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Nonlocal Corrections to Scalar Field Effective Action in de Sitter spacetime

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Based on the preprint: Cerne, W. and Suyama, T. "Nonlocal Corrections to Scalar Field Effective Action in de Sitter spacetime" (2026) arXiv:2601.22644

Overview

- Motivation: Quantum Theory of Inflation
- Previous Approaches and Their Limitations
- One-loop Effective Action in de Sitter Spacetime
- Physical Consequences

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The Quantum Theory of Inflation

- Quantum fluctuations stretching to superhorizon scales are what **seed the structure of our Universe** through their coupling to **metric perturbations**
- The superhorizon dynamics are predominantly classical with small quantum corrections.
- These quantum corrections leave imprints on observables and provide a **probe of physics at inflationary energy scales.**

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

Previous Works and their Limitations

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Boyanovsky et al. (2005)

Boyanovsky et al. “Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations”, arXiv:astro-ph/0503669 (2005)

- Semi-classical approach
- Limitations:
  - **Mean-field QFT approximation**
  - Leading order in fluctuations
  - **Ignores metric perturbations**

$$S[\phi] = \int d^4x \sqrt{-g} \left[\frac{1}{2}(\partial\phi)^2 - V(\phi) \right]$$

$\Downarrow$ split into background + fluctuations

$$\phi(\vec{x}, t) = \Phi_0(t) + \varphi(\vec{x}, t) \quad , \quad \Phi_0 = \langle \phi \rangle$$

$\Downarrow$ quantize fluctuations only

$$\varphi \to \hat{\varphi}$$

$\Downarrow$ vary action w.r.t. Φ_0

$$\ddot{\Phi}_0 + 3H\dot{\Phi}_0 + V'(\Phi_0) + \frac{1}{2}V'''(\Phi_0)\hat{\varphi}^2 + \cdots = 0$$

$\Downarrow$ take expectation value

$$\ddot{\Phi}_0 + 3H\dot{\Phi}_0 + V'(\Phi_0) + \frac{1}{2}V'''(\Phi_0)\langle\varphi^2\rangle = 0$$

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Herranen et al. (2015)

Herranen et al. "Quantum corrections to inflaton dynamics, the semi-classical approach and the semi-classical limit", arXiv:2011.12030 (2015)

- Tests validity of semi-classical approach
- Includes **scalar metric perturbations**
- Limitations:
  - **Mean-field QFT approximation**
  - Limited to **leading order radiative corrections**

One-loop Effective Action in de Sitter Spacetime

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My Work

Cerne, W. and Suyama, T. “Nonlocal Corrections to Scalar Field Effective Action in de Sitter spacetime”, arXiv:2601.22644 (2026)

- **Derives the full quantum-corrected mean field dynamics** through the Effective Action formalism
- Includes **higher order radiative corrections** which generate **nonlocal structure and dissipative effects**
- **Limitation:** neglects metric perturbations

Main Results:

The **renormalized one loop effective action** for the quantum expectation value of a **test scalar field** in the **de Sitter spacetime** and the basic **properties** of the **resultant equation of motion**

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Computation of the Effective Action

Action of a **real scalar field** in the **de Sitter spacetime:**

$$S[\phi] = \int d^4x \; a^3(t) \; \left[\frac{1}{2} g^{\mu\nu} \partial_\nu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - V(\phi)\right]$$

Separate into **mean field and fluctuations:**

$$\phi = \Phi + \chi \; ; \; \Phi = \langle \phi \rangle \; , \; \langle \chi \rangle = 0$$

Compute **effective action** using the **in-in formalism**:

$$\exp(i\Gamma[\Phi_+(x), \Phi_-(x)]) = \int_{\mathrm{1PI}} \mathcal{D}\chi_+ \mathcal{D}\chi_- e^{iS[\Phi_+ + \chi_+] - iS[\Phi_- + \chi_-]}$$

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After perturbative expansion about the interaction: $g_{\pm x} \equiv V''(\Phi_{\pm}(x))$

Zero-th order: Classical equation of motion

First order: Local correction to the effective potential (same as Boyanovsky (2005))

Second order: Non-local, quantum corrections to mean field behavior

$$
\begin{aligned}
i\Gamma_{1-\mathrm{loop}}\left[\Phi_{+}, \Phi_{-}\right] = & \int_{1\mathrm{PI-connected}} \mathcal{D}\chi_{+} \mathcal{D}\chi_{-} e^{iS_0[\chi_+] - iS_0[\chi_-]} \\
& \times \left(1 - \frac{i}{2} \int d^4x\ a^3(t) g_{+x} \chi_{+}^2(x) + \frac{i}{2} \int d^4x\ a^3(t) g_{-x} \chi_{-}^2(x)\right. \\
& \left. - \frac{1}{8} \int d^4x d^4y\ a^3(t_x) a^3(t_y) \left(g_{+x}\chi_{+}^2(x) - g_{-x}\chi_{-}^2(x)\right)\left(g_{+y}\chi_{+}^2(y) - g_{-y}\chi_{-}^2(y)\right)\right)
\end{aligned}
$$

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Second order in the interaction is **complex valued**

Switch Variables: $$\Phi_\pm(x) = \Phi(x) \pm \frac{1}{2}\Phi_\Delta(x)$$

EOM Condition: $$\left.\frac{\delta\Gamma}{\delta\Phi_\Delta}\right|_{\Phi_\Delta=0} = 0$$

$$\Gamma^{(2)}_{1-\mathrm{loop}}[\Phi,\Phi_\Delta] = -\int d^4x d^4y\, a^3(t_x)a^3(t_y)g'(\Phi(x))g(\Phi(y))\theta(t_x-t_y)\Phi_\Delta(x)\times\mathrm{Im}\Big(\langle\chi(x)\chi(y)\rangle^2\Big)$$
$$+\frac{i}{4}\int d^4x d^4y\, a^3(t_x)a^3(t_y)g'(\Phi(x))g'(\Phi(y))\Phi_\Delta(x)\Phi_\Delta(y)\times\mathrm{Re}\Big(\langle\chi(x)\chi(y)\rangle^2\Big)$$

Imaginary part can be interpreted as
noise with Gaussian correlation

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Renormalized Effective Action

Tree level

$$\Gamma_{\rm eff}[\Phi(x),\Phi_\Delta(x)] = +\int d^4x\, a^3(t)\,\frac{\partial S[\Phi]}{\partial\Phi}\,\Phi_\Delta(x)$$

Local term (Boyanovsky (2005))

New higher order correction (Cerne (2026))

$$+\frac{1}{2}\int d^4x\, a^3(t)\,\Phi_\Delta(x)\times\Big[V_R'''(\Phi(x))\,\langle\chi^2(x)\rangle_{\rm fin} + \big(V_R''(\Phi(x))\big)^2\,\big(I(x)-I_{\rm min}(x)\big)\Big]$$

$$+\int d^4x\, a^3(t)\int^t d^4x'\, a^3(t')\,V_R'''(\Phi(x))\times\Big[V_R''(\Phi(x'))-V_R''(\Phi(x))\Big]\,{\rm Im}\big(\langle\chi(x)\chi(x')\rangle^2\big)$$

$$-\int d^4x\, a^3(t)\,\Phi_\Delta(x)\,\xi(x)\,.$$

New, fully renormalized memory term (Cerne (2026))

Noise term (Cerne (2026))

$$\langle\xi(x)\xi(y)\rangle \equiv \frac{1}{2}a^3(t_x)a^3(t_y)V'''(\Phi(x))V'''(\Phi(y)){\rm Re}\Big(\langle\chi(x)\chi(y)\rangle^2\Big)$$

$$I(x) \equiv \int^t d^4x'\; a^3(t'){\rm Im}\left(\langle\chi(x)\chi(x')\rangle^2\right)$$

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Assumptions made:

1. The scalar field's energy density is small enough to not influence the evolution of the background spacetime. In this sense, it is treated as a "test field" evolving on a fixed background.
2. The self-interactions of the field are sufficiently weak to justify a perturbative treatment of the effective action.

Physical Consequences

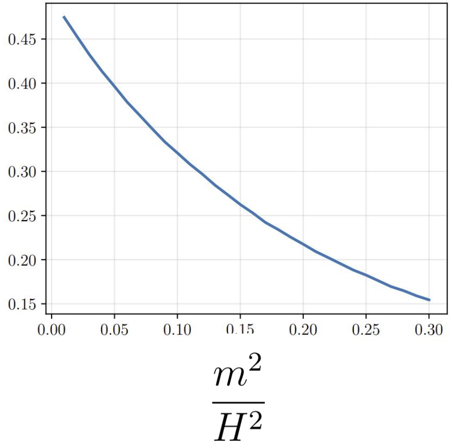
Localization of Equation of Motion

- Non-local nature of equation of motion makes it difficult to analyze.
- The field is assumed to vary slowly over time.

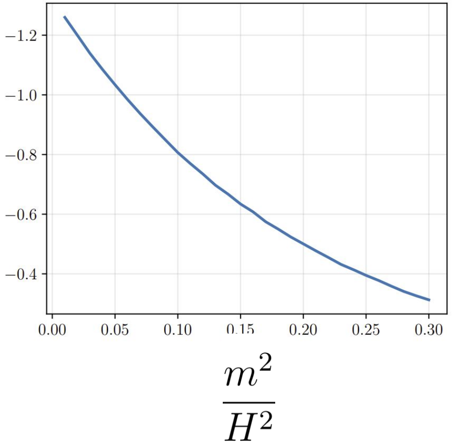
$$(\Box+m^2)\Phi(x)+V_R'(x)+\frac{1}{2}V_R'''(\Phi(x))A + V_R''(\Phi(x))V_R'''(\Phi(x))B + (V_R'''(\Phi(x)))^2\,\dot{\Phi}(x)C = \xi(x)$$

$$A \approx 0.038 \times \frac{H^4}{m^2}$$

B



C



Comparison to Stochastic Inflation

- Stochastic Inflation:
  a. calculates probability distribution of super-horizon modes by treating the contribution from sub-horizon modes as a classical random walk problem.
- My work:
  a. Calculates the behavior of the quantum expectation value of the background field on all scales
- In order to compare the two, I make some modifications to my work. It is important to note that these were not derived from first principles.

Comparison to Stochastic Inflation

Changes: restrict quantum corrections to only subhorizon contributions and artificially inject standard S.I noise. **The dissipative noise can be neglected compared to the standard noise.**

Result: Equation that governs the probability distribution of the field value (Fokker-Planck equation)

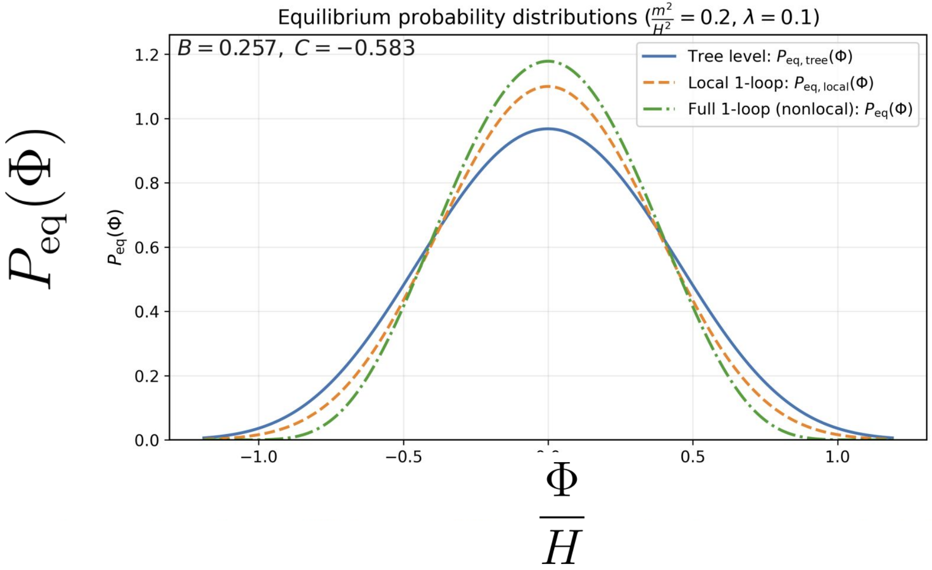
$$\frac{\partial P(\Phi)}{\partial t} = \frac{1}{3H}\frac{\partial}{\partial \Phi}\left[\frac{V'_{\mathrm{eff}}}{1+\frac{C}{3H}(V'''_{\mathrm{eff}})^2}\times P(\Phi)\right] + \frac{H^3}{8\pi^2}\frac{\partial^2 P(\Phi)}{\partial \Phi^2}$$

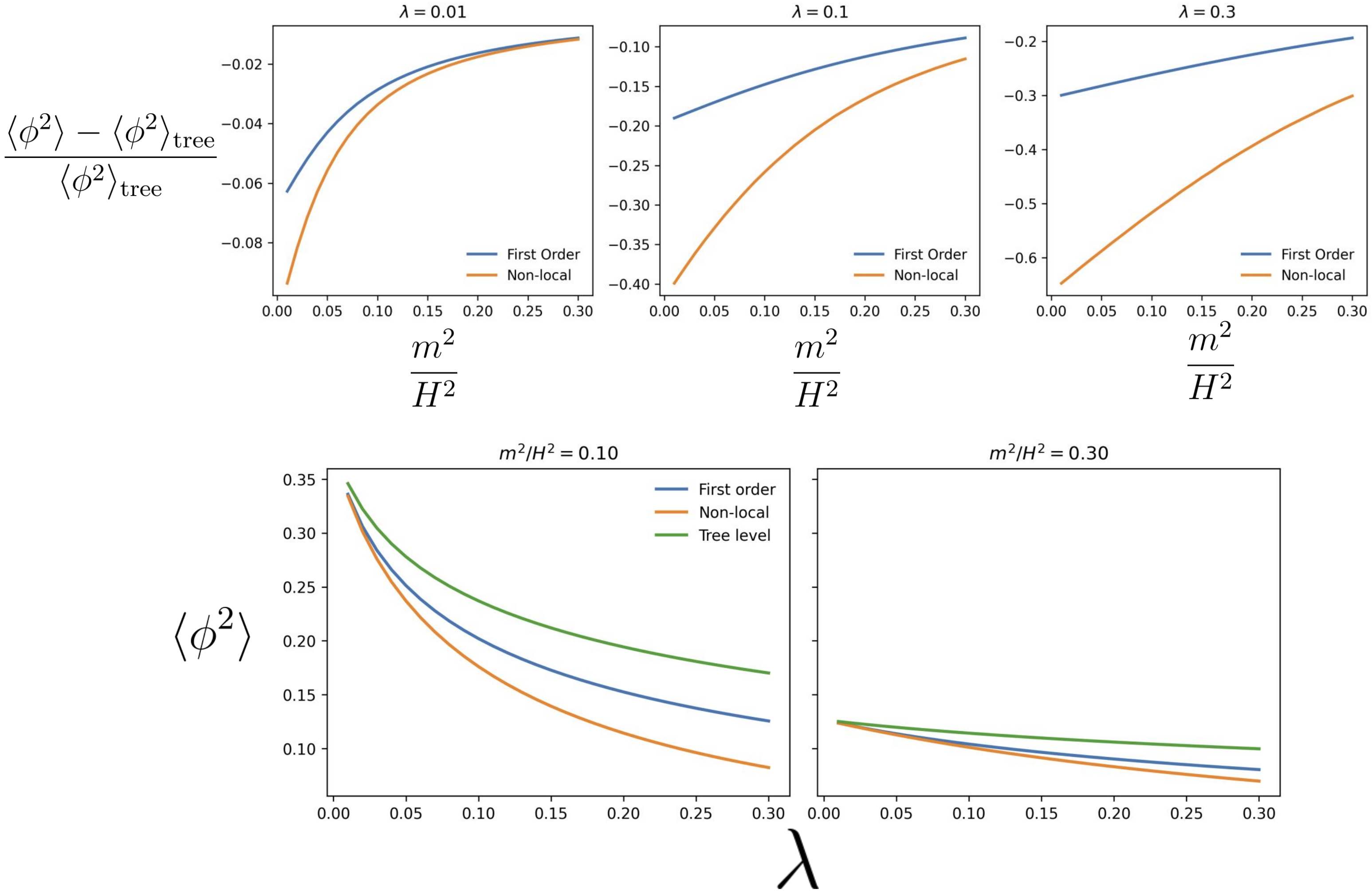
$$V_{\mathrm{eff}} = \frac{1}{2}m^2\Phi^2 + V + \frac{A_{\mathrm{sub}}}{2}V'' + \frac{B}{2}\left(V''\right)^2$$

Case Study: Massive Quartic Potential

$$P_{\rm eq}^{(\rm nonlocal)}(\Phi) \propto \exp\left[\frac{8\pi^2}{3H^4}\left(\mu\ln(1+\gamma\Phi^2)+\frac{\beta}{2\gamma}\Phi^2\right)\right]$$

$$\mu \equiv \frac{-m_{\rm eff}^2\gamma-\beta}{2\gamma^2}, \qquad \beta \equiv -\lambda - 18\lambda^2 B, \qquad \gamma \equiv 12\lambda^2\left(1+18B\lambda\right)^2\frac{C}{H}$$




$\lambda = 0.01$
$\lambda = 0.1$
$\lambda = 0.3$
$\frac{\langle\phi^2\rangle - \langle\phi^2\rangle_{\text{tree}}}{\langle\phi^2\rangle_{\text{tree}}}$
First Order
Non-local
$\frac{m^2}{H^2}$
$m^2/H^2 = 0.10$
$m^2/H^2 = 0.30$
First order
Non-local
Tree level
$\langle\phi^2\rangle$
λ

Summary of physical consequences

- Reduction of field friction
  - **Independent of potential shape**
- For a quartic potential:
  - Decrease of variance
  - Friction is still reduced, implying the second order effects are **dominated by local contributions**

References

Baumann (2009)

Boyanovsky et al. (2006)

Glavan & Prokopec (2023)

Vicentini et al. (2019)

Markkanen et al. (2018, 2012)

Herranen et al. (2014, 2015)

Janssen et al. (2009)

Starobinsky & Yokoyama (1994)

Coleman & Weinberg (1973)

Calculations / Appendix

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Computation of the Effective Action

Expand about the fluctuations:

$$S = \int d^4x \; a^3(t) \left[\frac{1}{2}\partial^\mu \left(\Phi + \chi\right) \partial_\mu \left(\Phi + \chi\right) - \frac{1}{2}m^2\phi^2 - V(\Phi) - V'(\Phi)\chi - \frac{1}{2}V''(\Phi)\chi^2 - \frac{1}{6}V'''(\Phi)\chi^3 + \dots \right]$$

Put into effective action:

$$\exp\left(i\Gamma_{\text{1-loop}}\left[\Phi_+, \Phi_-\right]\right) = \int_{\text{1PI}} \mathcal{D}\chi_+ \mathcal{D}\chi_- e^{iS_0[\chi_+] - iS_0[\chi_-] - \frac{i}{2}\int d^4x \;\; a^3(t) g(\Phi_+(x))\chi_+^2(x) + \frac{i}{2}\int d^4x \;\; a^3(t) g(\Phi_-(x))\chi_-^2(x)}$$

$$S_0[\phi] \equiv \int d^4x \; a^3(t) \left[\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2}m^2\phi^2 \right]$$

$$g(\Phi(x)) \equiv V''(\Phi(x))$$

Key Idea: I assume that the potential follows the slow-roll conditions, enabling a *perturbative expansion* of the above action.

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Computation of the Effective Action

Attach path integrations to fluctuation fields:

First Order:

$$\Gamma^{(1)}_{1-\text{loop}}[\Phi_+, \Phi_-] = -\frac{1}{2}\int d^4x\, a^3(t)\langle\chi^2(x)\rangle\,(g_{+x} - g_{-x})$$

Second Order:

$$\Gamma^{(2)}_{\text{1-loop}}[\Phi_+, \Phi_-] = \frac{i}{8}\int d^4x\, d^4y\, a^3(t_x)\, a^3(t_y)\left(g_{+x}g_{+y}\,\langle T\chi^2(x)\chi^2(y)\rangle + g_{-x}g_{-y}\,\langle \bar{T}\chi^2(x)\chi^2(y)\rangle - 2g_{-x}g_{+y}\,\langle\chi^2(x)\chi^2(y)\rangle\right)$$

$$\langle\cdots\rangle = \int \mathcal{D}\chi_+\mathcal{D}\chi_-\;(\cdots)\,e^{iS_0[\chi_+]-iS_0[\chi_-]} \qquad g_{\pm x} \equiv g(\Phi_\pm(x))$$

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Computation of the Effective Action

Use Wick's Theorem on the Second Order Term:

$$\Gamma^{(2)}_{1-\text{loop}}[\Phi_+, \Phi_-] = \frac{i}{4}\int d^4x d^4y\ a^3(t_x)a^3(t_y)\left(g_{+x}g_{+y}\langle \mathrm{T}\chi_x\chi_y\rangle^2 + g_{-x}g_{-y}\langle \bar{\mathrm{T}}\chi_x\chi_y\rangle^2 - 2g_{-x}g_{+y}\langle \chi_x\chi_y\rangle^2\right)$$

Issue: This Action is Complex-valued

The complex value of the action comes from

dissipative effects

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Computation of the Effective Action

Switch Variables: $$\Phi_\pm(x) = \Phi(x) \pm \frac{1}{2}\Phi_\Delta(x)$$

EOM Condition: $$\left.\frac{\delta\Gamma}{\delta\Phi_\Delta}\right|_{\Phi_\Delta=0} = 0$$

$$\Gamma^{(2)}_{1-\mathrm{loop}}[\Phi,\Phi_\Delta] = -\int d^4x d^4y\; a^3(t_x)a^3(t_y)g'(\Phi(x))g(\Phi(y))\theta(t_x-t_y)\Phi_\Delta(x) \times \mathrm{Im}\Big(\langle\chi(x)\chi(y)\rangle^2\Big)$$
$$+\frac{i}{4}\int d^4x d^4y\; a^3(t_x)a^3(t_y)g'(\Phi(x))g'(\Phi(y))\Phi_\Delta(x)\Phi_\Delta(y) \times \mathrm{Re}\Big(\langle\chi(x)\chi(y)\rangle^2\Big),$$

Imaginary part can be interpreted as *noise with Gaussian correlation*

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Renormalization of the Effective Action

Renormalize UV divergences

Flat Space Divergence

Divergence unique to curved space

$$\Gamma^{(1)}_{\text{1-loop}}[\Phi,\Phi_\Delta] = -\frac{1}{2}\int d^4x\, a^3(t) V'''(\Phi(x))\langle\chi^2(x)\rangle_{\text{Min}}\Phi_\Delta(x) - \frac{1}{2}\int d^4x\, a^3(t)V'''(\Phi(x))\left(\frac{R}{48\pi^2}\ln\Lambda\right)\Phi_\Delta(x)$$
$$-\frac{1}{2}\int d^4x\, a^3(t)V'''(\Phi(x))A(x)\Phi_\Delta(x).$$

Remainder

Renormalization of the Effective Action

Renormalize UV divergences

Local Contribution to Effective Potential

$$\Gamma^{(2)}_{\text{eff}}[\Phi(x), \Phi_\Delta(x)] = -\frac{1}{2}\int d^4x\, a^3(t)\, \Phi_\Delta(x) \times \left[\left(V_R''(\Phi(x))\right)^2 \left(I(x) - I_{\min}(x)\right)\right]$$

$$-\int d^4x\, a^3(t) \int^t d^4x'\, a^3(t')\, V_R'''(\Phi(x)) \times \left[V_R''(\Phi(x')) - V_R''(\Phi(x))\right] \mathrm{Im}\left(\langle\chi(x)\chi(x')\rangle^2\right)$$

Finite non-local contribution

$$I(x) \equiv \int^t d^4x'\; a^3(t')\mathrm{Im}\left(\langle\chi(x)\chi(x')\rangle^2\right) \;,\; I_{\text{Min}}(x) \equiv \int^t d^4x'\; a^3(t')\mathrm{Im}\left(\langle\chi(x)\chi(x')\rangle^2_{\text{Min}}\right)$$