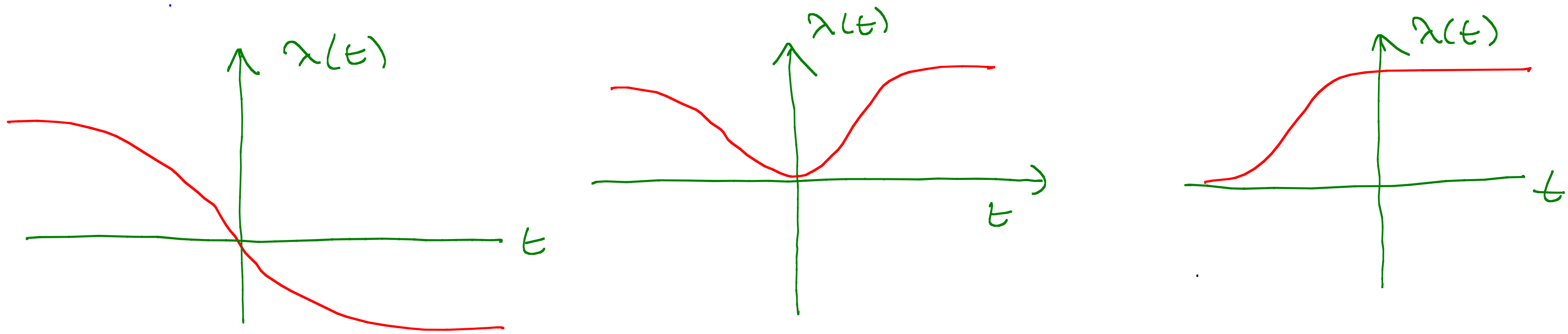
SMOOTH AND INSTANTANEOUS QUENCHES IN FIELD THEORY AND HOLOGRAPHY

SUMIT R. DAS

CONSIDER SOME HAMILTONIAN WITH A TIME DEP. PARAMETER $\lambda(t)$ WHICH INTERPOLATES BETWEEN CONSTANT VALUES



REGARDLESS OF THE SPEED : CALL THIS QUANTUM QUENCH

THIS TALK WILL, IN FACT, DEAL WITH THE QUESTION OF DEPENDENCE OF VARIOUS QUANTITIES ON QUENCH RATE

QUANTUM QUENCH IS PARTICULARLY INTERESTING WHEN THE PROTOCOL INVOLVES A CRITICAL POINT

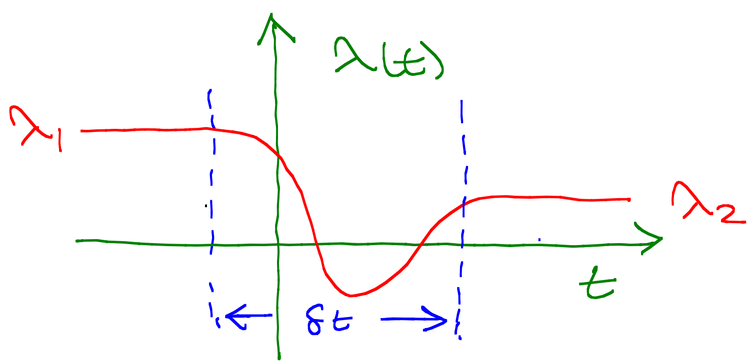
ONE THEN EXPECTS THAT TIME EVOLUTION CARRIES UNIVERSAL SIGNATURES OF THE CRITICAL POINT

OUR SETTING INVOLVES AN ACTION

$$S = S_{CFT} - \int d^{d-1}x \, dt \, \lambda(t) \, \mathcal{O}_\Delta(x,t)$$

Δ : DIMENSION OF RELEVANT OPERATOR $\mathcal{O}_\Delta$

$$\lambda(t) = \lambda_0 \, F(t/\delta t)$$



λ_0 : OVERALL SCALE

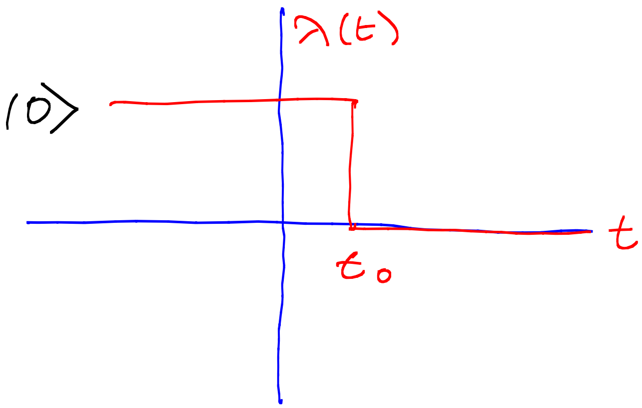
$(\delta t)^{-1}$: QUENCH RATE

$F \sim O(1)$ VALUES

UNIVERSAL RESULTS HAVE BEEN KNOWN FOR A WHILE IN TWO EXTREME LIMITS

- SUDDEN QUENCH (CALABRESE & CARDY)
- SLOW QUENCH (KIBBLE-ZUREK)

INSTANTANEOUS QUENCH



STATE AT $t = t_0 - \epsilon$ IS AN INITIAL CONDITION FOR FURTHER TIME EVOLUTION

IN 1+1 DIMENSIONS CALABRESE & CARDY DERIVED A SET OF UNIVERSAL RESULTS e.g. ONE POINT FUNCTION FOR SOME OPERATOR (WHICH IS NOT CONSERVED)

$$\langle A \rangle \sim \exp\left(-\frac{\pi x}{2\tau_0} t\right)$$

x : CONFORMAL DIMENSION

τ_0 : INITIAL LENGTH SCALE

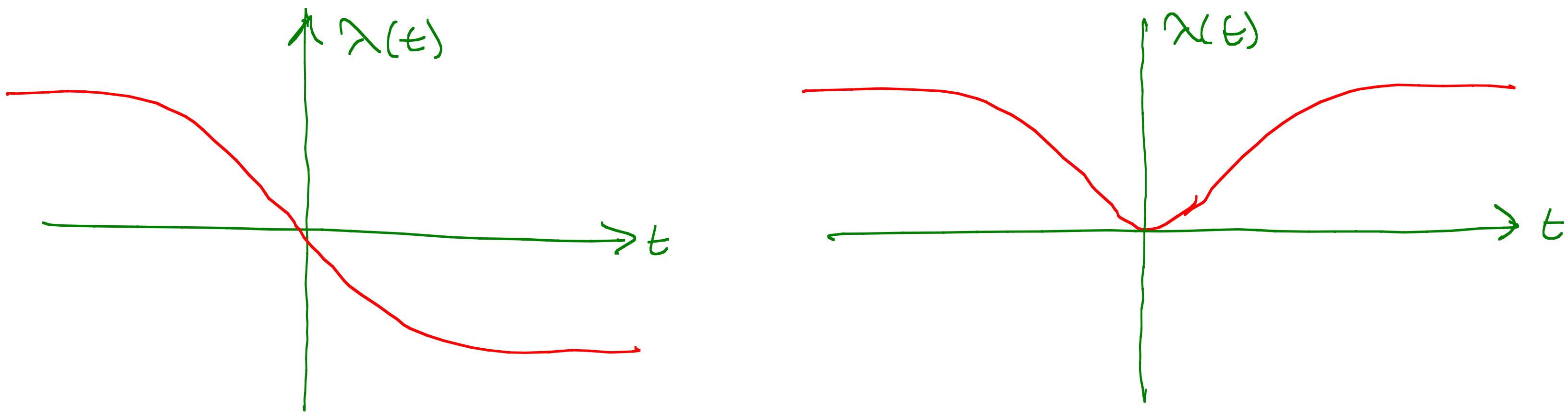
$\Rightarrow$ RATIOS OF RELAXATION TIMES UNIVERSAL

- OTHER RESULTS FOR CORR. FUNCTIONS
- TIME EVOLUTION OF ENTANGLEMENT ENTROPY

HOLOGRAPHIC REALIZATION : HARTMAN & MALDACENA

"SLOW" QUENCH

COUPLING STARTS FROM GAPPED PHASE SLOWLY AND CROSSES A CRITICAL POINT

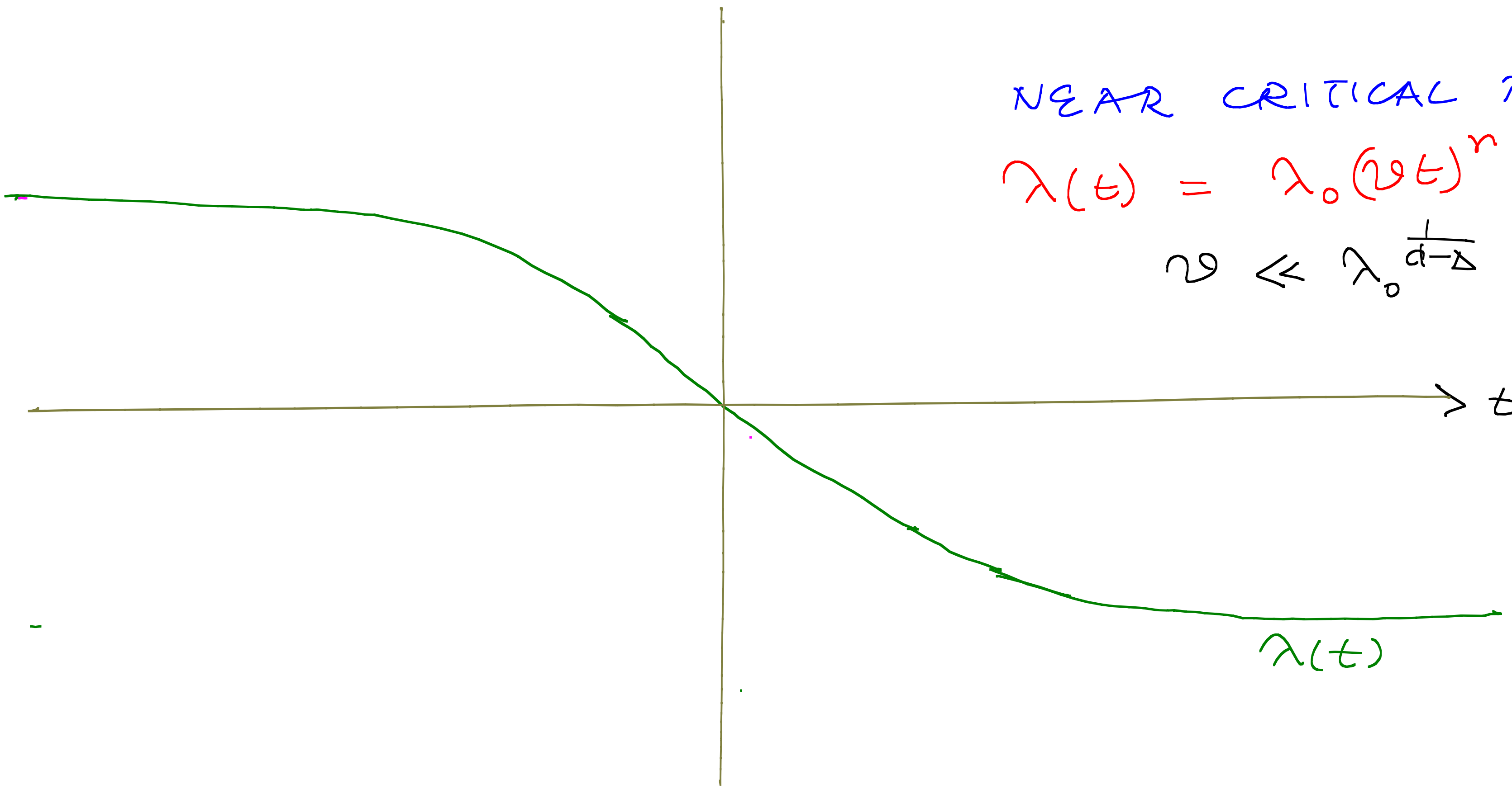


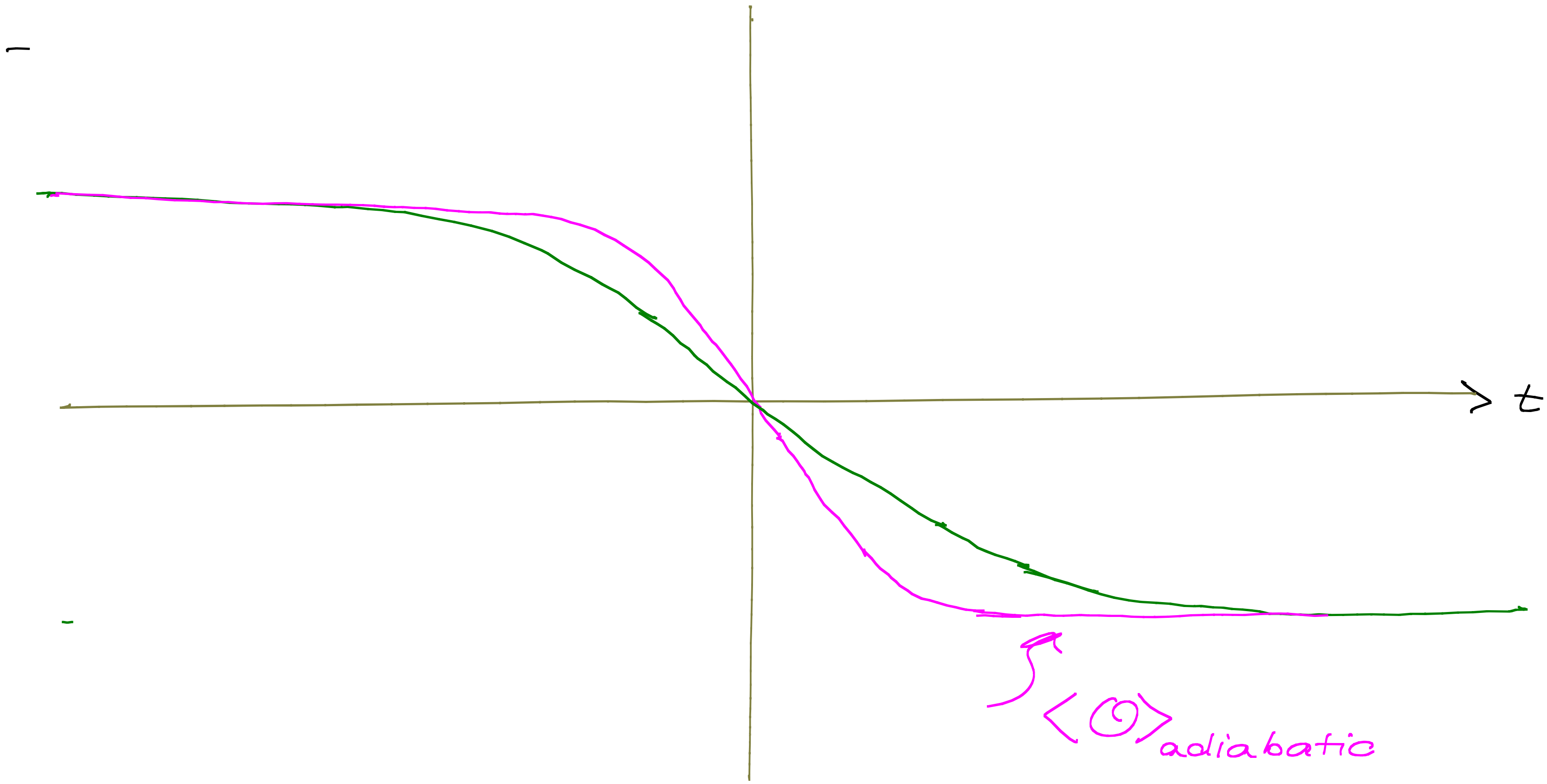
$\lambda = 0$ IS A CRITICAL THEORY

NEAR CRITICAL POINT

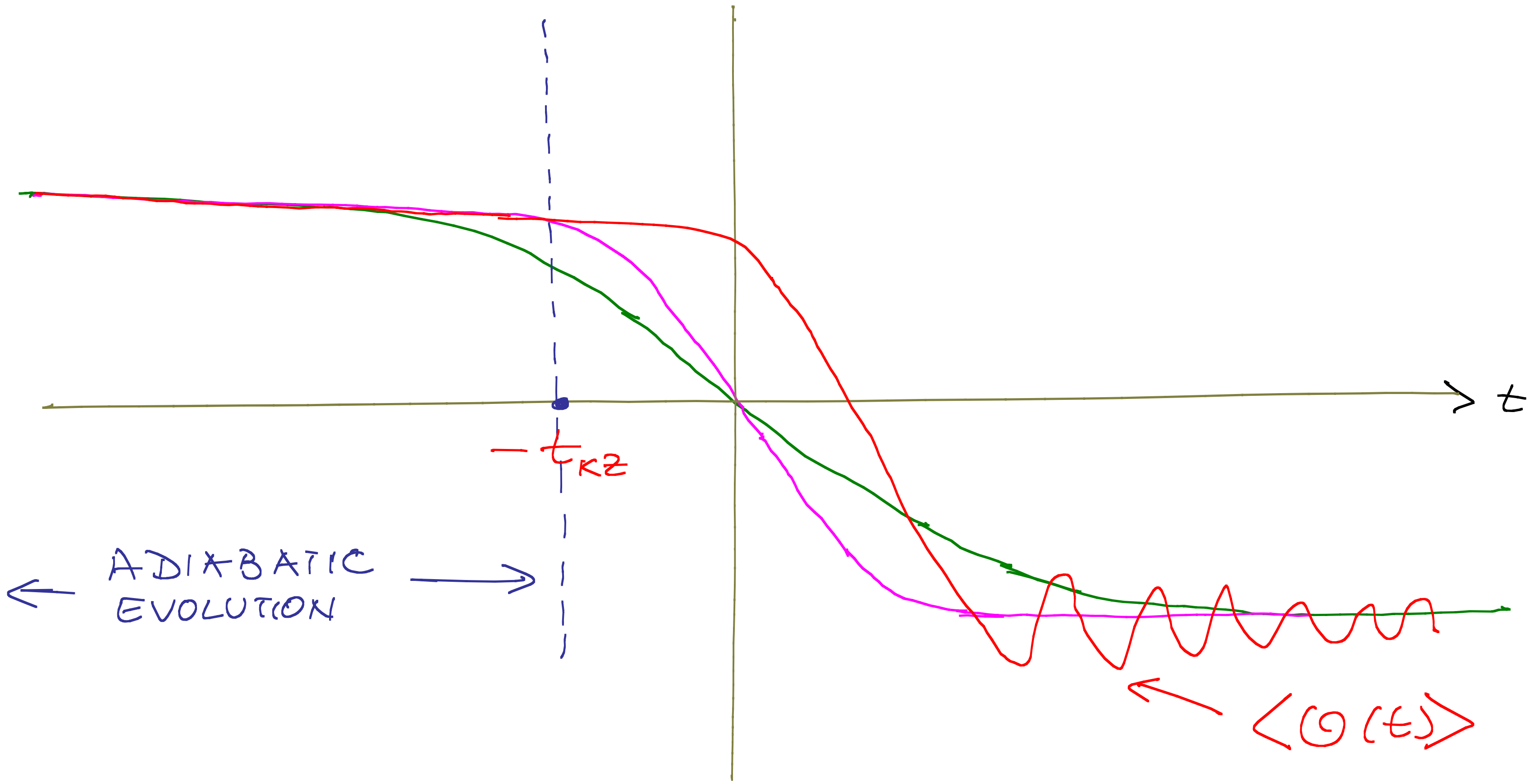
$$\lambda(t) = \lambda_0 (v t)^n$$

$$v \ll \lambda_0^{\frac{1}{d-\Delta}}$$





t
⟨O⟩adiabatic



t
$-t_{KZ}$
ADIABATIC
EVOLUTION
$\langle O(t) \rangle$

$$\langle \mathcal{O} \rangle \sim (\lambda_0 v^r)^{\frac{\Delta}{r+d-\Delta}}$$

$\lambda(t) \sim \lambda_0 (vt)^r$

Δ : CONF. DIMENSION

THIS IS A SPECIAL CASE OF KIBBLE-ZUREK SCALING

UNDERLYING ASSUMPTIONS NOT EASY TO UNDERSTAND FOR STRONGLY COUPLED THEORIES

- USING HOLOGRAPHY WE CAN GET SOME INSIGHT

P. BASU & S.R.D — JHEP 1201 (2012) 103

P. BASU, D. DAS, S.R.D & T. NISHIOKA — JHEP 1303 (2013) 146

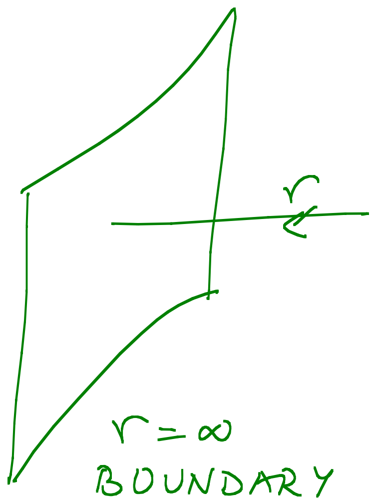
P. BASU, D. DAS, S.R.D & K. SENGUPTA — JHEP 1312 (2013) 070

S.R.D & T. MORITA — JHEP 1501 (2015) 084

HOLOGRAPHIC KIBBLE ZUREK

$$S = S_{eFT} + \int dt\, d^{d-1}x\ \lambda(t)\ \mathcal{O}_\Delta(x,t)$$

SCALAR OPERATOR $\mathcal{O}_\Delta$ DUAL TO $\phi(r,\vec{x},t)$



$$\phi(\vec{x},r,t) \xrightarrow[r\to\infty]{} r^{\Delta-d}\left[\lambda(t) + O(1/r^2)\right] + r^{\Delta}\left[A(t) + O(1/r^2)\right]$$

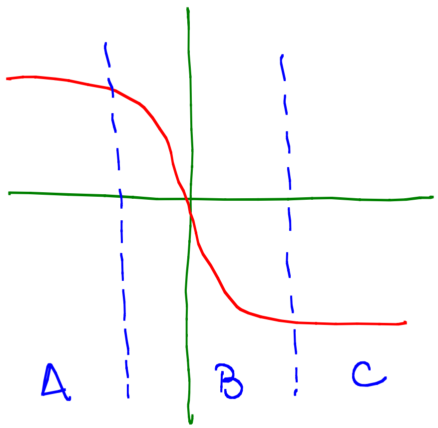
$$\langle \mathcal{O}_\Delta(\vec{x},t)\rangle = A(t)$$

$\Rightarrow$ SOLVE BULK EQUATIONS WITH A TIME DEP. BOUNDARY CONDITION

STRATEGY

- CONSIDER A BULK THEORY SUCH THAT THE BOUNDARY FIELD THEORY HAS AN ISOLATED CRITICAL POINT (e.g. HOLOGRAPHIC SUPERCONDUCTOR)
- INTRODUCE A TIME DEPENDENT SOURCE FOR THE CORRESPONDING ORDER PARAMETER WHICH CROSSES THIS CRITICAL POINT
- CALCULATE THE RESPONSE

THE RESULT FROM SEVERAL MODELS WITH HOLOGRAPHIC CRITICAL POINTS



$v \ll \lambda_0^{\frac{1}{d-\Delta}}$

REGION A : ADIABATIC EXPANSION VALID
– POWER SERIES EXPANSION IN v

REGION B : A DIFFERENT EXPANSION IN FRACTIONAL POWERS OF v, v^{α}
– POWER DETERMINED BY CRITICAL EXPONENTS

IN REGION B THE BULK SCALAR CAN BE EXPANDED IN EIGENMODES OF THE LINEAR FLUCTUATION OPERATOR $\mathcal{D}$

$$\phi(r,t) = \sum_n \varphi_n(r)\, \xi_n(t).$$

$$\mathcal{D}\varphi_n = \lambda_n \varphi_n$$

AT THE CRITICAL POINT $\lambda = 0$ THE OPERATOR $\mathcal{D}$ HAS A ZERO MODE $\Rightarrow$ ADIABATICITY BREAKDOWN

TO LOWEST ORDER IN THE v^α EXPANSION THE ZERO MODE DOMINATES THE DYNAMICS

BULK GRAVITY & GAUGE FIELDS DO NOT HAVE ZERO MODES

HOWEVER THE SCALAR ZERO MODE – THROUGH NON-LINEARITIES – CAUSES BREAKDOWN OF ADIABATICITY

(S.R.D & T.MORITA)

TO LEADING ORDER IN v^α EXPANSION THE BULK EQUATIONS HAVE SCALING SOLUTIONS

$$\mathcal{O}_\Delta(t, v) = v^\alpha f(t v^\beta)$$

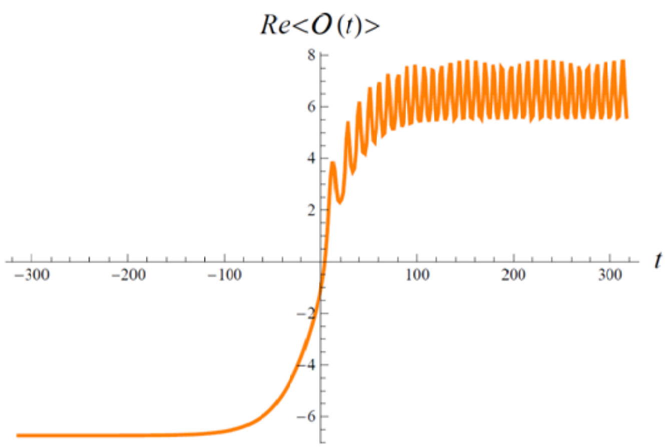
SIMILARLY FOR $\langle T_{\mu\nu} \rangle$ & $\langle J_\mu \rangle$

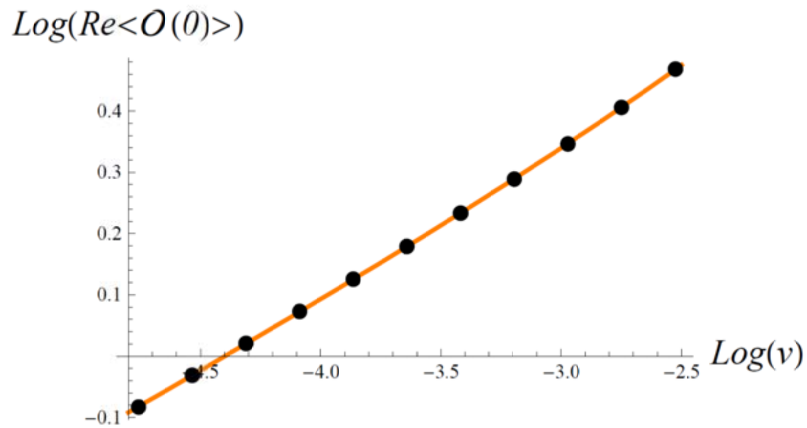
EXPONENTS CONSISTENT WITH KIBBLE ZUREK

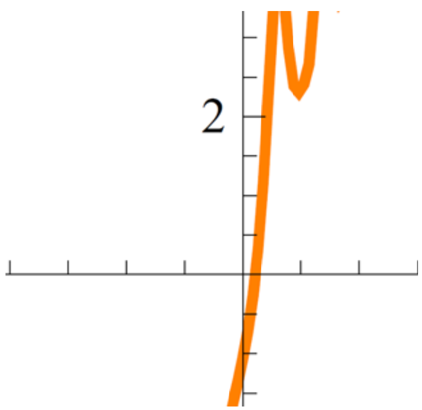
FRAMEWORK FOR CORRECTIONS – POWERS OF v^α

INHOMOGENITIES & DEFECTS

- SONNER, del CAMPO, ZUREK
- CHESLER, GARCIA-GARCIA & LIU







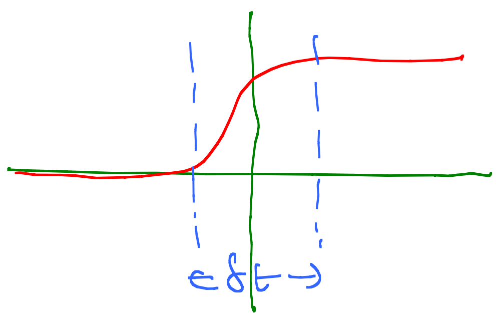
PROBE APPROXIMATION RESULTS FOR A T=0 HOLOGRAPHIC SUPERCONDUCTOR OF NISHIOKA-RYU-TAKAYANGI

(P. BASU, D. DAS, S.R. DAS & T. NISHIOKA)

HOLOGRAPHIC FAST QUENCH

BUCHEL, LEHNER, MYERS & VAN NIEKERK STUDIED QUENCHES HOLOGRAPHICALLY

$$S = S_{CFT} - \int \lambda(t)\, \mathcal{O}_\Delta(x,t)$$



$$\delta t \ll \lambda_0^{-\frac{1}{d-\Delta}}$$

USING STANDARD TECHNIQUES OF HOLOGRAPHIC RENORMALIZATION

$$\langle \mathcal{O}_\Delta \rangle_{ren} \sim (\delta t)^{d-2\Delta}$$

$$\langle \mathcal{E} \rangle_{ren} \sim (\delta t)^{d-2\Delta}$$

← AT TIMES SOON AFTER THE QUENCH

CONSISTENT WITH WARD IDENTITY

$$\frac{d\langle \mathcal{E} \rangle}{dt} = \frac{d\lambda}{dt} \langle \mathcal{O}_\Delta \rangle$$

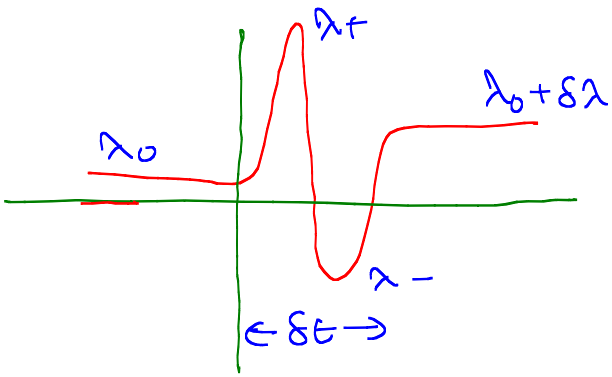
THESE RESULTS ARE SOMEWHAT PUZZLING.

$\langle O_\Delta \rangle \sim (\delta t)^{d-2\Delta} \Rightarrow$ NO SMOOTH LIMIT $\delta t \to 0$ WHEN $d > \Delta > d/2$

NEVERTHELESS THERE IS A PERFECTLY WELL DEFINED ABRUPT QUENCH

COULD THIS BE A SPECIAL FEATURE OF STRONGLY COUPLED FIELD THEORIES WHICH HAVE GRAVITY DUALS ?

WE WILL ARGUE THAT THIS IS A GENERAL RESULT FOR ANY DEFORMED CFT IN ANY NUMBER OF DIMENSIONS



$$\delta t \ll \lambda_i^{\frac{1}{\Delta-d}}\,,\ (\lambda_0+\delta\lambda)^{\frac{1}{\Delta-d}}$$

FOR TIMES $t \lesssim \delta t$

$$\langle O_\Delta \rangle \sim (\delta\lambda)(\delta t)^{d-2\Delta}$$

FOR TIMES AFTER QUENCH IS OVER

$$\langle \mathcal{E} \rangle \sim (\delta\lambda)^2(\delta t)^{d-2\Delta}$$

S.R.D, D.GALANTE & R.MYERS

PRL 112 (2014) 171601

JHEP 1502 (2015) 167

arxiv 1505.05224

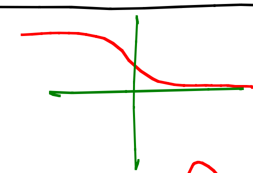
WE WILL ALSO DISCUSS IN SOME DETAIL HOW THE $\delta t \to 0$ RESULTS RELATE TO THOSE OF INSTANTANEOUS QUENCH

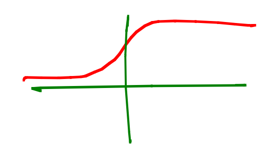
FAST QUENCH IN FREE FIELDS

SMOOTH QUENCHES IN SOLVABLE FREE FIELD THEORY WITH TIME DEPENDENT MASS

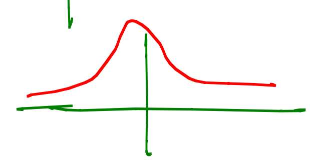
$$S = \int d^{d-1}x \, dt \, \left[(\partial\varphi)^2 - m^2(t)\varphi^2 \right]$$

(I) $m^2(t) = A + B \tanh(t/\delta t)$





(II) $m^2(t) = m_0^2 + \dfrac{m^2}{\cosh^2(t/\delta t)}$



PROBLEM CAN BE EXACTLY SOLVED FOR ALL δt FOR THESE MASS PROFILES

THE (HEISENBERG PICTURE) STATE IS THE "IN" VACUUM

$$\phi(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[a(k)\, u_k(x,t) + h.c. \right]$$

$$a(k)|0\rangle = 0$$

THE (HEISENBERG PICTURE) STATE IS THE "IN" VACUUM

$$\Phi(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[a(k)\, u_k(x,t) + h.c. \right] \qquad a(k)|0\rangle = 0$$

$$u(k) = \frac{1}{\sqrt{2\omega_{in}}}\, {}_2F_1\!\left(a, b; c; \tfrac{1}{2}\left(1+\tanh\frac{t}{\delta t}\right)\right) e^{i\left(\vec{k}\cdot\vec{x} - \omega_+ t - \omega_- \delta t \log\left(2\cosh\frac{t}{\delta t}\right)\right)}$$

$$a = 1 + i\omega_-\delta t\,, \quad b = i\omega_-\delta t\,, \quad c = 1 - i\omega_{in}\delta t$$

$$\omega_{in} = \sqrt{k^2+m_{in}^2} \qquad \omega_{out} = \sqrt{k^2+m_{out}^2} \qquad \omega_\pm = \frac{1}{2}\left(\omega_{out} \pm \omega_{in}\right)$$

$$u_k \to \frac{1}{\sqrt{2\omega_{in}}} \exp\left[i\left(\vec{k}\cdot\vec{x} - \omega_{in} t\right)\right] \quad \text{as } t \to -\infty$$

THERE IS ALSO A SET OF "OUT" MODES

$$\phi = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[b_k v_k + h.c. \right]$$

BOGOLIUBOV :

$$u_k = \alpha_k v_k + \beta_k v^*_{-k}$$

$$\alpha_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - i\omega_{in}\delta t)\,\Gamma(-i\omega_{out}\delta t)}{\Gamma(-i\omega_+\delta t)\,\Gamma(1 - i\omega_+\delta t)}$$

$$\beta_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - i\omega_{in}\delta t)\,\Gamma(i\omega_{out}\delta t)}{\Gamma(i\omega_-\delta t)\,\Gamma(1 + i\omega_-\delta t)}$$

THE ONE POINT FUNCTION OF QUENCHED OPERATOR

$$ {}_{in}\langle 0 | \phi^2(x,t) | 0 \rangle_{in} = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\omega_{in}} |{}_2F_1|^2 $$

THIS QUANTITY IS OF COURSE UV DIVERGENT

AN EFFICIENT WAY TO RENORMALIZE THIS IS TO REGARD $m^2(t)$ AS A BACKGROUND FIELD

$$ S \rightarrow S_0(\phi, m^2, g_{\mu\nu}) - S_{ct}(m^2, g_{\mu\nu}, \Lambda) $$

$g_{\mu\nu}$ IS A BACKGROUND METRIC

EXPECTATION VALUES MAY BE COMPUTED FROM

$$Z(m^2, g_{\mu\nu}) = \int \mathcal{D}\phi_x \exp[-iS].$$

$$\langle \phi^2 \rangle_{ren} = -2i \left(\frac{1}{\sqrt{g}} \frac{\delta}{\delta m^2} \log Z \right)_{\substack{g_{\mu\nu}=\eta_{\mu\nu} \\ \Lambda \to \infty}}$$

$$\langle T_{\mu\nu} \rangle_{ren} = -2i \left(\frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \log Z \right)_{\substack{g_{\mu\nu}=\eta_{\mu\nu} \\ \Lambda \to \infty}}$$

COUNTERTERMS GENERALLY DEPEND ON $\partial_t m^2$; $\partial_t^2 m^2 \ldots$ IN ADDITION TO m^2

IT TURNS OUT THAT THE COUNTERTERM ACTION MAY BE OBTAINED FROM AN ADIABATIC EXPANSION

e.g.

$$\langle 0 | \phi^2 | 0 \rangle_{\text{adiabatic}} = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\Omega_k}$$

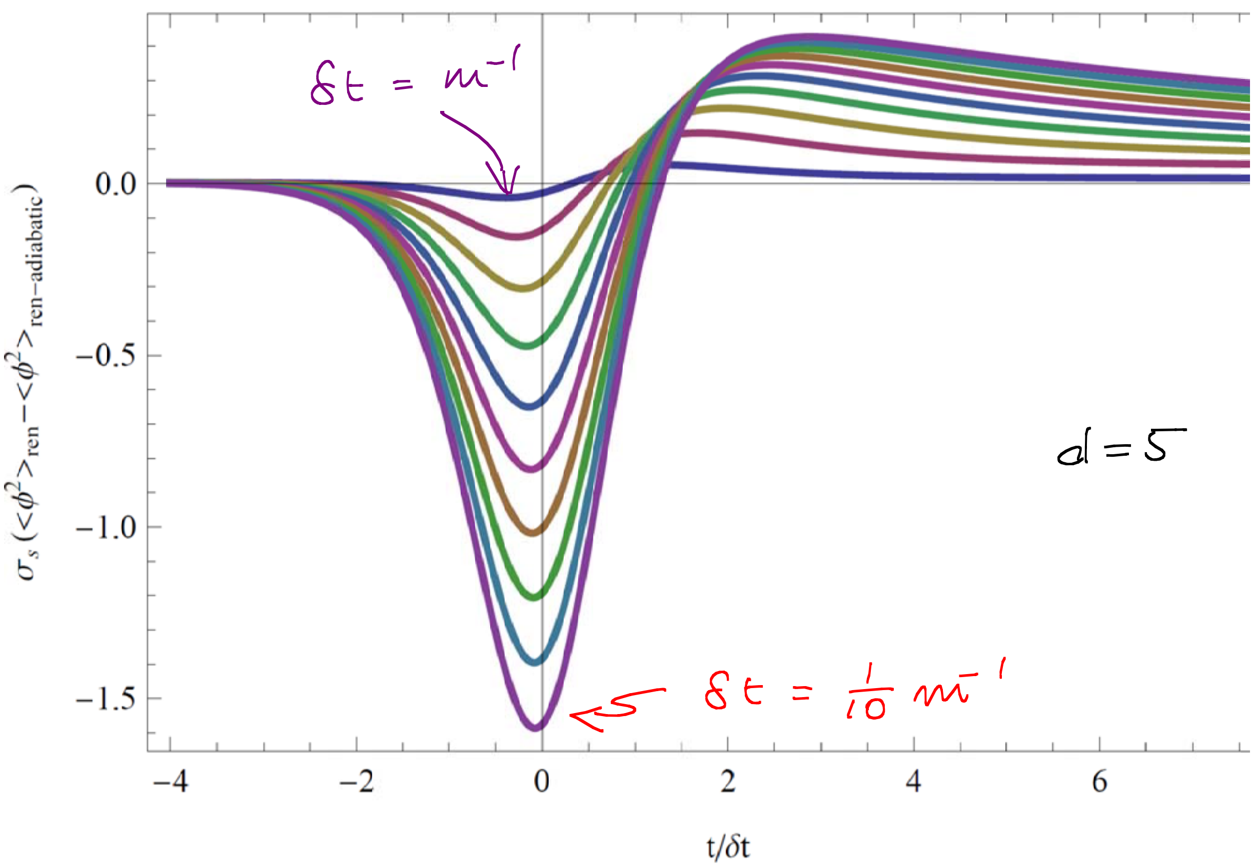
$$\Omega_k(t) = \omega_k(t) - \frac{1}{4\omega_k}\left(\frac{\ddot{\omega}_k}{\omega_k} - \frac{3}{2}\left(\frac{\dot{\omega}_k}{\omega_k}\right)^2\right) + \cdots$$

$$\omega_k(t) \equiv \sqrt{k^2 + m^2(t)}$$

THIS MAY APPEAR PUZZLING AT FIRST
ADIABATIC EXPANSIONS ARE GOOD WHEN THE RATE OF CHANGE IS SLOW — BUT WE ARE INTERESTED IN FAST QUENCHES

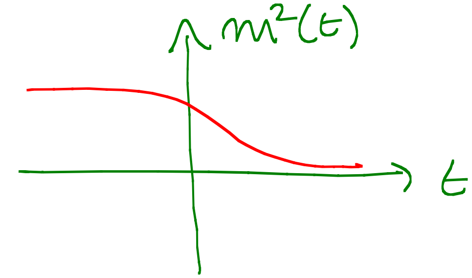
THE POINT IS THAT OUR QUENCH RATE IS FAST COMPARED TO PHYSICAL SCALES BUT SLOW COMPARED TO THE CUTOFF SCALE

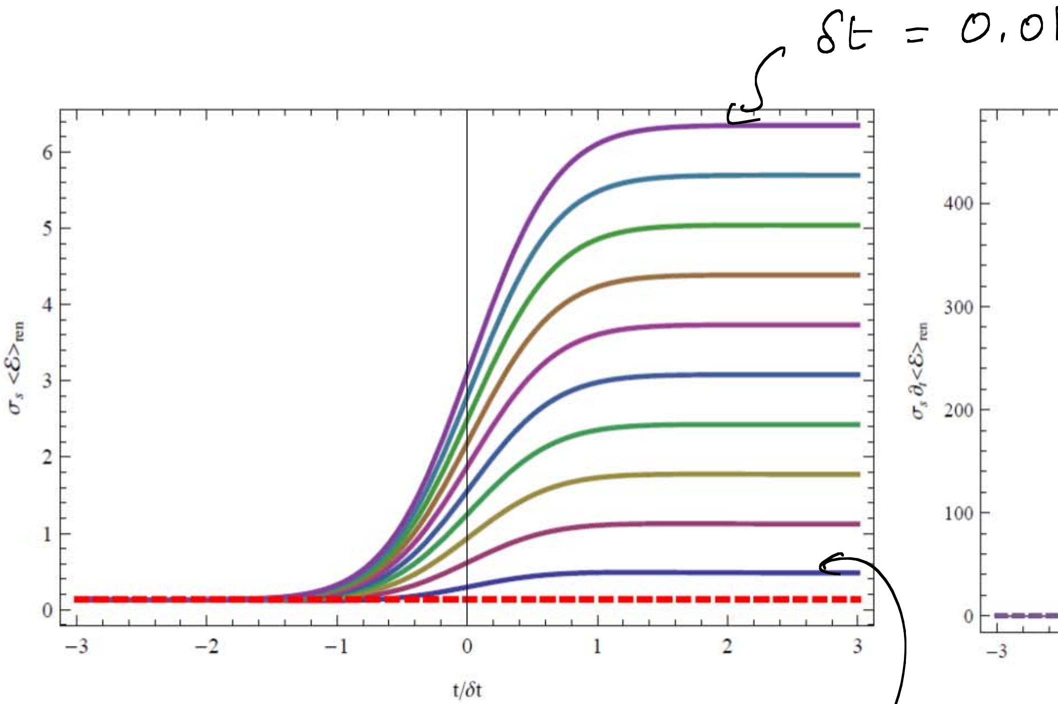
$\Rightarrow$ ADIABATIC EXPANSION IS GOOD FOR THE UV CONTRIBUTION



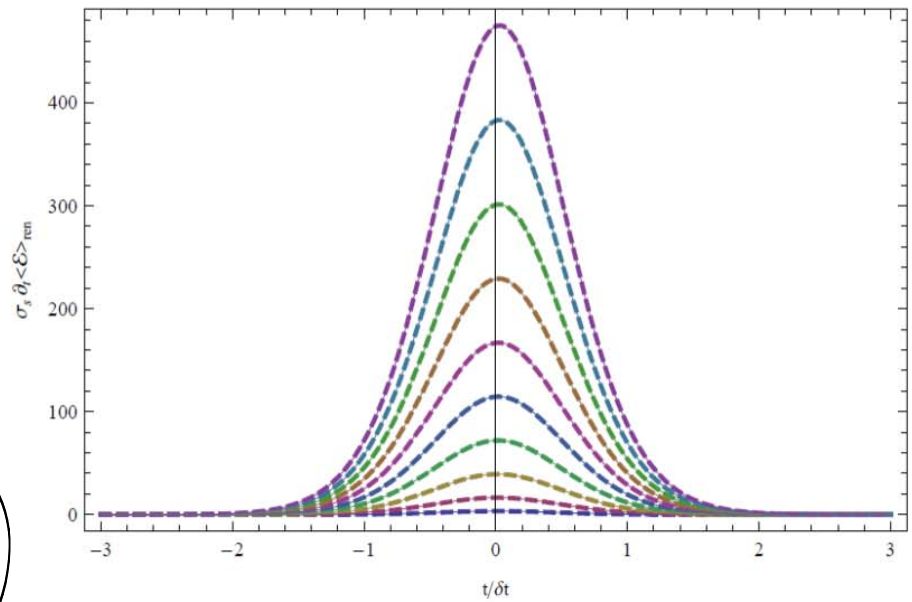
$$m^2(t) = \frac{1}{2} M_0^2 \left(1 - \tanh\frac{t}{\delta t}\right)$$

$d = 5$





$\langle \varepsilon \rangle$ AS FUNCTION OF $t/\delta t$



$\partial_t \langle \varepsilon \rangle$ AS FUNCTION OF $t/\delta t$

WE CAN NOW CONSIDER FAST QUENCH LIMIT

$$m_0 \delta t \ll 1$$

IN THIS LIMIT ONE RECOVERS ANALYTIC EXPRESSIONS

ODD d, $d \geq 5$

$$\langle \phi^2 \rangle_{ren} = (-1)^{\frac{d-1}{2}} \frac{\pi}{2^{d-2}} \partial_t^{d-4} m^2 + O(\delta t^{6-d})$$

$d = 3$

$$\langle \phi^2 \rangle_{ren} = -\frac{m_0}{4} - \frac{m_0^2 \delta t}{16} \log\left[\frac{1}{2}\left(1 - \tanh\frac{t}{\delta t}\right)\right]$$

$$\left(m^2(t) = \frac{1}{2} m_0^2 \left(1 - \tanh\frac{t}{\delta t}\right) \right)$$

FOR EVEN d THE RESULTS ARE

$d > 4$

$$\langle \phi^2 \rangle_{ren} = \frac{(-1)^{d/2}}{2^{d-3}} \log(\mu \delta t)\, \partial_t^{d-4} m^2(t) + \cdots$$

$d = 4$

$$\langle \phi^2 \rangle_{ren} = \frac{m_0^2}{4} \left(1 + \tanh \frac{t}{\delta t}\right) \log(\mu \delta t) + \cdots$$

THESE RESULTS IMPLY THAT, UPTO logs

$$\langle \phi^2 \rangle \sim m_0^2 (\delta t)^{4-d}$$

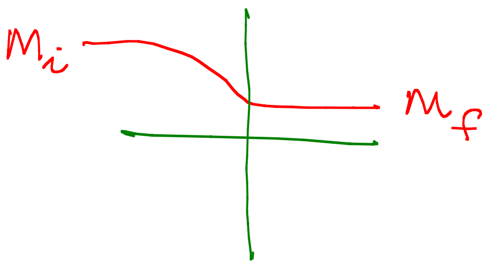
SINCE FOR THIS OPERATOR $\Delta = d-2$

$$\langle \phi^2 \rangle \sim m_0^2 (\delta t)^{d-2\Delta}$$

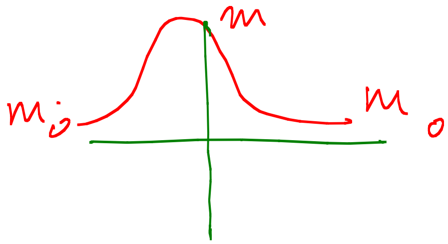
THIS IS EXACTLY AS FOUND IN HOLOGRAPHY

– LOG CORRECTIONS ALSO FOUND THERE

RESULTS GENERALIZE TO GENERIC MASS PROFILES



$m_i \delta t \ll 1$ $m_f \delta t \ll 1$



$m_0 \delta t \ll 1$ $m \delta t \ll 1$

SIMILAR RESULTS FOR DIRAC FERMIONS

THIS SCALING BEHAVIOR FOR FAST QUENCH SEEMS TO HOLD FOR

- STRONGLY COUPLED THEORIES WITH GRAVITY DUALS
- FREE FIELD THEORIES

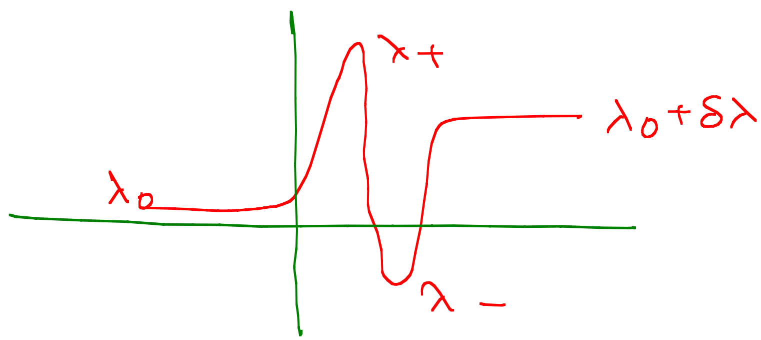
COULD THIS BE A GENERAL RESULT?

THE GENERAL RESULT

CONSIDER NOW A GENERAL INTERACTING THEORY

$$S = S_{CFT} - \int d^{d-1}x \, dt \, \lambda(t) \, \mathcal{O}_\Delta(x,t)$$

LET US NOW COMPUTE $\langle \mathcal{O}_\Delta \rangle$ PERTURBATIVELY FOR



QUENCH STARTS AT $t = 0$

$$\langle \mathcal{O}_\Delta \rangle = \langle \mathcal{O}_\Delta \rangle_{\lambda_0} + \int_0^t dt' \int d^{d-1}x' \, \lambda(t') \, G_R(x-x'; t-t')$$

$$+ \int dt' dt'' dx' dx'' \lambda(t') \lambda(t'') K(x, x', x''; t, t', t'')$$

FIRST TERM IS STANDARD LINEAR RESPONSE

$$G_R(x-x', t-t') \equiv \Theta(t-t') \langle 0 | [\mathcal{O}_\Delta(x,t), \mathcal{O}_\Delta(x',t')] | 0 \rangle_{\lambda_0}$$

WE NOW EXAMINE THIS TERM IN SOME DETAIL

$$\int_0^t dt' \int d^{d-1}x' \; \lambda(t') \, G_R(x-x'; t-t')$$

- (x', t') LIES IN PAST LIGHT CONE OF (x, t)
- IF $t \lesssim \delta t$ THE SPATIAL SEPARATION $|x - x'| \lesssim \delta t$
- HOWEVER IF δt IS MUCH SMALLER THAN ALL OTHER LENGTH SCALES, INCL. $(\lambda_0)^{-\frac{1}{d-\Delta}}$ CORRELATORS AT DISTANCES SHORTER THAN δt ARE BASICALLY CFT CORRELATORS

THUS RENORMALIZED $\langle \mathcal{O}_\Delta \rangle$ DEPENDS ONLY ON $\delta\lambda$ AND δt

IN TERMS OF A DIMENSIONLESS COUPLING

$$g \equiv (\delta\lambda)(\delta t)^{d-\Delta}$$

WE HAVE AN EXPANSION

$$\langle \mathcal{O} \rangle - \langle \mathcal{O} \rangle_{\lambda_0} = (\delta t)^{-\Delta} \left[a_1\left(\frac{t}{\delta t}\right) \cdot g + a_2\left(\frac{t}{\delta t}\right) g^2 + \cdots \right]$$

HOWEVER FAST QUENCH MEANS

$$g \ll 1$$

THUS THE LEADING ANSWER IS

$$\delta\langle \mathcal{O}_\Delta \rangle \sim (\delta t)^{-\Delta} g \; a_1\left(\frac{t}{\delta t}\right) = (\delta\lambda)(\delta t)^{d-2\Delta} \; a_1(t/\delta t)$$

$\Rightarrow$ "EARLY TIME" QUANTITY HAS THIS UNIVERSAL FORM IN ANY THEORY

— SIMILAR ARGUMENT FOR $\langle \mathcal{E} \rangle_{ren}$

THE ABRUPT LIMIT

- DOES $\delta t \to 0$ LIMIT REPRODUCE ABRUPT QUENCH ?
- TO INVESTIGATE THIS CONSIDER THE EXPLICIT RESULTS FOR FREE BOSONIC FIELDS
- OUR RESULTS ARE FOR RENORMALIZED QUANTITIES
  - THESE ARE RELEVANT ONLY IF THE QUENCH RATE IS MUCH BELOW THE CUTOFF

$$\Lambda_{UV}^{-1} \ll \delta t \ll (\delta\lambda)^{-\frac{1}{d-\Delta}}$$

THIS IS WHY ADIABATIC EXPN. GIVES COUNTERTERMS

AN ABRUPT QUENCH HAS A RATE FASTER THAN ALL SCALES – HAS TO BE OF CUTOFF SCALE

⇒ CLEARLY LOCAL OBSERVABLES IN FAST QUENCH WOULD BE, IN GENERAL, DIFFERENT FROM THOSE IN ABRUPT QUENCH

TO EXAMINE THIS IN MORE DETAIL, LOOK AT UV FINITE QUANTITIES –

EXCESS ENERGY

ONE SUCH QUANTITY IS THE EXCESS ENERGY DENSITY AT LATE TIMES

$$\Delta \mathcal{E} = \mathcal{E} - \mathcal{E}_{ground}$$

WHERE $\mathcal{E}_{ground}$ IS THE GROUND STATE ENERGY OF THE FINAL HAMILTONIAN

$$\mathcal{E}_{ground} = \frac{1}{2} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} (\omega_{out})$$

THIS IS IN FACT THE LEADING ADIABATIC RESULT

THE EXPRESSION FOR THIS IS

$$\Delta E = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \, \omega_{out} \, |\beta_k|^2$$

WHERE β_k IS THE BOGOLIUBOV COEFFICENT

THIS IS A UV FINITE QUANTITY

WE SAW THAT COUNTERTERMS WHICH RENDER $E(t)$ FINITE DEPEND ON DERIVATIVES OF m^2 IN ADDITION TO m^2 — BUT DERIVATIVES VANISH AS $t \to \infty$

THUS SUBTRACTION OF E_{ground} IS SUFFICIENT

$d = 2$ $$\Delta \mathcal{E}^{\delta t \to 0} = \frac{m^2}{16\pi} + c_1 m^4 (\delta t)^2 + \cdots$$

$d = 3$ $$\Delta \mathcal{E}^{\delta t \to 0} = \frac{m^3}{24\pi} + c_2 m^4 (\delta t) + \cdots$$

$d = 4$ $$\Delta \mathcal{E}^{\delta t \to 0} = c_3 m^4 \log (m \delta t) + O(\delta t)$$

$d \geq 5$ $$\Delta \mathcal{E}^{\delta t \to 0} \sim m^4 (\delta t)^{4-d}. + \cdots$$

FOR $d=2,3$ THIS IS FINITE AS $\delta t \to 0$. THIS FINITE ANSWER IS IN FACT THE RESULT FOR INSTANTANEOUS QUENCH

$$\Delta \mathcal{E}^{inst} = \frac{\Omega_{d-2}}{(2\pi)^{d-1}} \int dk \, . \, k^{d-1} \, \frac{\left(\sqrt{k^2+m^2} - k\right)^2}{4k\sqrt{k^2+m^2}}$$

FOR $d \geq 4$ EXCESS ENERGY DIVERGES AS $\delta t \to 0$

LIKEWISE $\Delta \mathcal{E}^{inst}$ IS UV DIVERGENT

WHILE THESE EXPLICIT RESULTS ARE FOR FREE FIELDS WE EXPECT SIMILAR RESULTS TO HOLD FOR ARBITRARY INTERACTING THEORIES

$\Delta < d/2$ $\quad \Delta \mathcal{E}$ IS FINITE AS $\delta t \to 0$

SUBLEADING PIECE DISPLAYS UNIVERSAL SCALING $\sim (\delta t)^{d-2\Delta}$

$\Delta > d/2$ $\quad \Delta \mathcal{E}$ DIVERGES AS $(\delta t)^{d-2\Delta}$

CORRELATION FUNCTIONS

ANOTHER CLASS OF UV FINITE QUANTITIES ARE CORRELATION FUNCTIONS AT FINITE SPATIAL SEPARATIONS

$$C(\vec{r},t) \equiv {}_{in}\langle 0|\, \phi(\vec{r},t)\, \phi(0,t)\, |0\rangle_{in}$$

LATE TIME CORRELATORS

$$C_{smooth}(t,r) = \int \frac{[d\vec{k}]}{2k} e^{i\vec{k}\cdot\vec{r}} \left\{ |\alpha_k|^2 + |\beta_k|^2 + \alpha_k \beta_k^* e^{2ikt} + \alpha_k^* \beta_k e^{-2ikt} \right\}$$

$$C_{abrupt}(t,r) = \int \frac{[d\vec{k}]}{2k} e^{i\vec{k}\cdot\vec{r}} \left\{ \frac{k^2 + m^2 \sin^2(kt)}{k^2 \sqrt{k^2+m^2}} \right\}$$

(δt) IS CONTAINED IN THE BOGOLIUBOV COEFFICIENTS α_k, β_k.

THE INTEGRANDS AGREE IN TWO LIMITS

- $m\delta t \ll 1 \qquad |\vec{k}|\delta t \ll 1$
- $m\delta t \ll 1 \qquad |\vec{k}|/m \gg 1 \qquad |\vec{k}|\delta t = \text{arbitrary}$

THE SECOND IMPLIES THAT THE LEADING SHORT DISTANCE SINGULARITIES OF THE TWO CORRELATORS ARE THE SAME – BUT THE SUBLEADING TERMS ARE NOT

IN FACT, THE SHORT DISTANCE EXPANSION FOR SMOOTH QUENCH CORRELATOR IS

$$C(t,r) \xrightarrow[r\to 0]{} \frac{a_1}{r^{d-2}} + \frac{a_2\, m^2(t)}{r^{d-4}} + \frac{a_3\left(3\, m^4(t) + \partial_t^2 m^2(t)\right)}{r^{d-6}} + \cdots$$

THESE TERMS ARE IN 1-1 CORRESPONDENCE WITH THE COUNTERTERMS FOR $\langle \phi^2 \rangle$ PREDICTED BY ADIABATIC EXPANSION

NOTE: FOR $d \geq 6$ DIVERGENT TERMS (as $r \to 0$) INVOLVE DERIVATIVES OF MASS.

THE SECOND REGIME WHERE THERE IS AGREEMENT IS $m\delta t \ll 1$ $|\vec{k}|\delta t \ll 1$

THUS LONG DISTANCE CORRELATORS SHOULD AGREE, SINCE FOR $r \gg \delta t$ WE EXPECT CONTRIBUTIONS FOR $k \lesssim r^{-1} \Rightarrow k \ll (\delta t)^{-1}$

– INDEED THEY DO

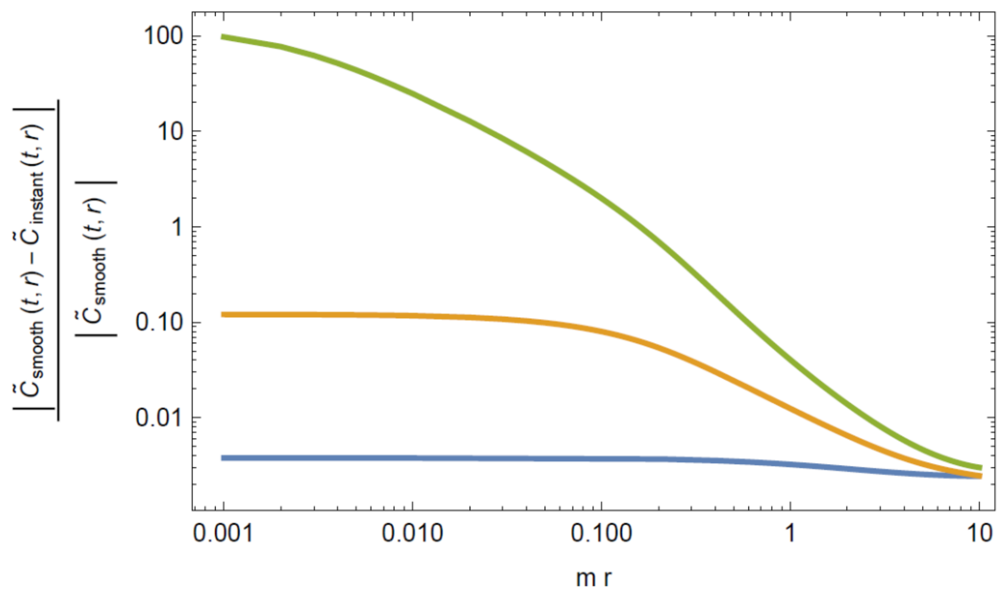


Figure 5. (Colour online) Difference between the late-time correlators for smooth and instantaneous quenches as a function of the separation distance r. The blue line corresponds to the $d = 3$ case while the yellow one belongs to $d = 5$ and $d = 7$ is shown in green. We are using $mt = 10$ with $m\delta t = 1/20$. In $d = 3$ and $d = 5$, the difference remains small for any value of r, while in $d = 7$, it seems to diverge as $r \to 0$.

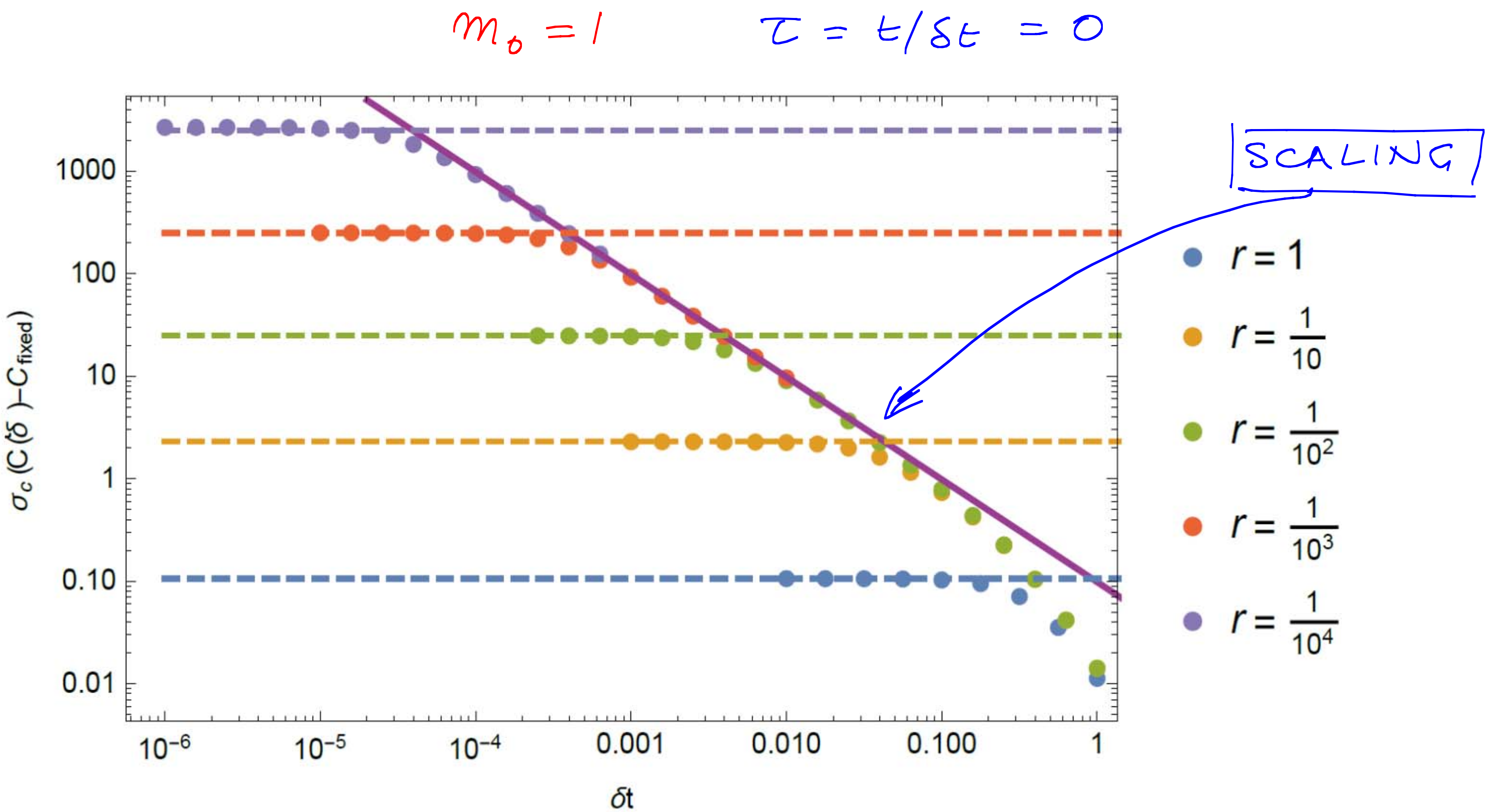
CROSSOVER BEHAVIOR

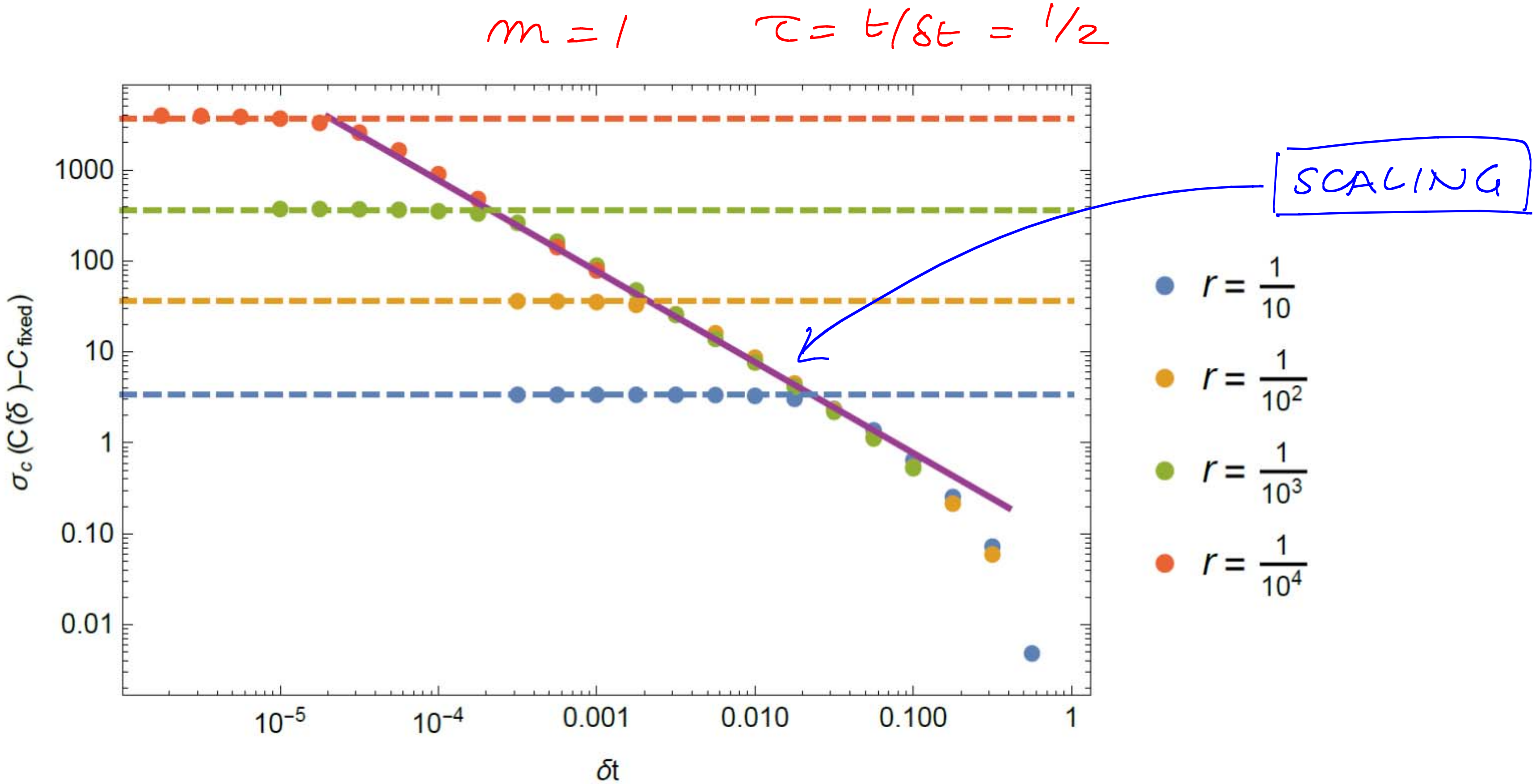
PERHAPS THE MOST INTERESTING BEHAVIOR APPEARS IN CORRELATORS AT EARLY TIMES.

RECALL: THE SCALING OF LOCAL OBSERVABLES APPEAR AT EARLY TIMES AS WELL.

SHOULD EXPECT THAT FOR SOME REGIME OF $|\vec{r}|$ WE SHOULD RECOVER SCALING

FOR LARGER $|\vec{r}|$ EXPECT SATURATION


m_0 = 1
τ = t/δt = 0
SCALING
1000
100
10
1
0.10
0.01
10^-6
10^-5
10^-4
0.001
0.010
0.100
1
δt
σ_c (C(δ) − C_fixed)
r = 1
r = 1/10
r = 1/10^2
r = 1/10^3
r = 1/10^4



m = 1
τ = t/δt = 1/2
SCALING
1000
100
10
1
0.10
0.01
10^{-5}
10^{-4}
0.001
0.010
0.100
1
δt
$\sigma_c\,(C(\delta\,) - C_{\text{fixed}})$
$r = \frac{1}{10}$
$r = \frac{1}{10^2}$
$r = \frac{1}{10^3}$
$r = \frac{1}{10^4}$

THUS, FOR AN INTERMEDIATE REGIME OF $|\vec{r}|$ WE INDEED SEE UNIVERSAL SCALING

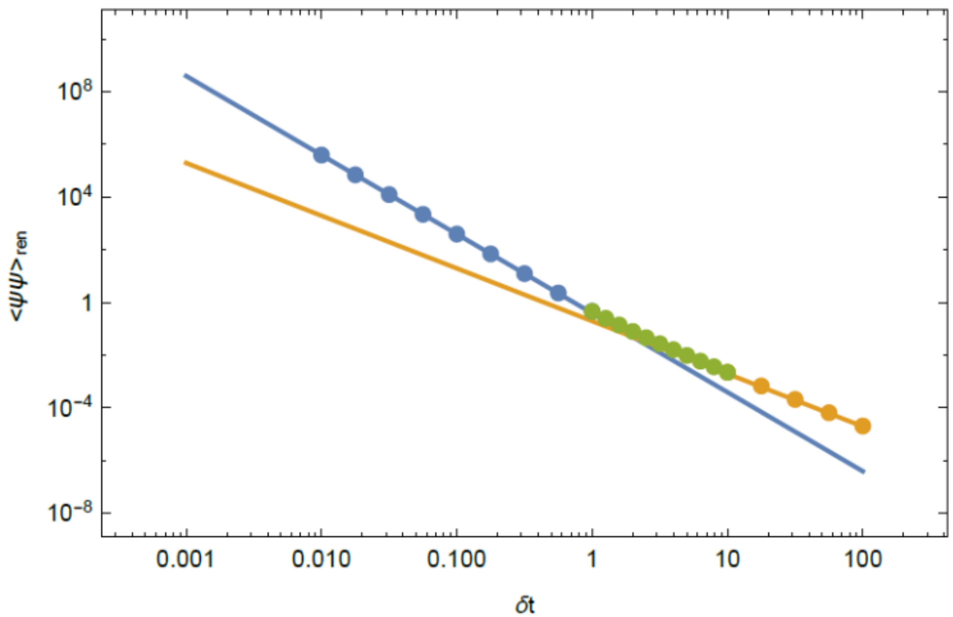
WE EXPECT THE SAME TO HOLD FOR INTERACTING THEORIES AS WELL

FINITE CUTOFFS ?

FOR THEORIES WITH A FINITE LATTICE SPACING a
OUR RESULTS INDICATE

THERE IS AN INTERMEDIATE QUENCH RATE
FOR WHICH THESE SCALING LAWS HOLD.

FOR APPROPRIATE PROTOCOLS, STUDY OF CROSSOVER
FROM KIBBLE ZUREK TO THIS SCALING CAN
BE STUDIED – COULD BE INTERESTING FOR
EXPERIMENTS!



10^8
10^4
1
10^{-4}
10^{-8}
0.001
0.010
0.100
1
10
100
δt
$\langle\psi\psi\rangle_{ren}$

OUR RESULTS INDICATE THAT FAST QUENCHES WITH OPERATORS WITH CONFORMAL DIMENSIONS $\Delta > d/2$ ARE QUALITATIVELY DIFFERENT FROM THOSE WITH $\Delta < d/2$

THIS MAY HAVE IMPLICATIONS FOR USING A CALABRESE - CARDY STATE

$$e^{-\tau_0 H} |B\rangle$$

AS A GOOD APPROXIMATION TO A PHYSICAL FAST QUENCH

どうもありがとうございました