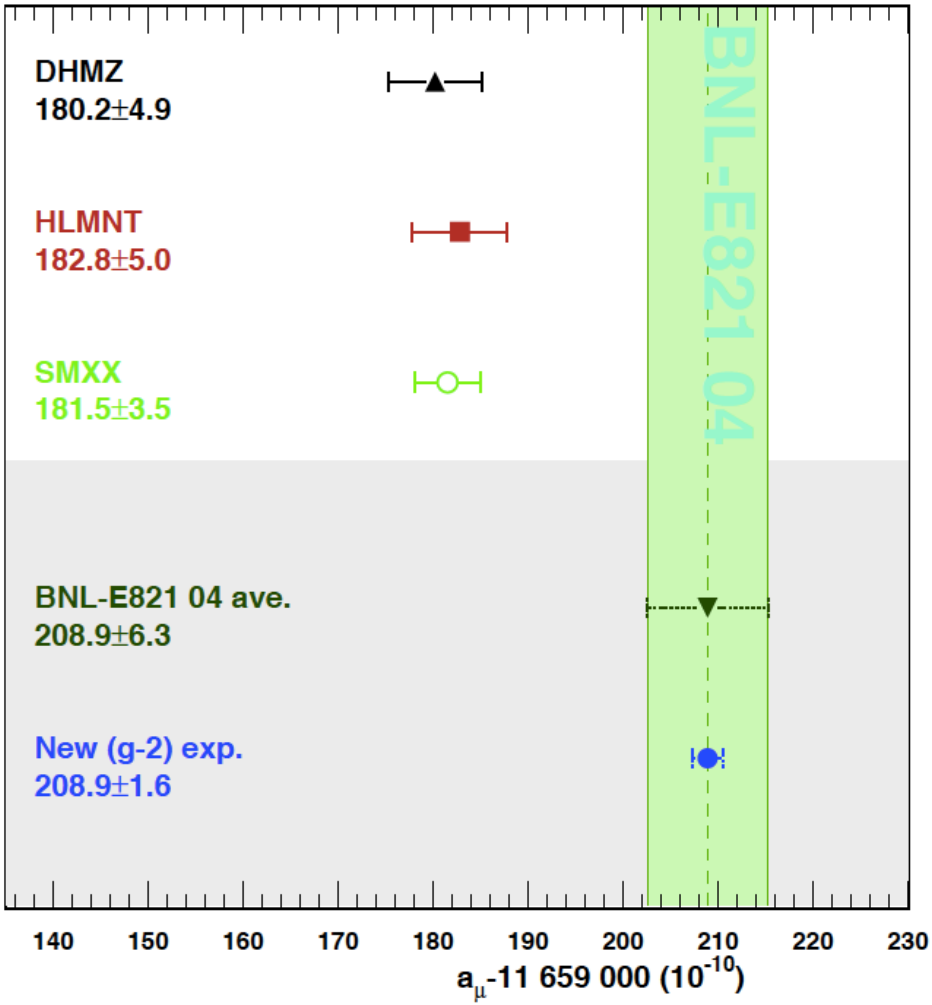
a_μ: Charge from new EXPs for the TH prediction

Future picture:

- if mean values stay and with no a_μ^{SM} improvement: 5σ discrepancy

- if also EXP+TH can improve a_μ^{SM} `as expected' (consolidation of L-by-L on level of `Glasgow consensus', about factor 2 for HVP): NP at 7-8σ

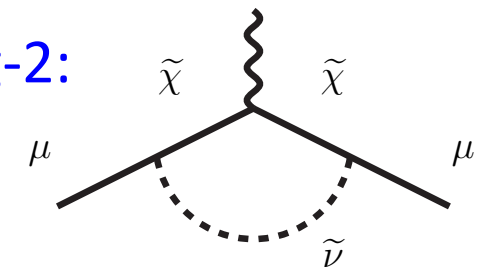
- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...

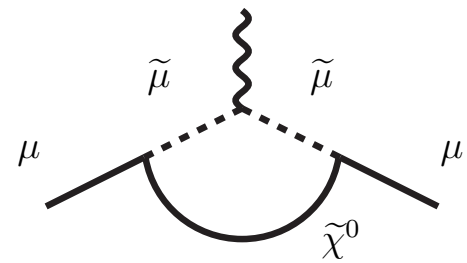


New Physics? (Non-minimal) SUSY?

SUSY could easily explain g-2:

Main 1-loop contributions:





Simplest case:

$$a_\mu^{\rm SUSY} \simeq sgn(\mu)\, 130 \times 10^{-11} \tan\beta \left(\frac{100\,{\rm GeV}}{\Lambda_{\rm SUSY}}\right)^2$$

- Needs μ>0, `light' SUSY-scale Λ and/or large tan β to explain 260 x 10^{-11}

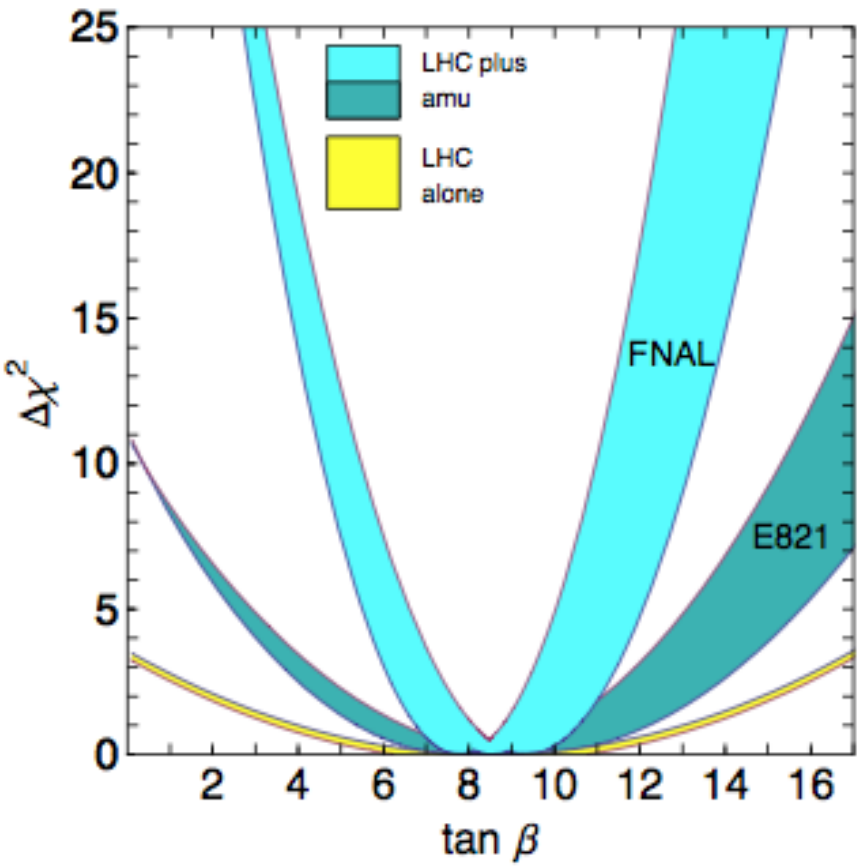
- This is already excluded by LHC searches in the simplest SUSY scenarios (like CMSSM); causes large χ^2 in simultaneous SUSY-fits with LHC data and g-2

- However note: SUSY does not have to be minimal (w.r.t. Higgs), could have large mass splittings (with lighter sleptons), or corrections (to g-2 and Higgs mass) different from simple models, or not be there at all

- g-2 constrains params, distinguishes between NP models `degenerate' for LHC

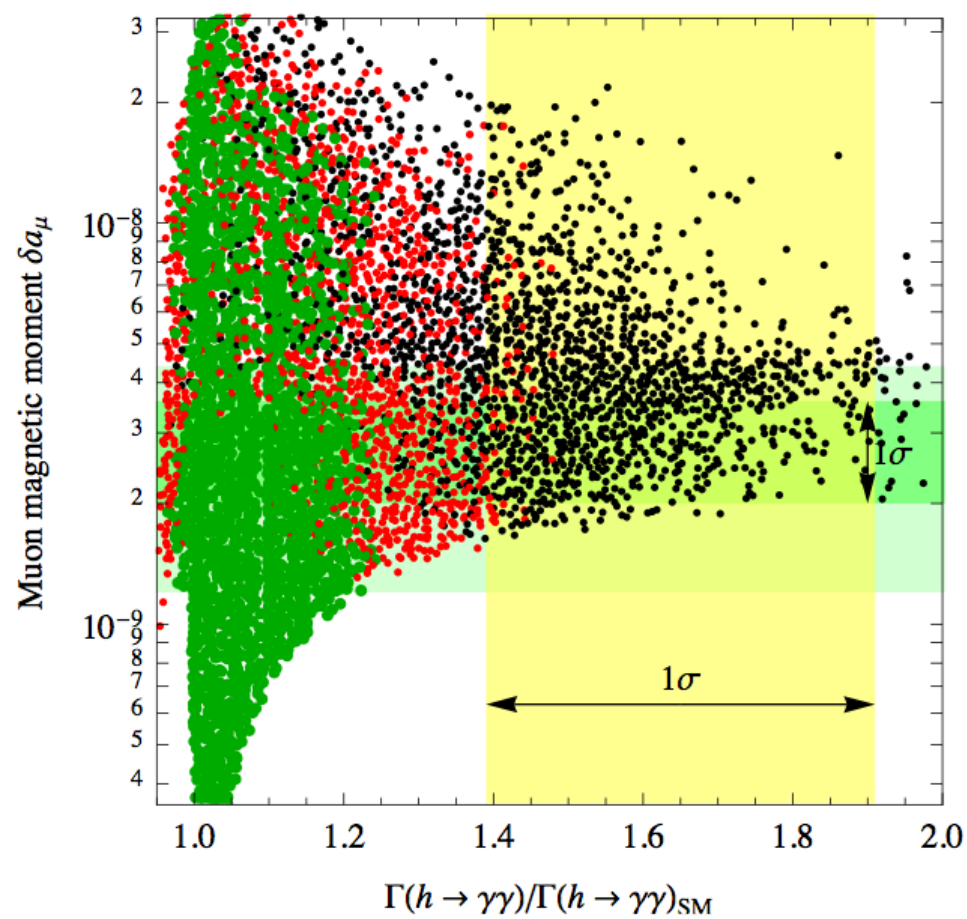
Precision $(g\text{-}2)_\mu$ constrains SUSY

LHC with (100 fb^{-1}) can determine tan(β) to 50%, with g-2 to 10%



Miller, de Rafael, Roberts and Stöckinger, Ann. Rev. Nucl. Part. Sci. 62 (2012) 237

g-2 complements LHC data selecting in the vast SUSY (param/model) space



Guidice, Paradisi, Strumia JHEP 1210, 186

a_μ^{SM}: overview and status

$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{hadronic}} + a_\mu^{\mathrm{NP?}}$$

Have to make progress soon soon as new experiments are becoming reality:

E989 Collaboration Nov. 2014

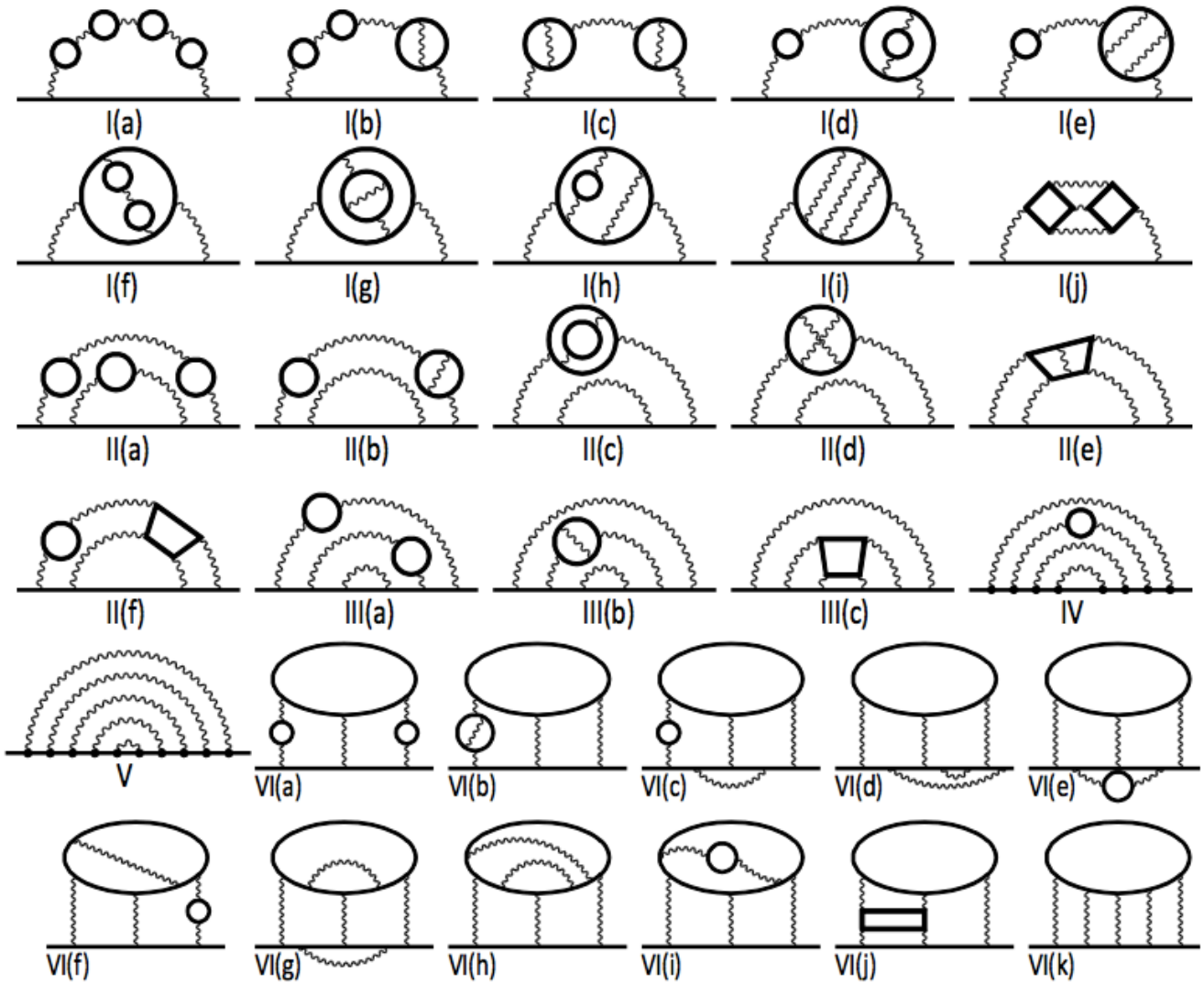
… and soon after

a_μ^{QED} Kinoshita et al: g-2 at 5-loop order

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

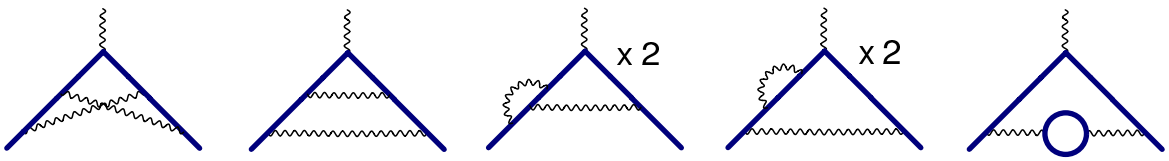
A triumph for perturbative QFT and computing!

10th
12672
diagrams



a_μ^{QED}

- Schwinger 1948: 1-loop $a = (g-2)/2 = \alpha/(2\pi) = 11\,614\,097 \times 10^{-10}$
- 2-loop graphs:

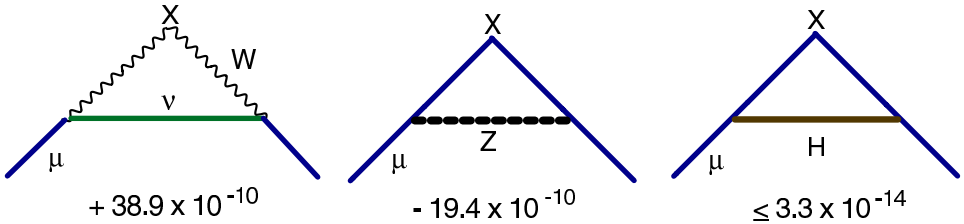


- 72 3-loop and 891 4-loop diagrams …
- Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams)
- Some graphs known analytically (Laporta; Aguilar et al.)
- Recently several independent checks of specific 4-loop and 5-loop diagrams: Steinhauser et al. [NPB 879 (2014) 1] and Baikov et al. [NPB 877 (2013) 647] confirm Kinoshita's results
- Ongoing attempt to check all 4-loop graphs independently
- So far no surprises, QED very accurate and stable: $a_\mu^{\rm QED} = C_\mu^{2n} \sum_n \left(\frac{\alpha}{\pi}\right)^n$ ✓

$$C_\mu^{2,4,6,8,10} = 0.5,\ 0.765857425(17),\ 24.05050996(32),\ 130.8796(63),\ 753.29(1.04)$$

a_μ Electro-Weak

- Electro-Weak 1-loop diagrams:



$a_\mu^{EW(1)} = 195\times10^{-11}$

- known to 2-loop (1650 diagrams, the first EW 2-loop calculation):
  Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

- agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{EW(1+2)} = (154\pm2)\times10^{-11}$

- Higgs mass now known, update by Gnendiger+Stoeckinger+S-Kim, PRD88(2013)053005

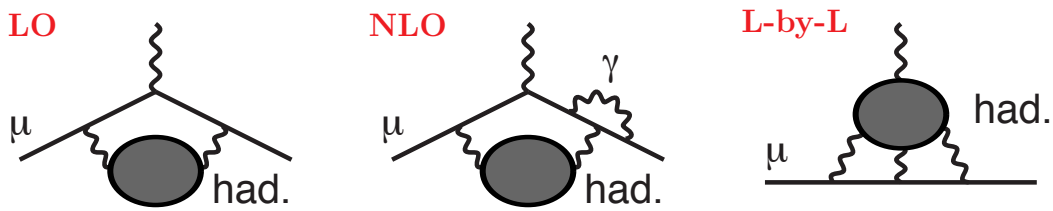
$$a_\mu^{EW(1+2)} = (153.6\pm1.0)\times10^{-11} \quad ✓$$

compared with $a_\mu^{QED} = 116\ 584\ 718.951\ (80)\ \times10^{-11}$

a_μ^{SM}: Hadronic Light-by-Light (I)

- Hadronic: the limiting factor in the SM prediction

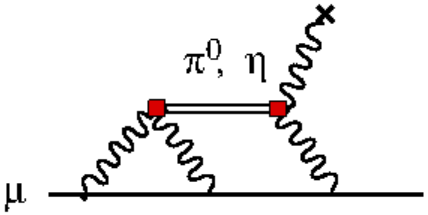
$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$



- L-by-L: $\gamma \to hadrons \to \gamma^*\gamma^*\gamma^*$ non-perturbative, impossible to fully measure ✗

- so far use of model calculations, based on large N_c limit, Chiral Perturbation Theory, plus short distance constraints from OPE and pQCD

- meson exchanges and loops modified by form factor suppression, but with limited experimental information:



- in principle off-shell form-factors (π^0, η, η', $2\pi \to \gamma^* \gamma^*$) needed
- at most possible, experimentally: π^0, η, η', $2\pi \to \gamma\gamma^*$

- additional quark loop, double counting? theory not fully satisfying conceptually ☹

a_μ^{SM}: Hadronic Light-by-Light (II)

Status: Not as bad as sometimes claimed…

- several independent evaluations, different in details, but good agreement for the leading N_c (π^0 exchange) contribution, differences in sub-leading bits

- Mostly used recently:

  - `Glasgow consensus' by Prades+deRafael+Vainshtein:
    $a_\mu^{had,L\text{-}by\text{-}L} = (105 \pm 26) \times 10^{-11}$
  - compatible with Nyffeler's $a_\mu^{had,L\text{-}by\text{-}L} = (115 \pm 40) \times 10^{-11}$

- also agrees with several constituent-quark based estimates
  (no room for a much larger contribution)

- calculations based on Dyson-Schwinger methods indicate possibility for increased L-by-L contribution, but so far no complete result and method regarded as problematic by many (`exact' but after specific truncation to ladder-like diagrams)

a_μ^{SM}: Hadronic Light-by-Light (III)

Prospects: difficult to predict, but…

- Transition FFs can be measured by KLOE-2 and BESIII using small angle taggers:

$$e^+e^- \to e^+e^-\gamma\gamma^* \to \pi^0,\ \eta,\ \eta',\ 2\pi$$

+ better $\pi^0 \to \gamma\gamma$ life-time measurements (also from PrimEx at JLab),

➔ will lead to better model constraints, $\Delta a_\mu^{\mathrm{had,L-by-L},\pi^0} \sim 1 \times 10^{-11}$

- New dispersive approach for L-by-L promising [Vanderhaegen et al.]

- Ultimately: `First principles' full prediction from lattice QCD+QED
  - first results encouraging, successful proof of principle (Blum et al.)
  - several groups: USQCD, UKQCD, ETMC, … much increased effort and resources
  - within 3-5 years a 10% estimate may be possible, 30% would already be useful

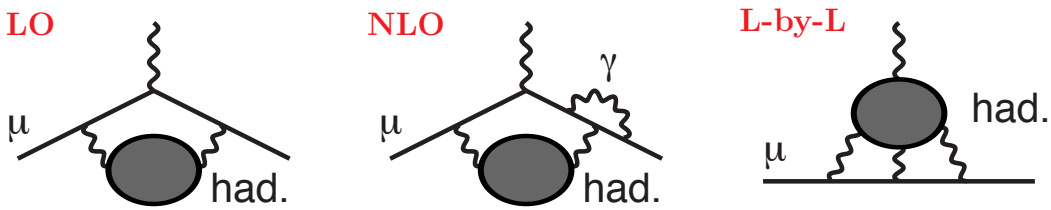
- Conservative prediction: we will at least be able to defend/confirm the error estimate of the Glasgow consensus, but possibly bring it down significantly. ✓

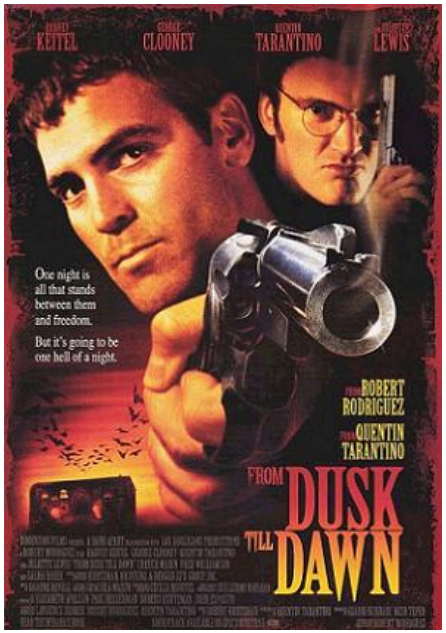
a_μ^{SM}: overview; Hadronic Vacuum Polarisation

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

- QED: ✓
- EW: ✓
- Hadronic: the limiting factor of the SM prediction ✗

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$





HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and a dispersion integral

- done at LO and NLO (see graphs)
- now even at NNLO [Steinhauser et al., PLB734(2014)144] $a_\mu^{\rm HVP,\ NNLO} = +\ 1.24 \times 10^{-10}$

- alternative: lattice QCD, but: need also QED corrections; systematics?

a_μ^{SM}: overview, numbers

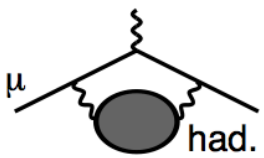
- Several groups have produced hadronic compilations over the years.
- Here: Hagiwara+Liao+Martin+Nomura+T
- Many more precise data in the meantime and more expected for near future
- At present HVP still dominates the SM error

QED contribution	11 658 471.808 (0.015) $\times 10^{-10}$	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contribution		
LO hadronic	**694.9 (4.3)** $\times 10^{-10}$	HLMNT11
NLO hadronic	−9.8 (0.1) $\times 10^{-10}$	HLMNT11
light-by-light	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 182.8 (4.9)** $\times 10^{-10}$	
Experiment	**11 659 208.9 (6.3)** $\times 10^{-10}$	world avg
Exp − Theory	**26.1 (8.0)** $\times 10^{-10}$	**3.3 σ discrepancy**

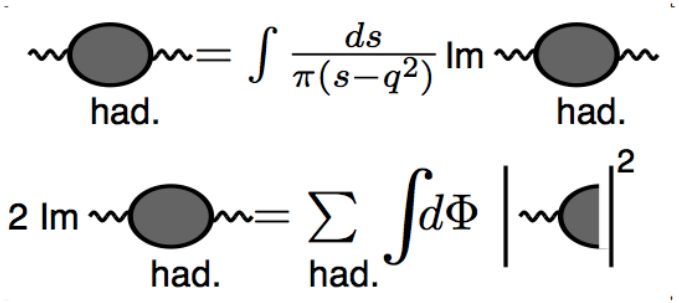
(Numbers taken from HLMNT11, arXiv:1105.3149)

Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$= \int \frac{ds}{\pi(s-q^2)} \, \mathrm{Im}$$

$$2\,\mathrm{Im}$$

$$= \sum \int d\Phi \left| \quad \right|^2$$

$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\mathrm{th}}}^{\infty} ds \, \frac{1}{s} \hat{K}(s) \sigma_{\mathrm{had}}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

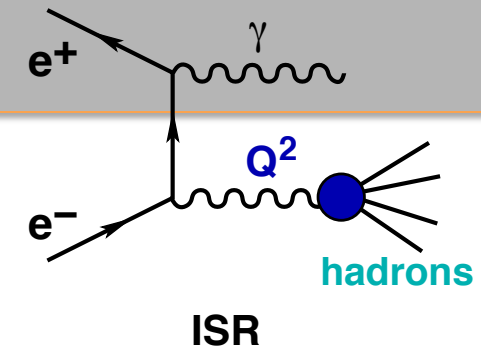
$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

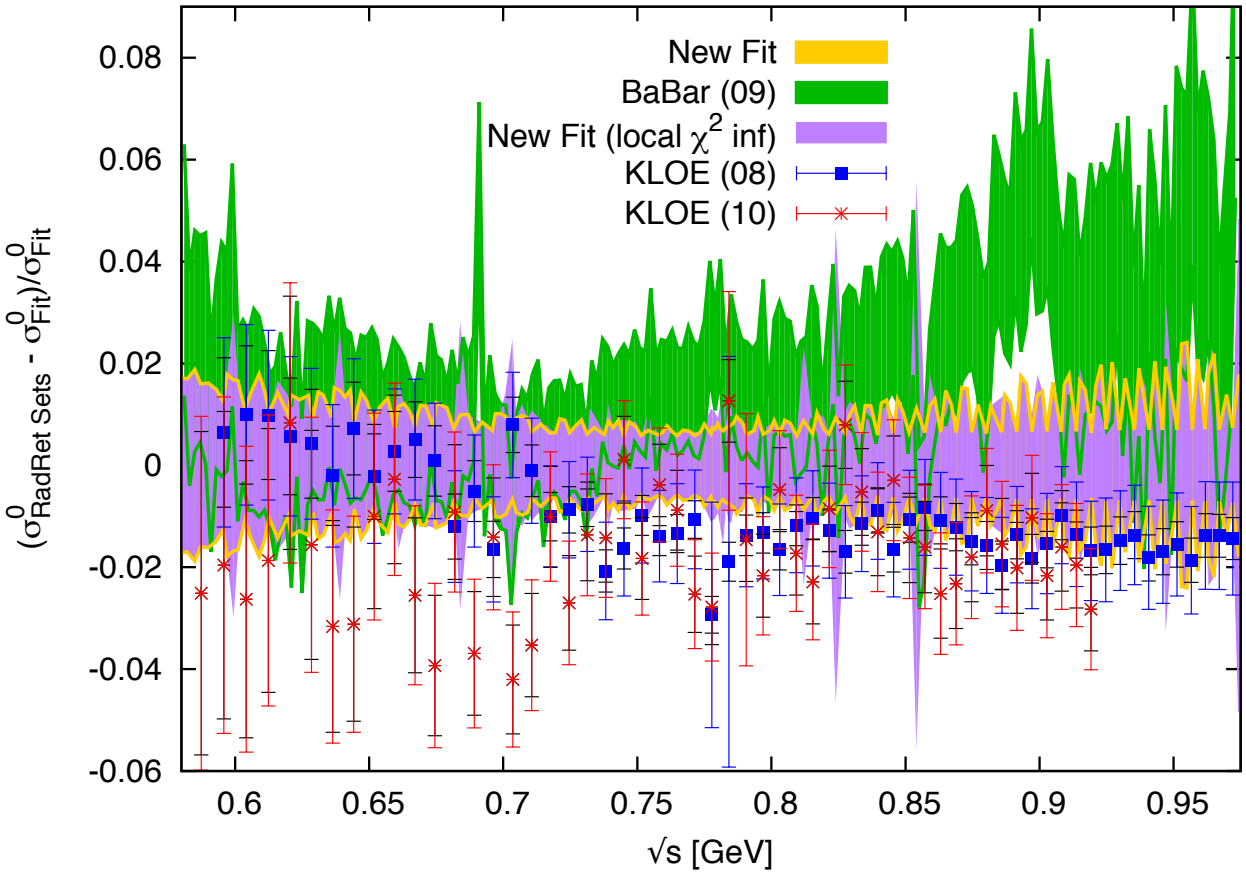
How to get the most precise σ^0_{had}? e^+e^- data:

- Low energies: sum ~ 25 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, ..., use iso-spin relations for missing channels
- Above ~1.8 GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)
- Challenge of data combination (locally in √s): from many experiments, in different energy bins, errors from different sources, correlations; sometimes inconsistencies/bias
- σ^0_{had} means `bare' σ, but WITH FSR: RadCorrs [HLMNT: $\delta a_\mu^{\mathrm{had,\ RadCor\ VP+FSR}} = 2 \times 10^{-10}$!]
- traditional `direct scan' (tunable $e^+e^-$ beams) vs. `Radiative Return' [+ τ spectral functions]

HVP: Data `puzzle' in the $\pi^+\pi^-$ channel



Radiative Return data in the combined fit of HLMNT 11



➔ 2π fit: overall χ^2_{min}/d.o.f. ~ 1.5 needs error inflation, limited gain in error

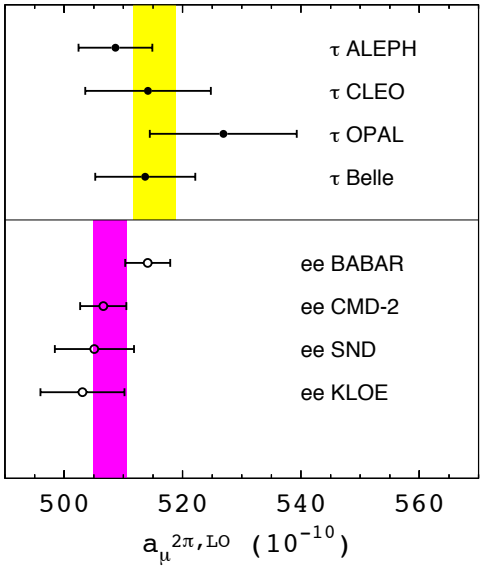
➔ Latest KLOE12 data confirm this tension

Note: $a_\mu^{\pi\pi,\ \text{w/out Rad Ret}} = 498.7 \pm 3.3$ BUT $a_\mu^{\pi\pi,\ \text{with Rad Ret}} = 504.2 \pm 3.0$

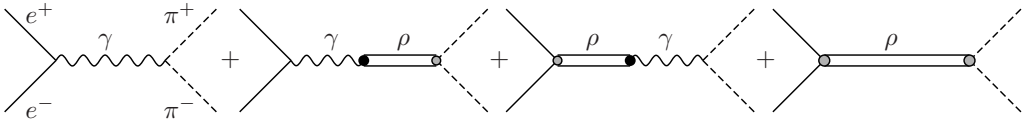
➔ i.e. a shift of +5.5 in HLMNT [DHMZ: $a_\mu^{\pi\pi}$ even higher by 2.1 units]

Another `puzzle': Use of tau spectral function data?

- Use CVC (iso-spin symmetry) to connect $\tau^- \to \pi^0\pi^-\nu_\tau$ spectral functions to $e^+e^- \to \omega, \rho \to \pi^+\pi^-$ but have to apply iso-spin corrections
- Early calculations by Alemany, Davier, Hoecker: use of τ data complementing e^+e^- data originally resulted in an improvement w.r.t. use of e^+e^- data alone; discrepancy smaller with tau data; later increased tension between e^+e^- and τ
- Recent compilation by Davier et al (Fig. from PRD86, 032013):



- Jegerlehner+Szafron: crucial role of γ-ρ mixing:



- They found discrepancy gone but τ data improves e^+e^- analysis only marginally

- Analyses by Benayoun et al: combined fit of e^+e^- and τ based on Hidden Local Symmetry (HLS):
  no big tension betw. e^+e^- and τ, but w. BaBar, hence not used; increased $\Delta a_{\mu:}$ of >≈ 4.5σ

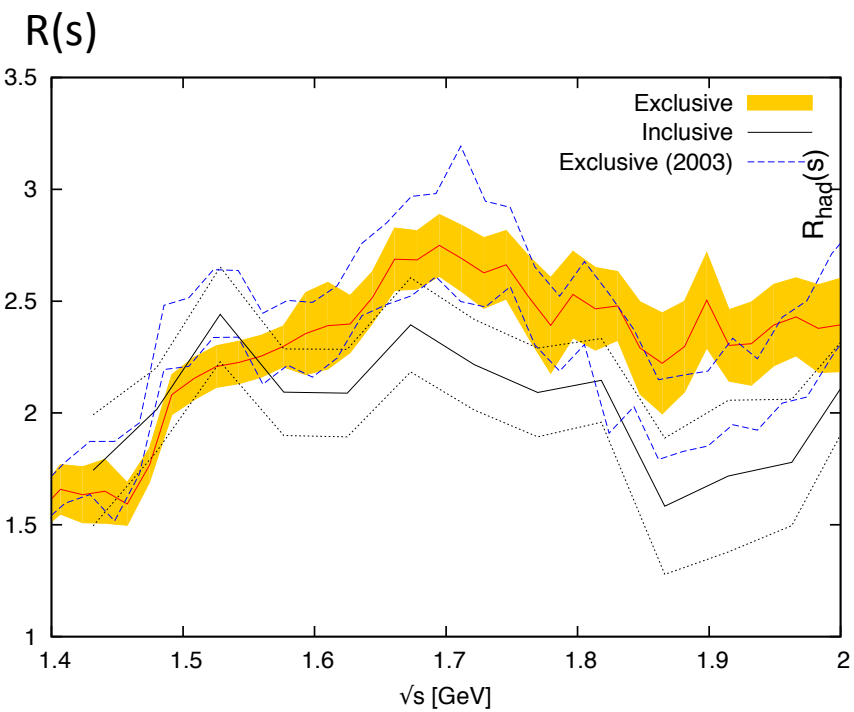
- Davier+Malaescu refute criticism, claim fair agreement betw. BaBar and their τ comp.

- HLMNT: stick to e^+e^- (and do not use τ data). With e^+e^- (incl. BaBar) discrepancy of 3-3.5σ

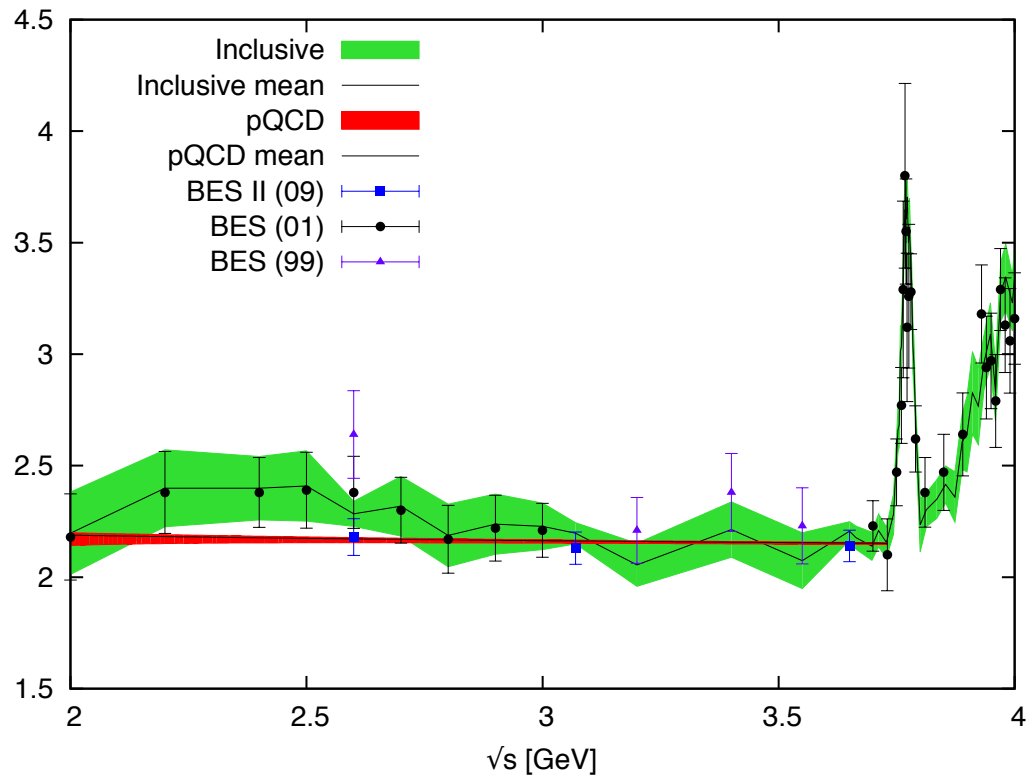
σ_{had} at higher energies; 2011 status to be improved soon

Inclusive vs. sum of exclusive

More from SND, CMD3, Belle, BaBar, BESIII



Inclusive data or perturbative QCD



More from BESIII soon!

- Exclusive data better now mainly due to many Radiative Return data from BaBar
- Latest BES data (blue markers) in perfect agreement w. pQCD; data-based $a_\mu^{incl} > a_\mu^{pQCD}$
- Different data and data vs. pQCD choices give slightly different a_μ (within errors)

$a_\mu^{HVP, LO}$ different comps. a_μ^{SM} status: Recent `history'

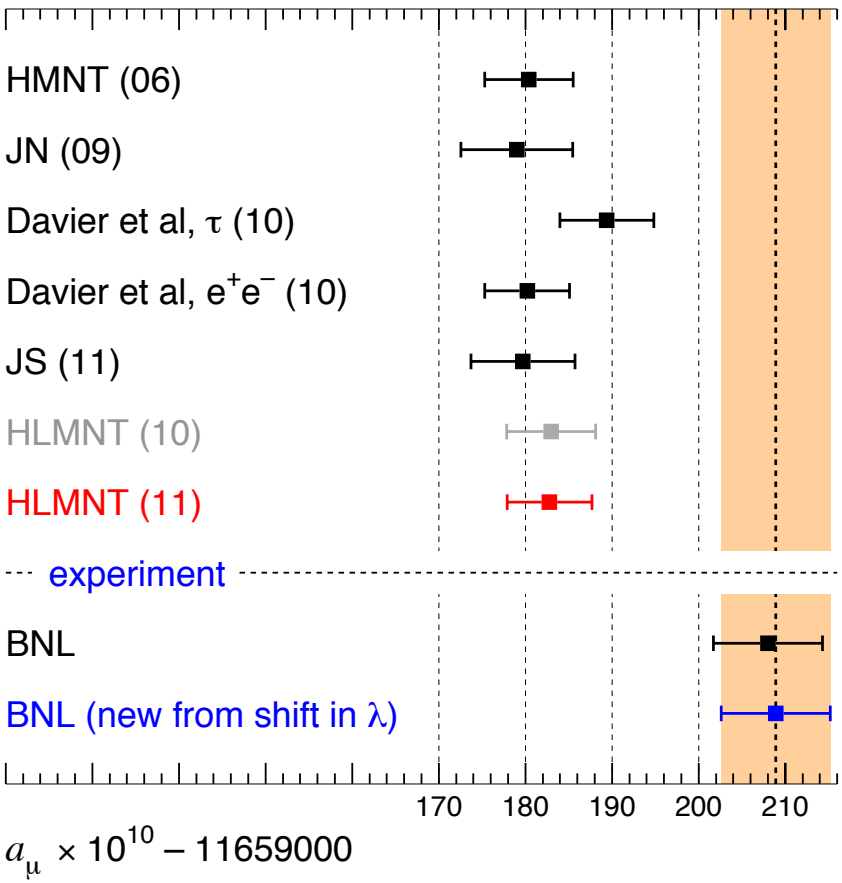
- Fair agreement between different e^+e^- analyses, including recent updates: (all numbers in 10^{-10})

HLMNT (11): 694.9 ± 3.7 (exp) ± 2.1 (rad)
Jegerlehner (11): 690.8 ± 4.7
Davier et al (11): 692.3 ± 4.2

- The `extremes' (both with τ data):

Davier et al (11): 701.5 ± 4.7 (+ ~ 1.5 shift from their 2013 τ re-analysis 1312.1501)
Benayoun et al (12): 681.2 ± 4.5

- New data available already do not shift the mean value strongly, but are incrementally improving the determination of a_μ^{HVP}

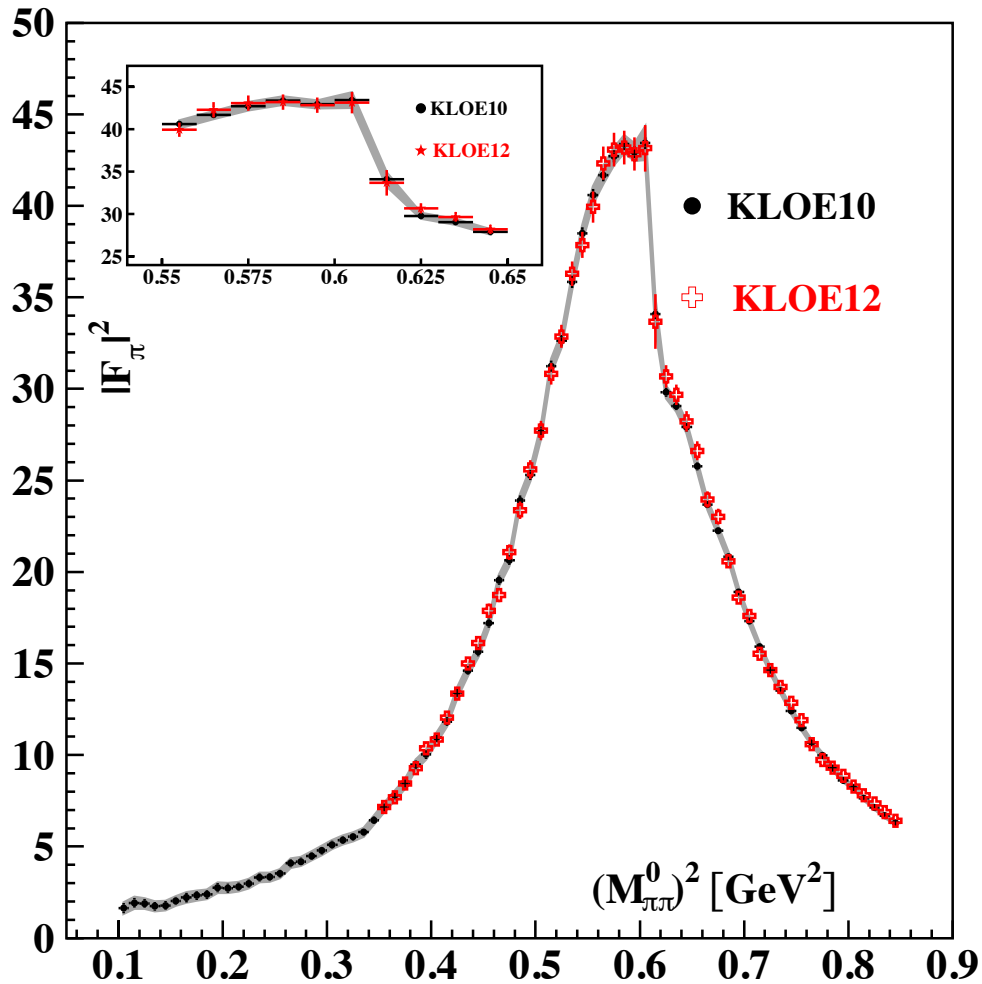


σ_{had}: recent new data

KLOE

$\pi^+\pi^-$ data with $\sigma_{\mu\mu}$ normalisation:

- confirm previous KLOE measurements
- will not decrease tension with BaBar once included in next round of `global' σ_{had} compilations, but slightly increase significance of KLOE
- Open question: Why are BaBar's data so different from KLOE's?
  Are there any issues with the MCs or analysis techniques used?



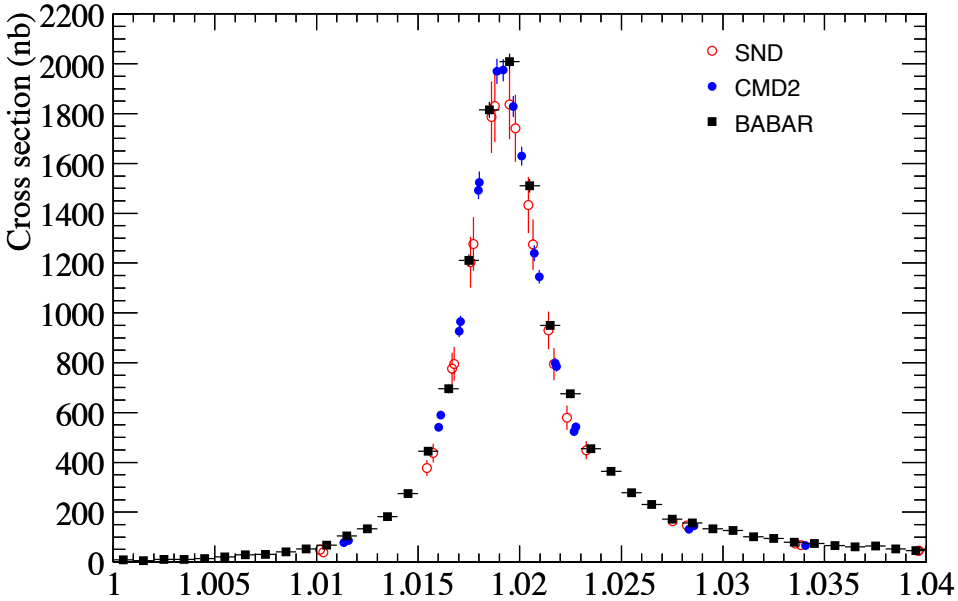
PLB720(2013)336

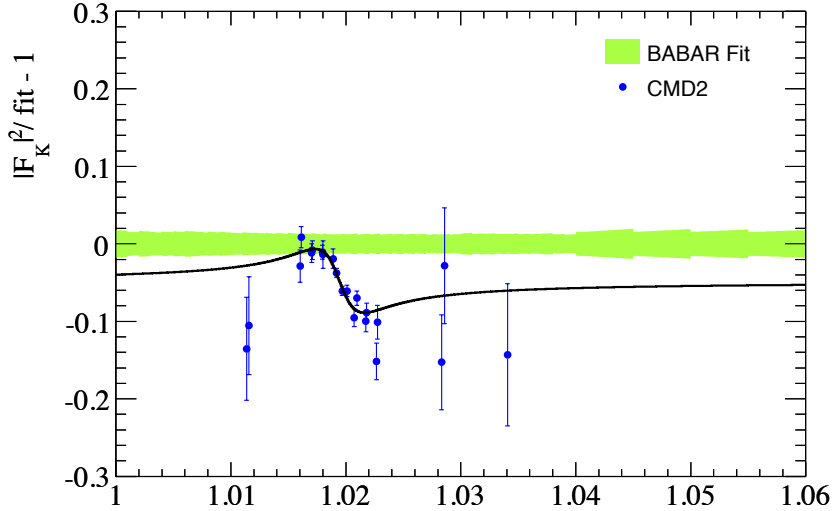
σ_{had}: recent new data: $K^+K^-(\gamma)$ from BaBar

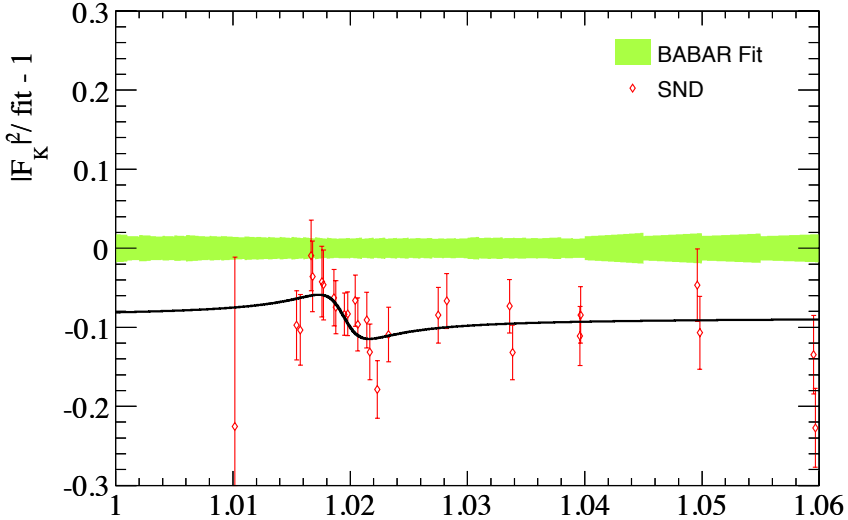
PRD 88(2013)3,032013

- a_μ = 22.94 ± 0.18 ± 0.22 up to 1.8 GeV vs. 21.63 ± 0.27 ± 0.68 for combined previous data
- significant shift up, error down?!
- may need to use mass shifts or ΔE for best combination; m_ϕ = ?

- Comp. plots BaBar vs Novosibirsk:



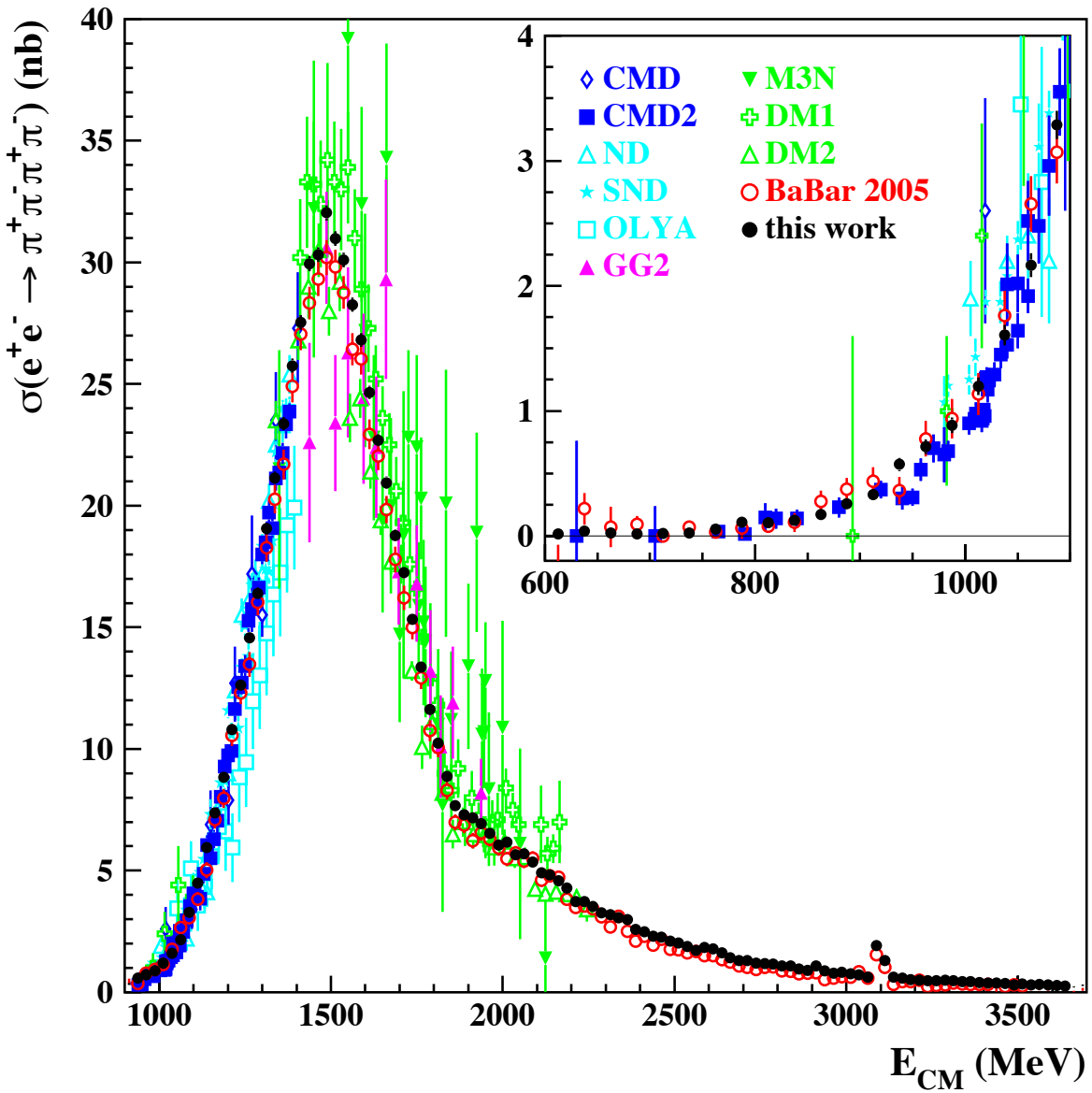




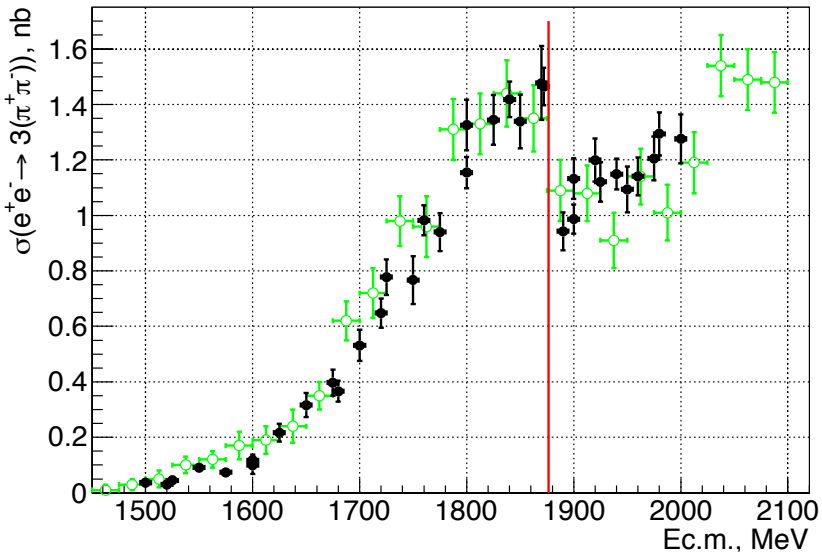
σ_{had}: recent new data: $2\pi^+2\pi^-(\gamma)$ from BaBar

PRD85(2012)112009

- shift of $+0.3 \times 10^{-10}$ for a_μ
- error down to a third
- combination?

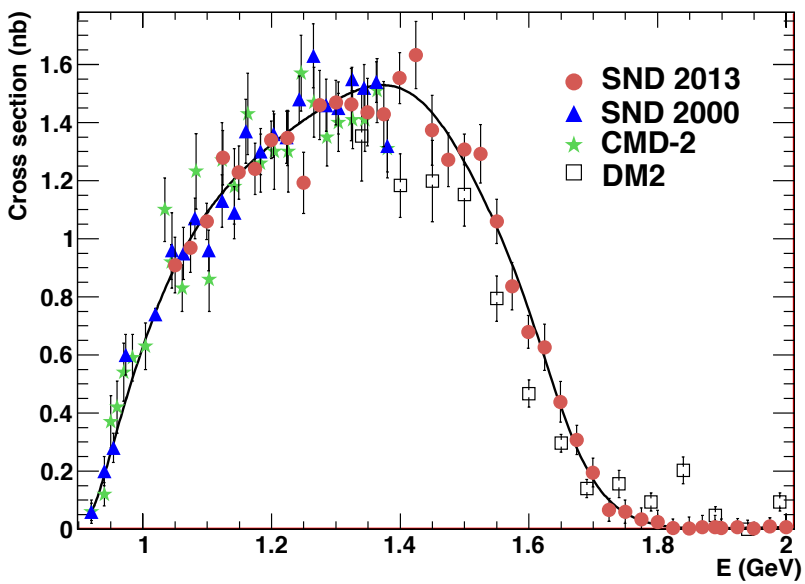


σ_{had}: recent new data from Novosibirsk



CMD-3 6π charged, PLB723(2013)82

- solid black: CMD-3, open green: BaBar
- full analysis will include $2(\pi^+\pi^-\pi^0)$



SND $\omega\pi^0$, PRD88(2013)054013

- many more analyses reported with preliminary results, incl. 3π, $4\pi(2n)$
- looking forward to rich harvest from SND and CMD-3

Future improvements for a_μ^{HVP}:

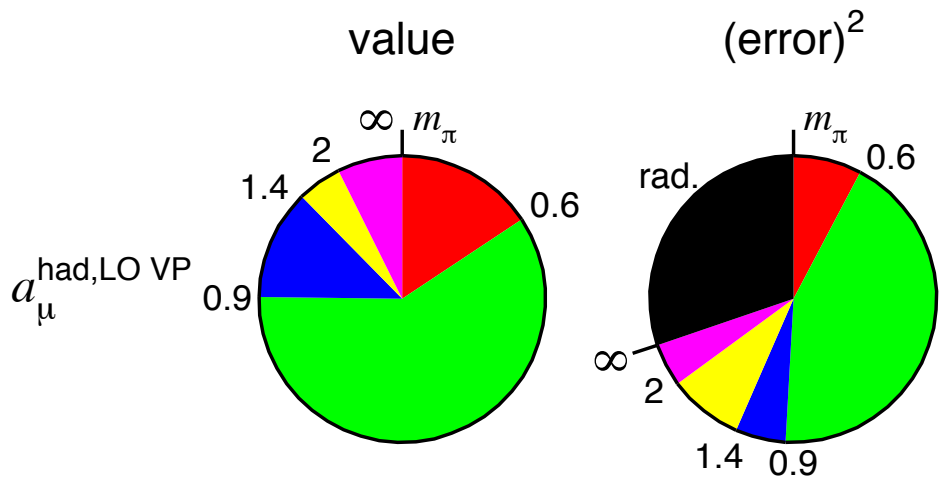
- Most important 2π:
  - close to threshold important; possible info also from space-like
  - better and more data this summer
  - understand discrepancy between sets, especially `BaBar puzzle'
  - possibility of direct scan & ISR in the same experiment(s)

- √s > 1.4 GeV:
  higher energies will improve with input from SND, CMD-3, BESIII, BaBar

- With channels more complete, test/ replace iso-spin corrections

- TH+Exp: squeeze the sizeable error (`rad') from Radiative Corrections

Pie diagrams from HLMNT 11:



Can expect significant improvements:

- 2π: error down by about 30-50%
- subleading channels: by factor 2-3
- √s > 2 GeV: by about a factor 2

➔ I believe we can half the HVP error in time for the new g-2 Exps.

Thank You Kaoru for Your collaboration and support!

Looking forward to more sessions from dusk till dawn …