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Emergence of Holographic Spacetime From Quantum Entanglement

``Surface/State Correspodence’’

Tadashi Takayanagi

Yukawa Institute for Theoretical Physics (YITP), Kyoto University

IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Based on

[1] arXiv:1503.08161 with Masamichi Miyaji (YITP, Kyoto)
``Surface/State Correspondence''

[2] arXiv:1412.6226 (to appear in JHEP)
with Masamichi Miyaji (YITP, Kyoto),
Shinsei Ryu (Illinois, Urbana–Champaign),
and Xueda Wen (Illinois, Urbana–Champaign).

[3] A paper in preparation with Masamichi Miyaji (YITP, Kyoto),
Tokiro Numasawa (YITP, Kyoto),
Noburo Shiba (YITP, Kyoto),
and Kento Watanabe (YITP, Kyoto).

Contents

① Introduction

String Theory ⇒ a unified theory of quantum gravity

It is still difficult to compute quantum corrections in cosmological spacetimes like big bang, de-Sitter etc.

However, a generalization of AdS/CFT (or holography) may be able to resolve this problem:

``Quantum Gravity = Quantum Many-body Systems''

on M_{d+2} on ∂M_{d+1}

For this, we need to understand the basic mechanism of AdS/CFT. ⇒ A key concept is **quantum entanglement**.

Entanglement Entropy (EE) $S_A = -\mathrm{Tr}[\rho_A \log \rho_A]$

⇒ The best measure of quantum entanglement

EE measures

(1) How much a many-body wave function is entangled.

(2) Active degrees of freedom. (~central charges)

(3) A quantum order parameter.

(4) An observable in numerical experiments.

(5) a ***geometry*** of quantum many-body system.

<u>Area law of EE</u> [Bombelli-Koul-Lee-Sorkin 86, Srednicki 93]

EE in QFTs includes UV divergences.

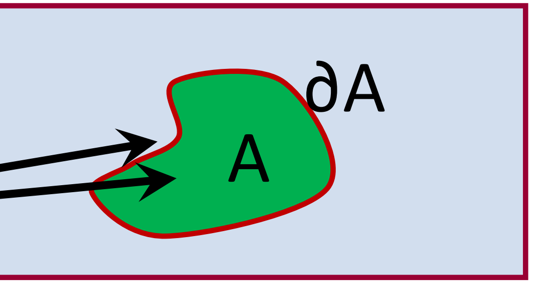
In a d+1 dim. QFT (d>1) with a UV relativistic fixed point,
the leading term of EE at its ground state behaves like

$$S_A \sim \frac{\mathrm{Area}(\partial A)}{\varepsilon^{d-1}} + (\text{subleading terms}),$$

where ε is a UV cutoff (i.e. lattice spacing). [d=1: log div.]

Intuitively, this property is understood like:

Most strongly entangled



Holographic Entanglement Entropy (HEE)

[Ryu-TT 06; derived by Casini-Huerta-Myers 09, Lewkowycz-Maldacena 13]

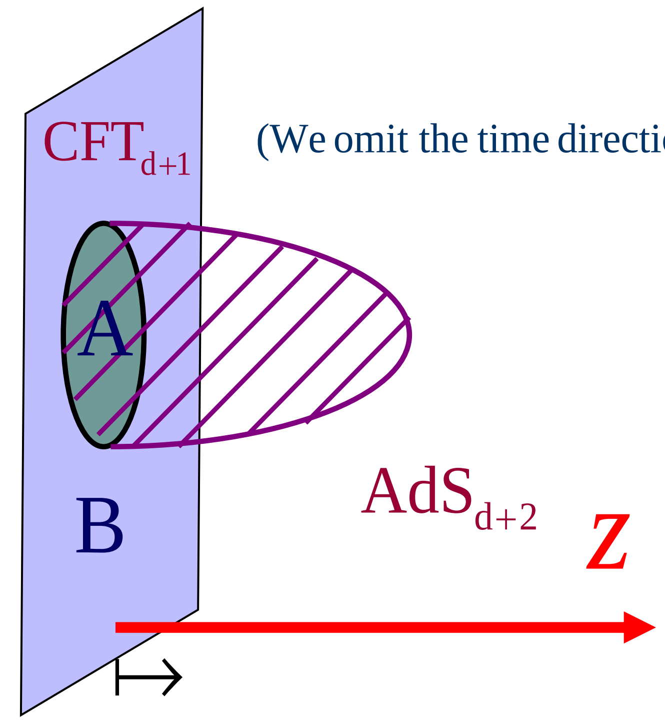
$$S_A = \underset{\substack{\partial\gamma_A = \partial A \\ \gamma_A \approx A}}{\mathrm{Min}} \left[\frac{\mathrm{Area}(\gamma_A)}{4G_N} \right]$$

γ_A is the minimal area surface (codim.=2) such that

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A \,.$$

homologous

(We omit the time direction.)



$z > \varepsilon$ (UV cut off)

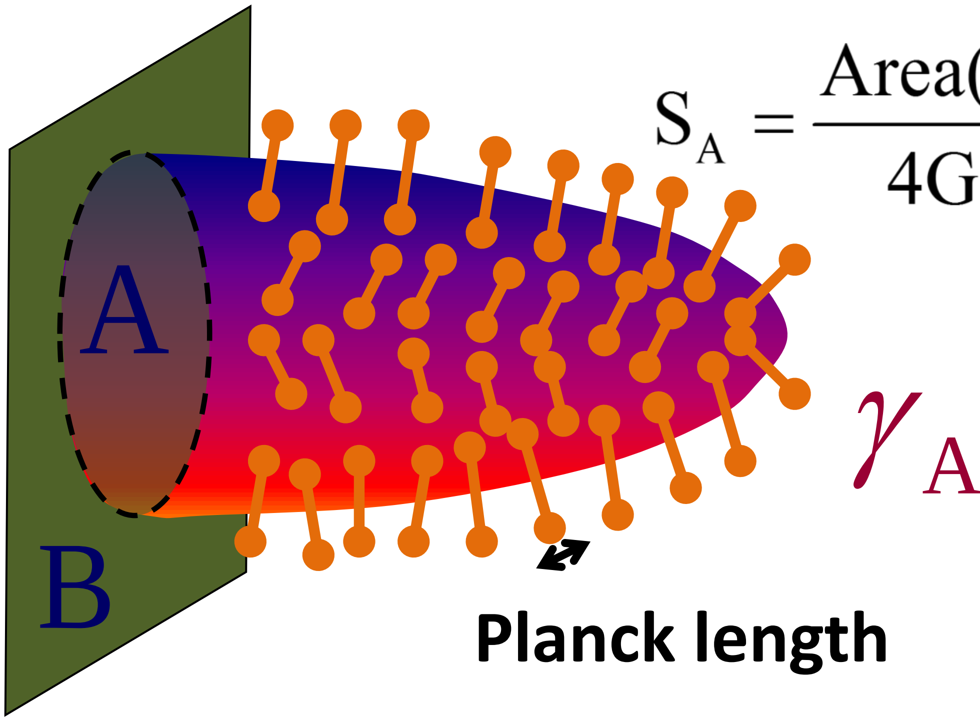
Note: In time-dependent spacetimes, we need to take extremal surfaces.

[Hubeny-Rangamani-TT 07]

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

The HEE suggests the following novel interpretation:

``A spacetime in gravity
= Collections of bits of quantum entanglement''

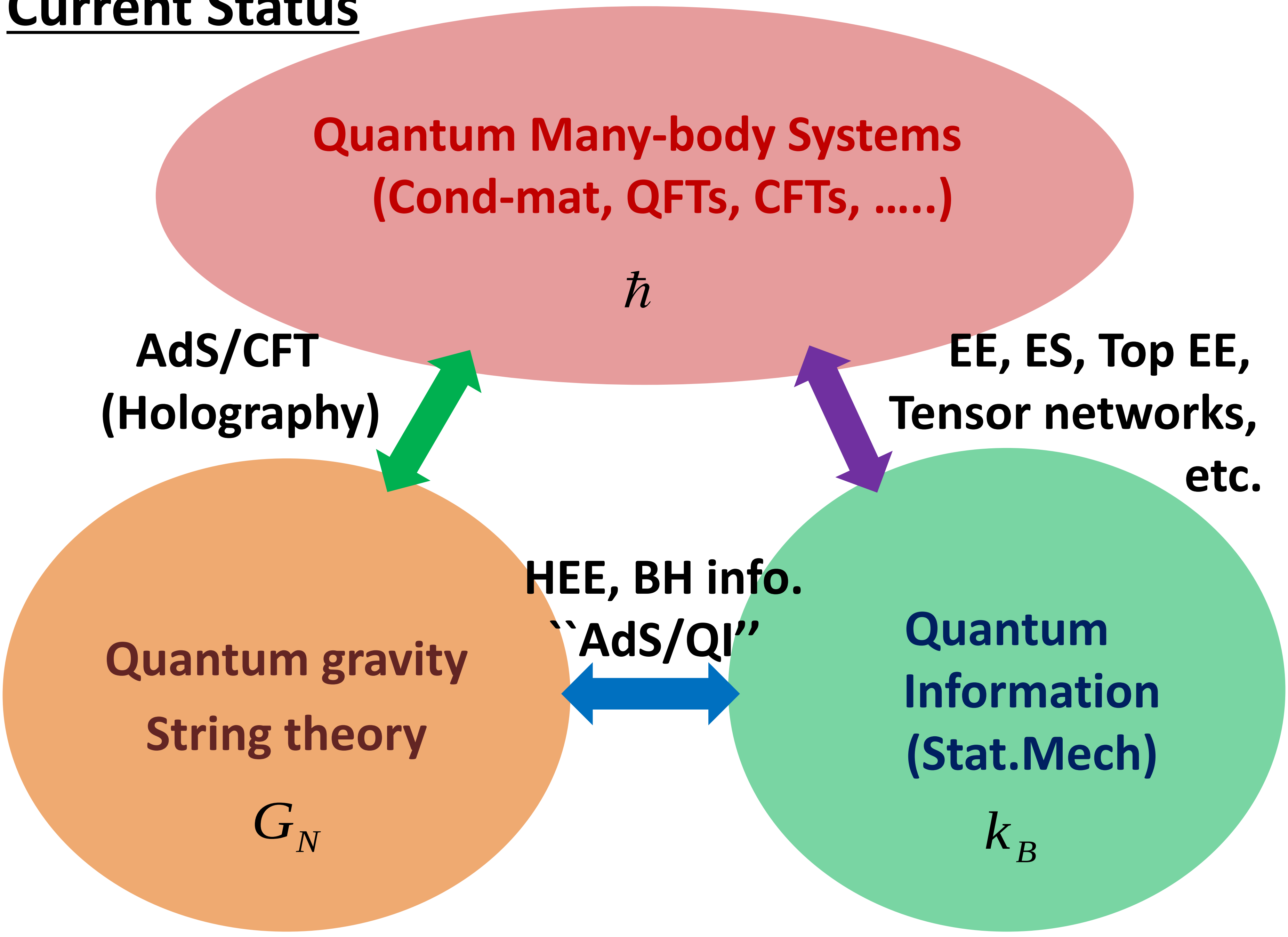


$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N} \approx \frac{\mathrm{Area}(\gamma_A)}{l_{pl}^2}.$$

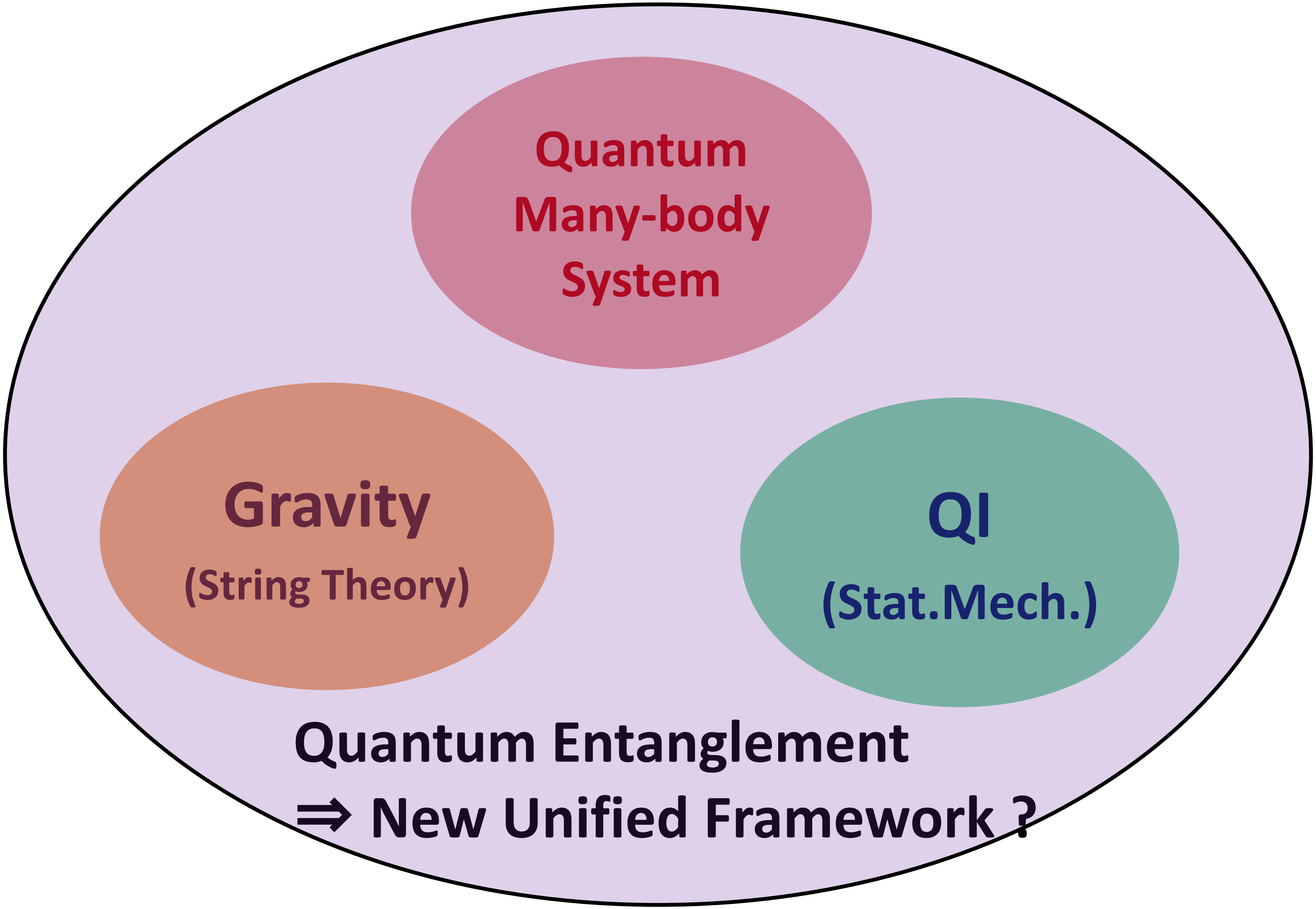
⇒ Manifestly realized in the recently found connection between AdS/CFT and tensor networks !

[Swingle 09, ……]

Current Status



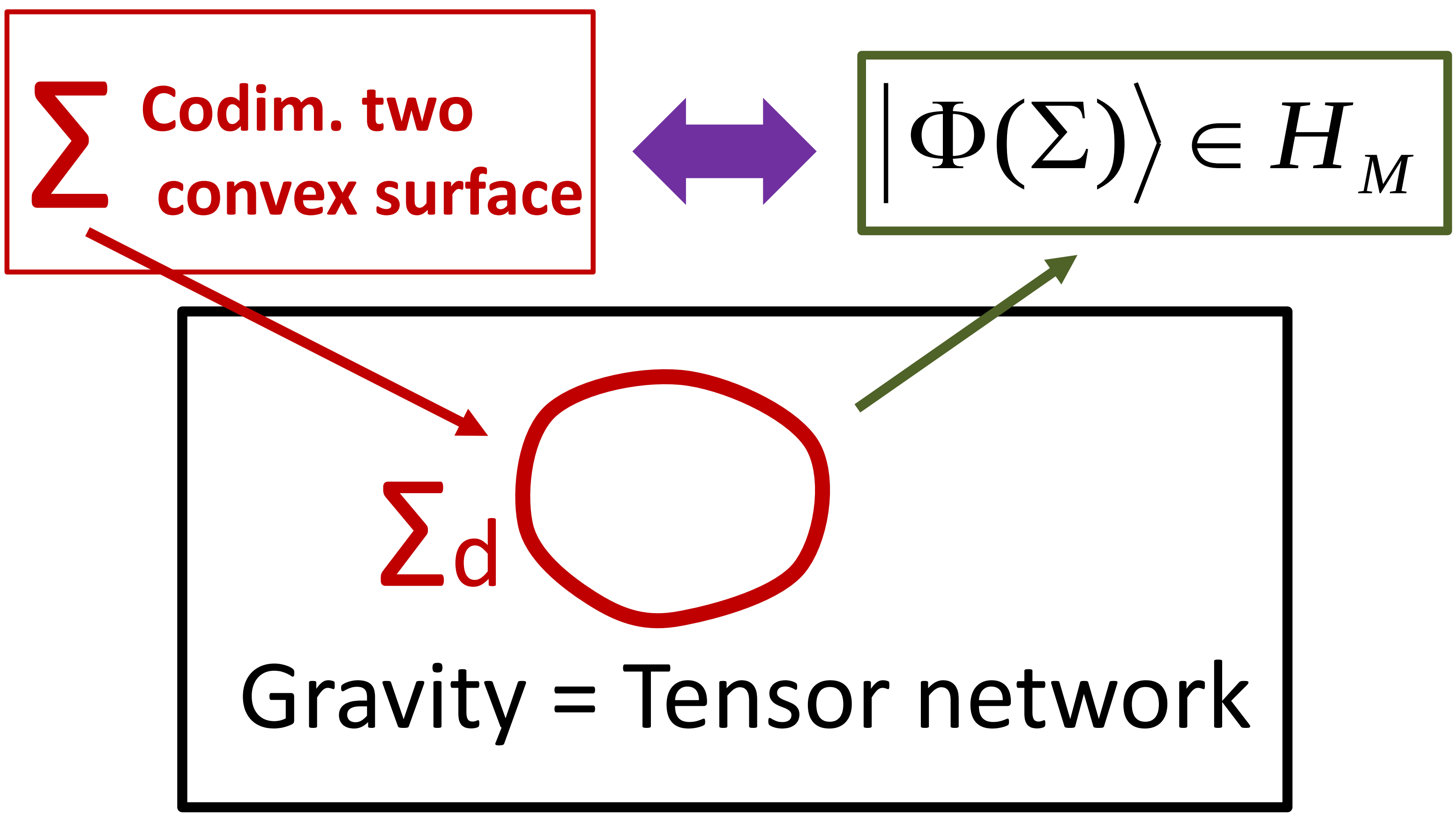
Our Hope?


Quantum
Many-body
System
Gravity
(String Theory)
QI
(Stat.Mech.)
Quantum Entanglement
⇒ New Unified Framework ?

In this talk we want to push a bit more in this direction

by proposing the **Surface / State correspondence**

as a new generalization of holography.



② AdS/CFT and Tensor Network

(2-1) Tensor Network [See e.g. Cirac-Verstraete 09(review)]

Tensor network states

= Efficient variational ansatz for the ground state wave functions in quantum many-body systems.

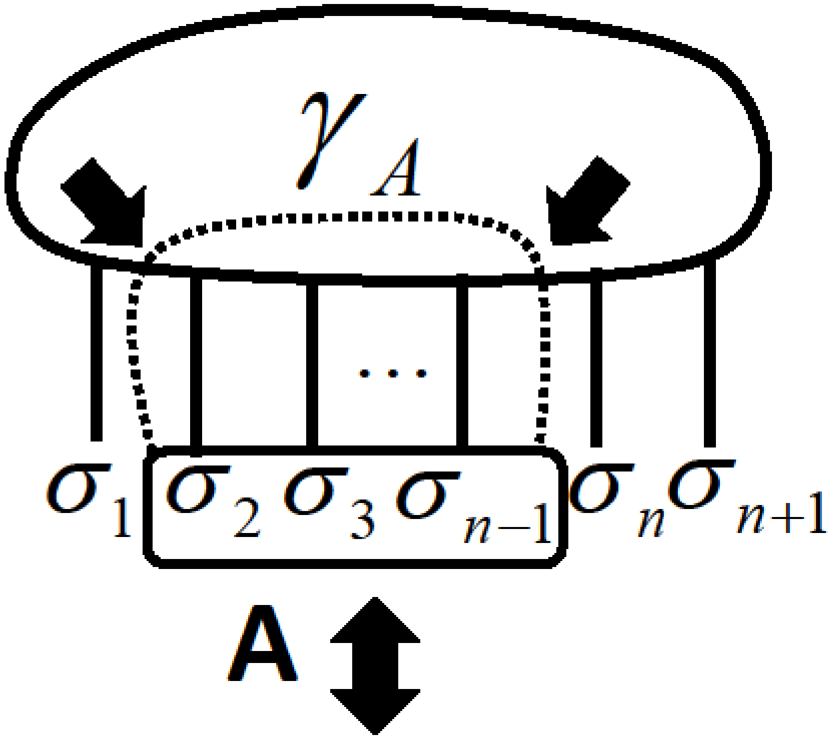
[A tensor network diagram = A wave function]

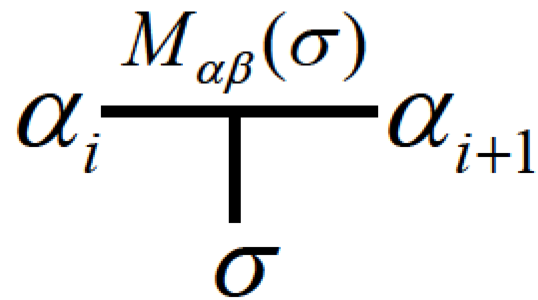
⇒ An ansatz should respect the correct **quantum entanglement** of ground state.

~Geometry of Tensor Network

Ex. Matrix Product State (MPS)

[DMRG: White 92,…, Rommer-Ostlund 95,..]





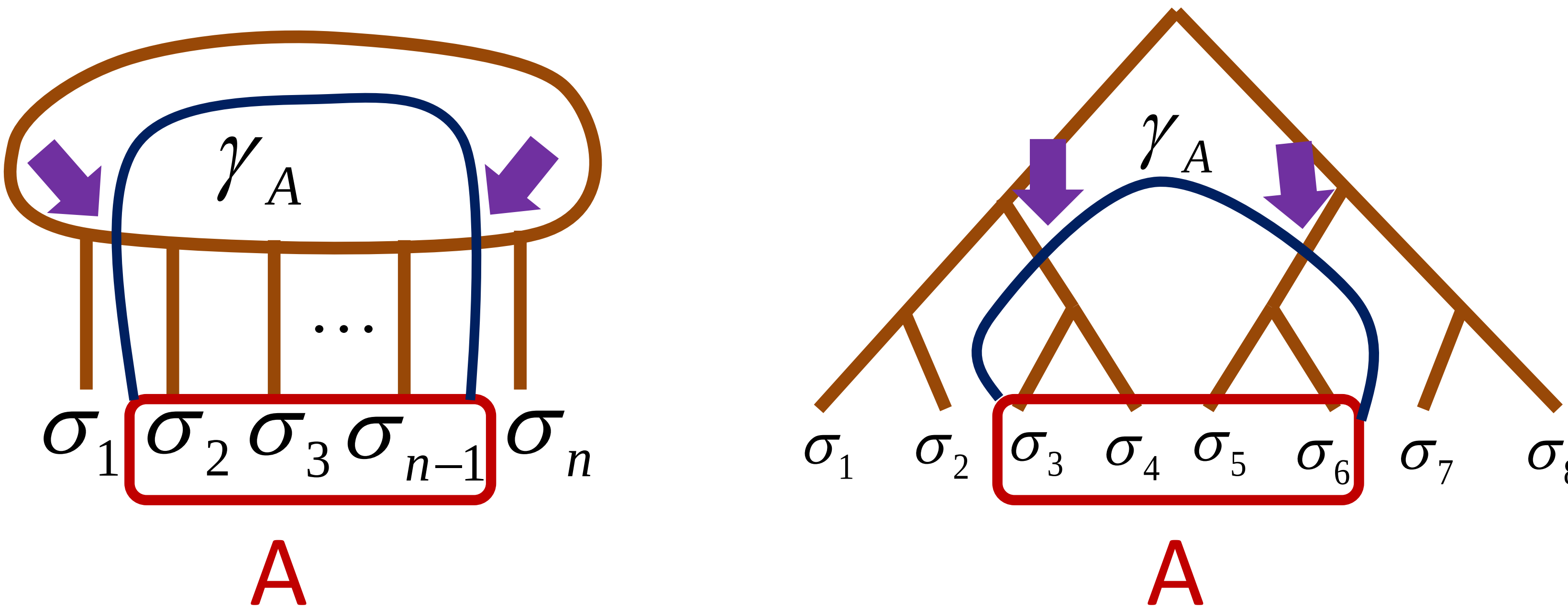
$\alpha_i = 1,2,\ldots,\chi,$

$\sigma_i = \uparrow$ or $\downarrow$.

$$\left|\Psi\right\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_n} \mathrm{Tr}[M(\sigma_1)M(\sigma_2)\cdots M(\sigma_n)]\, \underbrace{\left|\sigma_1,\sigma_2,\cdots,\sigma_n\right\rangle}_{n\ Spins}$$

MPS with finite χ does not have enough EE to describe 1d quantum critical points (2d CFTs) :

$$S_A \leq 2\log\chi \quad (<< \log L \sim S_A^{CFT}).$$



In general,

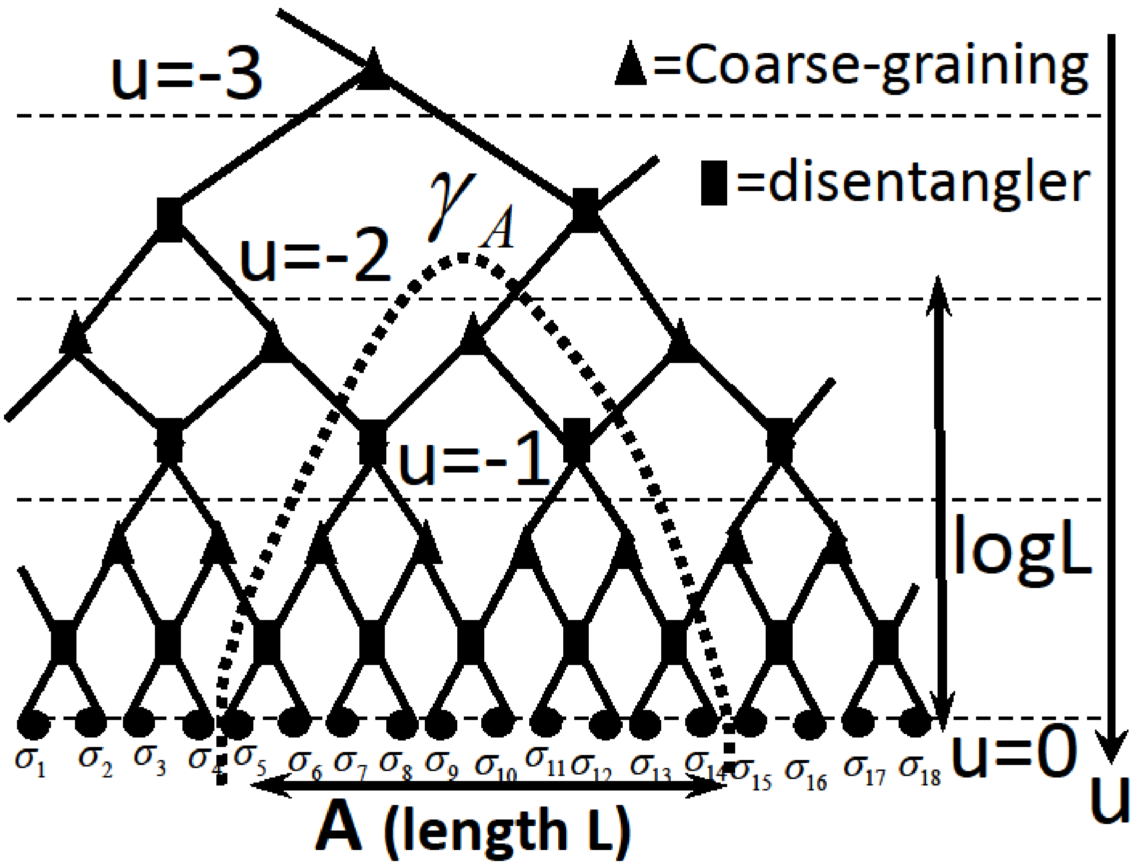
$$S_A \sim N_{\text{int}} \cdot \log\chi,$$
$$N_{\text{int}} \equiv \min[\#\,\text{Intersections of } \gamma_A].$$

(2-2) MERA

<u>MERA</u> (Multiscale Entanglement Renormalization Ansatz) [Vidal 05]

⇒ An efficient variational ansatz for CFT ground states.

To increase entanglement in a CFT, we add (dis)entanglers.

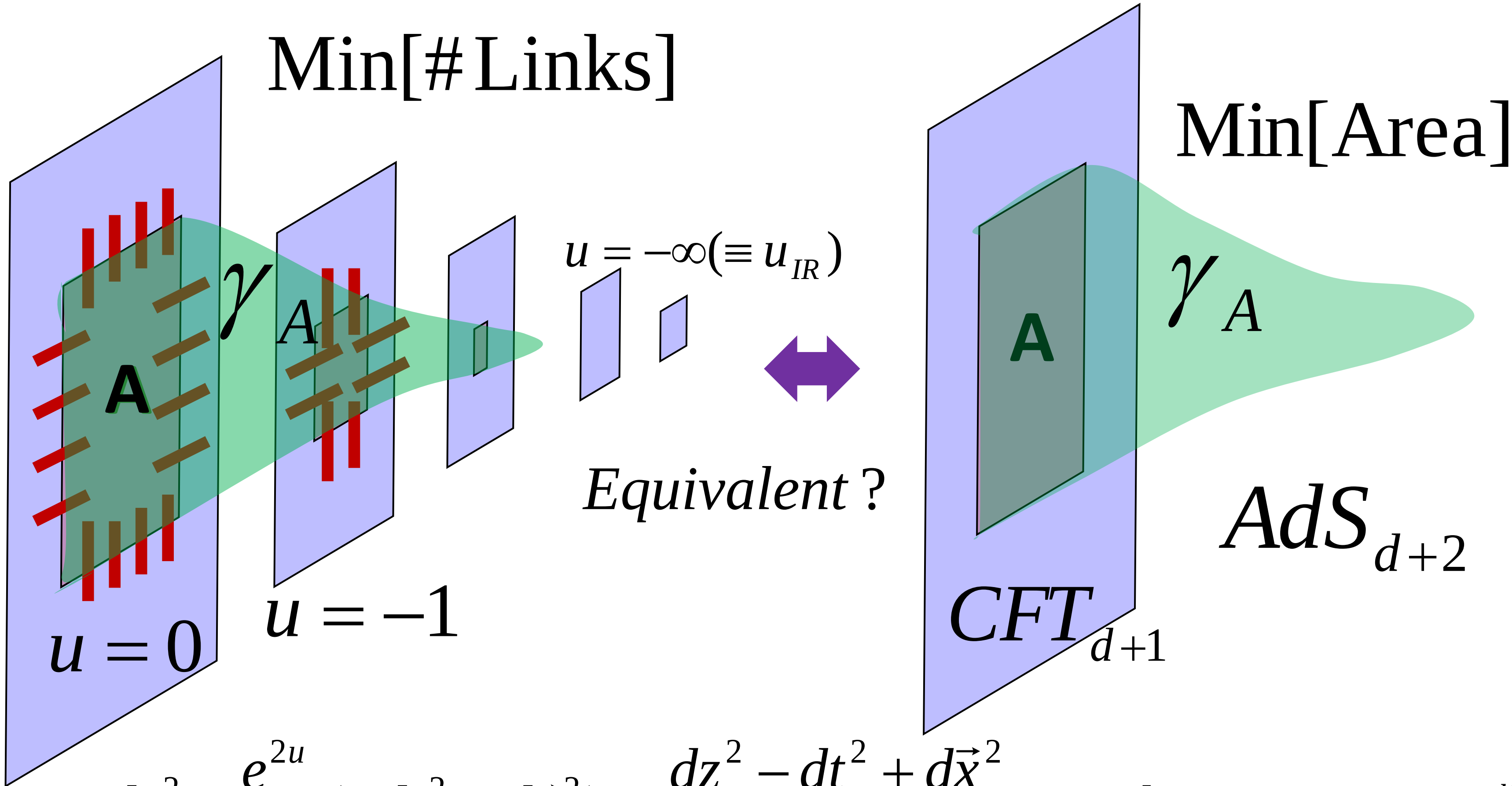


$$S_A \propto \mathrm{Min}[\#\,\mathrm{links}]$$

$$\propto \log L$$

⇒ agrees with results in 2d CFT !

A conjectued relation to AdS/CFT [Swingle 09]



$$ds^2 + \frac{e^{2u}}{\varepsilon^2}(-dt^2 + d\vec{x}^2) = \frac{dz^2 - dt^2 + d\vec{x}^2}{z^2}, \quad where \quad z = \varepsilon \cdot e^{-u} \quad .$$

However, MERA has lattice artifacts. ⇒ Continuum limit?

Other modifications: e.g.[Qi 13, Pastawski-Yoshida-Harlow-Preskill 15]

(2-3) cMERA [Haegeman-Osborne-Verschelde-Verstraete 11 reformulation and AdS/CFT interpretation: Nozaki-Ryu-TT 12]

To remove lattice artifacts, take a continuum limit of MERA:

$$\underbrace{|\Phi(u)\rangle}_{\text{State at scale u}} = P\cdot\exp\left(-i\int_{u_{IR}}^{u} ds\,\hat{K}(s)\right)\cdot\underbrace{|\Omega\rangle}_{\text{IR state}}.$$

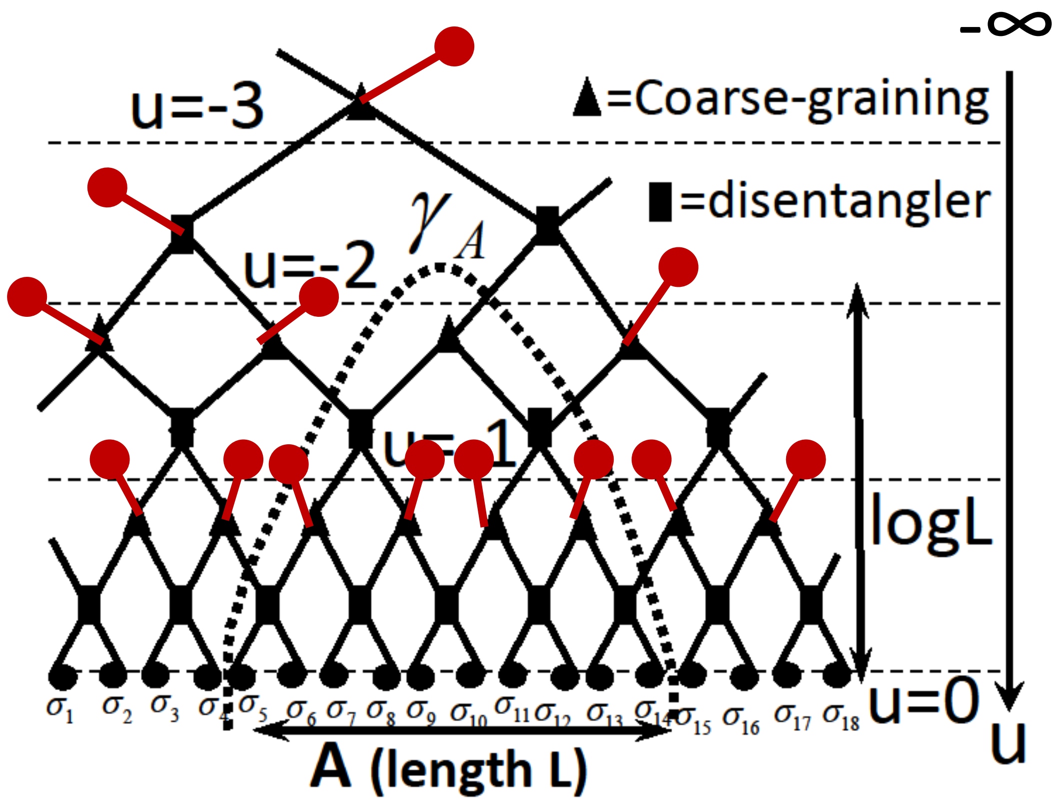
$$u_{IR} = -\infty$$

$\hat{K}(u)$: (dis)entangler at length scale $\sim \varepsilon\cdot e^{-u}$

$|\Omega\rangle$: unentangled state in real space

$\rightarrow S_A = 0$ for any A. ➡ What is this state ? Our next topic !

Relation to (discrete) MERA



By adding dummy states |0>  , we keep the dimension of Hilbert space for any u to be the same.

⇒ We can formally describe the real space RG by a **unitary transformation**.

③ Boundary State as Gravity Dual of Point-like Space

[Miyaji-Ryu-Wen-TT 14]

Q. A general construction of the IR states |Ω> in CFTs ?

Argument 1

We can realize disentangled states (IR states |Ω>)

⇔ Trivial (Point-like) spaces

by performing a (infinitely) massive deformation:

$$H_m = H_{CFT} + m^{d+1-\Delta_O} \int dx^d O(x),$$

$$\underset{m\to\infty}{\Longrightarrow} \left|\Omega\right\rangle = \text{the ground state of } H_m.$$

Now we apply the idea of ***quantum quenches***.

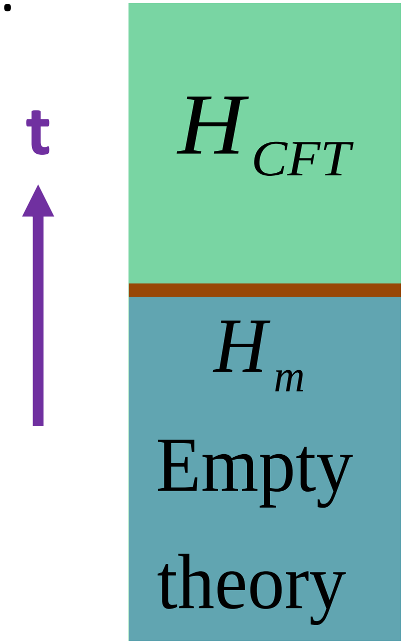
⇒ For t<0, we assume the ground state of the massive Hamiltonian H_m. Then at t=0, we suddenly change the Hamiltonian into H_{CFT} as in [Calabrese-Cardy 05].

In this setup, the state at t=0 is identified with the boundary state:

$$\left|\Psi_m(t=0)\right\rangle = \left|\Omega\right\rangle = \left|B\right\rangle.$$

We may introduce the UV cut off like

$$\left|\Omega_m\right\rangle \propto e^{-H/m} \cdot \left|B\right\rangle \ .$$



Boundary states in CFTs (assume 2d CFT)

A **boundary state** (Ishibashi state) : |B>

= A state which gives a conformally invariant boundary condition:

$$\left[L_n - \tilde{L}_{-n}\right]\left|B\right\rangle = 0.$$

In terms of the Virasoro algebra: $\left|B\right\rangle = \sum_{\vec{k}}\left|\vec{k}\right\rangle_L \left|\vec{k}\right\rangle_R$

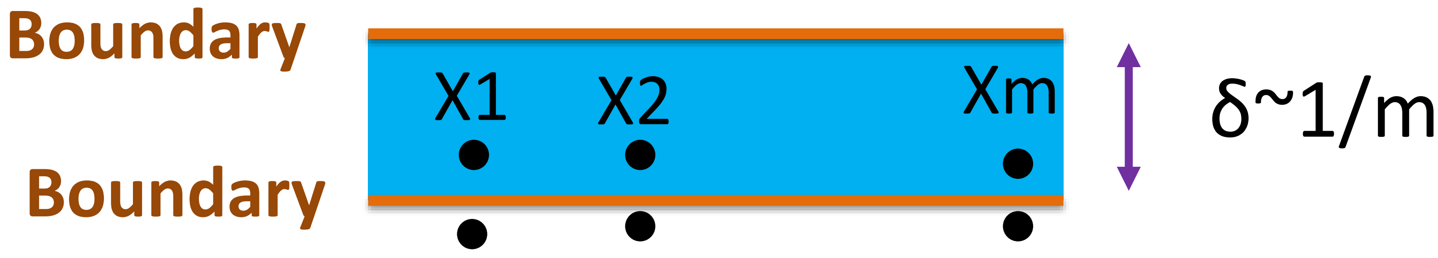
where $\vec{k} = (k_1, k_2, \ldots\ldots)$ represent

$$\left|\vec{k}\right\rangle = \sum (L_{-1})^{k_1} \cdot (L_{-2})^{k_2} \cdots \left|\Delta\right\rangle.$$

⇒ A maximally entangled state between left and right moving sectors !

⇒ But, the real space entanglement is quite suppressed !

Argument 2: Correlation functions of local operators



$$\frac{\langle\Omega|O(x_1)O(x_2)\cdots O(x_n)|\Omega\rangle}{\langle\Omega|\Omega\rangle} \approx \prod_{i=1}^{n}\langle O(x_i)\rangle.$$

⇒ When (xi-xj)>>δ, there is no correlations !

⇒ Disentangled !

Argument 3: Direct calculation of EE

For the regularized IR state $|\Omega\rangle = e^{-H\delta}|B\rangle,$

we can compute the EE explicitly in free fermion CFTs:

[Ugajin-TT 10]

$$S_A \approx \frac{c}{3}\log\frac{\delta}{\varepsilon} + [\text{Finite}], \quad (\delta \to 0).$$

Thus we can set $S_A \approx 0 \quad \text{when}\ \delta \approx \varepsilon.$

Note: Boundary states can still have non-zero finite *topological entanglement.*

④ Surface/State Correspondence [Miyaji-TT 15]

(4-1) Basic Principle

Consider Einstein gravity on a d+2 dim. spacetime **M**.

We argue the following correspondence:

Σ : an d dim. convex space-like surface in M which is closed and homologically trivial

↕

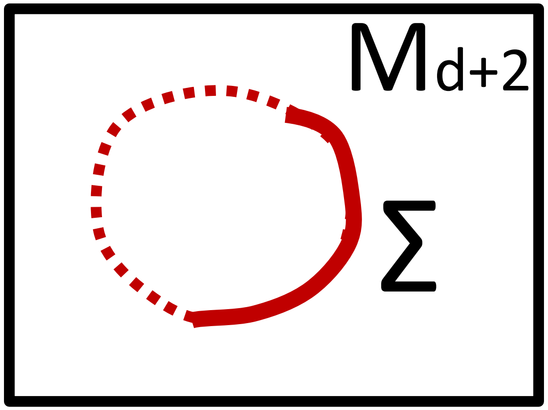
$$|\Phi(\Sigma)\rangle \in H_M$$

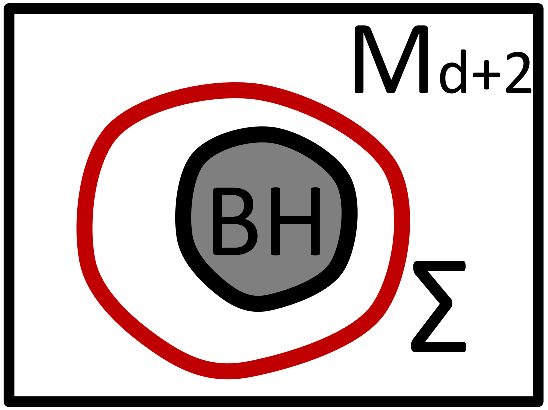
A pure state

More generally,
the quantum state dual to a convex surface Σ is

a **mixed state** $\rho(\Sigma)$

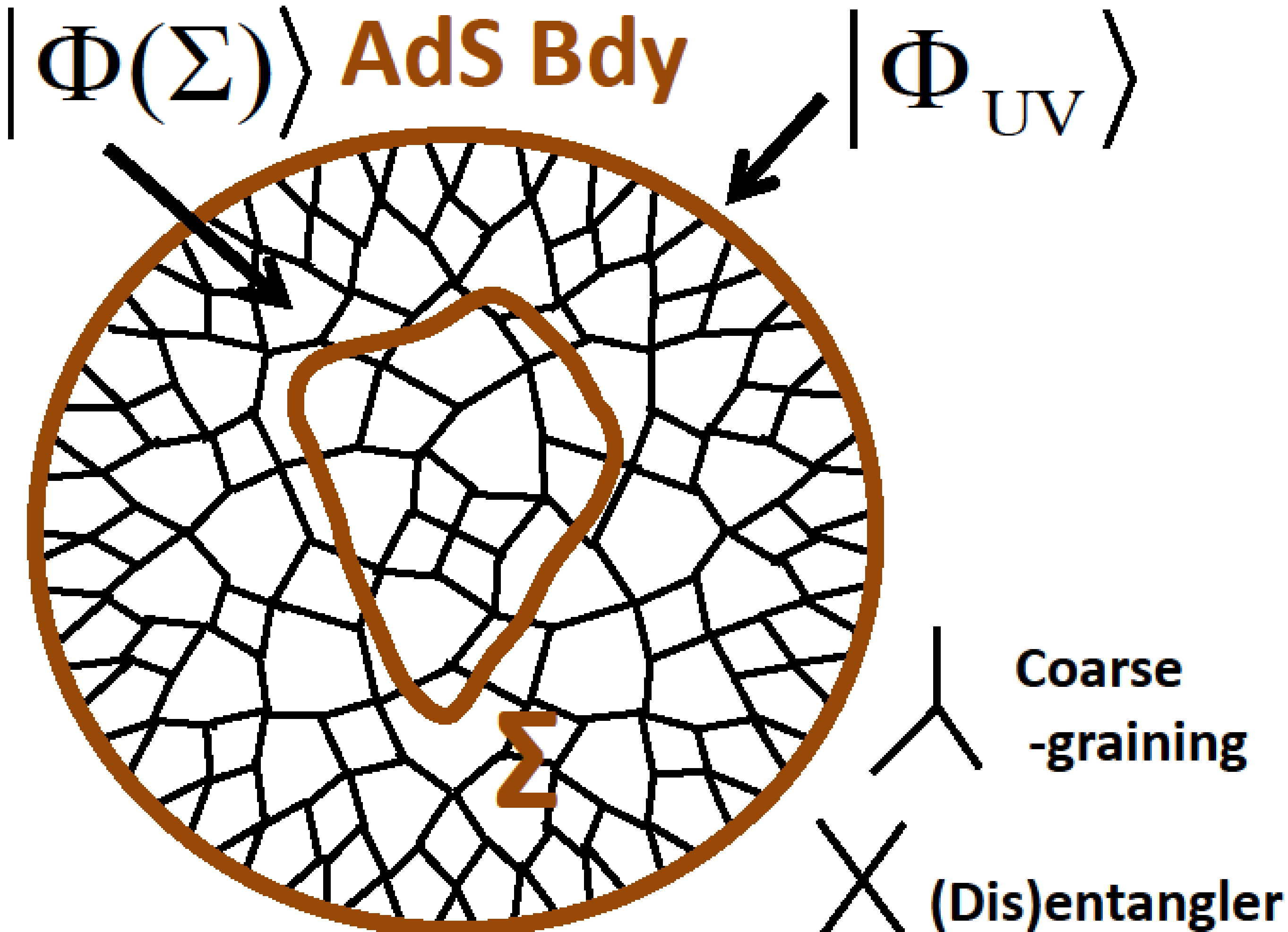
if Σ is open or topologically non-trivial.





On the other hand, the **zero size limit of Σ** corresponds to the **trivial state |Ω>** with no real space entanglement.

This surface/state correspondence is partially motivated by the tensor network description of holography.

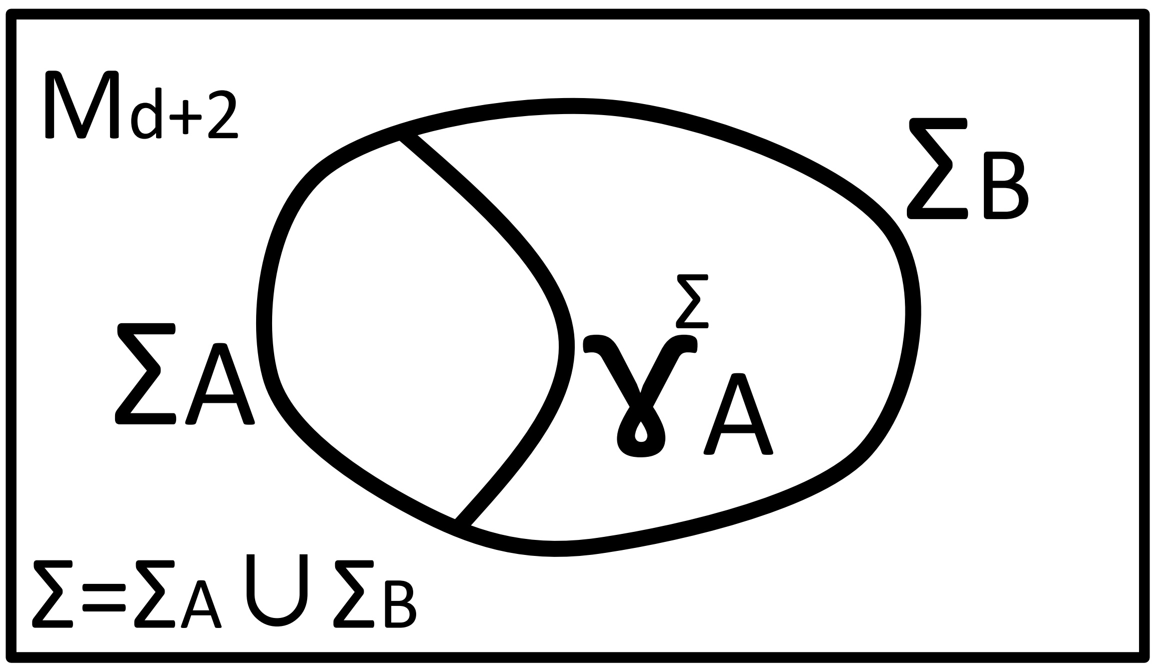


(4-2) Entanglement Entropy

We can naturally generalize HEE for our setup :

$$H_\Sigma = H_A \otimes H_B, \qquad \rho_A^\Sigma = \mathrm{Tr_B}[\rho(\Sigma)],$$

$$\Rightarrow \qquad S_A^\Sigma = \frac{\mathrm{Area}(\gamma_\mathrm{A}^\Sigma)}{4G_N}.$$



(4-3) Effective Dimension

By dividing the surface Σ into infinitesimally small pieces $\Sigma = \cup A_i$, we easily find:

$$\sum_i S^{\Sigma}_{A_i} = \frac{\mathrm{Area}(\Sigma)}{4G_N}.$$

⬇

We interpret this as the log of effective dim. for Σ

$$\log[\dim H^{eff}_{\Sigma}]$$

This is because $\rho^{\Sigma}_{A_i}$ is expected to be maximally entangled (except the dummy states).

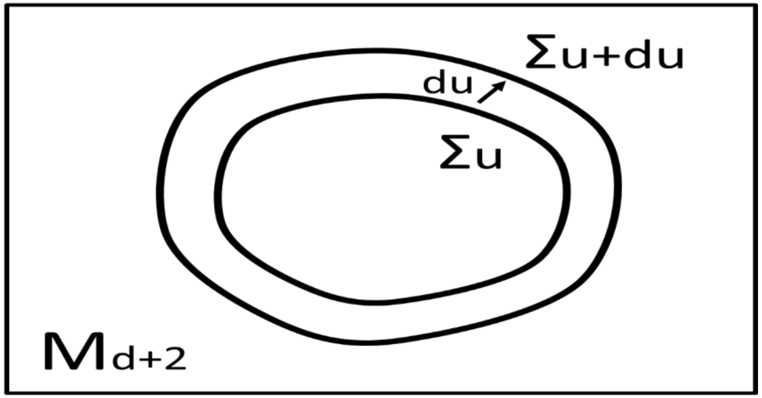
[cf. Differential entropy: Balasubramanian-Chowdhury-Czech-deBoer-Heller 13]

(4-4) Inner Products and Information Metric

Another intriguing physical quantity is an inner product $\langle \Sigma | \Sigma' \rangle$ between two surfaces.

Here focus on the two surfaces separated infinitesimally.

⇒ Consider an information distance between them



$$ds^2 = R^2 du^2 + g_{\mu\nu}(x,u) dx^\mu dx^\nu .$$

Remember the correspondence: $\Sigma_u \Leftrightarrow |\Phi(u)\rangle$.

The information metric is given by

$$1 - \left|\langle \Phi(u) | \Phi(u+du) \rangle\right| = (du)^2 \cdot G_{uu}^{(B)}$$

Define the Hamiltonian H(u) s.t. $|\Phi(u)\rangle$ is its ground state.

Then the standard perturbation theory leads to

$$1 - \left|\langle \Phi(u) | \Phi(u+du) \rangle\right| = (du)^2 \cdot \sum_k \frac{\left|\langle k | \partial_u H(u) | 0 \rangle\right|^2}{(\Delta E_k)^2}.$$

This consideration leads to the following form:

$$G_{uu}^{(B)}=\frac{1}{G_N}\int_{\Sigma u} dx^d \sqrt{g(x)}\int_{\Sigma u} dy^d \sqrt{g(y)}\cdot P_{\mu\nu\alpha\beta}(x,y,u)\frac{\partial g^{\mu\nu}(x,u)}{\partial u}\frac{\partial g^{\alpha\beta}(y,u)}{\partial u}.$$

If the metric is x-independent, we have

$$G_{uu}^{(B)}\sim\frac{1}{G_N}\int_{\Sigma u} dx^d \sqrt{g(x)}\left(K_u\right)^2.$$

Example 1: a flat spacetime $\Rightarrow G_{uu}^{(B)}=0.$

[Translational inv. $\Rightarrow \left|\Phi(u+du)\right\rangle=\left|\Phi(u)\right\rangle$.]

Example 2: an AdS spacetime [Nozaki-Ryu-TT 12]:

$$G_{uu}^{(B)}=N_{\mathrm{deg}}\cdot\frac{V_d}{\varepsilon^d}\cdot e^{du}\Rightarrow \text{Agrees with cMERA for }\mathrm{CFT}_{\mathrm{d+1}}$$

Quantum Estimation Theory

A quantum version of ***Cramer-Rao bound*** argues

$$\left\langle (\delta u)^2 \right\rangle \geq \frac{1}{G^{(B)}_{uu}}.$$

[Helstrom 76]

Mean square error

In the case of AdS/CFT, this leads to

$$\left\langle \frac{\delta z^2}{z^2} \right\rangle = \left\langle (\delta u)^2 \right\rangle \geq \frac{G_N}{\mathrm{Area}(\Sigma)} \sim \frac{1}{\log[\dim H^{eff}_{\Sigma}]} \propto N^{-2}.$$

In the large N limit, this error is highly suppressed.

⇒ Locality of the bulk in the large N limit ?

⇒ Some uncertainty principle of surfaces in QG ?

$$\left\langle \left(\delta \mathrm{Area}(\Sigma)\right)^2 \right\rangle \geq G_N \cdot \mathrm{Area}(\Sigma) \quad .$$

⑤ SS-correspondence in AdS/CFT

[Miyaji-Numasawa-Shiba-Watanabe-TT, in preparation]

We will show that the formulation of cMERA can be generalized so that it describes the SS-duality.

Let us focus on a AdS3/CFT2 setup. It is useful to start with the symmetry of global AdS3 space:

$$ds^2 = R^2(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2),$$

whose isometry $SL(2,R) \times SL(2,R)$ is generated by

$$L_0 = i\partial_+, \quad \tilde{L}_0 = \partial_-,$$

$$L_{\pm 1} = ie^{\pm ix^+} \left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_+ - \frac{1}{\sinh 2\rho}\partial_- \mp \frac{i}{2}\partial_\rho \right],$$

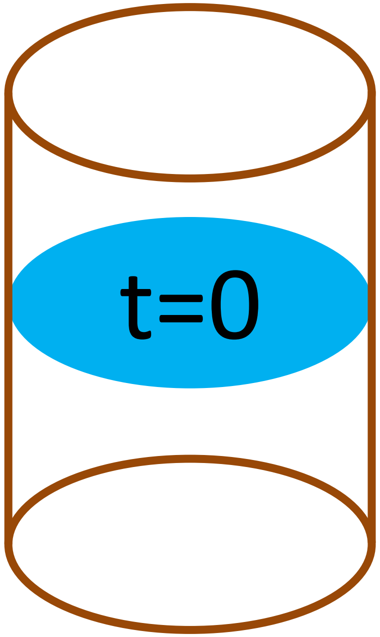
$$\tilde{L}_{\pm 1} = ie^{\pm ix^-} \left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_- - \frac{1}{\sinh 2\rho}\partial_+ \mp \frac{i}{2}\partial_\rho \right].$$

In particular, we are interested in the SL(2,R) subgroup which preserves the time slice t=0 (i.e. H2) of the AdS3.

They are generated by $l_n = \tilde{L}_{-n} - L_n, \quad (n = 0, \pm 1)$, which annihilates the boundary states.

The SL(2,R) action which maps ρ=0 to the point (ρ, ϕ) is given by

$$g(\rho,\phi) = e^{i\phi l_0} e^{\frac{\rho}{2}(l_{-1} - l_1)}.$$



cMERA for the ground state of CFT2 is formulated as:

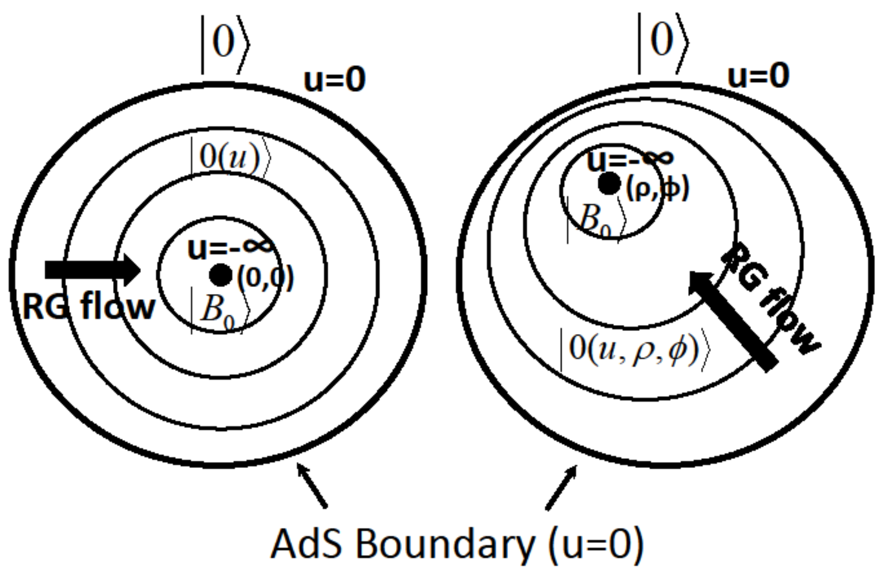
$$|0\rangle = P\exp\left(-i\int_{-\infty}^{0}\hat{K}(u)du\right)|B_0\rangle.$$

← boundary (Ishibashi) state for the identity sector

If we act the SL(2,R) transformation $g(\rho,\phi)$ we find

$$|0\rangle = P\exp\left(-i\int_{-\infty}^{0}\hat{K}_{(\rho,\phi)}(u)du\right)|B_0\rangle,$$

where $\hat{K}_{(\rho,\phi)}(u) = g(\rho,\phi)\cdot\hat{K}(u)\cdot g(\rho,\phi)^{-1}.$



More generally, we can describe the **diffeomorphism** by taking into account $l_n = \tilde{L}_{-n} - L_n, \quad (|n| = 2,3,..)$:

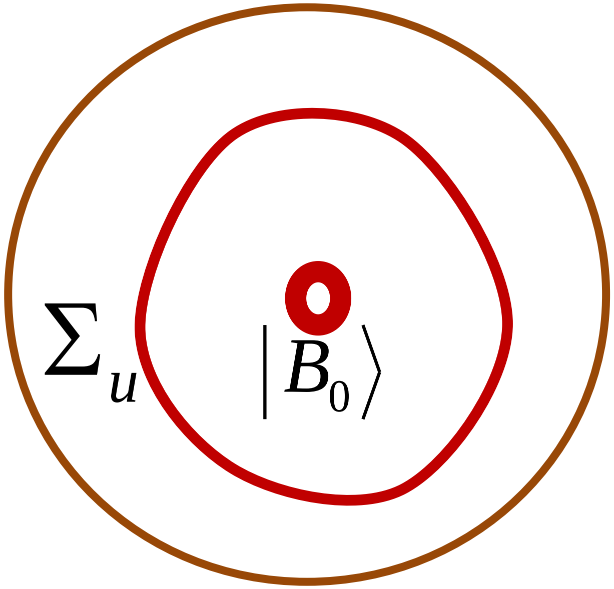
$$|0\rangle = P\exp\left(-i\int_{-\infty}^{0} \hat{K}_g(u)du\right)|B_0\rangle,$$

$$\hat{K}_g(u) = \hat{g}(u)\hat{K}(u)\hat{g}(u)^{-1} + \partial_u g(u)\cdot g(u)^{-1},$$

$$\text{where} \quad g(u) = \exp\left[\sum_n \xi_u(u)\, l_n\right] \text{ with } \quad \xi_n(0) = 0.$$

The dual state of a surface Σ_u expected in SS-correspondence is given by the form:

$$|\Phi(\Sigma_u)\rangle = P\exp\left(-i\int_{-\infty}^{u} \hat{K}_g(s)ds\right)|B_0\rangle.$$



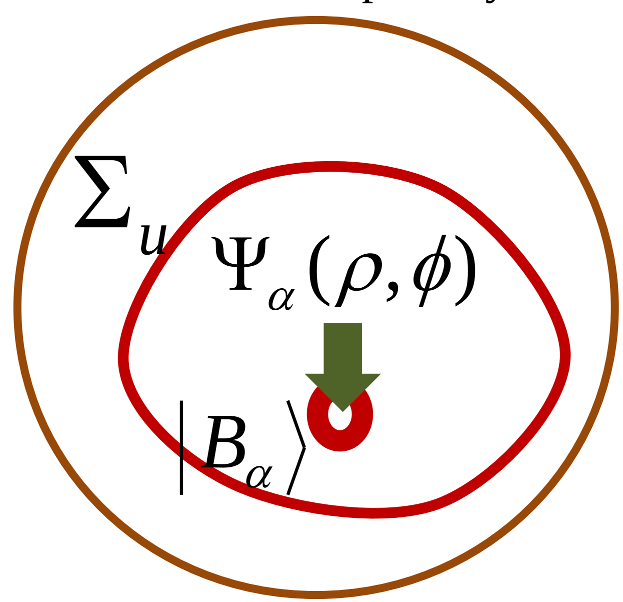
How to describe the bulk excitation ?

We argue the identification:

$$\underbrace{\Psi_\alpha(\rho,\phi)}_{\text{Bulk local operator}} \left|0\right\rangle_{Bulk}$$

$$\Leftrightarrow \quad \left|\Psi_\alpha(\rho,\phi)\right\rangle_{CFT} = P\exp\left(-i\int_{-\infty}^{0}\hat{K}_{(\rho,\phi)}(s)ds\right) \underbrace{\left|B_\alpha\right\rangle}_{\substack{\text{Ishibashi state} \\ \text{for primary } \alpha}} \quad .$$

This is because the local operator insertion does not change the bulk metric (= entanglement).



We argue this state is evaluated as

$$\left|\Psi_\alpha(\rho,\phi)\right\rangle_{CFT} \approx g(\rho,\phi)\cdot e^{\frac{\pi}{2}i(L_0+\tilde{L}_0)} \cdot \underbrace{e^{-\varepsilon H}}_{\text{some UV cut off}} \cdot \underbrace{\left|J_\alpha\right\rangle}_{\substack{SL(2,R) \\ \text{Ishibashi State}}} .$$

This satisfies the correct EOM:

$$\Box_{\text{AdS3}}\left|\Psi_\alpha(\rho,\phi,t)\right\rangle_{CFT} = 0.$$

We can compute the information metric:

$$|\left\langle\Psi_\alpha(\rho,\phi)\middle|\Psi_\alpha(\rho+\delta\rho,\phi+\delta\phi)\right\rangle| = 1 - G_{ab}dx^a dx^b,$$

$$ds^2 = \frac{1}{\varepsilon^2}(d\rho^2 + \sinh^2\rho\, d\phi^2).$$

$\approx c^2$ (as in AdS/CFT) by choosing $\varepsilon \approx c^{-1}$.

⑥ Conclusions

- Quantum entanglement represents a geometry of quantum state in many-body systems.

Ex.1 HEE ⇒ EE probes the metric of hol. geometry

Ex.2 AdS/TN(MERA,…) ⇒ Entanglement = geometry

Ex.3 Boundary state ⇔ trivial (point-like) space

- This gravity/entanglement duality looks more general than AdS/CFT and even than holography.

⇒ We proposed the surface/state correspondence.

The SS-duality argues

Top. trivial convex surface	⇔	a pure state
Top. non-trivial surface	⇔	a mixed state
Zero size surface	⇔	Boundary states
Area of surface	=	log[Eff. Dimension]
(Extrinsic curvature)^2	=	Information metric

Future problems

- Derivation of Einstein eq.
- Application to spacetime without (time-like) boundary e.g. de-Sitter spaces.
- Analysis of compact directions e.g. S5 in AdS5 × S5.