Inflation

Part II

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3. Non-Gaussian Curvature Perturbation

Possible Origins

- self-interactions of inflaton/non-trivial “vacuum”

  quantum physics, subhorizon scale during inflation

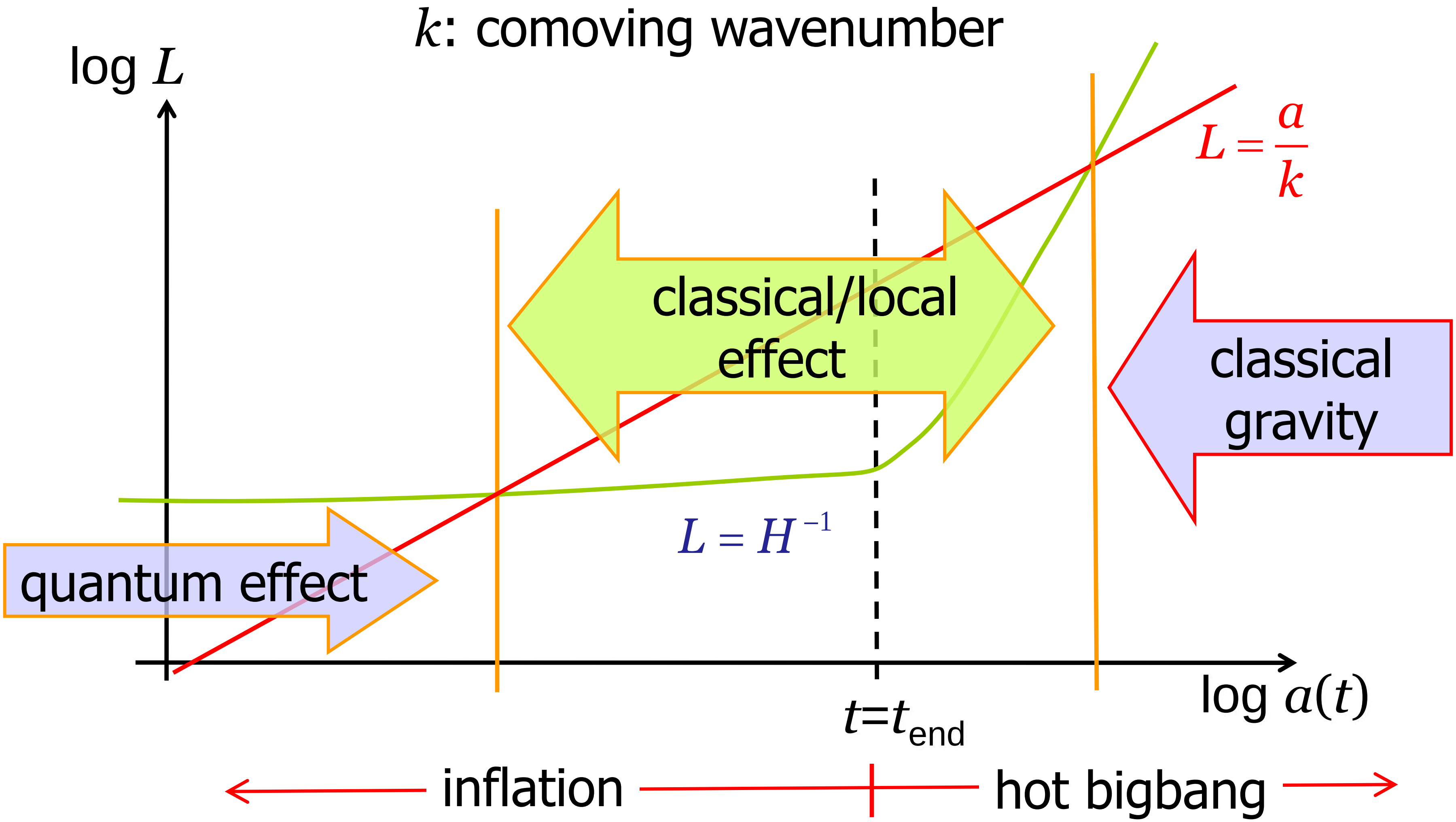
- multi-field

  classical physics, nonlinear coupling to gravity
  superhorizon scale during and after inflation

- nonlinearity in gravity

  classical general relativistic effect,
  subhorizon scale after inflation

Origins of NG and cosmic scales



Origin 1 : self-interaction/non-trivial vacuum

Non-Gaussianity generated on subhorizon scales

(quantum field theoretical)

- conventional self-interaction by potential is ineffective

ex. chaotic inflation — Maldacena ('03)

$$V = \frac{1}{2} m^2 \phi^2 \quad \cdots \text{ free field!}$$

(grav. interaction is Planck-suppressed) $\sim O(1/M_{Pl}^2)$

$$V = \lambda \phi^4 \quad \rightarrow \lambda \sim 10^{-15}$$

extremely small!

- need unconventional self-interaction

$\rightarrow$ non-canonical kinetic term can generate large NG

K-Inflation

ex: DBI inflation

Silverstein & Tong (2004),...

kinetic term: $K \sim f^{-1}(\phi)\sqrt{1-f(\phi)\dot{\phi}^2} \equiv f^{-1}\gamma^{-1}$

~ (Lorenz factor)$^{-1}$

perturbation expansion $\left(\delta\gamma = \frac{1}{2}\gamma^3\delta X; \quad X \equiv f\dot{\phi}^2\right)$

$$K = K_0 + \delta_1 K + \delta_2 K + \delta_3 K + \cdots$$

$\delta_1 K = 0$, $\delta_2 K \propto \gamma^3$, $\delta_3 K \propto \gamma^{3+2}$

$$\Rightarrow \quad \delta\phi \sim \delta\phi_0 + \gamma^2\delta\phi_0^2 + \cdots$$

large NG for large γ

Bi-spectrum (3pt function) in DBI inflation (K-inflation)

$$\left\langle \mathcal{R}_C(p_1)\mathcal{R}_C(p_2)\mathcal{R}_C(p_3)\right\rangle$$

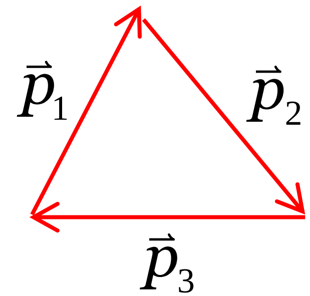
Alishahiha et al. ('04)

$$\sim \delta(\sum_j p_j) f_{NL}(p_1, p_2, p_3)\left(P_{\mathcal{R}}(p_1)P_{\mathcal{R}}(p_2) + \text{cyclic}\right)$$

⇩ $f_{NL} \sim \gamma^2$

f_{NL} large for equilateral configuration

$$|\vec{p}_1| \sim |\vec{p}_2| \sim |\vec{p}_3|$$



$$\left(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0\right)$$

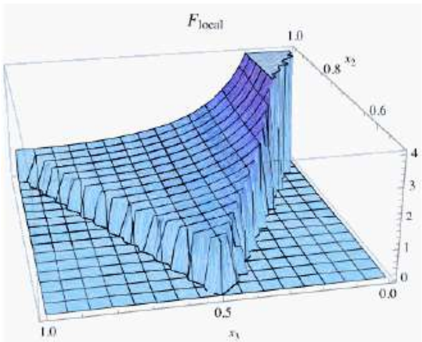
$$f_{NL} \Rightarrow f_{NL}^{\text{equil}}$$

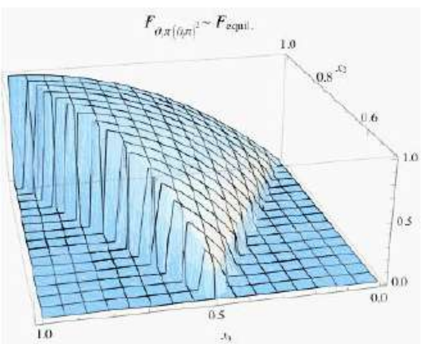
Planck 2018: $-73 < f_{NL}^{\text{equil}} < 21$ (68% CL)

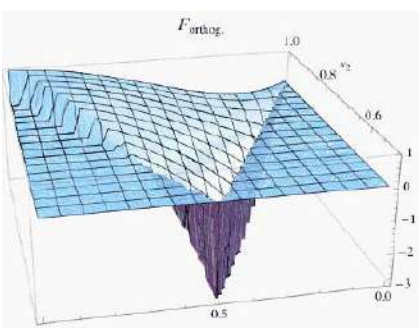
templates for bispectra

(figs from Senatore, Smith & Zaldarriaga '11)

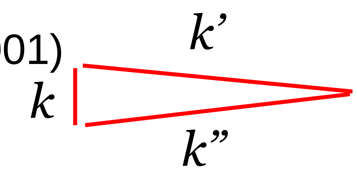
$$P_\zeta(k) = \mathcal{P}/k^3, \quad B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}(k_1,k_2,k_3)\big(P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\big)$$



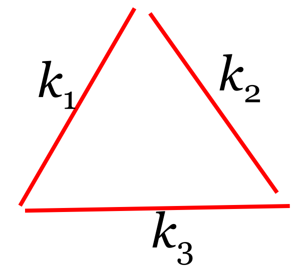




- squeezed type (Komatsu&Spergel 2001)
  - local in real space (f_{NL}=constant)
  - max for squeezed triangles: k<<k',k''



$$B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}^{\text{local}} \mathcal{P}^2 \left(\frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right)$$



- equilateral type (Creminelli et al 2005)
  - peaks for $k_1 \sim k_2 \sim k_3$

$$B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}^{\text{equil}} \mathcal{P}^2 \left(\frac{3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{k_1^3 k_2^3 k_3^3} \right)$$

- orthogonal type (Senatore et al 2009)

$$B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}^{\text{orthog}} \mathcal{P}^2 \left(\frac{81}{k_1 k_2 k_3 (k_1+k_2+k_3)^3} \right)$$

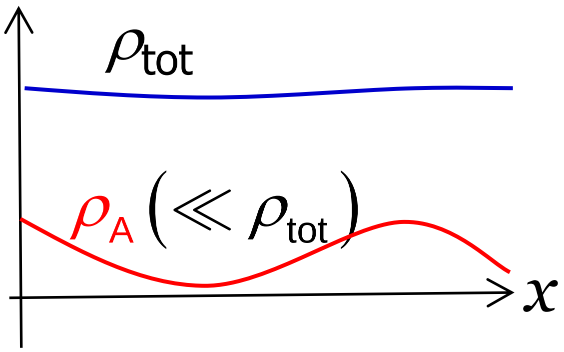
Origin 2 : Generation on superhorizon scales

- NG may appear if $T^{\mu\nu}$ depends nonlinearly on $\delta\phi$, even if $\delta\phi$ itself is Gaussian.

This effect is small in single-field slow-roll models
(⇔ linear approximation is valid to high accuracy)

Salopek & Bond ('90)

- For multi-field models, contribution to $T^{\mu\nu}$ from each field can be highly nonlinear.



NG is of local type:

$$f_{NL}(p_1, p_2, p_3) \to f_{NL}^{\text{local}} = \text{const.}$$

Planck 2018: $-6.0 < f_{NL}^{\text{local}} < 4.2$ (68% CL)

δN formalism for this type of NG

Origin 3: Nonlinearity in gravity

ex. post-Newtonian metric in asymptotically flat space

$$ds^2 = -\left(1 + 2\Psi - 2\Psi^2 + \cdots\right)dt^2 + \left(1 - 2\Psi + 2\Psi^2 + \cdots\right)dr^2 + \cdots$$

Newton potential

NL (Post-Newton) terms

(in both local and nonlocal forms)

- important when scales have re-entered Hubble horizon

distinguishable from NL matter dynamics?

- effect on CMB bispectrum may not be negligible

$f_{NL,PN} \sim O(5)$? Pitrou et al. (2010)

(for both squeezed and equilateral types)

δN formalism

- δN is the perturbation in # of e-folds counted backward in time from a fixed final time t_f

  therefore it is nonlocal in time by definition

- t_f should be chosen such that the evolution of the universe has become unique by that time: "adiabatic limit"

  isocurvature perturbation that persists until today must be dealt separately

- δN is equal to conserved NL comoving curvature perturbation on superhorizon scales at $t>t_f$

- δN formalism is valid independent of gravity theory

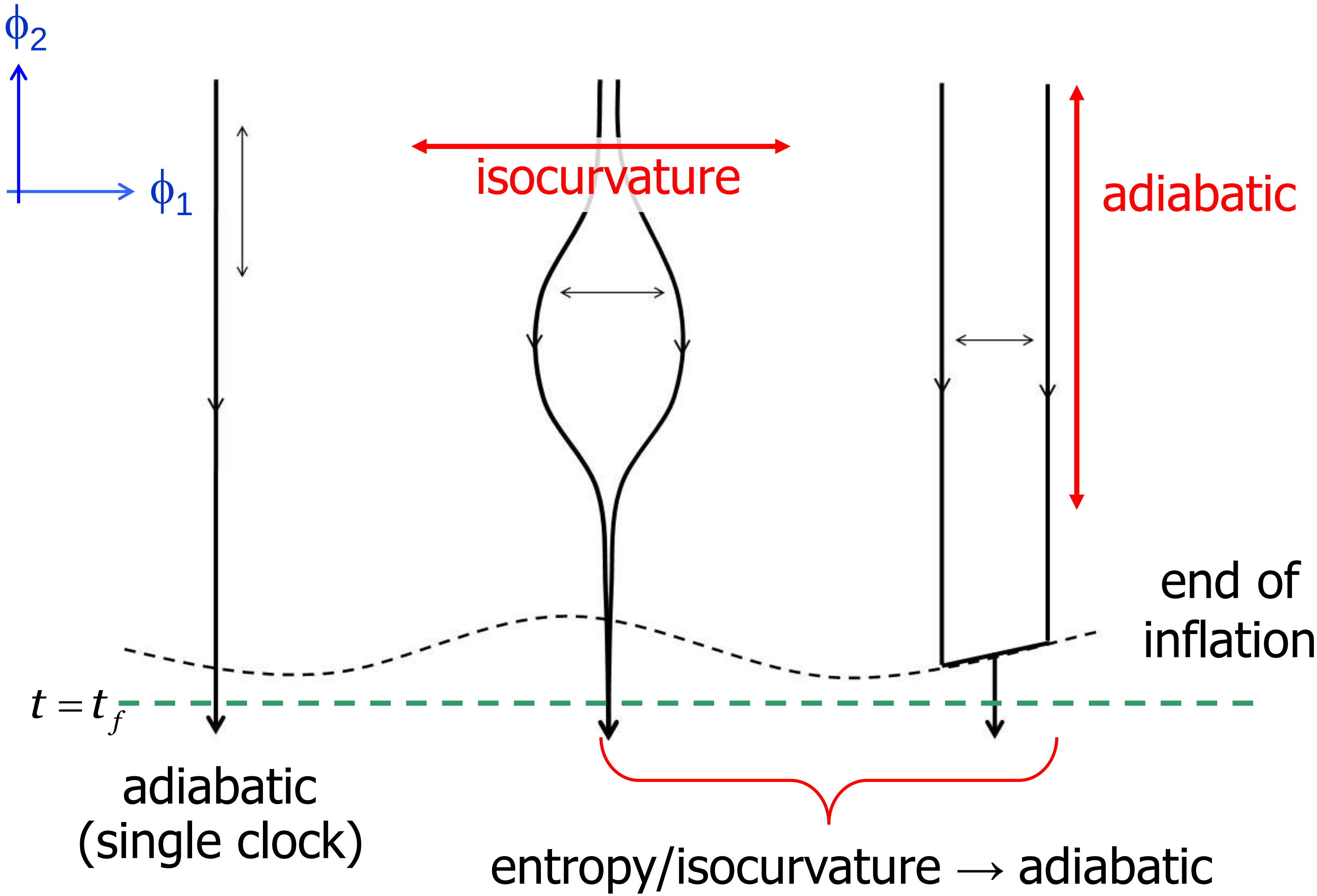
  (but I won't discuss modified gravity)

- intermission: advertisement -

δN Formalism in Cosmological Perturbation Theory

https://www.worldscientific.com/worldscibooks/10.1142/10953?srsltid=AfmBOooY-DShE4DKqjo6H5S1GYLhnDBPT6GvRZ7hHjDyRAvf9LwuOyPr#t=aboutBook

3 types of δN



ϕ_2
ϕ_1
isocurvature
adiabatic
end of
inflation
$t = t_f$
adiabatic
(single clock)
entropy/isocurvature → adiabatic

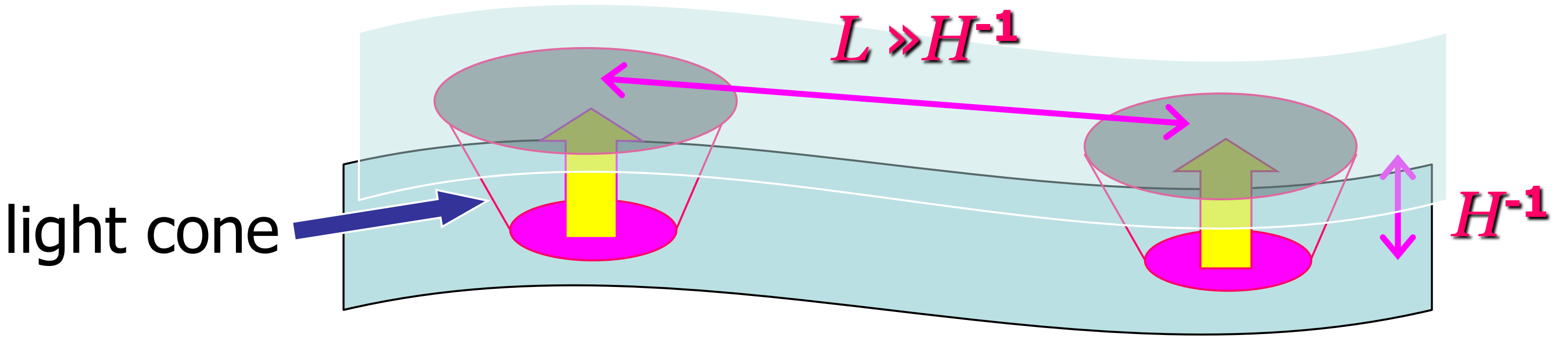
Separate Universe approach
(~ spatial gradient expansion)

- On superhorizon scales, spatial gradient expansion is valid:

$$\left|\frac{\partial}{\partial x^i}Q\right| \ll \left|\frac{\partial}{\partial t}Q\right| \sim HQ;\ \ H \sim \sqrt{G\rho}$$

Belinski et al. '70, Tomita '72, Salopek & Bond '90, …

This is a consequence of causality:



- At lowest order, no signal propagates in spatial directions.

Field equations reduce to ODE's

metric on superhorizon scales

- gradient expansion:

$$\partial_i \to \varepsilon \partial_i \,, \quad \varepsilon = \text{expansion parameter}$$

- metric:

$$ds^2 = -\mathcal{N}^2 dt^2 + e^{2\alpha} \tilde{\gamma}_{ij} \left(dx^i + \beta^i dt \right) \left(dx^j + \beta^j dt \right)$$

$$\det \tilde{\gamma}_{ij} = 1, \quad \beta^{\mathrm{i}} = O(\varepsilon)$$

the only non-trivial assumption

contains GW (~ tensor) modes

$$e^{\alpha(t,x^i)} = a(t) e^{\mathcal{R}(t,x^i)}$$

$\mathcal{R}$: NL curvature perturbation

fiducial `background'

Local Friedmann eqn & δN formula

Lyth, Malik & MS ('05)

$$\tilde{H}^2(t,x^i) = \frac{8\pi G}{3}\rho(t,x^i) + O(\varepsilon^2)$$

$$\tilde{H} \equiv \frac{\partial}{\partial\tau}\alpha = \frac{\partial}{\mathcal{N}\partial t}[\ln a + \mathcal{R}]$$

··· geometrical def of "Hubble"

x^i : comoving (Lagrangean) coordinates.

$d\tau = \mathcal{N}\, dt$: proper time along fluid flow

exactly the same as the homogeneous background

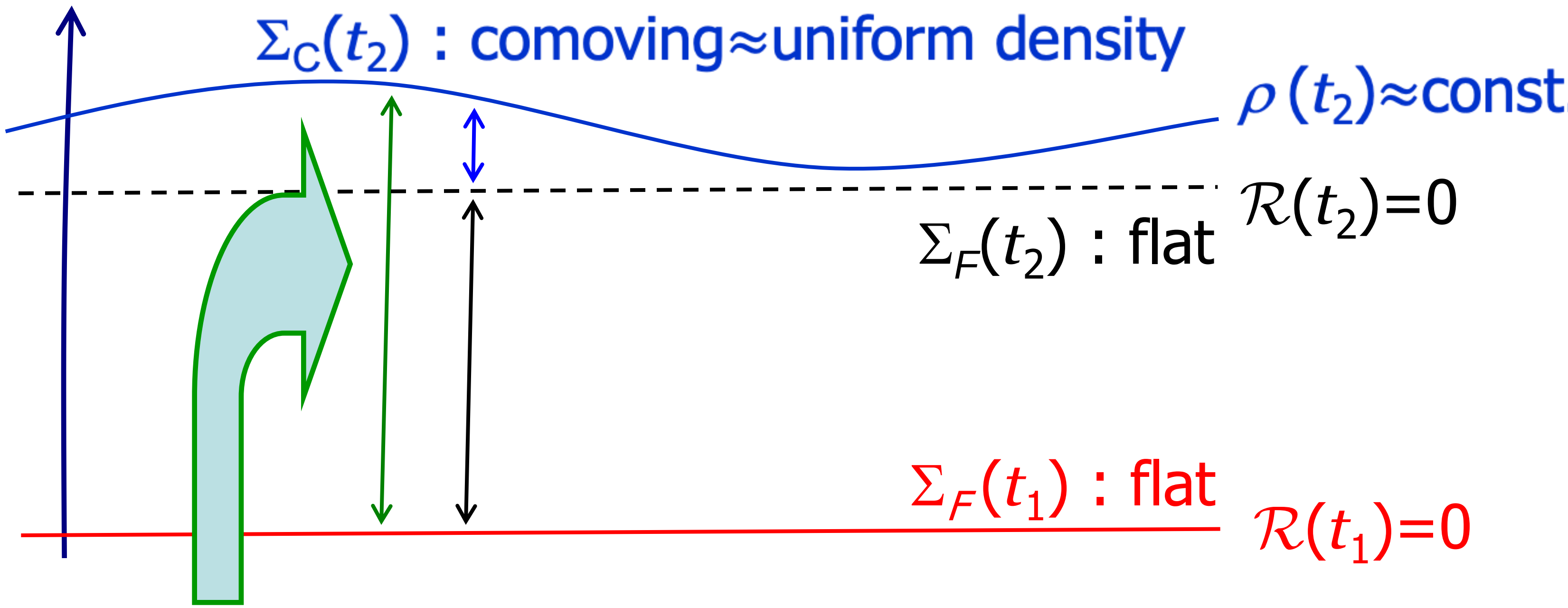
$$N(t_2,t_1) \equiv \int_{t_1}^{t_2} \tilde{H}\, d\tau = N_0(t_2,t_1) + \mathcal{R}(t_2,x^i) - \mathcal{R}(t_1,x^i)$$

$$N_0(t_2,t_1) \equiv \ln\left[a(t_2)/a(t_1)\right]$$

Nonlinear δN formula

Choose flat slice at $t = t_1$ [$\Sigma_F(t_1)$] and
comoving (=uniform density) at $t = t_2$ [$\Sigma_C(t_2)$] :

('flat' slice: $\Sigma(t)$ on which $\mathcal{R} = 0 \leftrightarrow e^{\alpha} = a(t)$)



$$N(t_2, t_1) \equiv N_0(t_2, t_1) + \delta N_F(t_2, t_1; x^i)$$

$$\boxed{\delta N_F(t_2, t_1; x^i) = \mathcal{R}_C(t_2, x^i)}$$

How do we relate δN to matter evolution?

need eqn relating ‘expansion’ with matter ‘evolution’

energy conservation!

$$\frac{d}{d\tau}\rho + 3\tilde{H}(\rho+p) = 0 \quad \Rightarrow \quad \tilde{H} \equiv -\frac{1}{3(\rho+p)}\frac{\partial}{\partial\tau}\rho$$

$$\Rightarrow \quad N(t_2,t_1) = -\int_{t_1}^{t_2} dt \frac{1}{3(\rho+p)}\frac{\partial\rho}{\partial t}$$

$$\delta N_F(t_2,t_1;x^i) = -\frac{1}{3}\int_{\Sigma_F(t_1)}^{\Sigma_C(t_2)} \left.\frac{\partial_t\rho}{\rho+P}\right|_{x^i} dt + \frac{1}{3}\int_{\Sigma_F(t_1)}^{\Sigma_C(t_2)} \left.\frac{\partial_t\rho}{\rho+P}\right|_{0} dt$$

x^i=0 : fiducial background trajectory

$\rho(x^i,t_2) = \rho(0,t_2)$ = uniform on $\Sigma_C(t_2)$

matter fluctuates only on the initial flat slice

Conservation of nonlinear $\mathcal{R}_{\rm NL}$

For adiabatic case ($p=p(\rho)$,or single-field slow-roll case),

$$N\left(t_2,t_1;x^i\right)=-\frac{1}{3}\int_{t_1}^{t_2}\frac{\partial_t\rho}{\rho+P(\rho)}dt$$

$$=-\frac{1}{3}\int_{\rho(t_1,x^i)}^{\rho(t_2,x^i)}\frac{d\rho}{\rho+P(\rho)}=\psi(t_2,x^i)-\psi(t_1,x^i)+\ln\left[\frac{a(t_2)}{a(t_1)}\right]$$

$$\Rightarrow\ \boxed{\mathcal{R}_{\rm NL}(x^i)\equiv\psi(t,x^i)+\frac{1}{3}\int_{\rho(t)}^{\rho(t,x^i)}\frac{d\rho}{\rho+P(\rho)}}$$ ···slice-independent

non-linear generalization of 'gauge'-invariant quantity ζ or $\mathcal{R}_c$

(ψ and ρ can be evaluated on any time slice)

ex.: slow-roll inflation

$$d\rho\approx V'd\phi,\quad \rho+P=\dot{\phi}^2\approx\frac{V'^2}{3V}\quad\Rightarrow\quad\frac{1}{3}\int_{\rho}^{\rho+\delta\rho}\frac{d\rho}{\rho+P}=\int_{\phi}^{\phi+\delta\phi}\frac{V}{V'}d\phi=\delta N$$

$$\Rightarrow\quad\mathcal{R}_{\rm NL}=\delta N\big|_{\psi=0}$$

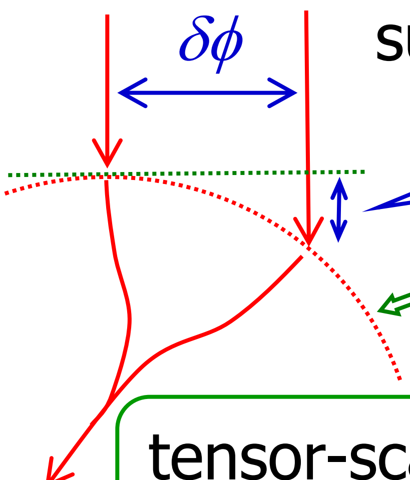
NG generation on superhorizon scales

two efficient mechanisms to convert
isocurvature to curvature perturbations:

- curvaton-type   Lyth & Wands ('01), Moroi & Takahashi ('01),...

  $\rho_{curv} << \rho_{tot}$ ⇔ highly nonlinear dep on $\delta\phi_{curv}$

- multi-brid type   MS ('08), Naruko & MS ('08),...



sudden change/transition in the trajectory

$$\delta N = \partial_a N \delta\phi^a + \frac{1}{2}\partial^2_{ab} N \delta\phi^a \delta\phi^b + \cdots$$

curvature of this surface determines sign of f_{NL}
Huang ('09)

tensor-scalar ratio r may be large in multi-brid models,
while it is always small in curvaton-type if NG is large.

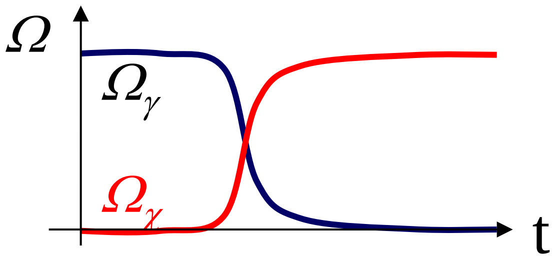
Curvaton model

2-field model: inflaton (ϕ) + curvaton (χ)

$$V = V(\phi) + \frac{1}{2} m_\chi^2 \chi^2 \qquad m_\chi^2 \ll H^2 \approx \frac{V}{3M_P^2}$$

- during inflation ϕ dominates.
- after inflation, χ begins to dominate.

$\rho_\phi = \rho_\gamma \propto a^{-4}$ and $\rho_\chi \propto a^{-3}$, hence $\Omega_\chi / \Omega_\gamma \propto a$



➢ final curvature pert amplitude depends on when χ decays.

- Before curvaton decay

$$\mathcal{R}_{\chi} = \psi + \frac{1}{3}\ln\left(\frac{\rho_{\chi}(t,x^i)}{\bar{\rho}_{\chi}(t)}\right) \qquad \mathcal{R}_{\gamma} = \psi + \frac{1}{4}\ln\left(\frac{\rho_{\gamma}(t,x^i)}{\bar{\rho}_{\gamma}(t)}\right)$$

$$\Longrightarrow \quad \rho_{\chi}(t,x^i) + \rho_{\gamma}(t,x^i) = \bar{\rho}_{\chi} e^{3(\mathcal{R}_{\chi}-\psi)} + \bar{\rho}_{\gamma} e^{4(\mathcal{R}_{\gamma}-\psi)}$$

- On uniform total density slices, $\psi = \zeta$ $\left(= \mathcal{R}_{NL}\right.$ on page 18$\left.\right)$

$$\rho_{\chi}(t,x^i) + \rho_{\gamma}(t,x^i) = \bar{\rho}_{\chi} e^{3(\mathcal{R}_{\chi}-\zeta)} + \bar{\rho}_{\gamma} e^{4(\mathcal{R}_{\gamma}-\zeta)} = \bar{\rho}_{\chi} + \bar{\rho}_{\gamma}$$

nonlinear version of $\zeta = \mathcal{R}_C = \sum_A \frac{(\rho_A + P_A)\mathcal{R}_A}{\rho + P}$

- With sudden decay approx, final curvature pert ζ satisfies

$$\boxed{\left(1-\Omega_{\chi}\right) e^{4(\mathcal{R}_{\gamma}-\zeta)} + \Omega_{\chi} e^{3(\mathcal{R}_{\chi}-\zeta)} = 1}$$

MS, Valiviita & Wands (2006)

Ω_{χ} : density fraction of χ at the moment of its decay

Q3. Derive the above formula.

Multi-brid inflation

"multi"-field hy"brid" inflation

$$L_\phi = -\frac{1}{2}\sum_A g^{\mu\nu}\partial_\mu\phi_A\partial_\nu\phi_A - V(\phi)$$

MS ('08) MS & Naruko ('09)

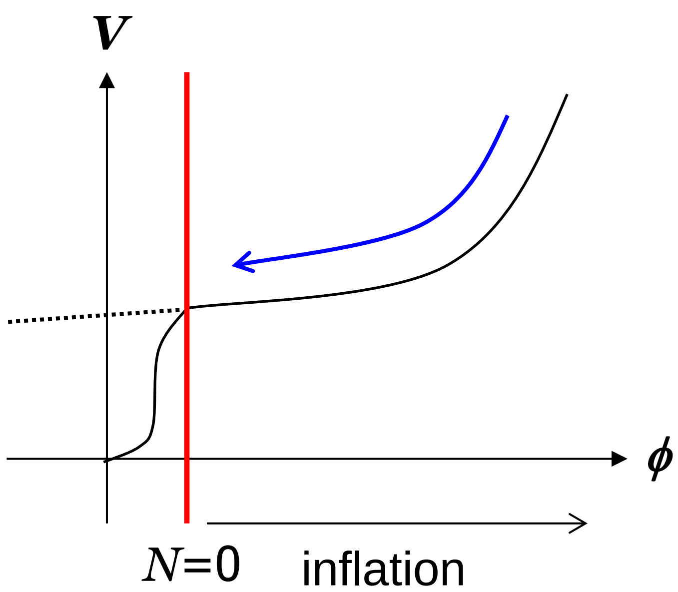
- slow-roll eom $(8\pi G = M_{\rm Planck}^{-2} = 1)$

$$\frac{d\phi_A}{dt} = -\frac{1}{3H}\frac{\partial V}{\partial\phi_A}\,, \quad 3H^2 = V$$

N as a time variable: $dN = -Hdt$

$$\Rightarrow \quad \frac{d\phi_A}{dN} = \frac{1}{3V}\frac{\partial V}{\partial\phi_A}$$

$$\Rightarrow \quad \phi_A = \phi_A(N, \phi_A^0)$$



… slow-roll ends at $F(\phi_A)=0$.

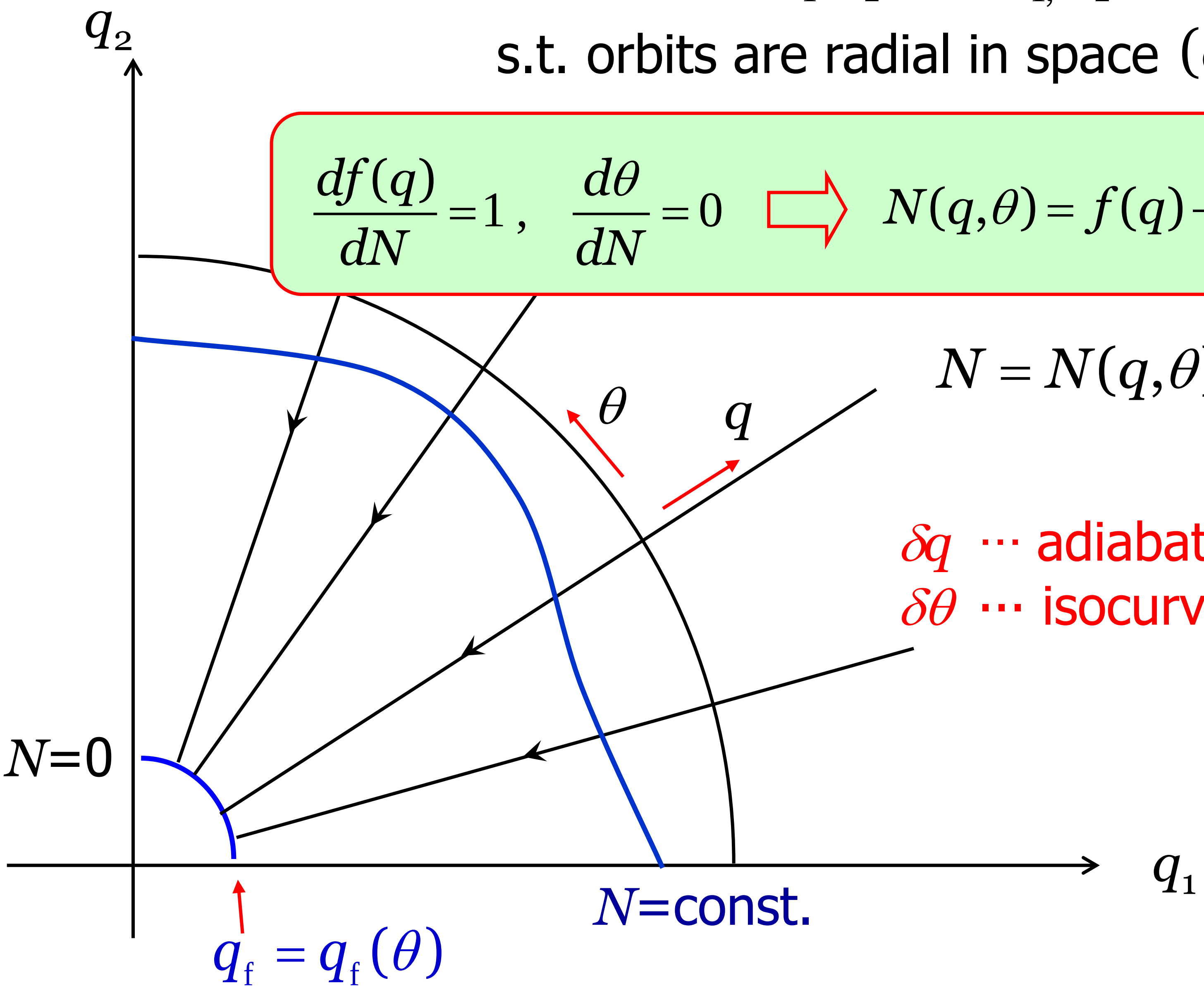
2-dim case:

coord trans $(\phi_1,\phi_2) \rightarrow (q_1, q_2)$

s.t. orbits are radial in space (q_1, q_2)

$$\frac{df(q)}{dN} = 1\,, \quad \frac{d\theta}{dN} = 0 \quad \Rightarrow \quad N(q,\theta) = f(q) - f(q_f(\theta))$$

$$N = N(q,\theta) = N(\phi_1,\phi_2)$$

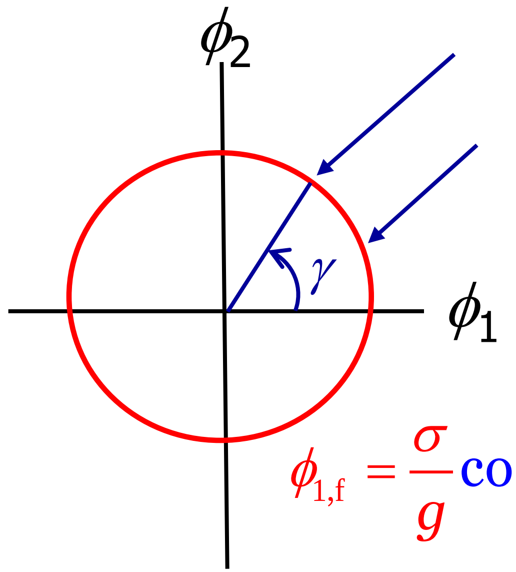


δq ⋯ adiabatic pertn

$\delta\theta$ ⋯ isocurvature pertn

analytical multi-brid model

- Exponential potential: $V = V_0 \exp\left[m_1\phi_1 + m_2\phi_2\right]$



Inflation ends at $g^2\left(\phi_1^2 + \phi_2^2\right) = \sigma^2$

realized by a waterfall field χ :

$$V_0 = \frac{1}{2} g^2\left(\phi_1{}^2 + \phi_2{}^2\right)\chi^2 + \frac{\lambda}{4}\left(\chi^2 - \frac{\sigma^2}{\lambda}\right)^2$$

$$\phi_{1,\mathrm{f}} = \frac{\sigma}{g}\cos\gamma, \quad \phi_{2,\mathrm{f}} = \frac{\sigma}{g}\sin\gamma$$

trajectory specified by "γ"

- δN to 2nd order in δϕ :

$$\delta N = \frac{\delta\phi_1\cos\gamma + \delta\phi_2\sin\gamma}{m_1\cos\gamma + m_2\sin\gamma} + \frac{g}{2\sigma}\frac{(m_2\delta\phi_1 - m_1\delta\phi_2)^2}{(m_1\cos\gamma + m_2\sin\gamma)^3}$$

$$\Rightarrow \quad \delta N = \delta_L N + \frac{3}{5} f_{NL}^{\mathrm{local}} \left(\delta_L N + S\right)^2$$

linear isocurvature perturbation contributes at 2nd order

$$\delta_L N \equiv \frac{\delta\phi_1 \cos\gamma + \delta\phi_2 \sin\gamma}{m_1 \cos\gamma + m_2 \sin\gamma}, \quad S \equiv \frac{\delta\phi_1 \sin\gamma - \delta\phi_2 \cos\gamma}{m_2 \cos\gamma - m_1 \sin\gamma}$$

"true" isocuvature perturbation

- curvature perturbation spectrum

$$P_S(k) = \frac{1}{(m_1 \cos\gamma + m_2 \sin\gamma)^2} \left(\frac{H}{2\pi}\right)^2 \Bigg|_{k=Ha}$$

spectral index: $n_s = 1 - (m_1^2 + m_2^2)$

tensor/scalar: $r \equiv \dfrac{P_T(k)}{P_S(k)} = 8(m_1 \cos\gamma + m_2 \sin\gamma)^2$

non-Gaussianity: $f_{NL}^{\mathrm{local}} = \dfrac{5g}{6\sigma} \dfrac{(m_2 \cos\gamma - m_1 \sin\gamma)^2}{m_1 \cos\gamma + m_2 \sin\gamma}$

just for fun ...

$$1 = M_{Pl} = (8\pi G)^{-1/2} = 2.43\times 10^{18}\,\mathrm{GeV}$$

model parameters: $m_1^2 \sim 0.005,\ \ m_2^2 \sim 0.035$

assume $m_1 \cos\gamma \gtrsim m_2 \sin\gamma \quad (\Leftrightarrow \gamma \ll 1)$

outputs:

$$n_s = 1 - (m_1^2 + m_2^2) \sim 0.96$$

$$r \approx 8 m_1^2 \sim 0.04$$

indep. of waterfall field

$$3H^2 = \sigma^4/4\lambda \sim 1.5\times 10^{-9} \quad \left(\Leftrightarrow P_S(k) \sim 2.5\times 10^{-9}\right)$$

$$\Rightarrow \quad \sigma^2 \sim \lambda^{1/2}\times 10^{-4}$$

$$\Rightarrow \quad f_{NL}^{\mathrm{local}} \approx \frac{5 g m_2^2}{6\sigma m_1} \sim 40\frac{g}{\lambda^{1/4}}$$

Nonlinear δN for multi-component slow-roll inflation

Lyth & Rodriguez '05

$$\delta N = N\left(\phi^A + \delta\phi^A\right) - N\left(\phi^A\right)$$
$$= \sum_n \frac{1}{n!} \frac{\partial^n N}{\partial\phi^{A_1}\partial\phi^{A_2}\cdots\partial\phi^{A_n}} \delta\phi^{A_1}\delta\phi^{A_2}\cdots\delta\phi^{A_n}$$

where $\delta\phi = \delta\phi_F$ is fluctuation on initial flat slice at or after horizon-crossing.

$\delta\phi_F$ may contain non-Gaussianity from subhorizon (quantum) interactions

- If slow-roll conditions are violated, δN may depend also on momentum of ϕ :

$$\delta N = \delta N(\phi, \pi) \qquad (\pi \sim \dot{\phi})$$

NG summary

- 3 origins of NG in curvature perturbation

1. subhorizon ··· quantum origin
2. superhorizon ··· classical (local) origin
3. NL gravity ··· late time classical dynamics

NG from inflation

- K-inflation type: origin 1.

f_{NL}^{equil} may be large

- non adiabatic vacuum: origin 1.

any type of f_{NL} may be large $f_{NL}^{\text{equil}}, f_{NL}^{\text{local}}, f_{NL}^{\text{orth}}$

- multi-field model: origin 2.

f_{NL}^{local} may be large.

curvaton-type models $r \ll 1$.
Multi-brid type may give $r \sim 0.1$.

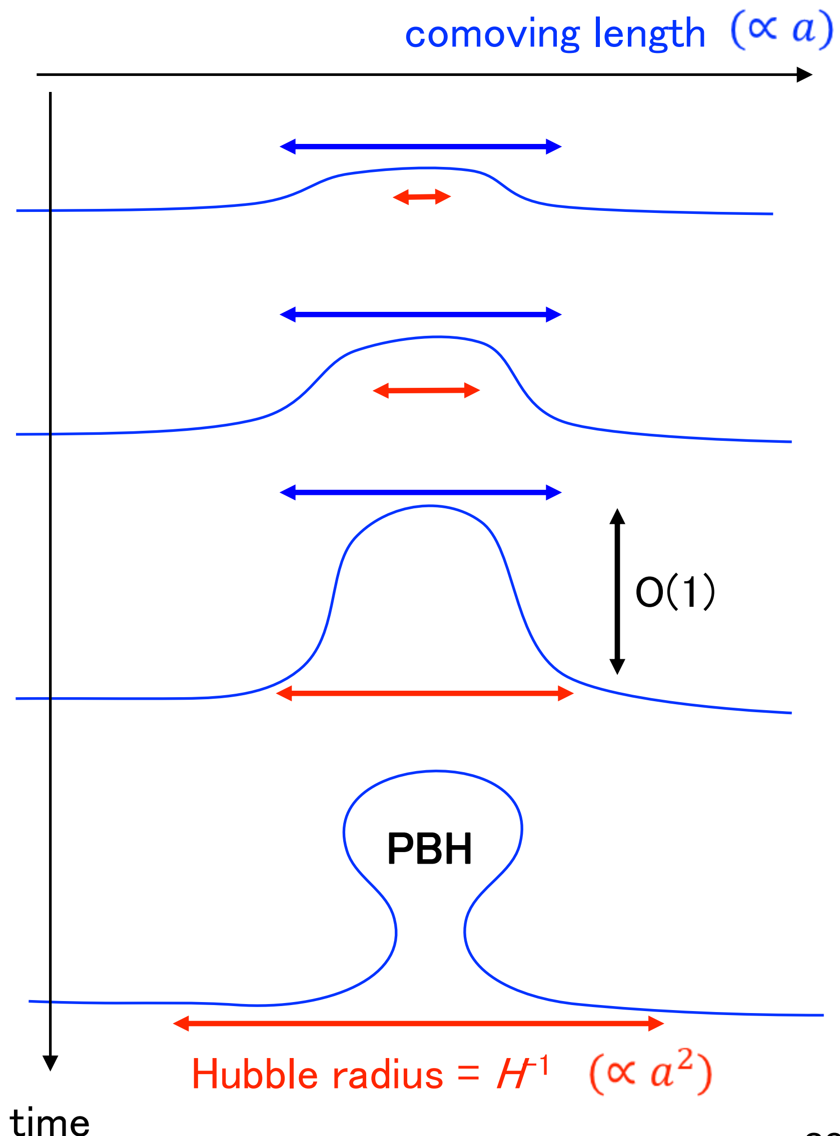
4. Primordial Black Holes

Hawking (1971), Carr & Hawking (1974), …

- Primordial Black Holes (PBHs) are those formed in the very early universe, conventionally when the universe was radiation-dominated.
- Presumably they originate from a large positive curvature perturbation produced during inflation (which hence should be a rare event).
- For a BH to form during radiation dominance, the perturbation must be O(1) on the Hubble horizon scale.

$$M_{\mathrm{PBH}} \sim M_{\mathrm{horizon}}$$

$$\sim \left(\frac{100\mathrm{MeV}}{T}\right)^2 M_\odot \sim \left(\frac{\ell}{1\mathrm{pc}}\right)^2 M_\odot$$



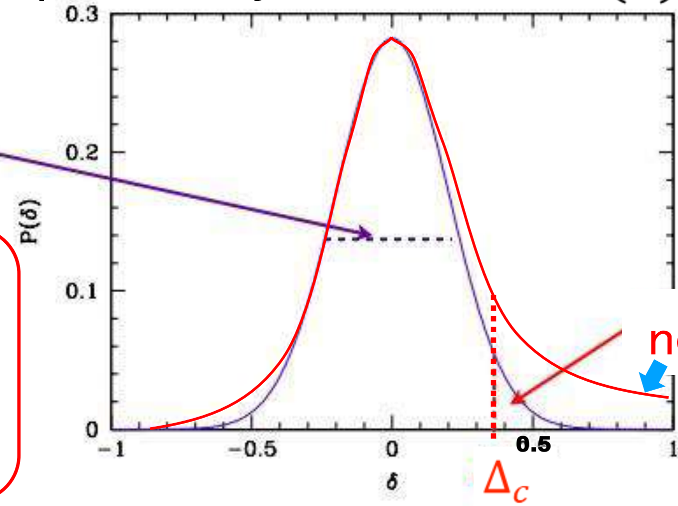
β: fraction of ρ that turns into PBHs

for Gaussian probability distribution $P(\delta)$

σ(M_H) (mass variance)
typical size of fluctuations

$$^{(3)}R \approx 6H^2 \frac{\delta\rho}{\rho}$$

$$^{(3)}R = O(H^2) \Longleftrightarrow \frac{\delta\rho}{\rho} = O(1)$$



PBH forming fluctuations

non−Gaussian tail

$$\delta \equiv \frac{\delta\rho}{\rho}$$

- When σ(M)<< Δ_c, β can be approximated by exponential:

$$\beta = \int_{\Delta_c}^{\infty} P(\delta)\, d\delta \approx \frac{\sigma(M)}{\Delta_c} \exp\left(-\frac{\Delta_c^2}{2\sigma(M)^2}\right) \quad \Delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

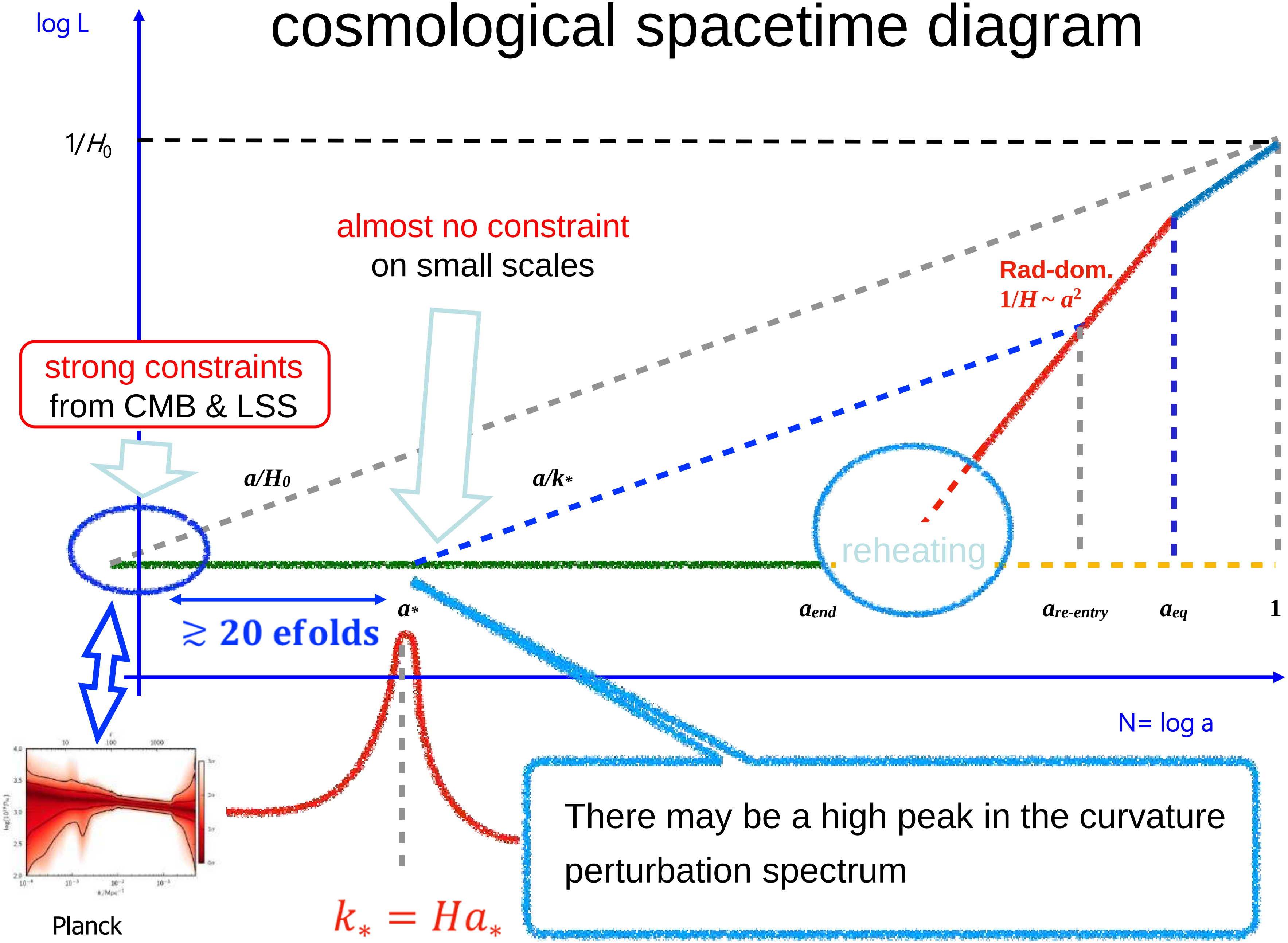
Carr, ApJ 201, 1 (1975), …

NB: Criterion using compaction function (C~GM/R) may be more relevant

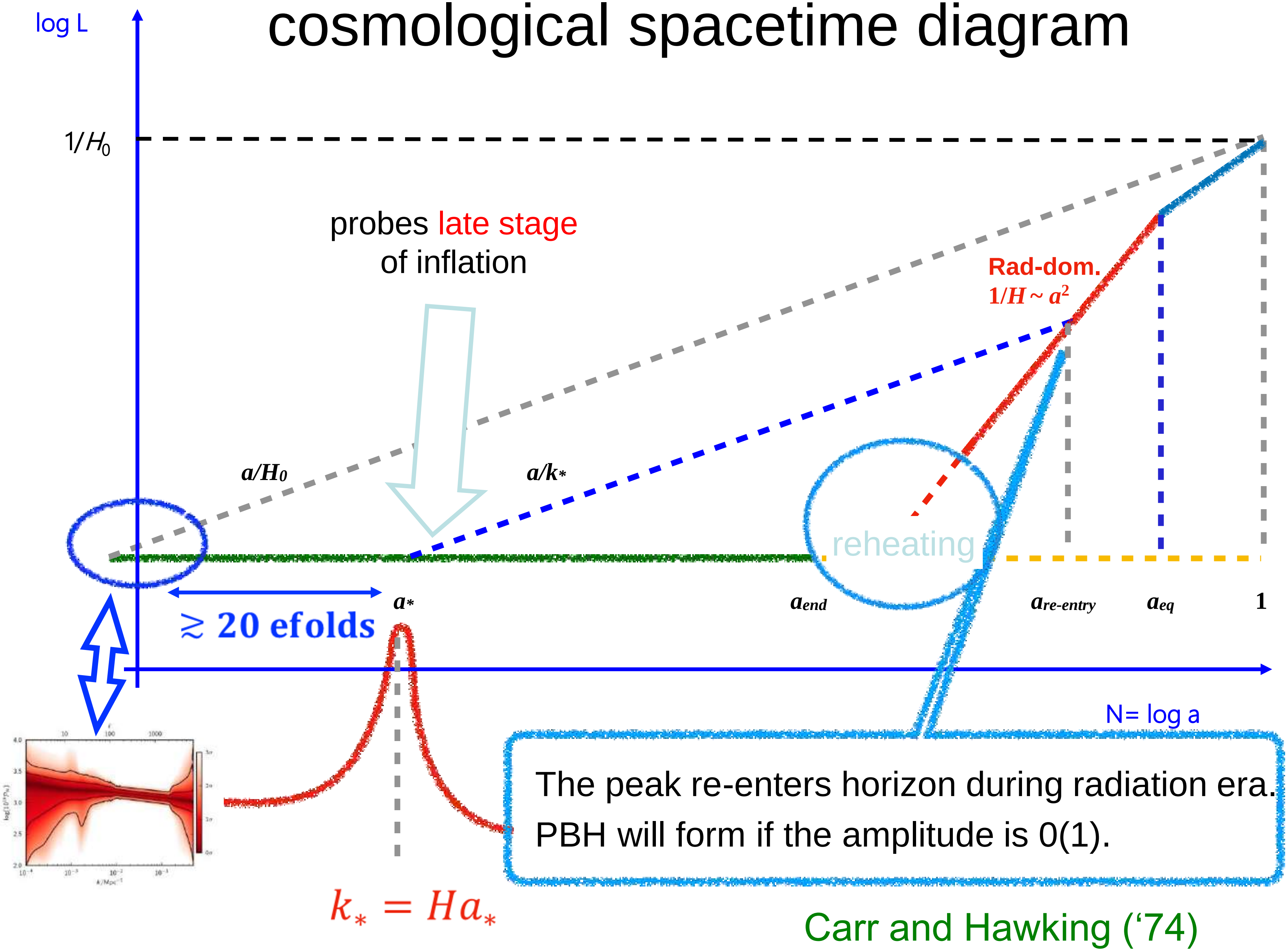
Escriva, Germani & Sheth, 2007.05564, Musco, et al., 2011.03014,…

Non−Gaussian tail may significantly affect β

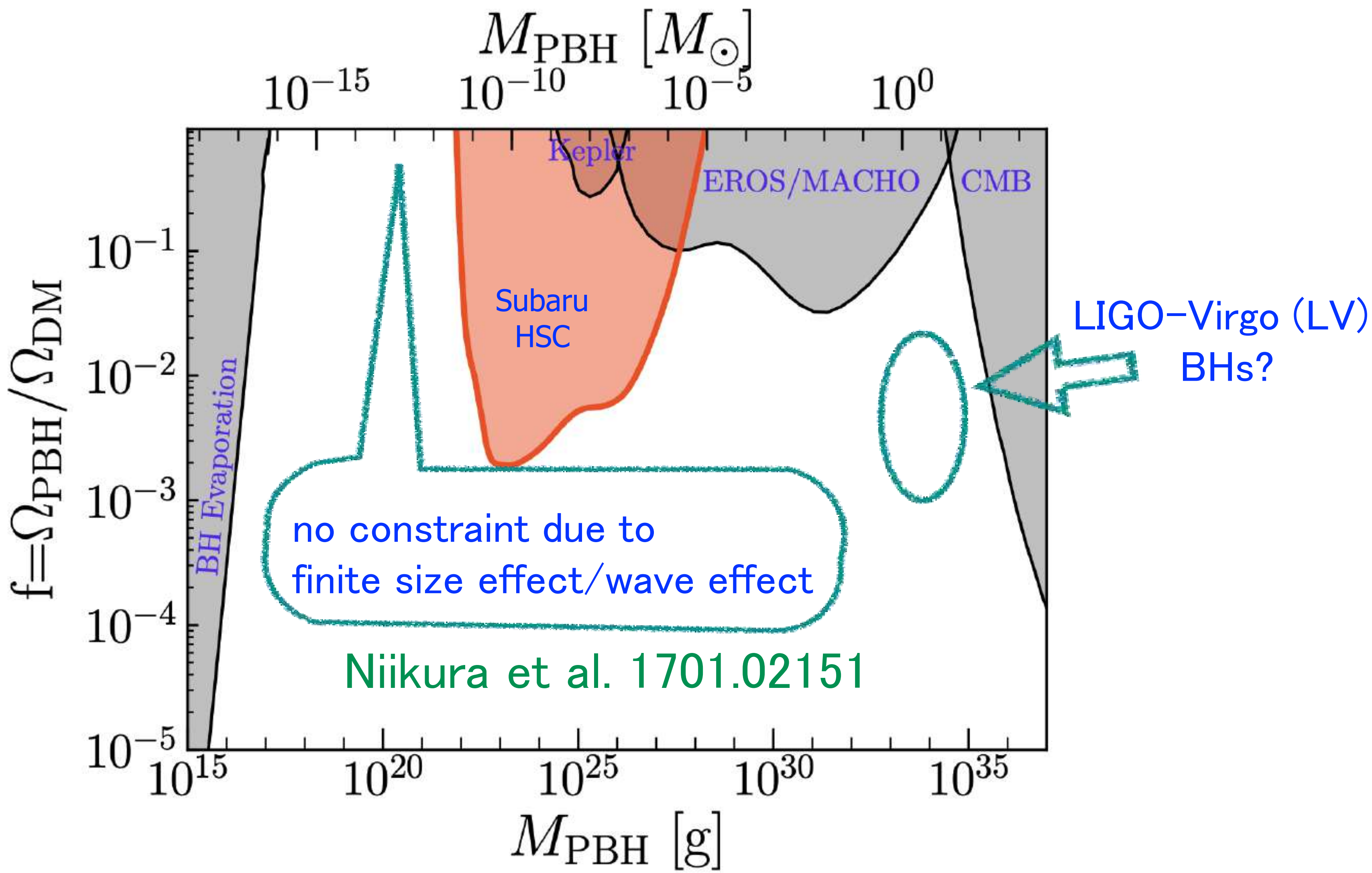
cosmological spacetime diagram



cosmological spacetime diagram



observational constraints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$g ⟷ $T_{\text{re-entry}} \sim 10^4$ -10^8 GeV

constraints on single-field case

- slow-roll case

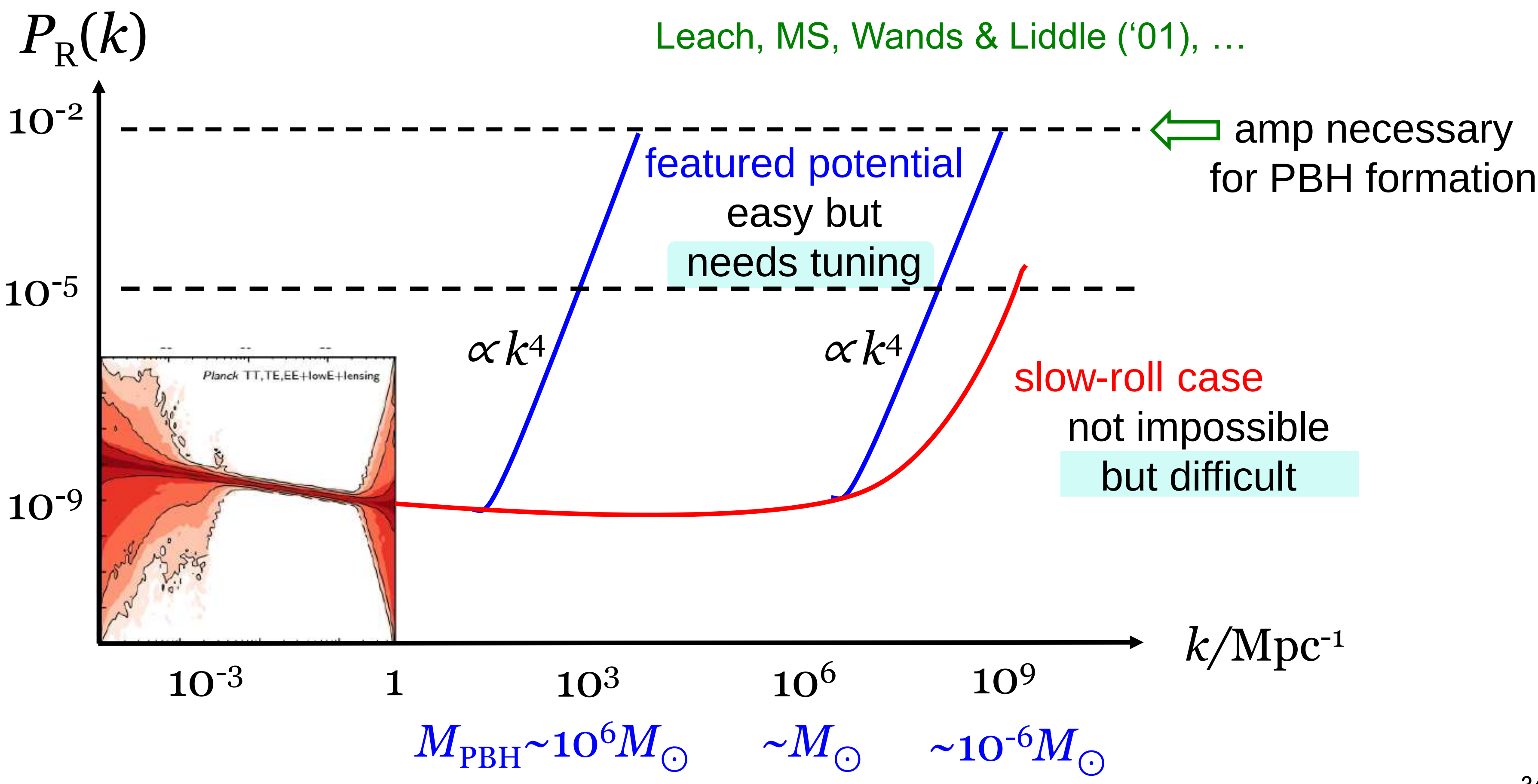
Kohri, Lyth & Melchiorri, 0711.5006

$$P_{\mathcal{R}}(k) = P_{\mathcal{R}}(k_*)\exp[(n_s - 1)\ln(k/k_*) + n_s' \ln^2(k/k_*)]$$

$n_s \approx 0.9649,\ n_s' \lesssim 0.013$ Planck 2018 X, 1807.06211

- a feature in the potential leads to the spectrum $\propto k^4$

Leach, MS, Wands & Liddle ('01), …



2-stage inflation model

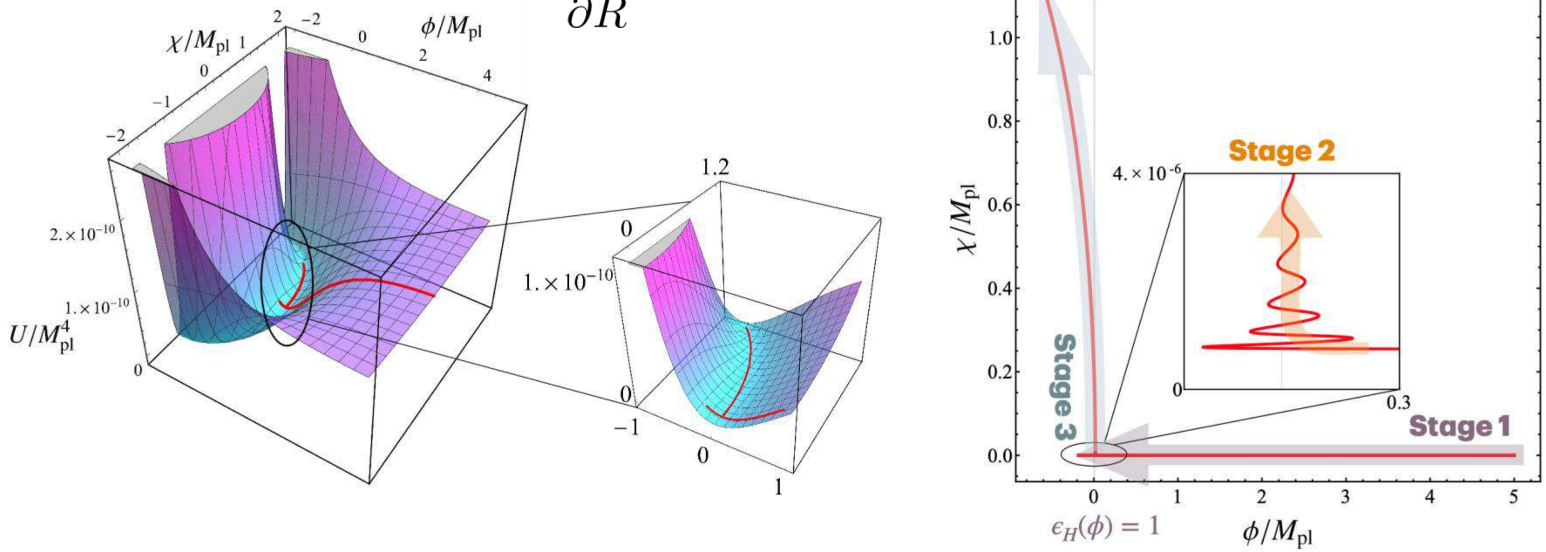
Pi, Zhang, Huang & MS, 1712.09896
Wang, Zhang & MS,2404.02492

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} f(R) - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right]$$

$$f(R) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{pl}^2} (\chi - \chi_0)^2$$

~ Starobinsky (scalaron) + curvaton

$$\frac{\partial f(R)}{\partial R} \equiv e^{\sqrt{2/3}\,\phi/M_{pl}}$$



- scalaron ϕ becomes massive at the end of Stage 1
- many 2-stage models can lead to PBH formation

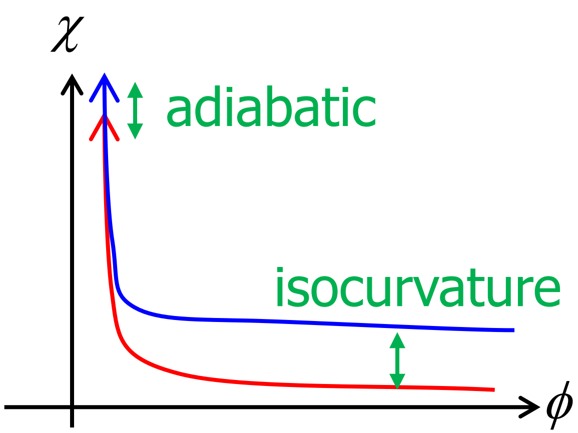
eg, Kawasaki et al., 1606.07631

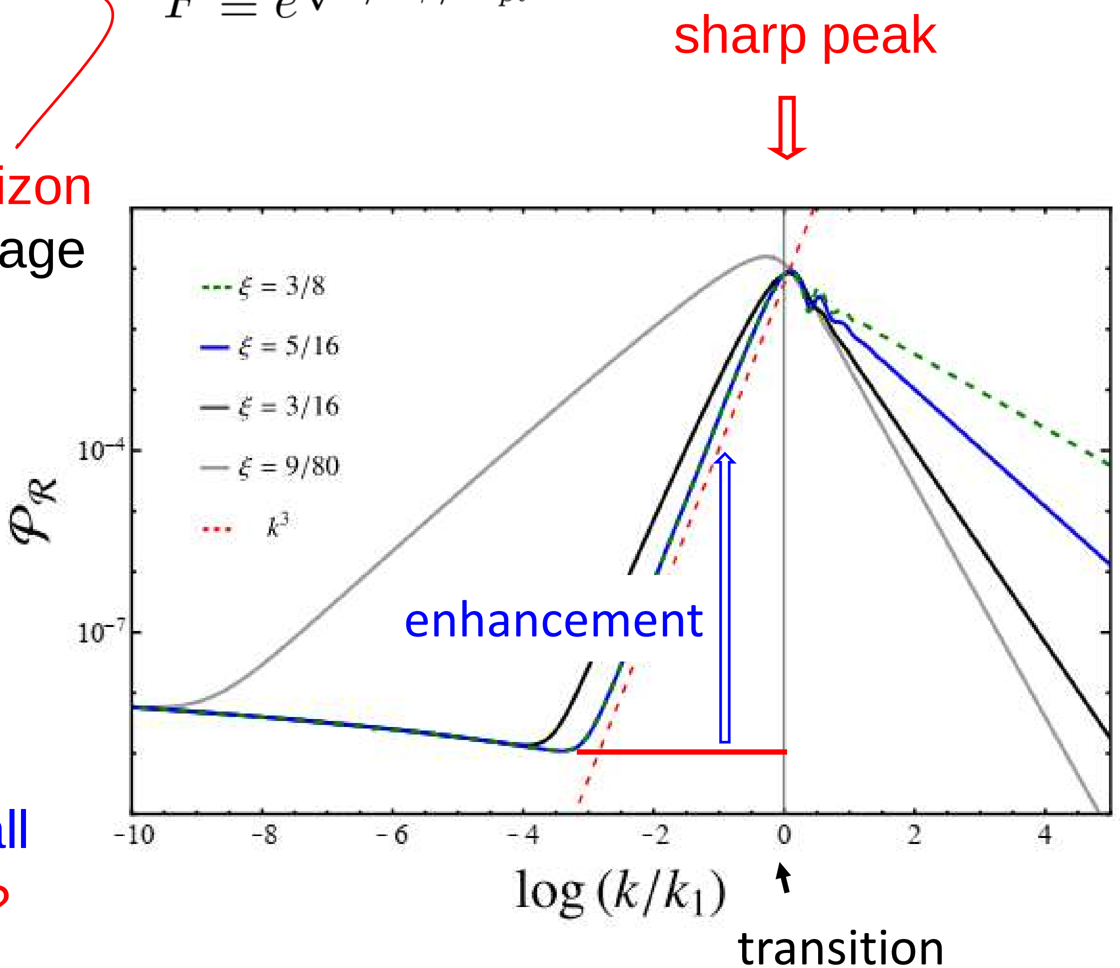
Superhorizon enhancement of curvature perturbation

curvature perturbation on comoving slices

$$\mathcal{R} = H\frac{\dot{\phi}\delta\phi + F^{-1}\dot{\chi}\delta\chi}{\dot{\phi}^2 + F^{-1}\dot{\chi}^2} \qquad F \equiv e^{\sqrt{2/3}\,\phi/M_{pl}}$$

- $\mathcal{R}$ is enhanced on superhorizon scales near the end of 1st stage



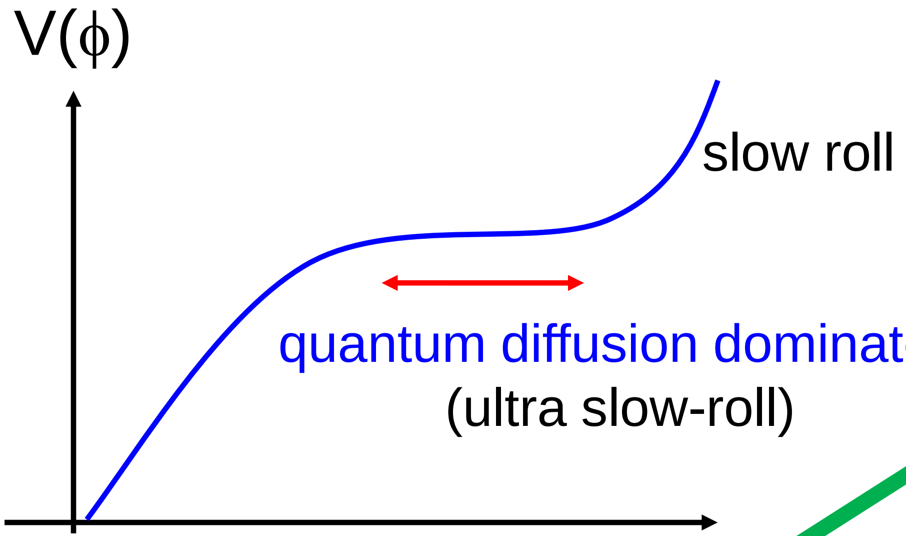


- non-Gaussianity seems small

  common in 2-stage models?

Single-field with inflection potential

Pattison, Vennin, Wands & Assadullahi, 2101.05741, …



$$\delta\phi \gtrsim \frac{\dot{\phi}}{H}$$

$\frac{H}{2\pi}$ vacuum fluctuations

classical background motion

$$\mathcal{R} \approx -\frac{H}{\dot{\phi}}\delta\phi = O(1)$$

tail of the PDF can be exponentially enhanced: $e^{-c\mathcal{R}}$ instead of $e^{-c\mathcal{R}^2}$

Gaussian PDF

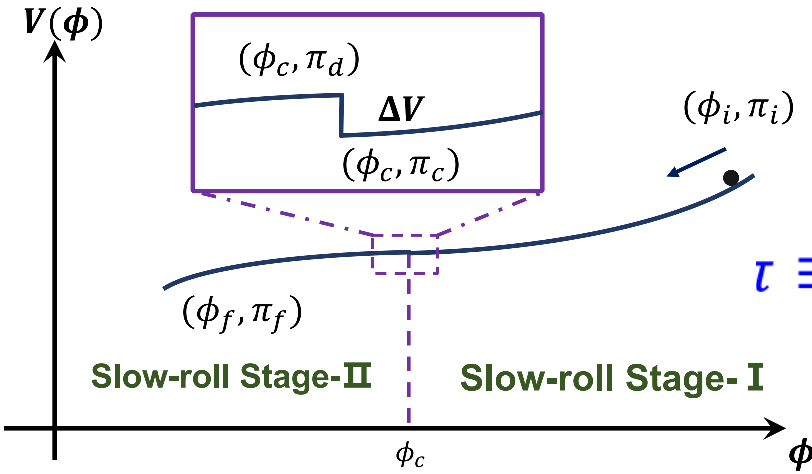
➡ non-Gaussian PBH formation

exponential tail is quite common in featured potentials

Pi & MS, 2211.13932

Upward-step model

Cai, Ma, MS, Wang & Zhou, 2112.13836; 2207.11910
Kawaguchi, Fujita & MS, 2305.18140



$$\tau \equiv \frac{d\phi}{Hdt}$$

- energy conservation at the step:

$$\pi_d = -\sqrt{\pi_c^2 - 6\Delta V/V}$$

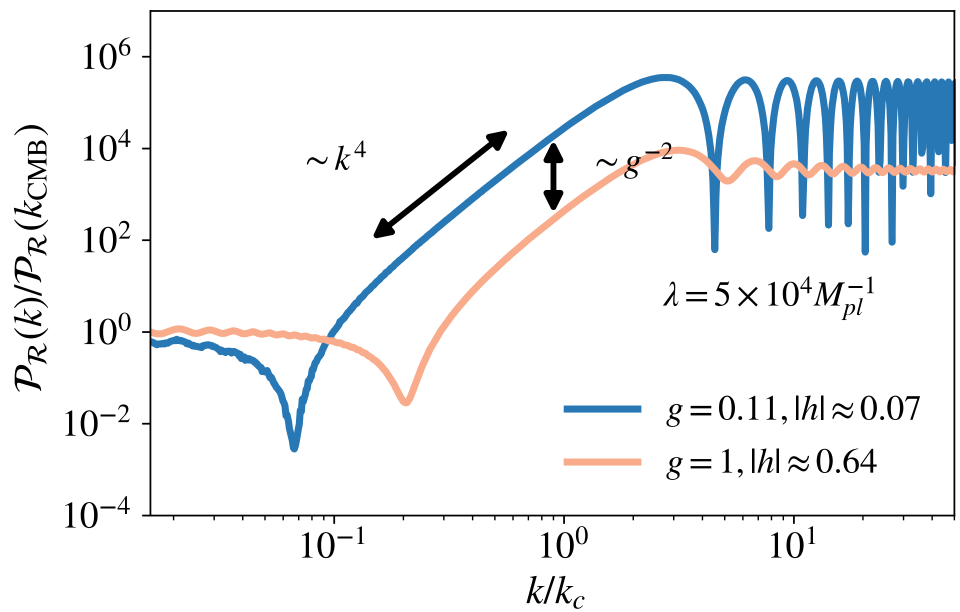
(in $M_{\rm Planck} = 1$ units)

$$\pi_c = -\sqrt{2\varepsilon_c}$$

ε_c: SR parameter at $\phi = \phi_c$

even for a tiny step, $\Delta V \ll V$,
$P_R(k)$ is enhanced by $1/g^2$ if

$$g \equiv \frac{\pi_d}{\pi_c} \ll 1$$



non-perturbative non-Gaussianity

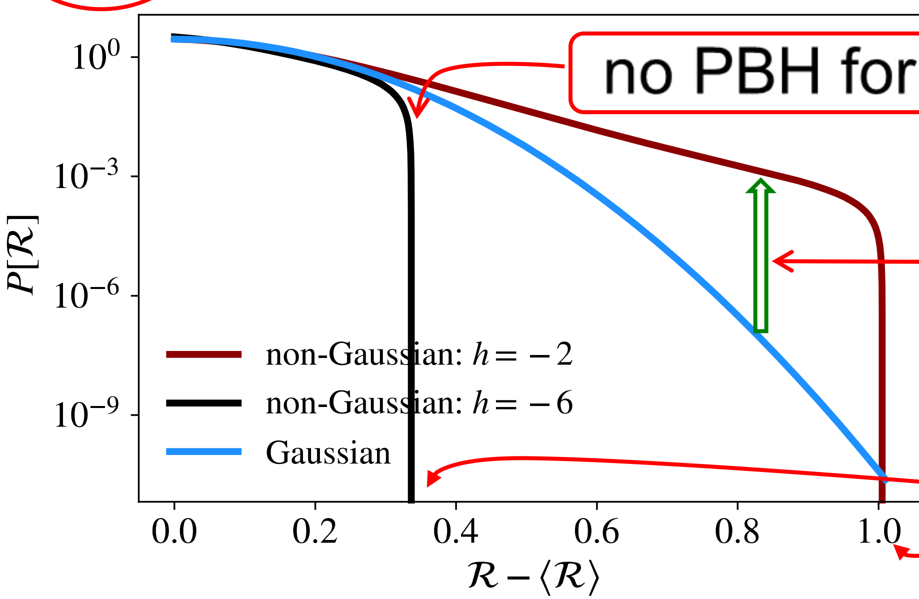
- curvature pert'n: $\mathcal{R} \simeq \frac{2}{|h|}\left(1 - \sqrt{1 - |h|\mathcal{R}_G}\right)$
- perturbative non-Gaussianity is small if $|h| \equiv \frac{6\sqrt{2\varepsilon_V}}{|\pi_d|} \ll 1$

$$\mathcal{R} = \mathcal{R}_G + \frac{|h|}{4}\mathcal{R}_G^2 + \frac{|h|^2}{8}\mathcal{R}_G^3 + \cdots \Longrightarrow \mathcal{P}(k) \approx \mathcal{P}_G(k)$$

power spectrum is given by Gaussian part

- tail of PDF is extremely non-Gaussian

$$\left|\frac{d\mathcal{R}_G}{d\mathcal{R}}\right| \qquad P[\mathcal{R}] = \frac{2 - |h|\mathcal{R}}{\Omega} \exp\left[-\frac{\mathcal{R}^2(4 - |h|\mathcal{R})^2}{32\sigma_{\mathcal{R}}^2}\right]$$

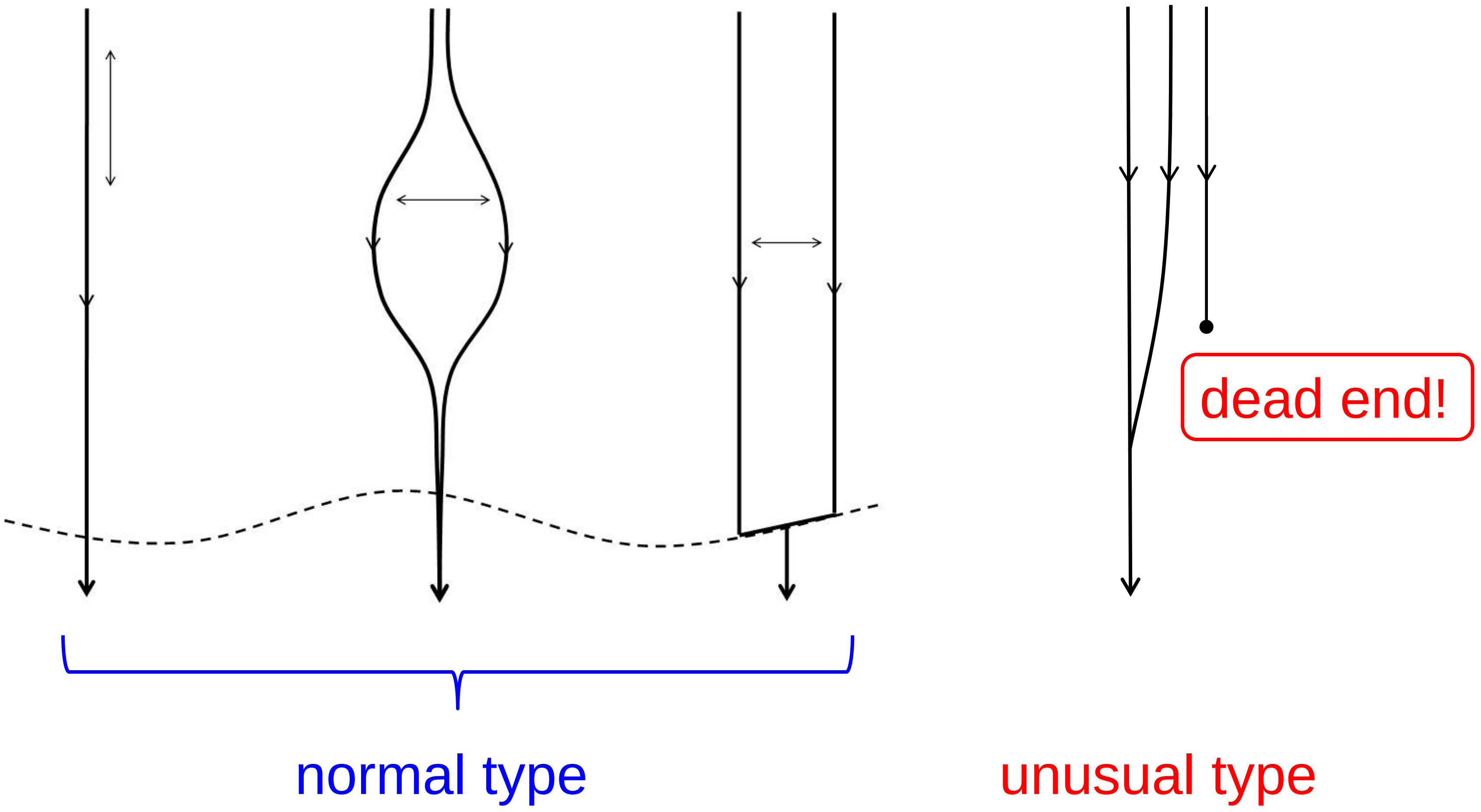


no PBH formation if $\mathcal{R}_{\rm cut} < \mathcal{R}_{\rm crit} \sim 0.5$

exponential enhancement for $|h|$<O(1)

$\mathcal{R}_{\rm cut} \equiv \frac{2}{|h|}$ ・・・ ∃cutoff

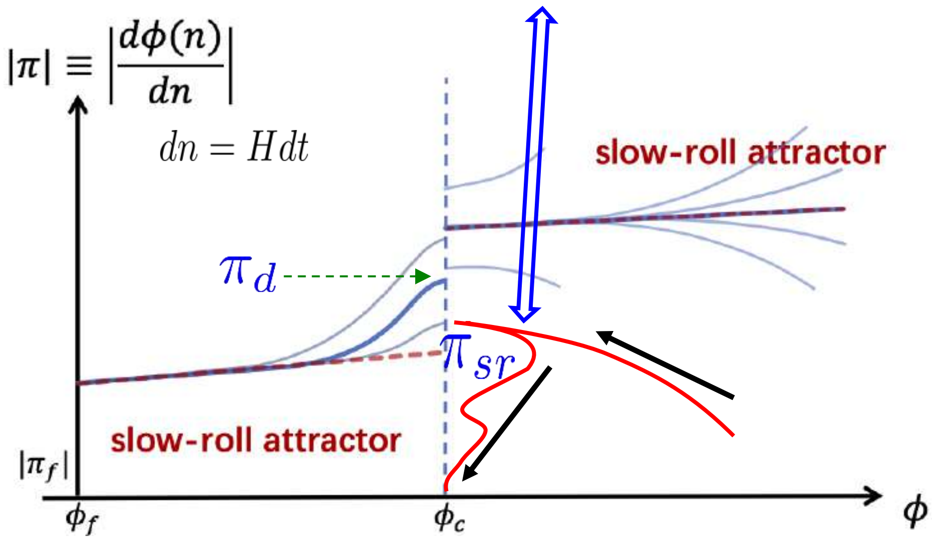
unusual type of field space trajectories

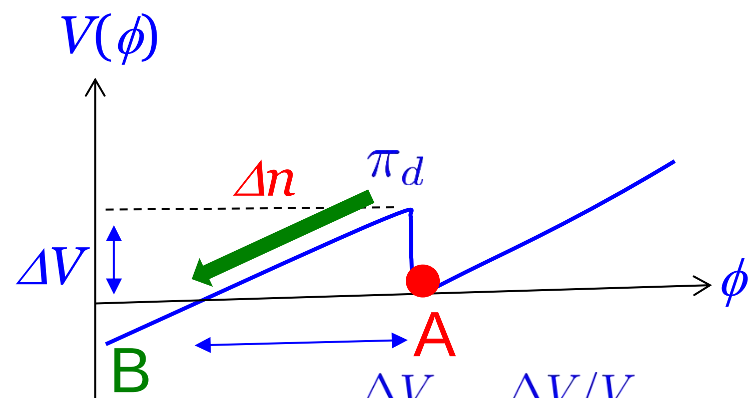


PBH formation during inflation

$$\mathcal{R} \simeq \frac{2}{|h|}\left(1-\sqrt{1-|h|\mathcal{R}_G}\right)$$ PDF cutoff at $\mathcal{R} = \mathcal{R}_{\rm cut} \equiv \frac{2}{|h|}$

⇨ trajectories that can't climb the step



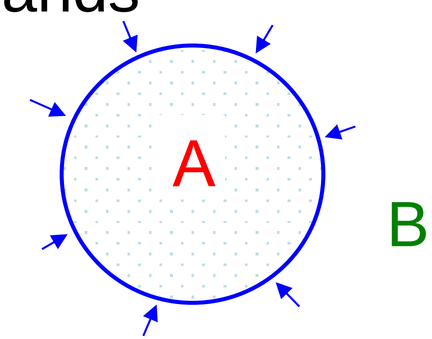


$$\Delta\phi = \frac{\Delta V}{V'} = \frac{\Delta V/V}{\sqrt{2\epsilon}}$$

$$\Delta n \simeq \frac{\Delta\phi}{|\pi_{sr}|} = \frac{\Delta V/V}{\pi_{sr}^2}$$

⇧

# of e-folds region A expands



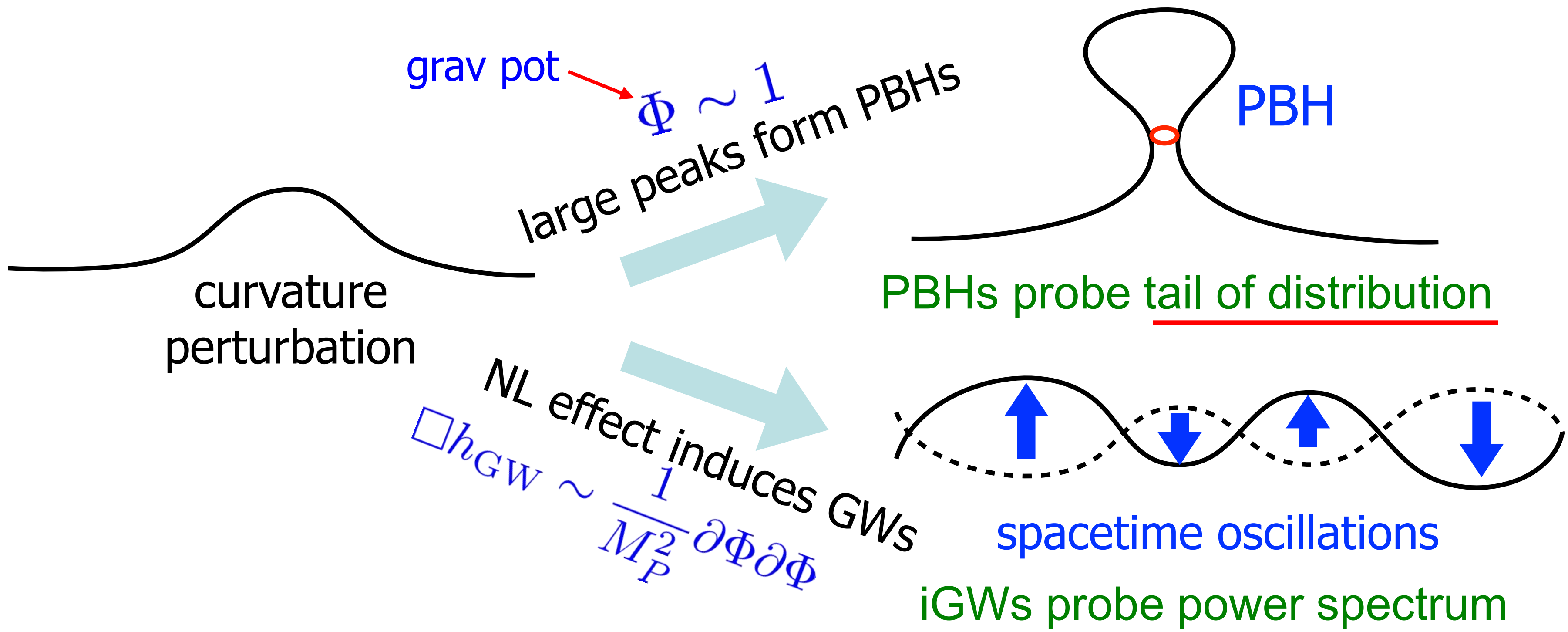
region stuck at $\phi=\phi_c$ will become PBH!

region A expands until $V(\phi)$ surrounding it becomes smaller than $V(\phi_A)=V_0$

$$M_{\rm BH} \simeq H^{-1}e^{\alpha\Delta n}$$ $\alpha = 2 \sim 3$

Deng & Vilenkin,1710.02865

Induced GWs and PBHs



PBHs = CDM with $M_{PBH} \sim 10^{21}$g generates GWs with $f \sim 10^{-3}$ Hz ⇨ GWs at LISA band

PBHs =LVK BHs with $M_{PBH} \sim 10 M_\odot$ generates GWs with $f \sim 10^{-8}$ Hz ⇨ GWs at PTA band

Q4. Derive the M_{PBH}-frequency relation.

Induced GWs and PBHs

continued

- The fraction of energy turned into PBHs must be very small:

$$\Omega_{\mathrm{PBH}} < 1 \quad \Rightarrow \quad \beta = \int_{\Delta_c}^{\infty} P(\delta)\, d\delta < 10^{-8} \left(\frac{100\mathrm{MeV}}{T}\right)$$

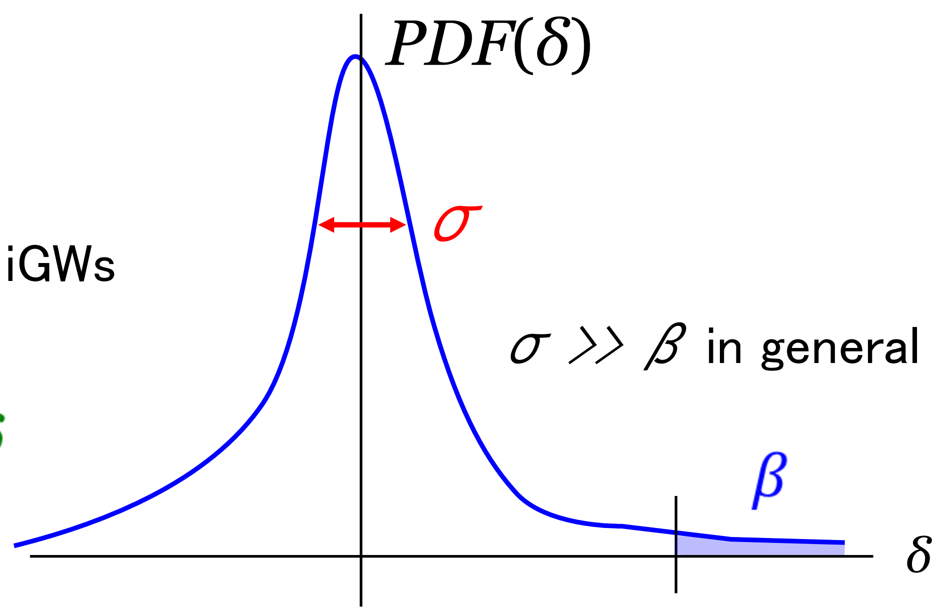
$$M_{\mathrm{PBH}} \sim \left(\frac{100\mathrm{MeV}}{T}\right)^2 M_{\odot}$$

- iGWs are determined by the power spectrum = σ^2 of $PDF(\delta)$.

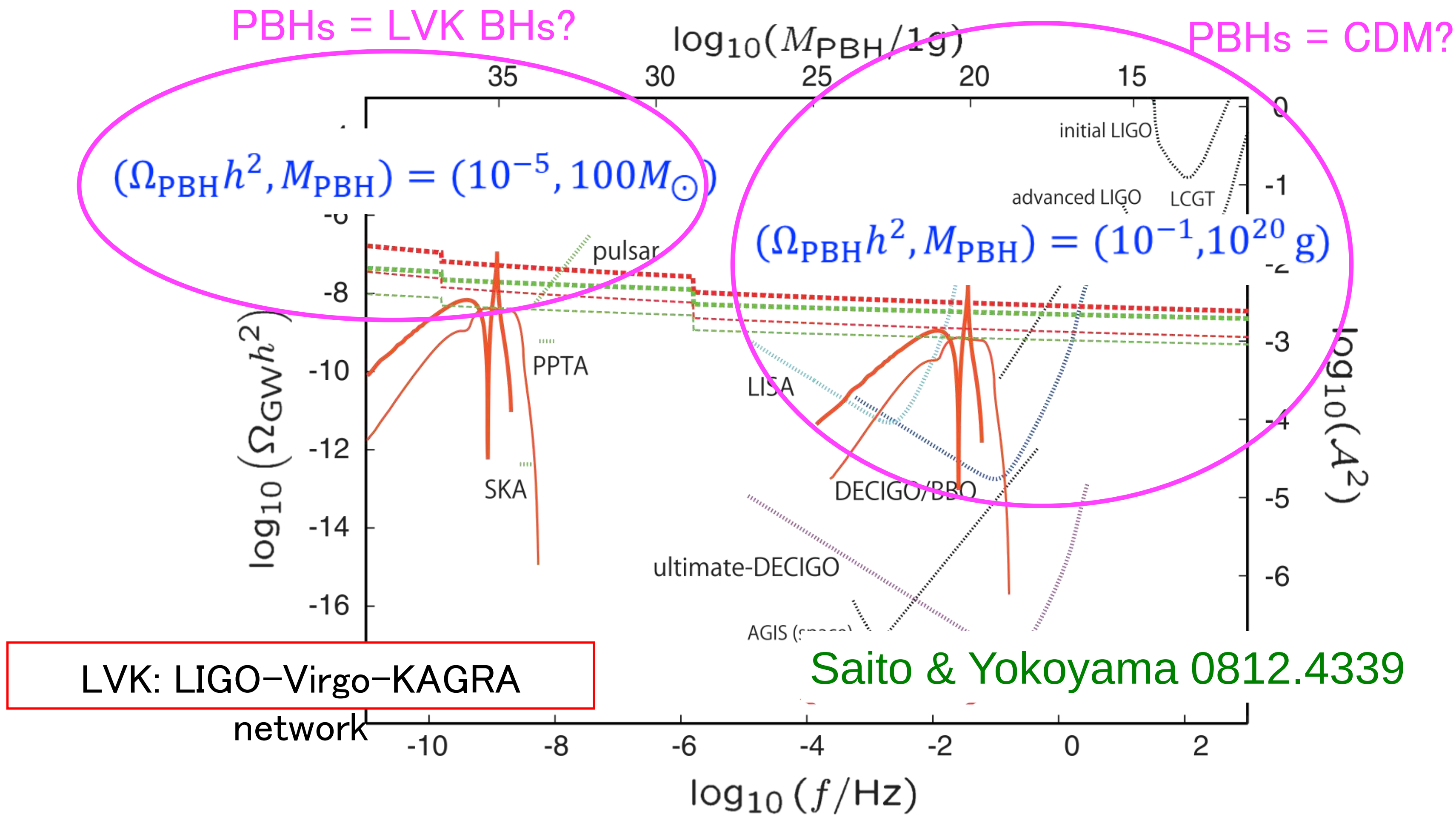
$$\sigma^2 = \int \delta^2 PDF(\delta)\, d\delta$$

NB. for large kurtosis $S_4 \gg \sigma^4$, iGWs may be substantially enhanced

$$S_4 = \int (\delta^4 - 3\sigma^4) PDF(\delta)\, d\delta$$



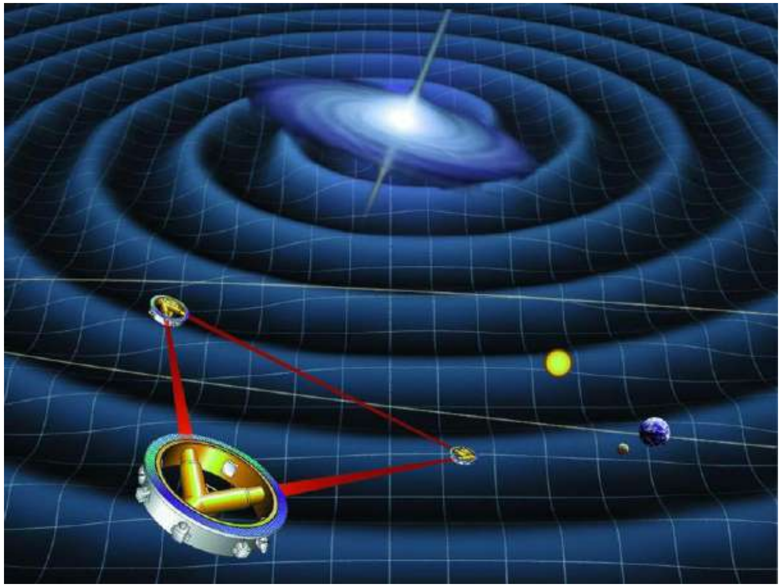
GWs can test PBH scenarios!



➢ PBHs = LVK BHs scenario is already constrained by NANOGrav(PTA)

Cai, Pi, Wang & Yang 1907.06372

GW observatories in space

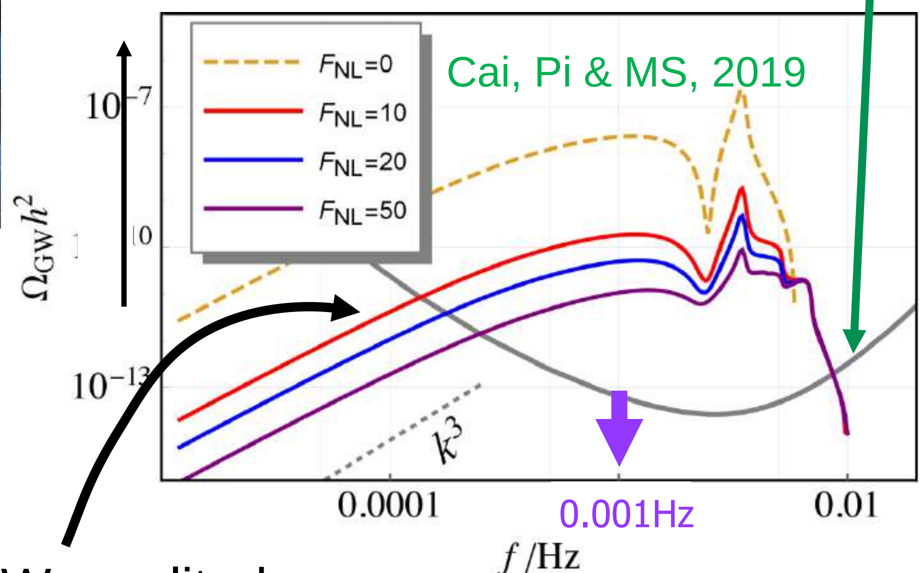


LISA 2035? (ESA+NASA)
arm length: 5,000,000 km

Taiji/TianQin 203X ? (China)
arm length: 3,000,000 km
1,000,000 km

GW amplitude

LISA sensitivity curve



expected GW amplitude

frequency

LISA will prove/disprove PBH=CDM scenario

summary

- Inflation has become the standard model of the early Universe
- Simple polynomial models are excluded by Planck
- Tensor (GW) perturbation needs to be detected
- Non-gaussianities offer important clues to model selection
- Late stage of inflation can be probed by PBHs and the associated secondary/induced GWs
- PBHs may be formed during inflation (super-critical BHs)

Era of observational/experimental inflationary cosmology