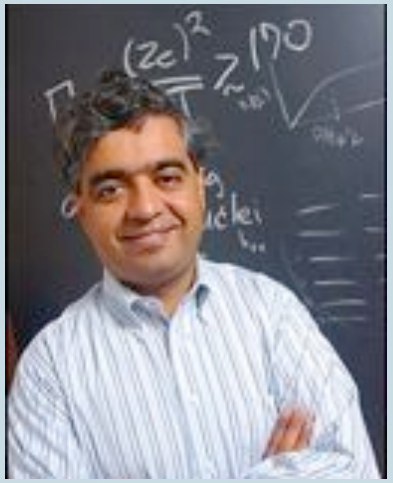
Dynamical & entanglement properties of CFTs
[2+1D]

William Witczak-Krempa
Perimeter Institute, Canada

AdS/CFT & CM workshop, IPMU, 29/05

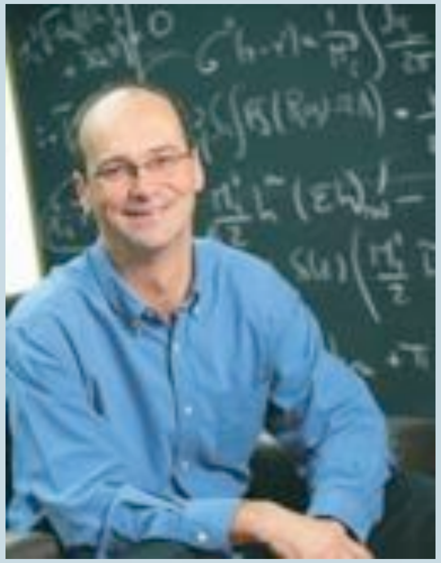


S. Sachdev
@Harvard / PI

E. Sorensen
@McMaster

E. Katz
@Boston U.

P. Bueno
@Madrid



R. Myers
@PI

Punch lines

focus: CFTs

- Super-universality of **entanglement entropy** from corners
- **Non-perturbative** results for dynamics at $T > 0$
- **Asymptotics** and **sum rules** of observables (conductivity)
- Concrete statements for the **superfluid insulator** quantum critical point in 2+1D [incl. large-scale Monte Carlo]

Plan

- Brief intro to quantum criticality
- Entanglement entropy (corner contribution)
- Quantum dynamics at $T > 0$

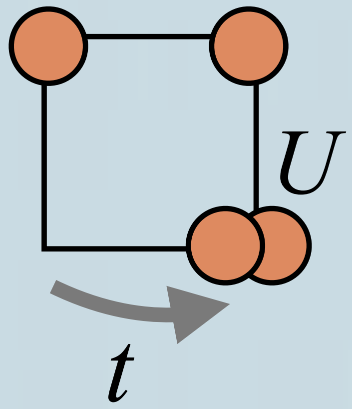
Quantum critical fluids

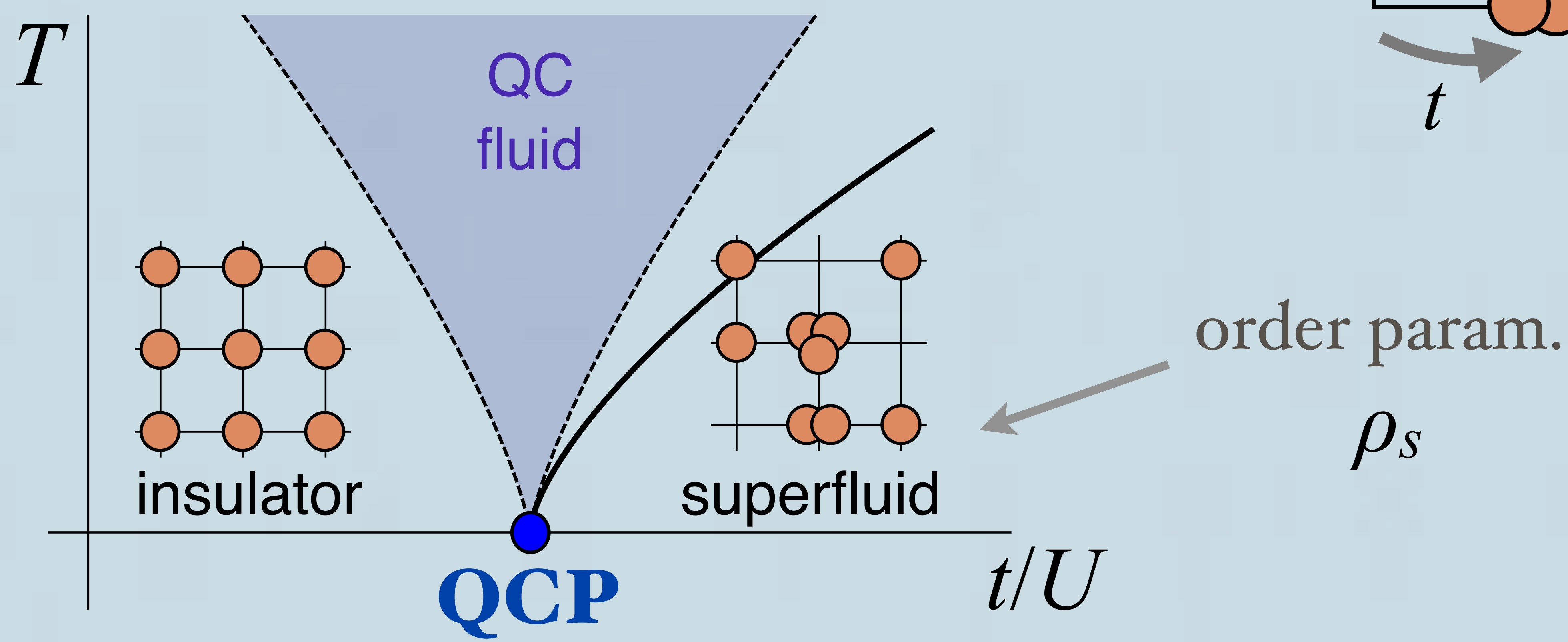
- (quasi)particles: ∞ lifetime (poles)

- Systems *without* quasiparticles:
  - Quantum critical phase transition
  - Strongly coupled gauge theories (e.g. gapless spin liquids), etc
- Focus on **Conformal Field Theories** ($z = 1$)

Superfluid-insulator transition

$$H = -t\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i - 1)$$



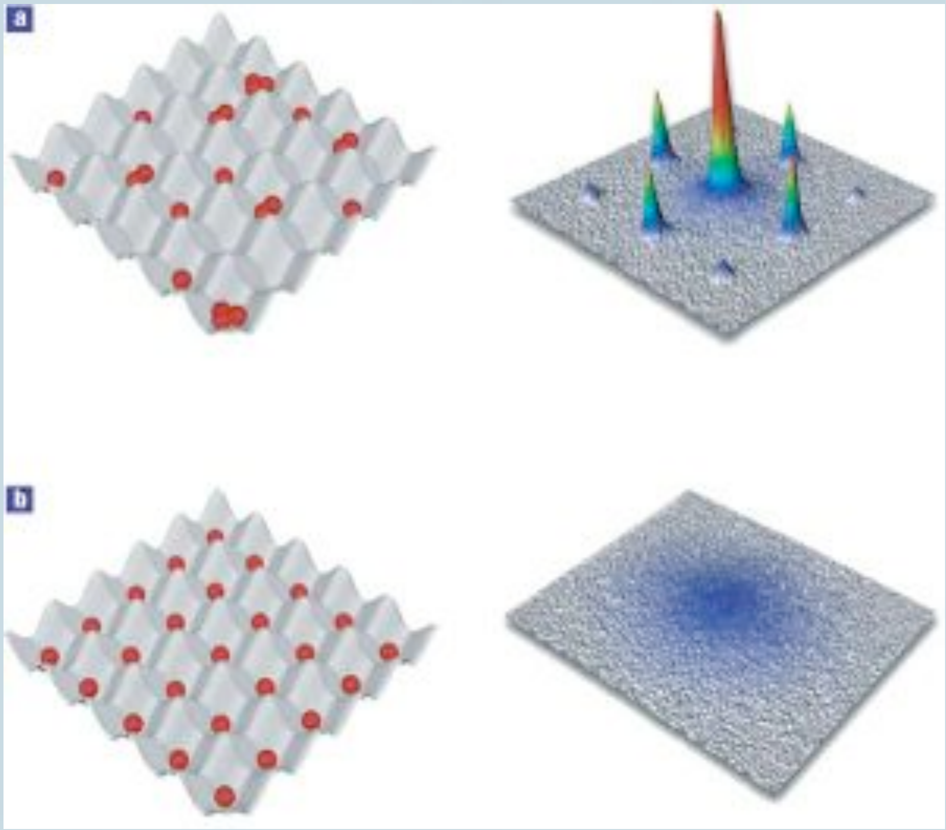


❖ O(2) **universality class**: XY spins, arrays of Josephson junctions, cold atoms, etc

- SSB of O(2) order parameter

$$\mathcal{L} = (\partial_t \vec{\phi})^2 + (\nabla \vec{\phi})^2 + m^2 \phi^2 + u \phi^4$$

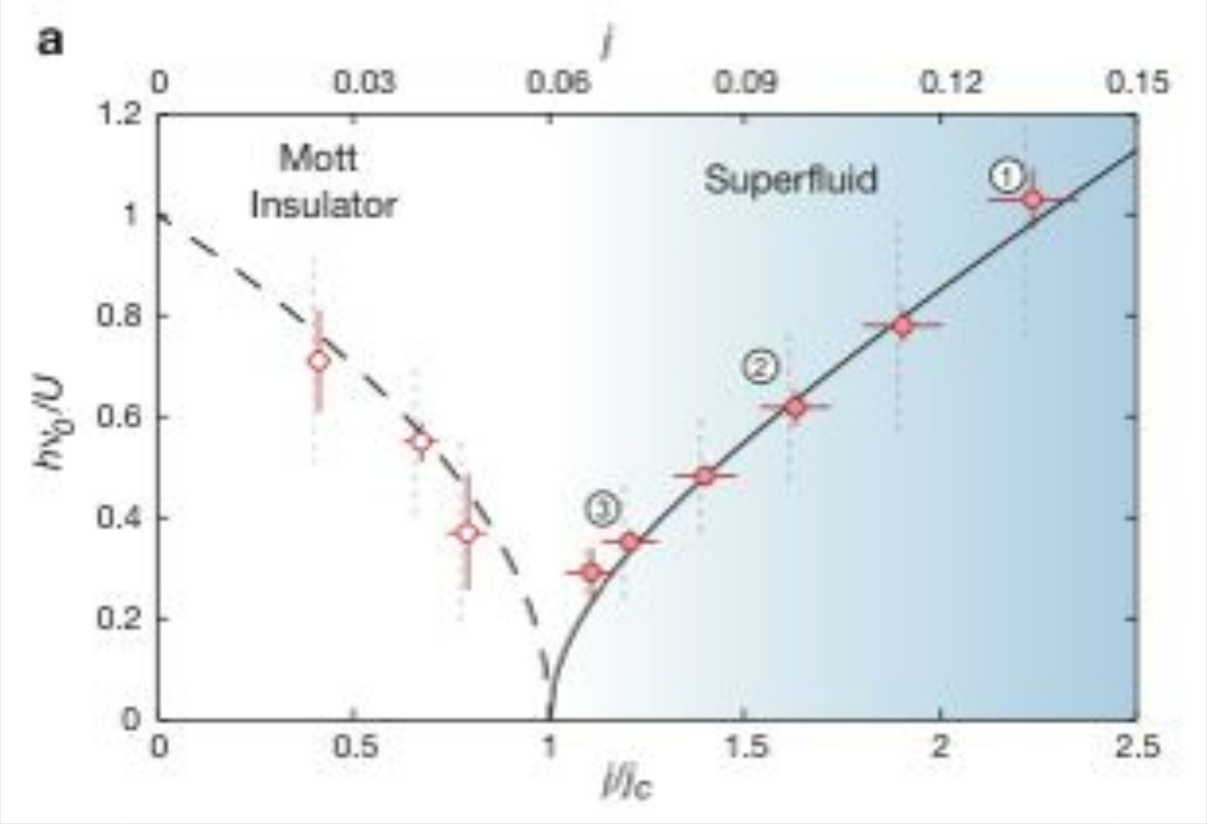
strongly coupled fixed point
in 2+1D (Wilson-Fisher)



Superfluid

Insulator

[Bloch]



^{87}Rb

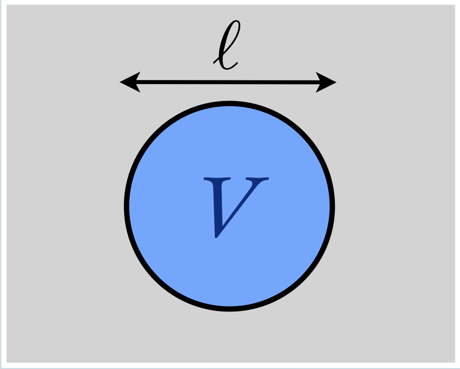
[Endres *et al*]

Entanglement
$T = 0$

P. Bueno, R. Myers, WK, 1505.04804

Counting non-quasiparticles

$$S = -\mathrm{Tr}\left(\rho_V \ln \rho_V\right)$$

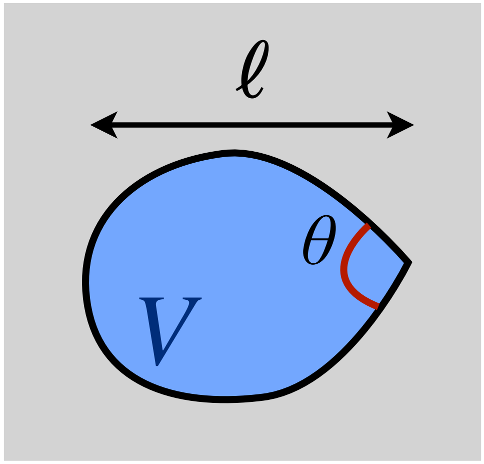


- Entanglement entropy
- RG monotone for CFTs (analog of *c* of 2D CFTs):
  [Myers *et al* ; Casini, Huerta]

$$S = B\,\frac{\ell}{\delta} - F + O(\delta/\ell)$$

- Hard to compute in many-body simulations,
  e.g. O(2) QCP

Corner entanglement

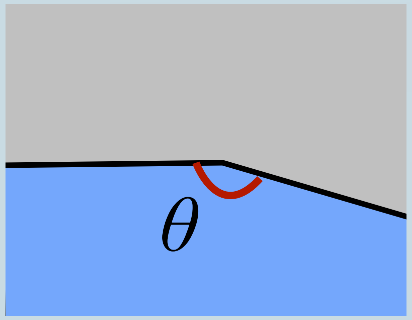


- V has corner w/ angle θ
- EE has **universal** log:

$$S = B\,\frac{\ell}{\delta} - a(\theta)\,\ln(\ell/\delta) + \text{const}$$

- $a(\theta)$ good measure of dof
  - free CFTs: $a = N\,a_{\text{scalar}}(\theta)$ [Casini, Huerta]
  - large-N super-conformal gauge th.: $a\,(\theta) \propto N^{3/2}$ [Hirata, Takayanagi]
- Numerics for O(N) Wilson-Fisher QCPs [Melko *et al*, Helmes *et al* ; Devakul *et al* ; etc]

Corner super-universality



- **Smooth** limit, $\theta \simeq \pi$

$$a(\theta \to \pi) = \sigma \, (\theta - \pi)^2$$

- Super-universality

$$\boxed{\sigma = \frac{\pi^2}{24} C_T}$$

$$\langle T_{\mu\nu}(x) T_{\lambda\rho}(0) \rangle = \frac{C_T}{x^6} \mathcal{I}_{\mu\nu\lambda\rho}(x)$$

- smooth limit determined by **local** correlator

Evidence for conjecture

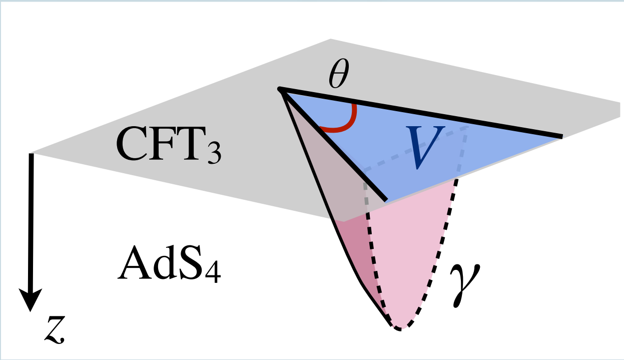
$$\sigma = \frac{\pi^2}{24} C_T$$

- Holds exactly for free theories
  ➞ predict new result (numerically checked)

$$\sigma_{\text{scalar}} = \frac{1}{256} = \sigma_{\text{Dirac}}/2$$

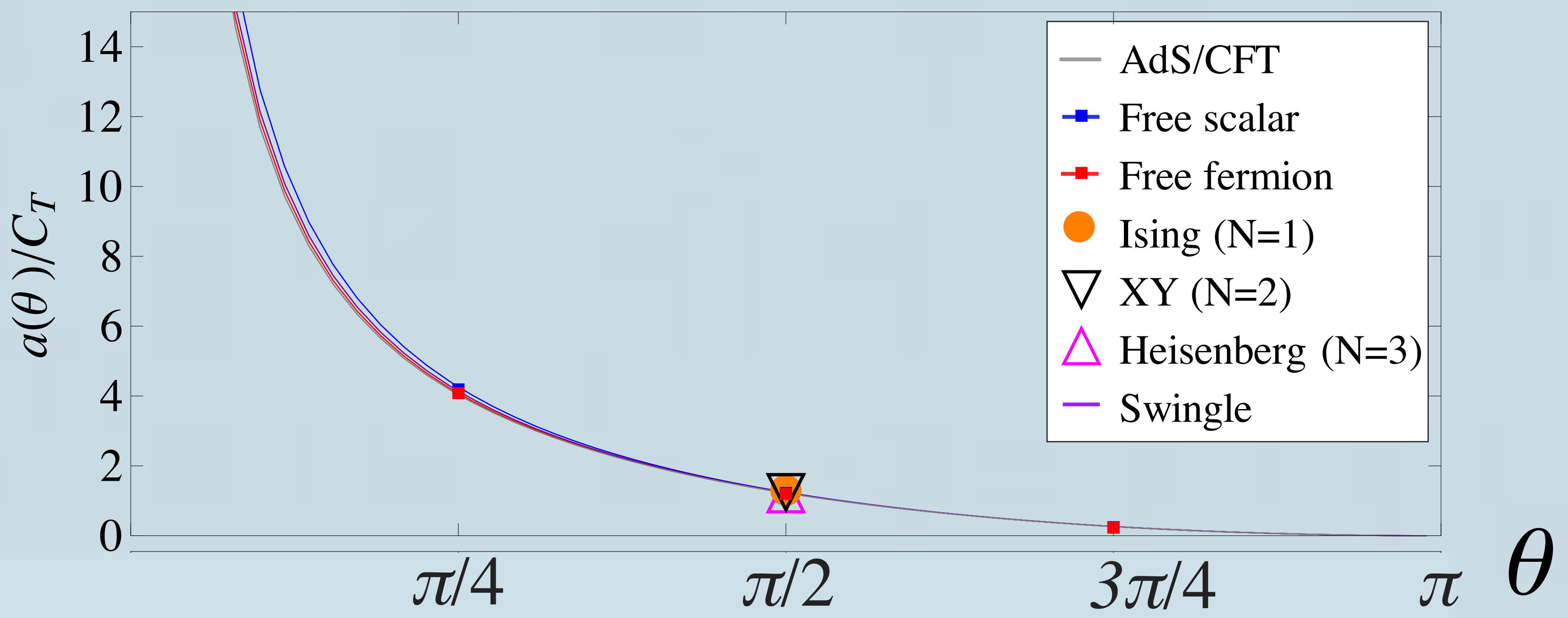
- Holds in holographic models with higher curvature terms [*à la* Ryu-Takayanagi]

$$S_{\text{bulk}} = \int_x \left[\frac{6}{L^2} + R + \lambda_1 R \mathcal{X}_4 + \lambda_2 \mathcal{X}_4^2 \right]$$



Beyond smooth limit

- Near super-universal behavior for many different CFTs



Beyond smooth limit

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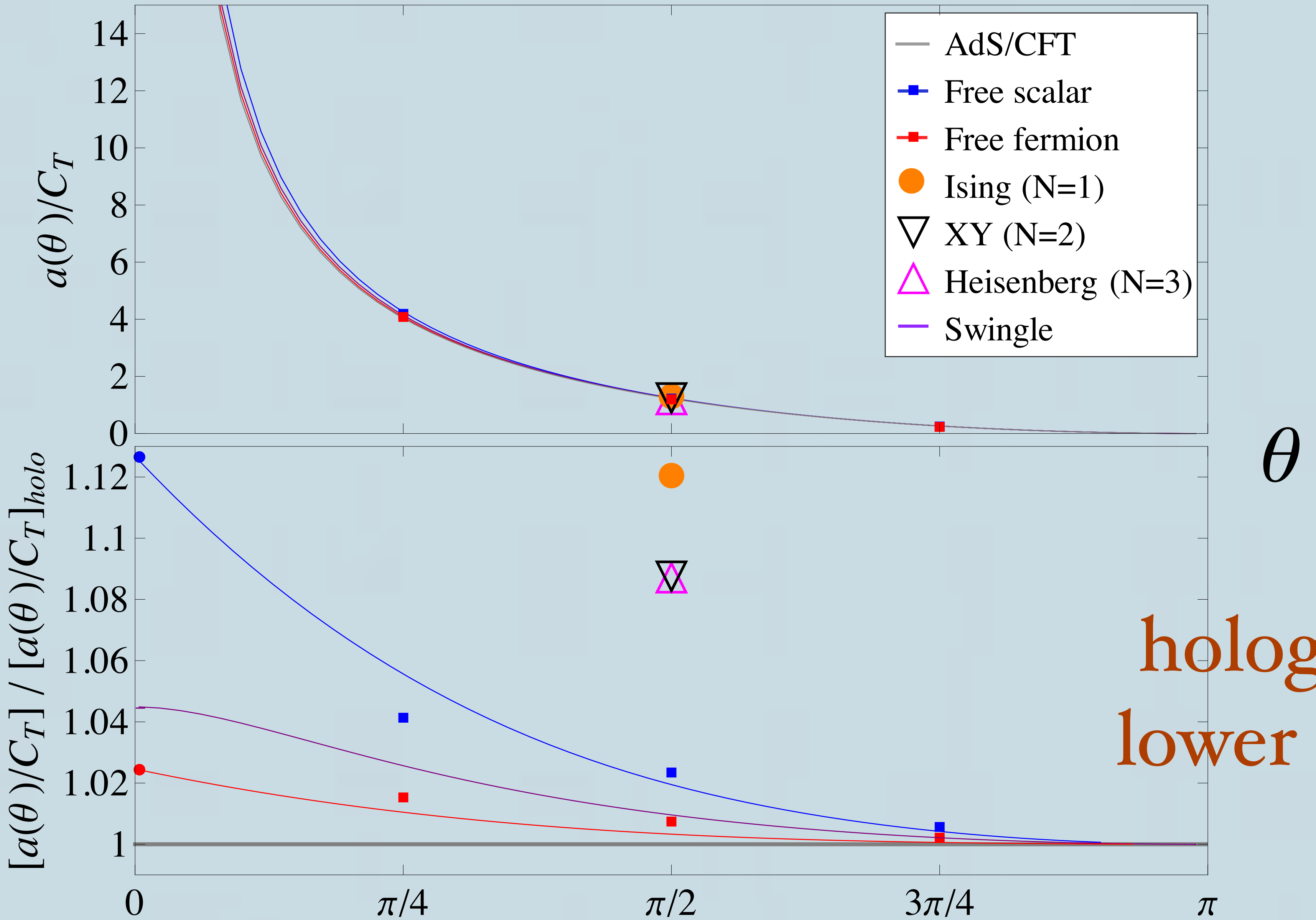


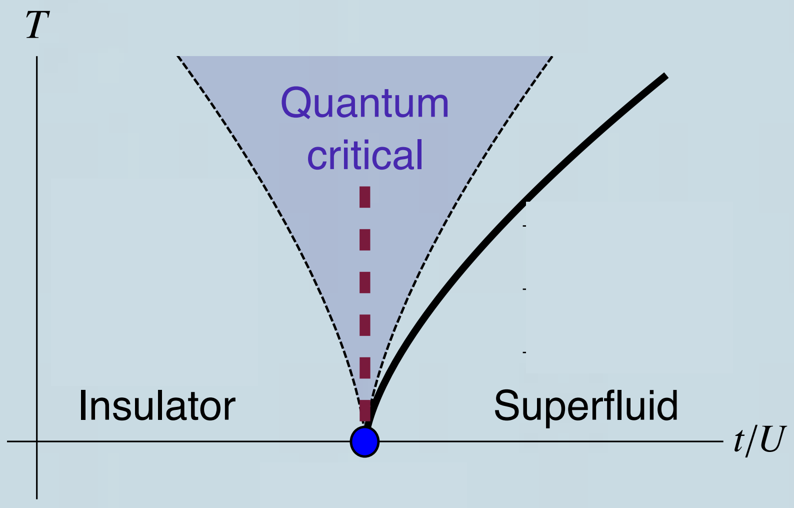
holographic lower bound!?

Quantum dynamics at $T > 0$

Universal conductivity

$$\partial_\mu J^\mu = 0$$

$$\sigma(\omega) = \frac{1}{i\omega}\langle J_x(\omega) J_x(-\omega)\rangle$$



Universal scaling function:

$$\sigma(\omega) = \Phi\left(\frac{\omega}{T}\right)$$

??

[early work: Damle, Sachdev]

Short & long times

- Characteristic time scale: $1 / T$
- **Short** times $\omega \gg T$: probing near vacuum
- **Long** times $\omega \gg T$: excitations interact strongly with thermal background

Short times via Operator Product Expansion

- A scalar primary op $O(x)$, w/ scaling dim Δ:

$$\langle O(x)\, O(0)\rangle = 1\, /\, x^{2\Delta}$$

- OPE:

$$\mathcal{O}(x)\,\mathcal{O}(0) \to \sum_n \frac{\mathcal{O}_n(0) + \text{desc.}}{x^{2\Delta-\Delta_n}}$$

[Wilson ; Polyakov; Ferrara *et al* ; etc]

Current OPE

$$J_\mu(x)J_\nu(0) = \frac{I_{\mu\nu}1}{x^{2\cdot 2}} + \mathcal{C}_{JJ\mathcal{O}}\frac{\tilde{I}_{\mu\nu}\,\mathcal{O}(0)}{x^{4-\Delta}} + \mathcal{C}_{JJT}\frac{T_{\mu\nu}(0)}{|x|} + \cdots$$

O(N) CFT:
relevant
scalar $\mathcal{O} \sim \varphi^2$

Stress tensor

- Get OPE coefficients from $\langle JJO \rangle_{T=0}$

Asymptotic conductivity

[WK, Sorensen, Sachdev, Katz]

- Fourier & take expectation value of OPE:

thermal average $\langle \mathcal{O} \rangle_T = B T^{\Delta}$

$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_O \left(\frac{T}{\omega_n}\right)^{\Delta} + b_T \left(\frac{T}{\omega_n}\right)^{3} + \cdots$$

relevant scalar O — stress tensor

Dominant operator?

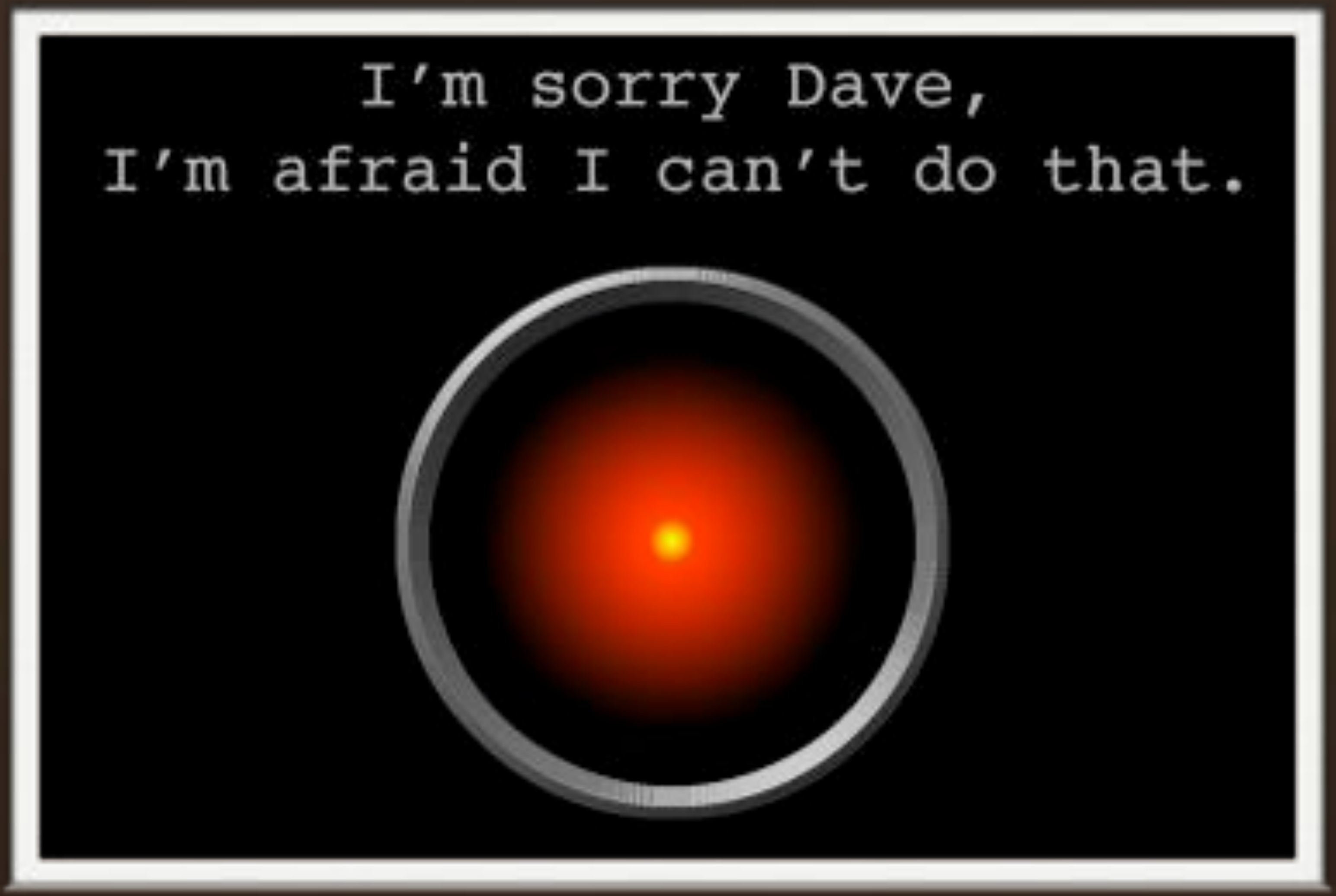
- O(N) Wilson-Fisher, it's the **scalar** O

$$O(x) = \varphi^2(x) \qquad \text{since} \qquad \Delta = 3 - 1/\nu$$

- $N = \infty$: $\Delta = 2$
  $N = 2$: $\Delta = 1.51$

- **All** CFTs have $1/\omega^3$ term because ***there's always a stress tensor***
  - sometimes dominates (Dirac CFT, QED3, etc)

Ask the computer



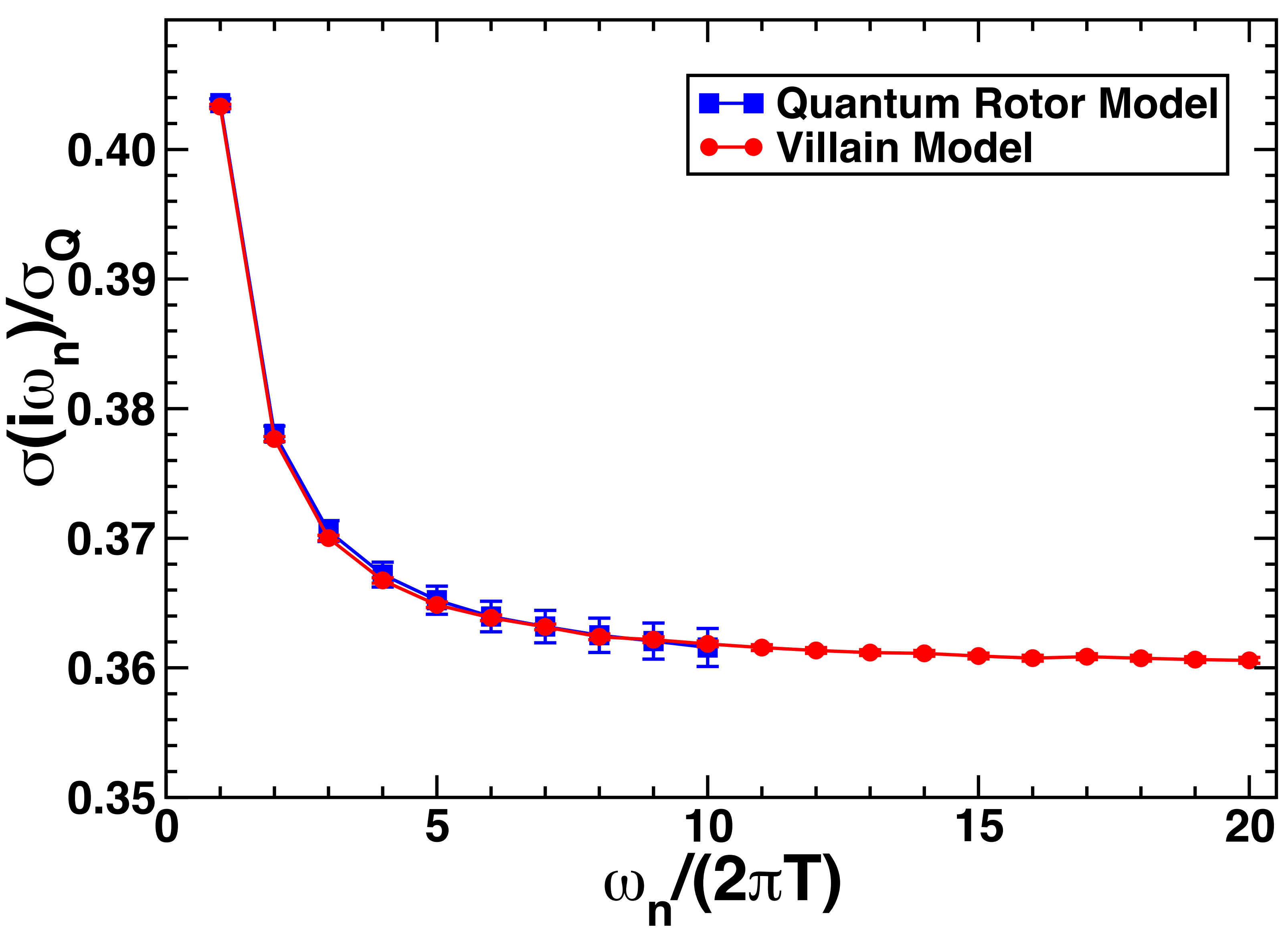
Quantum Monte Carlo

[WK, **Sorensen**, Sachdev, Nat. Phys. 2014]

- Large-scale simulations of $O(2)$ QCP
  - loop-current model (Villain)
  - quantum rotors
- Finite-T but **imaginary time**…
- Analytic continuation difficult!

1st case of universal quantum critical **dynamics**!

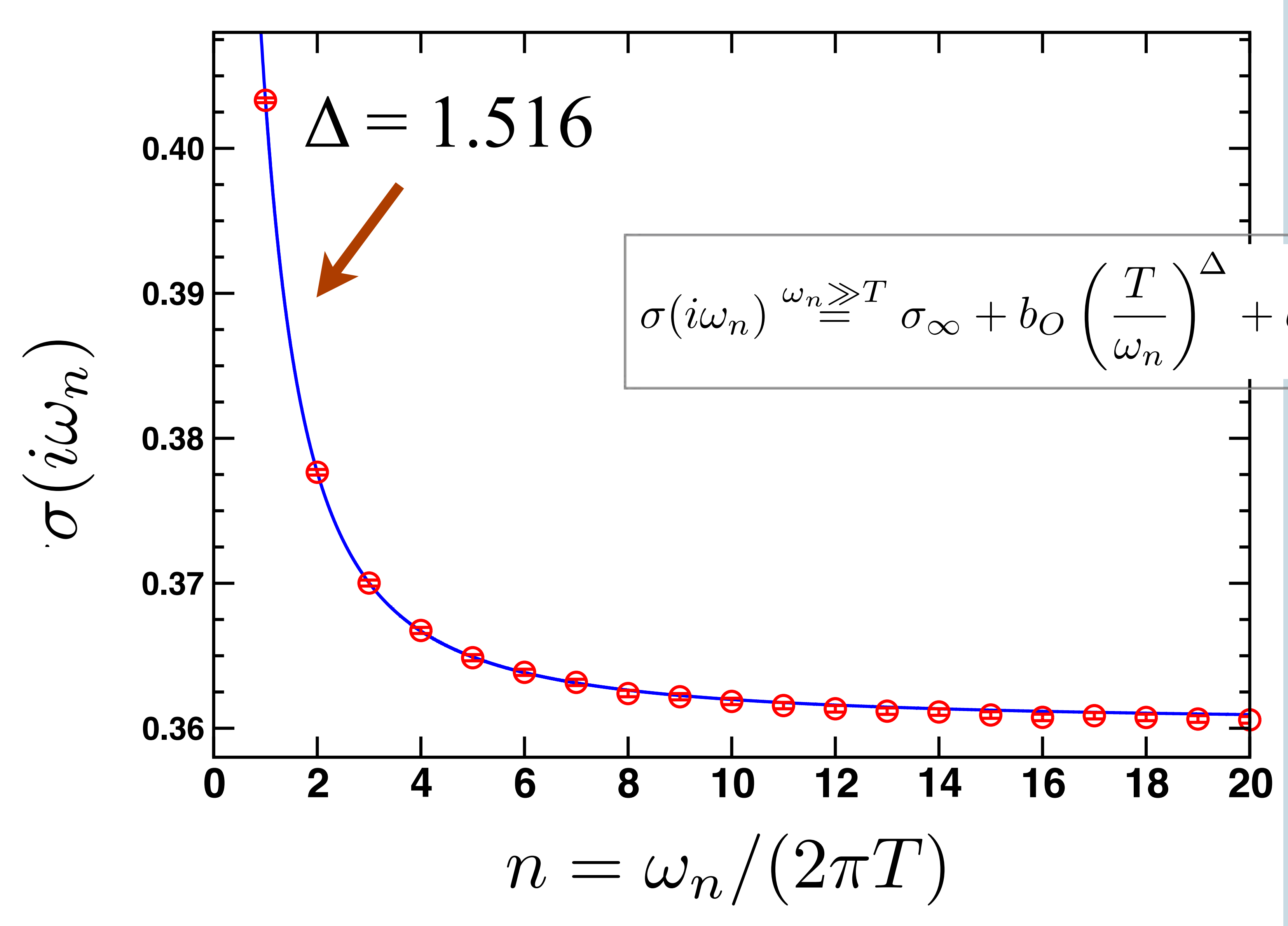
[WK, **Sorensen**, Sachdev, Nat. Phys. 2014]



[also Chen *et al*]

Quantum Monte Carlo II

[WK, **Sorensen**, Sachdev, Katz]



$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_O \left(\frac{T}{\omega_n}\right)^\Delta + b_T \left(\frac{T}{\omega_n}\right)^3 + \cdots$$

$\sigma_\infty = 0.36$

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Presence of Quantum Diffusion in Two Dimensions: Universal Resistance at the Superconductor-Insulator Transition

Matthew P. A. Fisher and G. Grinstein
IBM Research Division, T.J. Watson Research Center, Yorktown Heights, New York 10598

S. M. Girvin
Physics Department, Swain Hall West 117, Indiana University, Bloomington, Indiana 47405

$$\boxed{\sigma_\infty = 0.36}$$

- Determined after 25 years!

- Conformal bootstrap : $\sigma_\infty = 0.3554(8)$
  [Kos *et al* 2015]

[also Chen *et al;*
Gazit *et al*]

Sum rules

Sum rules

$$\int_0^\infty d\omega \, \mathrm{Re}\, \sigma(\omega) = \frac{\pi n e^2}{2m}$$

independent of T,
even of superconductivity!
{Kramers; Ferrell-Glover-Thinkham}

Robust, super-universal statements useful to:

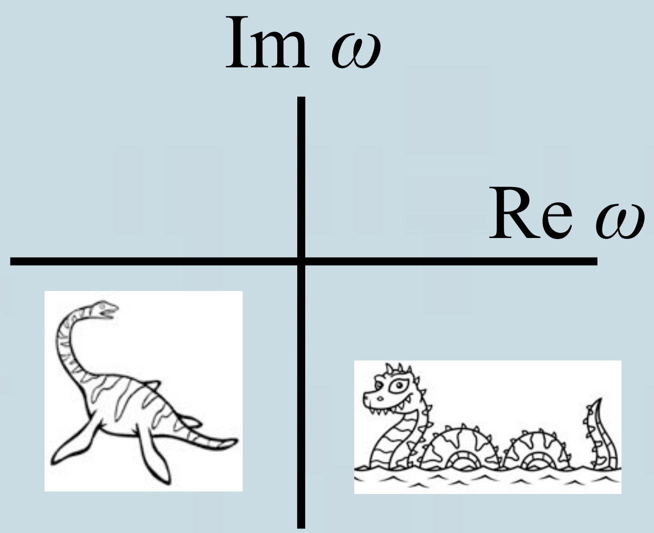
- Assess approximations
- Constrain numerical results
- Help with the analysis of experimental data

Building sum rules

- 2 ingredients:

    1. Kramers-Kronig (retarded)

    2. **Asymptotics** via OPE



Sum rule w/out quasiparticles

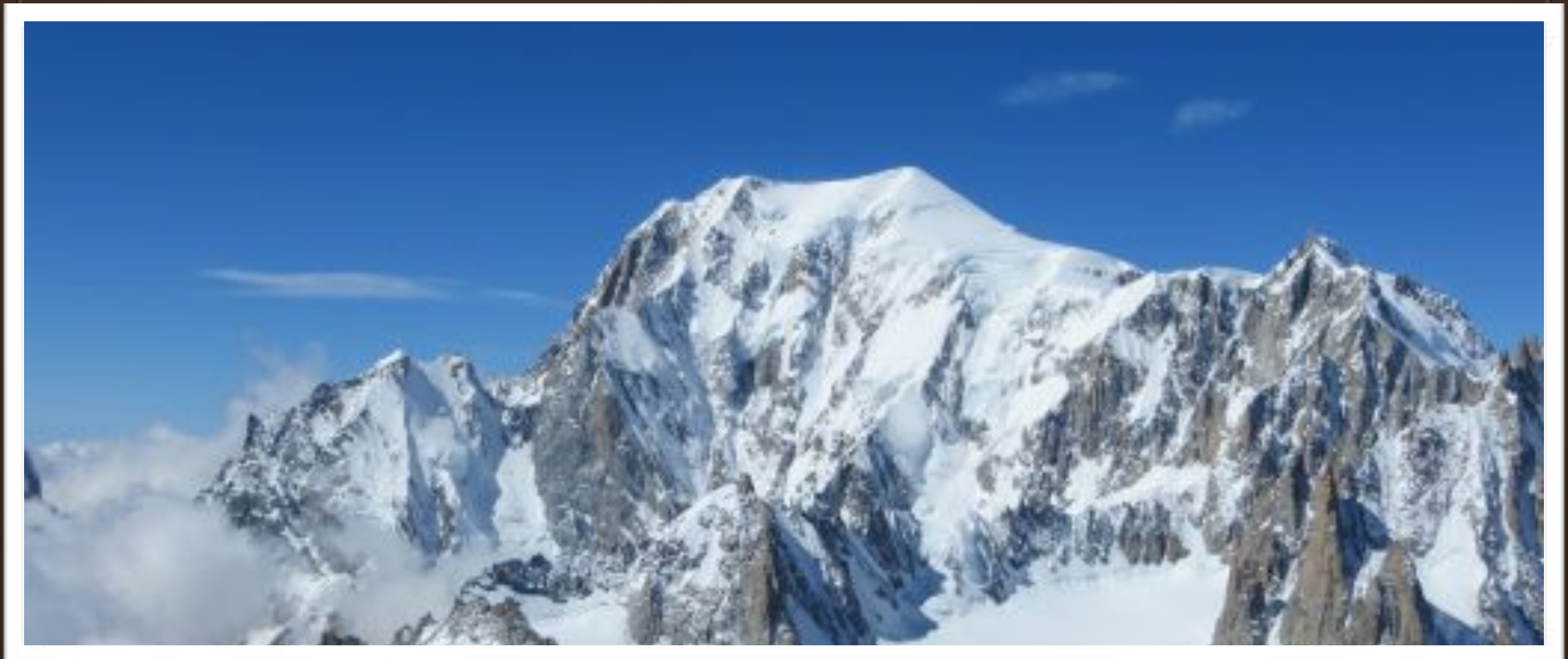
[WK, Sachdev ; Gulotta *et al* ;
Katz, Sachdev, Sorensen, WK]

$$\int_0^\infty d\omega \; [\mathrm{Re}\, \sigma(\omega) - \sigma_\infty] = 0$$

- General proof via OPE

$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_O \left(\frac{T}{\omega_n}\right)^\Delta + b_T \left(\frac{T}{\omega_n}\right)^3 + \cdots$$

- Check:
  - ✔ Maximally SUSY in large-N
  - ✔ O(N) CFT, N=∞
  - ✔ Dirac CFT



Long times & analytic continuation

★Use **AdS/CFT** to generate a family of physically motivated (constrained) variational functions $\sigma(\omega/T)$

Why dabble w/ black holes?

- Physical properties:
  - ***Tailored*** holographic description to match asymptotics
  - **Sum rules**

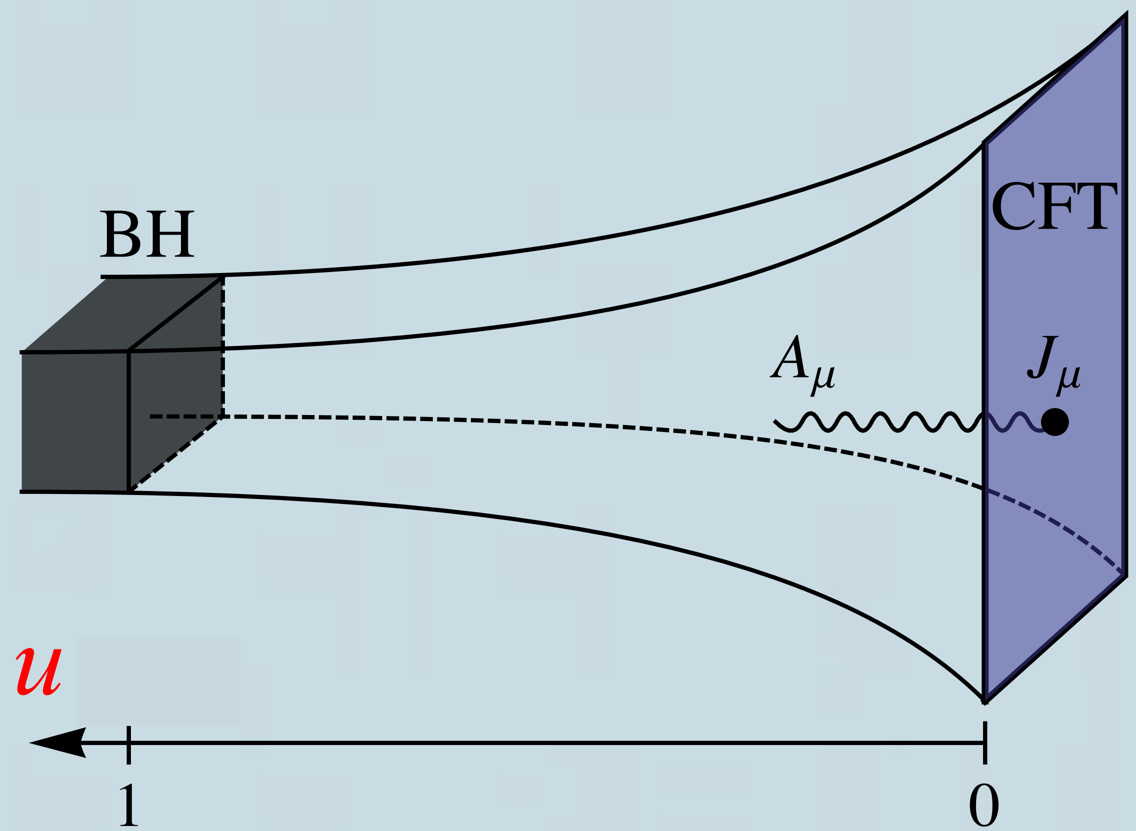
$$\int_0^\infty d\omega\, \mathrm{Re}[\sigma(\omega/T) - \sigma(\infty)] = 0$$

  - **Analytic properties** in common with field theory results
  - **Practical**: real-time, small number of params, fast numerics

σ via AdS/CFT

[Maldacena *etc*]

spacetime:
B-Hole in AdS$_4$



$$A_\mu(t,x,y;u) \leftrightarrow J_\mu^{\mathrm{CFT}}(t,x,y)$$

- Solve **classical** EoM for A_μ $\longrightarrow$ get J-correlator in **CFT**

$$\sigma(\omega/T) = \frac{T}{\omega}\left.\frac{\partial_u A_y(\omega,\vec{0};u)}{A_y(\omega,\vec{0};u)}\right|_{u=0}$$

Bulk action

$$S_{\text{bulk}}[A_\mu] = \int_x \frac{1}{g_4^2} \left[1 + \alpha\, \varphi(x)\right] F_{ab} F^{ab}$$

- Add **scalar field** (dilaton)

CFT	AdS
O	φ

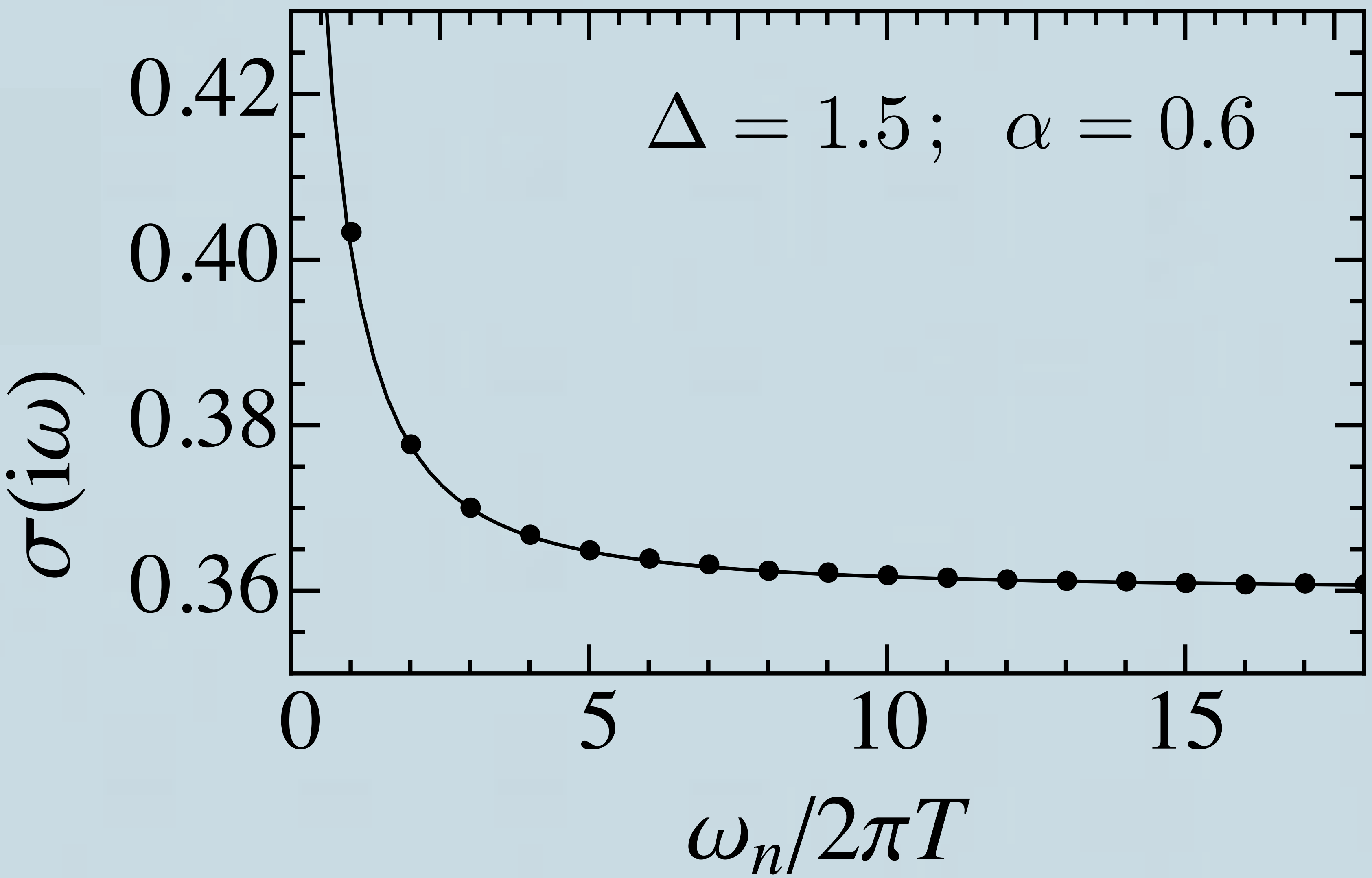
- Simplest **Ansatz**:
  fix profile using OPE of O(2) Wilson-Fisher CFT

$$\varphi(u) = u^{\Delta}$$

[Katz, Sachdev, Sorensen, WK]

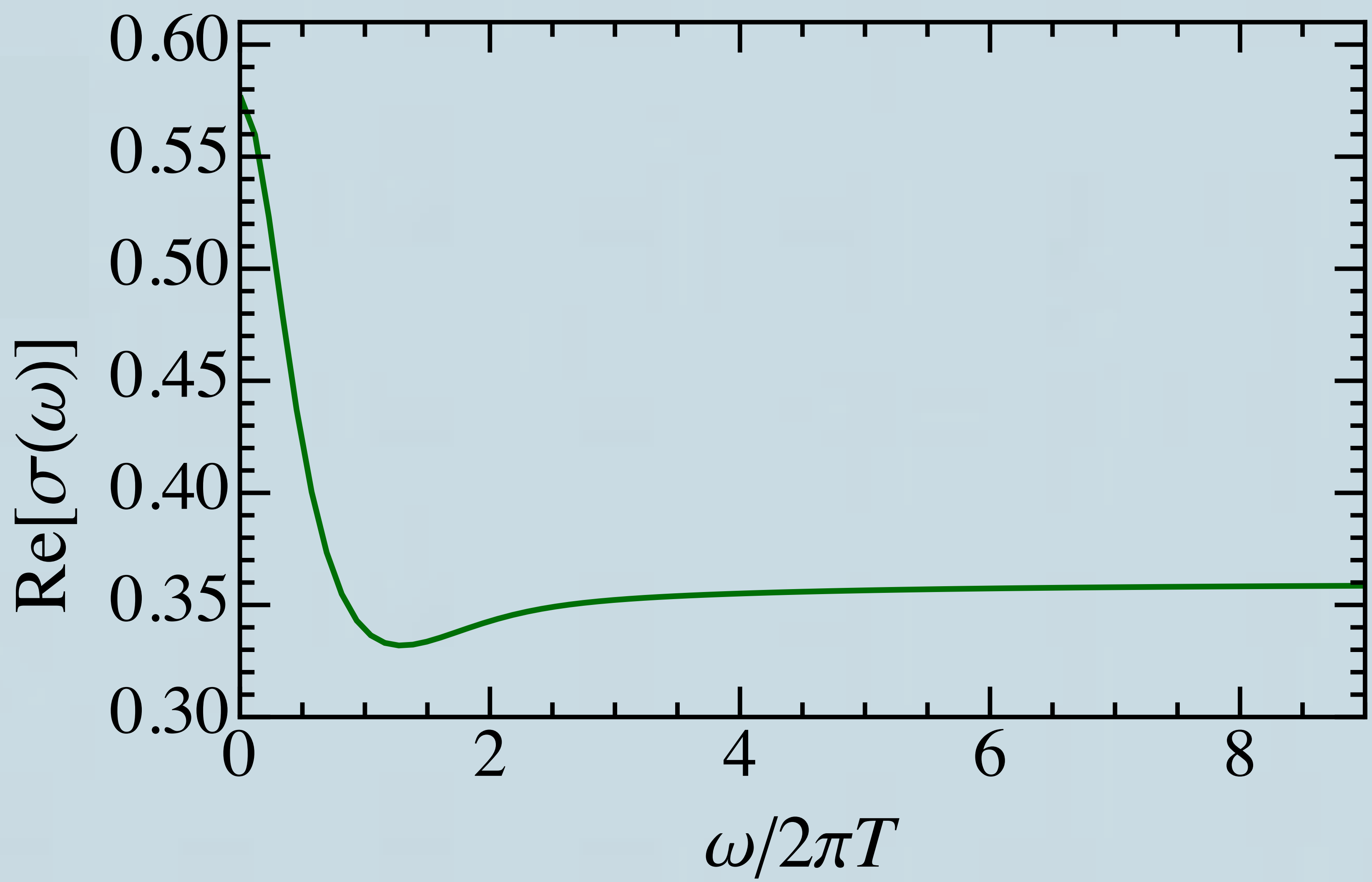
Holographic fit

[Katz, Sachdev, Sorensen, WK]



Real frequencies

[Katz, Sachdev, Sorensen, WK]



Conclusions

- **Corner** entanglement is a useful measure of dof

- In **smooth limit**, $a(\theta)$ fixed by $T_{\mu\nu}$ central charge

$$a(\theta \to \pi) = \sigma\ (\theta - \pi)^2 \qquad \boxed{\sigma = \frac{\pi^2}{24}\, C_T}$$

- Does pure Einstein holography give a **lower bound** for $a(\theta)/C_T$?

Conclusions

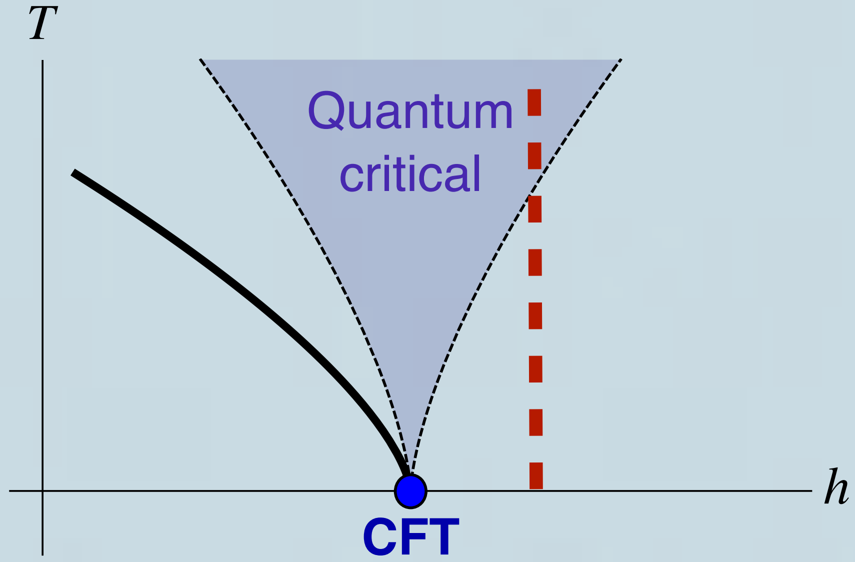
- Quantum critical dynamics at finite T, (CFTs in 2+1D)
- OPE to constrain *short time* physics
  - large ω conductivity

$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^\Delta + b_2 \left(\frac{T}{\omega_n}\right)^3 + \cdots$$

- Input OPE data of CFT into simple holographic ansatz
- Use holography to analytically continue Monte Carlo data for conductivity at superfluid/insulator QCP

Outlook

- Apply this program (OPE, sum rules) to other correlators & CFTs   {WK, Phys. Rev. Lett. 2015}
- Go beyond simplest holographic Ansatz
  {in prog. w/ Sierens & Myers}
- Finite detuning from QCP
  {in prog. w/ Gazit & Podolsky}



ありがとう

wkrempa@pitp.ca