Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Analogue spacetimes.

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Condensed matter physics meets adS/CFT

Kavli Institute — IPMU — Tokyo University

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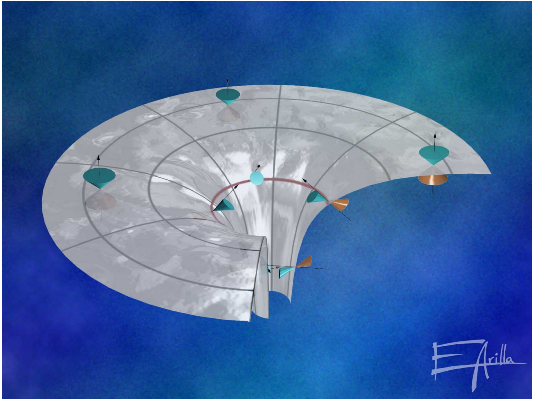
- Analogue spacetimes, (as opposed to general relativity spacetimes), arise when applying the mathematics of differential geometry to generic physical systems.
- As long as the perturbations have finite propagation speed, then the causal structure can be summarized by propagation cones, similar to the light cones of general relativity; thereby defining a conformal structure.
- Often one can go further and define an analogue Lorentzian metric.
- Sometimes these techniques can be used to gain a clearer understanding of general relativity.
- Sometimes the logic flows the other way — and then the tools of general relativity can be used to gain insight into radically different physical systems.

Abstract:



See for example:

- Carlos Barceló, Stefano Liberati, and Matt Visser,
  Analogue gravity
  Living Reviews in Relativity **8** (2005) 12
  Living Reviews in Relativity **14** (2011) 3

Outline:

1. Introduction
2. Emergent Einstein gravity?
3. Acoustic spacetimes
4. Surface wave spacetimes
5. Optical spacetimes
6. BEC spacetimes
7. Meta-material spacetimes
8. Analogue spacetimes
9. Information puzzle/Unitarity
10. Conclusions

Introduction

- Why are "analogue spacetimes" interesting?
- One reason is this:
  - Analogue spacetimes provide one with many specific, physically well-defined and physically well-understood, concrete models for many of the individual phenomena that seem to be part of the yet incomplete theory of "quantum gravity", or perhaps more accessibly, for "quantum gravity phenomenology".

Introduction

- For example "analogue spacetimes" provide concrete physical models of "emergence", (the effective low-energy theory can be radically different from the high-energy microphysics).
- For example "analogue spacetimes" provide controlled models of "Lorentz symmetry breaking", extensions of the usual notions of Lorentzian geometry: "rainbow spacetimes", pseudo-Finsler geometries, and more...
- I will provide an overview of the key items of "unusual physics" that arise in analogue spacetimes, and argue that they provide us with hints of what we should be looking for in any putative theory of "quantum gravity".

Emergent Einstein gravity?

Emergent Einstein gravity?

- The word "emergence" is being tossed around an awful lot lately.....
- But what does it really mean?
  - — Anderson's "more is different"?
  - — The sum is greater than its parts?
  - — Universality?
  - — Mean field?
- Usage within the physics community is not entirely consistent.
- Perhaps the safest interpretation is this:
  - Short-distance physics is often radically different from long-distance physics...

- Prime example: **Fluid dynamics**
- Long distance physics:
  - Euler equation — (generic).
  - Continuity equation — (generic).
  - Equation of state — (specific).
- Short distance physics:
  - Quantum molecular dynamics.
- You cannot hope to derive quantum molecular dynamics by quantizing fluid dynamics...
- Suggestion:
  (Repeatedly suggested from within many different sub-fields):

  Maybe you cannot hope to derive quantum gravity by quantizing classical gravity?

Emergent Einstein gravity?

Two key questions:

- 1) Can we get an "analogue spacetime"? — (generic).
- 2) Can we get "analogue Einstein's equations"? — (specific).

Possible implications:

- *IF* Einstein gravity is "emergent",
  THEN it makes absolutely no sense to "quantize gravity",
  at least not as a "fundamental" theory...
- (Note that's a big *IF*.)
- The best one could then hope for is some "effective theory"
  that has an ultraviolet completion to some uber-theory
  that approximately reduces to Einstein gravity in some limit.

Emergent Einstein gravity?

- The uber-theory would not necessarily be quantum... ['t Hooft]
- It must have (at the vey least) the approximate limits:
  - — Classical Einstein gravity...
  - — Non-relativistic quantum mechanics...
  - — Non-relativistic quantum field theory (many body physics)...
  - — Relativistic quantum field theory (Minkowski space)...
  - — Curved space QFT...
  - — Semiclassical quantum gravity...
- Analogue spacetimes are (among other things) baby steps in this direction...
- String-inspired models are (among other things) baby steps in this direction...

Acoustic spacetimes

The simplest "analogue spacetimes" are the "acoustic spacetimes"...

Theorem: Suppose you have a non-relativistic flowing fluid, governed by the Euler equation plus the continuity equation.

Suppose the fluid flow is barotropic, irrotational, and inviscid.

Suppose we look at linearized fluctuations.

Then the linearized fluctuations (*aka* sound waves, *aka* phonons) are described by a massless minimally coupled scalar field $\Delta\psi = 0$ propagating in a (3+1)-dimensional acoustic metric

$$g_{ab}(t, \vec{x}) \equiv \frac{\rho}{c_s} \left[\begin{array}{ccc} -(c_s^2 - v^2) & \vdots & -\vec{v} \\ \cdots\cdots\cdots\cdots & \cdot & \cdots\cdots \\ -\vec{v} & \vdots & I \end{array} \right].$$

Metric depends algebraically on speed of sound, 3-velocity of fluid, and density of fluid...

Inverse metric:

$$g^{ab}(t, \vec{x}) \equiv \frac{1}{\rho c_s} \left[\begin{array}{ccc} -1 & \vdots & -v^j \\ \cdots\cdots & \cdot & \cdots\cdots\cdots\cdots \\ -v^i & \vdots & (c_s^2\, \delta^{ij} - v^i\, v^j) \end{array} \right].$$

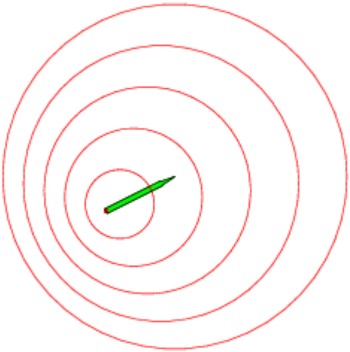
Line element:

$$\mathrm{d}s^2 \equiv g_{ab}\, \mathrm{d}x^a\, \mathrm{d}x^b.$$

$$\mathrm{d}s^2 = \frac{\rho}{c_s} \left\{ -c_s^2\, \mathrm{d}t^2 + \|\mathrm{d}\vec{x} - \vec{v}\, \mathrm{d}t\|^2 \right\}.$$

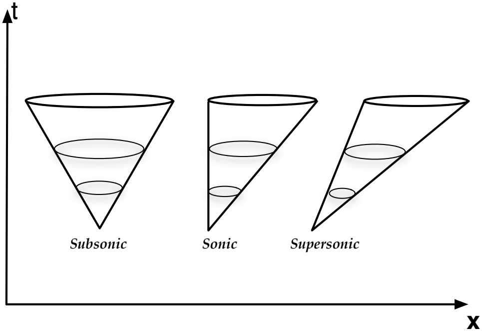
- If you move with the fluid, then the null cones spread out at the speed of sound.
- The conformal factor is required to get a nice minimally coupled d'Alembertian equation of motion for the velocity potential.

Moving with the fluid,
the null cones spread out at the speed of sound.



Acoustic spacetimes:

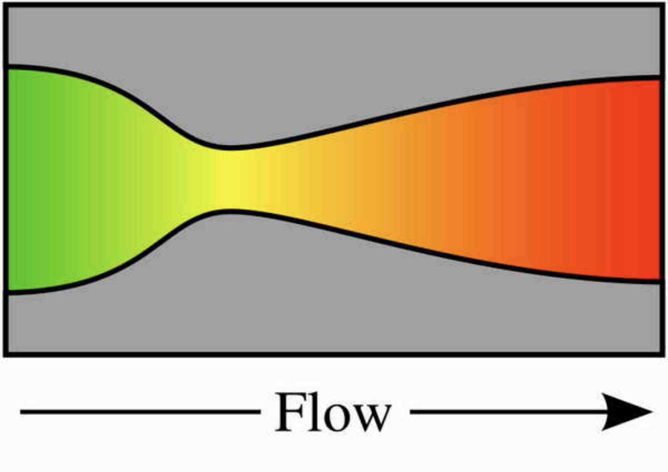
Can even get acoustic horizons when fluid speed goes supersonic...
[focus on the Mach number]...



Physically relevant for:

- Bondi–Hoyle accretion... (interstellar fluid onto a compact object)...
- Solar wind physics... (the Sun is an acoustic white hole)...
- The de Laval nozzle... (the physics is not what you expect)...

De Laval nozzle:



De Laval nozzle:

- Continuity:

$$\rho v A = const.$$

- Euler:

$$\rho v v' = -p' = -c_s^2 \rho';$$

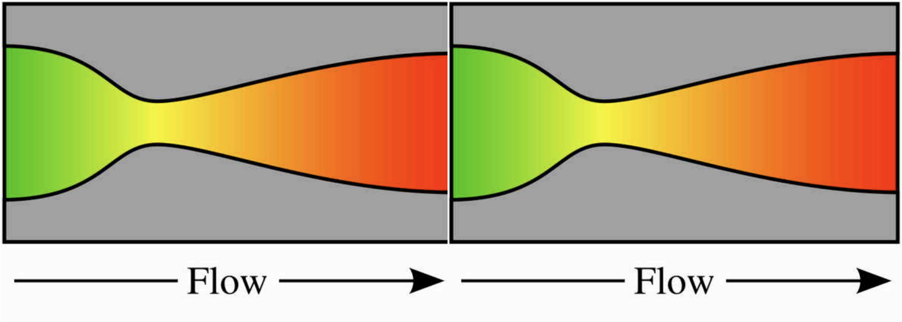
- Eliminate ρ:

$$(c_s^2 - v^2)\; v' = -v\; c_s^2\; \frac{A'}{A}.$$

- Note sign flip at Mach 1:

$$(1 - M^2)\; \frac{v'}{v} = -\frac{A'}{A}.$$

Paired de Laval nozzles:



Define the normal Mach number to a surface:

$$M_n = \frac{\hat{n} \cdot \vec{v}}{c_s}.$$

- Acoustic apparent/trapping horizon when $M_n = 1$.
- Acoustic event horizon not really meaningful.
- Surface gravity:

$$\kappa_H = c_s^2 \; ||\nabla M_n||_H.$$

- Hawking phonons with a temperature:

$$k_B T_H = \frac{\hbar \; \kappa_H}{2\pi \; c_s} = \frac{\hbar \;\; ||\nabla M_n||_H \;\; c_s}{2\pi}.$$

- Unruh 1981...

Relativistic generalizations (barotropic irrotational) are straightforward:

Theorem:

$$g_{ab}^{\text{acoustic}} = \Omega^2 \left(g_{ab}^{\text{usual}} + [1 - c_s^2] V_a V_b \right)$$

Metric depends algebraically on the usual metric, speed of sound,
4-velocity of fluid, and conformal factor
(which in turn depends on density of fluid).

Nomenclature:
Physicists now often call this a "disformal" transformation...
Aka "Gordon metric"...

Acoustic geometry for general relativistic barotropic irrotational fluid flow
Matt Visser and Carmen Molina-París
New Journal of Physics **12** (2010) 095014
e-Print: arXiv:1001.1310 [gr-qc]

Generalizations including vorticity are much messier...

... not just a simple d'Alembertian equation...

... now need additional structure.

Wave equation for sound in fluids with vorticity
Santiago Perez–Bergliaffa, Katrina Hibberd, Michael Stone, Matt Visser
Physica **D191** (2004) 121–136 [e-Print: cond-mat/0106255]

$$\Delta\psi = \nabla \cdot \left(\frac{c_s}{\rho_0} \, h(\xi) \right); \qquad (\mathcal{L}_{V_0})\xi = \; i_{h(\xi)} \, \omega_0.$$

In fact, once one thinks about it, in the ray acoustic limit we will always have either the Newtonian result

$$g_{ab}(t,\vec{x}) \propto \left[\begin{array}{ccc} -(c_s^2 - v^2) & \vdots & -\vec{v} \\ \cdots\cdots\cdots\cdots & \cdot & \cdots\cdots \\ -\vec{v} & \vdots & I \end{array} \right],$$

or its relativistic extension

$$g_{ab}^{\mathrm{acoustic}} \propto \left(g_{ab}^{\mathrm{usual}} + [1 - c_s^2] V_a V_b \right).$$

This generalizes to any system with mono-refringent propagation cones...

Rainbow spacetimes?

- $c_s \longrightarrow c_{\mathrm{group}}(\omega)$ for wave-packets;
- $c_s \longrightarrow c_{\mathrm{phase}}(\omega)$ for modes.

Surface wave spacetimes

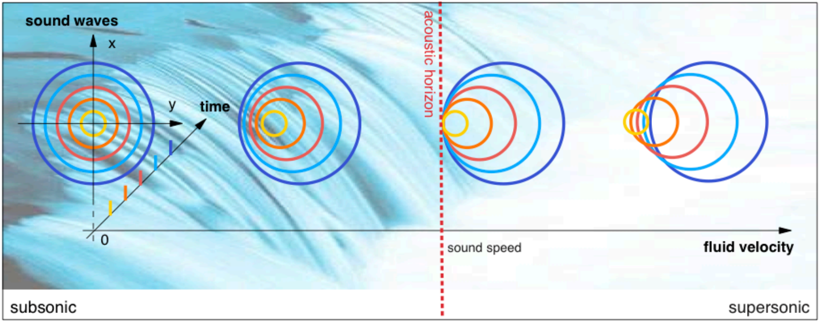
Surface wave spacetimes:

Qualitatively similar physics for surface waves on a fluid interface ...
[now consider the Froude number]...

Physically relevant for:

- Hydraulic jumps...
- Wave blocking $\longleftrightarrow$ blocking horizons...

Details are damnably messy...

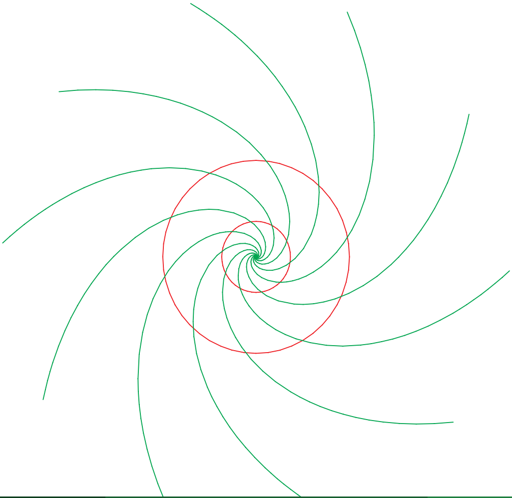


Hydraulic jump:

The black hole of Montecello:

For teaching purposes:

To distinguish a horizon from an ergo-surface...



Surface wave spacetimes:

In fact, once one thinks about it, even for surface waves, in the ray limit we will always have the Newtonian result

$$g_{ab}(t,\vec{x}) \propto \left[\begin{array}{ccc} -(c_s^2 - v^2) & \vdots & -\vec{v} \\ \cdots\cdots\cdots\cdots & \cdot & \cdots\cdots \\ -\vec{v} & \vdots & I \end{array} \right].$$

(Relativistic surface waves are unlikely to be of immediate interest. Perhaps relativistic surface waves on the surface of neutron stars?)

Define the normal Froude number to a line:

$$(\mathbf{Fr})_n = \frac{\hat{n} \cdot \vec{v}}{c_{\mathrm{surface}}}.$$

- Surface wave blocking horizon when $(\mathbf{Fr})_n = 1$.
- Surface wave event horizon not really meaningful.
- Surface gravity:

$$\kappa_H = c^2_{\mathrm{surface}} \; ||\nabla \mathbf{Fr}_n||_H.$$

- Hawking "ripplons" with a temperature:

$$k_B T_H = \frac{\hbar \; \kappa_H}{2\pi \; c_{\mathrm{surface}}} = \frac{\hbar \;\; ||\nabla \mathbf{Fr}_n||_H \;\; c_{\mathrm{surface}}}{2\pi}.$$

Note:

$$\frac{\hbar\,\omega}{k_B\,T_H} \qquad \longrightarrow \qquad \frac{2\pi\,\omega}{||\nabla \mathbf{Fr}_n||_H \;\; c_{\mathrm{surface}}}.$$

This quantity survives in the classical limit.

The classical limit of stimulated Hawking radiation has been observationally probed in the laboratory.

Measurement of stimulated Hawking emission in an analogue system
Silke Weinfurtner, Edmund W. Tedford, Matthew C. J. Penrice,
William G. Unruh, Gregory A. Lawrence
Physical Review Letters **106** (2011) 021302
e-Print: arXiv:1008.1911 [gr-qc]

The Boltzmann factor $\exp\left(-\frac{2\pi\ \omega}{||\nabla \mathbf{Fr}_n||_H\ \ c_{\text{surface}}}\right)$ has been observed...

See also:

Classical aspects of Hawking radiation verified in analogue gravity experiment
Silke Weinfurtner, Edmund W. Tedford, Matthew C. J. Penrice,
William G. Unruh, Gregory A. Lawrence.
Lecture Notes in Physics **870** (2013) 167–180

Bill Unruh has very strongly argued that observing the classical limit automatically verifies the physical reality of the quantum effect.

Has Hawking radiation been measured?
W.G. Unruh
Foundations of Physics **44** (2014) 532–545
e-Print: arXiv:1401.6612

Optical spacetimes

Theorist's toys:

- Index gradient methods:

$$g_{ij}(x) = n(x)^2 \;\; \delta_{ij}; \qquad\qquad \mathrm{d}s = n(x) \;\; \mathrm{d}\ell.$$

  Fermat's principle of least time $\Longleftrightarrow$ geodesics in conformally flat 3-space.

- Snell's law:

$$g_1(t_1, k) = g_2(t_2, k).$$

  Killing vector, tangent vector, optical metric...
  Snell's law $\Longleftrightarrow$ Killing's conservation equation applied to the two sides of the interface...

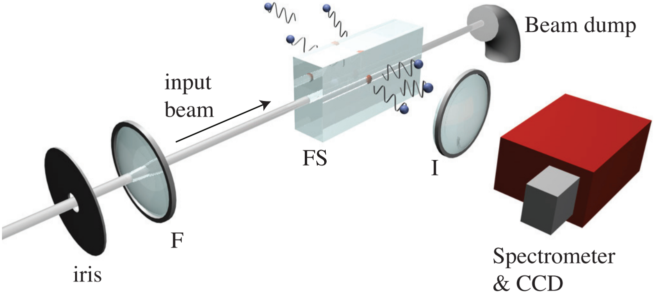
Optical spacetimes:

Experimentalist's toys:

Hawking radiation from ultrashort laser pulse filaments
F. Belgiorno, S.L. Cacciatori, M. Clerici, V. Gorini, G. Ortenzi,
L. Rizzi, E. Rubino, V.G. Sala, D. Faccio.
Physical Review Letters **105** (2010) 203901
e-Print: arXiv:1009.4634 [gr-qc]

Experimental evidence of analogue Hawking radiation
from ultrashort laser pulse filaments
E. Rubino, F. Belgiorno, S.L. Cacciatori, M. Clerici, V. Gorini, G. Ortenzi,
L. Rizzi, V.G. Sala, M. Kolesik, D. Faccio.
New Journal of Physics **13** (2011) 085005

Optical spacetimes:



Some controversies:

- Kinematic issues?
- Micro-fractures?

BEC spacetimes

Observation of self-amplifying Hawking radiation
in an analog black hole laser
Jeff Steinhauer.
Nature Physics **10** (2014) 864
e-Print: arXiv:1409.6550 [cond-mat.quant-gas]

"We thus observe self-amplifying Hawking radiation."

Acoustics in an atomic BEC:

- $c_s \approx 1$ mm/sec.
- $T_H \gtrsim T_{BEC}$.

People have been gearing up for this for over a decade...

Meta-material spacetimes

Exercise:

Show that the curved-space Maxwell equations in any static gravitational field are equivalent to those obtained by considering a particular dielectric medium (permittivity = permeability) on flat Minkowski space.

- This is an exercise in Landau & Lifshitz, "The classical theory of fields"...
- Rediscovered and elaborated on many many times...
- See Living Review article for many historical details...
- Note the result is one-way: If (permittivity $\neq$ permeability) then you don't get just a gravitational field; it's gravity plus other additional structure...

For a dielectric medium:

$$D^{ab} = E^{abcd}\; F_{cd}$$

with

$$E^{0i0j} = -\epsilon^{ij}; \qquad E^{ijkl} = \mathfrak{e}^{ijm}\; \mathfrak{e}^{kln}\; \mu^{-1}_{mn}.$$

When does

$$E^{abcd} \longleftrightarrow \sqrt{-g}\left(g^{ac}g^{bd} - g^{ad}g^{bc}\right)\,?$$

Then

$$LHS = 2 \times (6\ components) = 12\ components.$$

$$RHS = (conformal\ class\ of\ static\ metrics) = 6\ components.$$

Parameter count mismatch forces one to adopt:

$$(permittivity = permeability).$$

Note (permittivity $=$ permeability) is not "natural".

- Normal bulk matter has

$$1 \leq \mu \leq 1 + \mathcal{O}(10^{-6}),$$

  and

$$1 \leq \epsilon \leq \mathcal{O}(10).$$

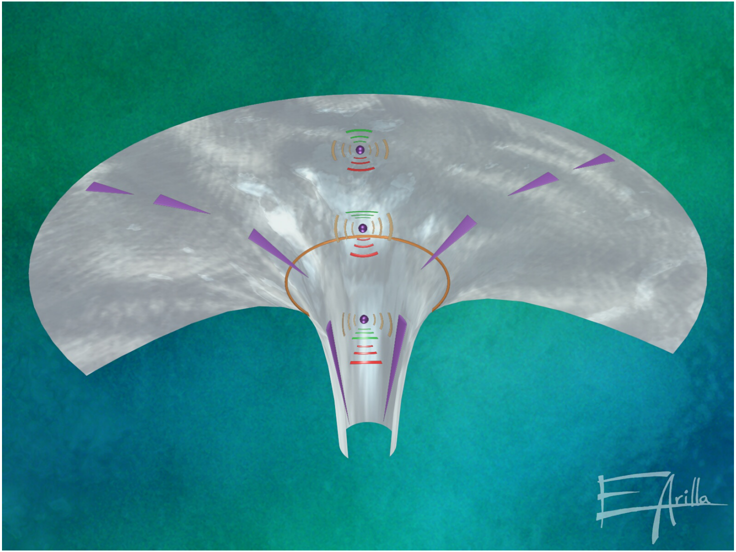
- Need "active elements" and/or "split ring resonators" to get interesting meta-materials....
- Deliberately engineered micro resonators...
- Now becoming technologically relevant...
  - "Cloaking devices"...
  - Transformation optics...
- Ray optics limit better behaved...

Analogue spacetimes

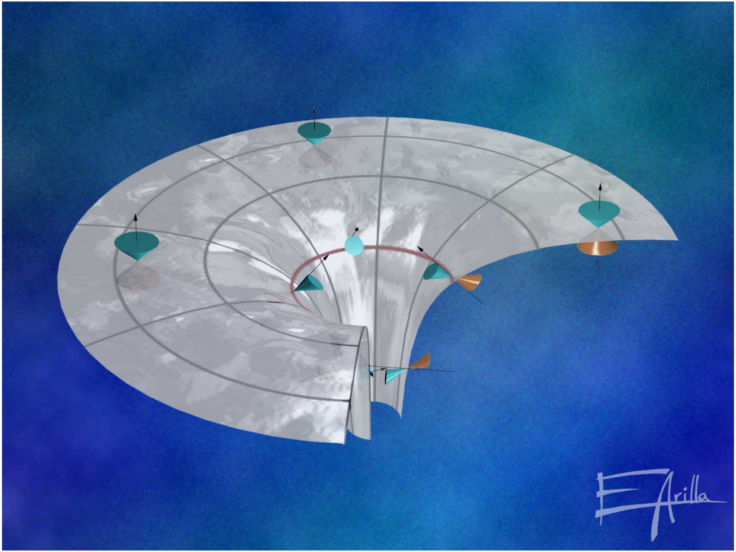
Gravity plus...

- This is one of the weaknesses of the analogue spacetime programme...
- It is distressingly easy to get multi-refringence...
- It is distressingly easy to get "extra structure"...
- It is damnably difficult to enforce the Einstein equivalence principle...
- Particle physics experiments (direct) show no signs of "extra structure" / multi-refringence at least out to LHC energies (10 TeV $= 10^4$ GeV)...
- Particle physics experiments (indirect) show no signs of "extra structure" / multi-refringence at least out to 10^{15} GeV...

It's more than just light-cones:



It's more than just light-cones:



Information puzzle/Unitarity

Information puzzle/Unitarity:

Key point:

There is a crucial difference between the "qualitative" and "quantitative" information loss problems.

- The "qualitative" problem is this:
  If a spacelike singularity forms (in the strict mathematical sense), then there will be a (strict mathematical) event horizon, and unavoidably some loss of unitarity associated with any matter that might cross the event horizon.
- The "quantitative" problem is this:
  How much information is lost behind the event horizon, (if it forms), and how much comes out in the Hawking radiation?

Thermality of the Hawking flux, arXiv:1409.7754 [gr-qc].

Key point:

- Hawking radiation is associated with the apparent/trapping horizon, and couldn't care less about the event horizon, (if present).
- Unitarity violation, (if present), is associated with the event horizon, (if present), and couldn't care less about the apparent/trapping horizon.

Only if you assume that the (strict mathematical) event horizon actually forms, and that it closely tracks the apparent/trapping horizon, is there ever any significant information loss.

Thermality of the Hawking flux, arXiv:1409.7754 [gr-qc].

Key point:

- Event horizons are not physically observable...
- Apparent/trapping horizons are physically observable...

Physical observability of horizons
Physical Review **D90** (2014) 127502
e-Print: arXiv:1407.7295 [gr-qc]

Key point:

- Hawking radiation depends on near-horizon physics...
- Unitarity violation depends on near-singularity physics....

Thermality of the Hawking flux, arXiv:1409.7754 [gr-qc].

Conclusions

Conclusions:

- There is a lot of very interesting work going on with these analogue spacetimes...
- No Einstein equations yet — though with Nordström gravity we are starting to get somewhat close...
- Some experiments already done, some do-able...
- Some extremely interesting matters of principle to consider...

Thank you.