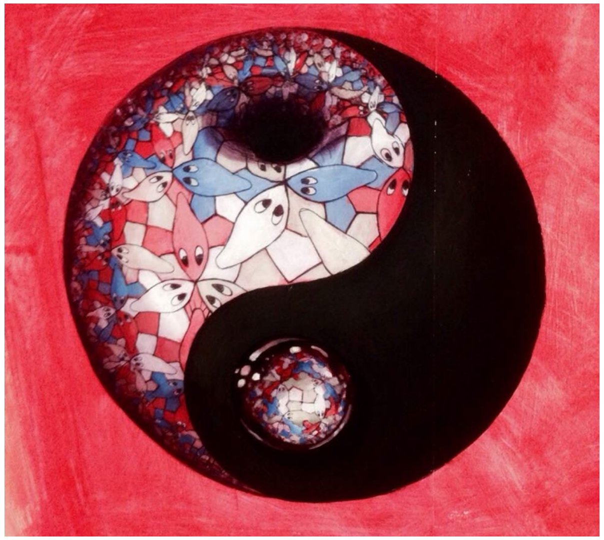
Condensed matter and holography: seductivity and resistance

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University



Netherlands Organisation for Scientific Research

$$J_{exp} = \sigma E_{theor}$$

σ : seductivity

ρ : resistance

$$\sigma = \frac{1}{\rho}$$

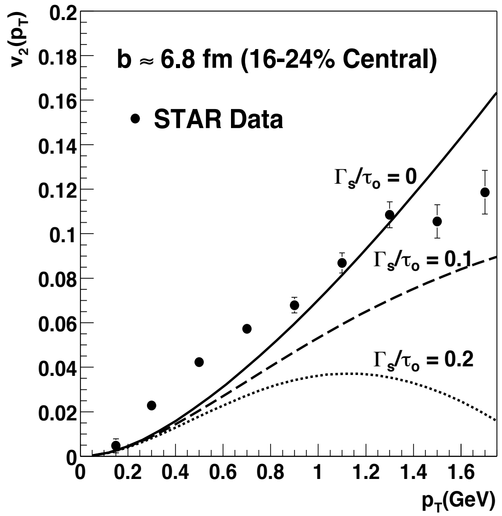
Can holography explain experiment?

Minimal viscosity and AdS/RHIC

- Policastro, Son, Starinets (2001)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Teaney (2003)



$$\Gamma_s = \frac{4}{3T}\frac{\eta}{s}$$

Seductivity in AdS/RHIC

- Quark Matter 2006 (McLerran)

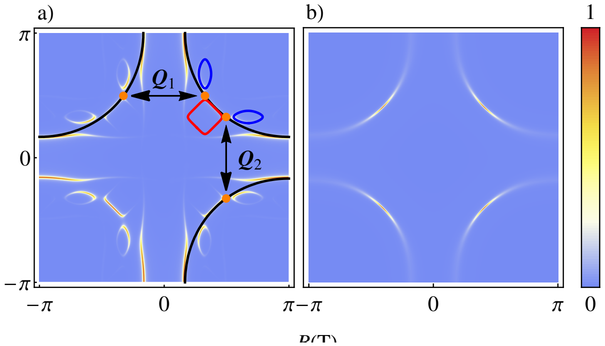
"AdS/CFT MUST be accountable to the same scientific standards as are other computations, or else it is not science."

- Quark Matter 2012 (Wiedemann)

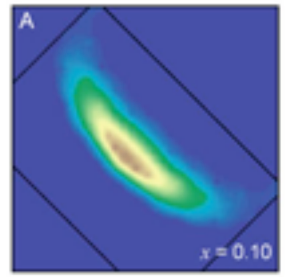
"It is worth recalling that QCD does not fall into the class of field theories with known gravity dual....In the light of this caveat, it is truly remarkable to what extent the AdS/CFT correspondence has offered a framework for understanding central open questions in the phenomenology of heavy ion collisions."

Quantum Oscillations

- Sebastian



CDW order in underdoped cuprates

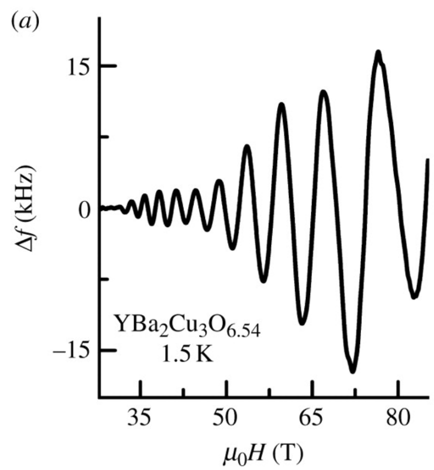


"hard" pseudogap at T=0

$$\chi_{\rm osc} \sim \cos\frac{A_F}{B}\sum_{n=0}^{\infty} e^{-c_n \frac{T A_F}{\mu B}}$$

Quantum Oscillations

- Sebastian



- Holography

Hartnoll, Hofman

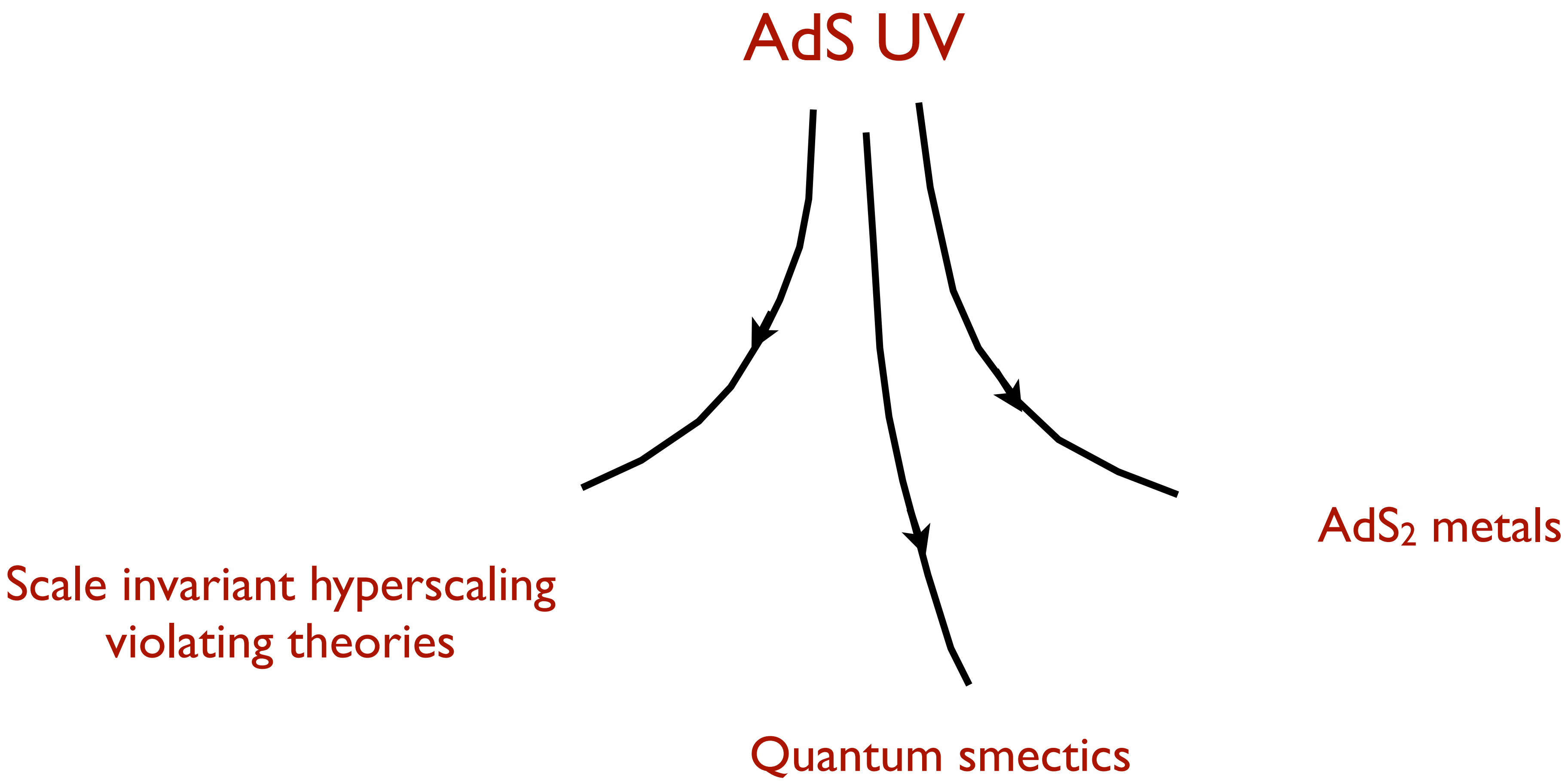
$$\chi_{\rm osc} \sim \cos\frac{A_F}{B}\sum_{n=0}^{\infty} e^{-c_n \frac{T A_F}{\mu B}\left(\frac{T}{\mu}\right)^{2\nu-1}}$$

$$G_F \sim \frac{1}{\omega - v_F k + \omega^{2\nu}}$$

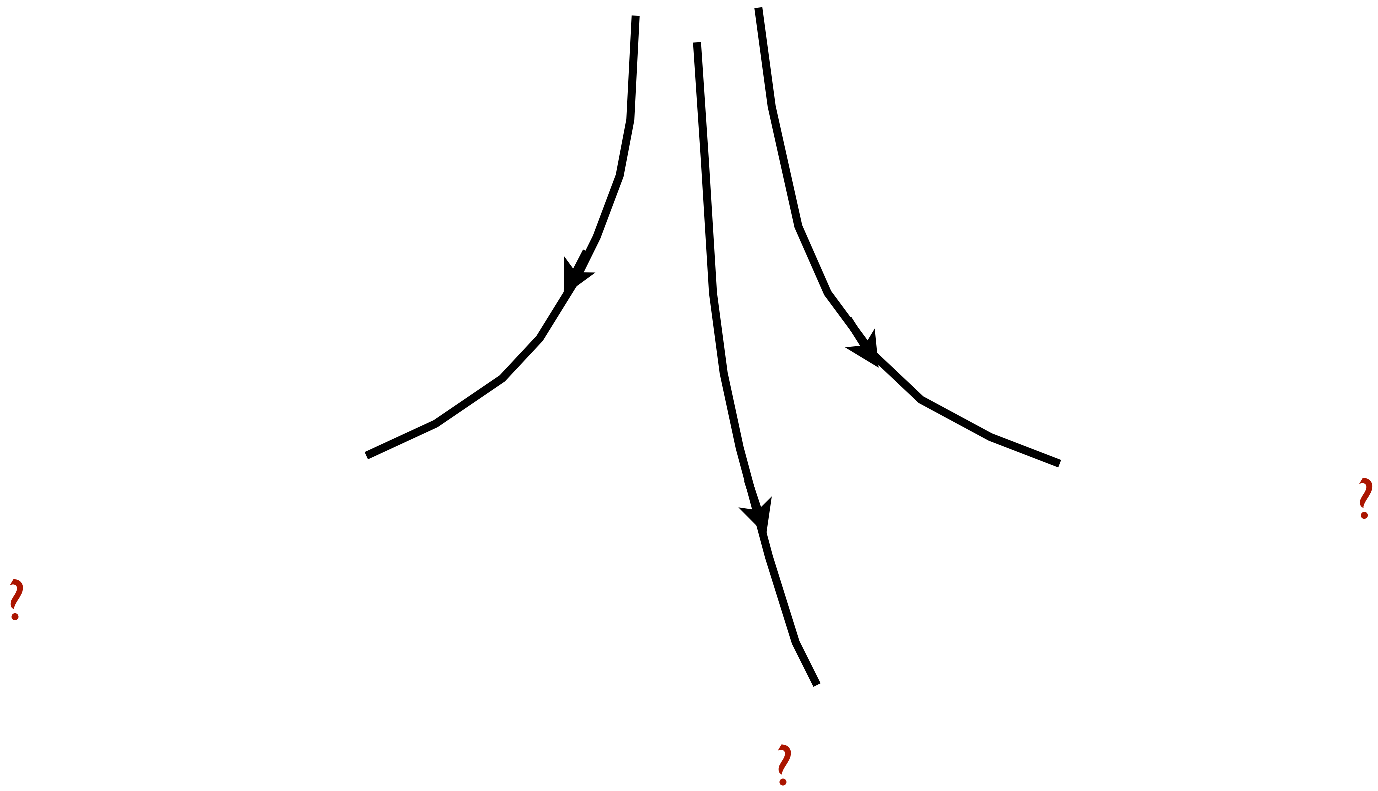
No evidence, so far

Why we can reasonably hope that AdS/CFT is relevant.

- AdS/CFT: new insight into strongly coupled systems at finite density:

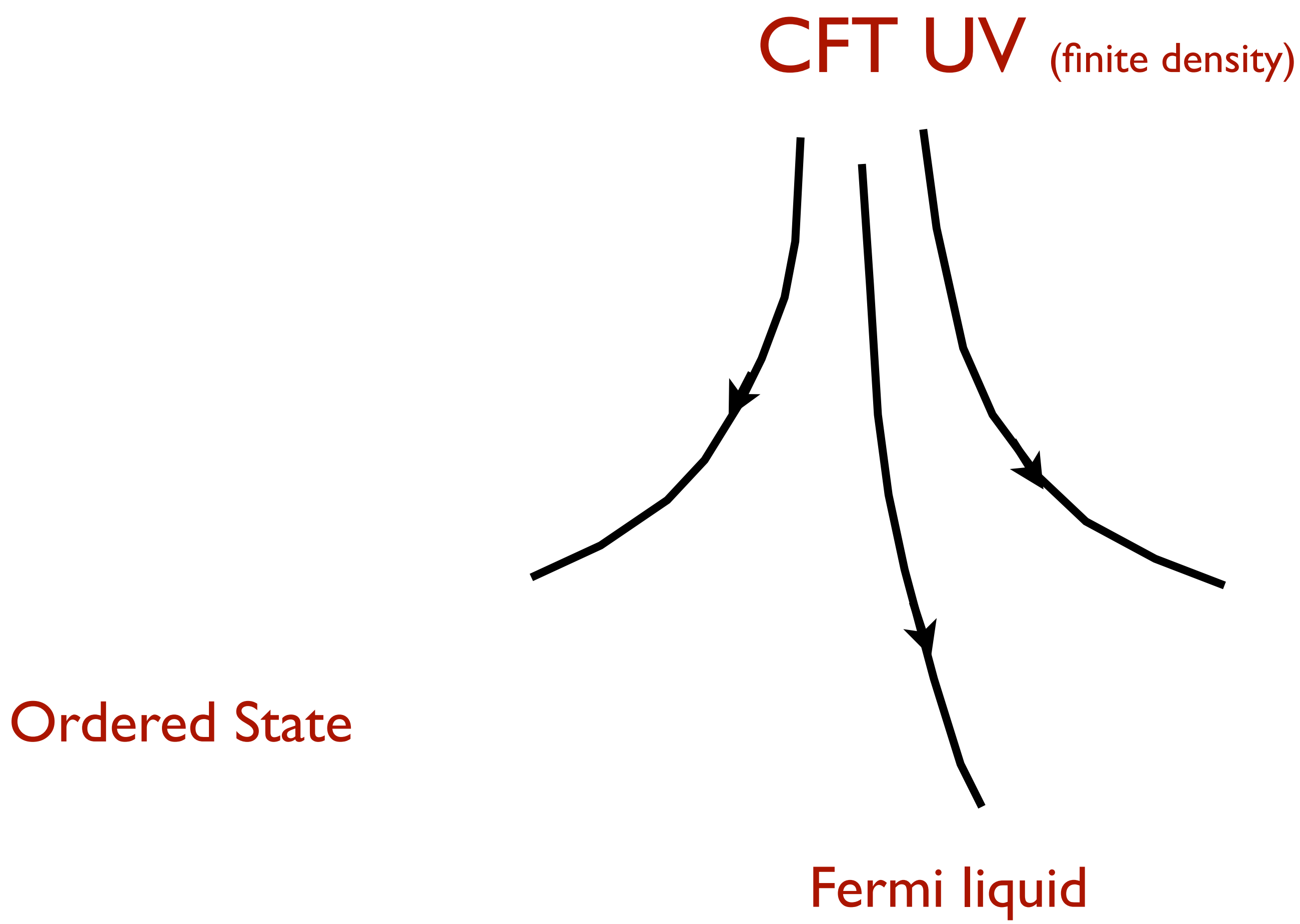


CFT UV (finite density)



Quantum Condensed Matter Physics - Lecture Notes

Chetan Nayak


CFT UV (finite density)
Ordered State
Fermi liquid
BCS SC

- AdS/CFT is very non-generic

Quantum Condensed Matter Physics - Lecture Notes

Chetan Nayak

Holographic Superconductor

Hartnoll, Herzog, Horowitz

(Quantum) Electron Star

Hartnoll, Tavanfar

...

Kachru, Karch, Yaida

Erdmenger et.al

- Erdmenger: Kondo effect in Holography

Kondo models from gauge/gravity duality

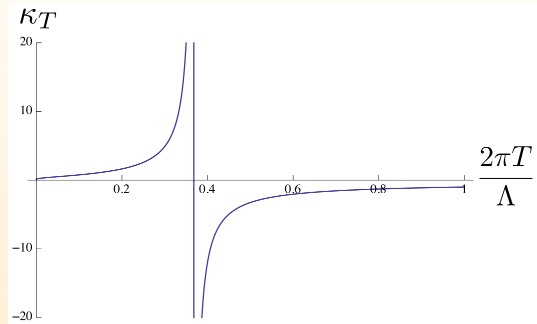
J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

Scale generation

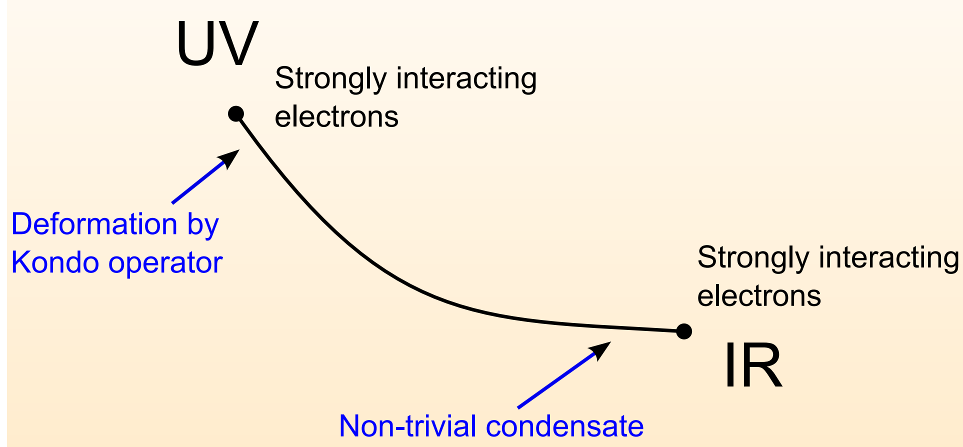


Divergence of Kondo coupling determines Kondo temperature T_K

Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

RG flow



Holography describes new states of matter

- A Holographic superconductor is novel
  - Non-canonical scaling dimension
  - Emergence from criticality

$$\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 L^2}$$

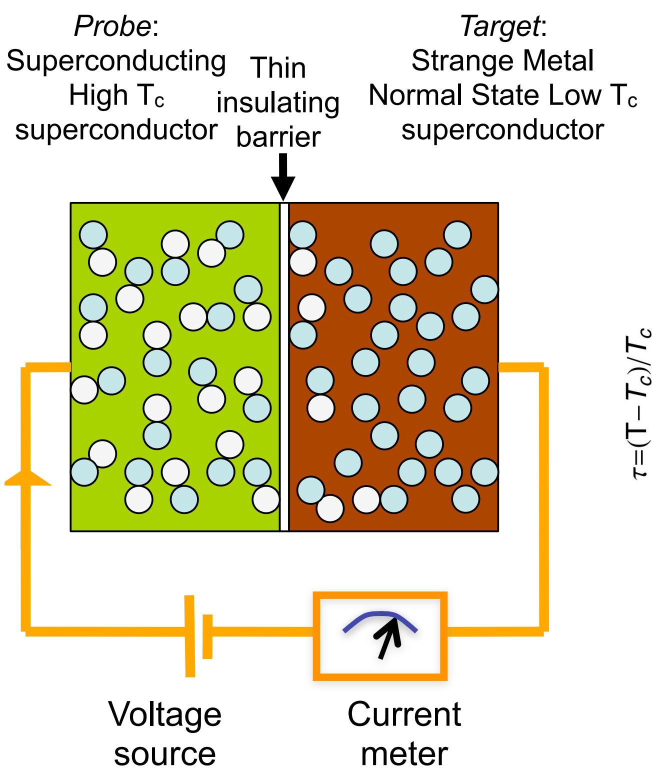
Holography describes new states of matter

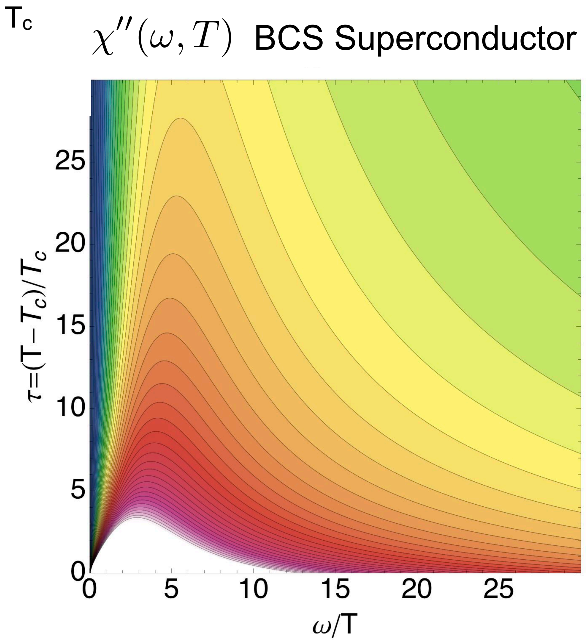
She, Overbosch, Sun, Liu, Mydosh, Zaanen, Schalm

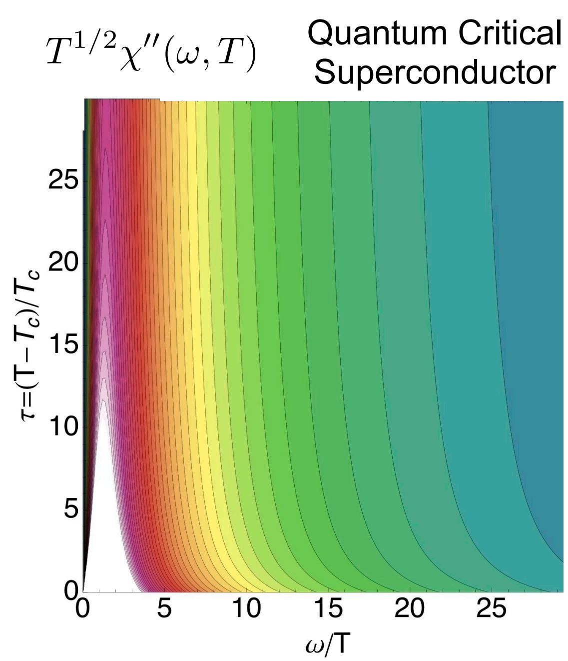
- Experimental: Order parameter susceptibility
  - Non-canonical scaling dimension
  - Emergence from criticality

$$\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 L^2}$$

$$\chi(\omega) = \mathrm{Im} G_R(\omega) = \langle \mathcal{O}^\dagger(\omega)\mathcal{O}(0)\rangle$$

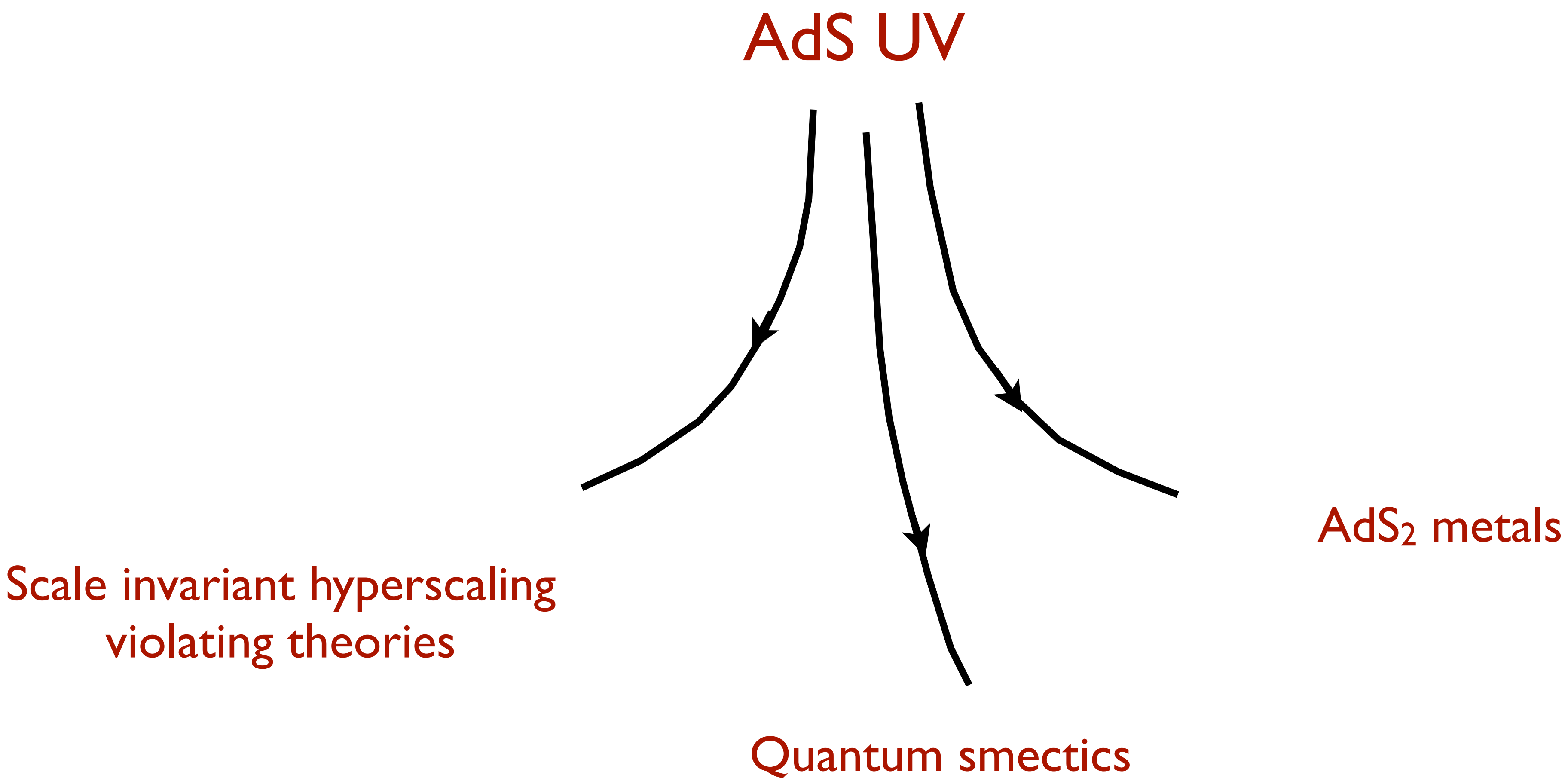






Why we can reasonably hope that AdS/CFT is relevant.

- AdS/CFT: new insight into strongly coupled systems at finite density:



Holography describes new states of matter

- Emergent scale invariant hyperscaling violating theories

$$ds^2 = \frac{L^2}{r^2}\left[r^{2\theta/(d-\theta)}dr^2 - r^{-2d(z-1)/(d-\theta)}dt^2 + dx^2\right] \qquad A_t = Q\ r^{\zeta - z}$$

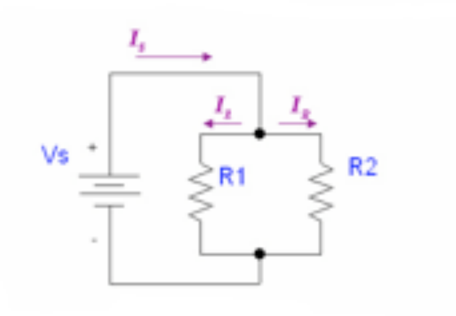
$$s \sim T^{(d-\theta)/z}$$

\

Supported by an ordered state

- Experimental signature: Thermoelectric response

$$\sigma = \sigma_{ccs} + \sigma_{relax}$$



Inverse Matthiessen law: two independent sectors

Thermoelectric response and Momentum relaxation

- Hall angle in cuprates

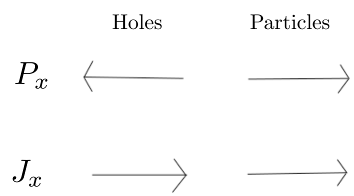
$$\sigma \sim \frac{1}{T} \qquad \theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

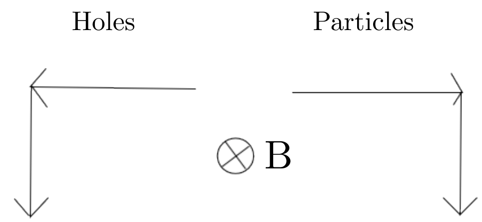
- Theory (e.g. Drude, memory matrix)

$$\sigma \sim \tau \qquad \theta_H \sim \tau$$

- Holography

Blake, Donos





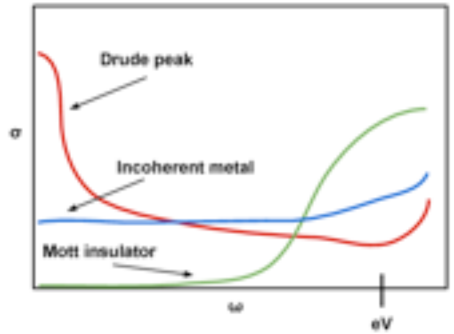
$$\sigma_{css} \text{ does not contribute to } \sigma_{xy}$$

$$\sigma = \sigma_{ccs} + \sigma_{relax} \qquad \sigma_{css} \sim \frac{1}{T}, \quad \sigma_{relax} \sim \frac{1}{T^2}$$

Coherent and Incoherent metals

Coherent Drude regime

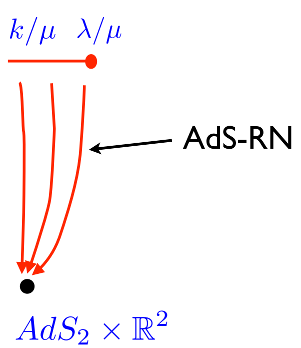
$\sigma_{ccs} \ll \sigma_{relax} \ , \qquad \tau_{relax} \gg \tau_{other}$

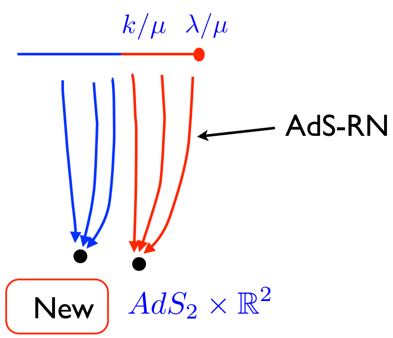


Incoherent metals

$\sigma_{ccs} \sim \sigma_{relax} \ , \qquad \tau_{relax} \sim \tau_{other}$

- Gauntlett





  - Navier-Stokes on the horizon determines holographic transport

$$\nabla_i v^i = 0$$

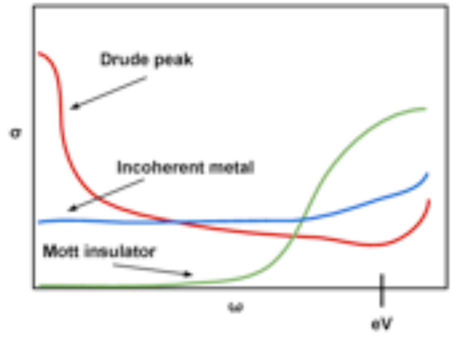
$$\nabla^2 w - v^i \nabla_i (a_t^{(0)}) = -\nabla_i E^i$$

$$\nabla^2 v_j + R_{ji} v^i - a_t^{(0)} \nabla_j w - \nabla_j p = 4\pi T \, \zeta_j + a_t^{(0)} E_j$$

Coherent and Incoherent metals

Coherent Drude regime

$\sigma_{ccs} \ll \sigma_{relax}$, $\tau_{relax} \gg \tau_{other}$



Incoherent metals

$\sigma_{ccs} \sim \sigma_{relax}$, $\tau_{relax} \sim \tau_{other}$

- Hartnoll

The Lorenz ratio

Fermi Dirac

$$L_0 = \frac{\pi^2}{3}$$

- **Matrix of conductivities:**

$$\begin{pmatrix} j \\ j^Q \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\alpha & T\overline{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- **Thermal conductivity at j = 0.**

$$\kappa = \overline{\kappa} - \frac{\alpha^2 T}{\sigma}$$

- **Lorenz ratio:**

$$L = \frac{\kappa}{\sigma T}$$

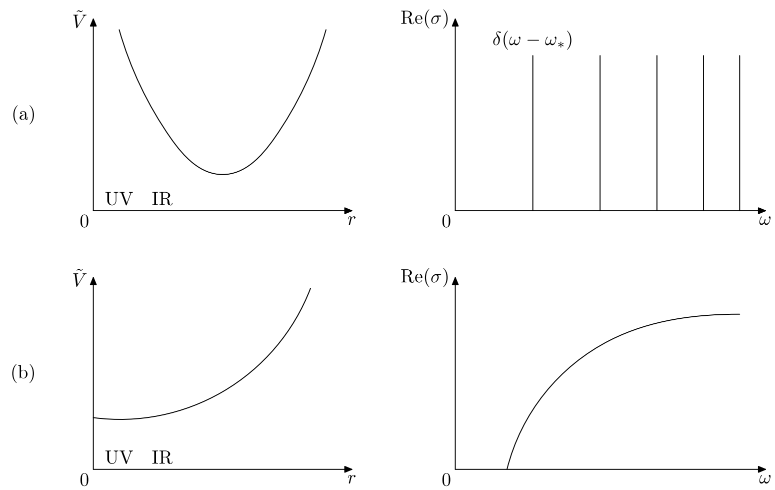
$$L \sim T^{-2\Phi/z}$$

$$n \sim T^{(2-\theta+\Phi)/z}$$

From metals to insulators

Novel insulators: strong dissipation

- Kiritsis

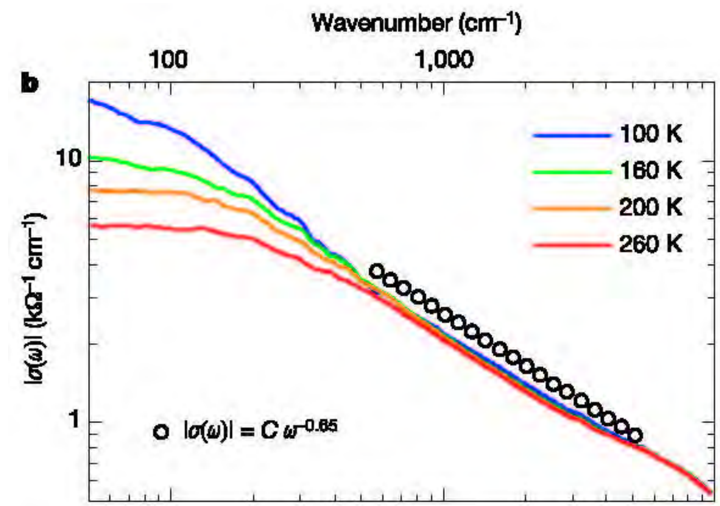


$$-\psi'' + V_{eff}\ \psi = \omega^2 \psi$$

$$\sigma = \sigma_{ccs} + \sigma_{relax}$$

$$\sigma_{ccs} \sim \omega^{-\alpha}$$

$$\sigma_{relax} \sim \frac{1}{T - i\omega}$$

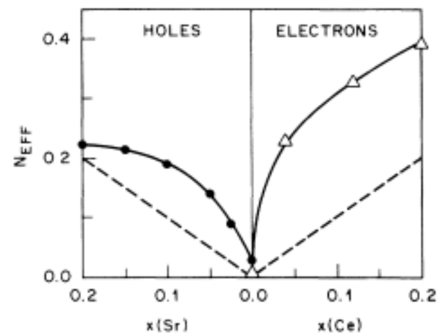


Van der Marel et al

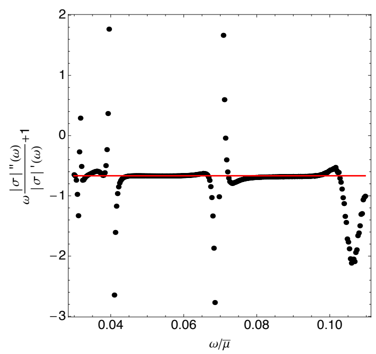
From metals to insulators

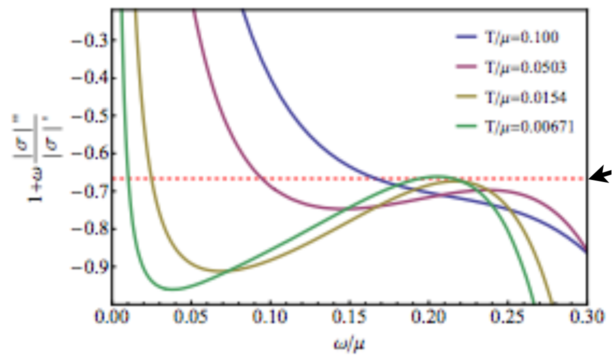
- Phillips
  - Mott insulator and doping

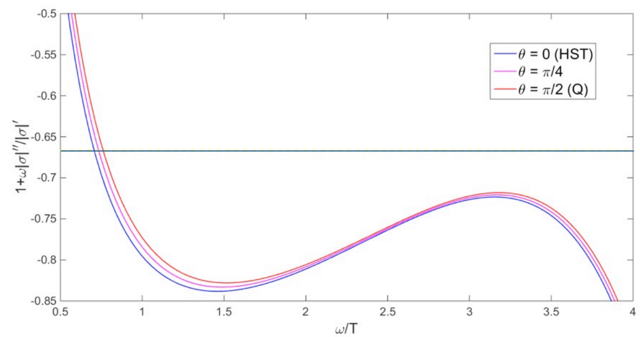
$$N_{\rm eff}(\Omega) = \frac{2mV_{\rm cell}}{\pi e^2}\int_0^{\Omega} \sigma(\omega)d\omega \qquad \frac{d\ln N_{\rm eff}}{d\ln x} \neq 1$$



  - Scaling in AC conductivity







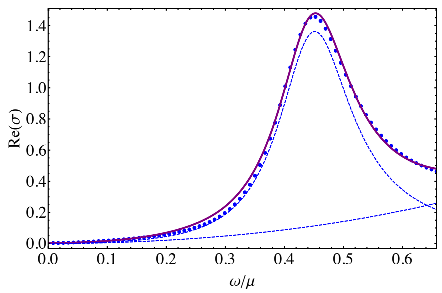
HST *DG* *Phillips et al*

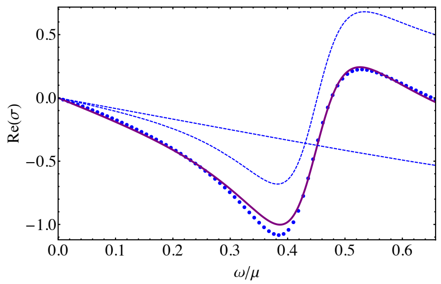
  - Anomalous conductivity requires anomalous physics

QFT Ward-Identity $\mathrm{Re}\sigma \sim L^{d-2}$

From metals to insulators

- Ling
  - Optical conductivity in a holographic CDW: MIT





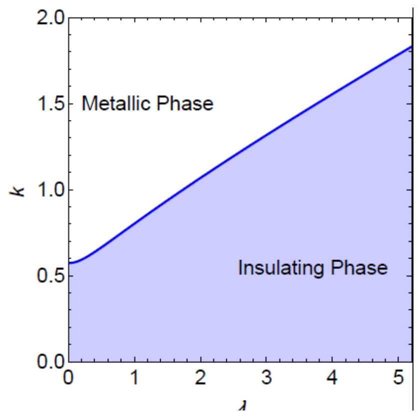
$$\sigma_{CDW}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2/\omega^2)}$$

  - MIT in a holographic Lattice

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R + 6 - \frac{1}{2}F_{ab}F^{ab} - |\partial\Phi|^2 - m^2|\Phi|^2]$$

$$\Phi = e^{ikx} z^{3-\Delta}\phi(z)$$

$$\phi(0) = \lambda$$



Erdmenger, Herwerth, Klug, Meyer, Schalm

Homes’ relation in holographic superconductors

MIT transitions in holography

- Holography with Bianchi VII helix geometry

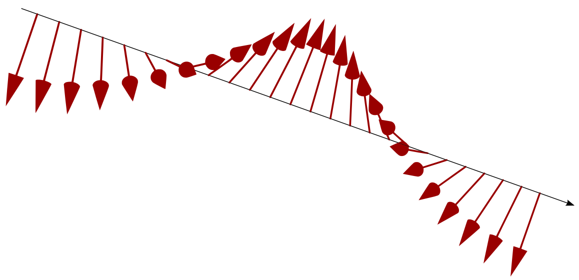
Donos, Hartnoll

$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{m^2}{2}B_\mu B^\mu\right) - \frac{\kappa}{2}\int B \wedge F \wedge W$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2v_1(r)}\omega_1^2 + e^{2v_2(r)}\omega_2^2 + e^{2v_3(r)}\omega_3^2$$

$$\omega_1 = dx_1\,, \qquad \omega_2 + i\omega_3 = e^{ipx_1}(dx_2 + idx_3)\,.$$

$$A^{(0)} = \mu\, dt\,, \qquad B^{(0)} = \lambda\,\omega_2\,.$$



period p; strength λ

MIT transitions in holography

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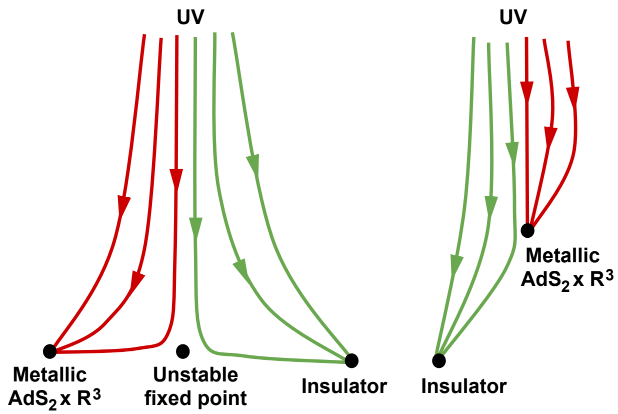


- The ground state is a quantum smectic

$$\sigma_{DC,xx} = 0\ , \quad \sigma_{DC,yy} = \infty$$

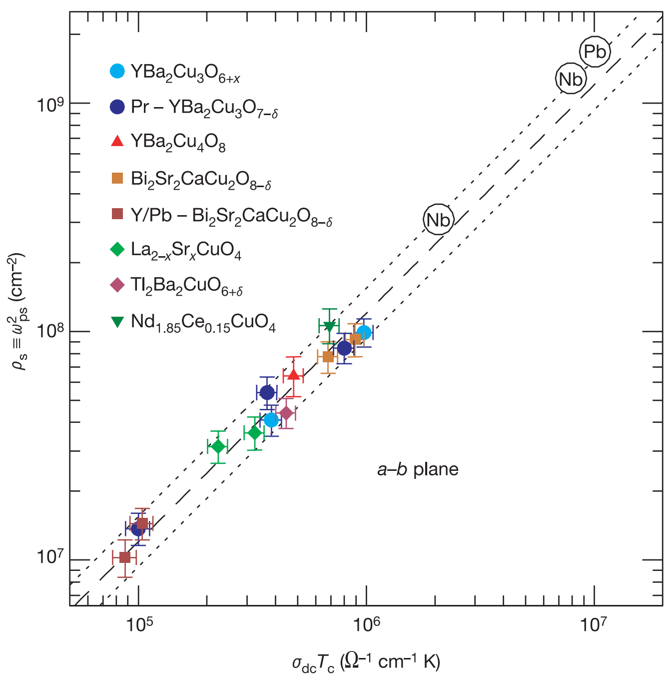
Universal relations in strange metals

- An analog of

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Homes' relation

$$\rho_s(0) = C\sigma_{DC}(T_c)T_c$$



Homes et al

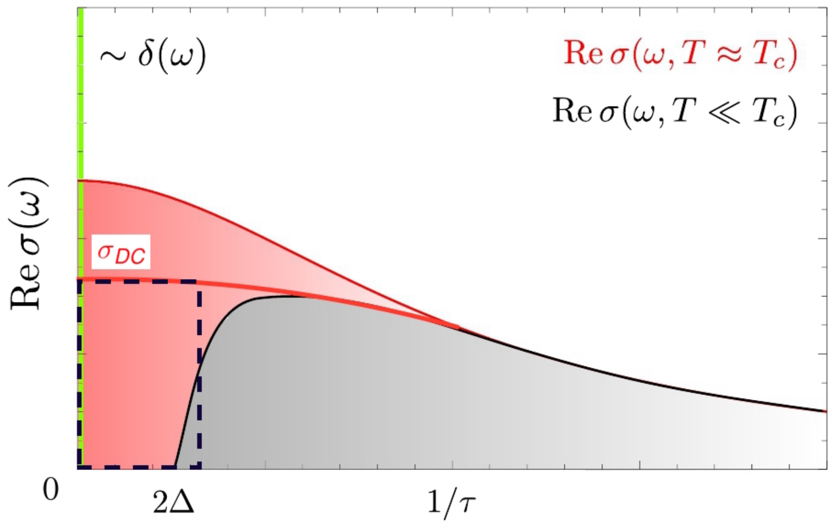
- Homes' relation

$$\rho_s(0) = C\sigma_{DC}(T_c)T_c$$

  - Naively

$$\rho_s(0) = \sigma_{DC} \cdot \Delta_{gap}$$

$$\Delta_{gap} = T_c$$



Homes' relation in holographic superconductors

- Homes' relation

Zaanen

$$\rho_s(0) = C\sigma_{DC}(T_c)T_c$$

Standard laws $\rho_s \sim \omega_{p,s}^2$

$$\sigma_{DC} \sim \omega_{p,n}^2 \tau$$

Tanner's law $\omega_{p,s}^2 \sim \omega_{p,n}^2$ (empirical)

Critical scaling $\tau \sim \frac{\hbar}{kT}$ gives $C \sim 1$

- Observed

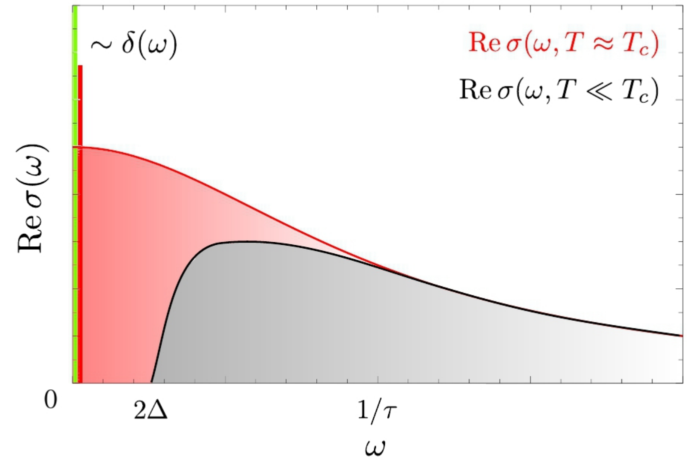
Homes et al

$$C_{exp} \sim 4.4$$

Homes' relation in holographic superconductors

- Canonical holographic superconductor: Two problems

  - Translation invariance



$$Re\sigma_n = K\delta(\omega) + \ldots\,, \quad Re\sigma_s = (K' + \rho_s)\delta(\omega) + \ldots$$

  - Scale invariant hyperscaling violating sector

    not all charge carriers are condensed at finite T

    Goldstone $\sim \mathcal{O}(1)$ Geometry $\sim \mathcal{O}(N^2)$

  - Needed: insulator with broken translation invariance

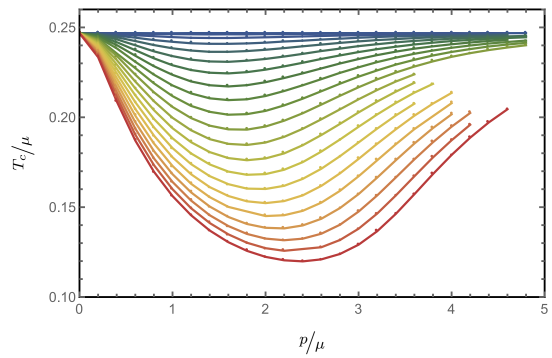
Homes' relation in holographic superconductors

- Donos Hartnoll plus charged scalar

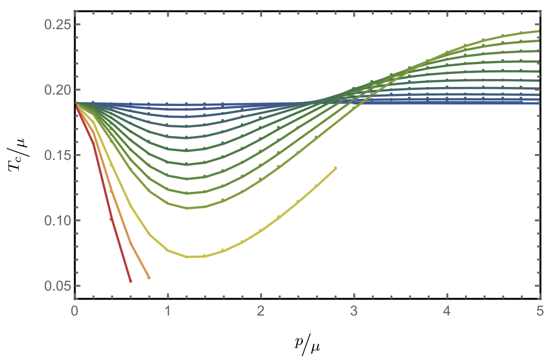
Erdmenger, Herwerth, Klug, Meyer, Schalm

$$S=\int d^5x\sqrt{-g}\left(R+12-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}W_{\mu\nu}W^{\mu\nu}-\frac{m^2}{2}B_\mu B^\mu\right)-\frac{\kappa}{2}\int B\wedge F\wedge W-|(\partial_\mu-2iqA_\mu)\eta)|^2$$

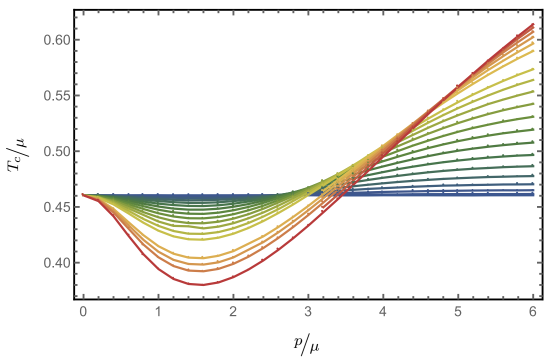
$\kappa = 0$ & $q = 6$



$\kappa = 1/\sqrt{2}$ & $q = 5$

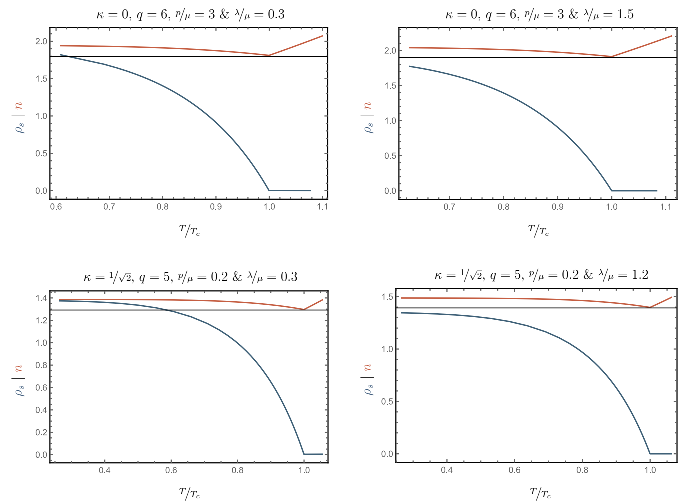


$\kappa = 1/\sqrt{2}$ & $q = 10$



Homes' relation in holographic superconductors

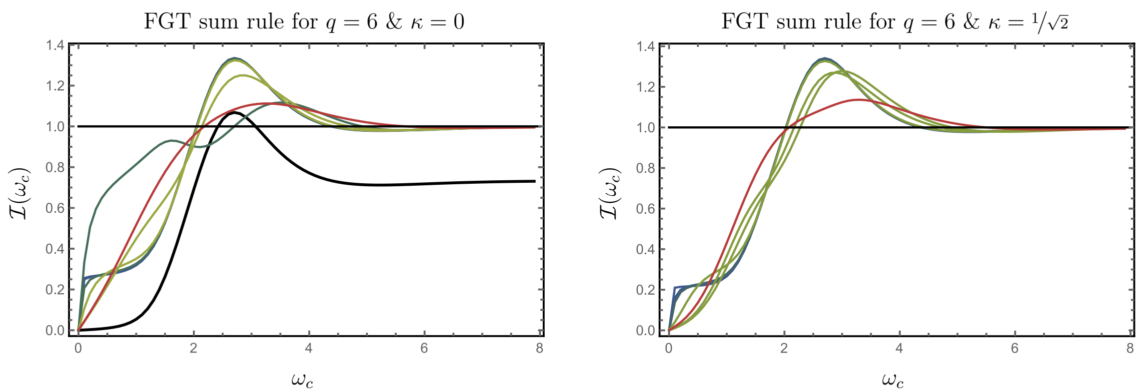
- All d.o.f. condense



Extracted from matching the optical conductivity to a two-fluid model

$$\mathrm{Re}\,\sigma(\omega) = \sigma^{\mathrm{reg}}(\omega) + \rho_s \delta(\omega) = \left(\chi_n(T) \frac{\tau}{1+\omega^2\tau^2} + \frac{\pi}{2} \chi_s(T) \delta(\omega) \right),$$

Sum rule holds



Homes' relation in holographic superconductors

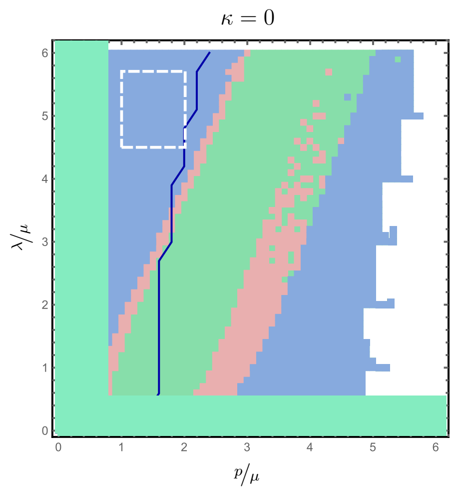
- Testing Homes' relation
  - Careful: never works with weak momentum relaxation

$$\rho_s(0) = C\sigma_{DC}(T_c)T_c$$

There is another scale; nothing universal

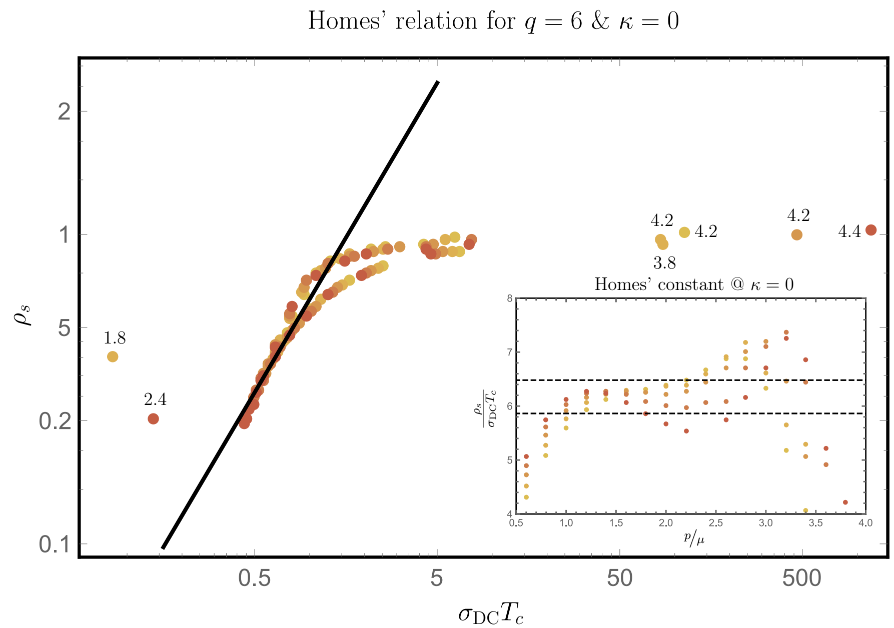
$$\sigma_{DC} \sim \tau_{relax}(\lambda, p)$$

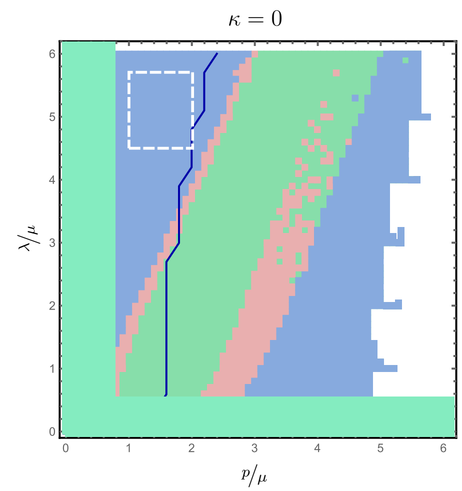
  - Careful: Bianchi VII model has some peculiarities



Homes' relation in holographic superconductors

- For a certain parameter regime Homes' relation holds

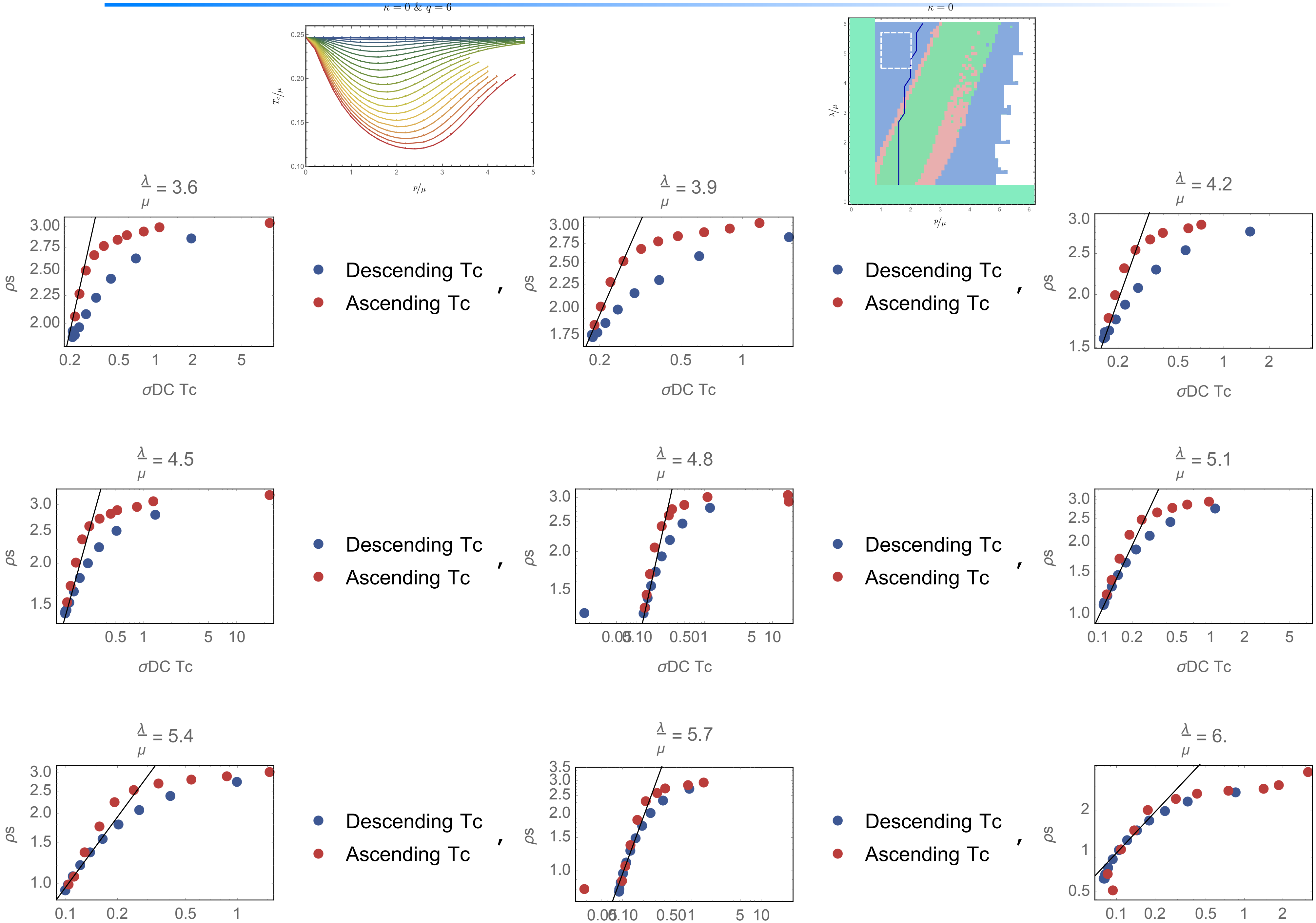




$$\rho_s(0) = C \sigma_{DC}(T_c) T_c$$

$$C \sim 6.2$$

Homes’ relation in holographic superconductors



- Conclusion

Homes' relation appears to hold in the regime of strong momentum relaxation (with some caveats)

Holographic evidence for universal physics?

- Increasing E_{theor}

$$J_{exp} = \sigma E_{theor}$$

$$\sigma : \text{ seductivity}$$

$$\rho : \text{ resistance}$$

$$\sigma = \frac{1}{\rho}$$

What is a holographic state of matter

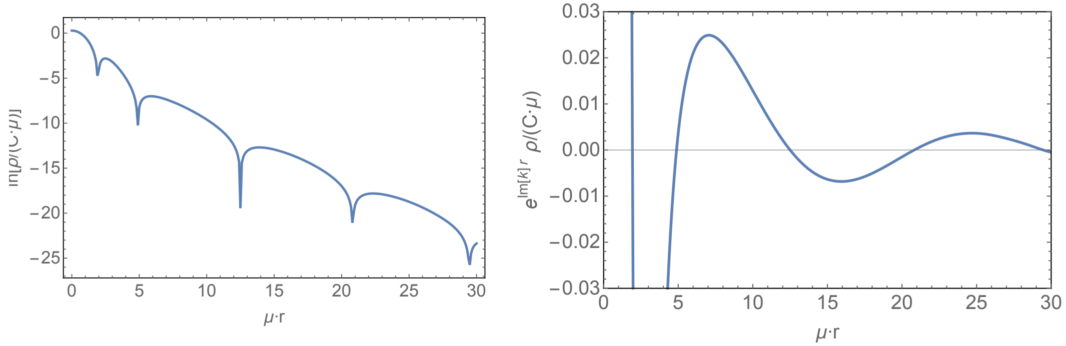
- Finite density holographic matter: Generalization of a Fermi liquid

Compressible $\partial_\mu F \sim \mu^\alpha$

Exhibits Fermi Surfaces $G \sim \dfrac{1}{\omega - v_F k + \omega^{2\nu}}$

Holographic Charge Oscillations

Blake, Donos, Tong



$$\delta\rho \sim \frac{e^{-r/\lambda}}{\sqrt{r}}\cos(r/\xi) \qquad r \gg R, T^{-1}, \mu^{-1}$$

$$\mu = \mu_0 + Ce^{-r^2/2R^2}$$

- Finite density holographic matter: Generalization of a Fermi liquid
  - Long range entangled

Huijse, Sachdev, Swingle

$$S_{EE} \sim (Lk_F)^{d-1} \ln(Lk_F)$$

Regular Fermi Liquid

$$S = Q^{\frac{d-1}{d}} A \ln(Q^{\frac{d-1}{d}} A)$$

Holography with $\theta = d - 1$

- Finite density holographic matter: Generalization of a Fermi liquid
  - Long range entangled

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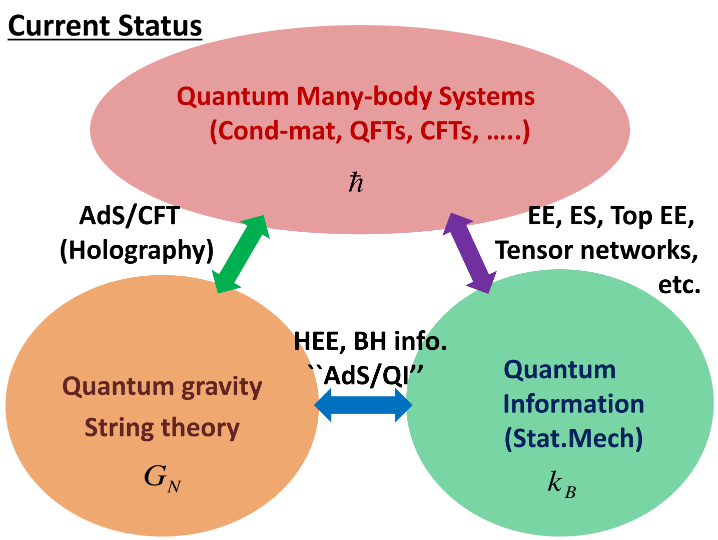
Holography with $\theta = d - 1$

- Takayanagi

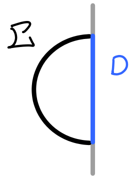


Entanglement and holography

- **Ooguri**

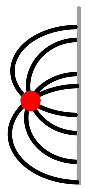
Entropy Inequalities in CFT

$\Longrightarrow$ Energy Conditions in Gravity



Radon Transform:

$$\int_\Sigma \varepsilon \sqrt{g_\Sigma} \Rightarrow S(\rho_1|\rho_0)$$



Inverse Radon transform:

$$\int \left(\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}-\frac{1}{R^2}\right) S(\rho_1|\rho_0)$$

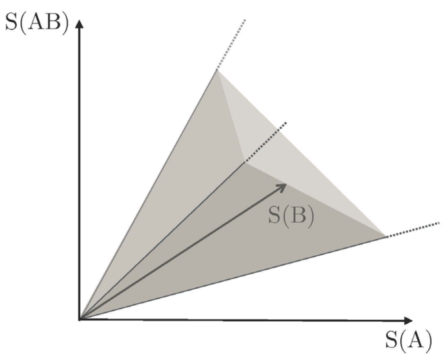
$$\Rightarrow \mathcal{E}(z,x)$$

☆ Bulk stress tensor near boundary can be diagnosed by boundary entanglement entropy.

☆ Entanglement inequalities on the boundary are (integrated) positive energy conditions in the bulk.

Smooth Holographic Dual

$\Longrightarrow$ Entropy Inequalities in CFT



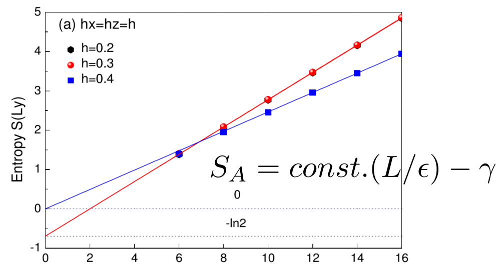
For any k and ℓ such that $n \geq 2k+\ell$

$$\sum_{i=1}^{n} S(A_i \cdots A_{i+k+\ell-1}) \geq \sum_{i=1}^{n} S(A_{i+\ell} \cdots A_{i+k+\ell-1}) + S(A_1 \cdots A_n)$$

- Ryu

  - Entanglement is a measure of a SPT

- (2+1)d Topologically ordered states

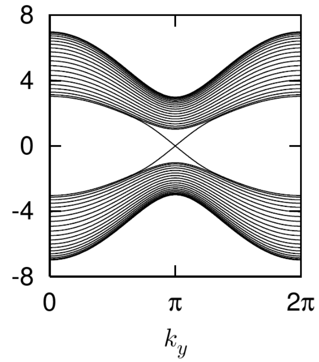


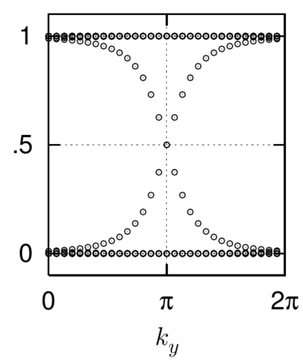
  - Entanglement spectrum

$$C_{IJ} = \langle \Psi_G | \psi_I^\dagger \psi_J | \Psi_G \rangle$$

$$\rho_L = \exp(-H_e) \qquad H_e = \sum_{IJ \in L} \psi_I^\dagger K_{IJ} \psi_J$$

$$K = \ln[(1 - C_L)/C_L]$$



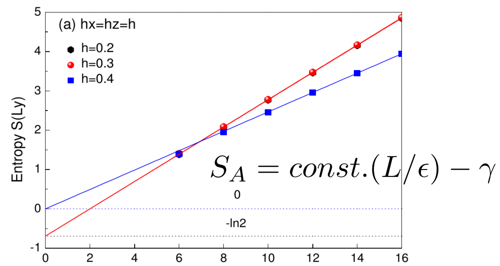


Physical spectrum v.s. (single-particle) ES

[SR-Hatsugai(06)]

- Ryu
  - Entanglement is a measure of a SPT

- (2+1)d Topologically ordered states



  - Entanglement spectrum and hidden susy

$$C_{IJ} = \langle \Psi_G | \psi_I^\dagger \psi_J | \Psi_G \rangle$$

$$\rho_L = \exp(-H_e) \qquad H_e = \sum_{IJ \in L} \psi_I^\dagger K_{IJ} \psi_J$$

$$K = \ln[(1 - C_L)/C_L]$$

- $C^2 = C$

$$\begin{aligned} C_L^2 - C_L &= -C_{LR} C_{RL}, \\ Q_L C_{LR} &= -C_{LR} Q_R, \\ C_{RL} Q_L &= -Q_R C_{RL}, \\ C_R^2 - C_R &= -C_{RL} C_{LR}, \end{aligned}$$

[Turner-Zhang-Vishwanath, Hughes-Prodan-Bervevig Chang-Mudry-SR]

$$Q_{IJ} := 1 - 2C_{IJ}$$

- Introduce

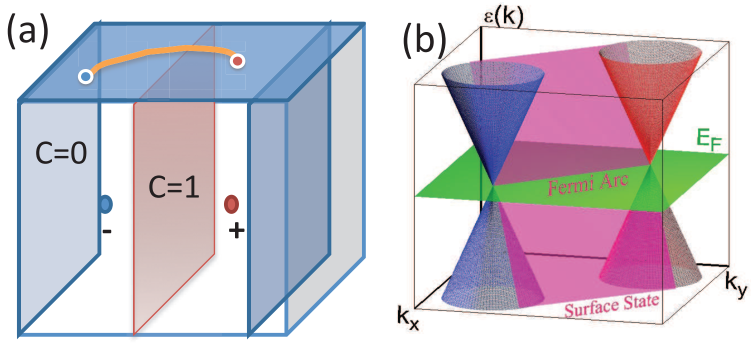
$$\mathcal{S} = \begin{pmatrix} 1 - Q_L^2 & 0 \\ 0 & 1 - Q_R^2 \end{pmatrix} \quad \mathcal{Q} = \begin{pmatrix} 0 & 2C_{LR} \\ 0 & 0 \end{pmatrix} \quad \mathcal{Q} = \begin{pmatrix} 0 & 0 \\ 2C_{RL} & 0 \end{pmatrix}$$

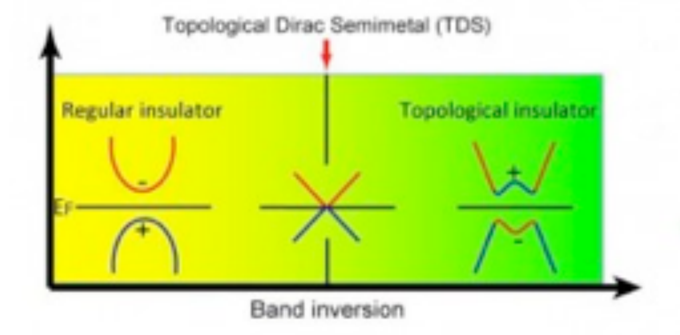
- SUSY algebra:

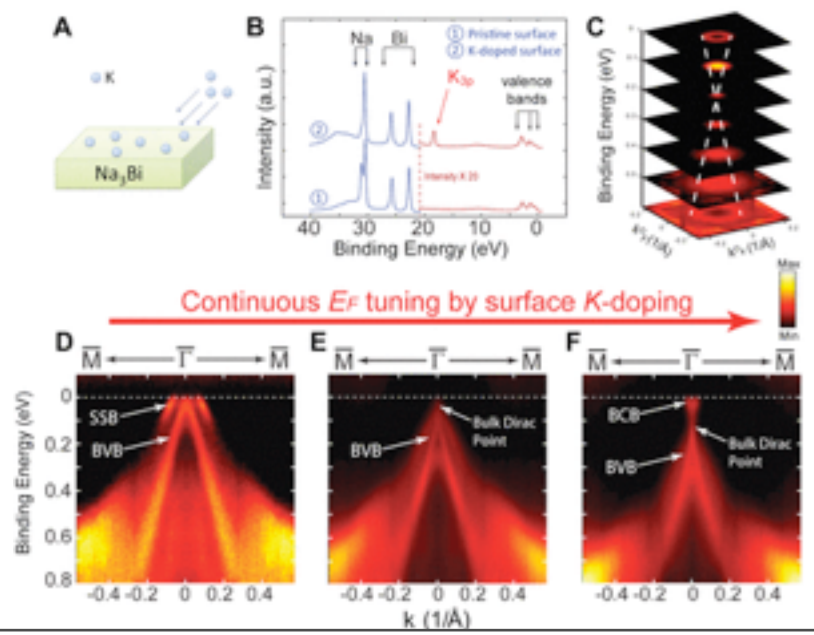
$$[\mathcal{S}, \mathcal{Q}] = [\mathcal{S}, \mathcal{Q}^\dagger] = 0,$$
$$\{\mathcal{Q}, \mathcal{Q}^\dagger\} = \mathcal{S}, \quad \{\mathcal{Q}, \mathcal{Q}\} = \{\mathcal{Q}^\dagger, \mathcal{Q}^\dagger\} = 0.$$

- Fang

Weyl semimetal







Magnetic monopole
in momentum space

Magnetic Monopoles:

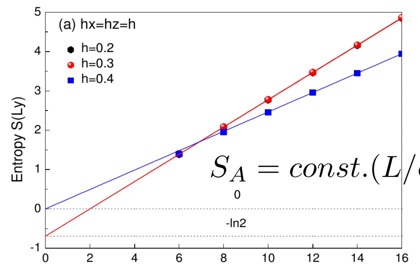
$$\vec{\Omega}(k) = \vec{\nabla}_k \times \vec{A}(k) = \pm \frac{\vec{k}}{2|k|^3} \qquad \vec{\nabla}\cdot\vec{\Omega} \neq 0$$

$$\frac{1}{2\pi}\oint_S \vec{\Omega}(k)\cdot dS(k) = Q$$ magnetic Charge

Fang, Science (2003).

- Ryu
  - Entanglement is a measure of a SPT

- (2+1)d Topologically ordered states



$$S_A = const.(L/\epsilon) - \gamma$$

  - Entanglement spectrum and hidden susy

$$C_{IJ} = \langle \Psi_G | \psi_I^\dagger \psi_J | \Psi_G \rangle$$

$$\rho_L = \exp(-H_e) \qquad H_e = \sum_{IJ \in L} \psi_I^\dagger K_{IJ} \psi_J$$

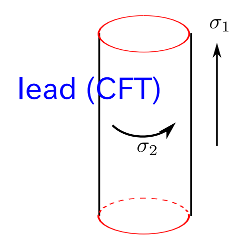
$$K = \ln[(1 - C_L)/C_L]$$

[Work in progress with Gil Young Cho and Andreas Ludwig]

- Ideal lead obeys B.C. set by SPT

$$\Phi(\sigma_2) - U \cdot \Phi(\sigma_2) = 0$$

$$[\Phi(\sigma_2) - U \cdot \Phi(\sigma_2)] |B\rangle = 0$$



- Symmetry G acts on fundamental fields

$$\mathcal{G} \cdot \Phi(\sigma_2) \cdot \mathcal{G}^{-1} = U_G \cdot \Phi(\sigma_2)$$

- B.C. is invariant under G:

$$\mathcal{G}\,[\Phi - U \cdot \Phi]\,\mathcal{G}^{-1} = U_G \cdot \Phi - U_G \cdot U \cdot \Phi$$

- But boundary state may not be:

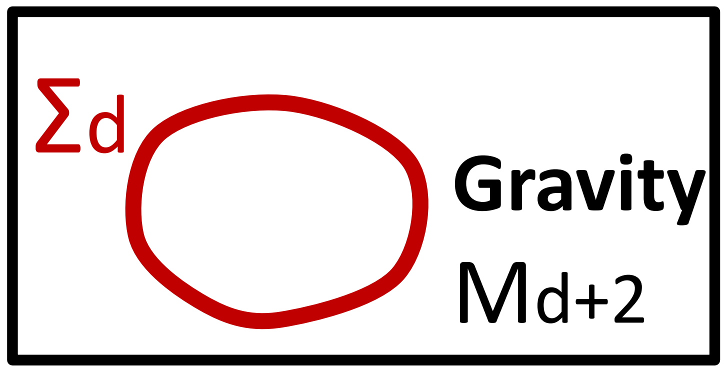
$$\mathcal{G} \cdot |B\rangle \neq |B\rangle$$

- Takayanagi

Σ : an d dim. convex space-like surface in M which is closed and homologically trivial

$$\left|\Phi(\Sigma)\right\rangle \in H_M$$

A pure state



- Takayanagi

$$\underbrace{|\Phi(u)\rangle}_{\text{State at scale u}} = P \cdot \exp\left(-i\int_{u_{IR}}^{u} ds\, \hat{K}(s)\right) \cdot \underbrace{|\Omega\rangle}_{\text{IR state}} .$$

$$u_{IR} = -\infty$$

$$|\Psi_m(t=0)\rangle = |\Omega\rangle = |B\rangle.$$

$$|\Omega_m\rangle \propto e^{-H/m} \cdot |B\rangle \;.$$

$$1 - |\langle\Phi(u)|\Phi(u+du)\rangle| = (du)^2 \cdot G_{uu}^{(B)}$$

Applied to AdS/CFT

$$ds^2 = \frac{c}{\varepsilon^3}(d\rho^2 + \sinh^2\rho d\phi^2).$$

$$\approx c^2 \quad \text{(as in AdS/CFT) if } \varepsilon \approx c^{-1/3}$$

- Takayanagi

$$\underbrace{\left|\Phi(u)\right\rangle}_{\text{State at scale u}} = P\cdot\exp\left(-i\int_{u_{IR}}^{u} ds\, \hat{K}(s)\right)\cdot \underbrace{\left|\Omega\right\rangle}_{\text{IR state}} .$$

$$u_{IR} = -\infty$$

$$\left|\Psi_m(t=0)\right\rangle = \left|\Omega\right\rangle = \left|B\right\rangle.$$

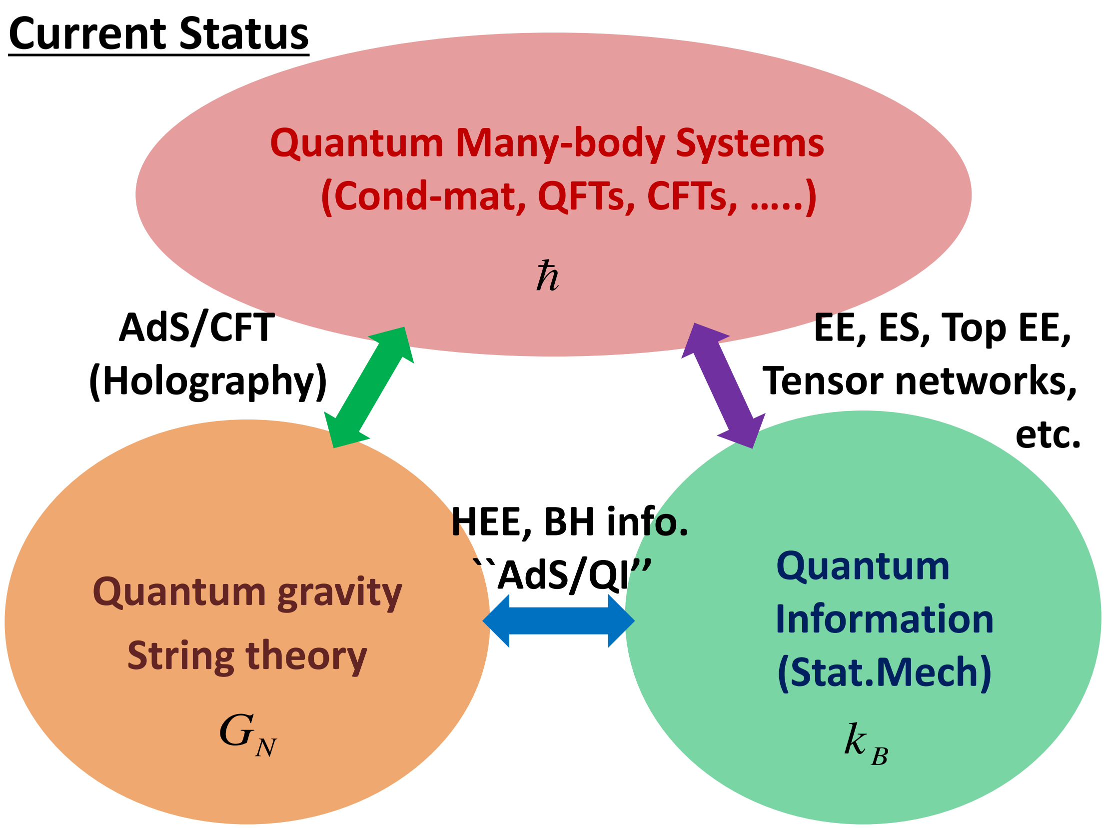
$$\left|\Omega_m\right\rangle \propto e^{-H/m}\cdot\left|B\right\rangle .$$

$$1 - \left|\left\langle\Phi(u)\right|\Phi(u+du)\right\rangle\right| = (du)^2\cdot G_{uu}^{(B)}$$

Quantum entanglement represents a geometry of quantum state in many-body systems.

- This gravity/entanglement duality looks more general than AdS/CFT and even than holography.

- Takayanagi

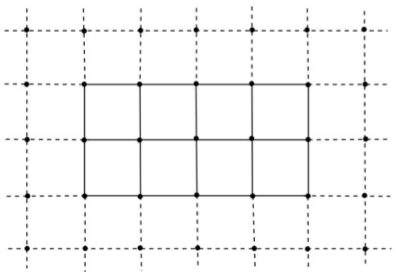


Current Status
Quantum Many-body Systems
(Cond-mat, QFTs, CFTs, ...)
$\hbar$
AdS/CFT
(Holography)
EE, ES, Top EE,
Tensor networks,
etc.
HEE, BH info.
``AdS/QI''
Quantum gravity
String theory
G_N
Quantum
Information
(Stat.Mech)
k_B

- Trivedi

Hilbert space of states does not admit a tensor product decomposition between $\mathcal{H}_{in}, \mathcal{H}_{out}$

Entanglement Entropy In A Z_2 Gauge Theory :



$$\rho = \sum_{\mathbf{k}} \rho^{\mathbf{k}}$$

$$\rho^{\mathbf{k}} = Tr_{\mathcal{H}^{\mathbf{k}}_{ginv,out}} |\psi >< \psi|$$

$$S_{EE} = -\sum_{\mathbf{k}} Tr_{\mathcal{H}^{\mathbf{k}}_{ginv,in}} \rho^{\mathbf{k}} \log(\rho^{\mathbf{k}})$$

k sums all possible flux configurations across the surface

- Can be readily extended to (non)-Abelian gauge theories

Analogue spacetimes

- Visser

Key point:

- Event horizons are not physically observable...
- Apparent/trapping horizons are physically observable...

- More seduction

$$J_{exp} = \sigma E_{theor}$$

$$\sigma : \text{ seductivity}$$

$$\rho : \text{ resistance}$$

$$\sigma = \frac{1}{\rho}$$

Holography for non-equilibrium physics

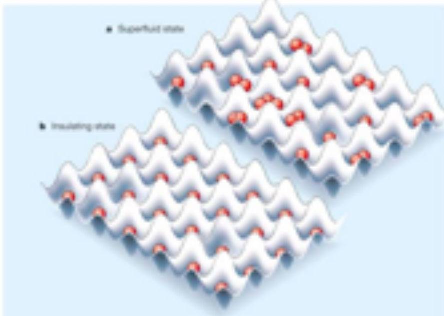
AdS/CFT has a unique ability to compute real-time physics (at finite temperature/density)

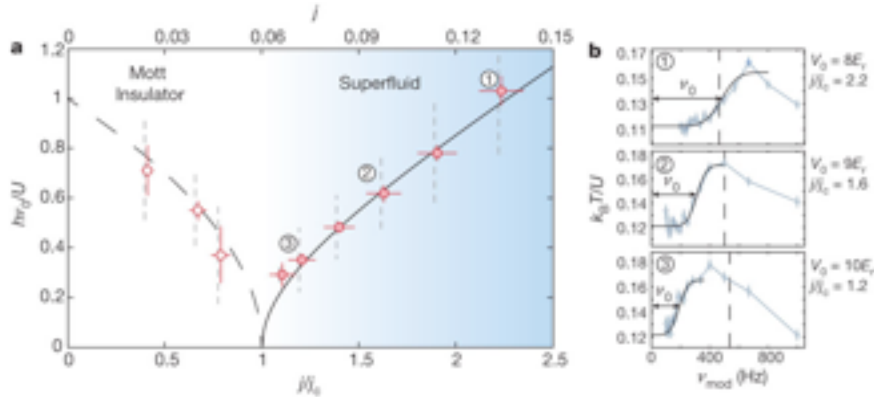
- *Real-time dynamics*
- *Full non-equilibrium and transition to hydro*
- *Strongly coupled systems, especially critical theories*

New organizing principles out of equilibrium

Cold Atom Experiments

- Takahashi
  - Superfluid-Insulator transition



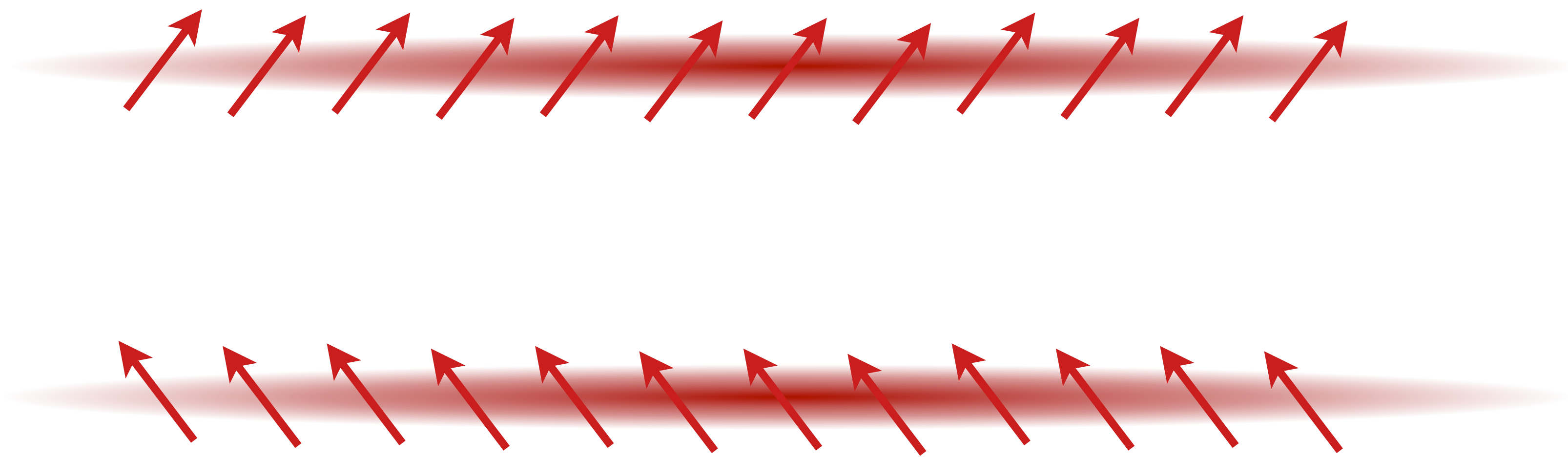


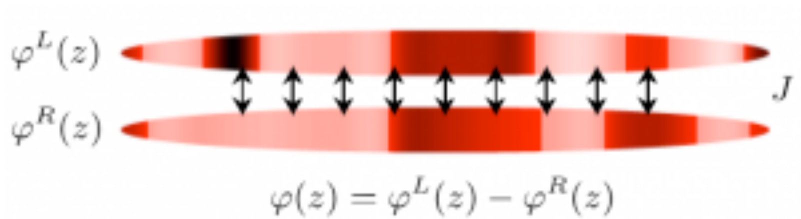
Bloch et al

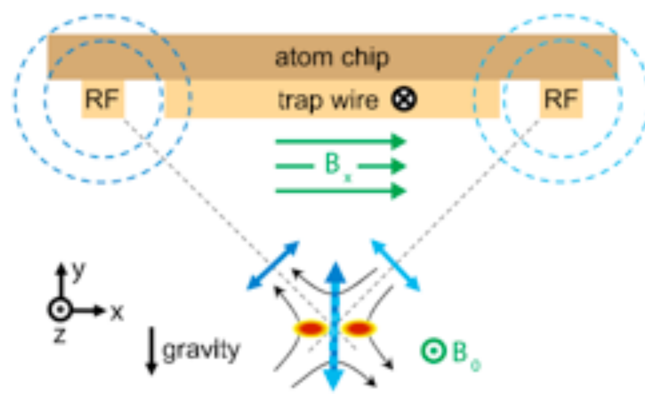
Cold Atom Experiments

- Phase quenches in a BEC

Schmiedmayer et al



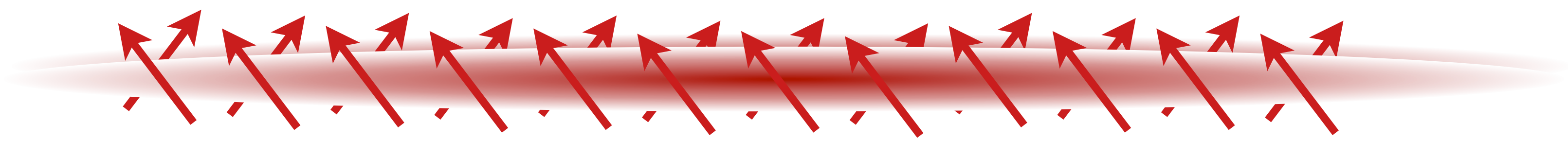


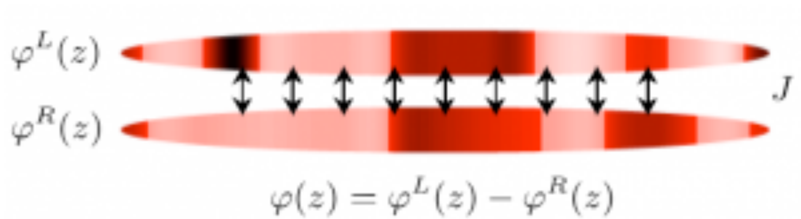


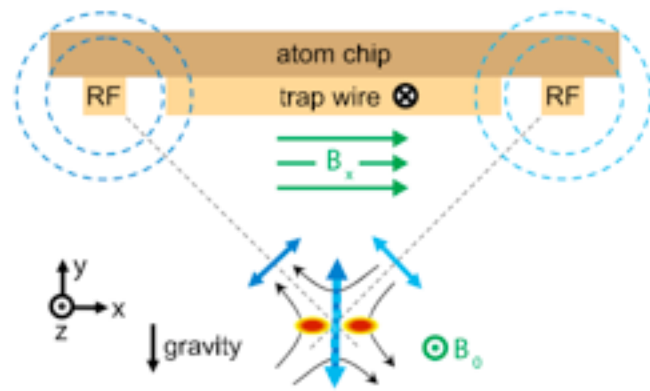
Cold Atom Experiments

- Phase quenches in a BEC

Schmiedmayer et al



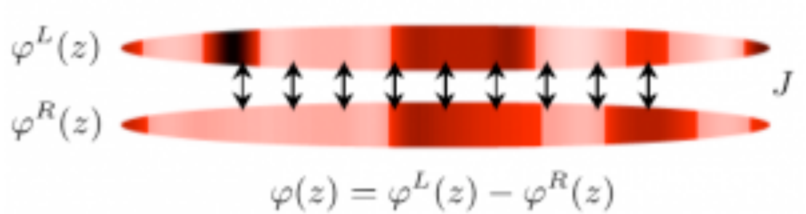


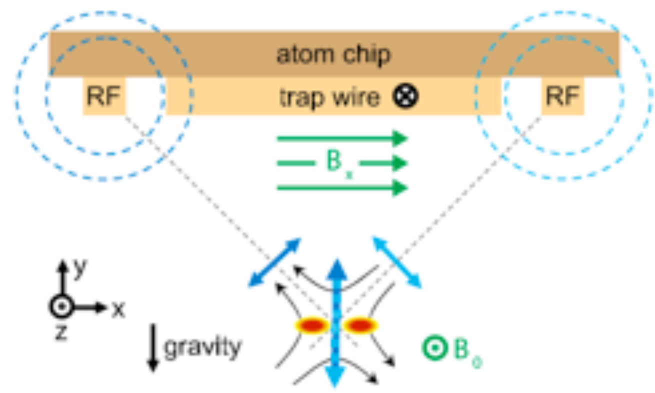


Cold Atom Experiments

- Phase quenches in a BEC

Schmiedmayer et al





Universal results in quench dynamics

- Das
    - slow (Kibble-Zurek) vs instantaneous (Calabrese Cardy) quench
    - fast quench has universal scaling

$$\delta t \ll \lambda_0^{-\frac{1}{d-\Delta}}$$

$$\langle \mathcal{O}_\Delta \rangle_{ren} \sim (\delta t)^{d-2\Delta}$$

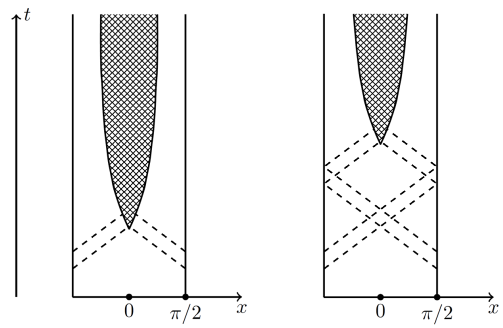
$$\langle \mathcal{E} \rangle_{ren} \sim (\delta t)^{d-2\Delta}$$

- First obtained in holography, now proven for general theories

$$\Lambda_{UV}^{-1} \ll \delta t \ll (\delta\lambda)^{-\frac{1}{d-\Delta}}$$

- Universal scaling regime should be detectable in correlation functions

- Craps
  - BH instabilities in AdS

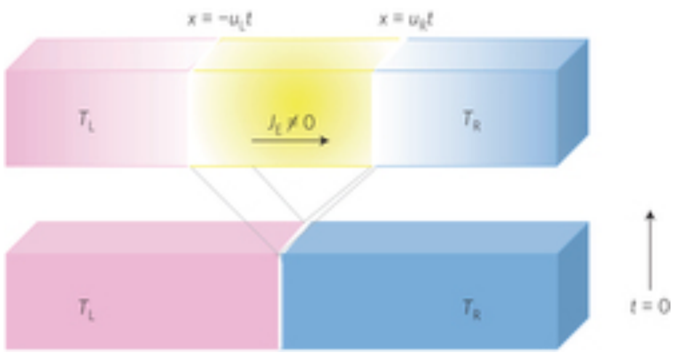


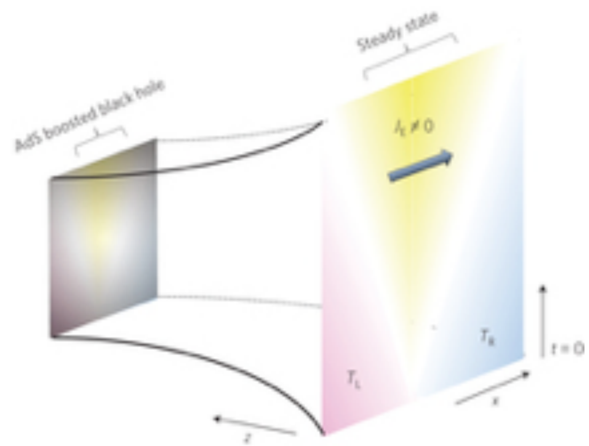
For initial conditions $\phi, \dot{\phi} \sim \epsilon$, time scale for collapse $\sim 1/\epsilon^2$

  - Secular terms invalidate perturbation expansion
  - Resummation leads to a controlled analytic framework
  - Answer the question of BH formation (or not!)

Universal results in far-from-equilibrium

- Bhaseen





  - Universal non-equilibrium steady state

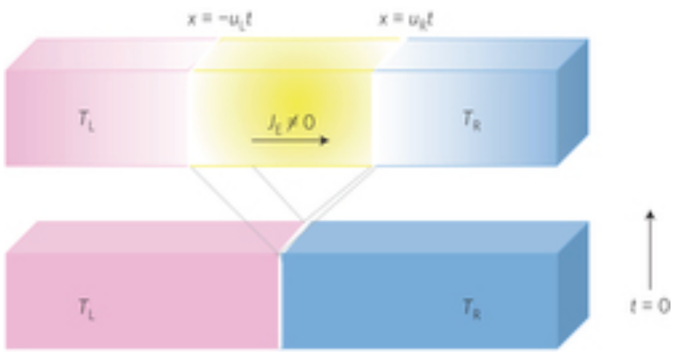
$$\langle T^{tx}\rangle = a_d\left(\frac{T_L^{d+1} - T^R_{d+1}}{u_L + u_R}\right) \qquad \chi = \left(\frac{T_L}{T_R}\right)^{\frac{d+1}{2}}$$

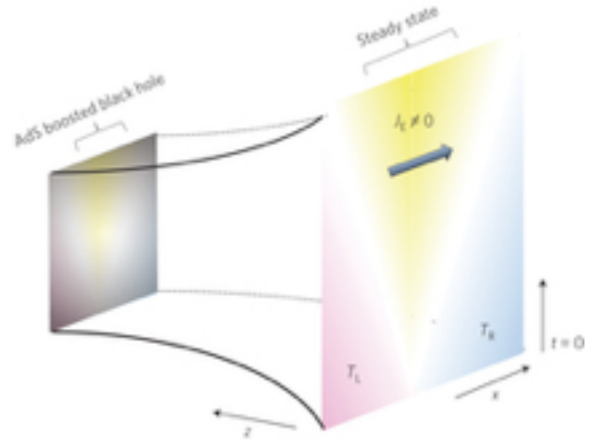
$$T_{ss} = \sqrt{T_L T_R} \qquad u_L = \frac{1}{d}\sqrt{\frac{\chi + d}{\chi + d^{-1}}} \qquad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}} \qquad \eta_{ss} = \frac{\chi - 1}{\sqrt{(\chi + d^{-1})(\chi + d)}}$$

  - Separated by non-linear sound waves with speeds $u_L,\ u_R$

Universal results in far-from-equilibrium

- Bhaseen





  - Universal Fluctuations

$$\langle J^{n+1} \rangle = \frac{d^n}{d\mu^n} \, J(\beta_L - \mu, \beta_R + \mu) \Big|_{\mu=0}$$

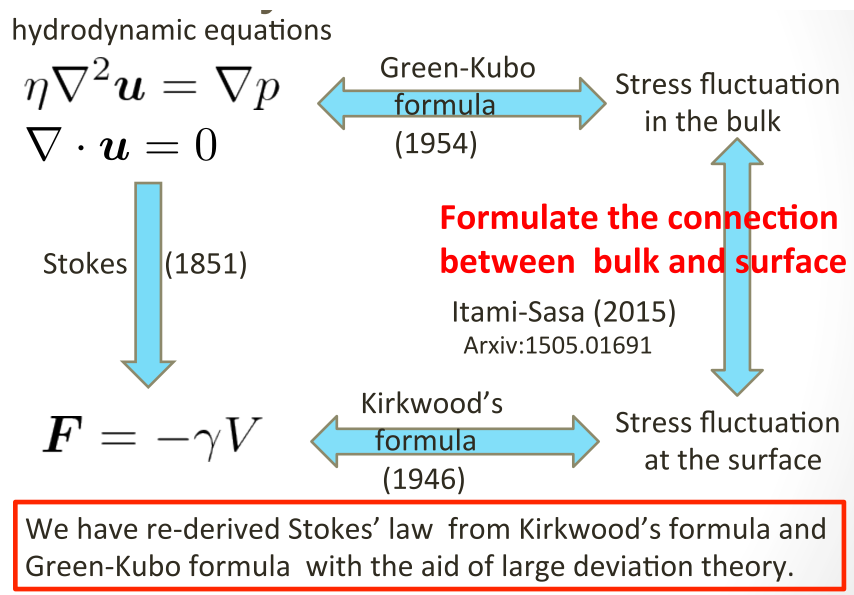
- Sasa
  - Large fluctuation theorem (beyond hydrodynamics)

$$P(\bar{F}^z; V) \simeq e^{-\tau I(\bar{F}^z; V)}$$

Symmetry

$$I(\bar{F}^z; V) - I(-\bar{F}^z; V) = \frac{\bar{F}^z V}{k_{\mathrm{B}} T}$$

  - Application to Stokes Law



Non-equilibrium holography as the benchmark

- Witczak-Krempa

❖ **Corner** entanglement is a useful measure of dof

❖ In **smooth limit**, fixed by $T_{\mu\nu}$ central charge

$$a(\theta \to \pi) = \sigma\,(\theta - \pi)^2 \qquad \boxed{\sigma = \frac{\pi^2}{24} C_T}$$

❖ Does pure Einstein holography give a **lower bound** for $a(\theta)/C_T$?

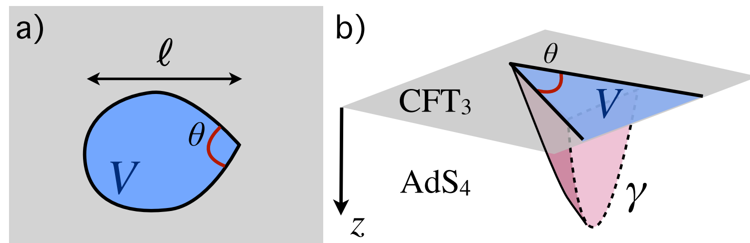
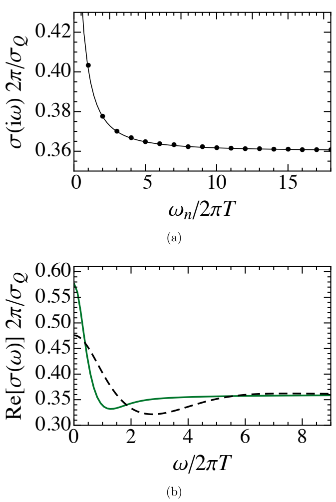
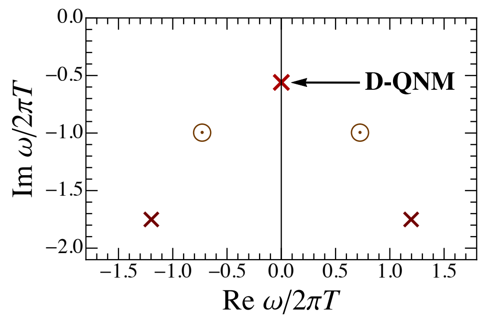


FIG. 1: a) An entangling region V of size ℓ with a corner; b) The holographic entangling surface γ for a region on the boundary of AdS_4 with a corner.





❖ Quantum critical dynamics (CFTs) in 2+1D

❖ OPE to constrain *short time* physics

  ❖ large ω conductivity $\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^{\Delta} + b_2 \left(\frac{T}{\omega_n}\right)^{3} + \cdots$

❖ Input OPE data of CFT into simple holographic ansatz

❖ Can match Monte Carlo data of O(2) CFT w/out unphysical tweaks

$$J_{exp} = \sigma E_{theor}$$

σ : seductivity

ρ : resistance

$$\sigma = \frac{1}{\rho}$$

- Physics Today 2013 (Anderson)

"As a very general problem with the AdS/CFT approach in condensed-matter theory, we can point to those telltale initials "CFT"—conformal field theory. Condensed-matter problems are, in general, neither relativistic nor conformal. Near a quantum critical point, both time and space may be scaling, but even there we still have a preferred coordinate system and, usually, a lattice. There is some evidence of other linear-T phases to the left of the strange metal about which they are welcome to speculate, but again in this case the condensed-matter problem is overdetermined by experimental facts."

0. Linear-in-T resistivity

1. Power law in AC conductivity

2. Hall angle vs conductivity scaling

3. Inverse Matthiessen law

4. Lots of power law scaling

- Physics Today 2013 (Anderson)

"[Where AdS/CFT fails]"

0. Linear-in-T resistivity

1. Power law in AC conductivity

2. Hall angle vs conductivity scaling

$$\sigma = \sigma_{ccs} + \sigma_{relax}$$

3. Inverse Matthiessen law

4. Lots of power law scaling

- IPMU 2015 (you)

5. Holographic states as an extension of topological states: Strange metals, black holes, long range entanglement

6. Holography: a unique window on non-equilibrium physics

Thank you

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