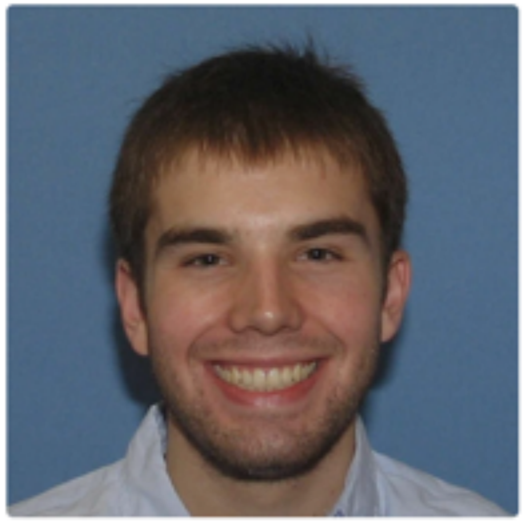
Optical Conductivity in the Cuprates: from Mottness to Scale Invariance

Thanks to: NSF, EFRC (DOE)

Brandon Langley



Garrett Vanacore

Kridsangaphong Limtragool

PHYSICAL REVIEW B VOLUME 43, NUMBER 10 1 APRIL 1991

Optical spectra of $La_{2-x}Sr_xCuO_4$: Effect of carrier doping on the electronic structure of the CuO_2 plane

S. Uchida
Engineering Research Institute, University of Tokyo, Yayoi, Tokyo 113, Japan

T. Ido and H. Takagi
Department of Applied Physics, University of Tokyo, Hongo, Tokyo 113, Japan

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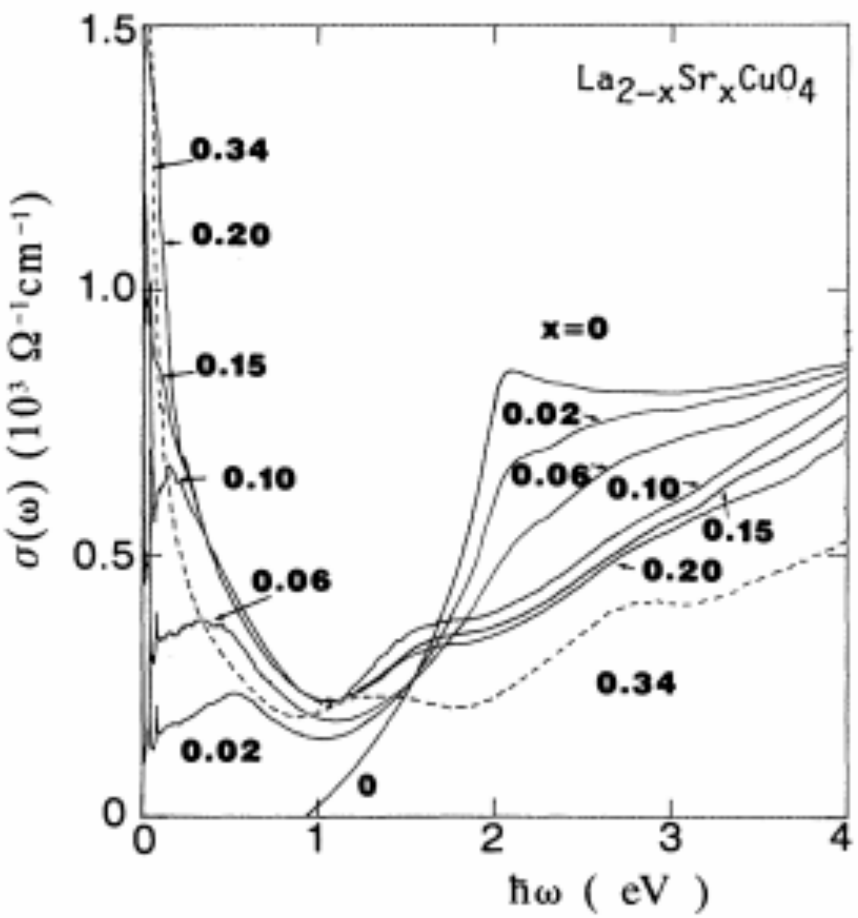
Department of Physics, University of Tokyo, Hongo, Tokyo 113, Japan

S. Tajima
Superconducting Research Laboratory, International Superconductivity Technology Center, Shinonome, Tokyo, Japan
(Received 30 August 1990)



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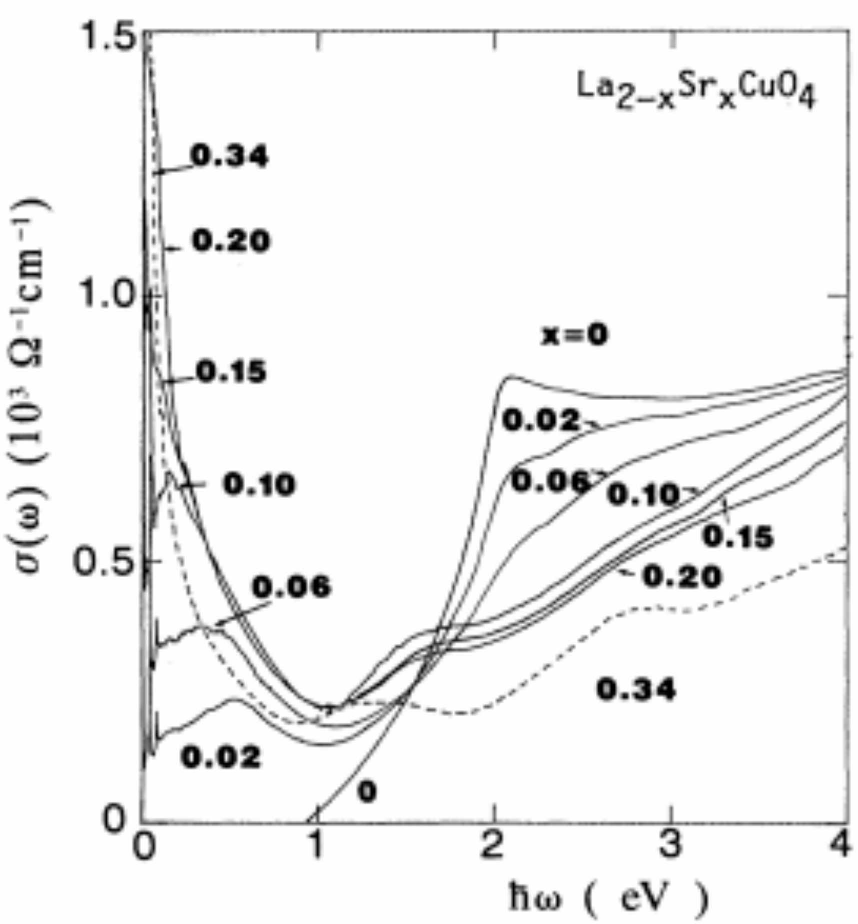
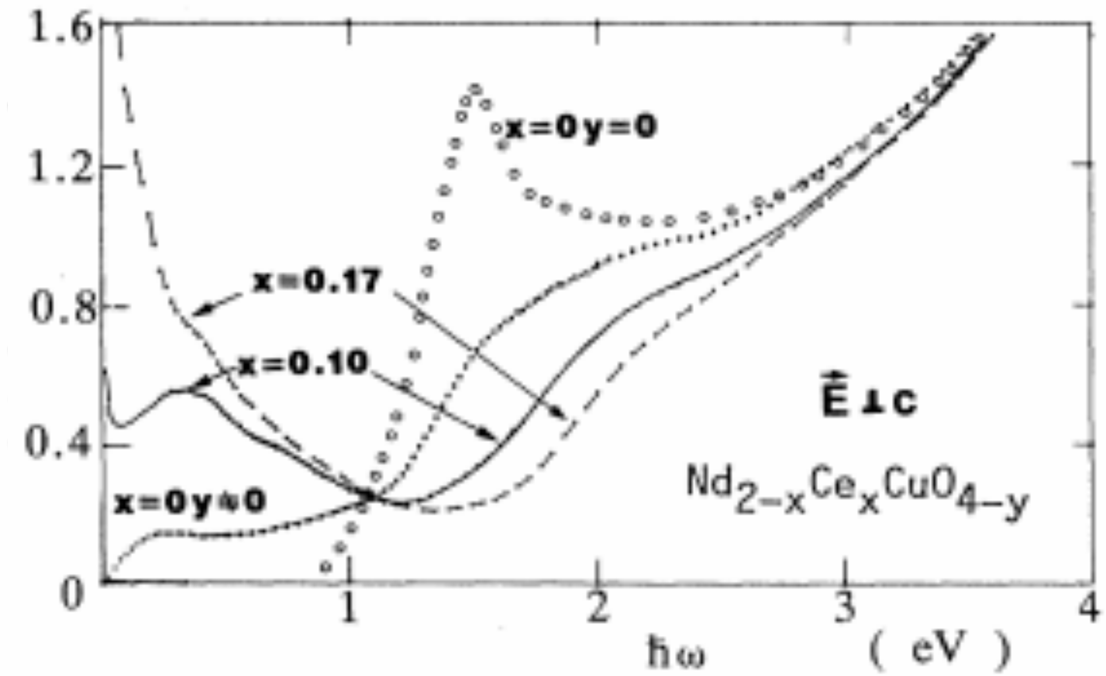
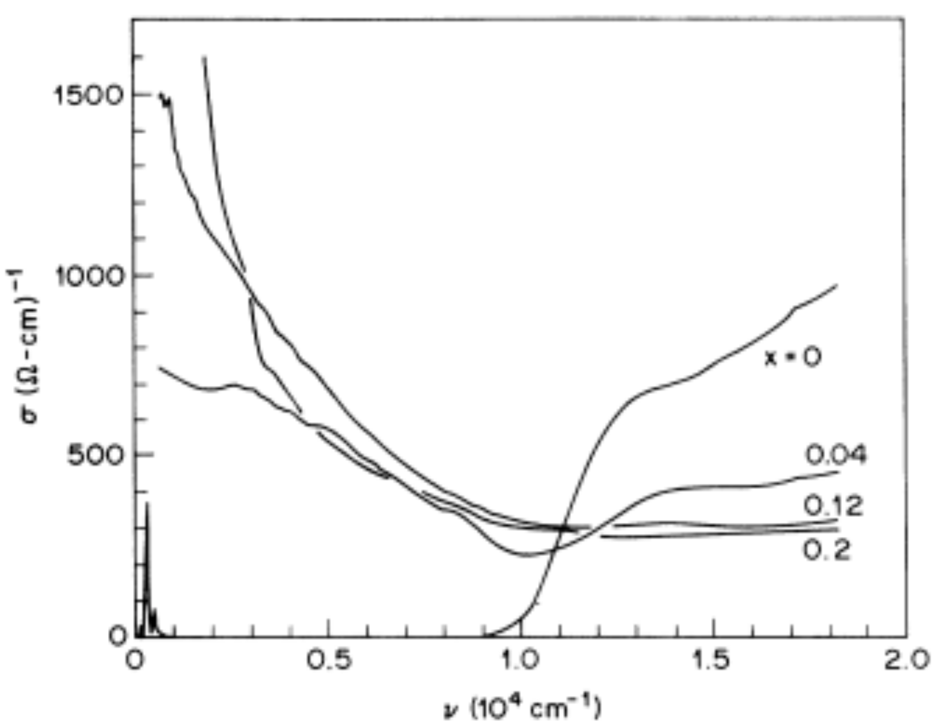




Growth of the optical conductivity in the Cu-O planes

S. L. Cooper, G. A. Thomas, J. Orenstein, D. H. Rapkine, A. J. Millis, S-W. Cheong, and A. S. Cooper
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

Z. Fisk
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 7 March 1990)



$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$

optical gap

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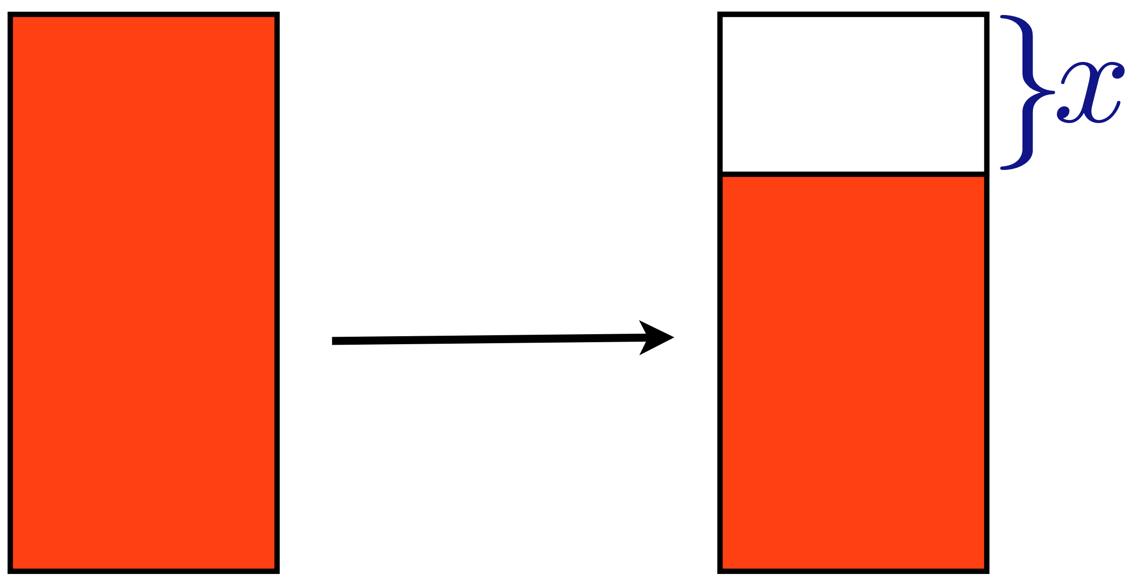

optical gap

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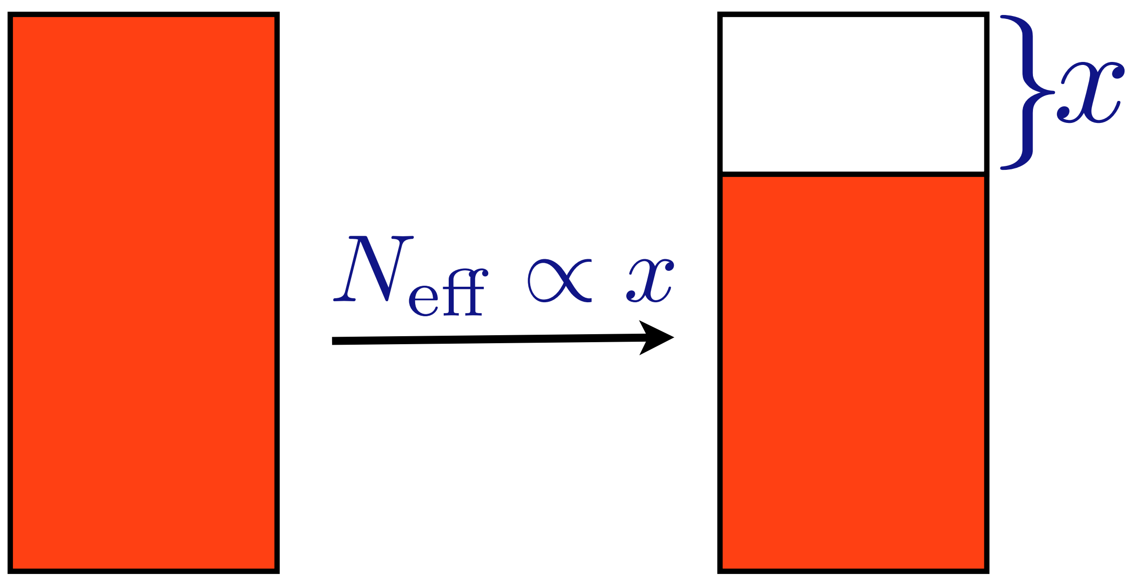
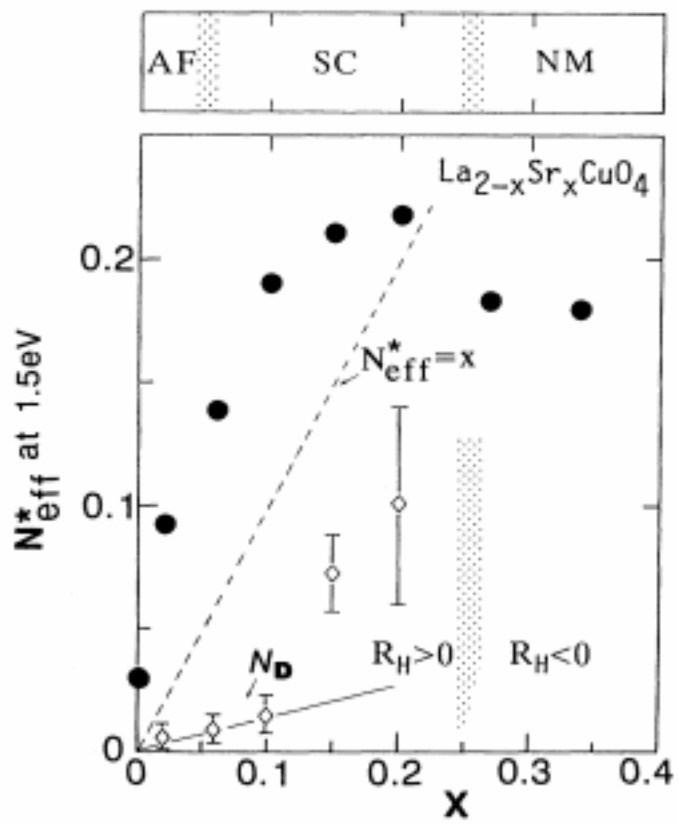
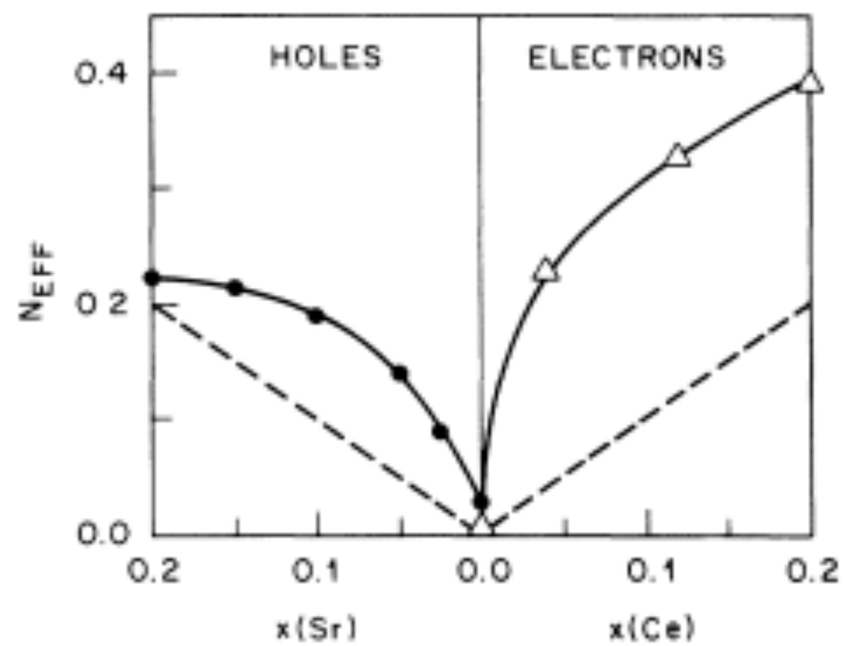
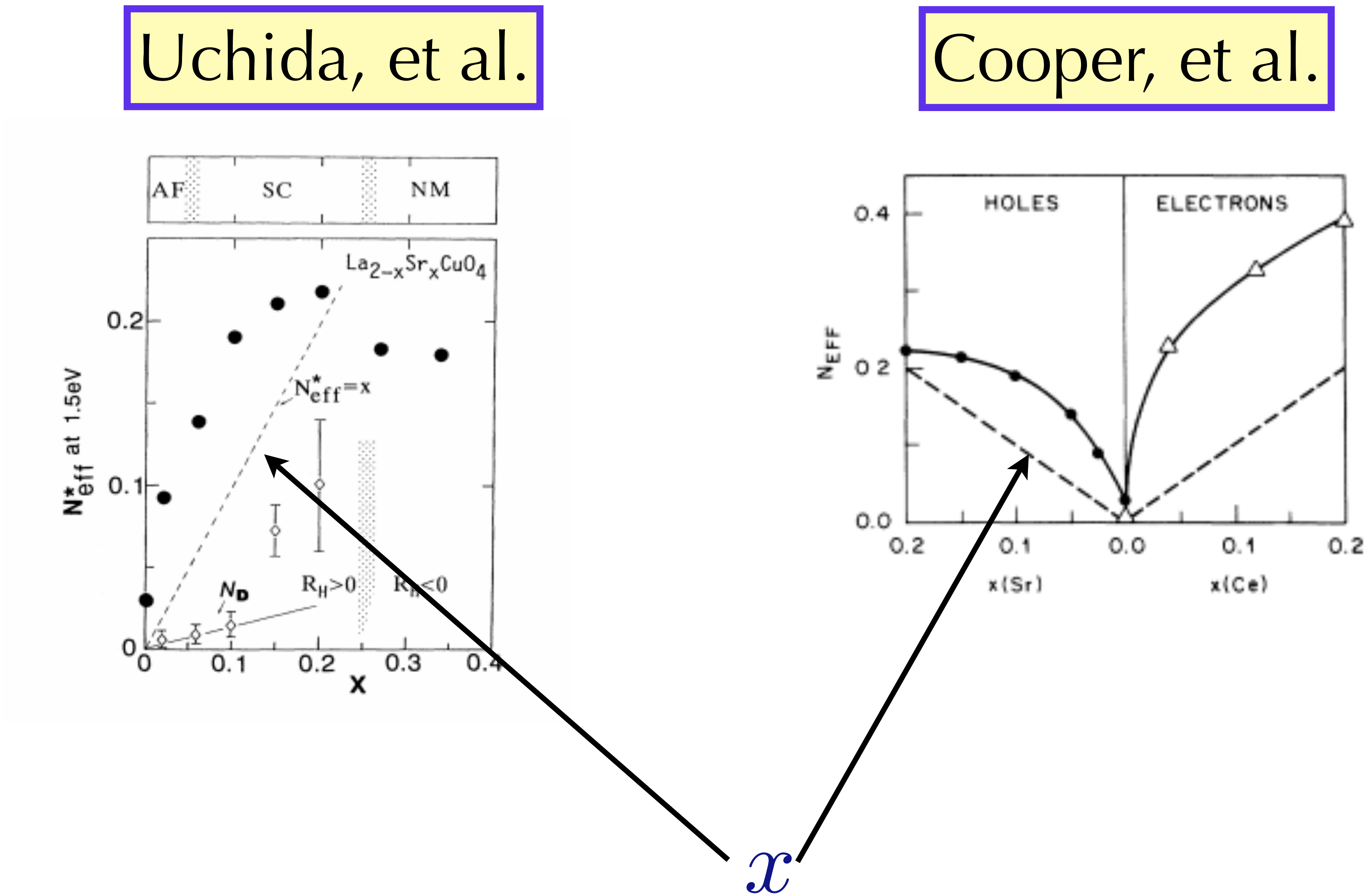


Uchida, et al.



Cooper, et al.

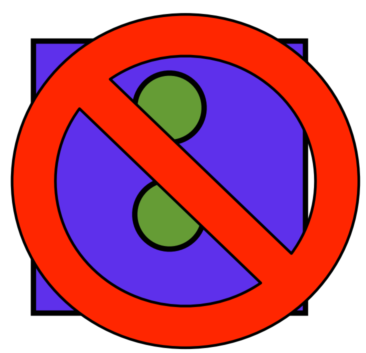


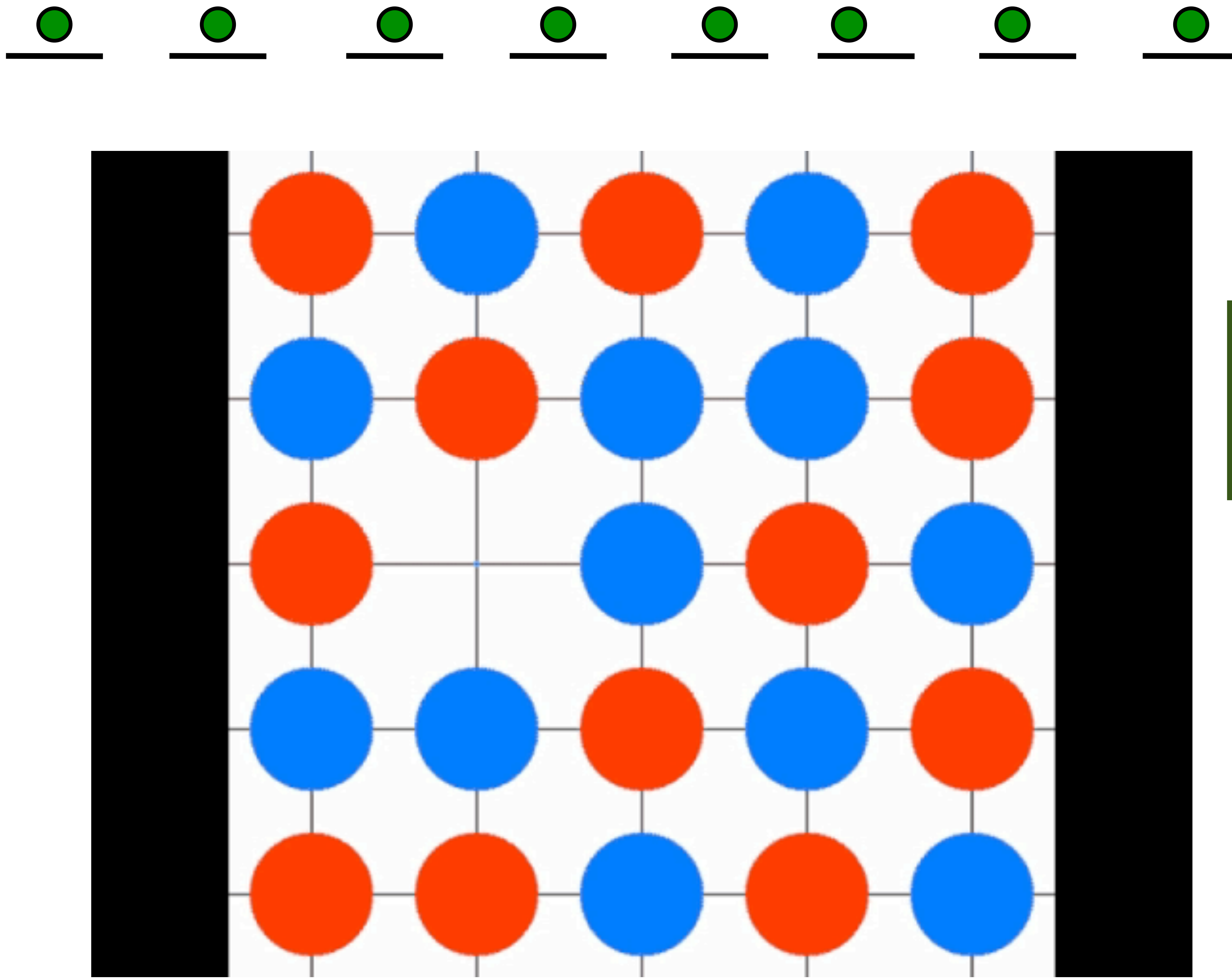

Uchida, et al.
Cooper, et al.
AF
SC
NM
$La_{2-x}Sr_xCuO_4$
N^*_{eff} at 1.5eV
$N^*_{eff}=x$
N_D
$R_H>0$
$R_H<0$
0
0.1
0.2
0.3
0.4
X
HOLES
ELECTRONS
N_{EFF}
0.0
0.2
0.4
x(Sr)
x(Ce)
x

excess carriers?

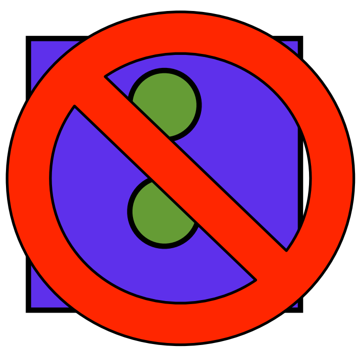
Mottness

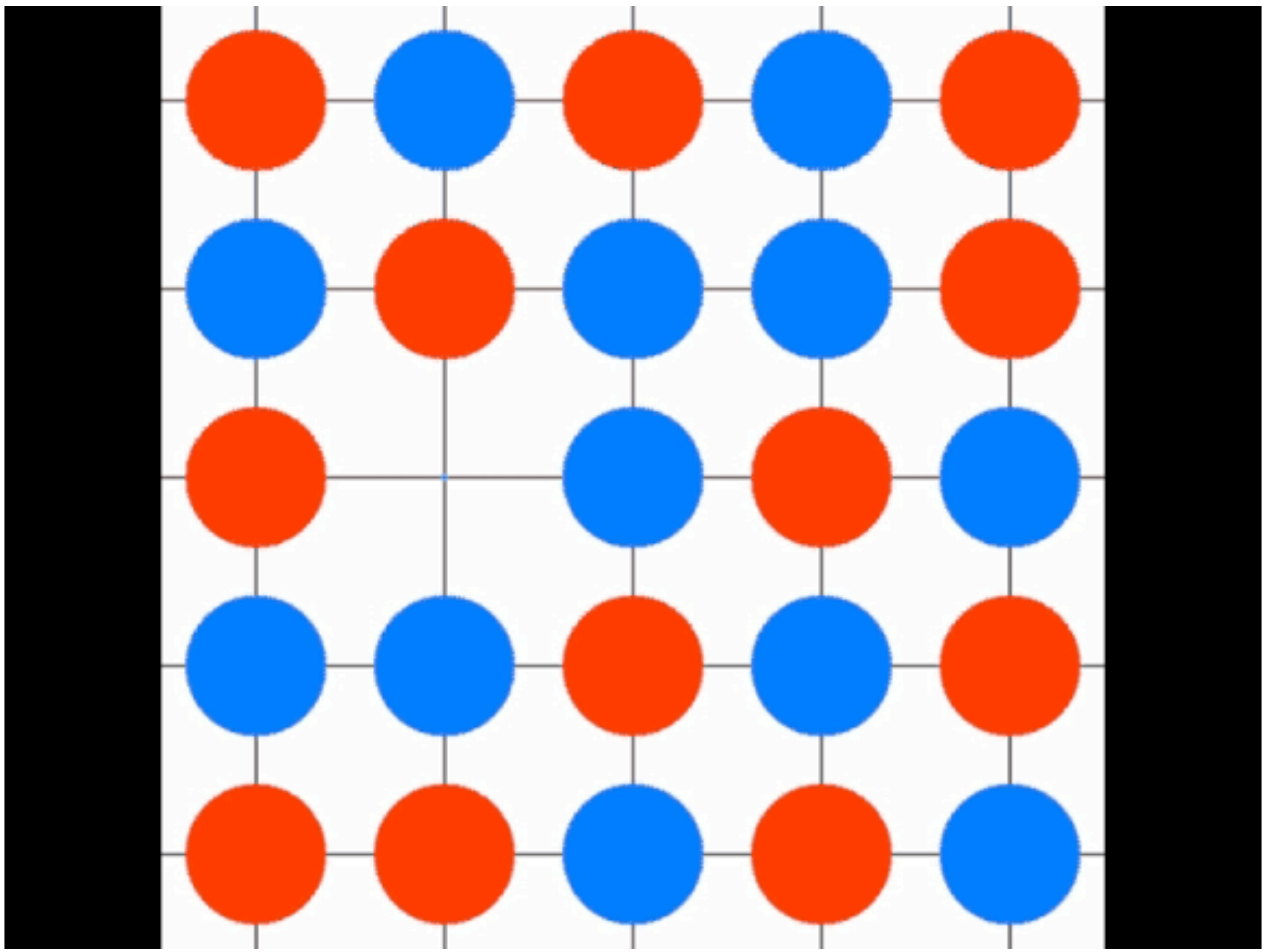
classical
(atomic) limit



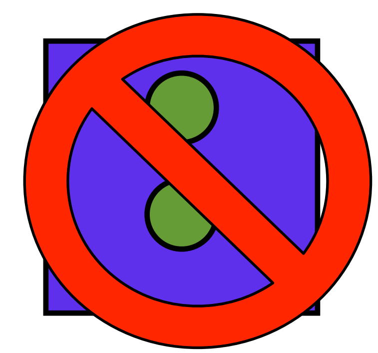


classical
(atomic) limit

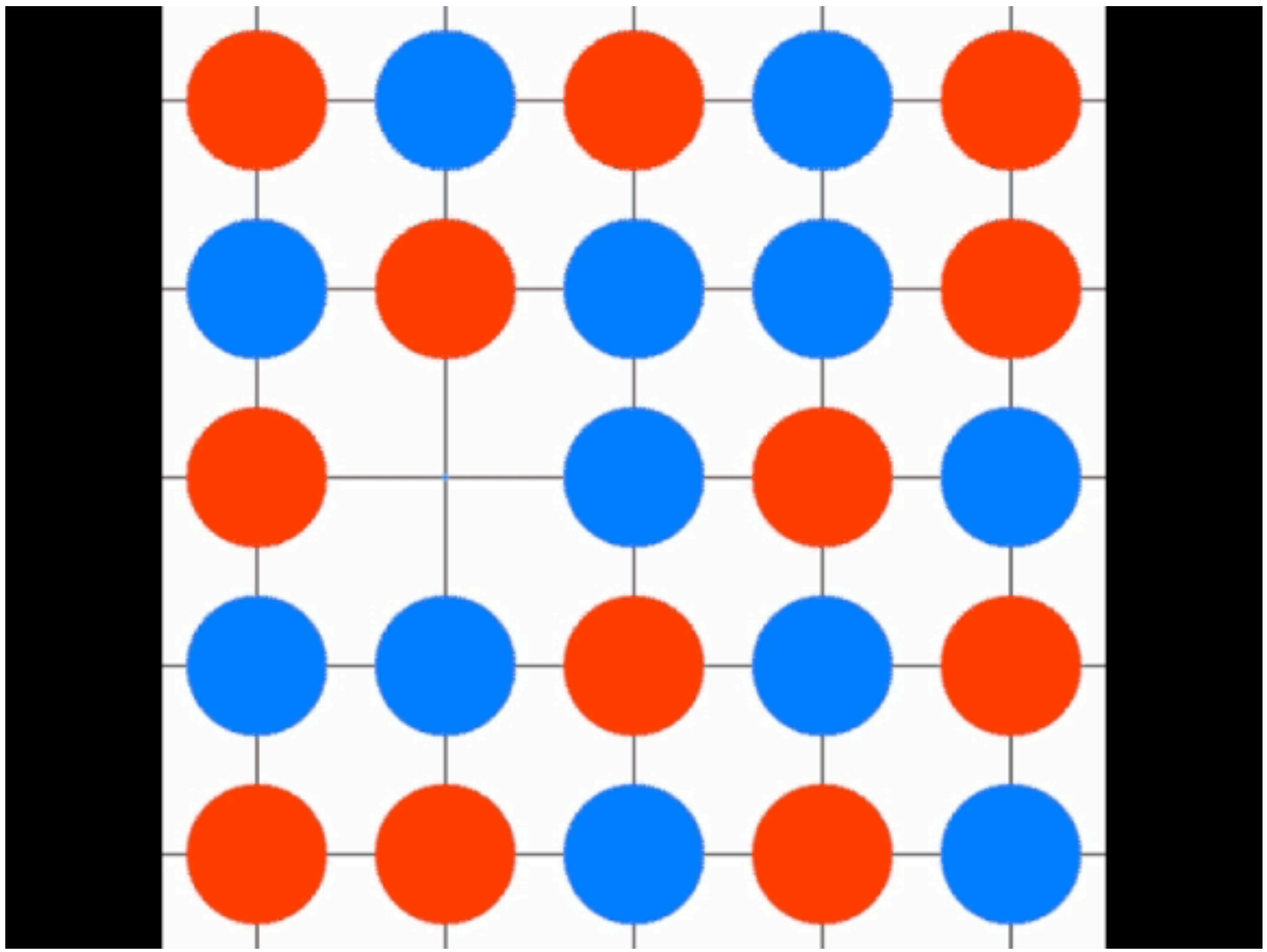




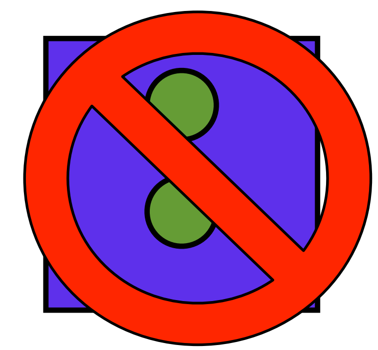
classical
(atomic) limit



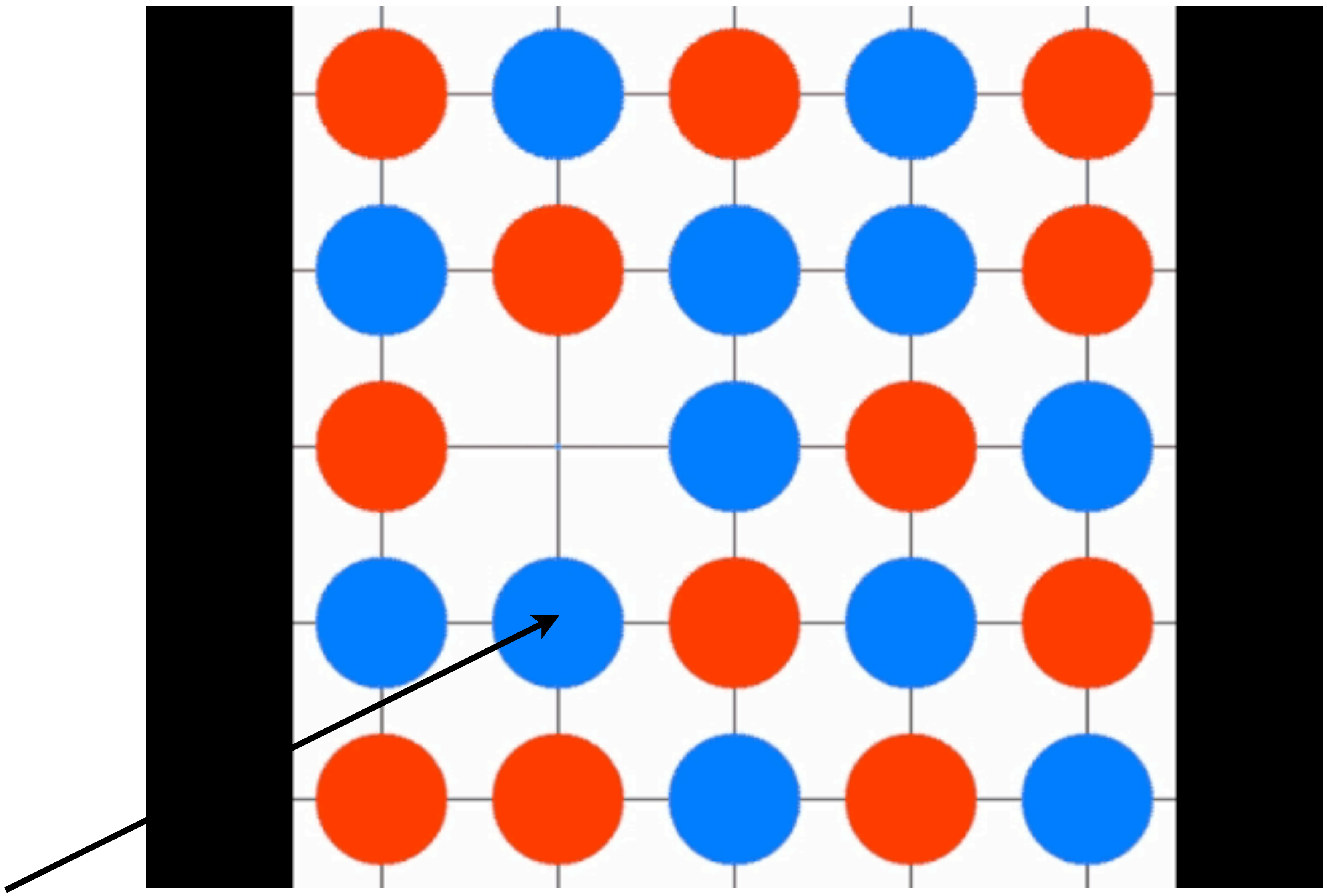
number of empty sites=x



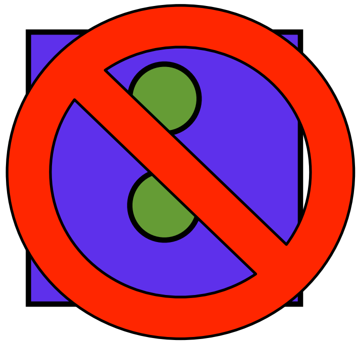
classical (atomic) limit



number of empty sites=x → $N_{\rm eff} \propto x$



classical (atomic) limit



2 states

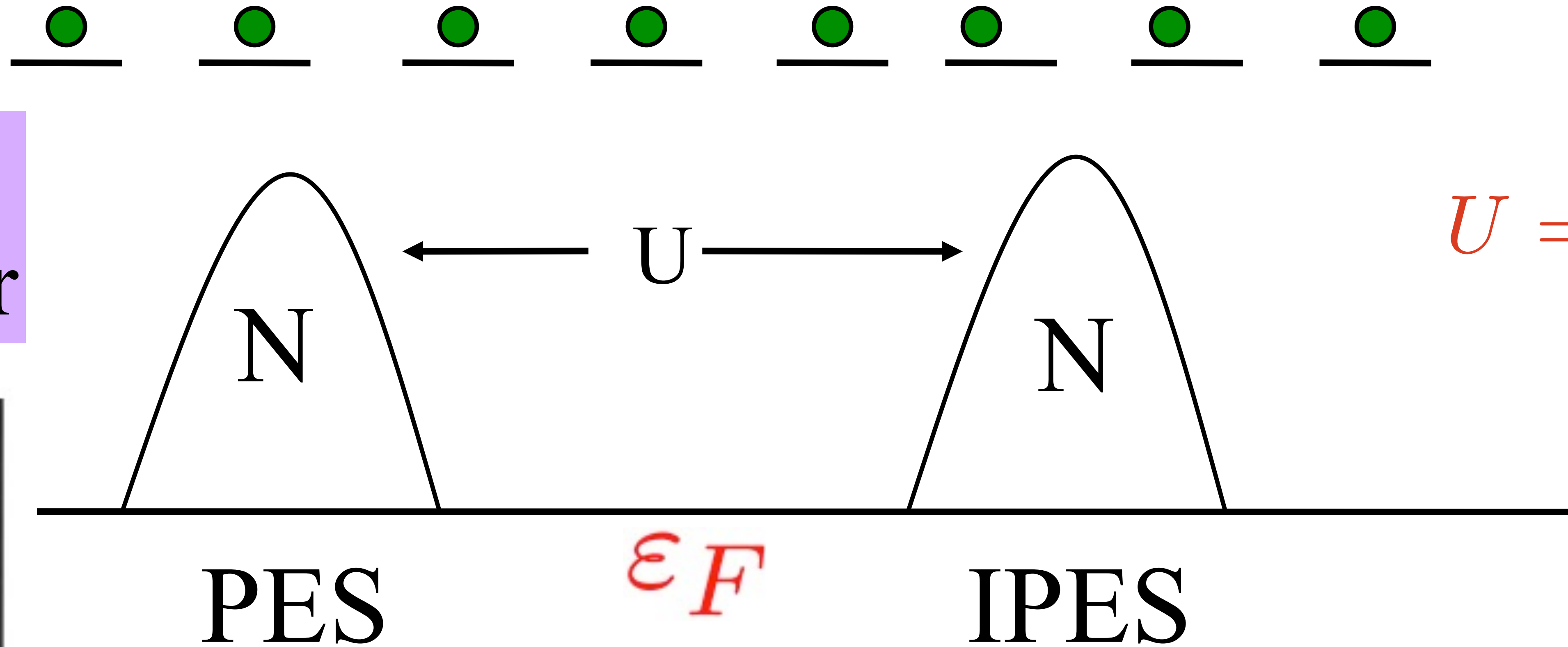
number of empty sites=x $\Rightarrow$ $N_{\text{eff}} \propto x$

Mott insulator

Mott
insulator

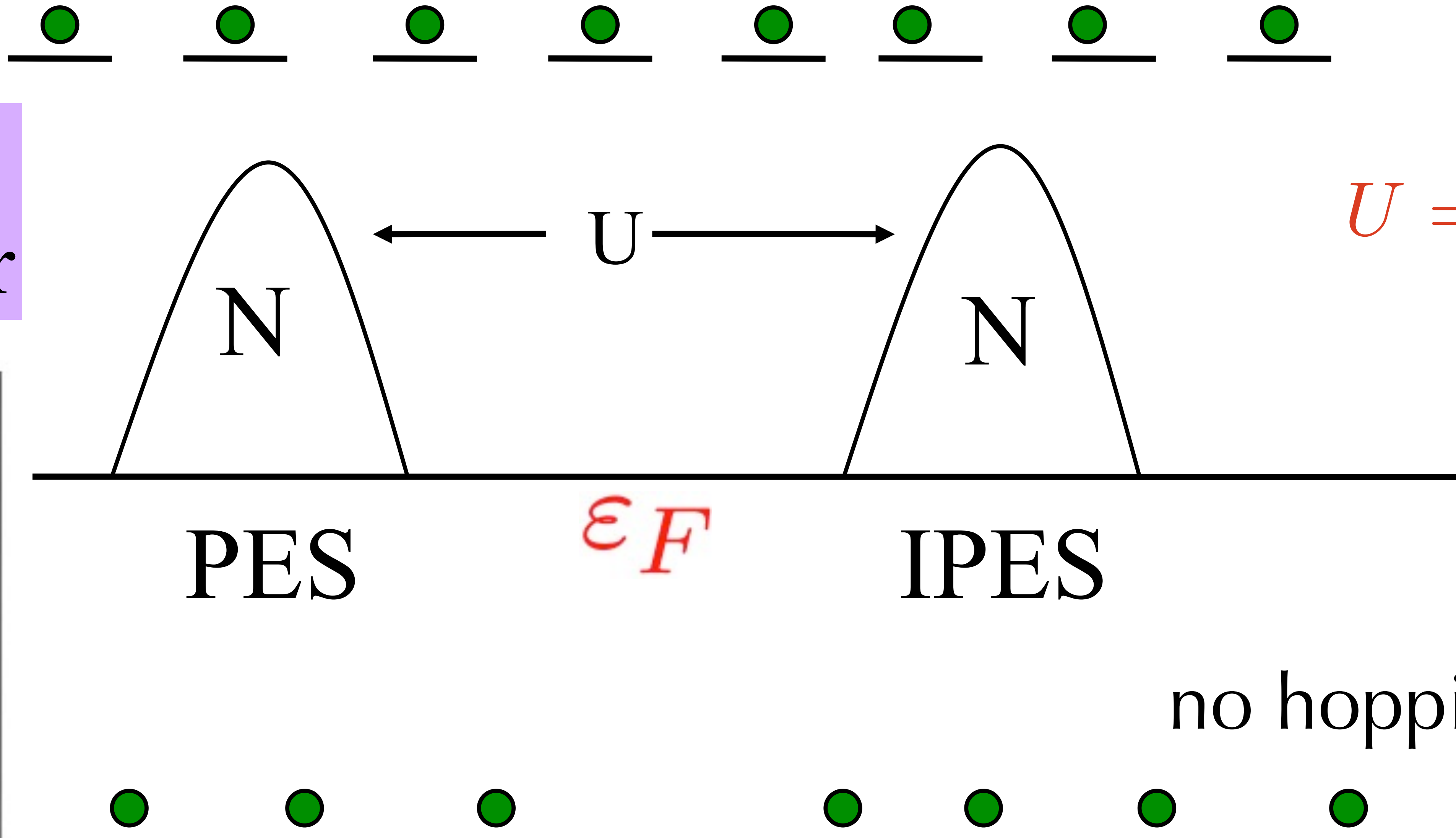
$U = \infty$

Mott insulator



$U = \infty$

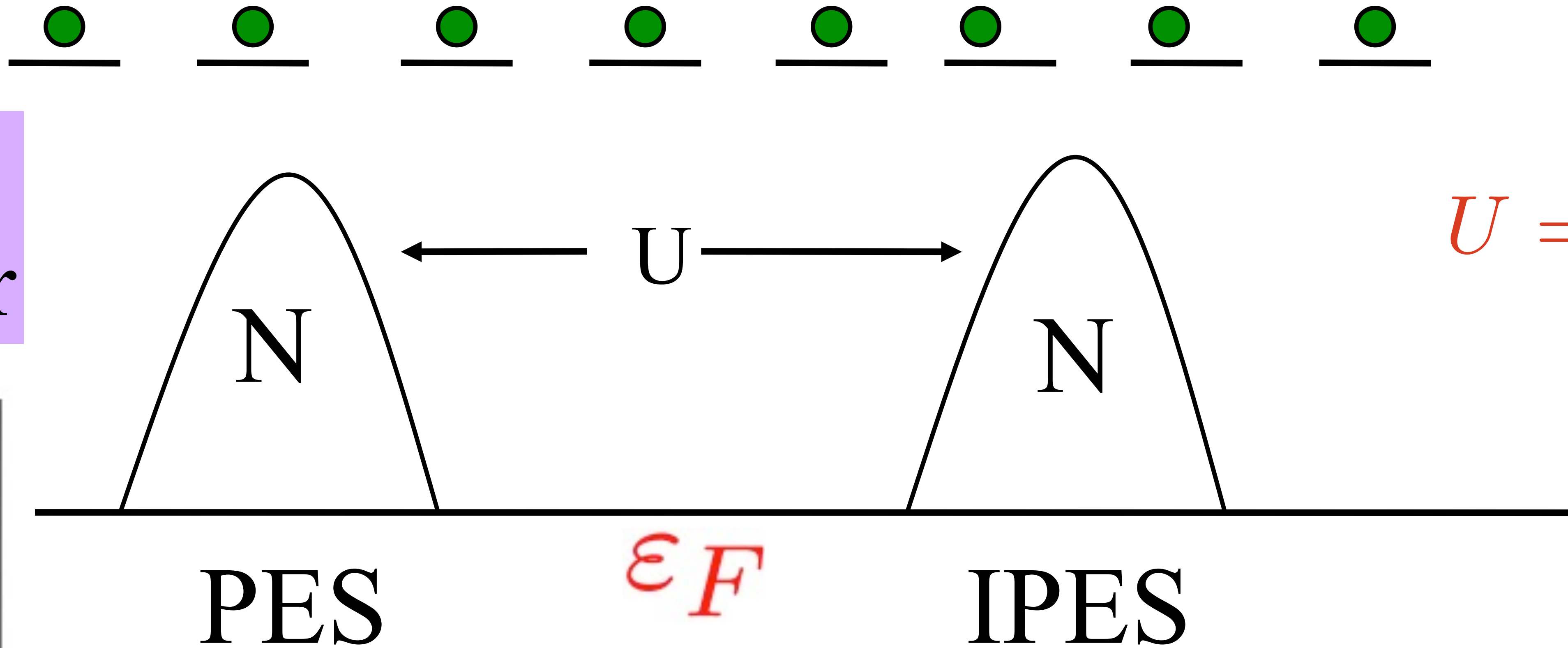
Mott insulator



$U = \infty$

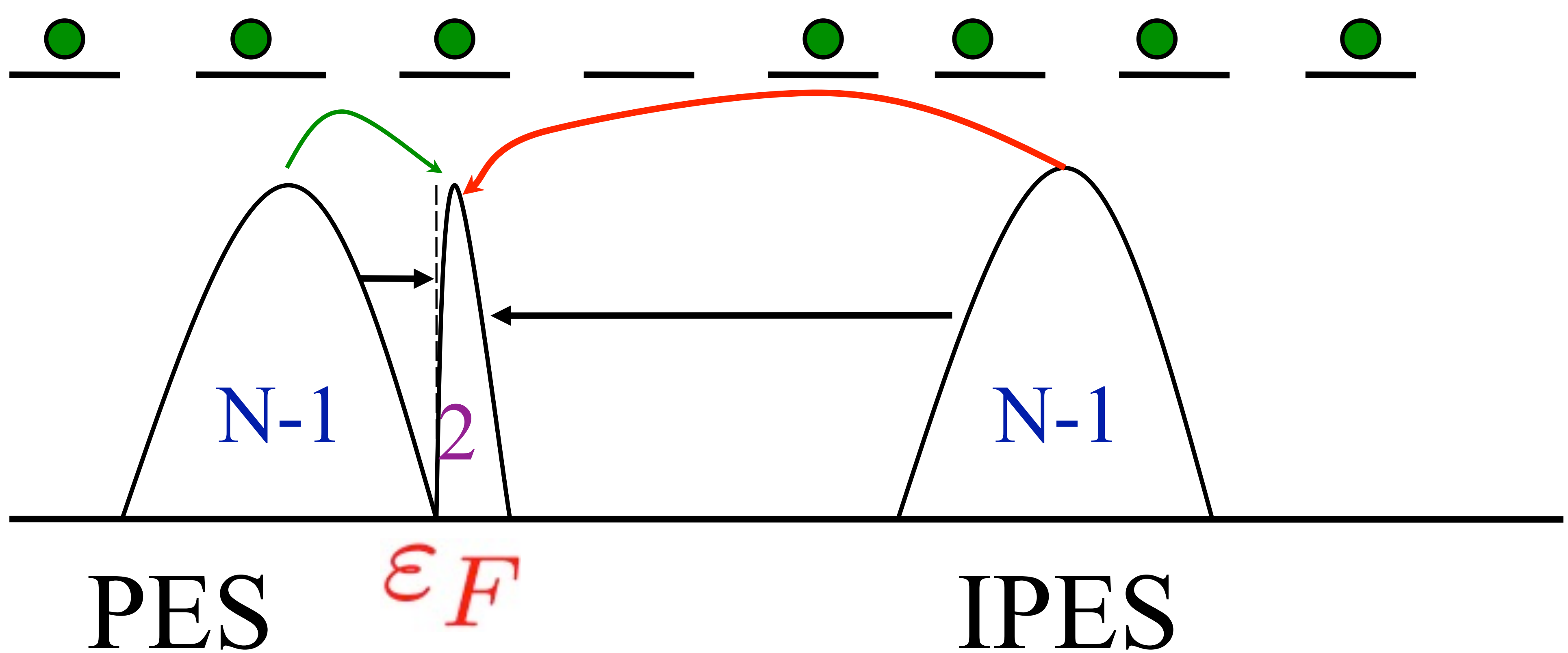
no hopping:

Mott insulator



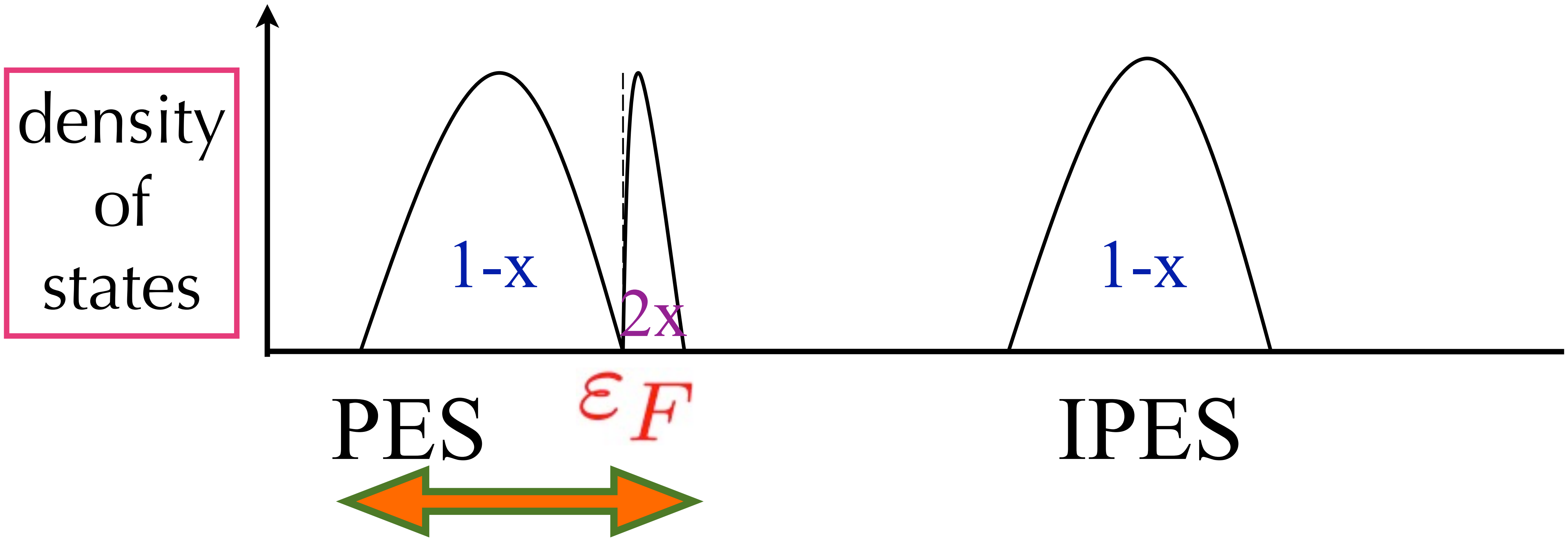
$U = \infty$

no hopping:

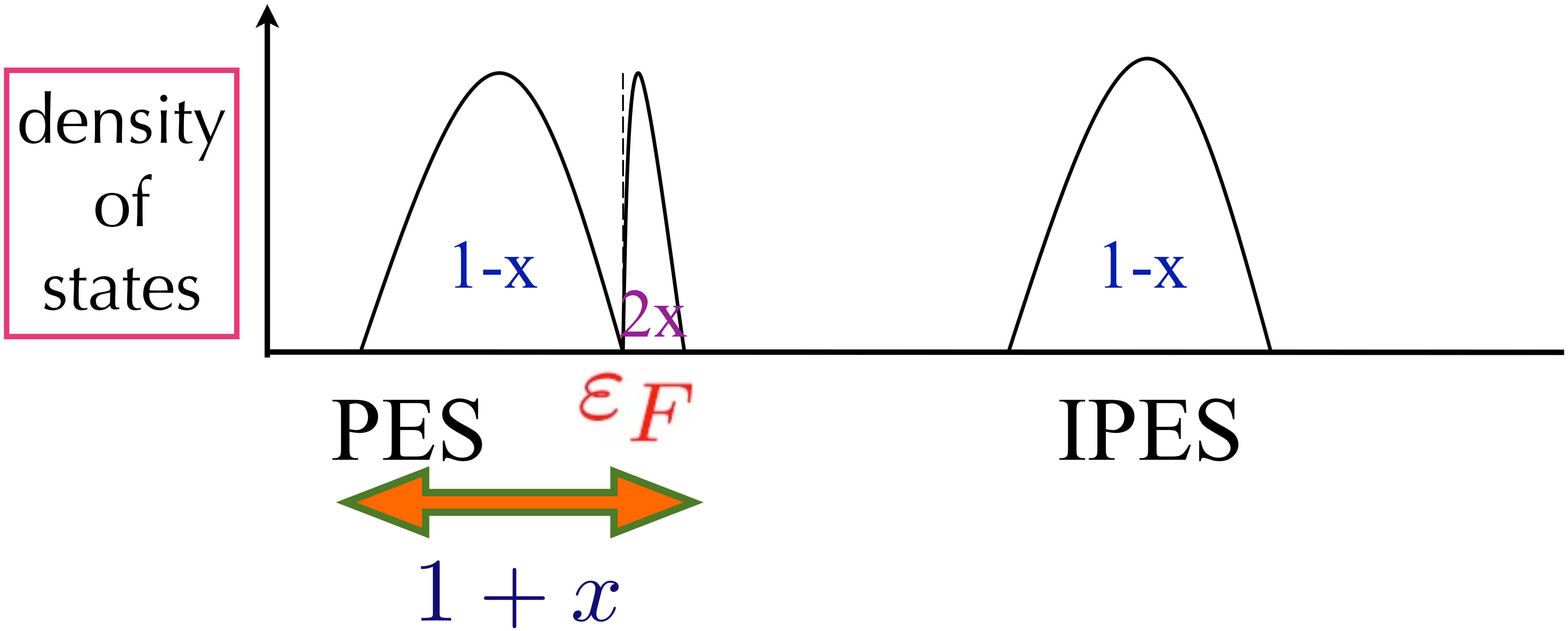


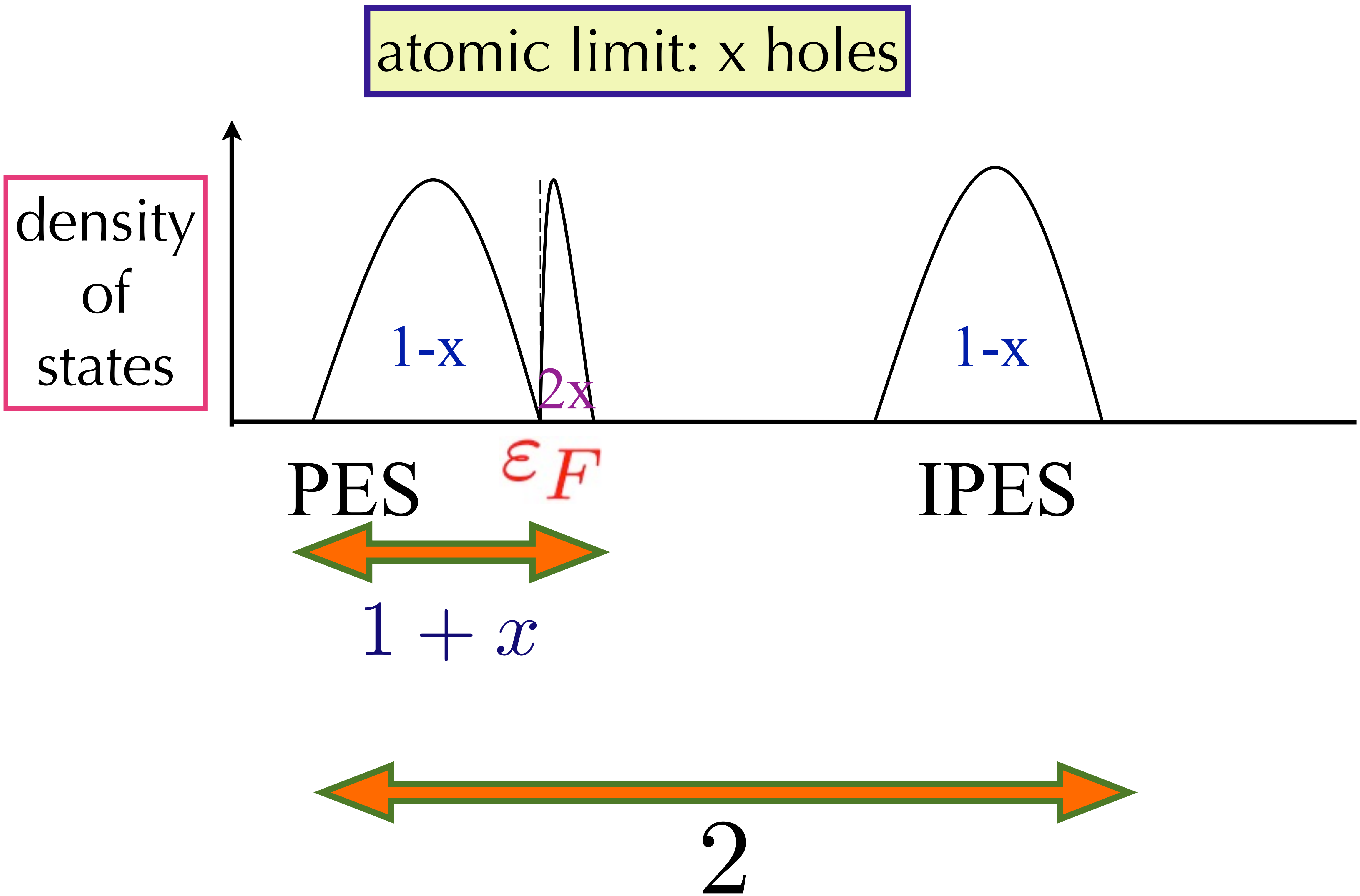
Sawatzky

atomic limit: x holes



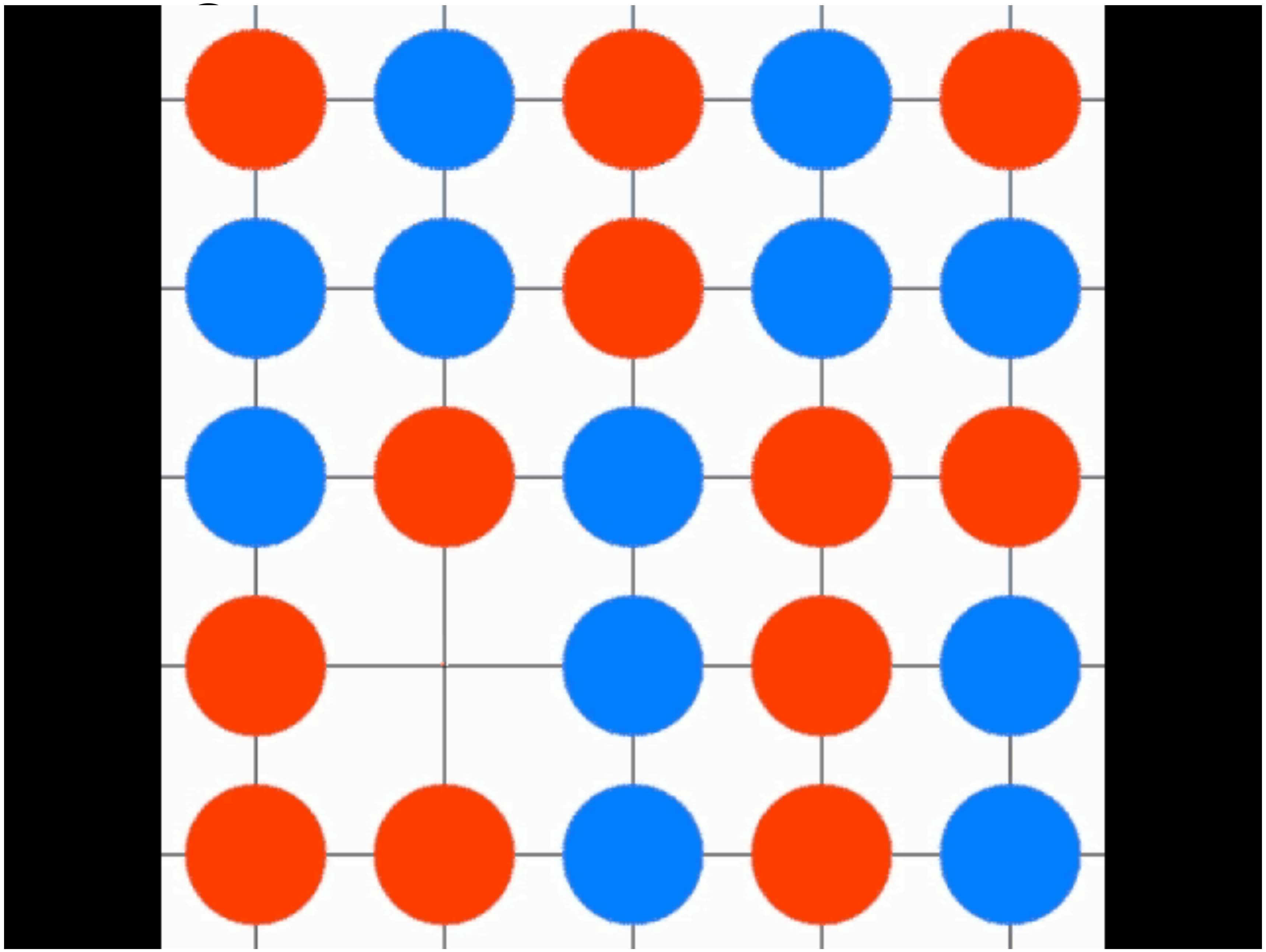
atomic limit: x holes



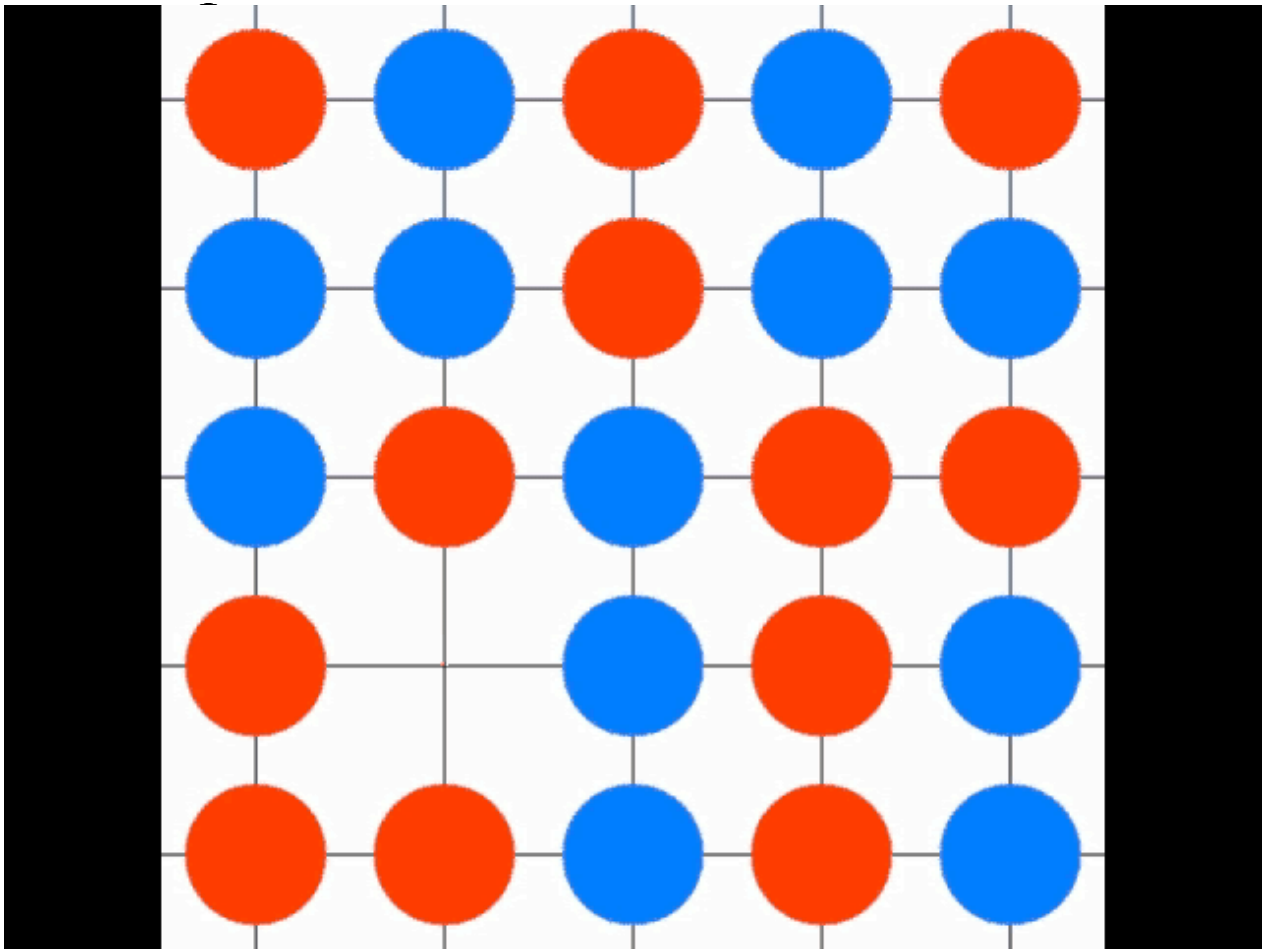


atomic limit: x holes
density of states
1-x
2x
1-x
PES
ε_F
IPES
$1+x$
2

U finite $U \gg t$

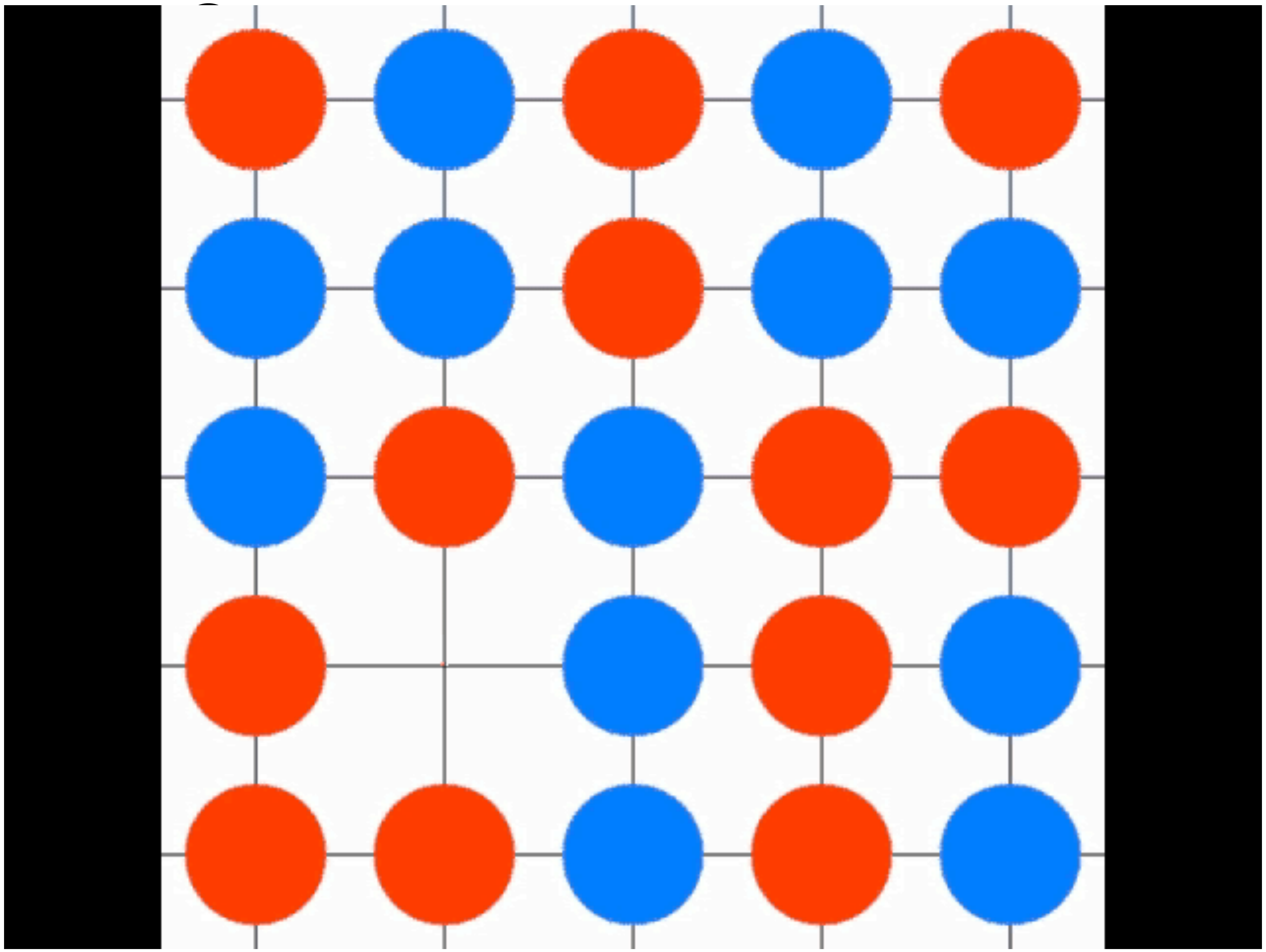


U finite $U \gg t$



double occupancy in ground state!!

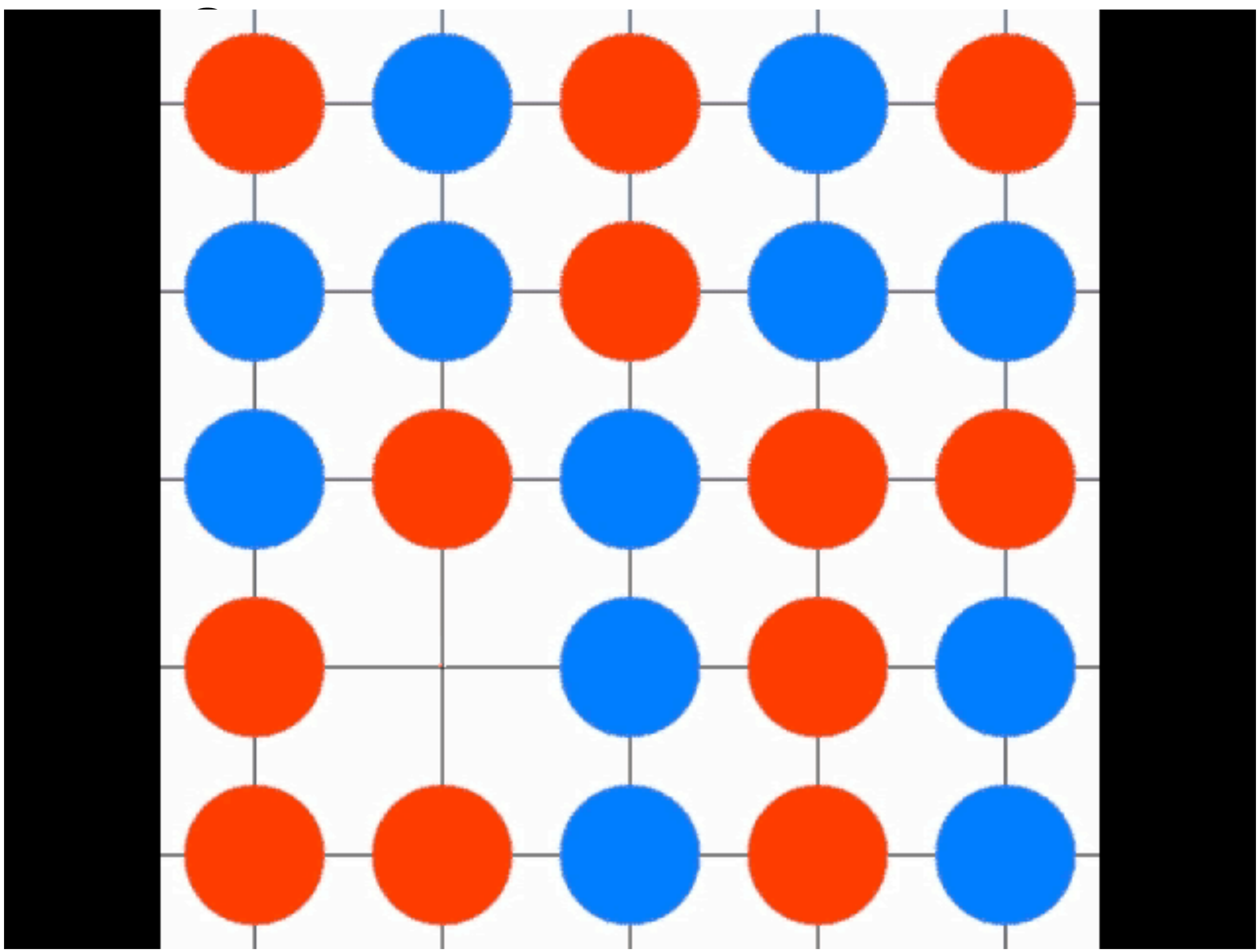
U finite $U \gg t$



double occupancy in ground state!!

$W_{\mathrm{PES}} > 1 + x$

U finite $U \gg t$



$N_{\text{eff}} \neq \#x$

double occupancy in ground state!!

$W_{\text{PES}} > 1 + x$

why is this a problem?

counting electron states

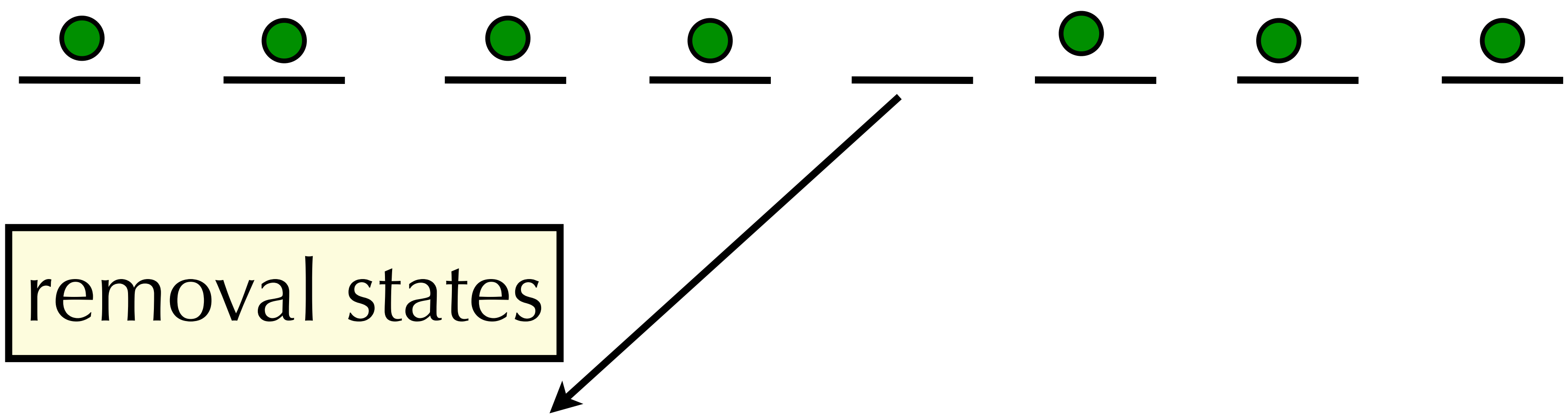
need to know: N (number of sites)

counting electron states

$$x = n_h/N$$

need to know: N (number of sites)

counting electron states



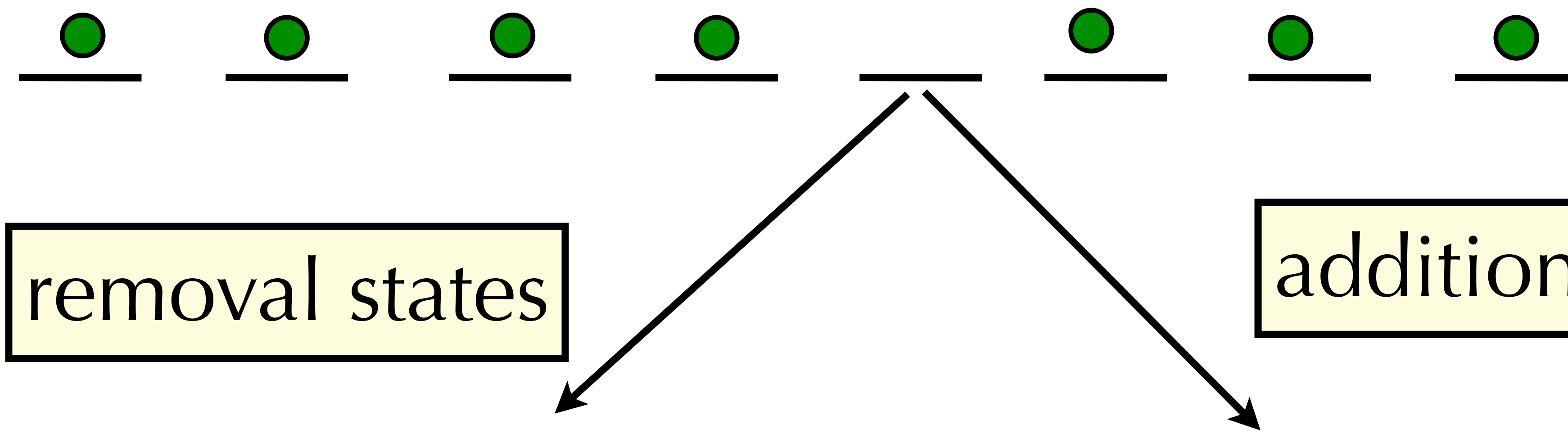
$x = n_h/N$

removal states

$1 - x$

need to know: N (number of sites)

counting electron states



$$x = n_h/N$$

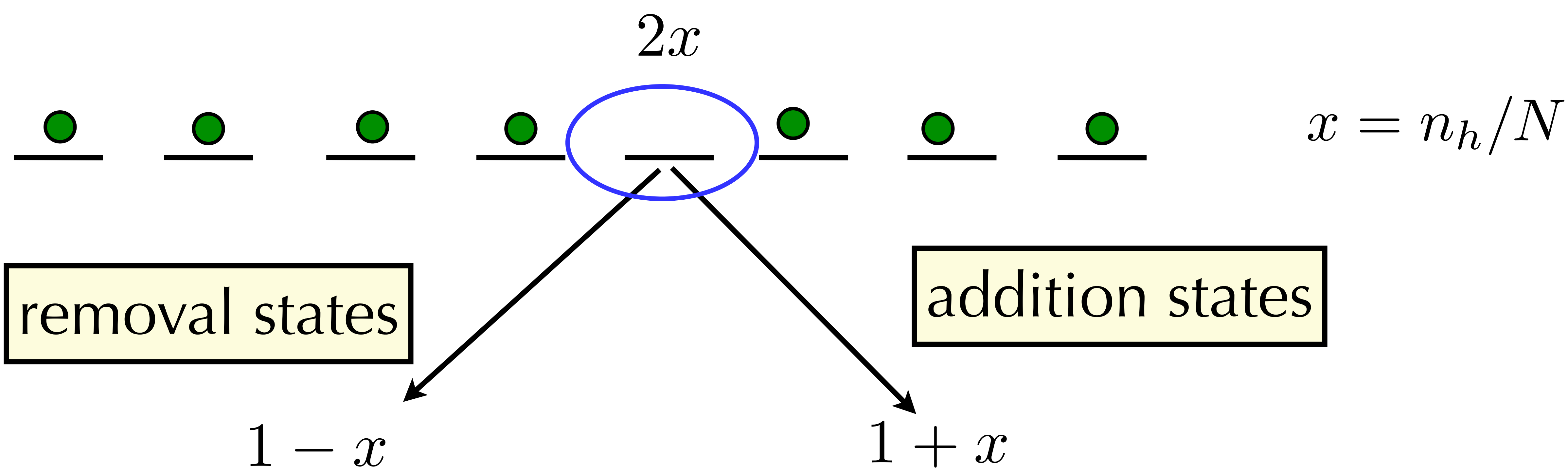
removal states

addition states

$1 - x$ $\qquad$ $1 + x$

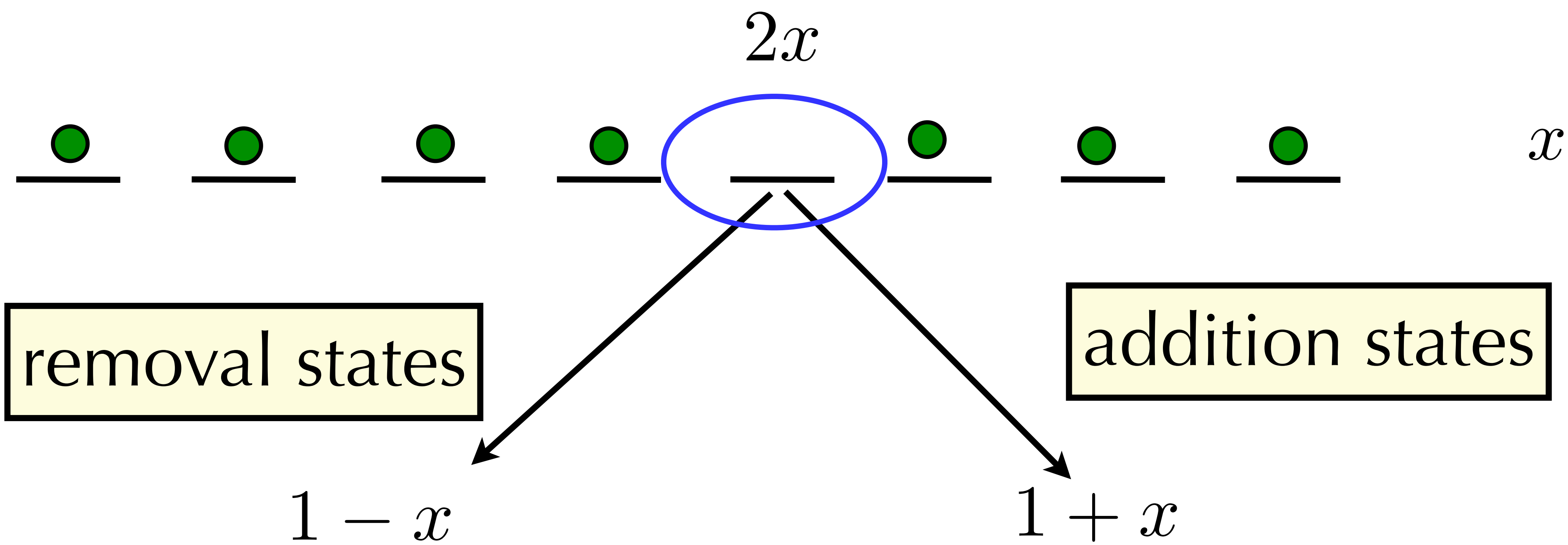
need to know: N (number of sites)

counting electron states



need to know: N (number of sites)

counting electron states



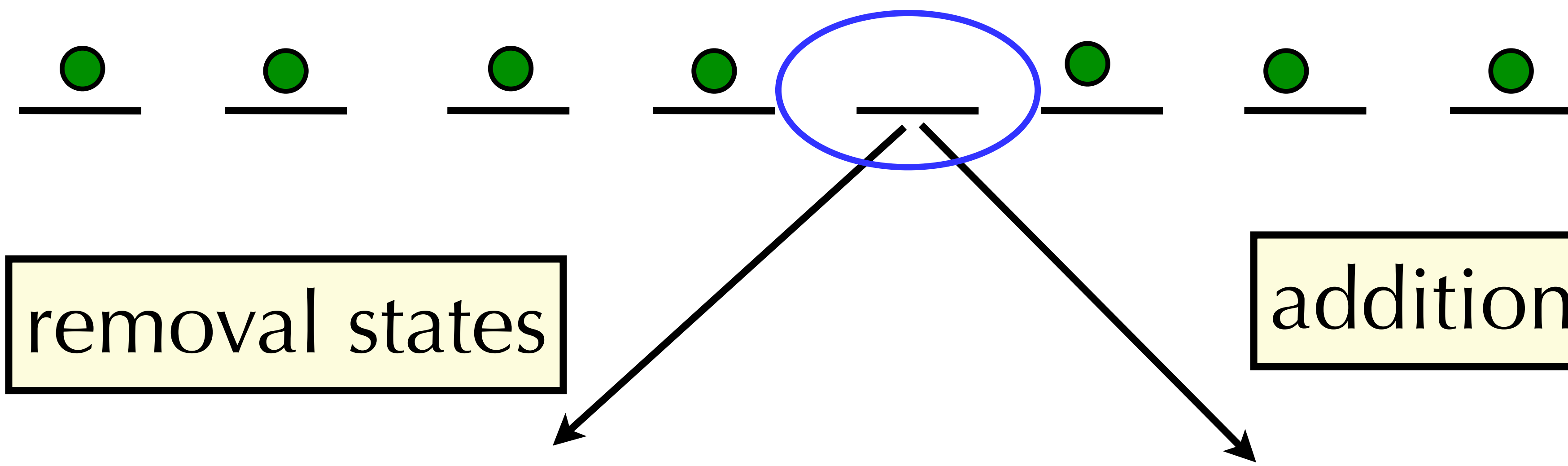
$x = n_h/N$

low-energy electron states

need to know: N (number of sites)

counting electron states

$2x$



$x = n_h/N$

removal states

addition states

$1 - x$ $\qquad$ $1 + x$

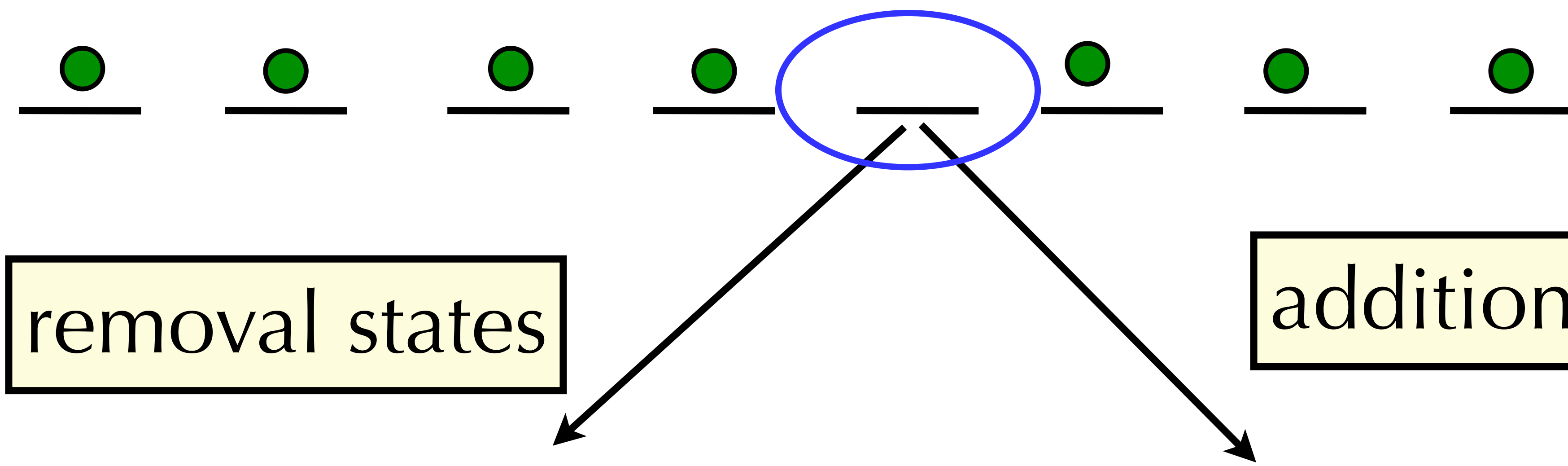
low-energy electron states

$$1 - x + 2x = 1 + x$$

need to know: N (number of sites)

counting electron states

$2x$



$x = n_h/N$

removal states

$1 - x$

addition states

$1 + x$

low-energy electron states

$1 - x + 2x = 1 + x$

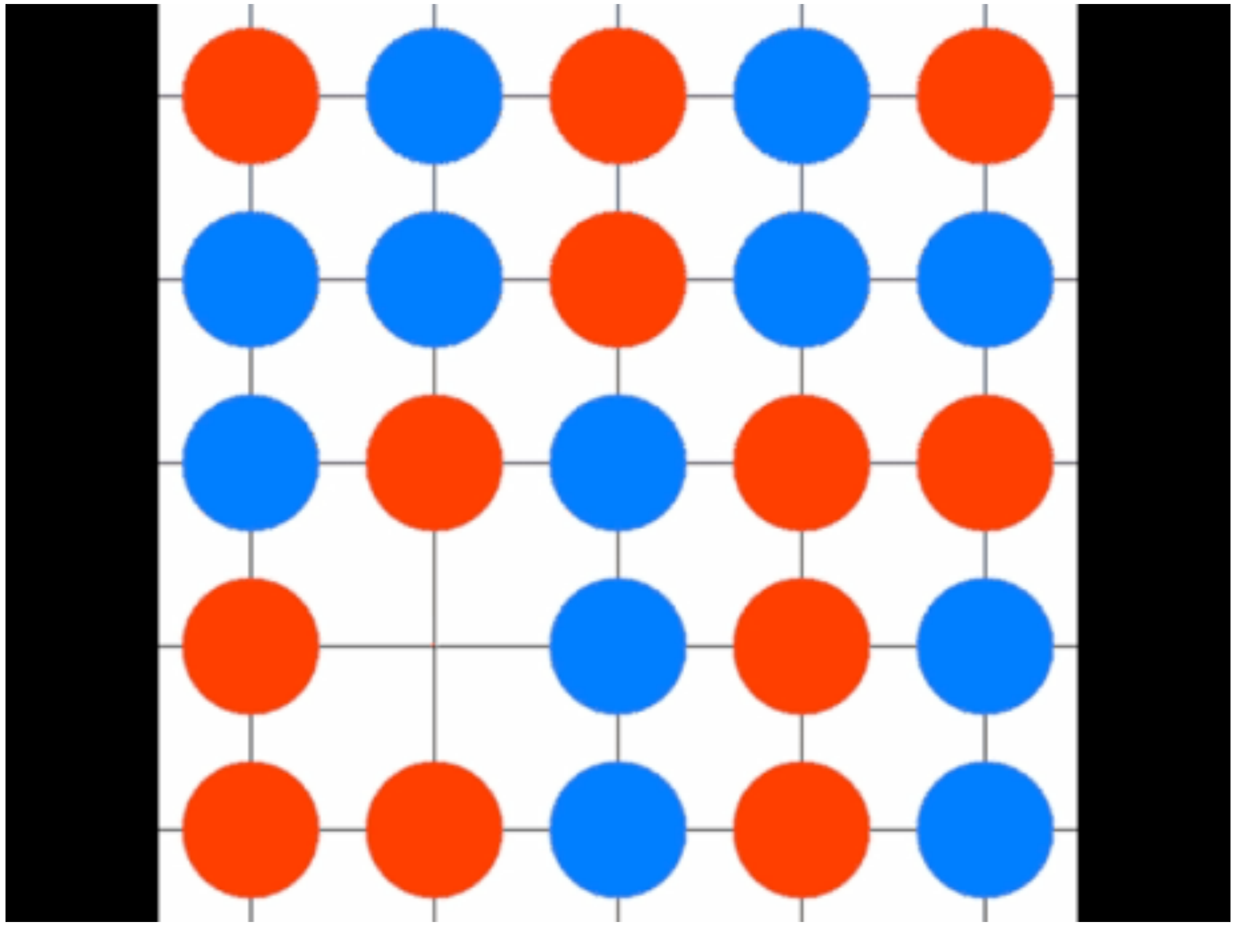
high energy

$1 - x$

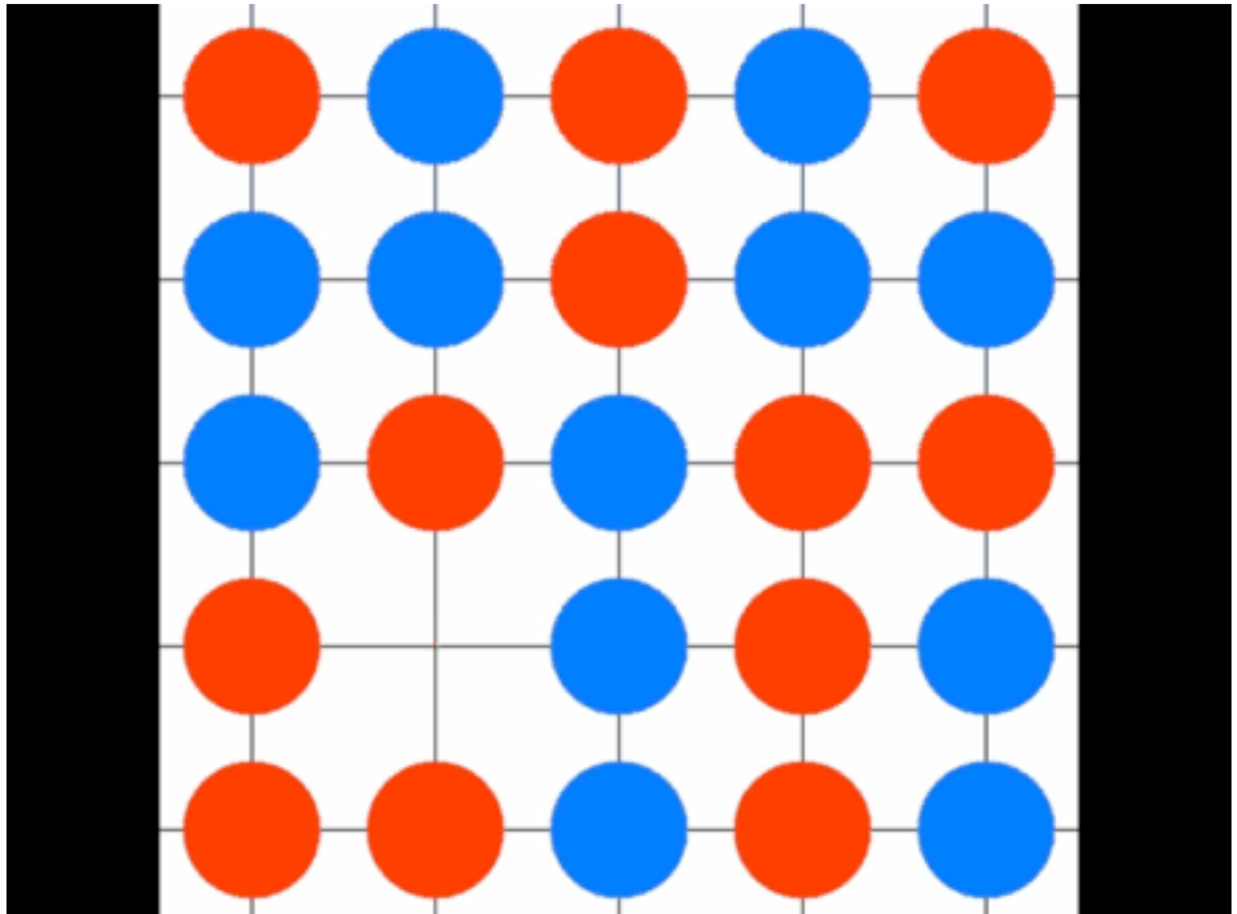
need to know: N (number of sites)

spectral function (dynamics)

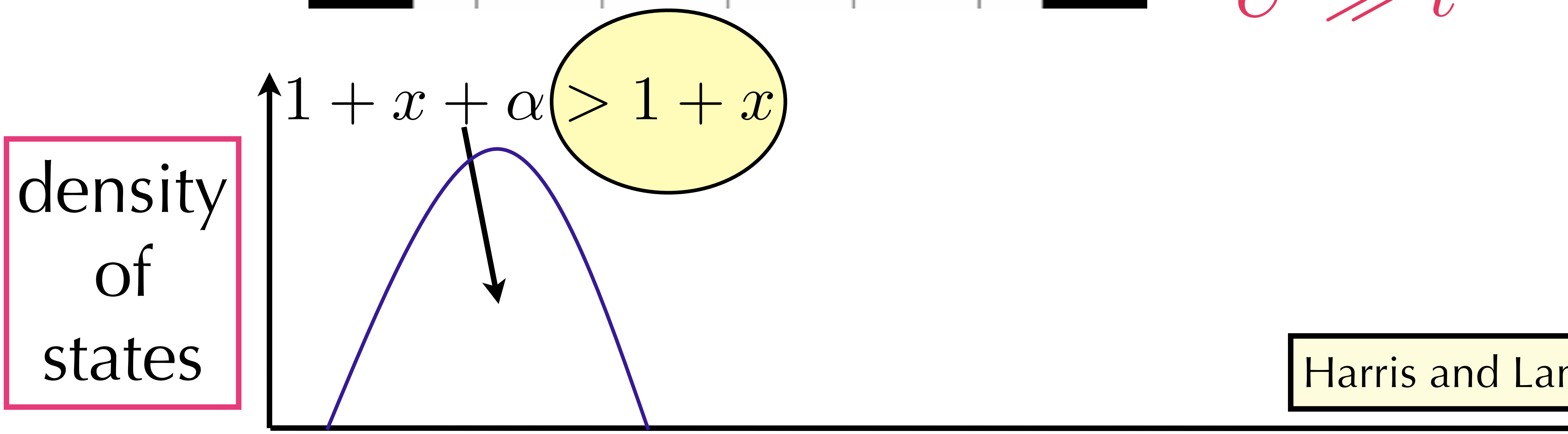
spectral function (dynamics)



spectral function (dynamics)

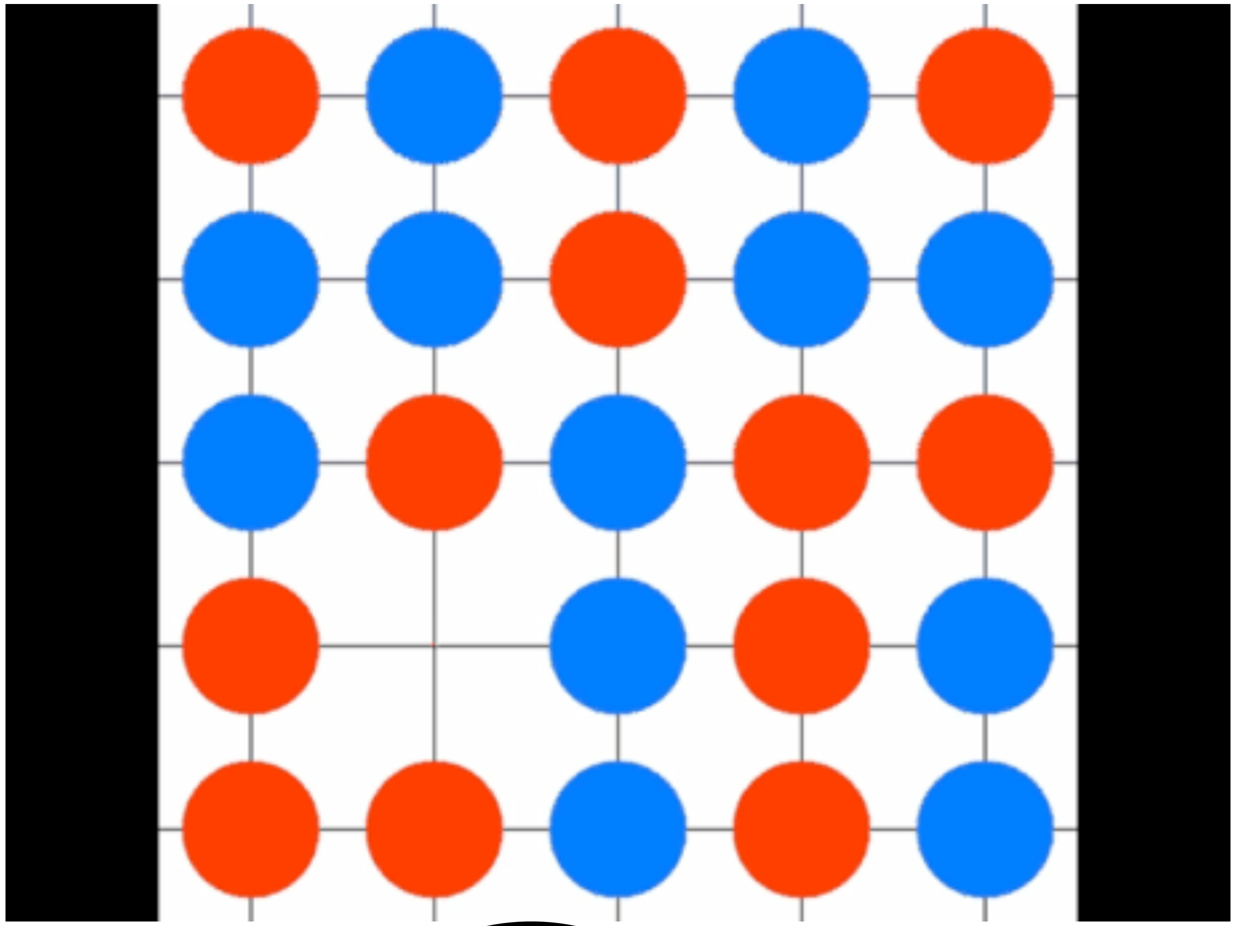


$U \gg t$

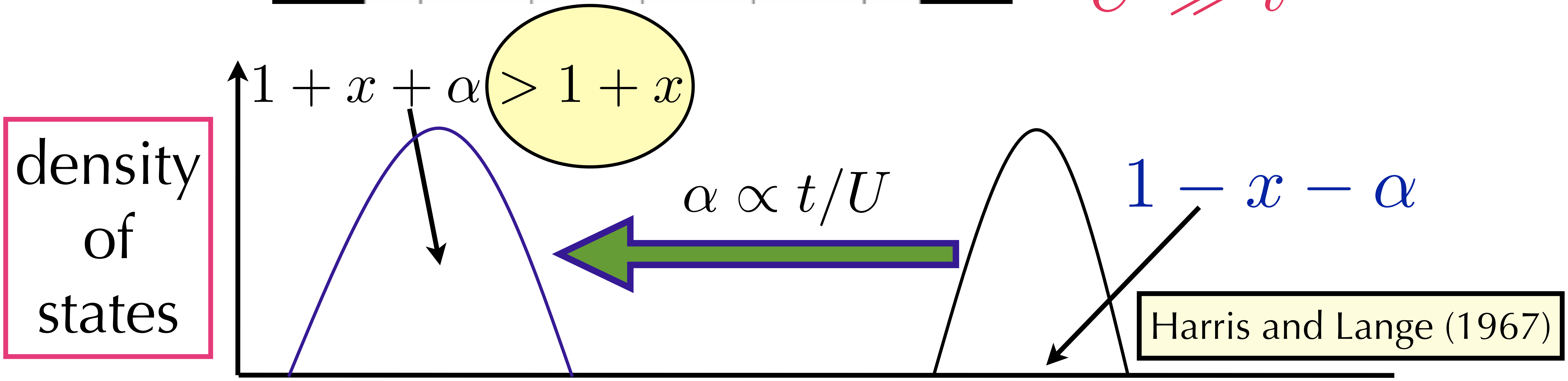


Harris and Lange (1967)

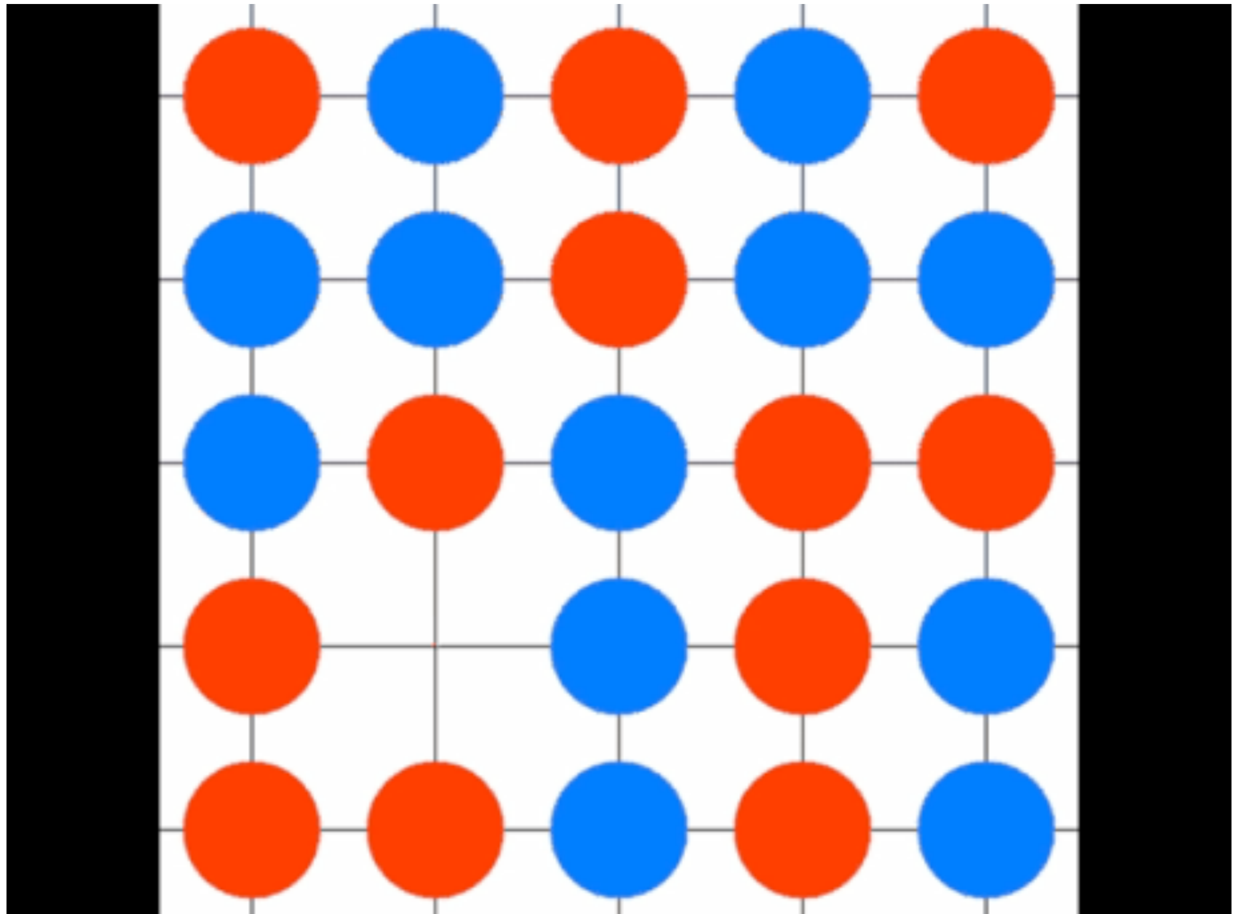
spectral function (dynamics)



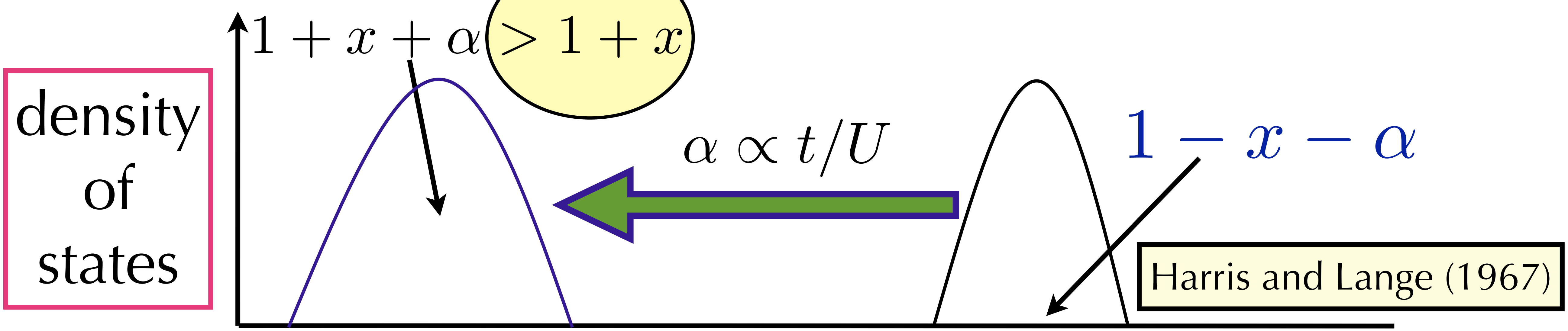
$U \gg t$



spectral function (dynamics)



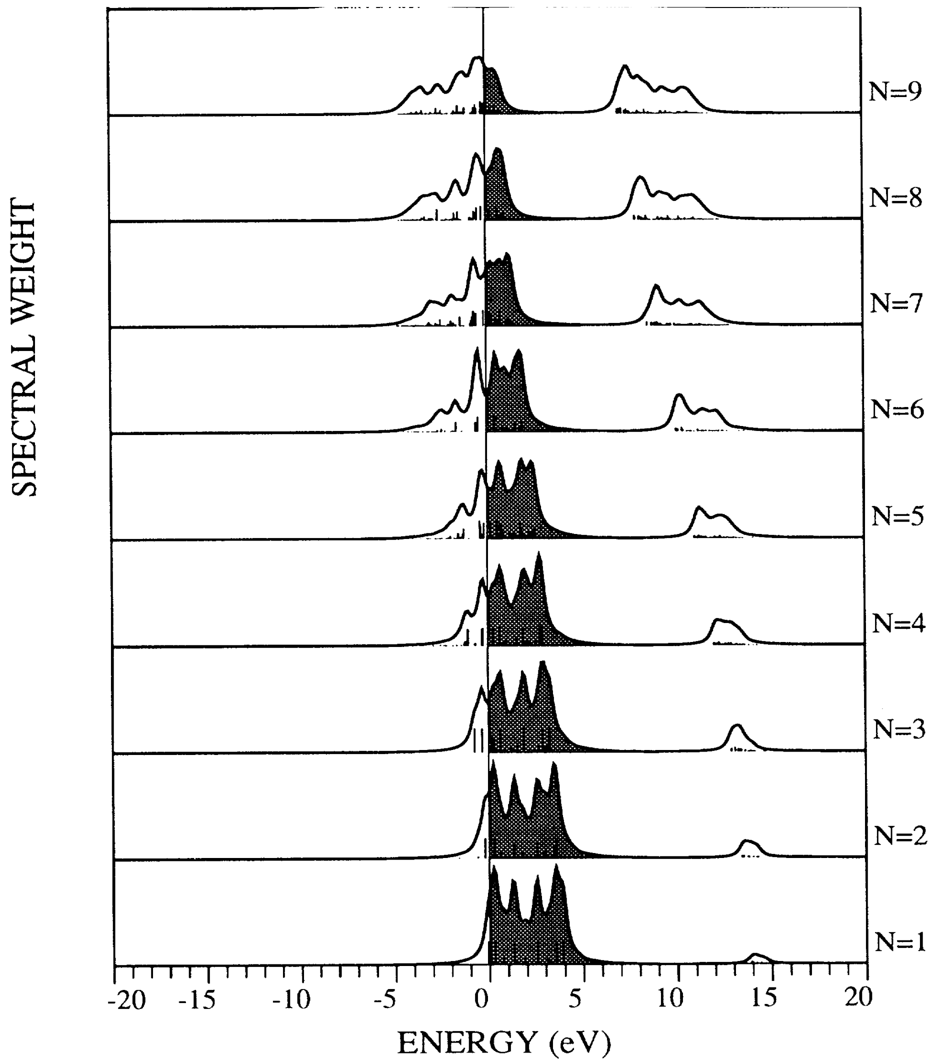
$U \gg t$



not exhausted by counting electrons alone

determination of N_{eff}

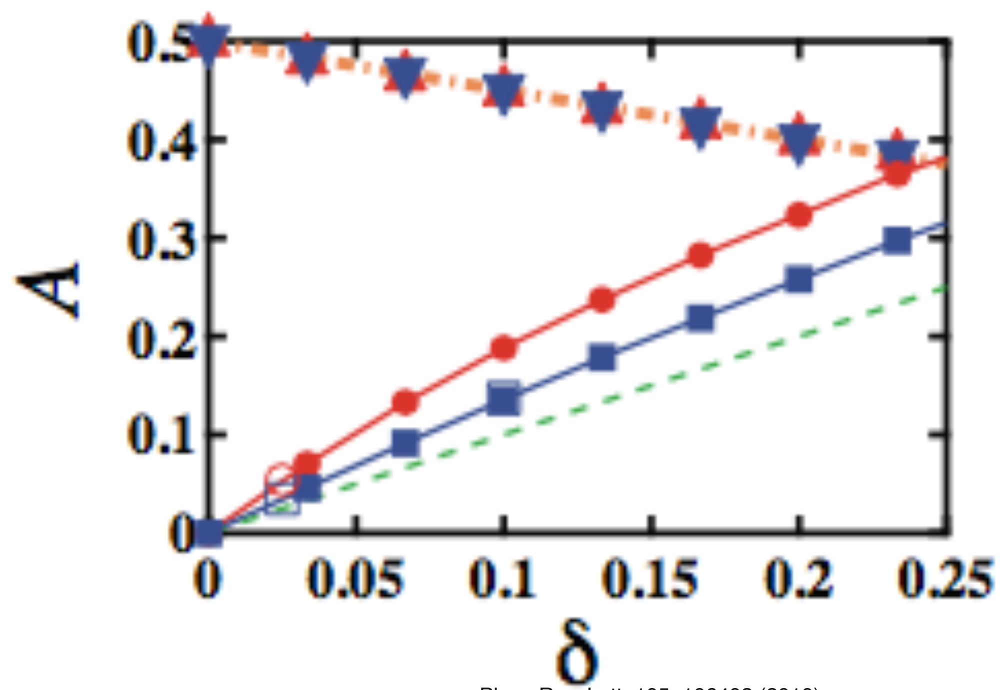
Hubbard model



x=0.1
x=0.2
x=0.3
x=0.4
x=0.5
x=0.6
x=0.7
x=0.8
x=0.9

Sawatzky

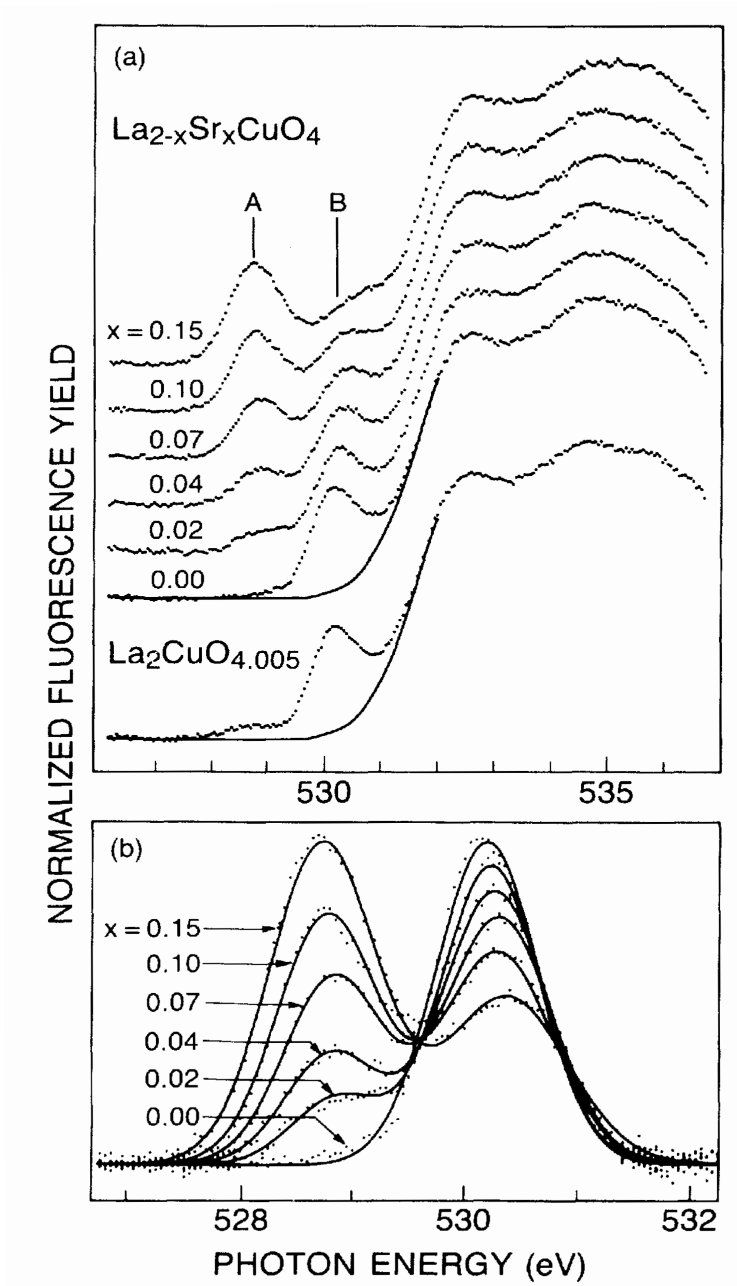
d=1

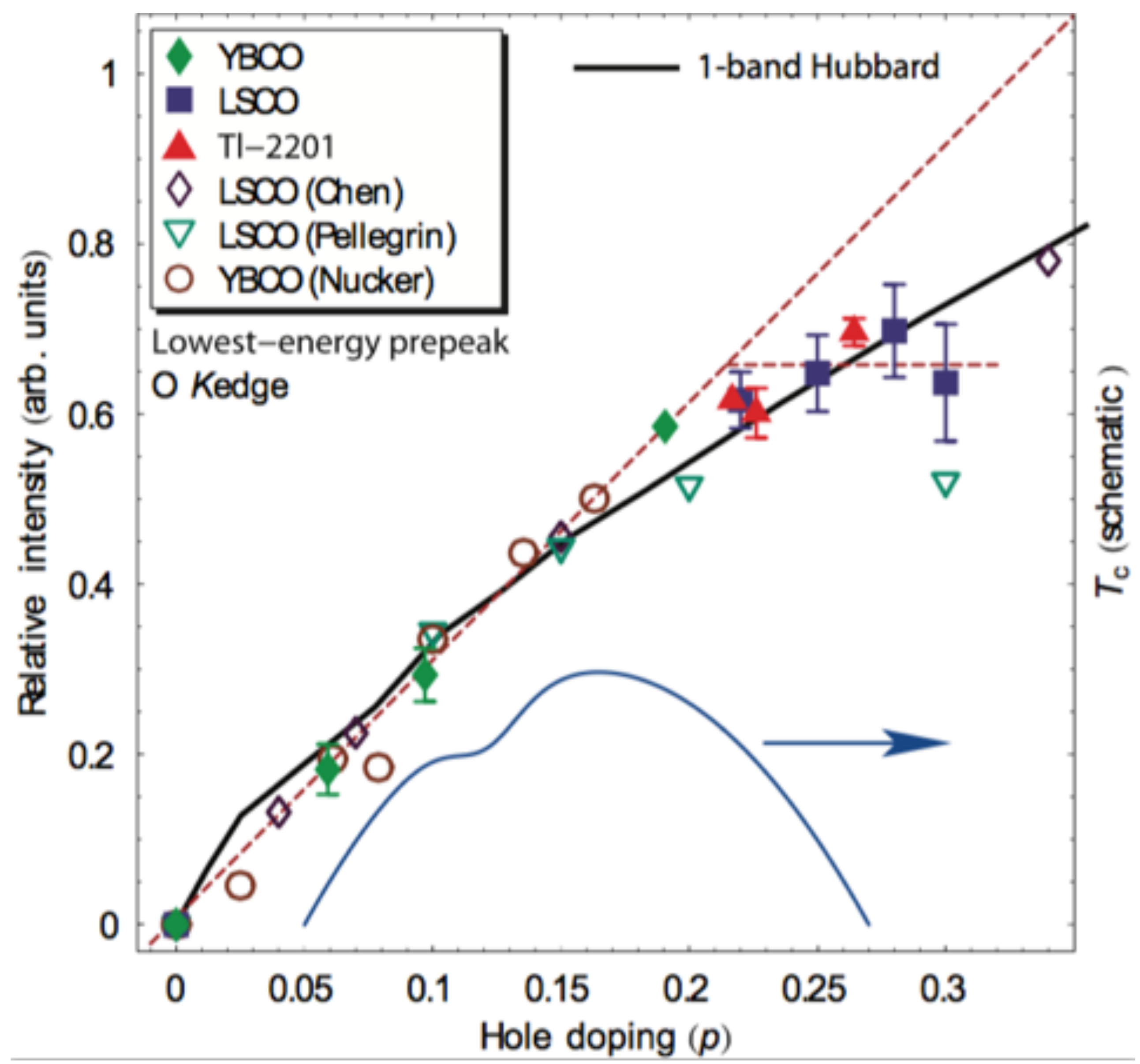


Phys. Rev. Lett. 105, 106402 (2010)

M. Kohno, PRL, 105, 106402

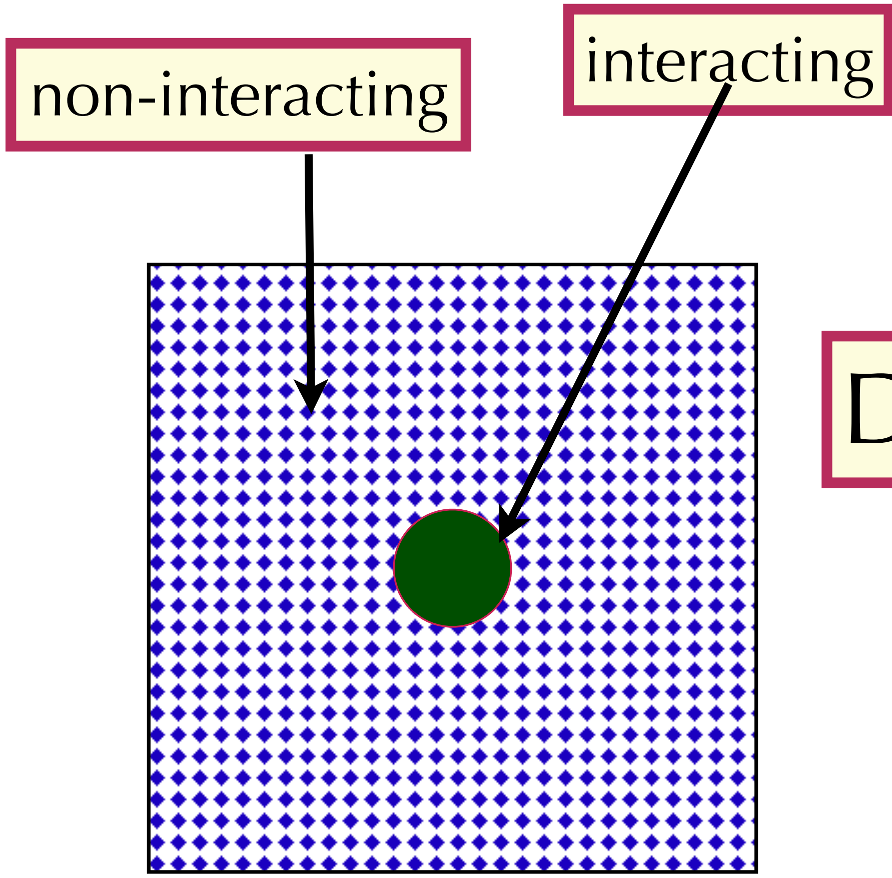
determination of N_{eff}





chen/Batlogg, 1990

DMFT on Hubbard model



DMFT

DMFT on Hubbard model

DMFT on Hubbard model

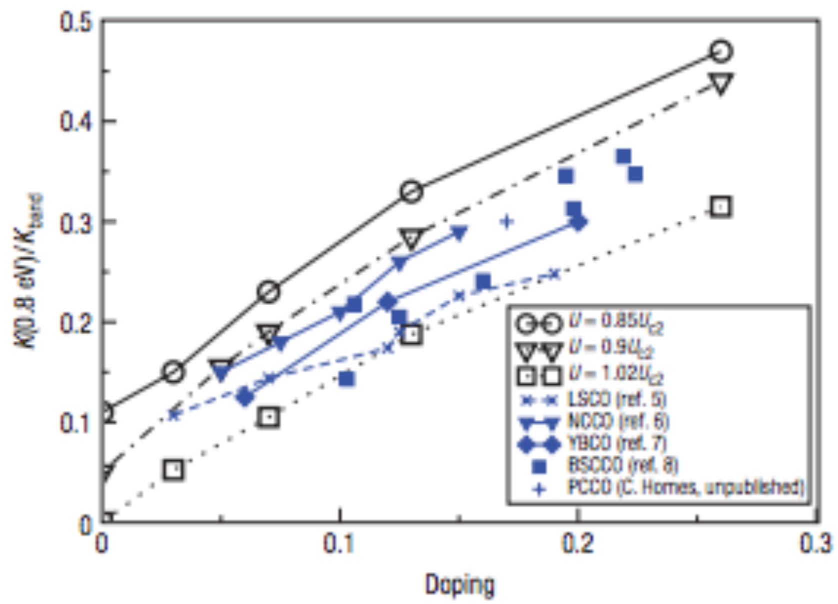
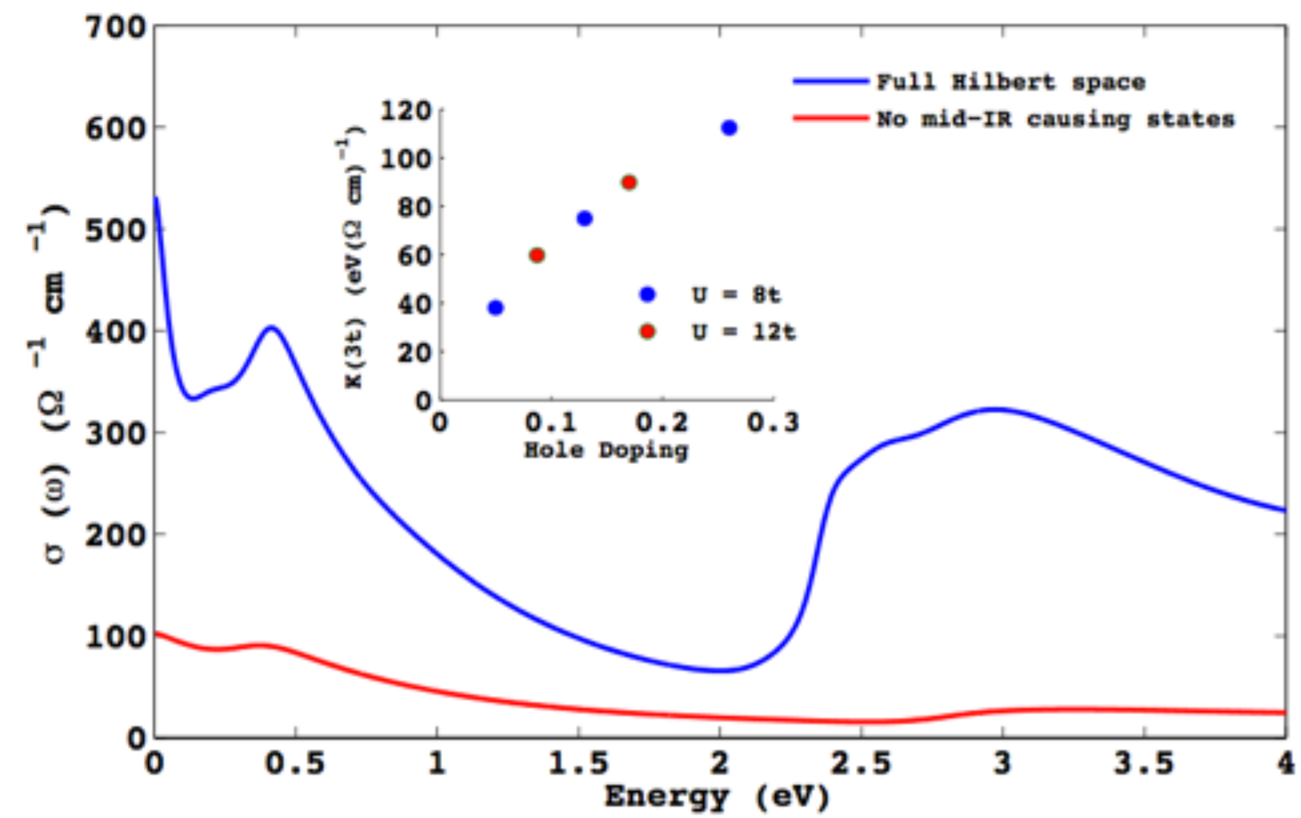
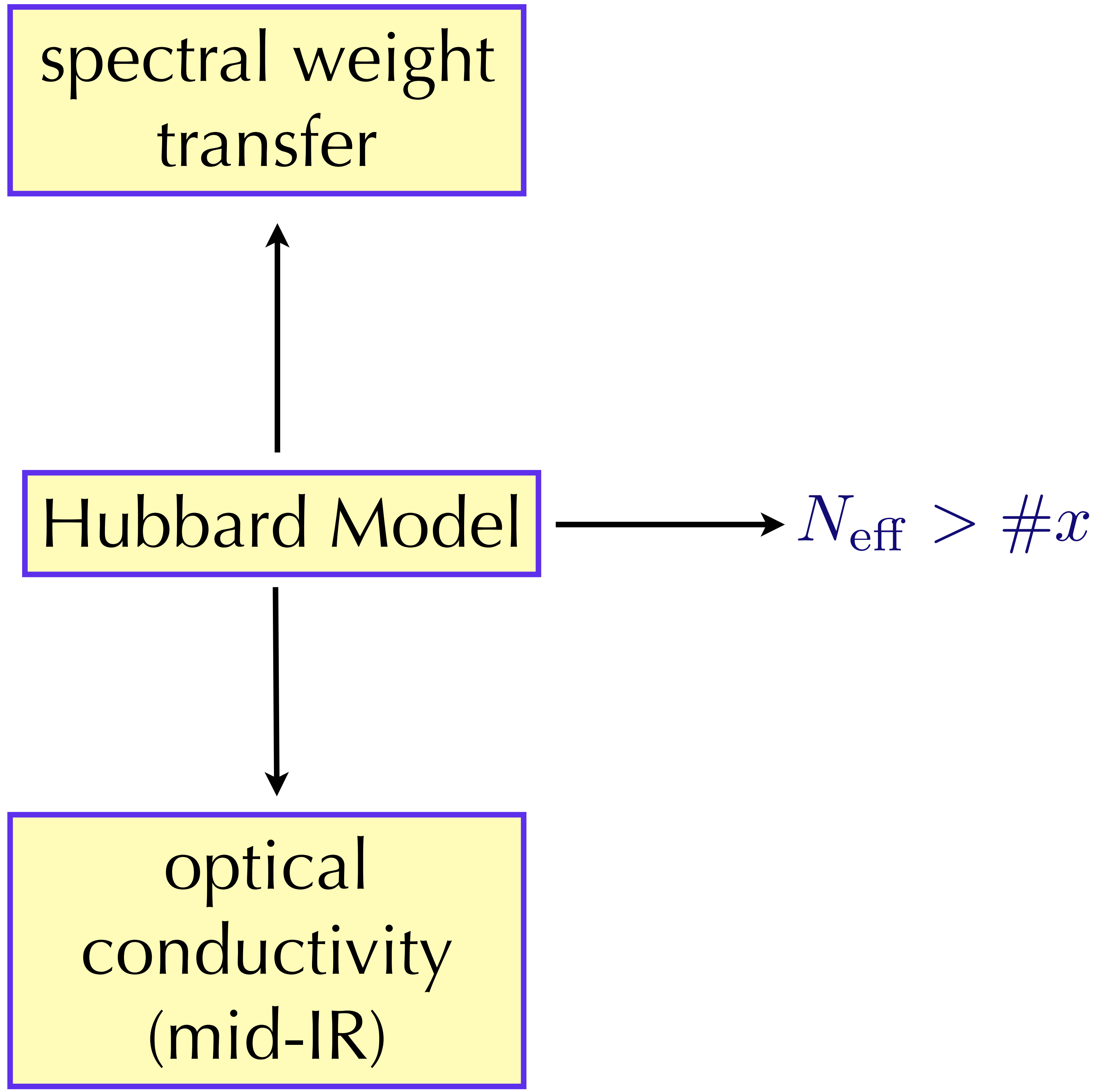


Figure 3 Comparison of measured and calculated optical spectral weight. Filled symbols: spectral weight obtained by integrating experimental conductivity up to 0.8 eV from references given. Open symbols: theoretically calculated spectral weight, integrated up to $W/4$. For $U = 0.85U_{c2}$ and $U = 0.9U_{c2}$, the band-theory estimate $W = 3$ eV is used to convert the calculation to physical units; for $U = 1.02U_{c2}$, the value $W = 2.25$ eV which reproduces the insulating gap is used.



Millis, 2008

chakraborty & Phillips, 2007


spectral weight transfer
Hubbard Model
$N_{\text{eff}} > \#x$
optical conductivity (mid-IR)

is there
anything else?

yes

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1*], H. J. A. Molegraaf[1*], J. Zaanen[2], Z. Nussinov[2*], F. Carbone[1*], A. Damascelli[3*], H. Eisaki[3*], M. Greven[3], P. H. Kes[2] & M. Li[2]

[1] *Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2] *Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3] *Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

yes

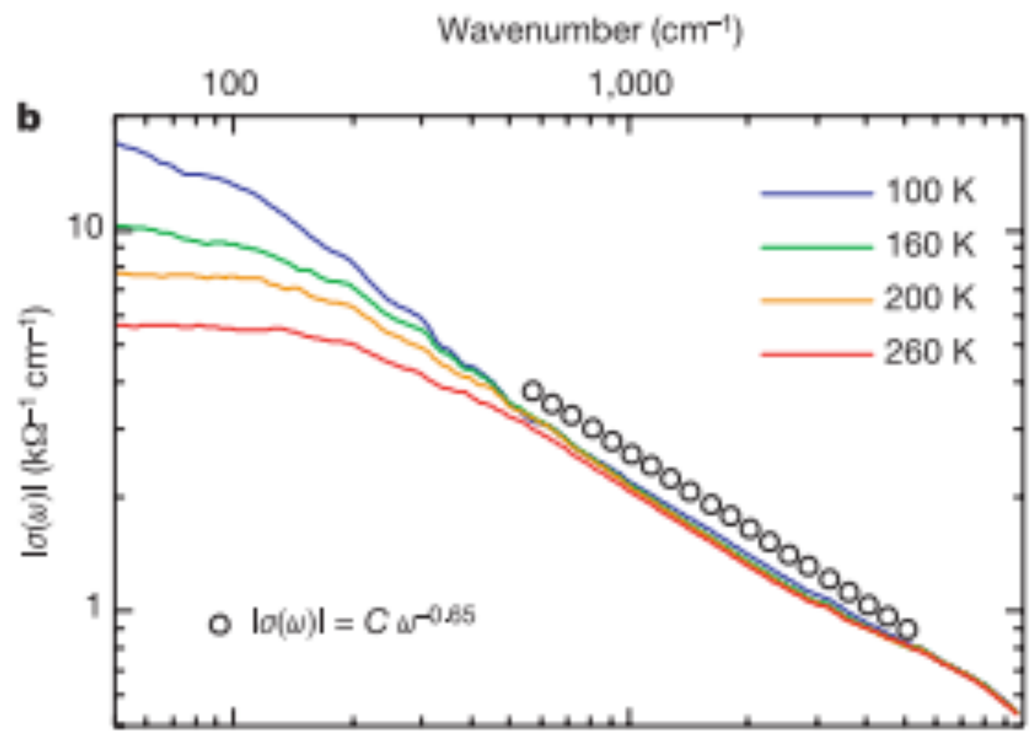
Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]

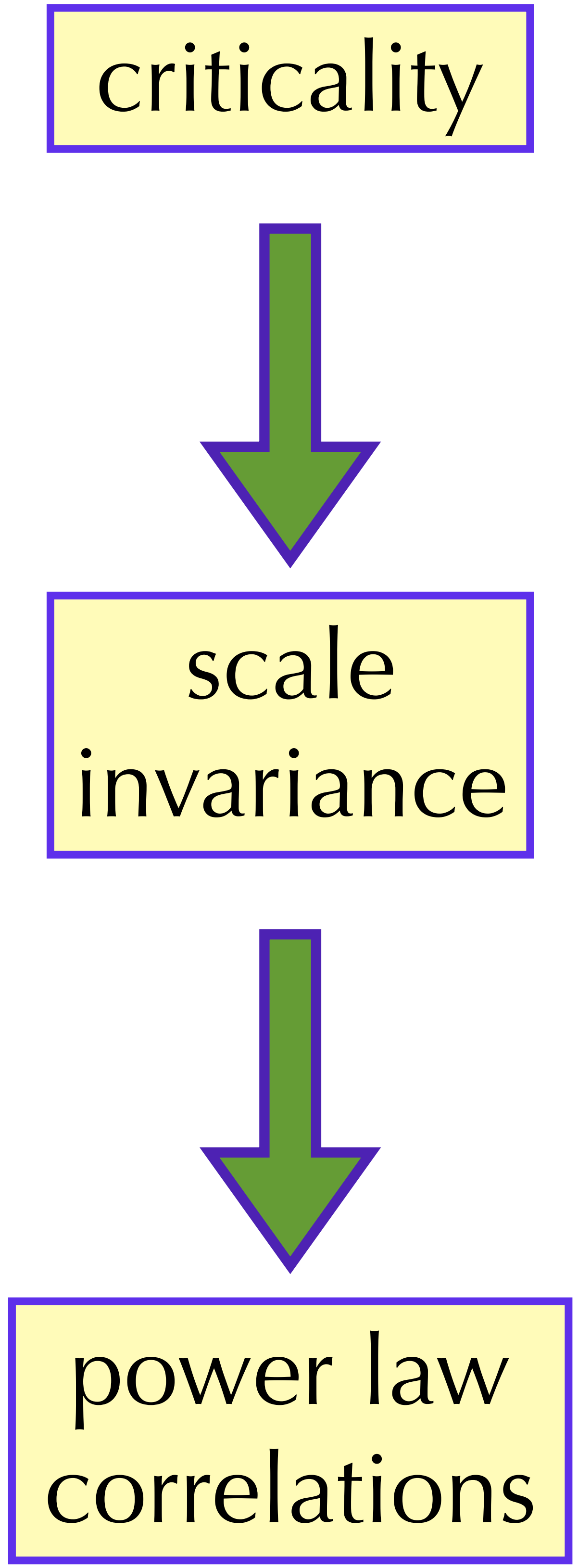
[1]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2]*Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$


criticality
scale invariance
power law correlations

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

Anderson: use Luttinger Liquid propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^{\eta}}$$

compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

$$\sigma(\omega) \propto \frac{1}{\omega}\int dx \int dt G^e(x,t)G^h(x,t)e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

problems

problems

1.) cuprates are not 1-dimensional

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

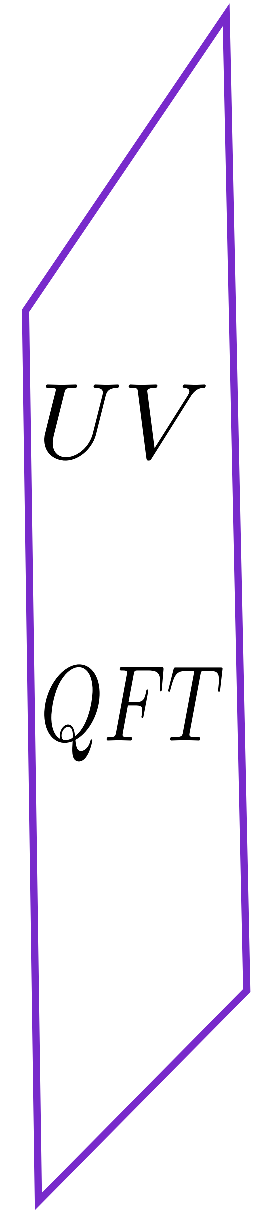
$$\left.\begin{aligned} &\sigma \propto G^2\Gamma^\mu\Gamma^{\mu\nu} \\ &[G] = L^{d+1-d_U} \\ &[\Gamma^\mu] = L^{2d_U-d} \\ &[\Gamma^{\mu\nu}] = L^{2d_U-d+1} \end{aligned}\right\} \quad [\sigma] = L^{3-d}$$

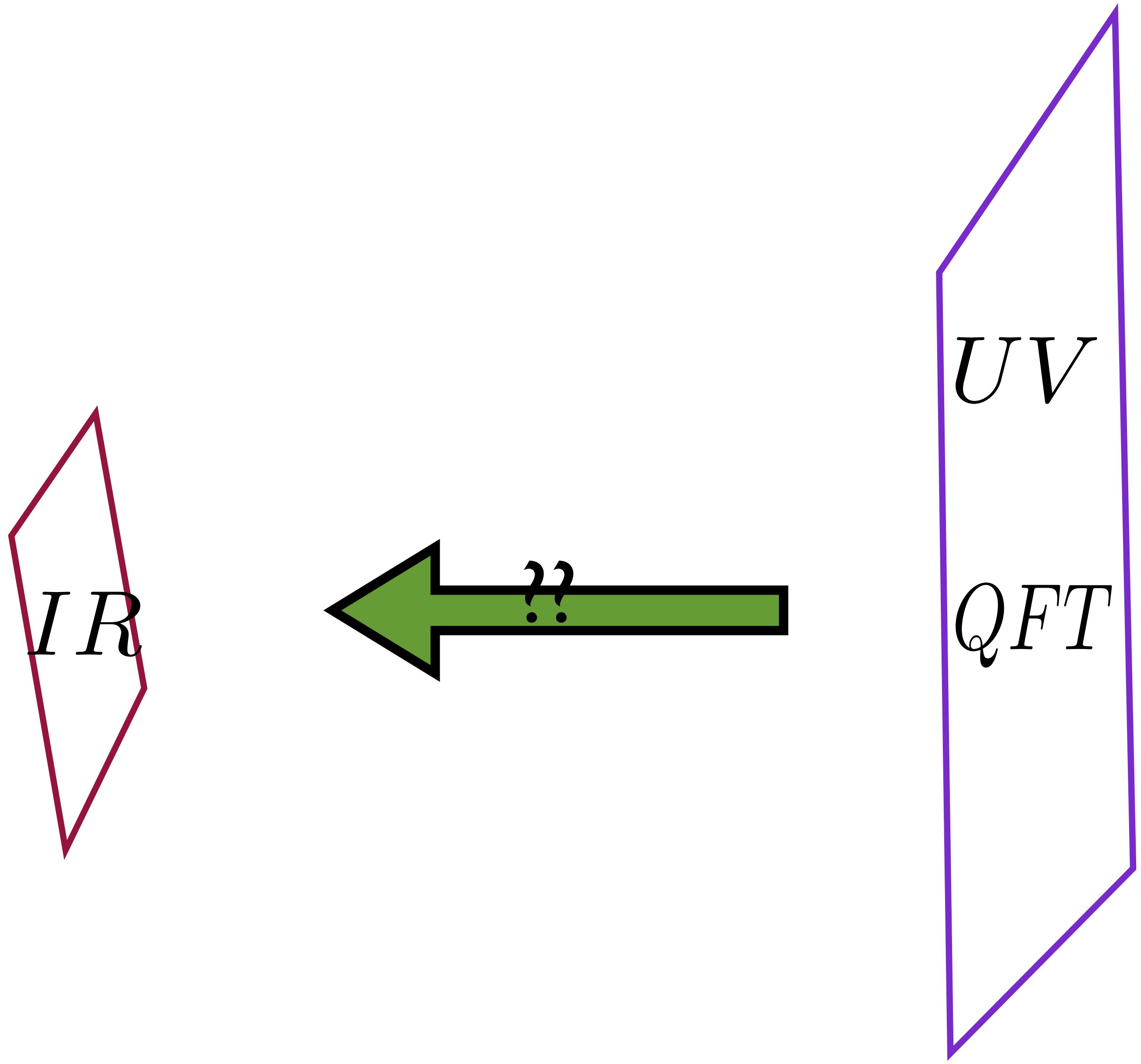
independent of d_U

power law?

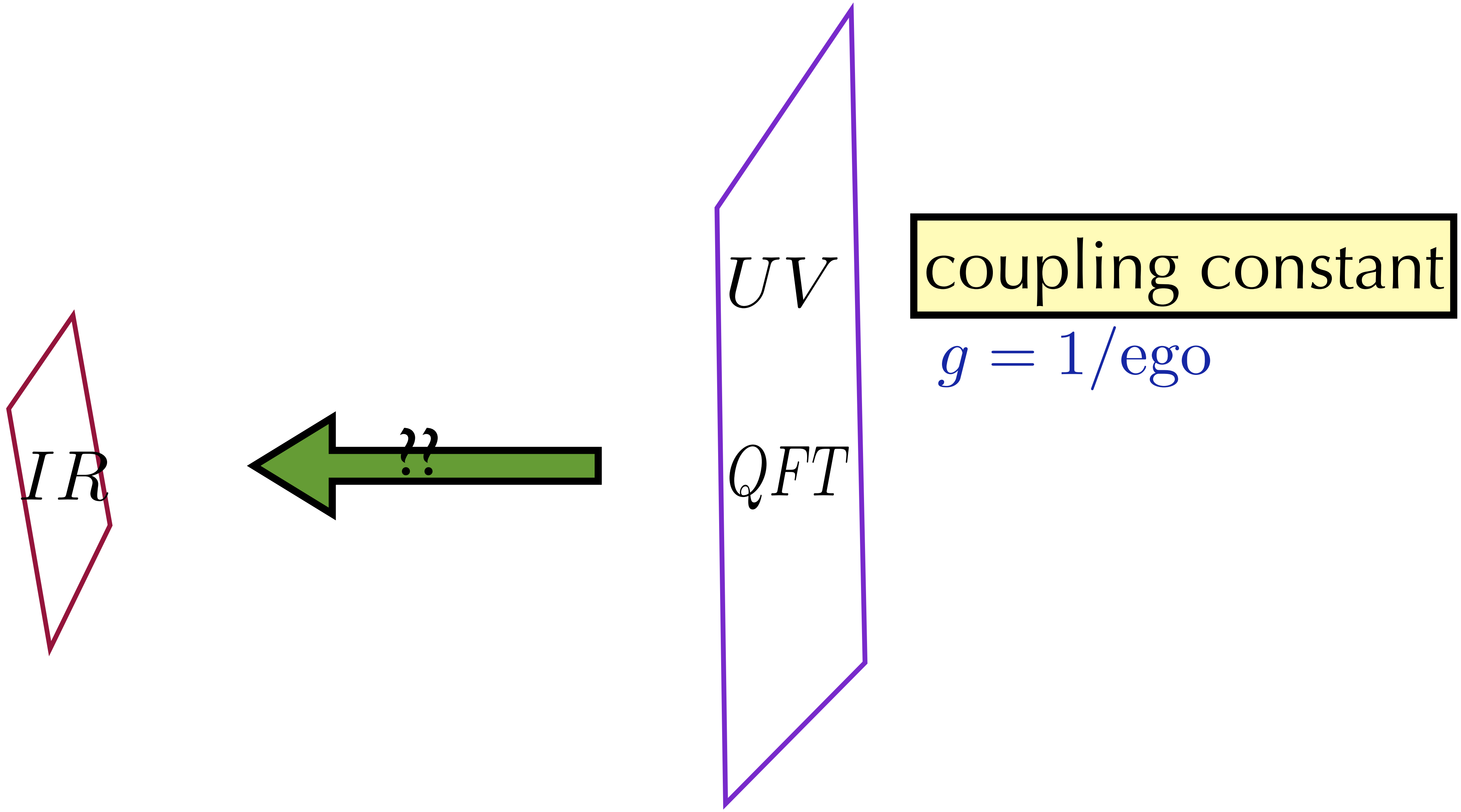
power law?

Could string theory be the answer?

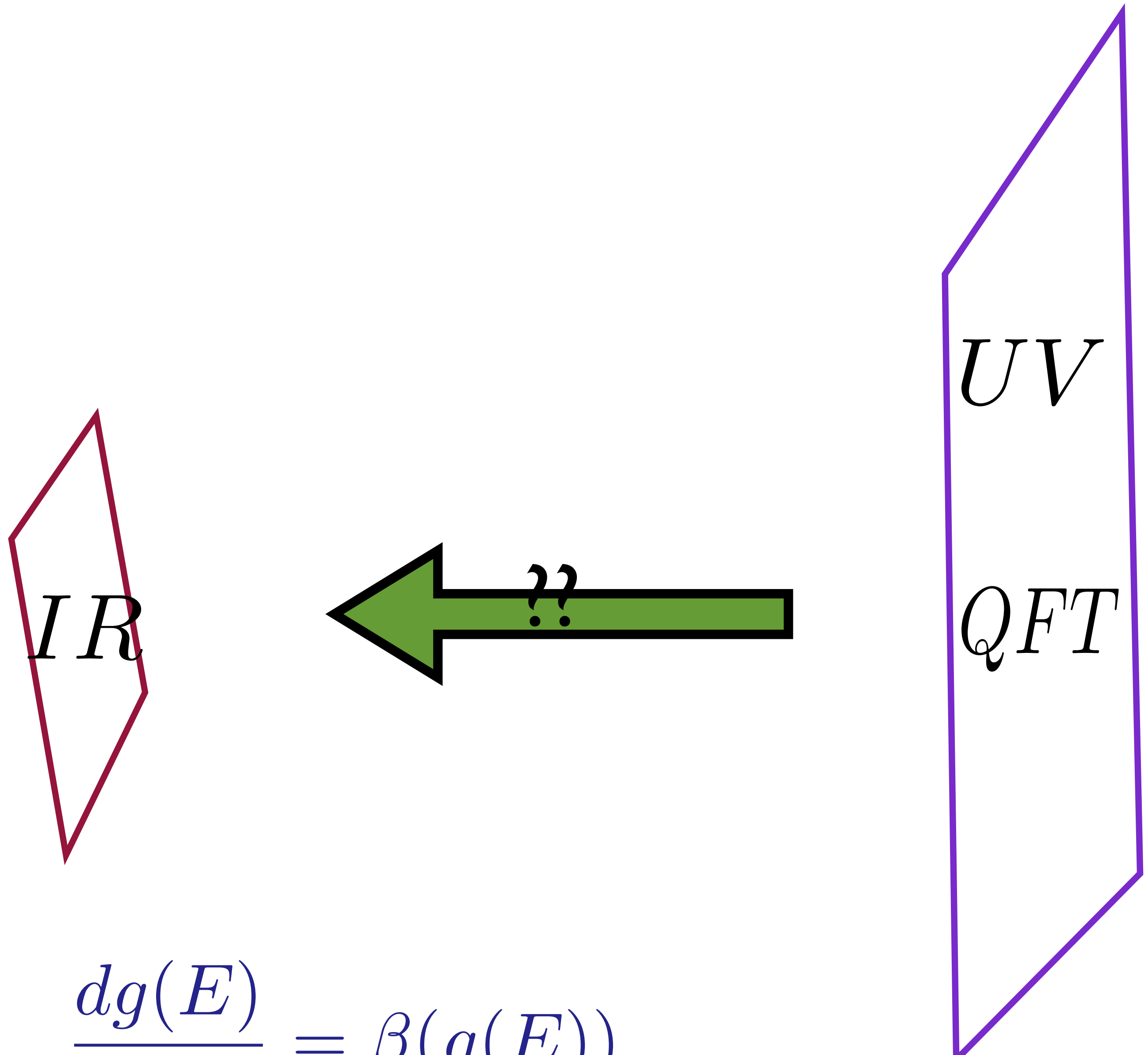

UV
QFT



IR
??
UV
QFT



IR
??
UV
QFT
coupling constant
$g = 1/\text{ego}$



coupling constant

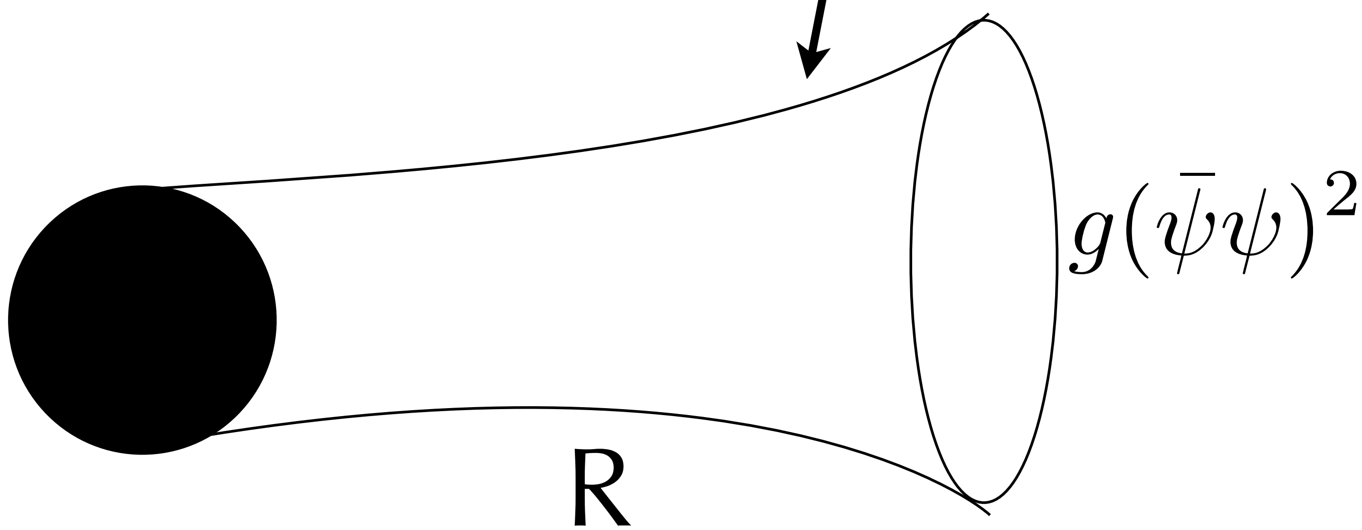
$g = 1/\text{ego}$

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

replace coupled theory
with geometry

scale-invariant
anti de-Sitter



$$R \propto \sqrt{g}$$

replace coupled theory
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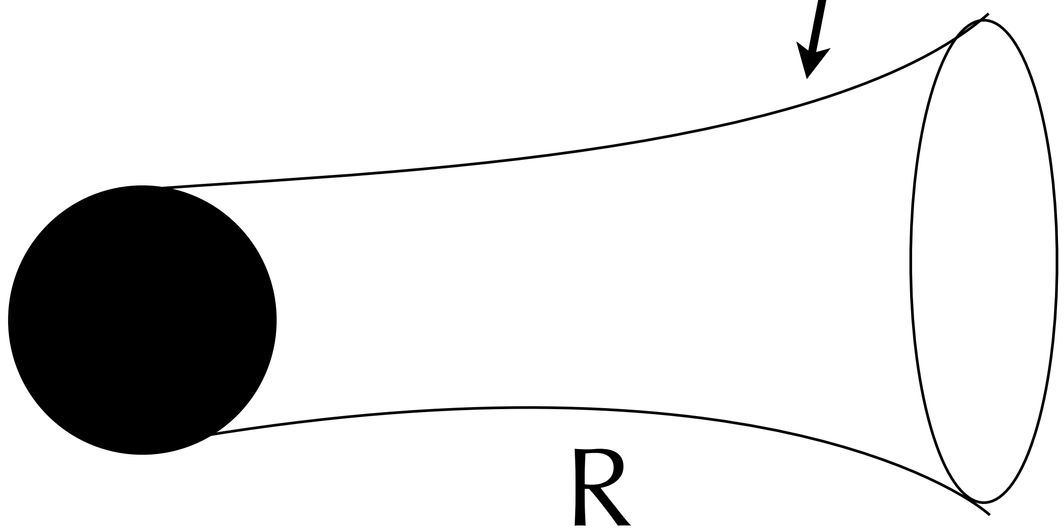
anti de-Sitter



RG=GR

$$R \propto \sqrt{g}$$

replace coupled theory
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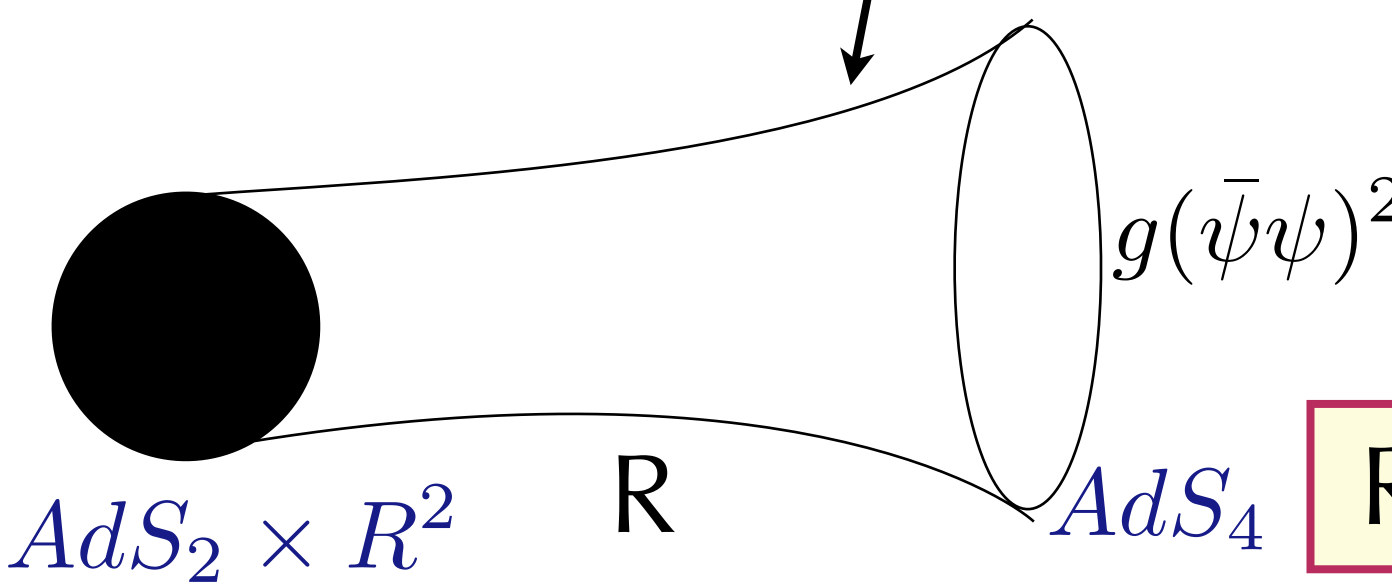

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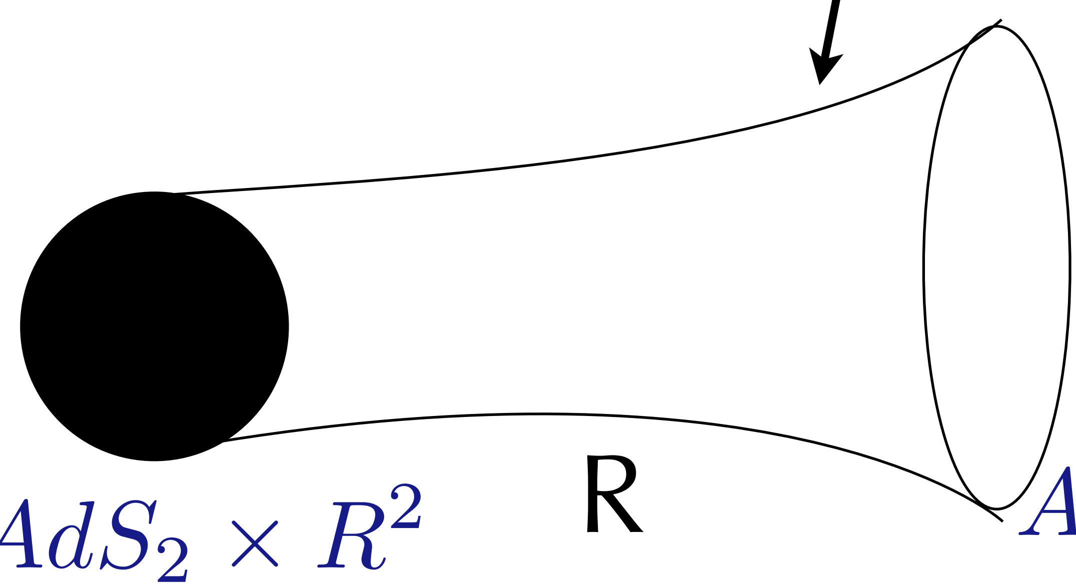


RG=GR

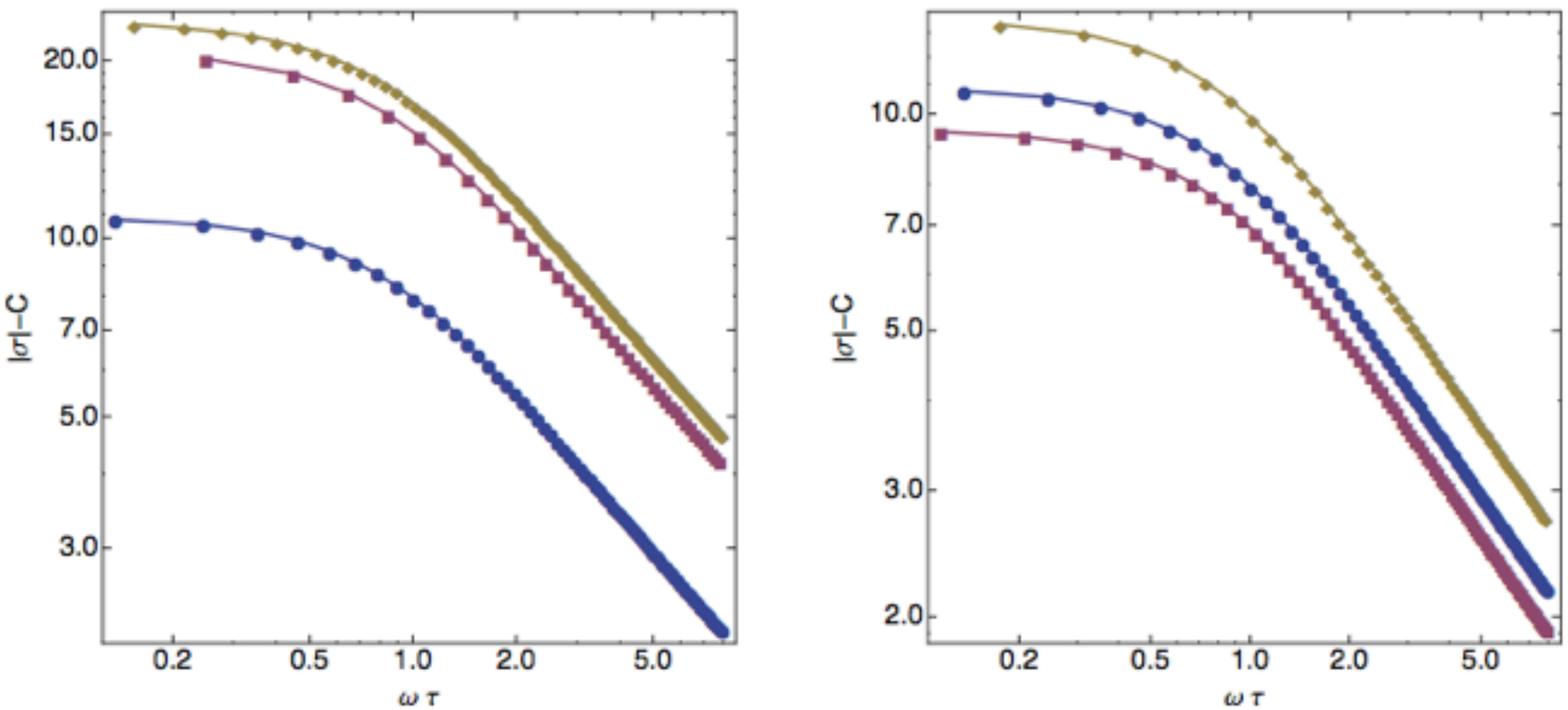
$R \propto \sqrt{g}$

cannot describe systems at g=0!

optical conductivity from a gravitational lattice

G. Horowitz *et al.*, Journal of High Energy Physics, 2012

optical conductivity from a gravitational lattice



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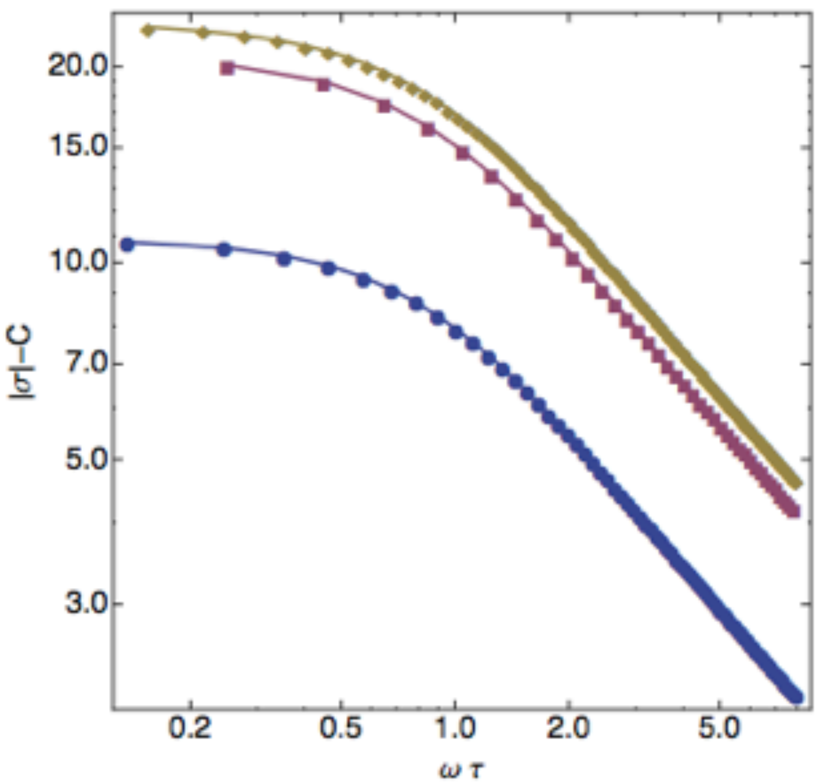

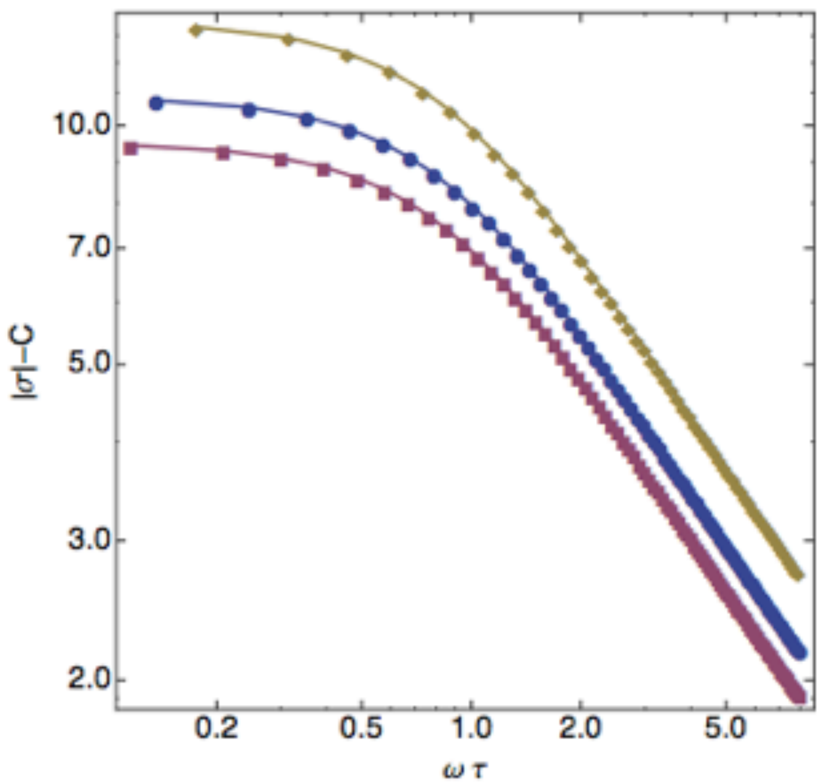

log-log plots for various parameters

G. Horowitz *et al*., Journal of High Energy Physics, 2012

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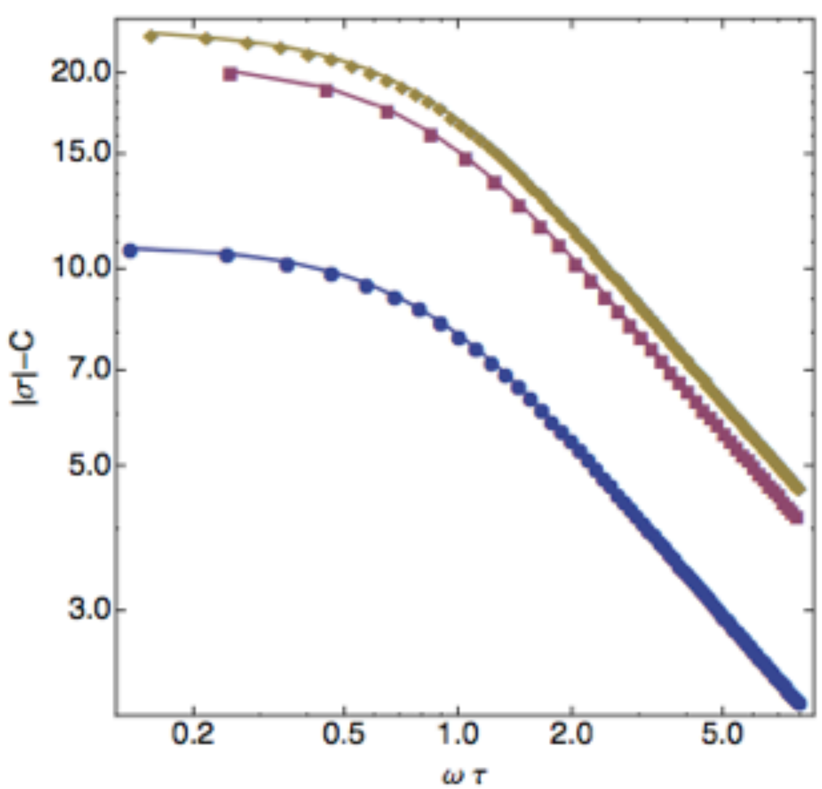

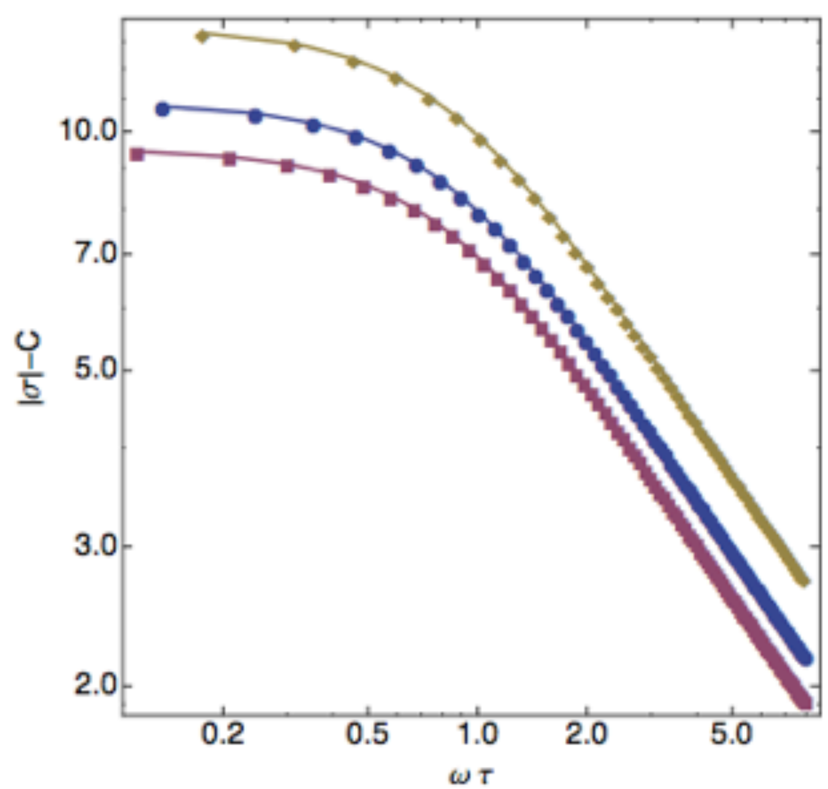

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz *et al*., Journal of High Energy Physics, 2012

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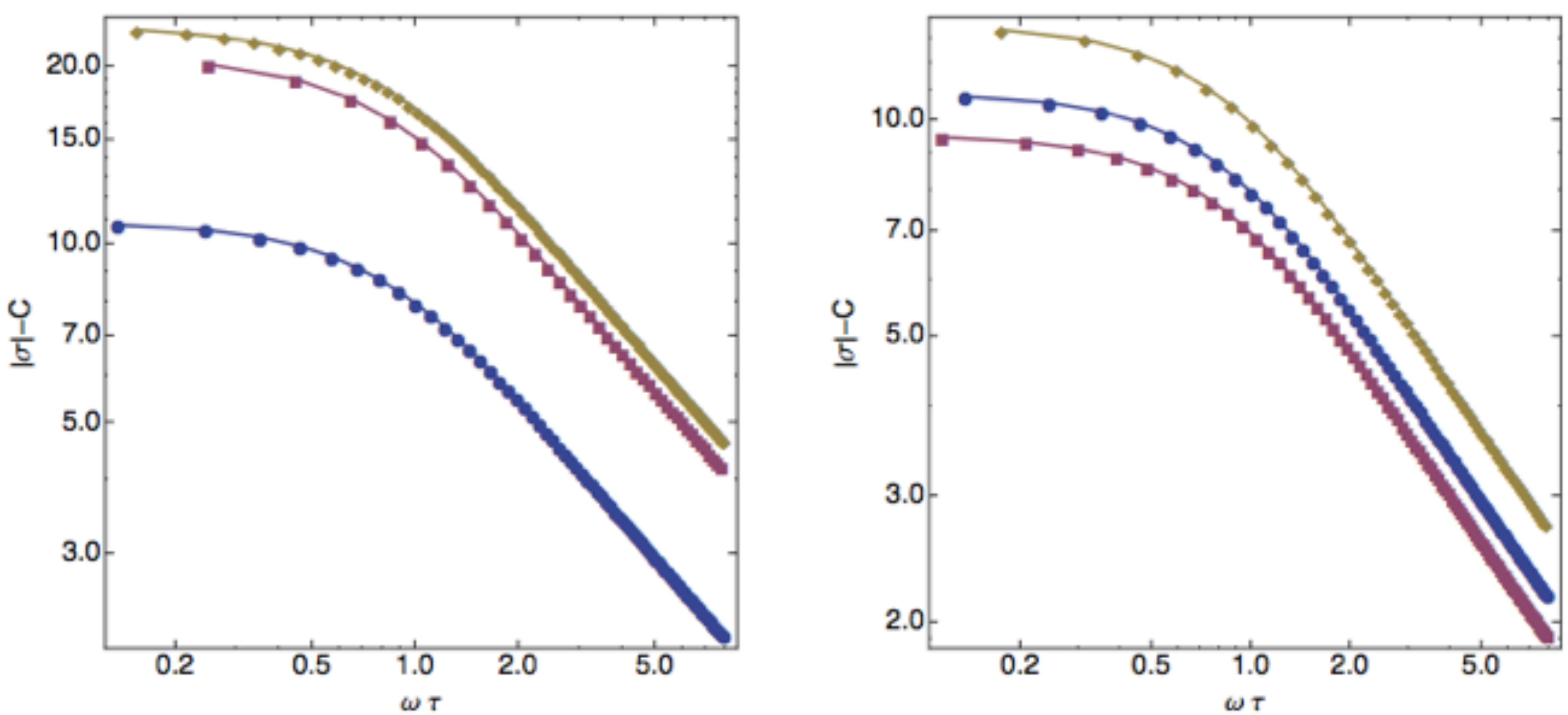


log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

a remarkable claim!
replicates features of the strange metal? how?

G. Horowitz *et al*., Journal of High Energy Physics, 2012

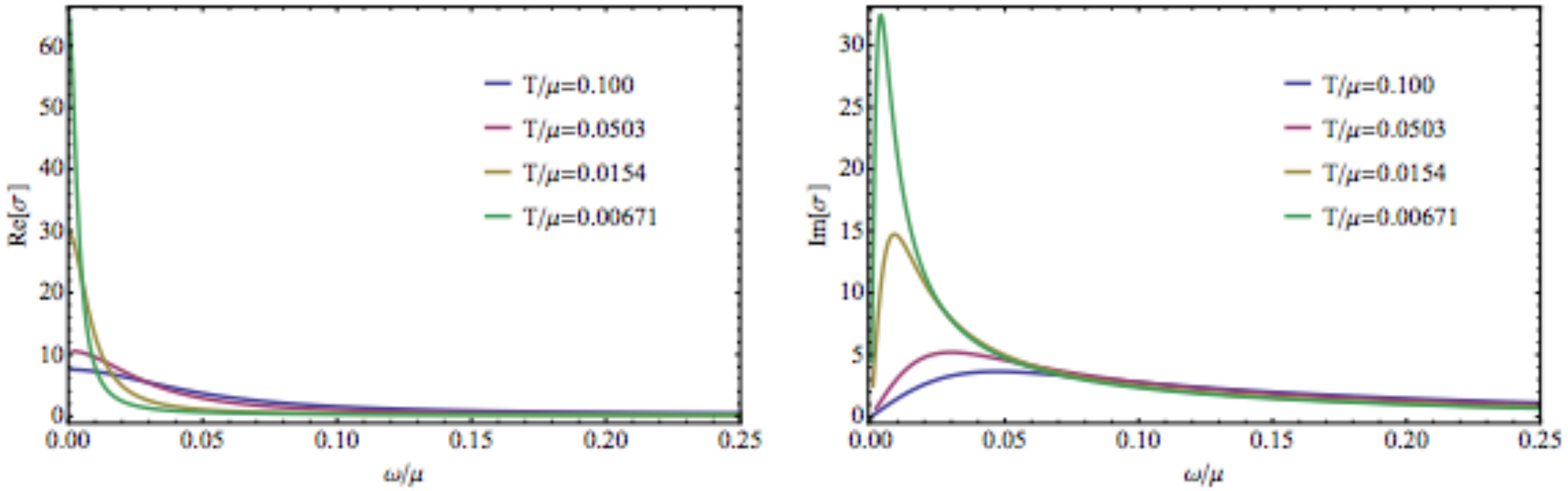
Einstein-Maxwell equations $+$ non-uniform charge density $= B\omega^{-2/3}$

not so fast!

Donos and Gauntlett
(gravitational crystal)

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



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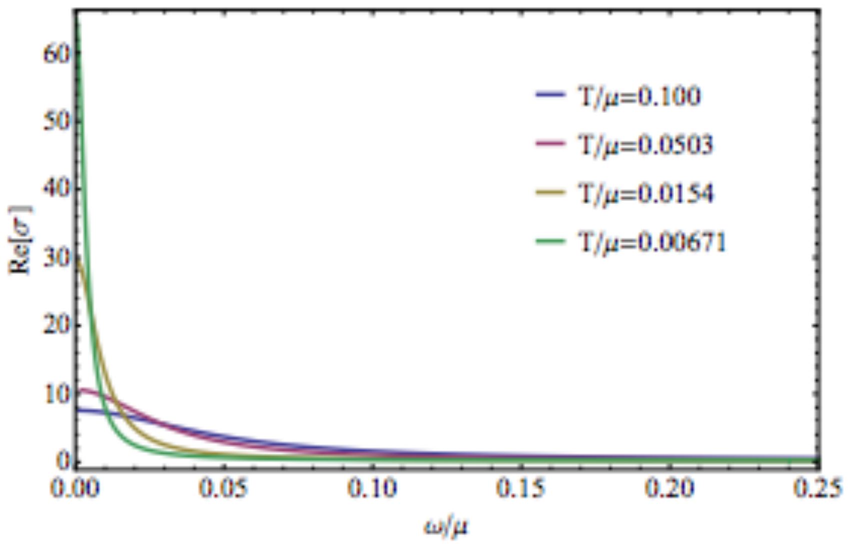

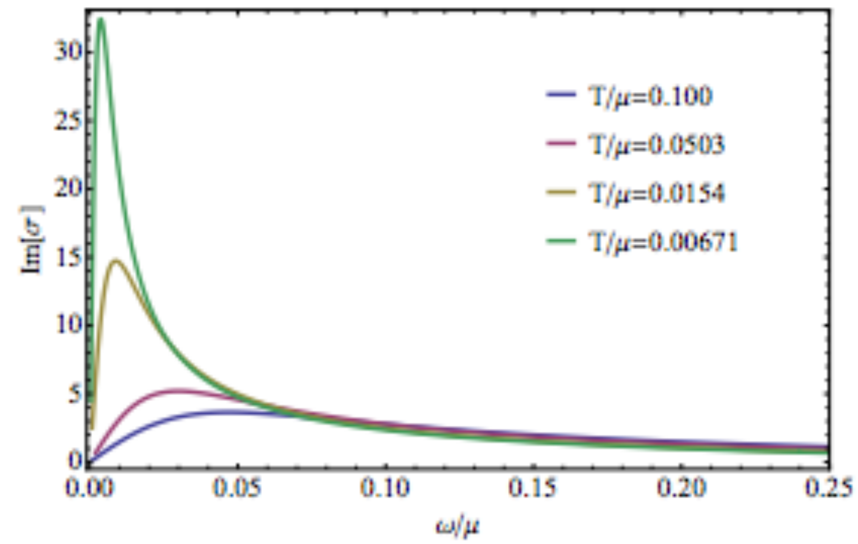

no power law!!

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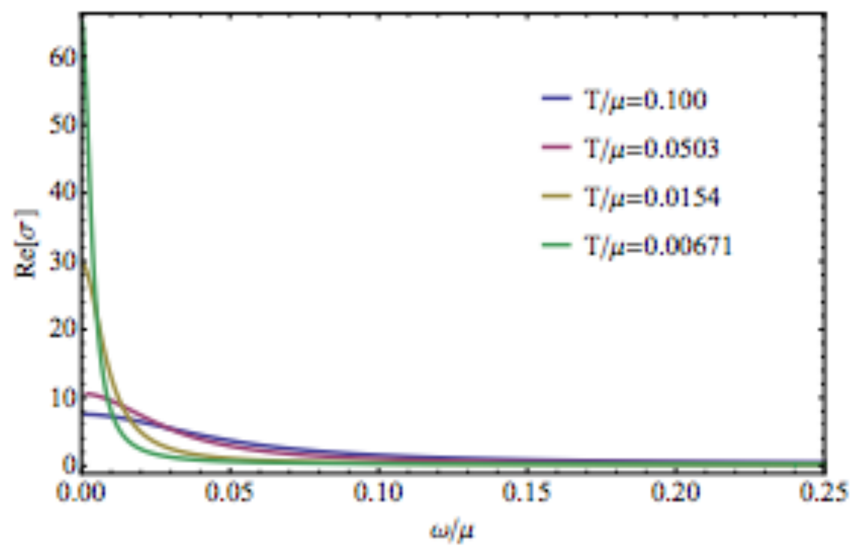

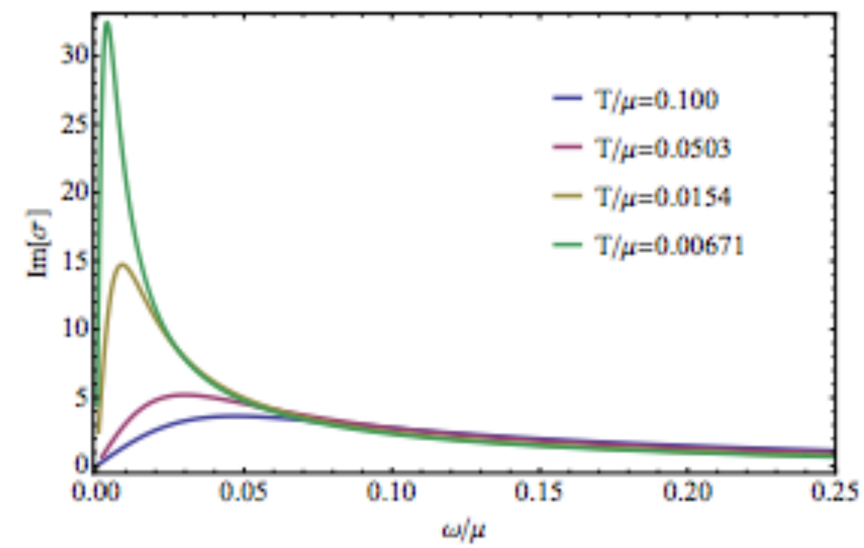

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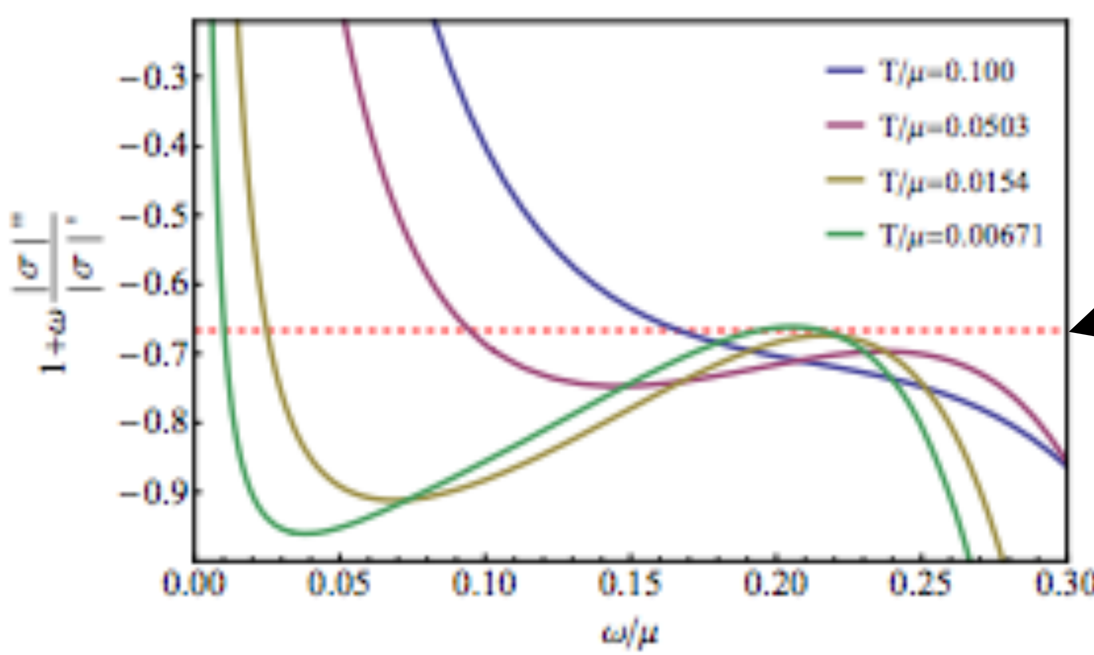


$B\omega^{-2/3}$

who is correct?

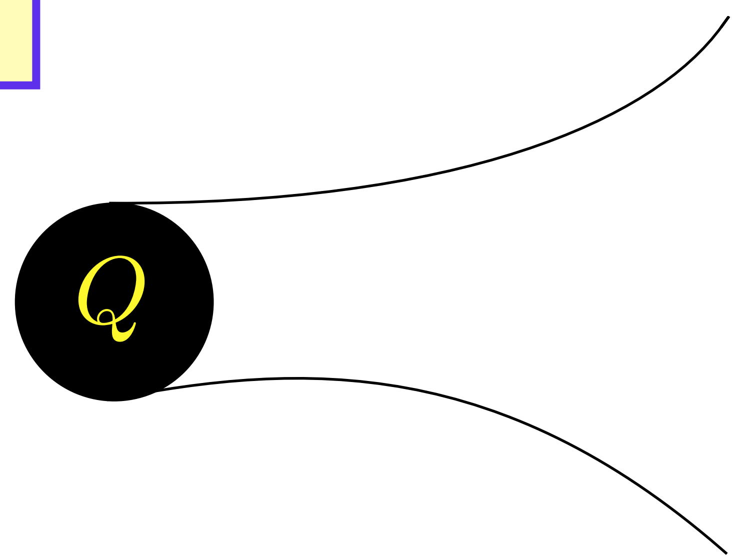
who is correct?

let's redo the calculation

conductivity within AdS

$$(g_{\mathrm{ab}}, V(\Phi), A_t)$$

$$(\mathrm{metric}\,, \mathrm{potential}\,, \mathrm{gaugefield})$$



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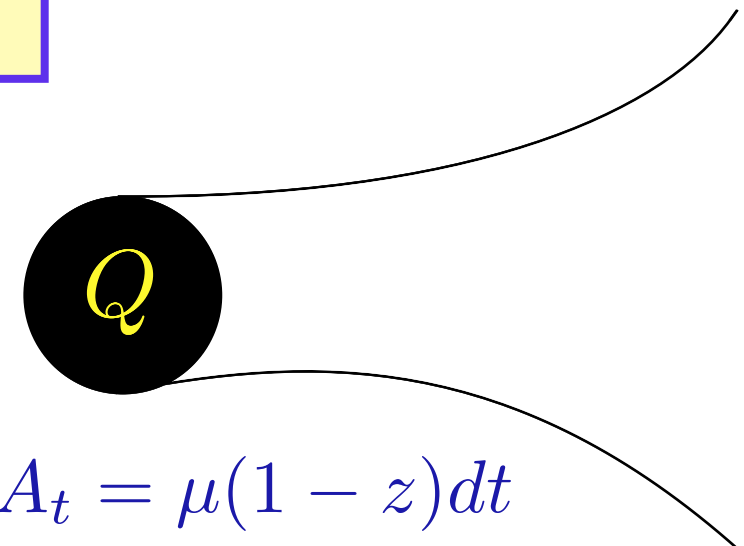

$$A_t = \mu(1-z)dt$$
$$\rho = \lim_{z\to 0} \sqrt{g} F^{tz}$$

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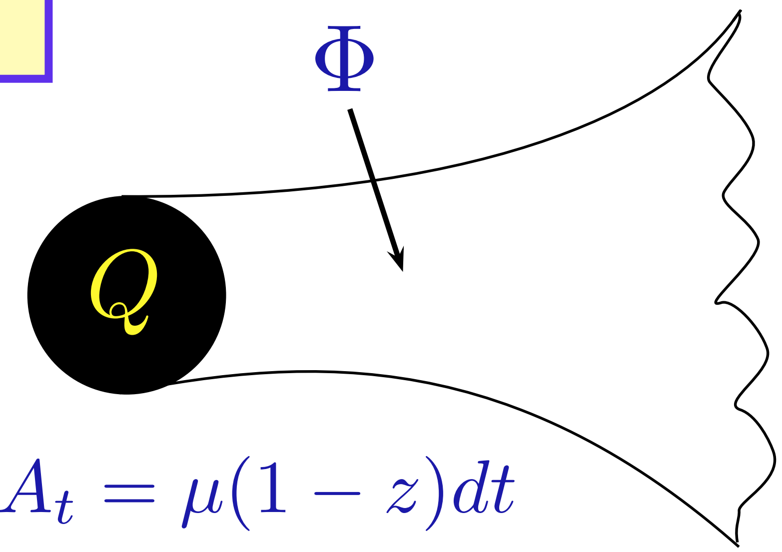

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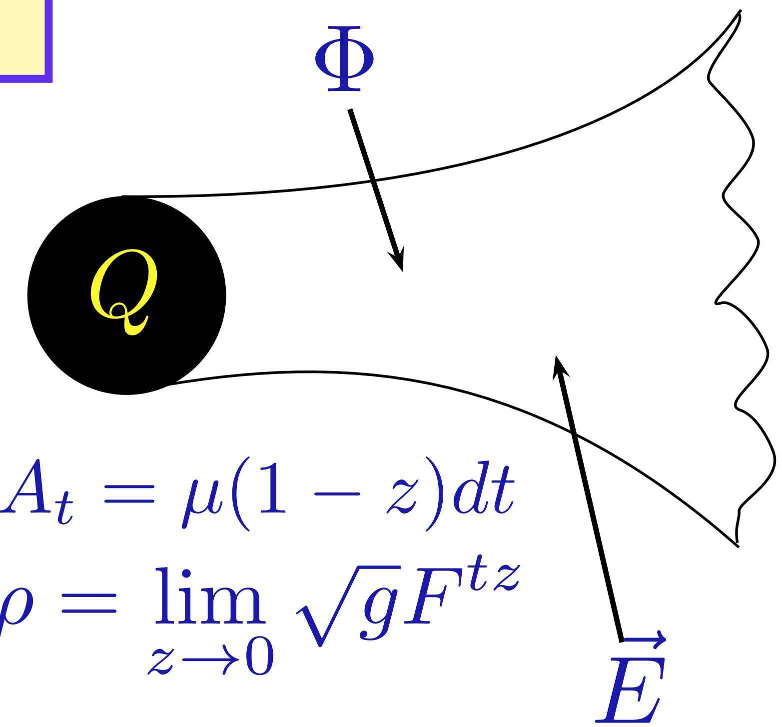
perturb with electric field



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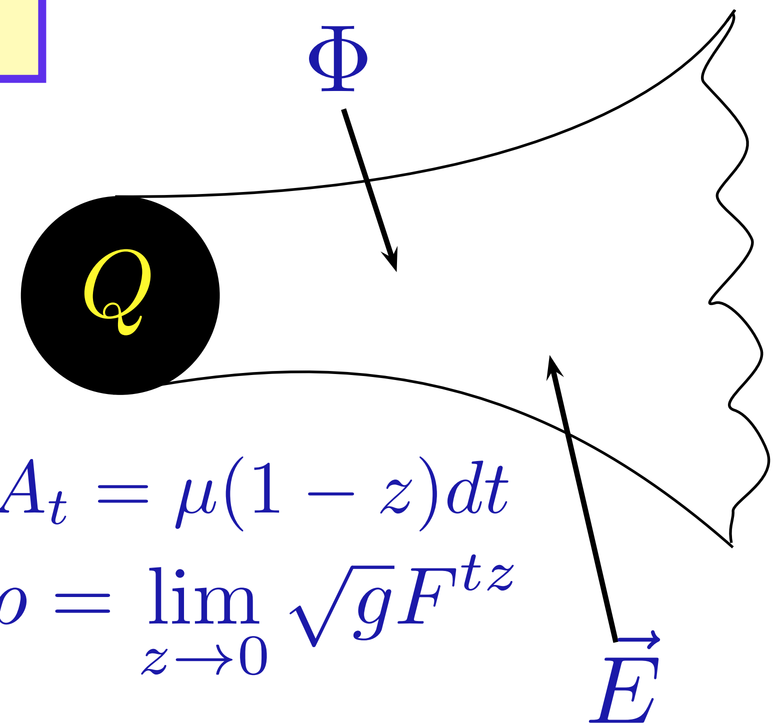

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perturb with
electric field

$$g_{\rm ab} = \bar{g}_{\rm ab} + h_{\rm ab}$$
$$A_a = \bar{A}_a + b_a$$
$$\Phi_i = \bar{\Phi}_i + \eta_i$$

conductivity within AdS

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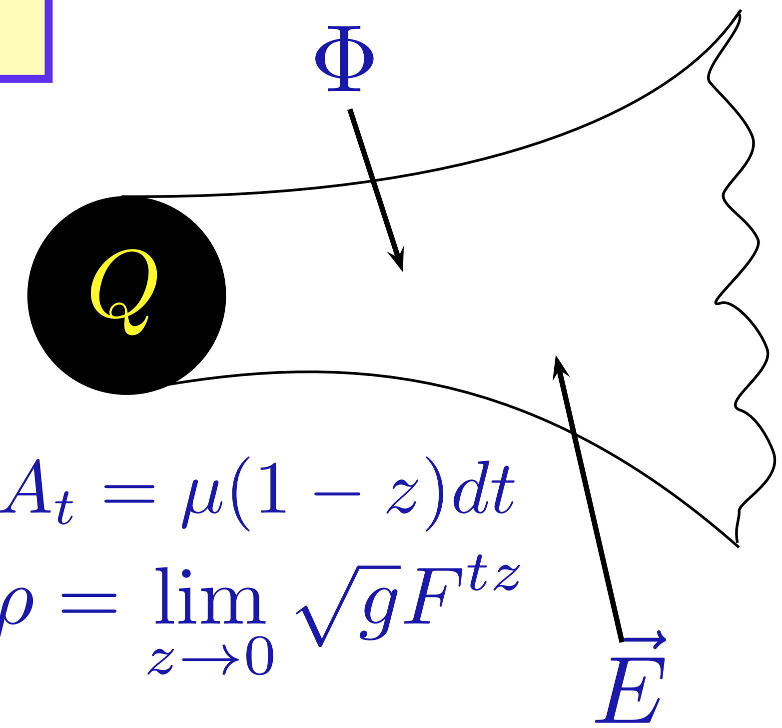

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$$\delta A_x = \frac{E}{i\omega} + J_x(x,\omega)z + O(z^2)$$

conductivity
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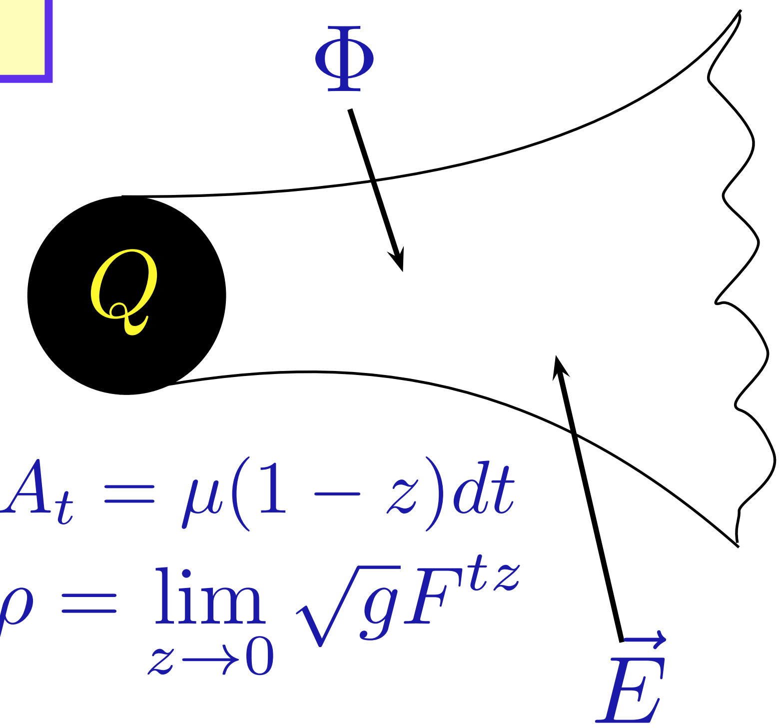

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solve equations of motion
with gauge invariance

conductivity within AdS

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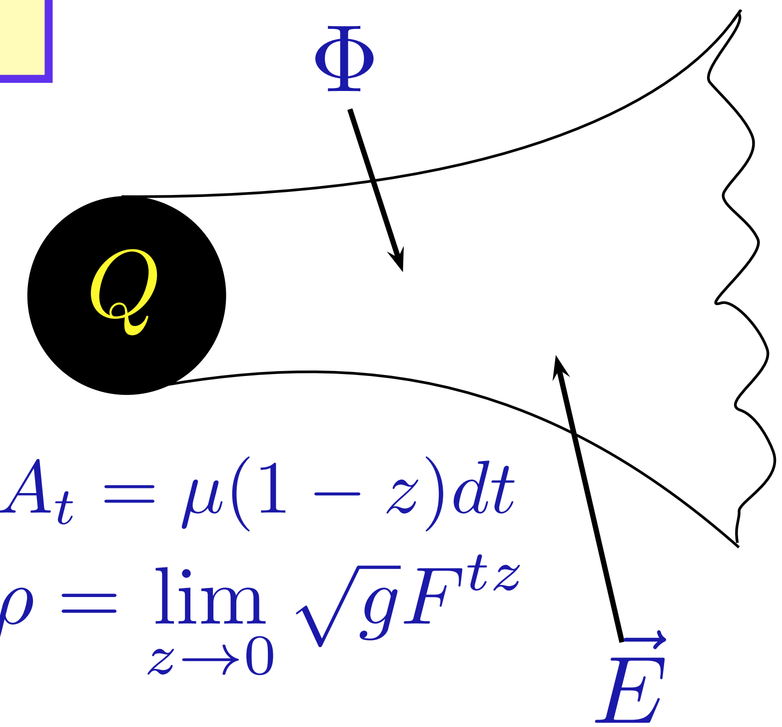

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perturb with electric field

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solve equations of motion with gauge invariance

$$\sigma = J_x(x,\omega)/E$$

model

RNAdS

$$ds^2 = \frac{L^2}{r^2 f\left(\frac{r_H}{r}\right)} dr^2 + \frac{r^2}{L^2}\left(-f\left(\frac{r_H}{r}\right) dt^2 + dx^2 + dy^2\right),$$

model

RNAdS $$ds^2 = \frac{L^2}{r^2 f\left(\frac{r_H}{r}\right)} dr^2 + \frac{r^2}{L^2}\left(-f\left(\frac{r_H}{r}\right) dt^2 + dx^2 + dy^2\right),$$

action $=$ gravity $+$ EM $+$ lattice

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action = gravity + EM + lattice

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}F^2\right),$$

model

RNAdS

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$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}F^2\right),$$

$$\mathcal{L}(\phi) = \sqrt{-g}[-|\partial\phi|^2 - V(|\phi|)]$$

HST vs. DG

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots\,,$$
$$\Phi^{(1)}(x) = A_0 \cos(kx)$$

inhomogeneous in x

$$m^2 = -2/L^2$$

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DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

$$m^2 = -3/(2L^2)$$

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radial gauge

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

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$$\Phi_2 = \qquad z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots\,, \quad \Phi_2^{(1)}(x) = A_0\cos\left(kx + \frac{\theta}{2}\right).$$

Our Model

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$\theta = 0$ → HST

Our Model

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$\theta = 0$ → HST

$\theta = \frac{\pi}{2}$ → DG

Einstein-De Turck EOM

$$G^H_{ab} = G_{ab} - \nabla_{(a}\xi_{b)},$$
$$\xi^a = g^{cd}\left(\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\bar{g})\right).$$

Einstein-De Turck EOM

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reference metric

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metric ansatz

reference metric

$$ds^2 = \frac{L^2}{z^2}\left[-(1-z)P(z)Q_{tt}dt^2 + \frac{Q_{zz}dz^2}{(1-z)P(z)} + Q_{xx}(dx + z^2 Q_{zx}dz)^2 + Q_{yy}dy^2\right],$$

$$P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2}z^3.$$

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RN-AdS when

$$Q_{tt} = Q_{zz} = Q_{yy} = 1 \qquad \Phi = 0 \qquad a_t = \mu_1 = \mu$$

Dirchlet boundary conditions

$$Q_{tt}(0,x) = Q_{zz}(0,x) = Q_{xx}(0,x) = Q_{yy}(0,x) = 1$$

$$Q_{zx}(0,x) = 0 \quad a_t(0,x) = \mu \quad \Phi(0,x) = \Phi^{(1)}(x)$$

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regularity at z=1

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regularity at z=1

Newton-Raphson on grid

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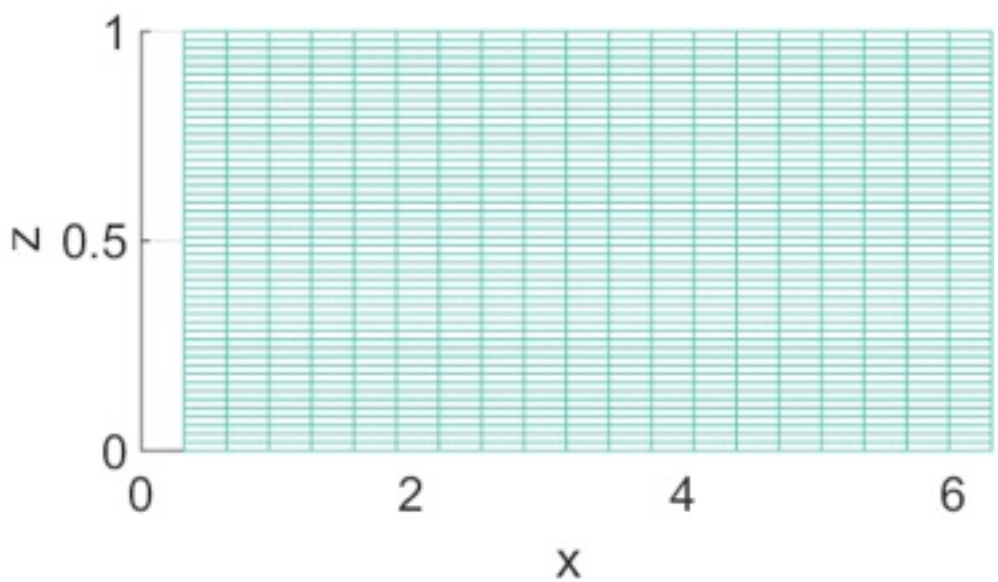
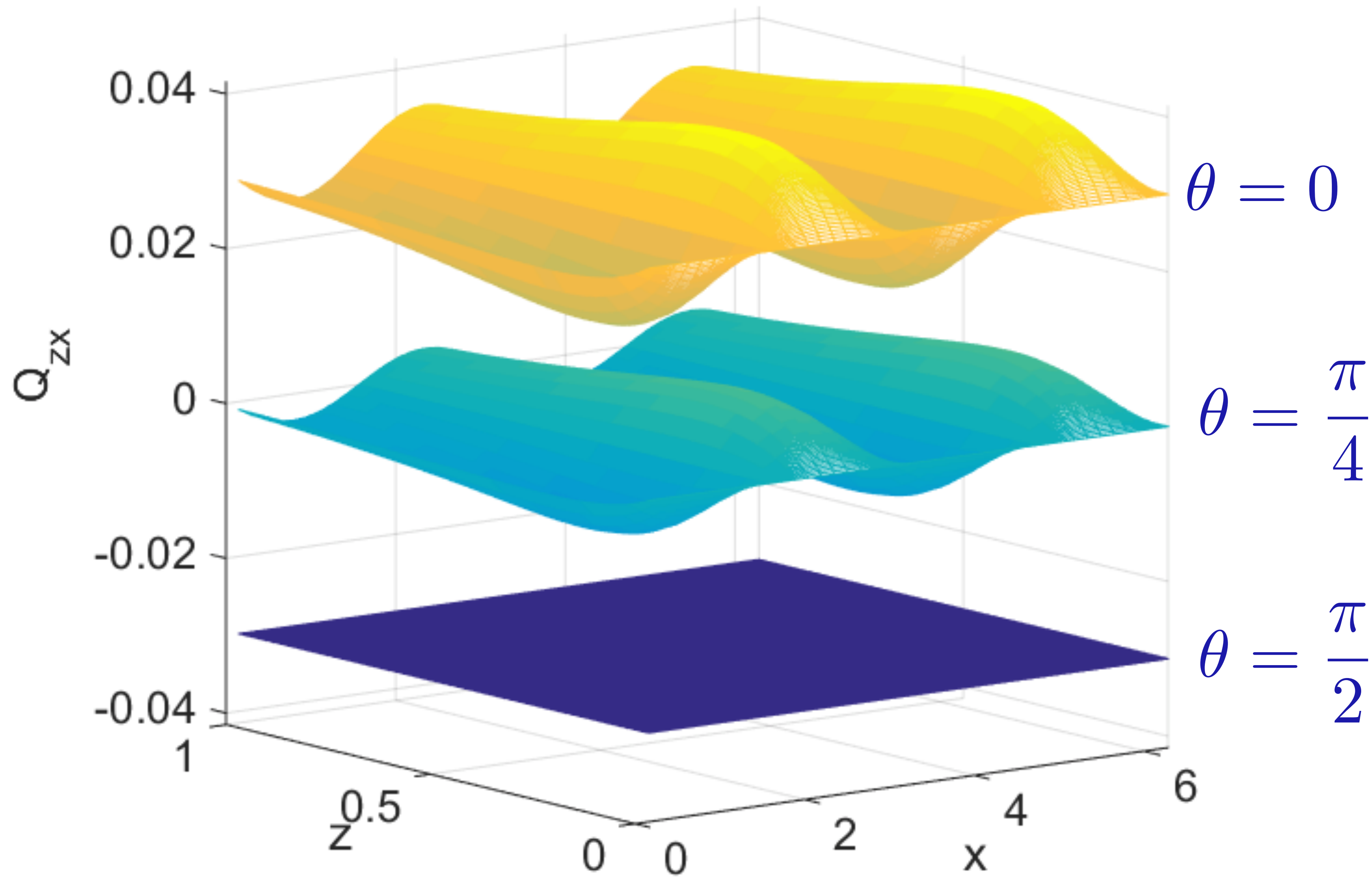
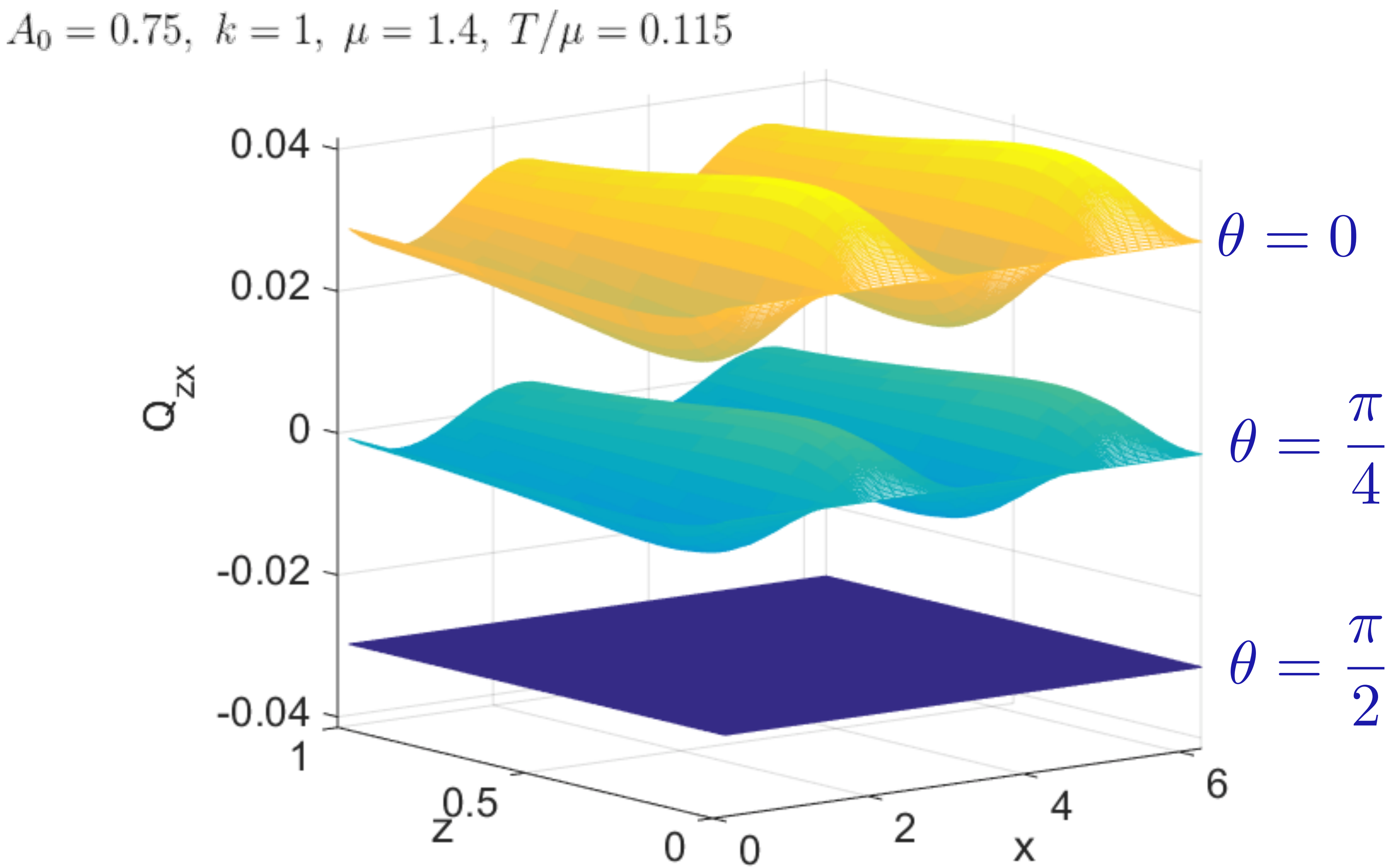
Newton-Raphson on grid



$$z \in [0,1], x \in (0, 2\pi)$$

$A_0 = 0.75, \ k = 1, \ \mu = 1.4, \ T/\mu = 0.115$

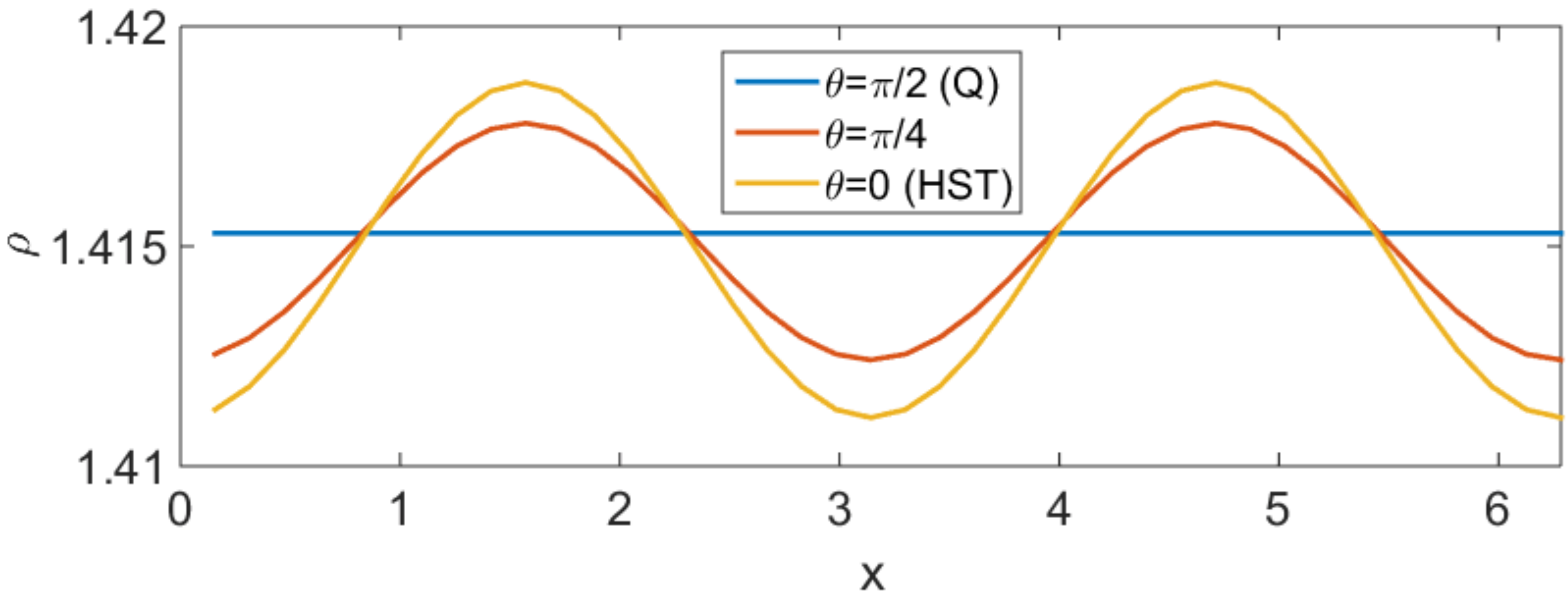




translational invariance is broken in metric in multiples of 2k

charge density

$$\rho = \lim_{z \to 0} \sqrt{-g} F^{tz}$$



perturb with electric field

$$g_{\text{ab}} = \bar{g}_{\text{ab}} + h_{\text{ab}}$$
$$A_a = \bar{A}_a + b_a$$
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gauge invariance

$$\delta g_{ab} + \mathcal{L}_\zeta \overline{g}_{ab} = 0,$$

$$\delta A_a + \mathcal{L}_\zeta \overline{A}_a + \nabla_a \Lambda = e^{-i\omega t} \mu_x^J,$$

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solve equations without mistakes!!

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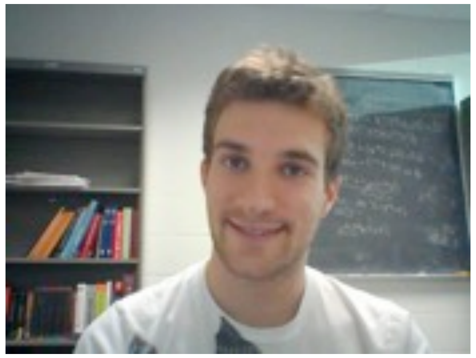
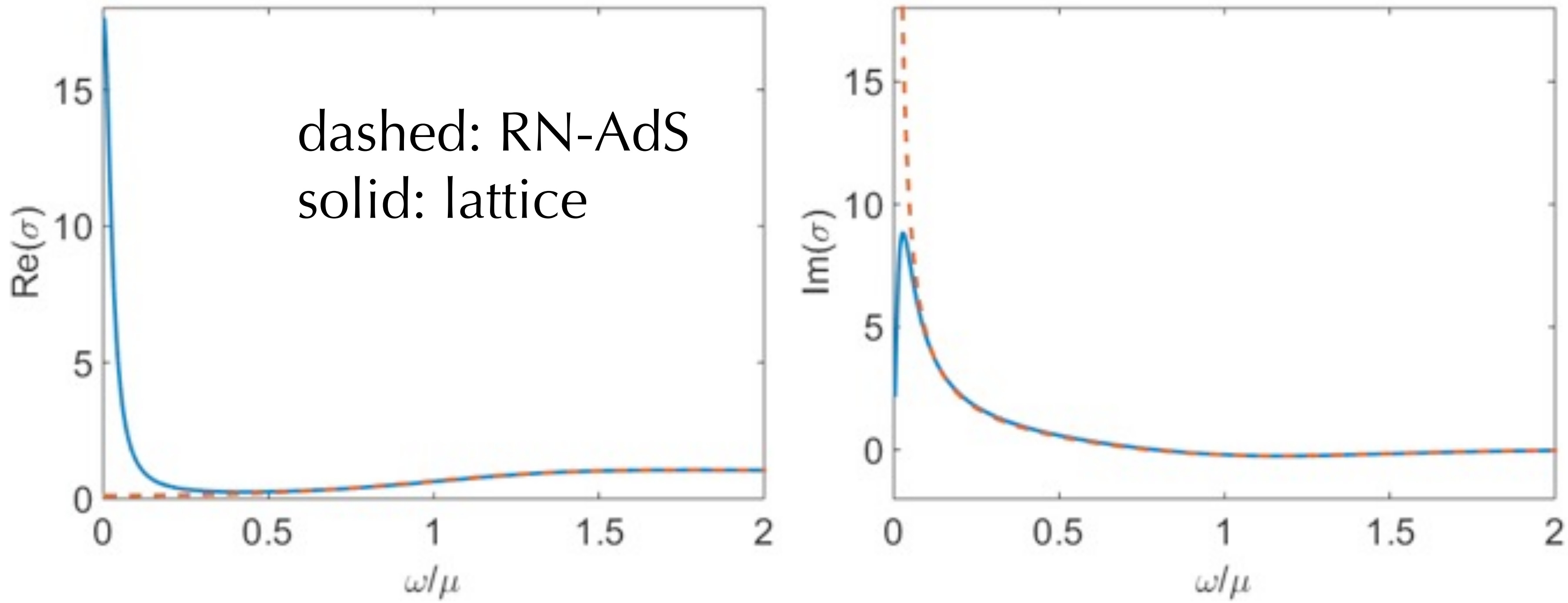


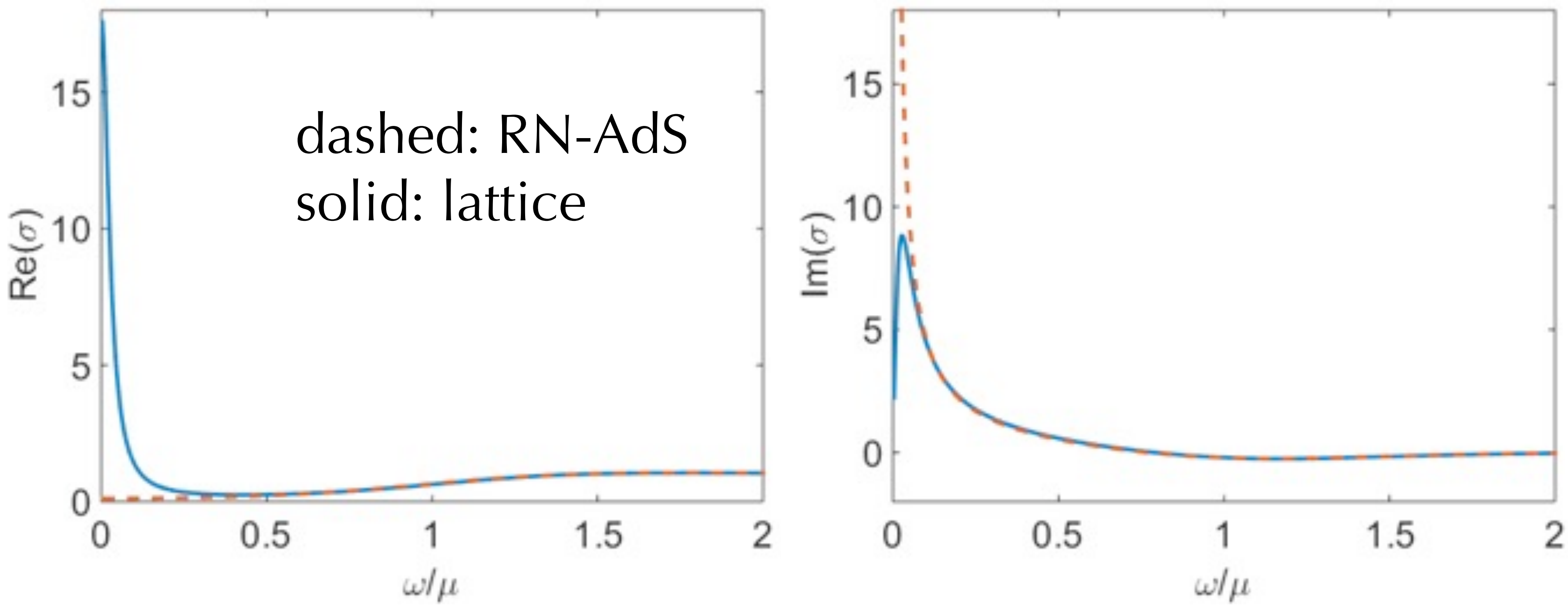
Brandon Langley

solve equations without mistakes!!



- high-frequency behavior is identical
- low-frequency RN has $\mathrm{Re}(\sigma) \sim \delta(\omega),\ \mathrm{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form

sample conductivity plots



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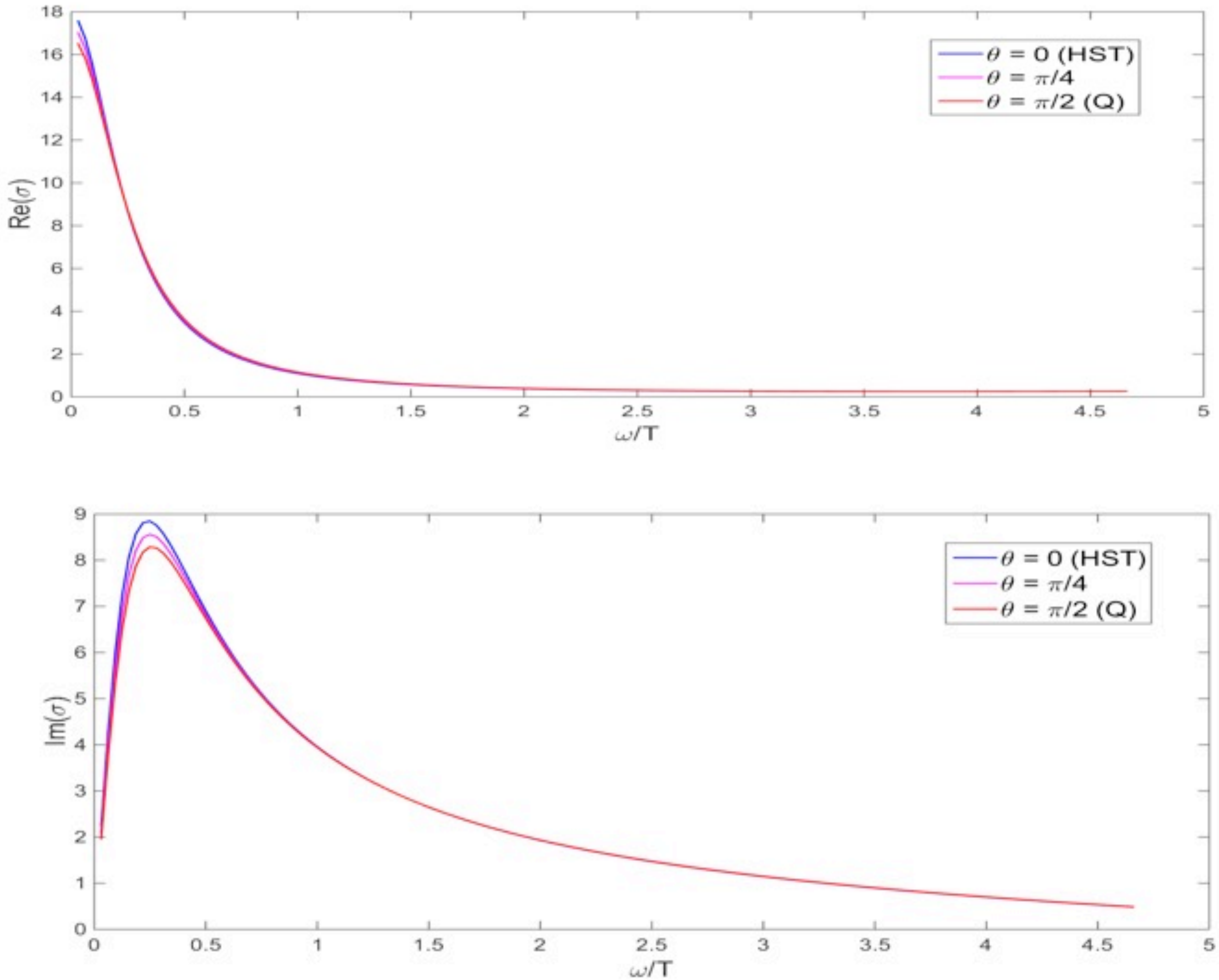
$$k = 1,\ A = 0.75/\sqrt{2},\ T/\mu = 0.115,\ \mu = 1.4$$



Results

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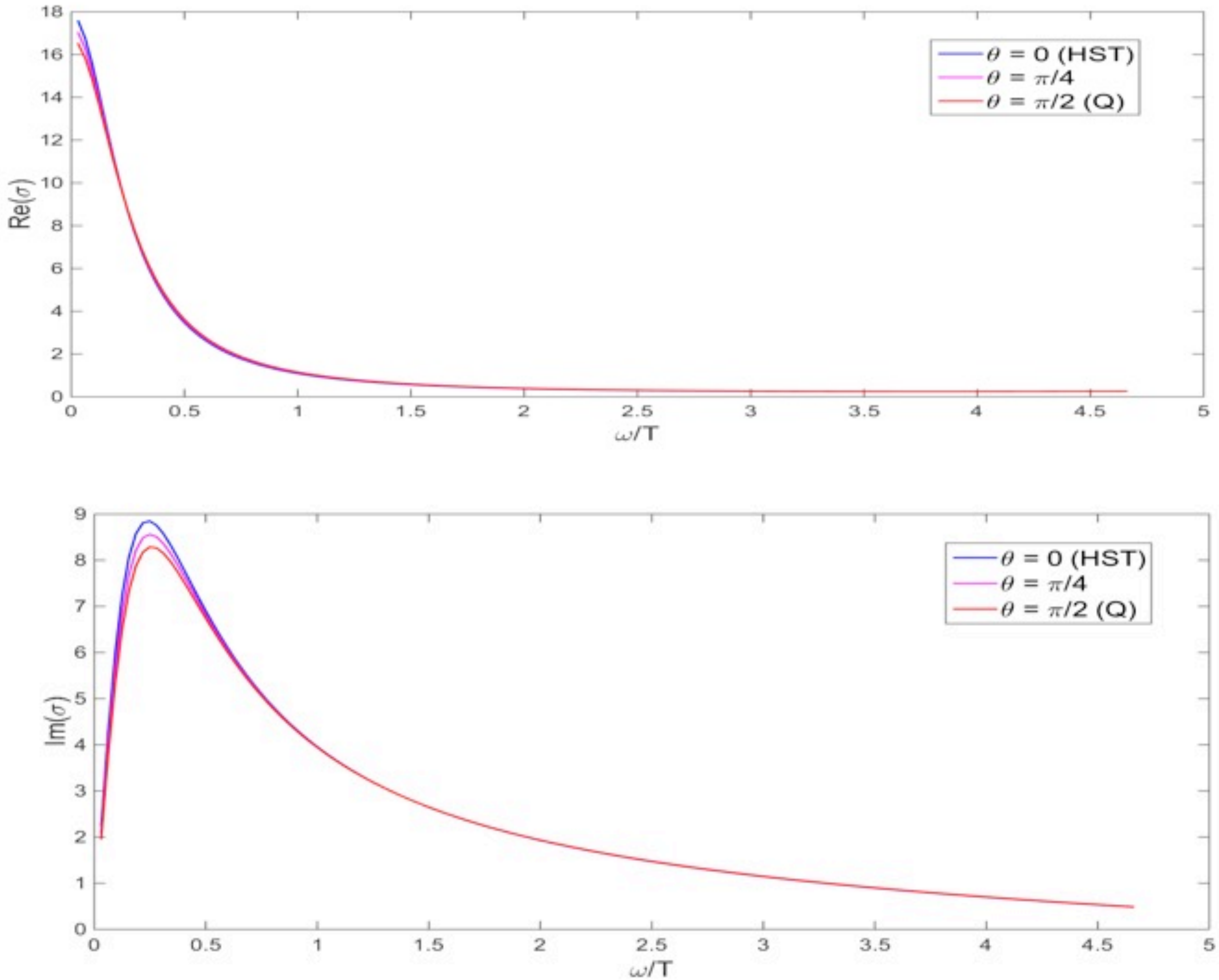

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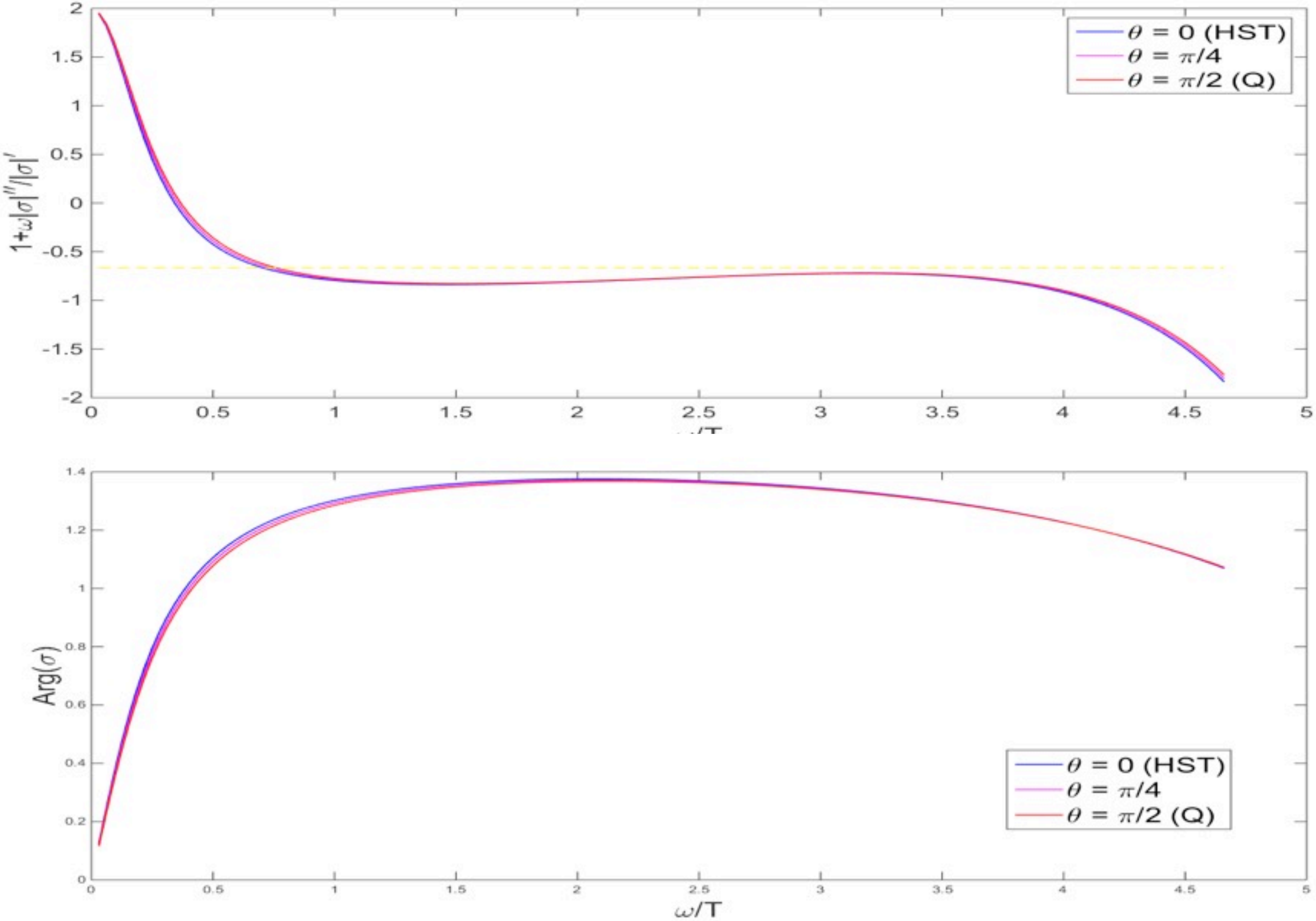

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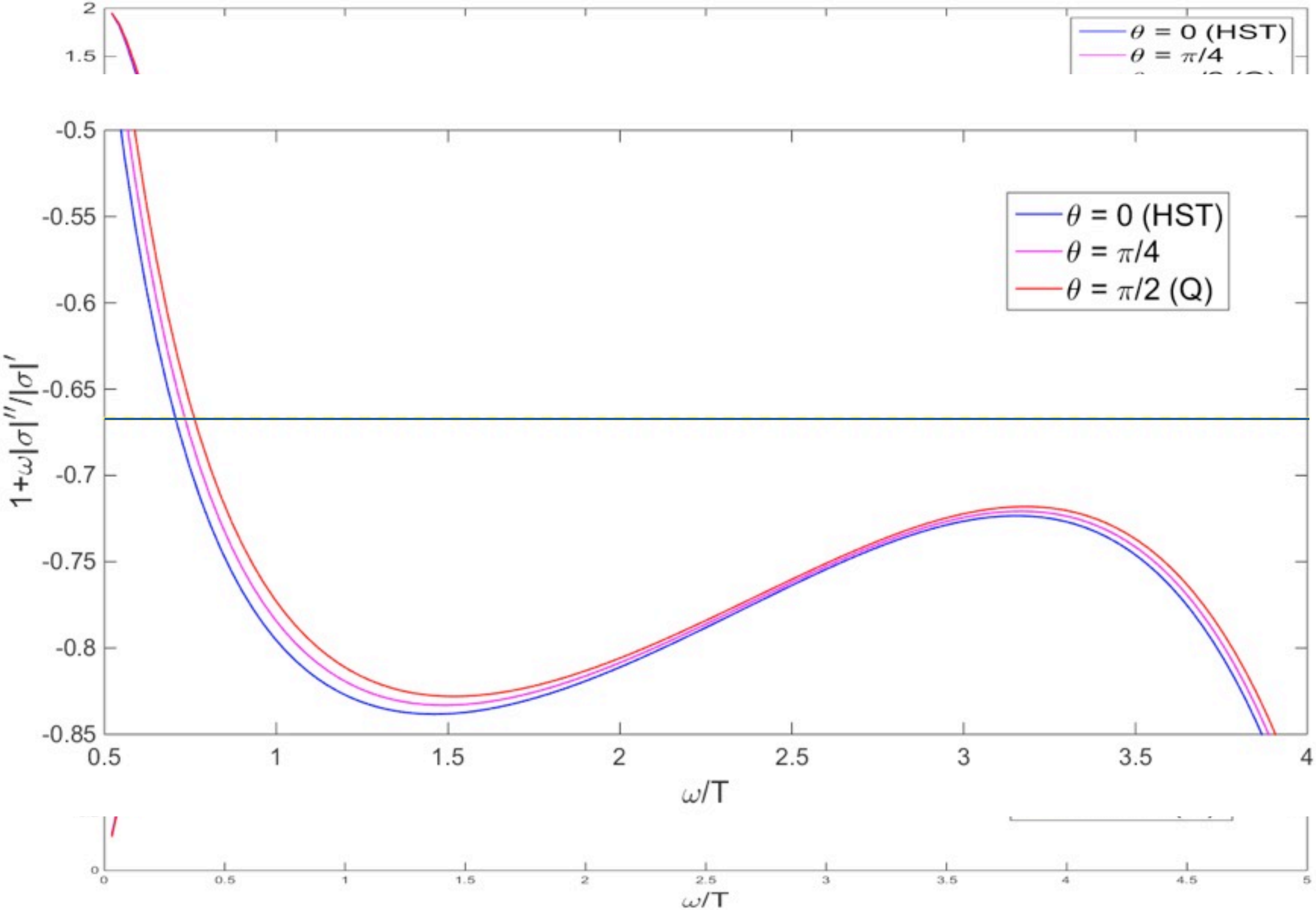

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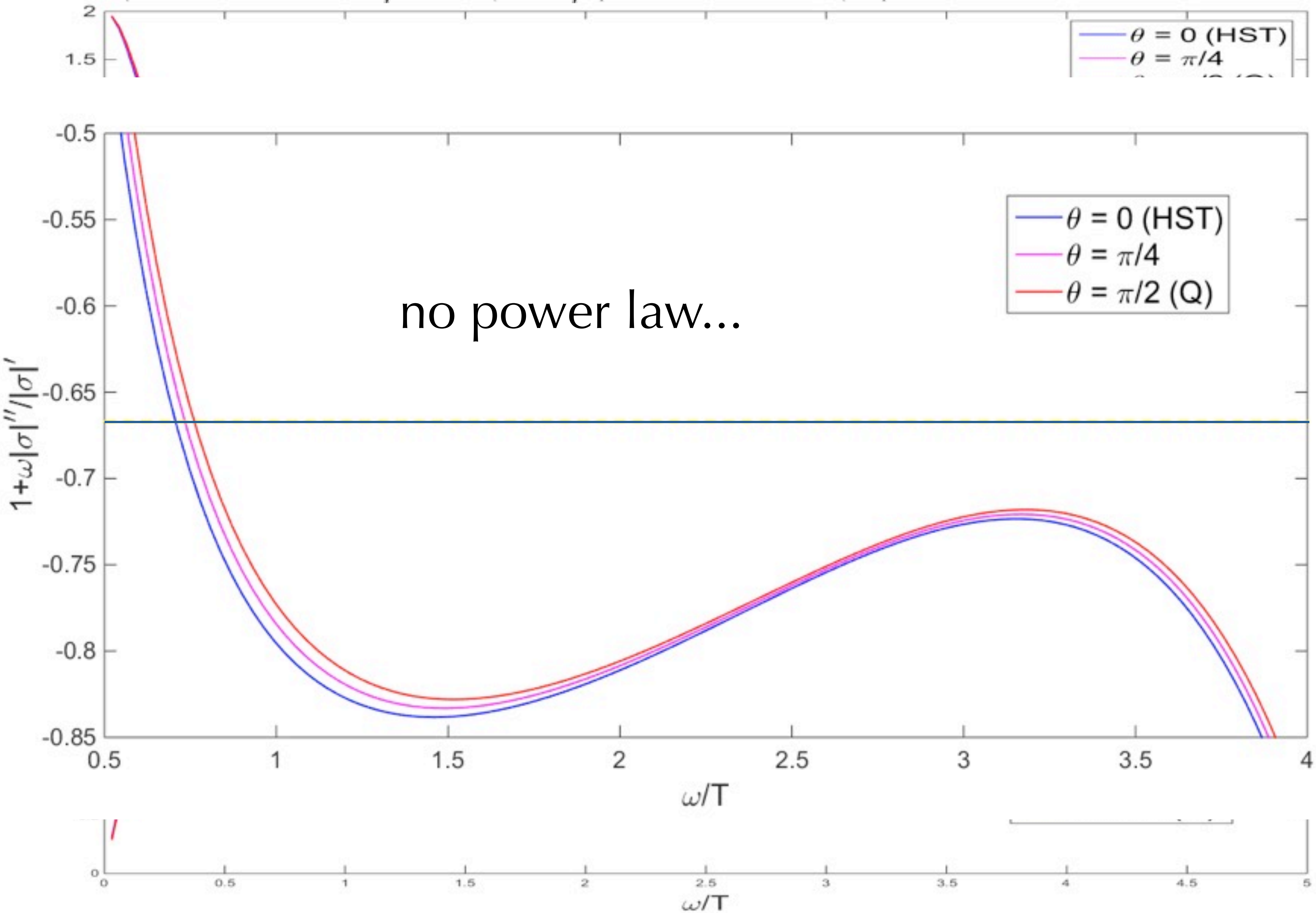
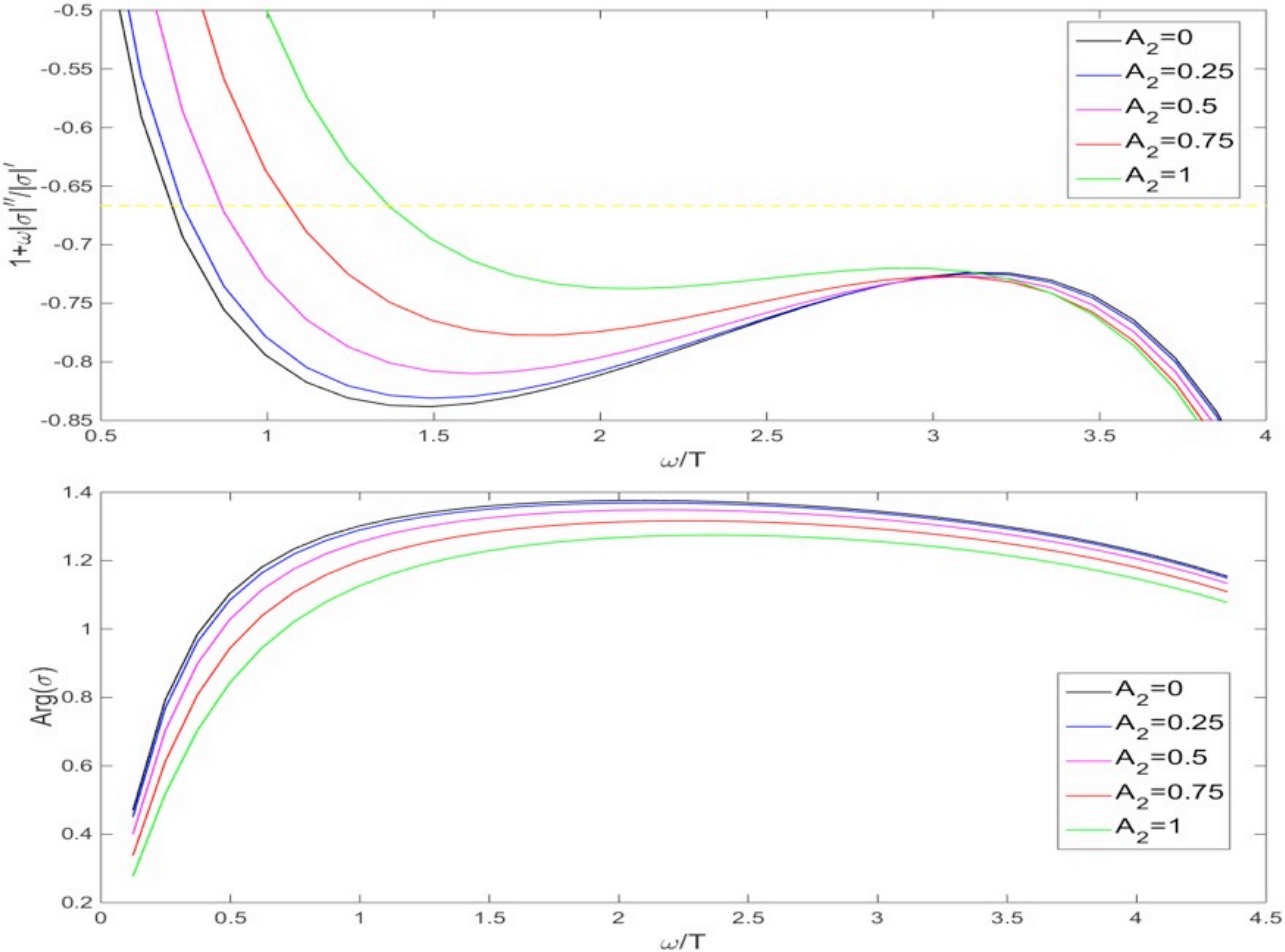


no power law...

$$\phi_i = A_i \cos(k_i x)\,,\ A_1 = 0.75,\ k_1 = 1,\ k_2 = 2$$



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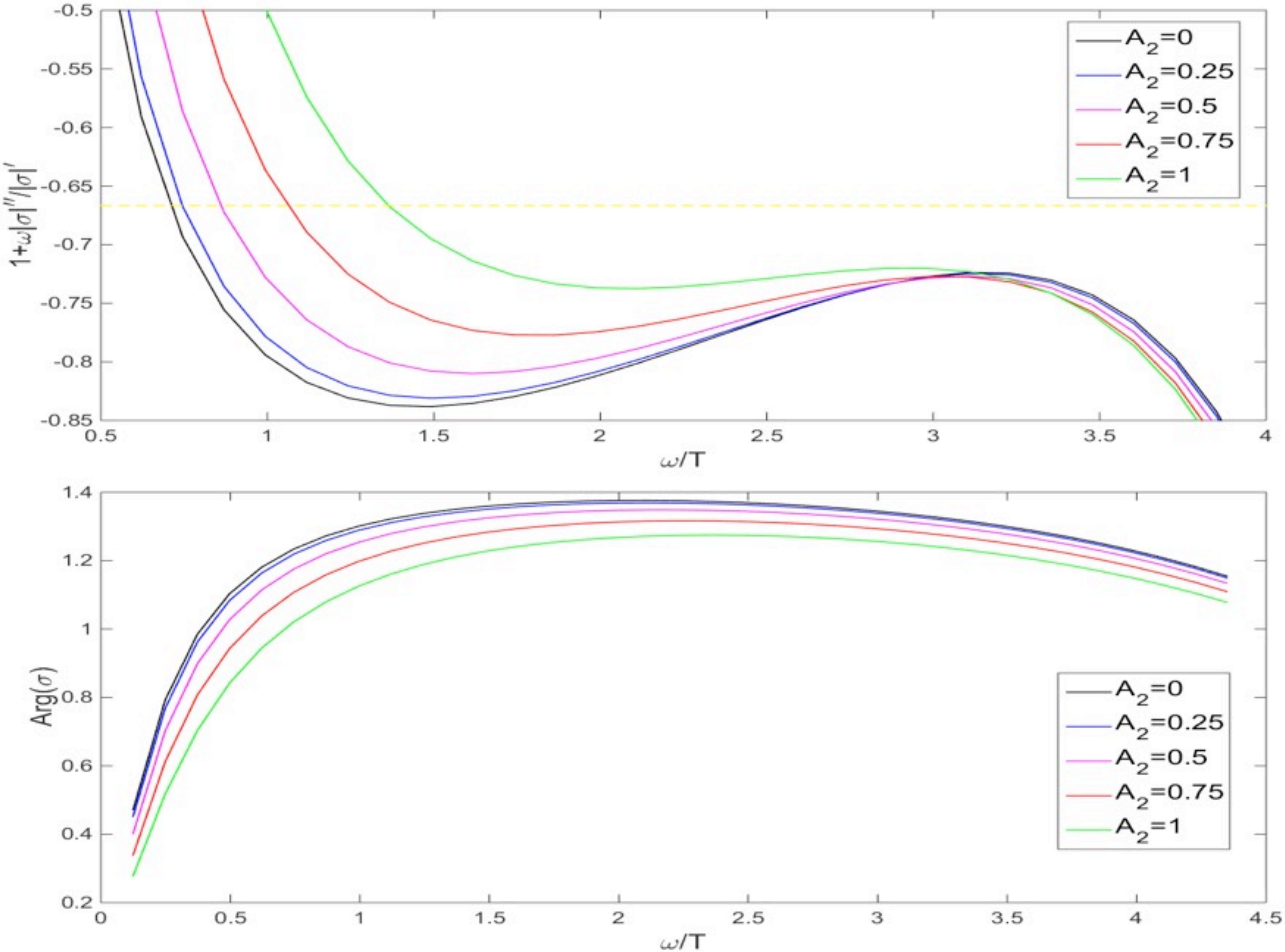

origin of power law?

origin of power law?

phenomenology

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scale-invariant propagators

$$(p^2)^{d_U - d/2}$$

origin of power law?

phenomenology

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no well-defined mass

$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

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incoherent stuff (all energies)

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

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$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

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$$m^2 \phi^2$$

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no scale
invariance

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu\phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big)\, dm^2$$

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theory with all possible mass!

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$$x \to x/\Lambda$$
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$$\phi \rightarrow \phi(x, m^2/\Lambda^2)$$
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$$\mathcal{L} \rightarrow \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

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not particles

unparticles

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propagator

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$$d_U - 2$$

continuous mass

$\phi(x, m^2)$

flavors

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

$\downarrow$

$$d_U - 2$$

continuous mass

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flavors

$e^2(m)$

Karch, 2005

multi-bands

propagator

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Karch, 2005

flavors

multi-bands

use unparticle propagators to calculate conductivity

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assume Gaussian action

$$S = \int d^{d+1}p \; \phi_U^\dagger(p) i G^{-1}(p) \phi_U(p)$$

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$$G(p) \sim \frac{i}{(-p^2 + i\epsilon)^{\frac{d+1}{2} - d_U}}$$

gauge unparticles

gauge unparticles

Wilson line

$$S = \int d^{d+1}x d^{d+1}y \; \phi_U^\dagger(x) F(x-y) W(x,y) \phi_U(y),$$

gauge unparticles

Wilson line

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vertices

gauge unparticles

Wilson line

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vertices

$$g\Gamma^\mu(p,q) = \frac{\delta^3 S}{\delta A^\mu(q) \delta\phi^\dagger(p+q) \delta\phi(p)}$$

1-gauge

$$g^2\Gamma^{\mu\nu}(p,q_1,q_2) = \frac{\delta^4 S}{\delta A^\mu(q_1) \delta A^\nu(q_2) \delta\phi^\dagger(p+q_1+q_2) \delta\phi(p)}$$

2-gauge

use Ward-Takahashi
identities to simplify vertices

use Ward-Takahashi identities to simplify vertices

$$-iq_\mu \Gamma^\mu(p,q) = G^{-1}(p+q) - G^{-1}(p)$$

$$q_{1\mu}\Gamma^{\mu\nu}(p,q_1,q_2) = \Gamma^\nu(p+q_1,q_2) - \Gamma^\nu(p,q_2)$$

use Ward-Takahashi identities to simplify vertices

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response function to an electric field

use Ward-Takahashi identities to simplify vertices

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response function to an electric field

$$K^{\mu\nu}_{-n,-n'}(-q,-q') = -\frac{(2\pi)^{2d}}{T^2}\mathcal{Z}^{-1}\frac{\delta^2}{\delta A_{\mu,n}(q)\delta A_{\nu,n'}(q')}\bigg|_{A=0}\mathcal{Z}[A]$$

compute conductivity

$$\sigma^{\mu\nu}(i\omega_n) = \lim_{q\to 0} \frac{1}{\omega_n} K_n^{\mu\nu}(q)!$$

compute conductivity

$$\sigma^{\mu\nu}(i\omega_n) = \lim_{q\to 0} \frac{1}{\omega_n} K_n^{\mu\nu}(q)!$$

no power law

$$\sigma(i\omega_n) = (\frac{d+1}{2} - d_U)\sigma_0(i\omega_n)$$

what went wrong?

what went wrong?

free field

$$\phi_U(x) = \int_0^\infty dm^2 \; f(m^2)\phi(x, m^2)$$

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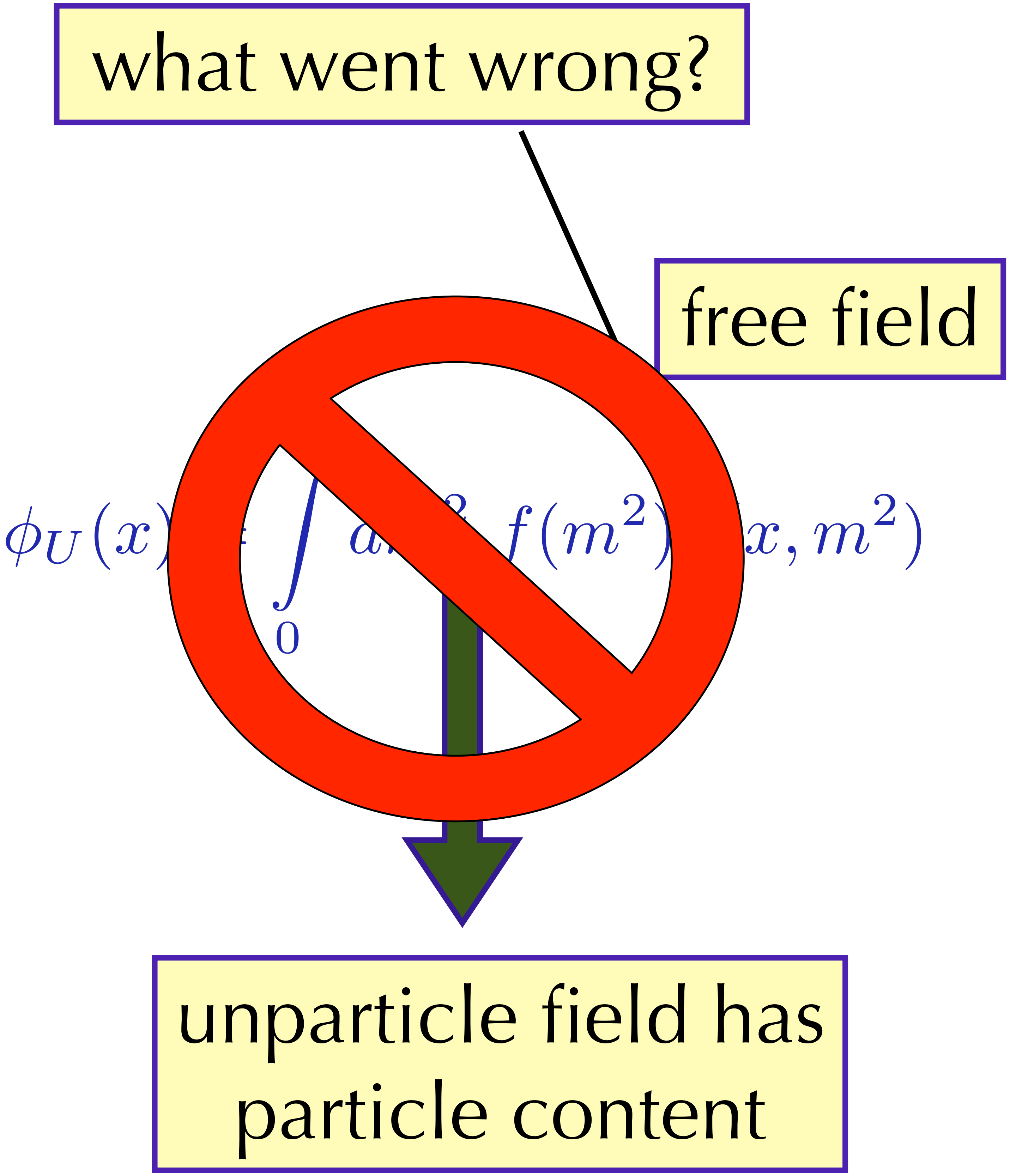
unparticle field has particle content


what went wrong?
free field
$\phi_U(x)$ … $\int_0$ … $f(m^2)$ … $x, m^2)$
unparticle field has particle content

continuous mass taken seriously

$$S = \sum_{i=1}^{N} \int d\tau \int d^d x (|D_\mu \phi_i^2| + m_i^2 |\phi_i|^2)$$

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$$\sigma(\omega) = \int_0^M dm \rho(m) e^2(m) f(\omega, m, T)$$

$$\propto \omega^\alpha \quad \alpha > 0 (\omega < 2M)$$

last attempt

last attempt

take experiments seriously

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$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

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continuous mass

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

Karch, 2015

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Karch, 2015

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$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega\tau_0 \frac{m^c}{M^c}}$$

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Karch, 2015

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Karch, 2015

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perform integral

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

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$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

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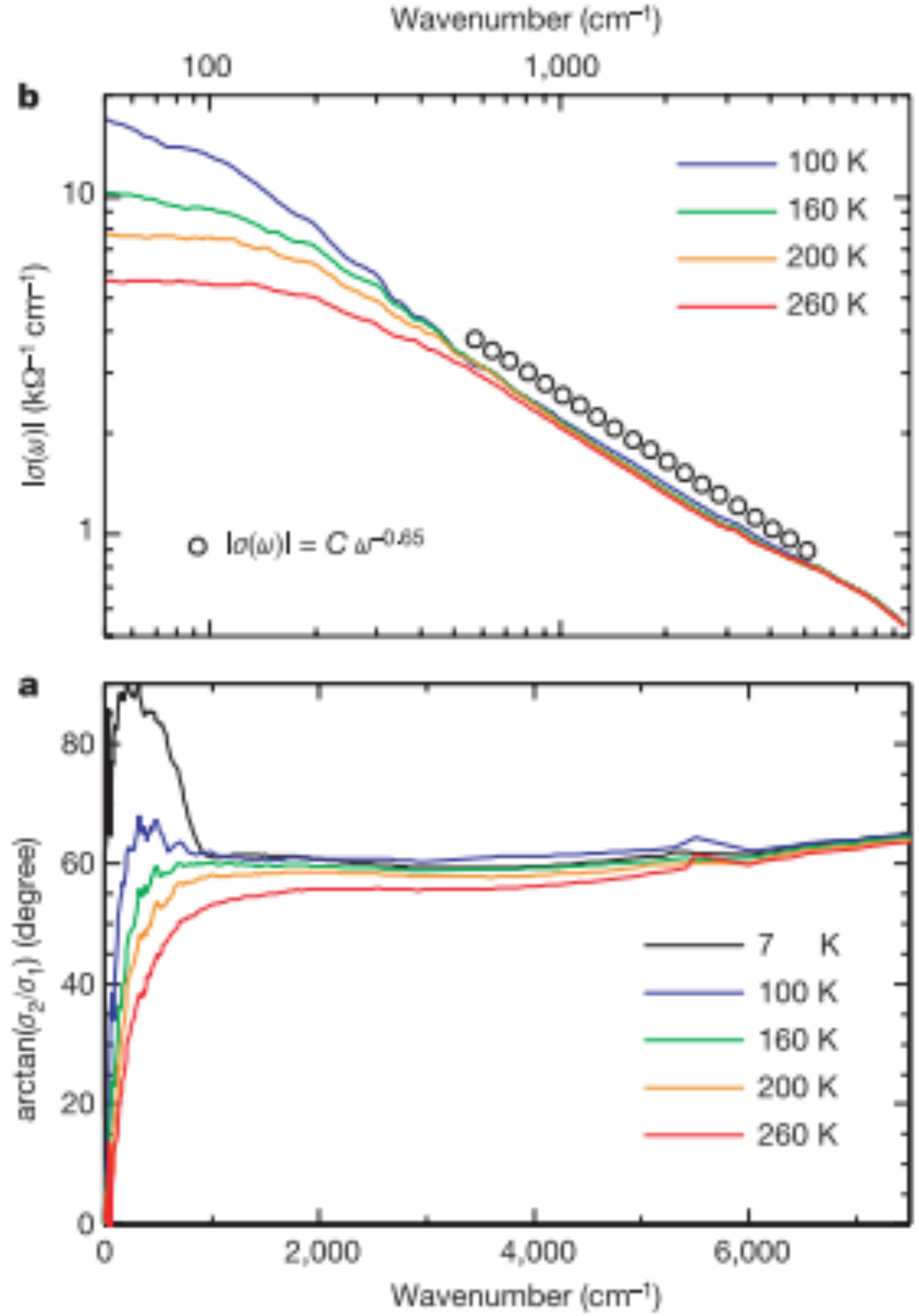
$$\tan\sigma = \sqrt{3}$$

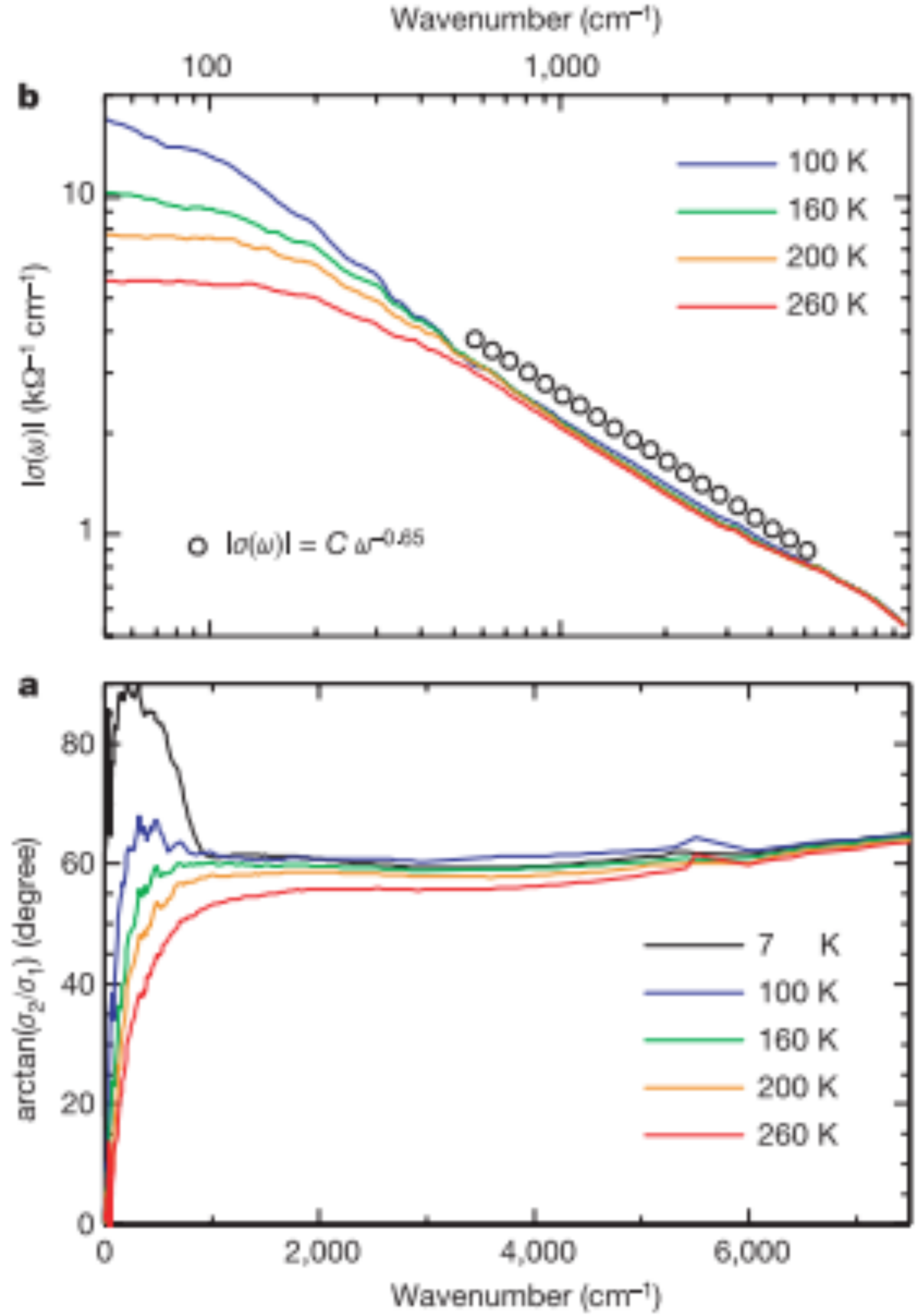
$$60^\circ$$

experiments



$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$
$$\gamma = 1.35$$

experiments



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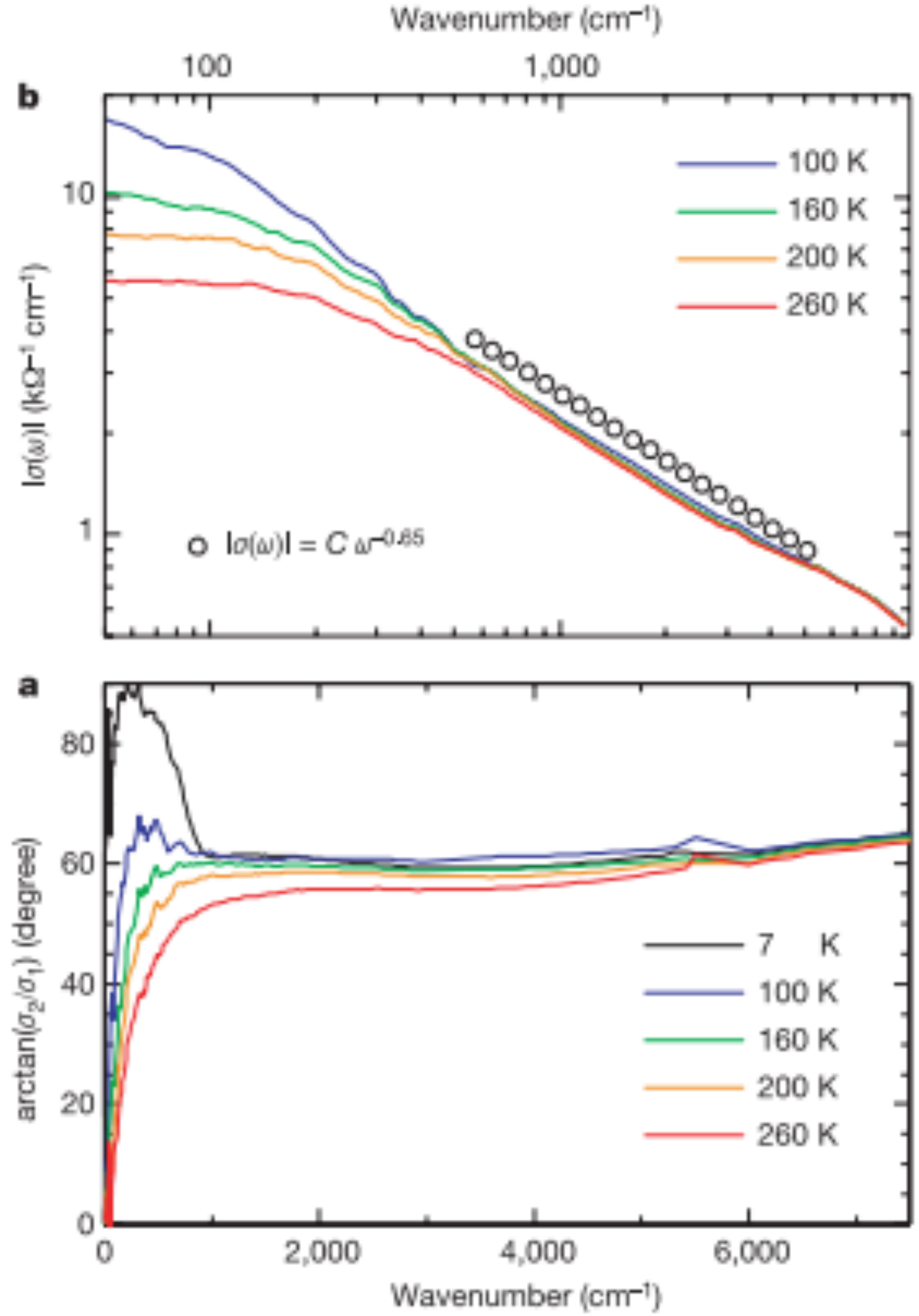

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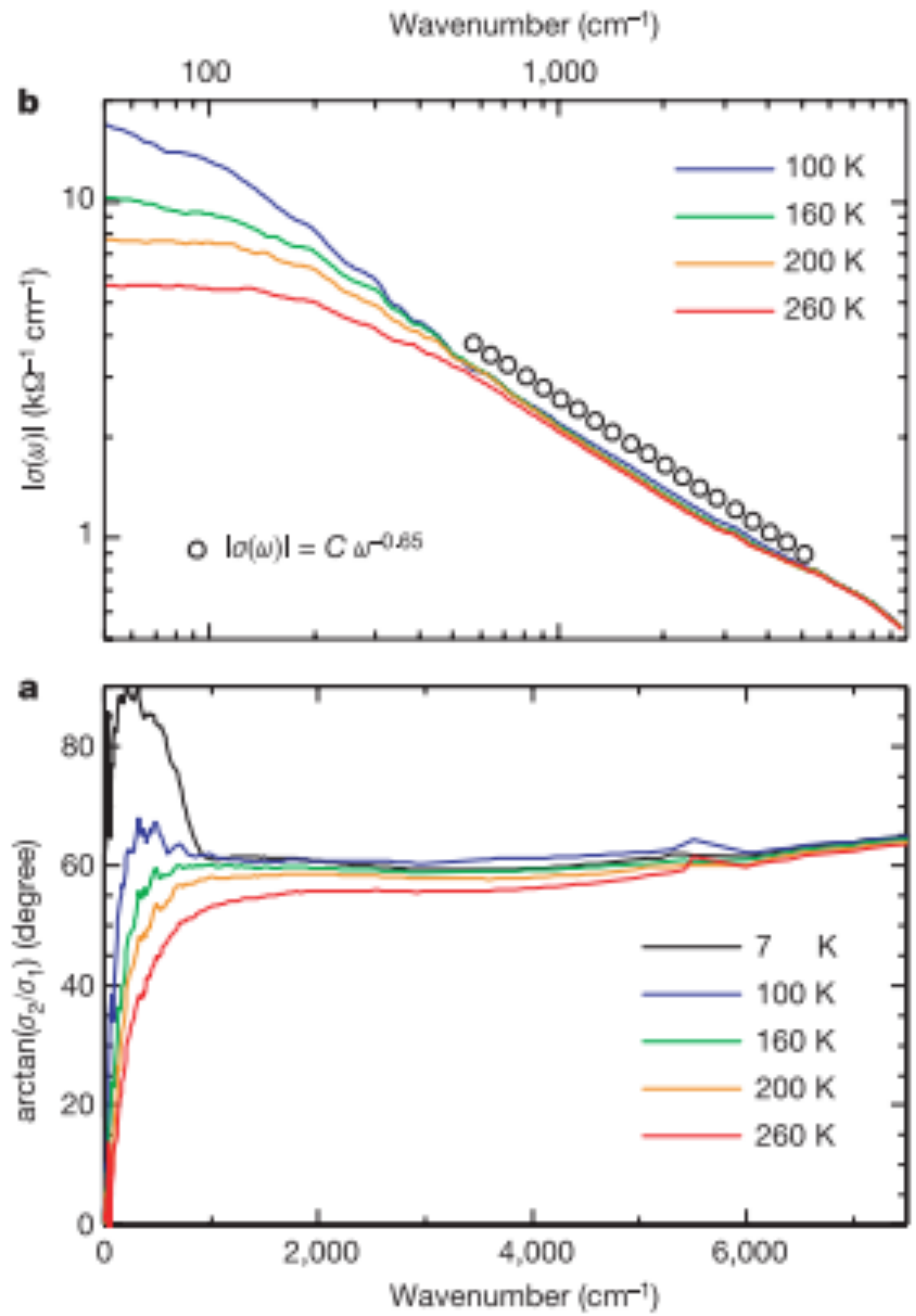

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victory!!

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are anomalous dimensions necessary

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are anomalous dimensions necessary

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hyperscaling violation

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$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

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$$c = 1$$
$$a + 2b = 2/3$$

$\Rightarrow$

$$b = 0$$
$$a = 2/3$$

No

but they are possible!

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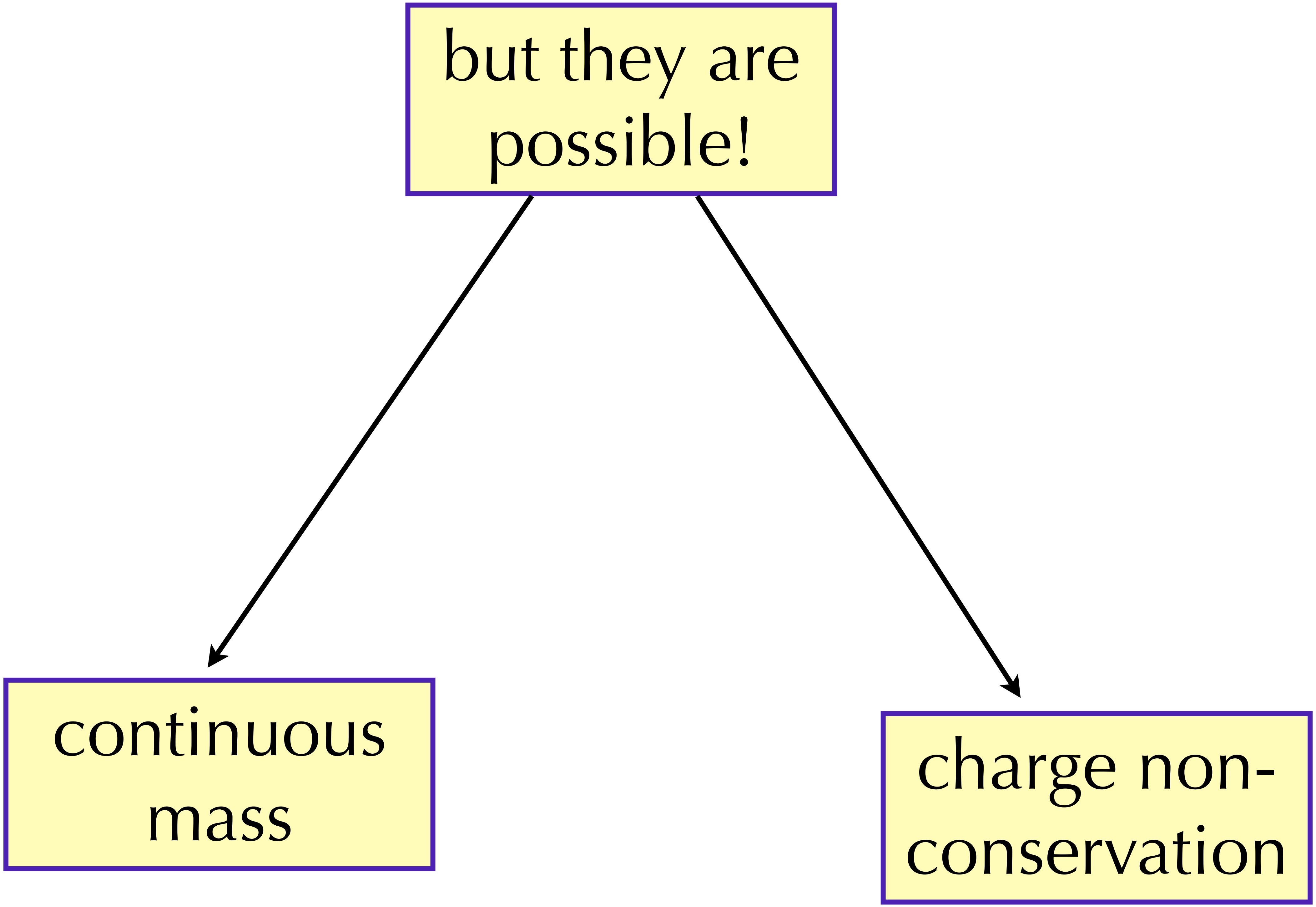
continuous mass


but they are possible!
continuous mass
charge non-conservation

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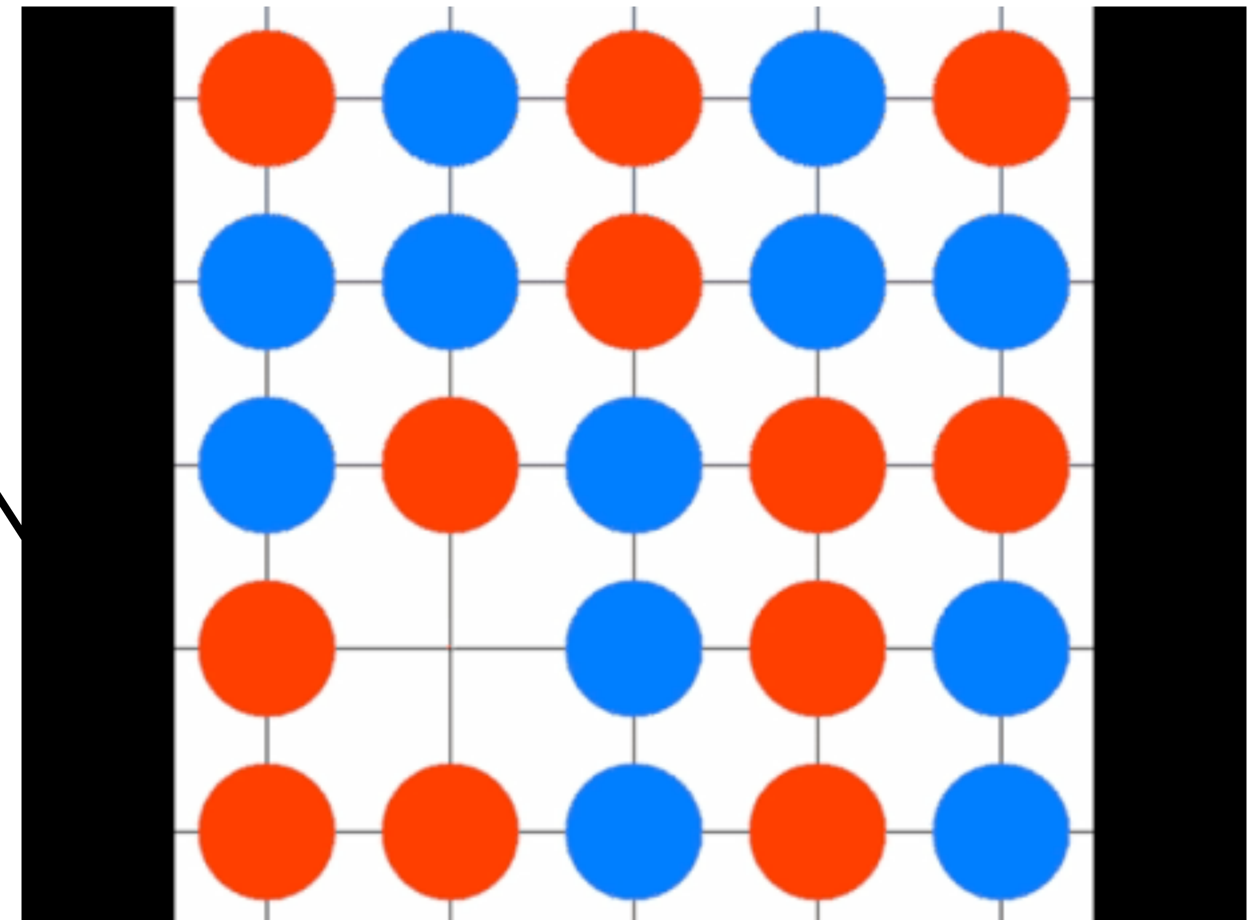
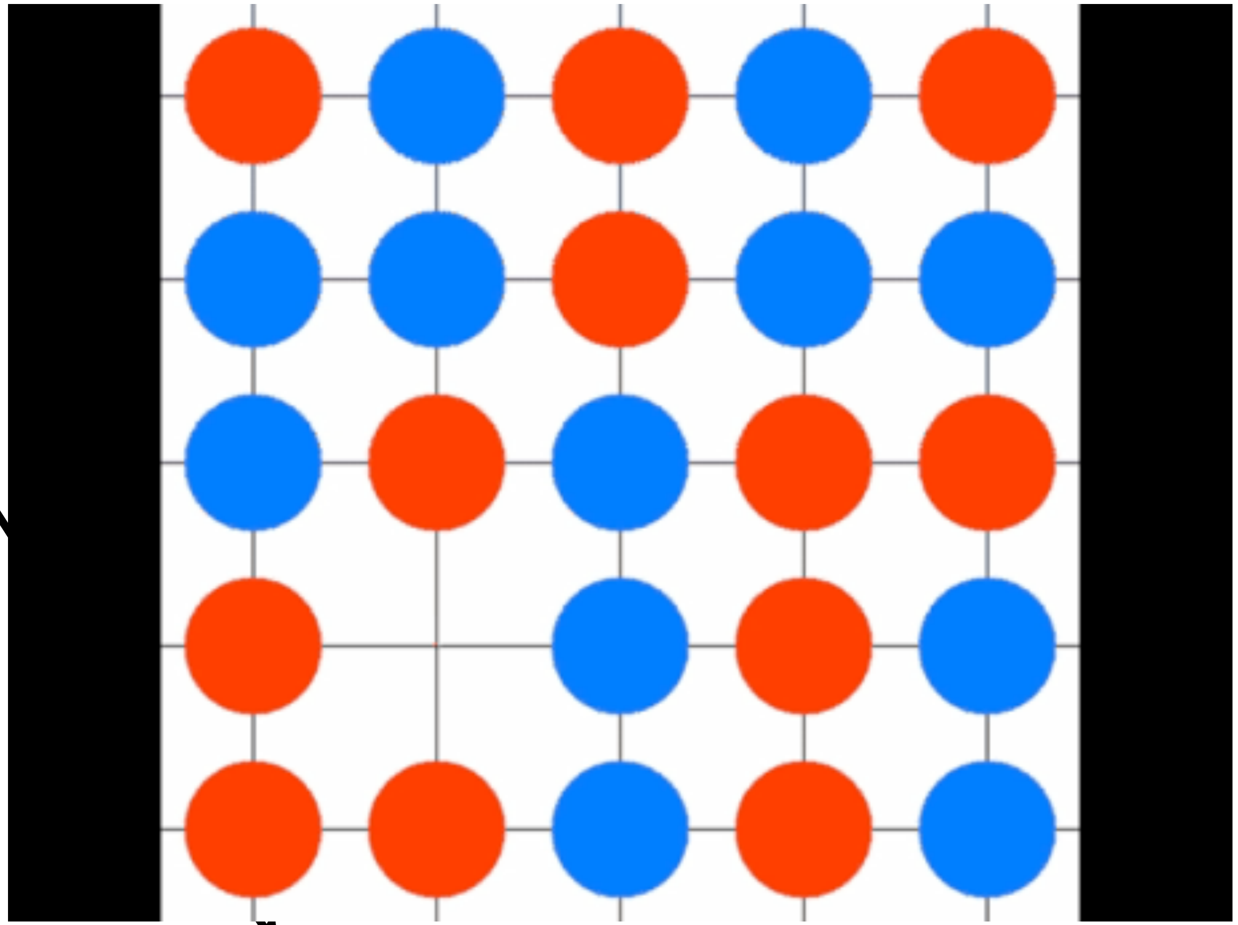
continuous mass



charge non-conservation

but they are possible!



continuous mass

charge non-conservation

unparticles