

# Inflation

## Part I

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### selected (highly biased) references

#### Basics on cosmological perturbations

- H. Kodama and MS, *Cosmological Perturbation Theory*  
Prog. Theor. Phys. Suppl. 78 (1984) 1-166.

#### Very good textbook on early universe cosmology including inflation

- V. Mukhanov, *Physical foundations of cosmology*  
Cambridge, UK: Univ. Pr. (2005)

#### Overview of these lectures

- MS, *Inflation and Birth of Cosmological Perturbations*  
arXiv:1210.7880 [astro-ph.CO]

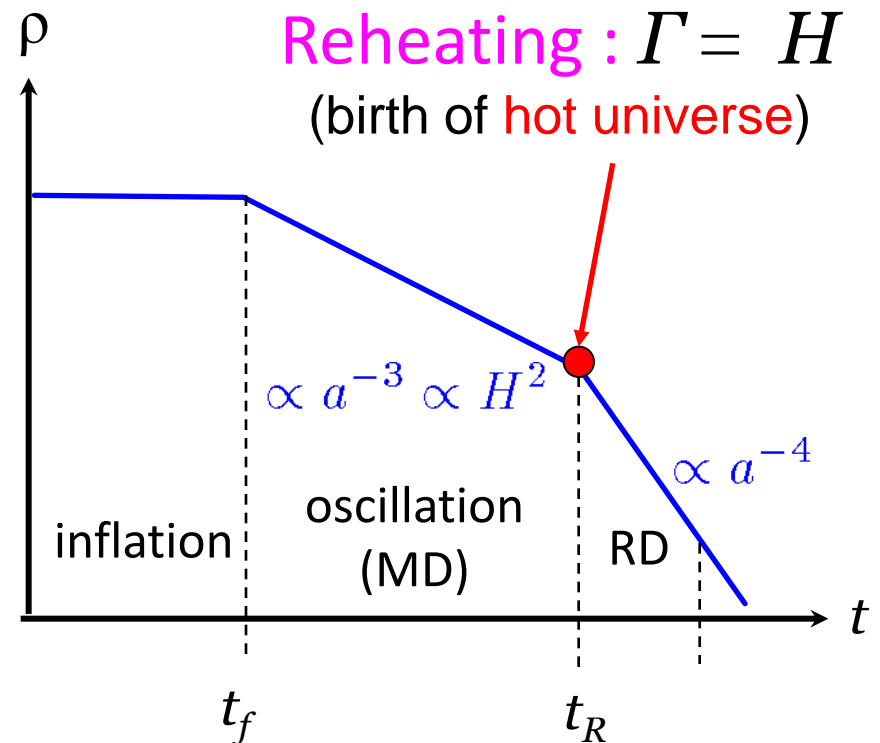
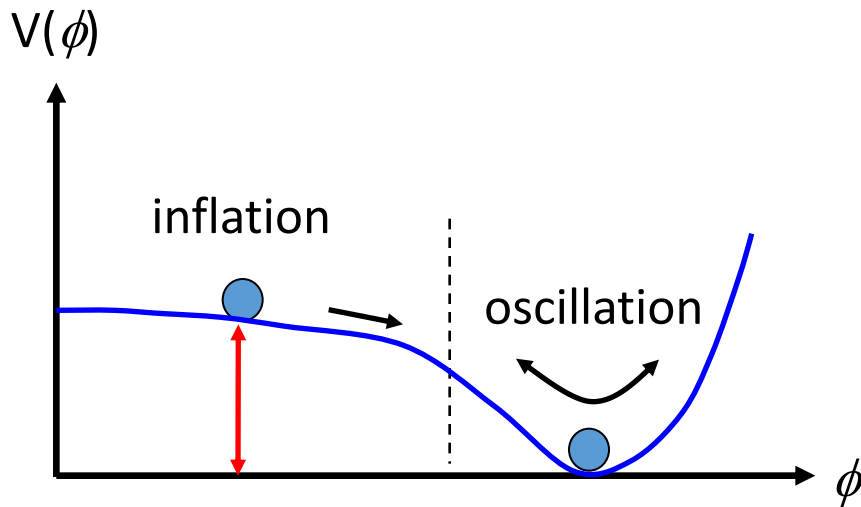
# 0. What is Inflation?

Brout, Englert & Gunzig '77, Starobinsky '79, Guth '81, Sato '81, ...

- Inflation is a **quasi-exponential expansion** of the Universe at its very early stage; perhaps at  $t \sim 10^{-35}$  sec.
- It was meant to solve **the initial condition (singularity, horizon & flatness, etc.) problems** in Big-Bang Cosmology:
  - if any of them can be said to be solved depends on precise definitions of the problems.
- **Quantum vacuum fluctuations** during inflation turn out to play the most important role. They give the initial condition for **all the structures in the Universe**.
- **Cosmic gravitational wave background** is also generated.

# From inflation to bigbang

After inflation, vacuum energy is converted to **thermal energy** (called “re”heating) and **hot Bigbang Universe** is realized.



# more on $\checkmark$ inflation

## the meaning of

1. Homogeneity and isotropy are the (most important) assumption, **NOT a consequence** of inflation.
2. Quasi-exponential expansion in the “Einstein frame”:  
**conformal invariant definition.**
3. At least 50-60 e-folds before the end of inflation:  
solving “**horizon problem**”
4. Don't care what happened before inflation:  
predictions are **almost independent** of initial conditions.

1 & 2: basic assumptions/definition of inflation

# 1. Inflationary Universe

- horizon problem

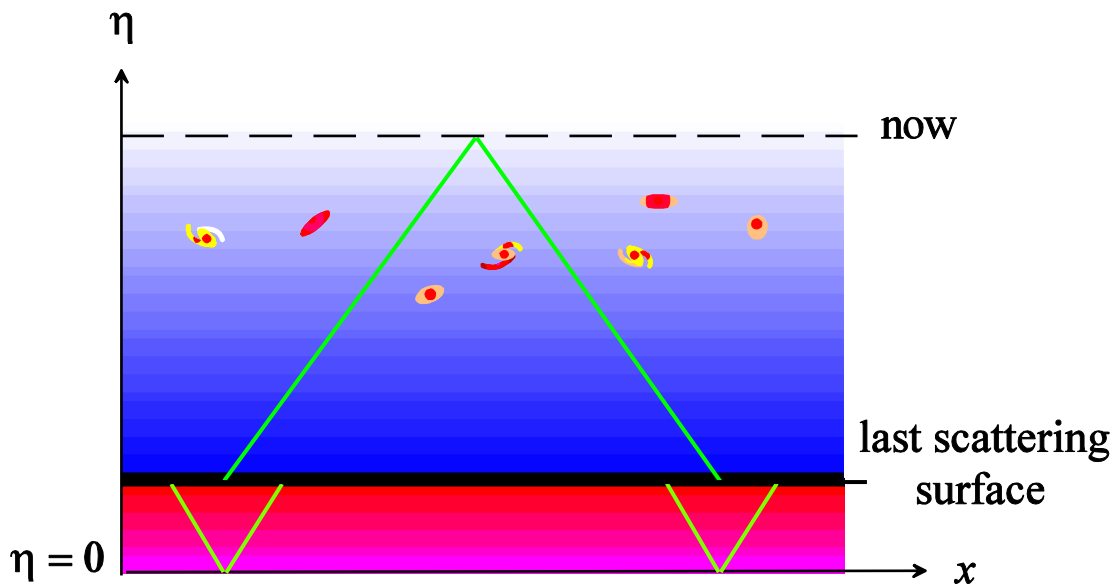
$$ds^2 = -dt^2 + a^2(t)d\sigma_{(3)}^2 \quad \ddot{a} = -\frac{4\pi G}{3}(\rho + 3P)a < 0 \text{ for } P > -\frac{\rho}{3}$$

if  $a \propto t^n$ ,  $n < 1$

gravity=attractive

$$ds^2 = a^2(\eta)(-d\eta^2 + d\sigma_{(3)}^2)$$

$$d\eta = \frac{dt}{a(t)} : \text{conformal time}$$



$$\int_{t_0 \rightarrow 0}^t \frac{dt}{a(t)} = \text{finite}$$

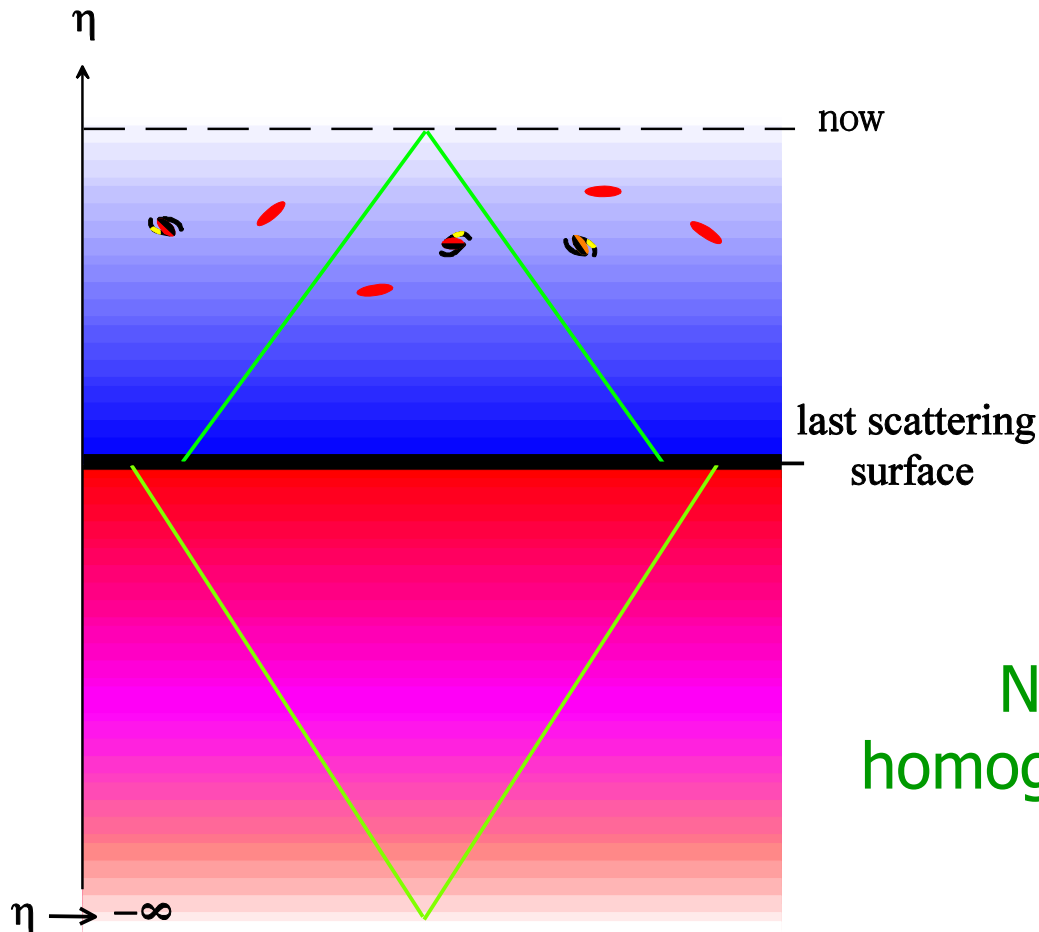
conformal time is **bounded** from below

$\exists$  particle horizon

- solution to the horizon problem

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3P)a > 0$$

for a sufficient lapse of time in the early universe



$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)} \xrightarrow{t_0 \rightarrow 0} \infty$$

or large enough to cover the present horizon size

NB: horizon problem  $\neq$  homogeneity & isotropy problem

- flatness problem (= entropy problem)

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}; \quad -\infty < K < +\infty$$

if  $\rho \propto a^{-4}$ ,  $\rho \gg \frac{|K|}{a^2}$  in the early universe.

conversely if  $\rho \approx |K|/a^2$  at an epoch in the early universe, the universe must have either **collapsed (if  $K > 0$ )** or become completely **empty (if  $K < 0$ )** by now.

alternatively, the problem is the existence of **huge entropy** within the curvature radius of the universe

$$S = T^3 \left( \frac{a}{\sqrt{|K|}} \right)^3 \approx T_0^3 \left( \frac{a_0}{\sqrt{|K|}} \right)^3 \gg T_0^3 H_0^{-3} \approx 10^{87}$$

(# of states = exp[S])  $\frac{|K|}{a_0^2} \ll H_0^2$

# solution to horizon & flatness problems

spatially homogeneous scalar field:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\Rightarrow \rho + 3P = 2(\dot{\phi}^2 - V(\phi)) < 0 \quad \text{if } \dot{\phi}^2 < V(\phi)$$

$$\Rightarrow \rho \approx -P \approx V(\phi) \quad \text{if } \dot{\phi}^2 \ll V(\phi) \quad \text{potential dominated}$$

$V \sim$  cosmological const./vacuum energy

$$\rho \approx \text{const.} \quad \frac{K}{a^2} \text{ decreases rapidly}$$

$$\Rightarrow H^2 \approx \frac{8\pi G}{3} \rho \approx \text{const.}$$

inflation

“vacuum energy” converted to radiation  
after sufficient lapse of time

solves horizon & flatness problems simultaneously

# slow-roll inflation

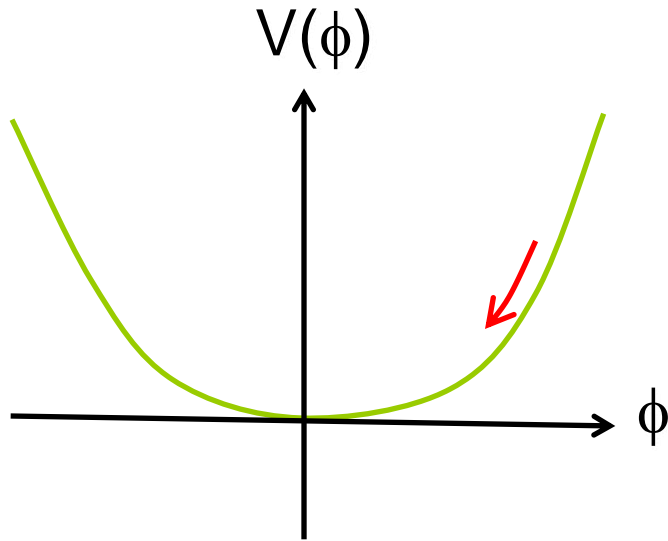
- single-field slow-roll inflation Linde '82, ...

metric:  $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$

field eq.:

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V'(\phi) = 0 \Rightarrow \dot{\phi} = -\frac{V'(\phi)}{3H}$$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2} \cancel{\dot{\phi}^2} + V(\phi) \right]$$



$$\Rightarrow -\frac{\dot{H}}{H^2} = \frac{\frac{3}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V} \approx \frac{3}{2} \frac{\dot{\phi}^2}{V} \ll 1 \quad \dots \text{slow variation of } H$$

$$a \sim e^{Ht}$$

inflation!

# slow-roll conditions

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\frac{3}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V} \approx \frac{3}{2} \frac{\dot{\phi}^2}{V} = \frac{M_P^2 V'^2}{2V^2} \ll 1$$

condition for quasi-de Sitter  
(inflationary) expansion

$$\delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}} = -\varepsilon + \frac{\dot{\varepsilon}}{2H\varepsilon} \approx -\varepsilon - \frac{M_P^2 V''}{V}; \quad |\delta| \ll 1$$

condition for friction-dominated  
(over-damped) evolution

sufficient condition on potential:

$$\varepsilon_V \equiv \frac{M_P^2 V'^2}{2V^2} \ll 1, \quad \eta_V \equiv \frac{M_P^2 V''}{V}; \quad |\eta_V| \ll 1$$

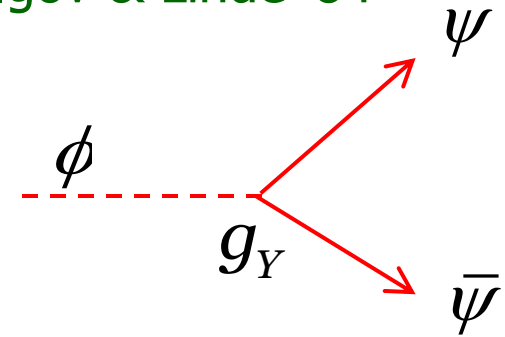
# reheating

Abbott & Wise '84, Dolgov & Linde '84

- standard scenario

e.g.  $L_{\text{int}} \sim g_Y \phi \bar{\psi} \psi$

decay rate:  $\Gamma \sim g_Y^2 m_\phi$ ;  $m_\phi \gg m_\psi$



effective equation of motion:  $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \dots$

when  $m_\phi \gg H > \Gamma$ , damped oscillation:

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0 \Rightarrow \phi \propto a^{-3/2} \cos(m_\phi t + \alpha)$$

effect of  $\Gamma \Rightarrow \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -\Gamma\dot{\phi}$

$$\Leftrightarrow \frac{d}{dt} \left( \frac{1}{2} \dot{\phi}^2 + V \right) = -(3H + \Gamma) \langle \dot{\phi}^2 \rangle$$

$$\Rightarrow \dot{\rho}_\phi + 3H\rho_\phi = -\Gamma\rho_\phi$$

## energy conservation eqns

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma\rho_\phi$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma\rho_\phi \quad \rho_r : \text{produced radiation}$$

- $\Gamma < H \sim t^{-1}$

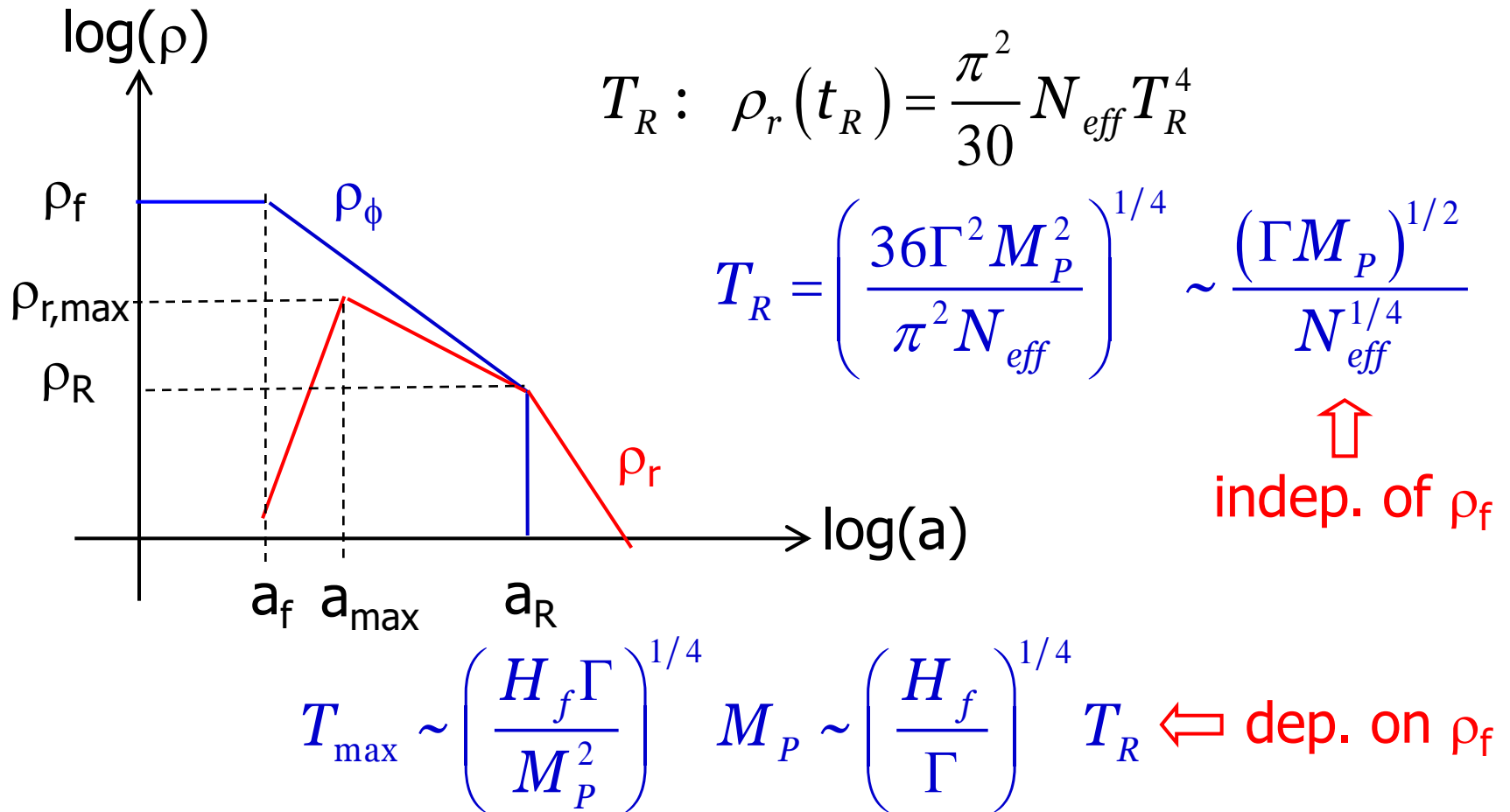
$$\rho_\phi = \rho_{\phi_f} \left( \frac{a}{a_f} \right)^{-3}, \quad \rho_r = \frac{2\Gamma}{5H_f} \rho_{\phi_f} \left( \frac{a}{a_f} \right)^{-4} \left( \left( \frac{a}{a_f} \right)^{5/2} - 1 \right)$$

$$\rho_r = \text{max at } \frac{a}{a_f} = \left( \frac{8}{3} \right)^{2/5} \approx 1.48$$

- $\Gamma > H \sim t^{-1}$

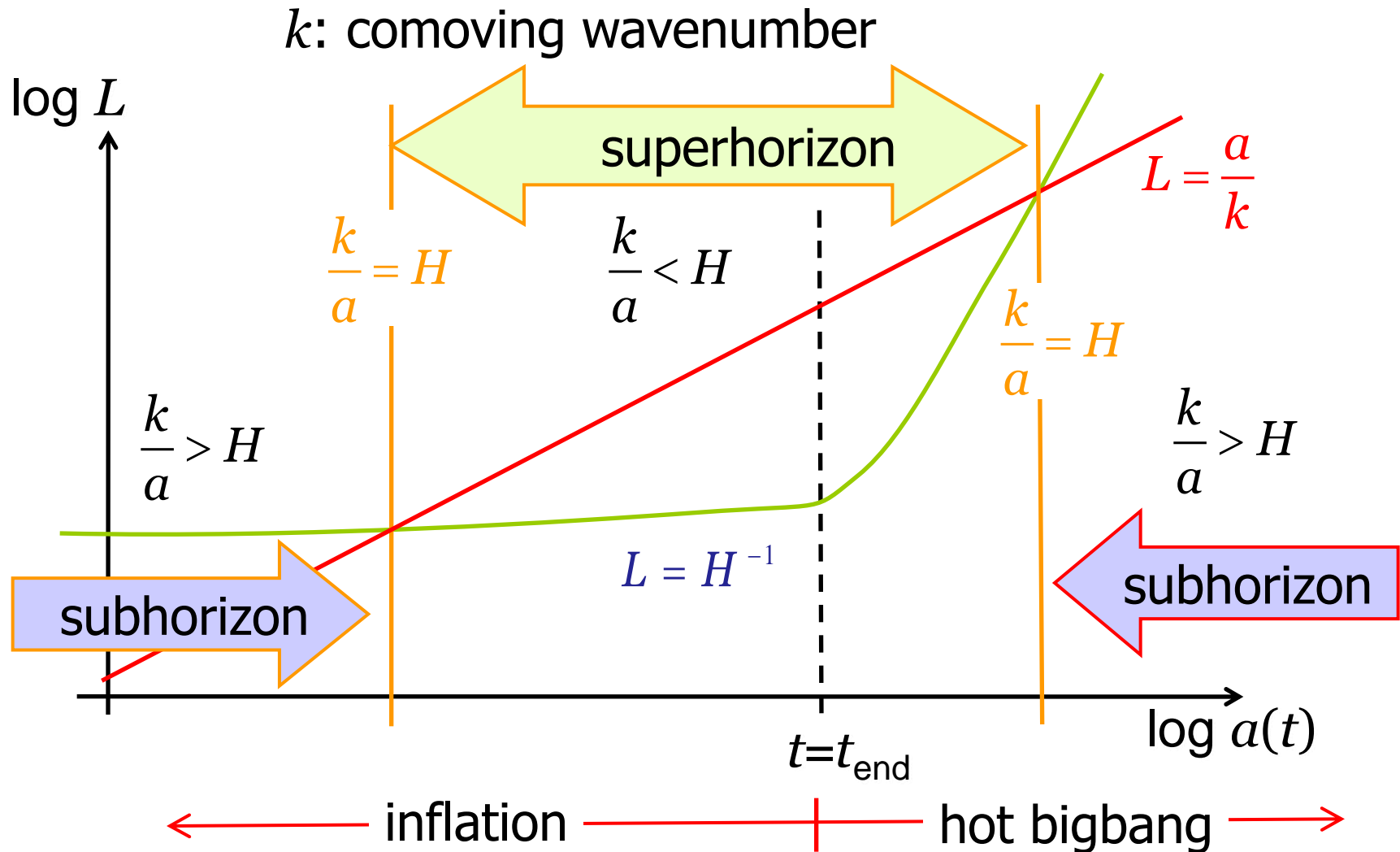
$$\rho_\phi = 0, \quad \rho_r = \rho_r(t_R) \left( \frac{a}{a_R} \right)^{-4} \quad t_R : \text{def by } H(t_R) = \Gamma$$

# reheating temp & max temp



$T_{max}$  is important for **thermal history**  
 $T_R$  is important for **horizon problem**

# comoving scale vs Hubble horizon radius



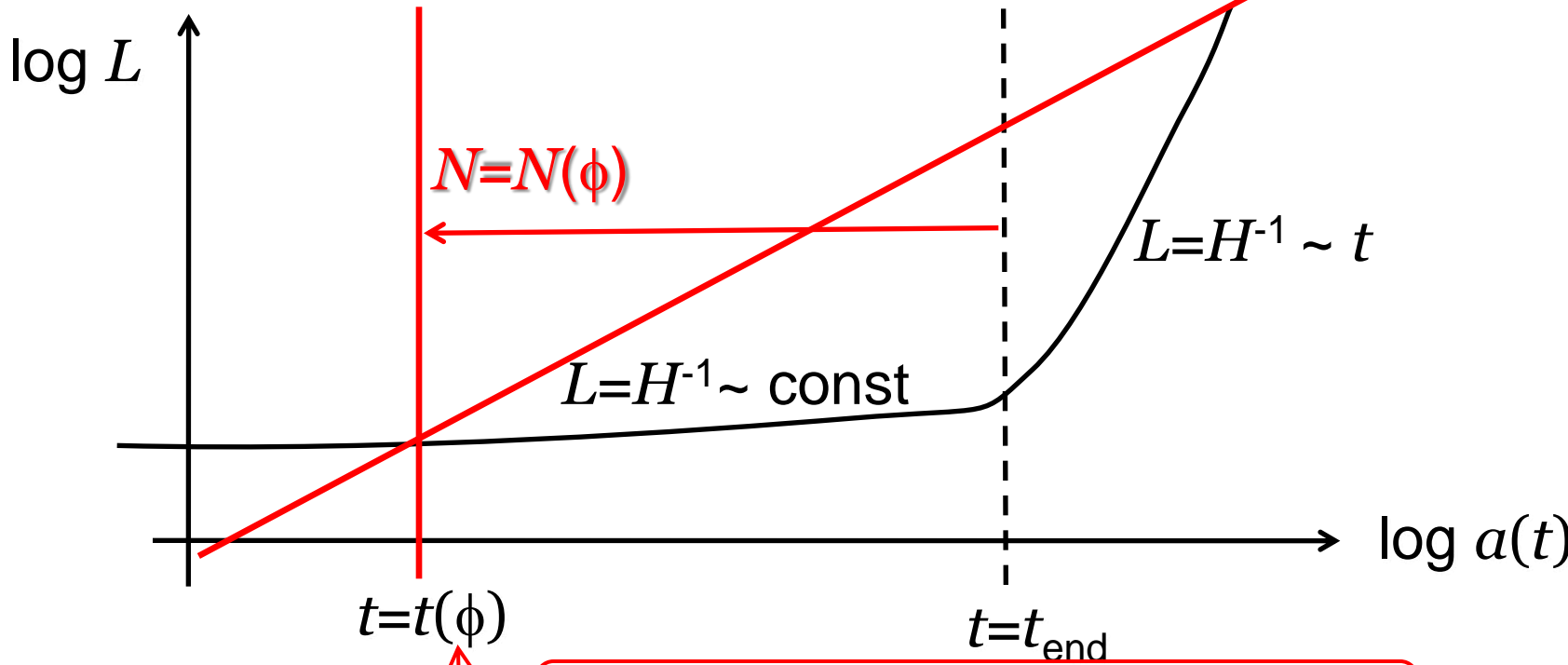
Hubble horizon=causal horizon for local physics

# e-folding number: $N$

# of e-folds from  $\phi = \phi(t)$  until the end of inflation

redshift  
↓

$$\frac{a(t_{\text{end}})}{a(t)} = \exp[N(t \rightarrow t_{\text{end}})] \Rightarrow N = N(\phi) = \int_{t(\phi)}^{t_{\text{end}}} H dt \sim \ln[1 + z(\phi)]$$



$\phi$  determines comoving scale  $k$

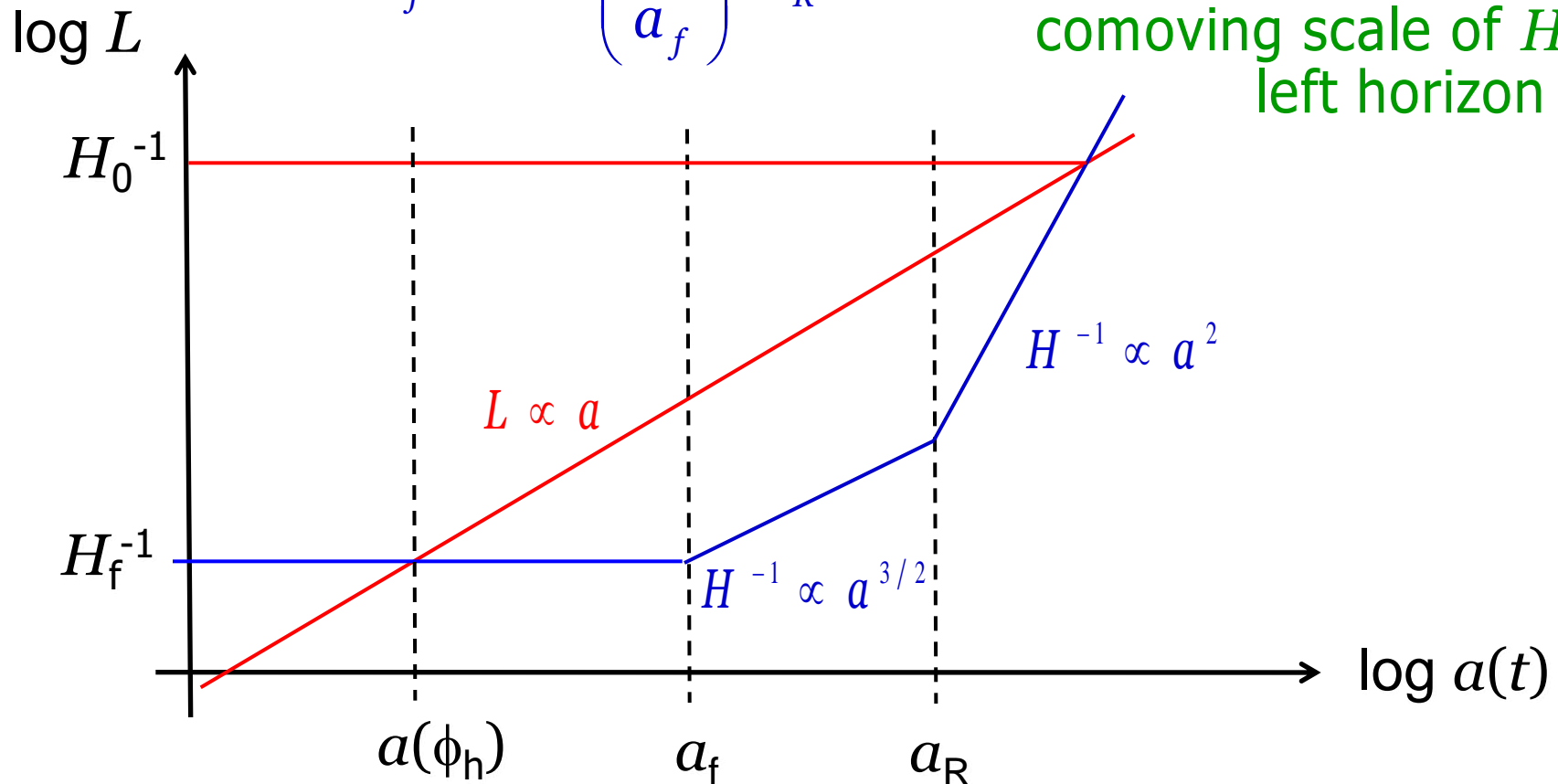
# condition on e-folding number

ignoring variation of  $H$  during inflation.

entropy generated within present Hubble volume:

$$S = H_f^{-3} e^{3N(\phi_h)} \left( \frac{a_R}{a_f} \right)^3 T_R^3$$

$\phi_h \equiv$  value of  $\phi$  at which  
comoving scale of  $H_0^{-1}$   
left horizon



$$\begin{aligned}
S &= H_f^{-3} e^{3N(\phi_h)} \left( \frac{a_R}{a_f} \right)^3 T_R^3 \sim \left( \frac{\rho_f}{M_P^2} \right)^{-3/2} e^{3N(\phi_h)} \left( \frac{\rho_f}{T_R^4} \right) T_R^3 \\
&\approx \left( \frac{M_P^3}{T_R \rho_f^{1/2}} \right) e^{3N(\phi_h)} > \left( \frac{T_0}{H_0} \right)^3 \sim 10^{87} \\
&\Rightarrow N(\phi_h) > 53 + \frac{2}{3} \ln \left[ \frac{\rho_f^{1/4}}{10^{15} \text{ GeV}} \right] + \frac{1}{3} \ln \left[ \frac{T_R}{10^{10} \text{ GeV}} \right]
\end{aligned}$$

- $N > 50-60$  solves horizon & flatness problems
- changing  $T_R$  by a factor 10 changes  $N$  only by 1

Q1. Show that conformal time  $\eta_h$  at  $\phi = \phi_h$  satisfies  $|\eta_h| > \eta_0$ , where  $\eta_0$  is the conformal time today.

# preheating

Kofman, Linde & Starobinsky '94

If  $\phi$  couples to other light scalar (boson) fields

$$\text{e.g. } L_{\text{int}} \sim g\phi^2 \chi^2, \quad m_\chi^2 \ll m_\phi^2$$

catastrophic  $\chi$ - particle creation can occur

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left( (k/a)^2 + g\phi^2 \right) \chi_k = 0$$

for  $m_\phi \Delta t \gg 1$  ( $H\Delta t \lesssim 1$ )

$$\phi^2 = \phi_f^2 (a_f/a)^3 \sin^2 m_\phi t$$

oscillating potential

$$\ddot{\chi}_k + \left( (k/a)^2 + g\phi^2 \sin^2 m_\phi t \right) \chi_k = 0$$

$$\Leftrightarrow \ddot{\chi}_k + \left( a - 2b \cos 2m_\phi t \right) \chi_k = 0 \quad \text{Mathieu eqn}$$

possible parametric amplification of  $\chi_k$

# instability bands

$$a = \left( \frac{k}{am_\phi} \right)^2 + 2b$$

$$b = \left( \frac{g\phi^2}{4m_\phi^2} \right)$$

For  $\frac{k}{a} \ll m_\phi$ ,  $a = 2b$

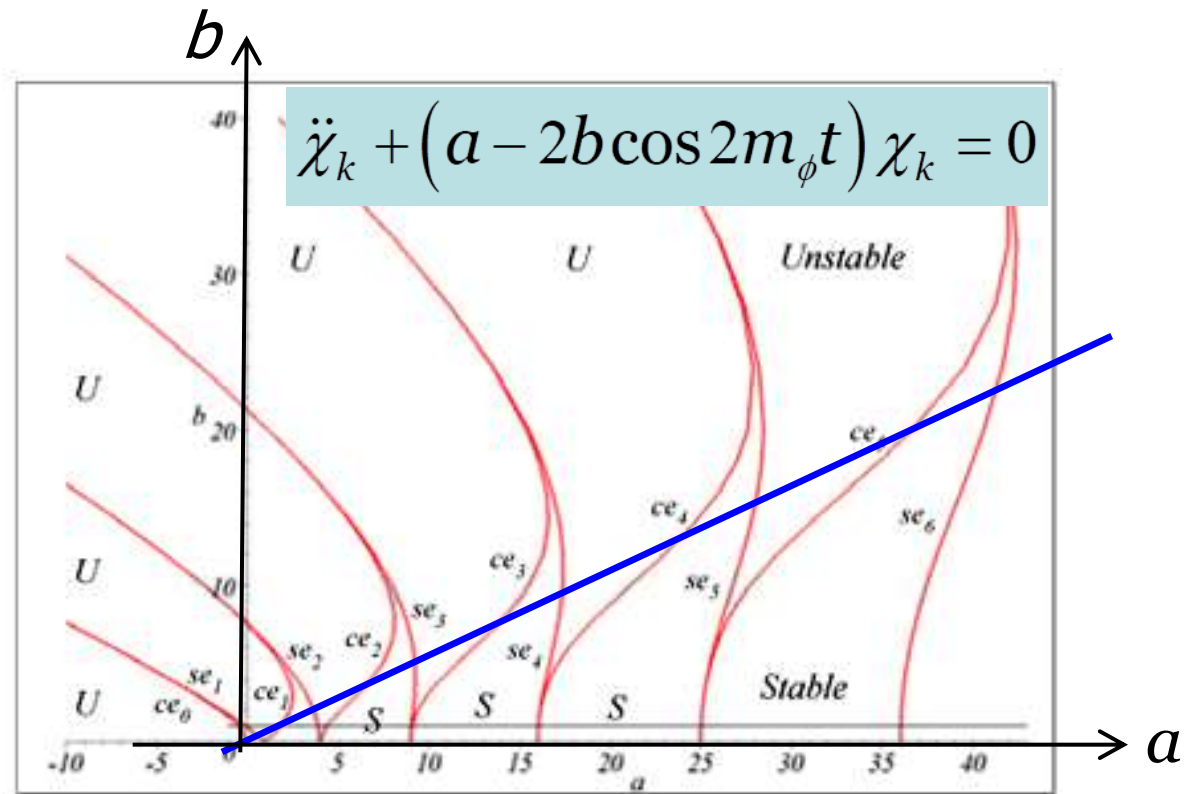


Figure 1. Mathieu stability chart based on the numerical values, generated by (McLachlan, 1947).

if  $b > 1$  initially, evolutionary path passes through unstable region

$\Rightarrow$  (almost) instantaneous reheating

# 2. Cosmological Perturbations from Inflation

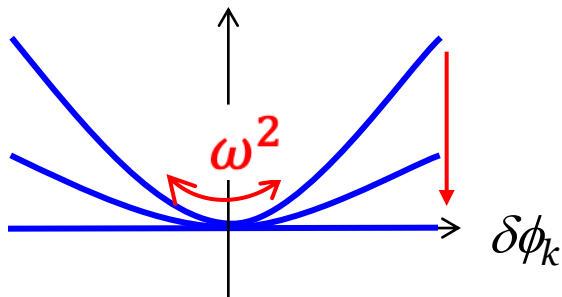
## ➤ curvature perturbation: intuitive derivation

zero-point (vacuum) fluctuations of  $\phi$ :  $\delta\phi = \sum_k \delta\phi_k(t) e^{ik \cdot x}$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega^2(t)\delta\phi_k = 0 ; \quad \omega^2(t) = \frac{k^2}{a^2(t)} \equiv \left( \frac{2\pi c}{\lambda(t)} \right)^2$$

physical wavelength  $\lambda(t) \sim a(t)$

harmonic oscillator with friction term and time-dependent  $\omega$



$\delta\phi_k \rightarrow \text{const.}$

... frozen when  $\lambda > c H^{-1}$   
(on superhorizon scales)

gravitational wave modes also satisfy the same eq.

- fluctuation amplitude (**vacuum fluctuations=Gaussian**)

$$\left| \langle \phi | \vec{k} \rangle \right|^2 = |\varphi_k|^2, \quad \varphi_k \sim \frac{1}{a^{3/2} \sqrt{2\omega_k}} e^{-i\omega_k t}; \quad \omega_k = \frac{k}{a} \gg H$$

**“adiabatic” vacuum**

$$\varphi_k(a \gg k/H) \approx \varphi_k(a = k/H) \approx \frac{H}{\sqrt{2k^3}} \Rightarrow \langle \delta\phi_k^2 \rangle = \left( \frac{H}{2\pi} \right)_{k/a \sim H}^2$$

**frozen at  $a = k/H$**

In the above, metric perturbations  $\delta g$  are ignored  
 $\sim$  a gauge in which  $\delta g$  is minimized (suppressed by  $M_P^{-2}$ )

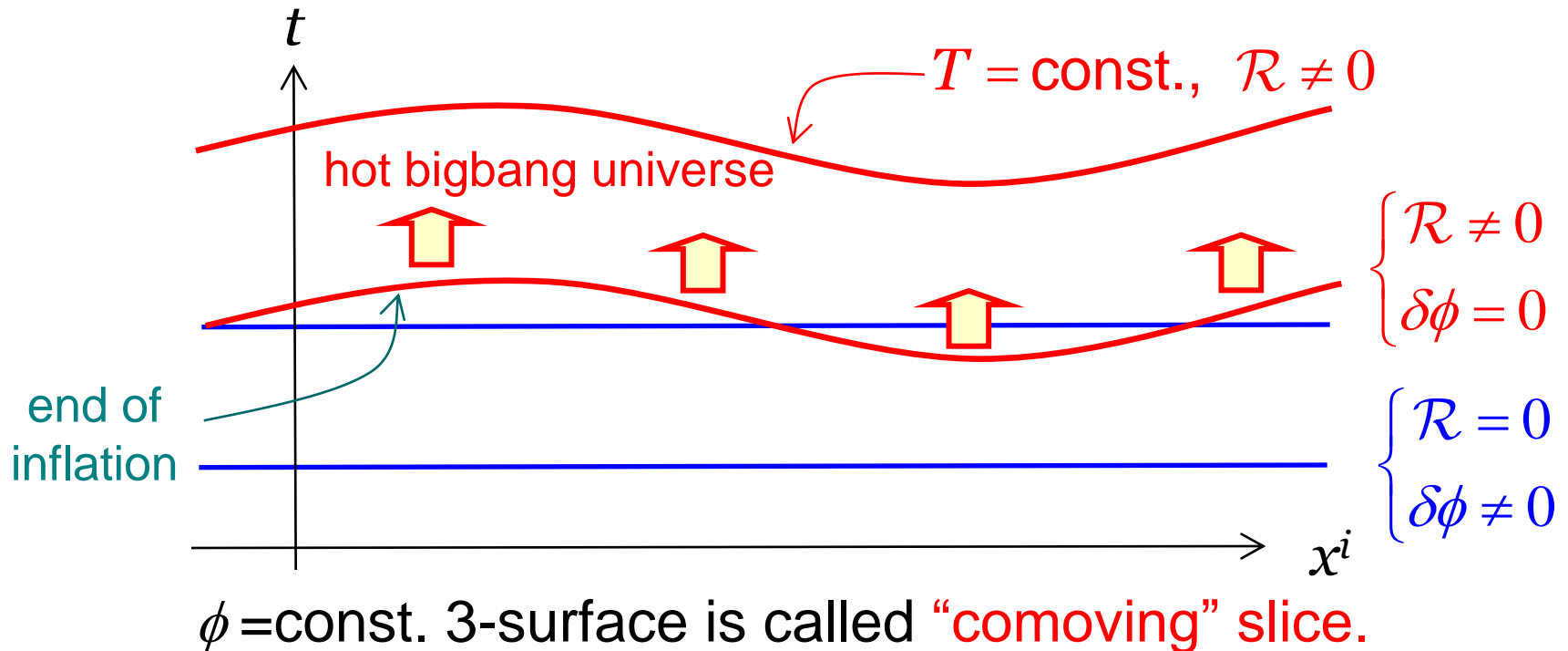
**= hypersurface on which  $\delta R^{(3)}=0$ : “flat” slice**

$$R^{(3)} = \frac{K}{6a^2}, \quad \delta R^{(3)} = \frac{4k^2}{a^2} \mathcal{R} \Rightarrow \delta K = \frac{2k^2}{3} \mathcal{R}$$

$\mathcal{R}$  : called curvature perturbation

# generation of “comoving” curvature perturbation

- $\delta\phi$  is frozen on “flat” ( $\mathcal{R}=0$ ) 3-surface ( $t = \text{const.}$  hypersurface)
- Inflation ends/damped osc starts on  $\phi = \text{const.}$  3-surface.



- curvature perturbation on comoving slices:

gauge transf.  $\implies \mathcal{R}_C = -\frac{H}{\dot{\phi}} \delta\phi$  ← evaluated on flat slice

# conservation of comoving curvature perturbation

Kodama & MS '84, ...

- eom

$$\mathcal{R}_C'' + \frac{(z^2)'}{z^2} \mathcal{R}_C' + k^2 \mathcal{R}_C = 0; \quad z^2 \equiv \frac{a^2 \dot{\phi}^2}{H^2} = 2\varepsilon a^2 M_P^2; \quad ' = \frac{d}{d\eta} = a \frac{d}{dt}$$

$$\Downarrow \quad k^2 \rightarrow 0$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+w), \quad w = \frac{P}{\rho}$$

$\varepsilon$  : slow-roll parameter

$$\frac{(z^2)'}{z^2} \mathcal{R}_C' + k^2 \mathcal{R}_C = 0$$

$\mathcal{R}_C' \propto 1/z^2$  : decaying mode

$\mathcal{R}_C = \text{const.}$  : "growing" mode

if  $\mathcal{R}_C$  becomes const., "adiabatic" limit is reached

$$\mathcal{R}_C|_{k \ll aH} \approx \mathcal{R}_C|_{k=aH} = -\left(\frac{H}{\dot{\phi}} \delta\phi\right)\Big|_{k=aH}$$

# Curvature perturbation spectrum

- spectrum 
$$P_{\mathcal{R}}(k) = \left( \frac{H^2}{2\pi\dot{\phi}} \right)_{k=aH}^2 = \frac{1}{2} \left( \frac{H}{2\pi M_P \epsilon^{1/2}} \right)_{k=aH}^2$$

Mukhanov ('85), MS ('86)

- spectral index

$$P_{\mathcal{R}}(k) = Ak^{n_s-1} ; n_s - 1 = M_P^2 \left( 2 \frac{V''}{V} - 3 \frac{V'^2}{V^2} \right) = 2\eta_V - 6\epsilon_V$$

Liddle & Lyth ('92)

- more elegantly derived a la Faddeev-Jackiw method

Garriga, Montes, MS & Tanaka ('98)

- generalized to k-inflation:  $L = P(X, \phi); X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

Garriga & Mukhanov ('99)

- generalized action for  $\mathcal{R}_C$

Garriga & Mukhanov ('99)

$$S = \int d\eta d^3x \frac{z^2}{2c_s^2} \left[ \mathcal{R}'_C{}^2 - c_s^2 k^2 \mathcal{R}_C^2 \right]; \quad z^2 = 3(1+w)a^2 M_P^2$$

$c_s$  = sound velocity (=1 for canonical case)

canonical quantization:

$$\pi_R = \frac{\delta S}{\delta \mathcal{R}'_C} = z^2 c_s^{-2} \mathcal{R}'_C \quad [\mathcal{R}_C, \pi_R] = i\hbar$$

adiabatic vacuum  
positive freq fcn

$$\mathcal{R}_C = a_{\vec{k}} r_k(\eta) + a_{-\vec{k}}^\dagger r_k^*(\eta); \quad r_k \rightarrow \frac{1}{\sqrt{2c_s k}} \frac{c_s}{z} e^{-ic_s k \eta} \quad (\eta \rightarrow -\infty)$$

$$\Rightarrow P_{\mathcal{R}}(k) = \frac{4\pi k^3}{(2\pi)^3} |r_k|_{c_s k|\eta|=1}^2 = \frac{1}{3c_s(1+w)} \left( \frac{H}{2\pi M_P} \right)_{c_s k=aH}^2$$

Q2. Derive the above spectrum by performing canonical quantization as outlined above.

- $\delta N$  - formula Starobinsky ('85)

$$N(\phi) = \int_{t(\phi)}^{t_{\text{end}}} H dt = \int_{\phi}^{\phi_{\text{end}}} \frac{H}{\dot{\phi}} d\phi$$

$$\Rightarrow \delta N(\phi) = \left[ \frac{\partial N}{\partial \phi} \delta\phi \right]_{k=aH} = \left[ -\frac{H}{\dot{\phi}} \delta\phi \right]_{k=aH} = \mathcal{R}_c$$

$$P_{\mathcal{R}}(k) = \left( \frac{H^2}{2\pi\dot{\phi}} \right)_{k=aH}^2 = \left( \frac{\partial N}{\partial \phi} \right)^2 |\varphi_k|_{|k|\eta|=1}^2; \quad |\varphi_k|^2 = \langle \delta\phi_k^2 \rangle = \left( \frac{H}{2\pi} \right)_{k=aH}^2$$

{

 geometrical justification  $\delta N = \sum_A \frac{\partial N}{\partial \phi^A} \delta\phi^A$  MS & Stewart ('96)  
 NL generalization Lyth, Malik & MS ('04)
}

only knowledge of background evolution is necessary

# Tensor (GW) Perturbation

Starobinsky ('79)

$$\partial^i h_{ij}^{TT} = \delta^{ij} h_{ij}^{TT} = 0 \quad : \text{transverse-traceless}$$

- canonically normalized tensor field

$$S \sim \int d^4x \sqrt{-g} \frac{1}{2} \left( \frac{\partial \phi_{ij}}{\partial t} \right)^2 + \dots$$

$$\phi_{ij} \equiv \frac{1}{\sqrt{32\pi G}} h_{ij}^{TT} = \frac{M_P}{2} h_{ij}^{TT}; \quad M_P \equiv \frac{1}{\sqrt{8\pi G}}$$

$$\phi_{ij}(k;t) = \sum_{\sigma=+,\times} a_k^\sigma P_{ij}^\sigma(k) \varphi_k(t) + h.c.$$

$\varphi_k(t)$ : same as massless scalar

- tensor spectrum

$$\sum_{\sigma} \left| \langle h_{ij}^{TT} | \vec{k}, \sigma \rangle \right|^2 = \frac{4}{M_P^2} \sum_{\sigma} \left| \langle \phi_{ij} | \vec{k}, \sigma \rangle \right|^2 = \frac{8 |\varphi_k|^2}{M_P^2} = 8 \left( \frac{H}{2\pi M_P} \right)^2$$

## ➤ Tensor-to-scalar ratio

$$\|\nabla N\|^2 \equiv H^{ab}(\phi) \frac{\partial N}{\partial \phi^a} \frac{\partial N}{\partial \phi^b}$$

- scalar spectrum:  $P_s(k) \frac{4\pi k^3}{(2\pi)^3} = \frac{H^2}{(2\pi)^2} \|\nabla N\|^2 \propto k^{n_s-1}$

- tensor spectrum:  $P_g(k) \frac{4\pi k^3}{(2\pi)^3} = \frac{8}{M_P^2} \frac{H^2}{(2\pi)^2} \propto k^{n_g}$

- tensor spectral index:  $-n_g = -\frac{2\dot{H}}{H^2} = \frac{\|\dot{\phi}\|^2}{M_P^2 H^2} = \frac{\|\dot{\phi}\|^2}{M_P^2 \|\dot{\phi} \cdot \nabla N\|^2}$

$$H = -\frac{dN}{dt} = -\dot{\phi}^a \nabla_a N \geq \frac{1}{M_P^2 \|\nabla N\|^2} = \frac{P_g}{8P_s}$$



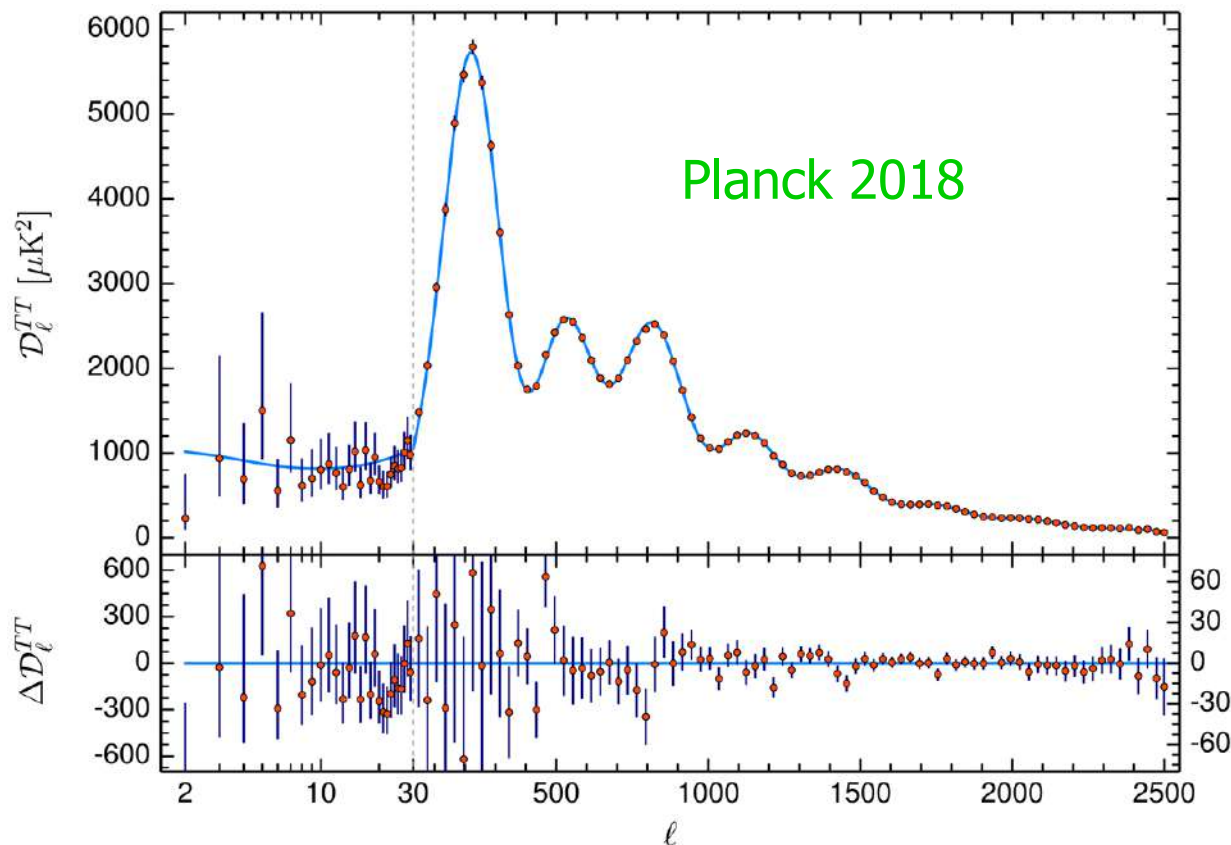
$$r \equiv \frac{P_g}{P_s} \leq 8 |n_g| = 2\varepsilon$$

... valid for all slow-roll models with canonical kinetic term

MS & Stewart ('96)

# Comparison with observation

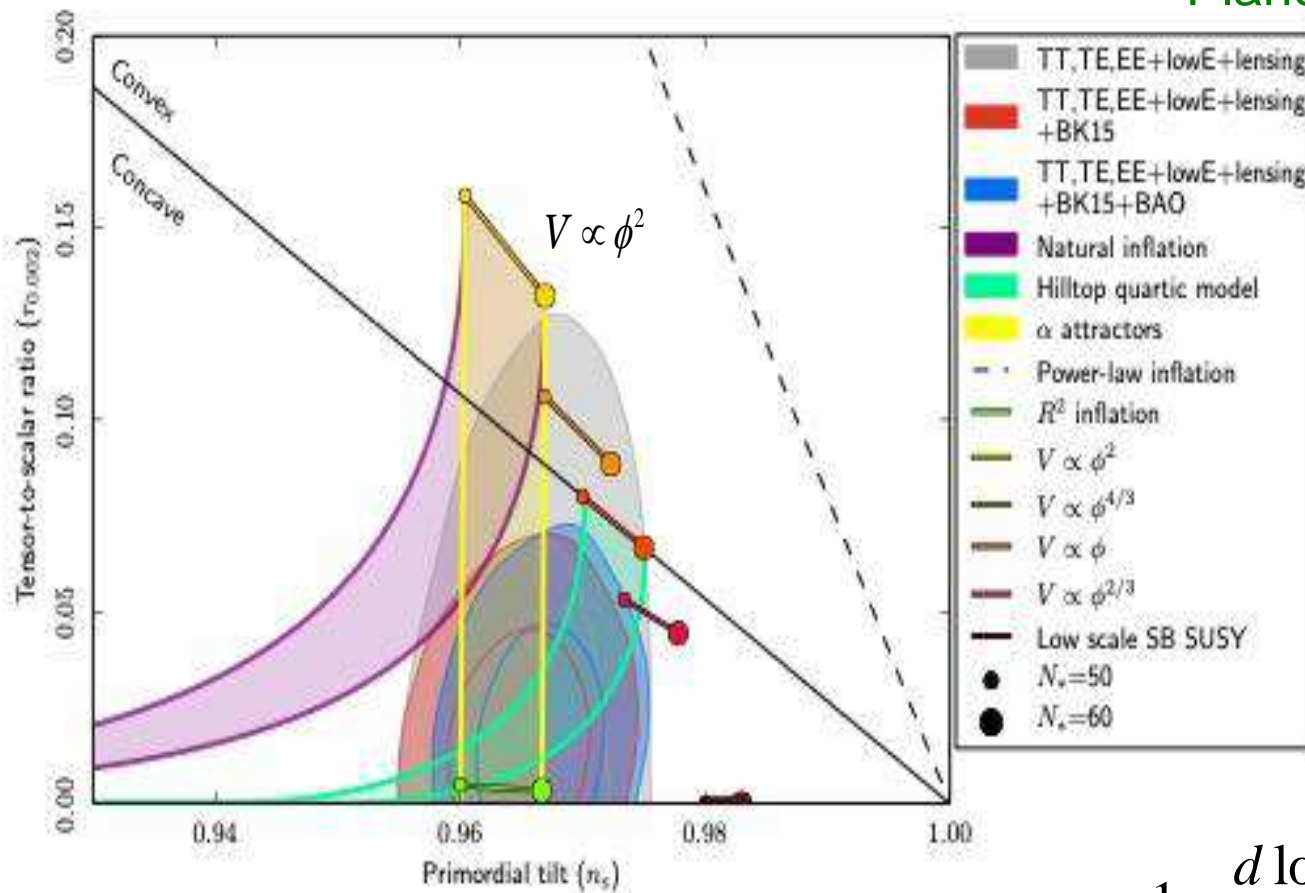
- Standard (single-field, slow-roll) inflation predicts scale-invariant **Gaussian** curvature perturbations.



- CMB (Planck) is consistent with the prediction.
- Linear** perturbation theory seems to be valid.

# Planck constraints on inflation

Planck 2018



- scalar spectral index:  $n_s \sim 0.965$  ( $> 0.971?$ )
- tensor-to-scalar ratio:  $r < 0.03$
- simplest  $V \propto \phi^2$  model is **excluded**

ACT DR6

$$n_s - 1 \equiv \frac{d \log [k^3 P_s(k)]}{d \log k}$$

$$r \equiv \frac{P_T(k)}{P_S(k)}$$

# Nothing seems to be conclusive....

- Inflation may be **non-conventional**  
multi-field, non-slowroll, K-inflation, extra-dim's, ...

- **Non-Gaussianity** may exist (on certain scales)

(comoving) curvature perturbation:

$$\mathcal{R}_C = \mathcal{R}_{\text{gauss}} + \frac{3}{5} f_{NL} \mathcal{R}_{\text{gauss}}^2 + \dots; \quad f_{NL} \gtrsim \mathcal{O}(1)?$$

↳ can be scale-dependent

- **B-mode (tensor)** may or may not be detected.

energy scale of inflation  $H^2 \gtrsim 10^{-10} M_{\text{Planck}}^2$  ?

modified (quantum) gravity? **NG signature?**

Quantifying NL/NG effects is important