# Comments on Holography and Unconventional transport

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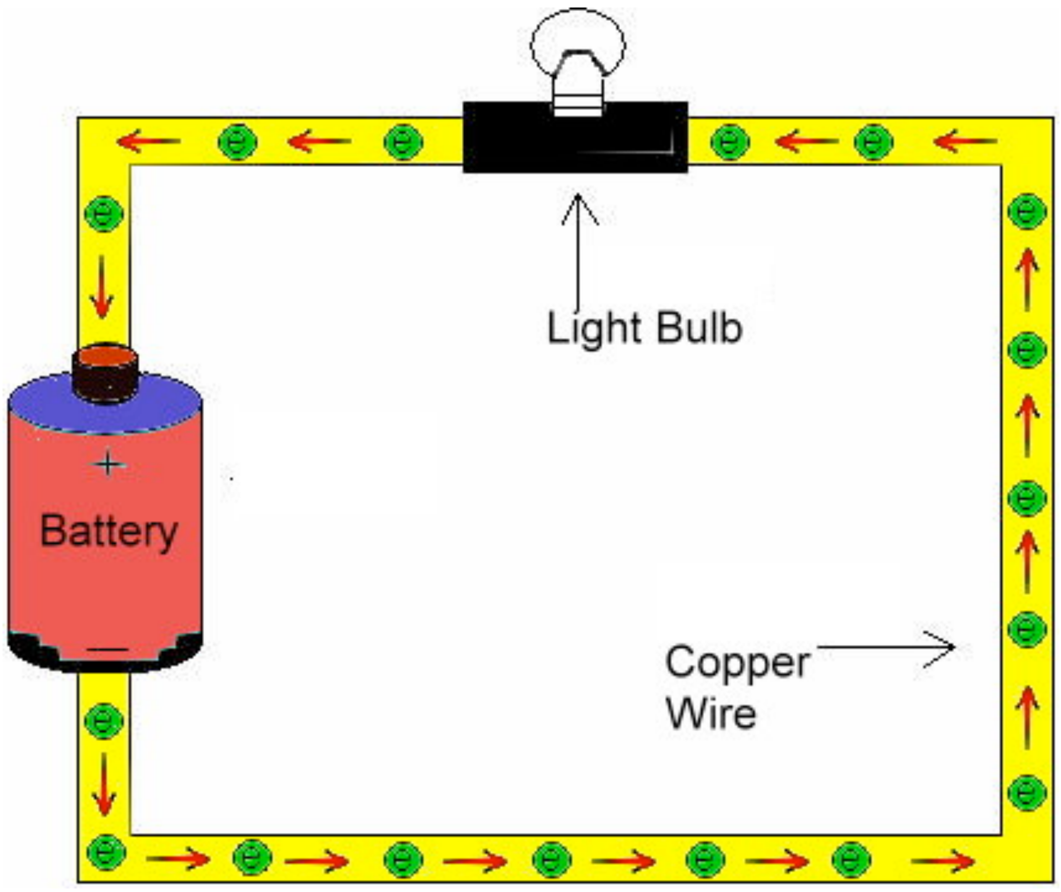
# Plan

- Conventional metals
- Unconventional metals: Facts
- Unconventional metals: Framework
- An Observable: The <u>Lorenz ratio</u>
- <u>Disorder</u> in holography

# I. Conventional metals

# Simple equations




$$j(\omega) = \sigma(\omega) E(\omega)$$

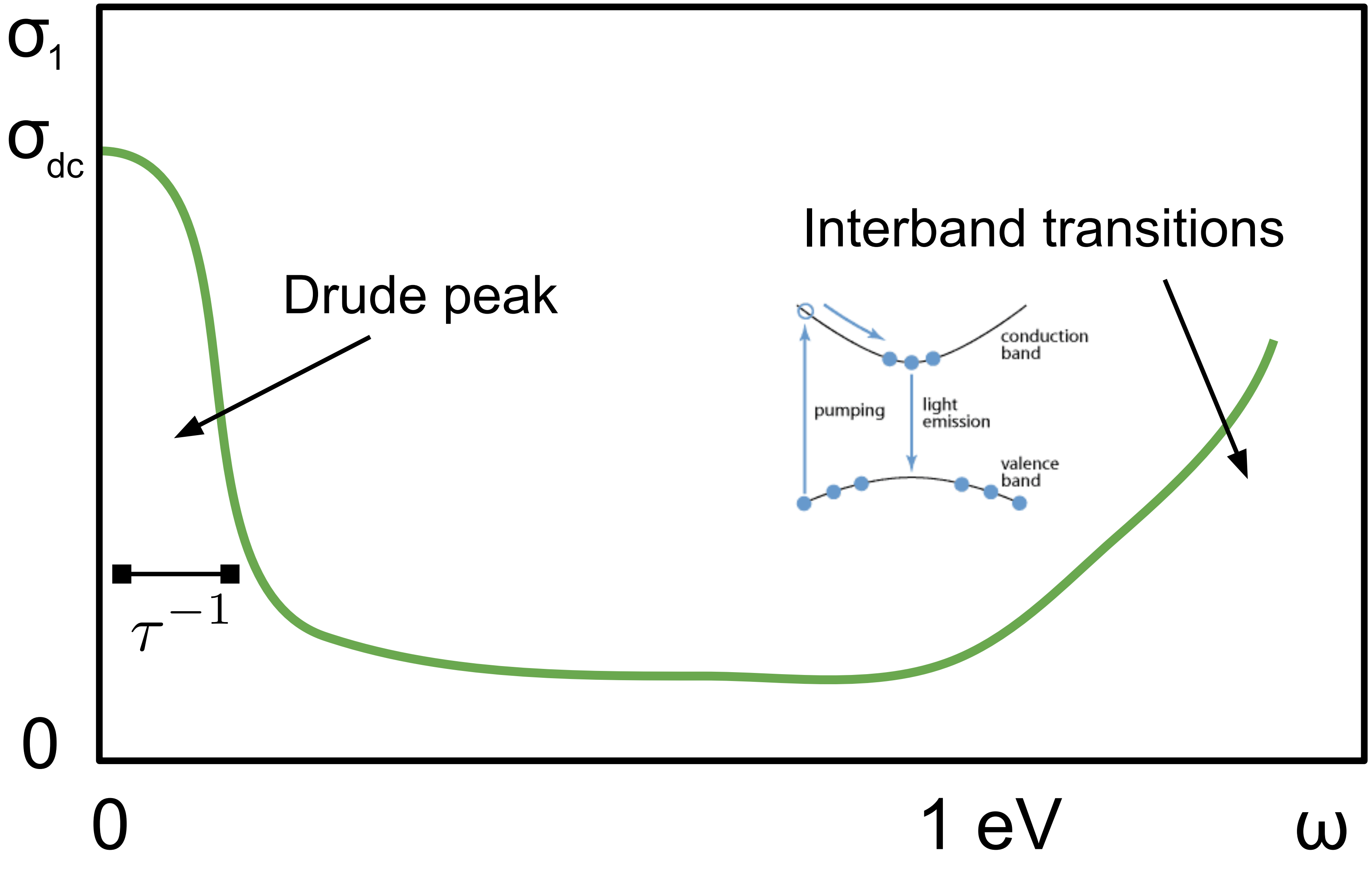
$$\sigma(\omega) = \sigma_1(\omega) + i\,\sigma_2(\omega)$$

$$P = \sigma_1(\omega) E(\omega)^2$$

$$\left( \sigma_1(\omega) = \frac{\operatorname{Im} G^R_{J^x J^x}(\omega)}{\omega} \right)$$

**Joule Heating**

# Optical conductivity



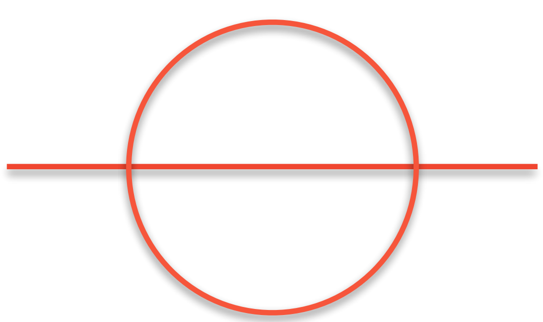
# Essential facts

- **The <u>quasiparticle lifetime $\tau$</u> is the longest timescale in the game.**
  **$\Rightarrow$sharp Drude peak.**

- **The dc conductivity (Drude, 1900):**

$$\sigma_{\rm dc} = \frac{ne^2\tau}{m_\star}$$

- **Electron-electron scattering gives:**



$$\tau \sim \frac{\hbar}{k_B T}\frac{E_F}{k_B T} \gg \frac{\hbar}{k_B T}$$

**(Landau Fermi Liquid)**

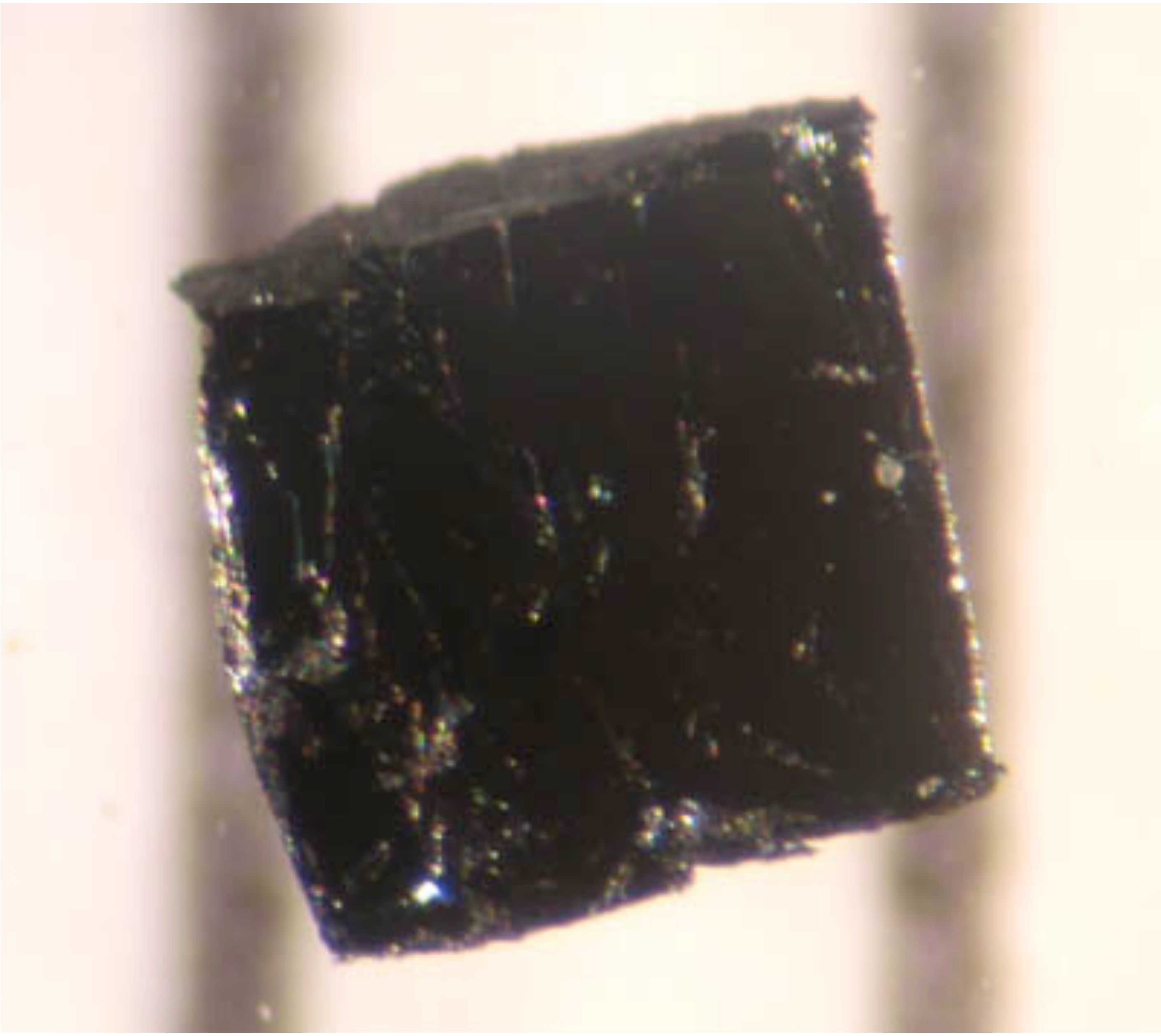
# Essential facts

- Computations are possible because the low energy effective field theory of a conventional metal has infinitely many almost conserved operators:

$$\delta n_k = c_k^\dagger c_k$$

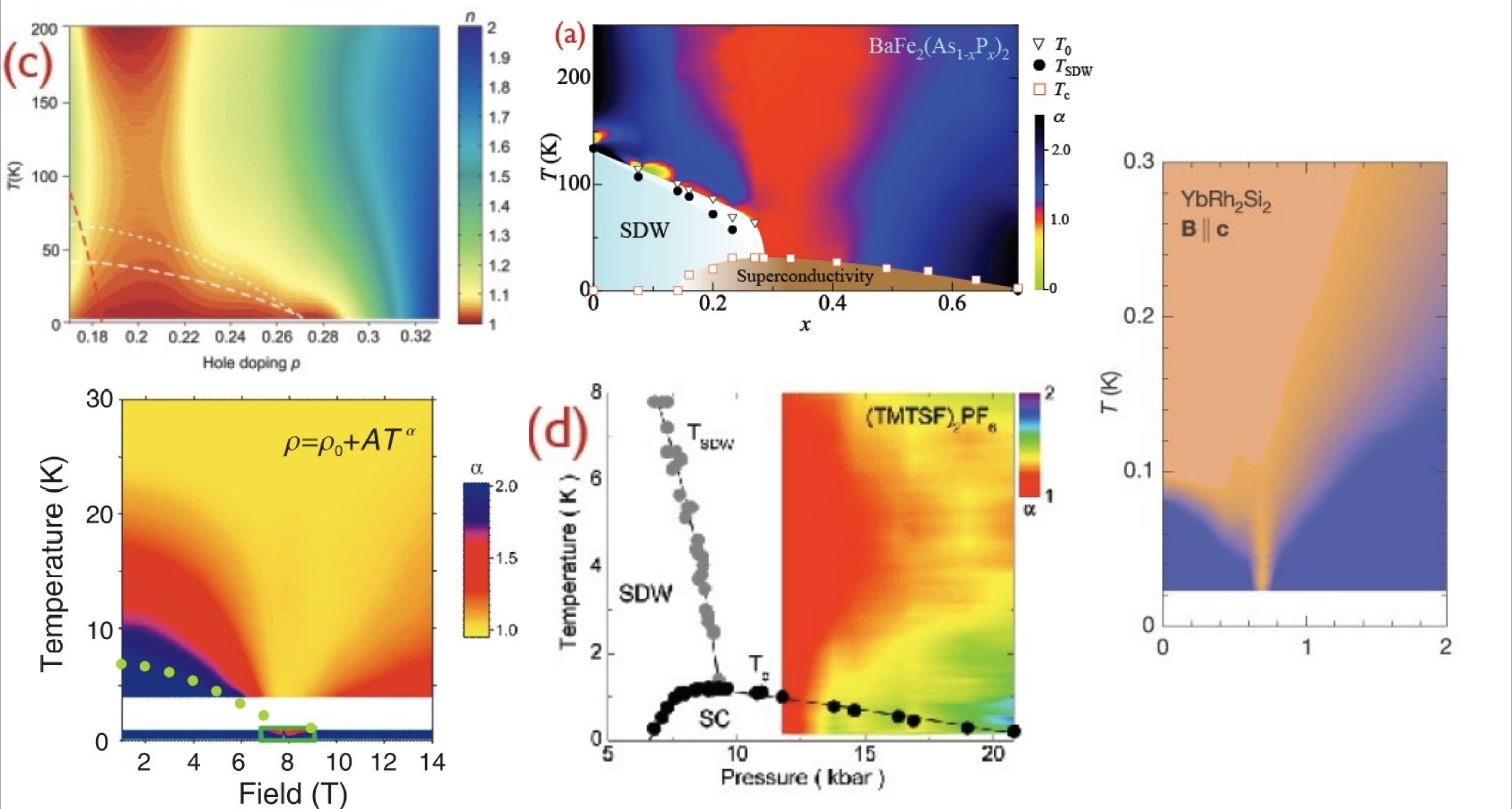
- "Almost conserved"
  = conserved up to irrelevant operators.
- Correct theoretical framework: Boltzmann equation.

# II. Unconventional metals: Facts

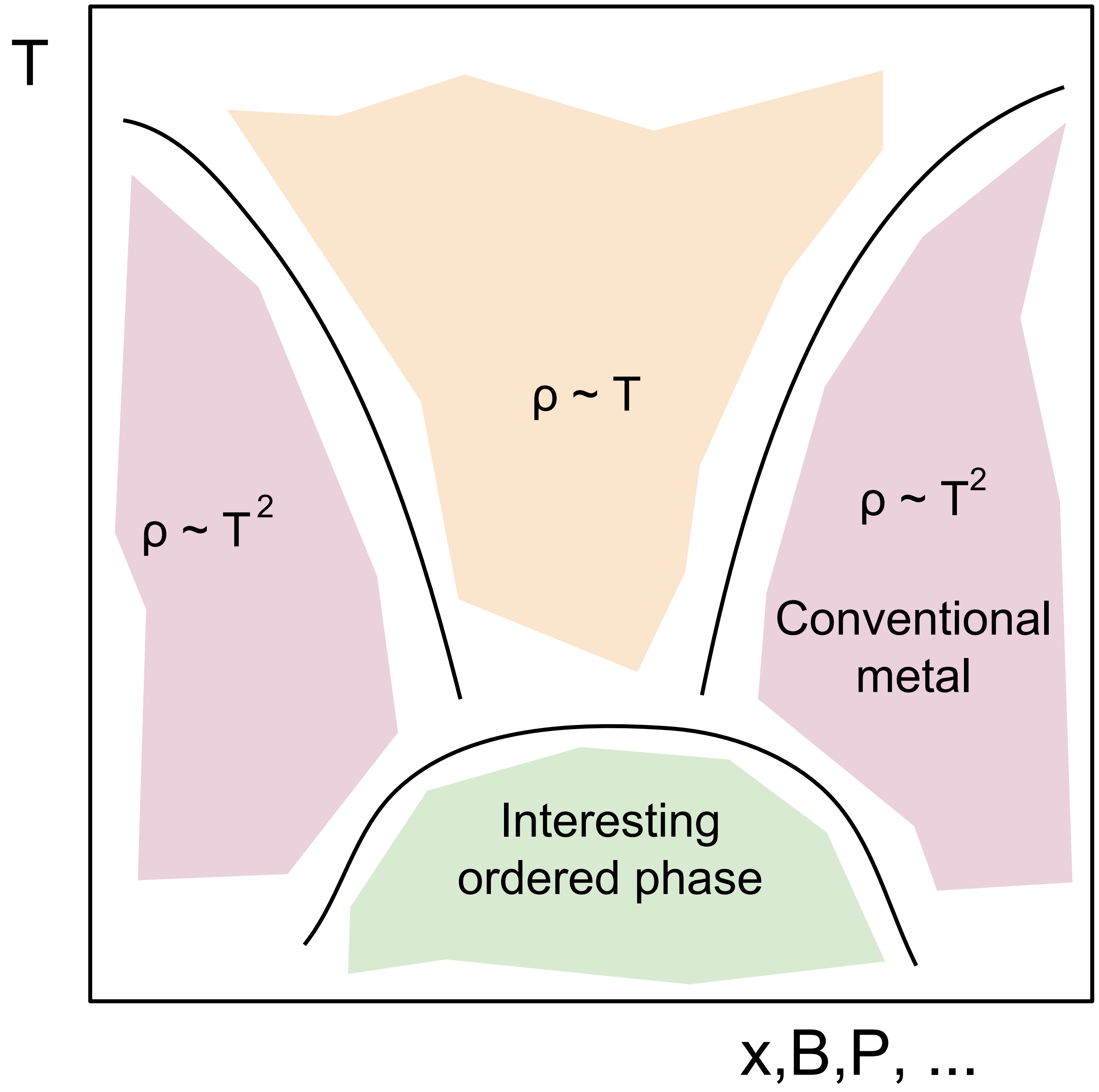


# T-linear resistivity

**Cuprates, pnictides, organics, ruthenates, heavy fermions … [After Sachdev-Keimer '11]**



# Cartoon phase diagram



# T-linear resistivity

- It is possible that a ‘mundane’ quasiparticle explanation of T-linearity exists.
  (cf. phonons)

- No compelling theory at the present time.
  (and it’s been ~ 30 years …)

- Similarity in phase diagrams across materials rather striking …

# Bad metals

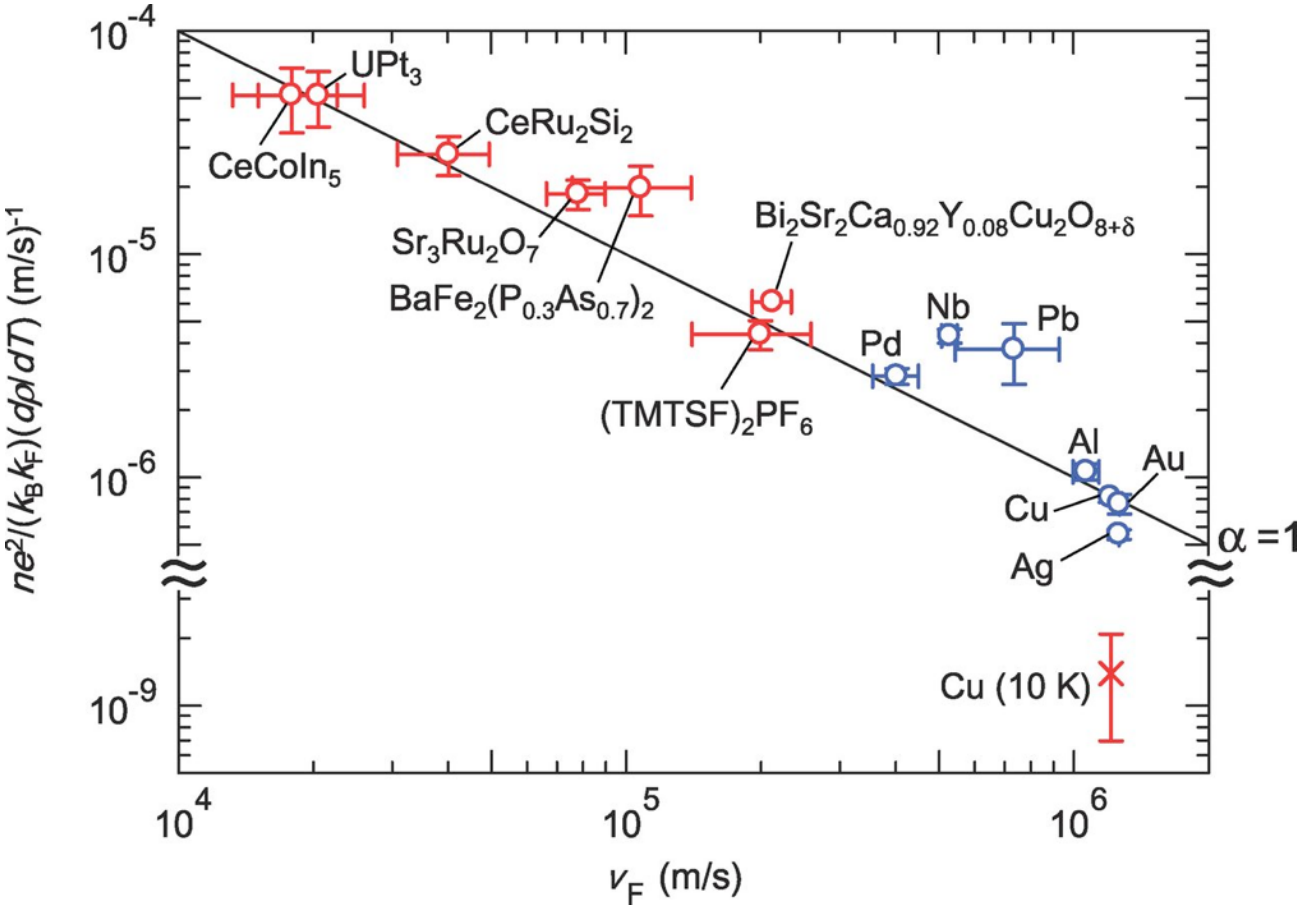
- **Some (not all) of these T-linear resistivities have magnitudes so large, Drude formula would require a mean free path shorter than the de Broglie wavelength. Not consistent.**

$$\ell_{\rm mfp} \sim v_F \tau \lesssim \ell_{dB}$$

- **Such 'bad metals' [Emery-Kivelson '95] probably require a non-quasiparticle based description ...**

# Universal bounds?

- **Very different magnitudes of resistivity.**
- **But, they share a universal timescale.**




**(Bruin et al. '13)**

$$\tau = \alpha \frac{\hbar}{k_B T} \qquad (\alpha \approx 1-2)$$

- **Saturation of some universal bound?**

**(Sachdev, Zaanen, SAH)**

**Holographic opportunity**

# III. Unconventional metals: Framework

# Conservation laws

- In the absences of quasiparticles, <u>symmetries</u> give us the key operators.
- Conservation of energy and charge:
  $\Rightarrow$ Electric and Heat currents: J and $J^Q$.
- These operators are directly probed by conductivities:

$$\sigma_{AB}(\omega) = \frac{G^R_{AB}(\omega)}{i\omega}$$

# “Coherent” metals

(Lucas-Sachdev ’15 ; Hartnoll-Hofman ’12; Hartnoll-Kovtun-Muller-Sachdev ’07)

- **If ∃ a long wavelength continuum QFT description of the underlying lattice system, then there is an emergent almost conserved momentum P.**
- **P is relaxed by irrelevant operators only and this dominates the conductivities:**

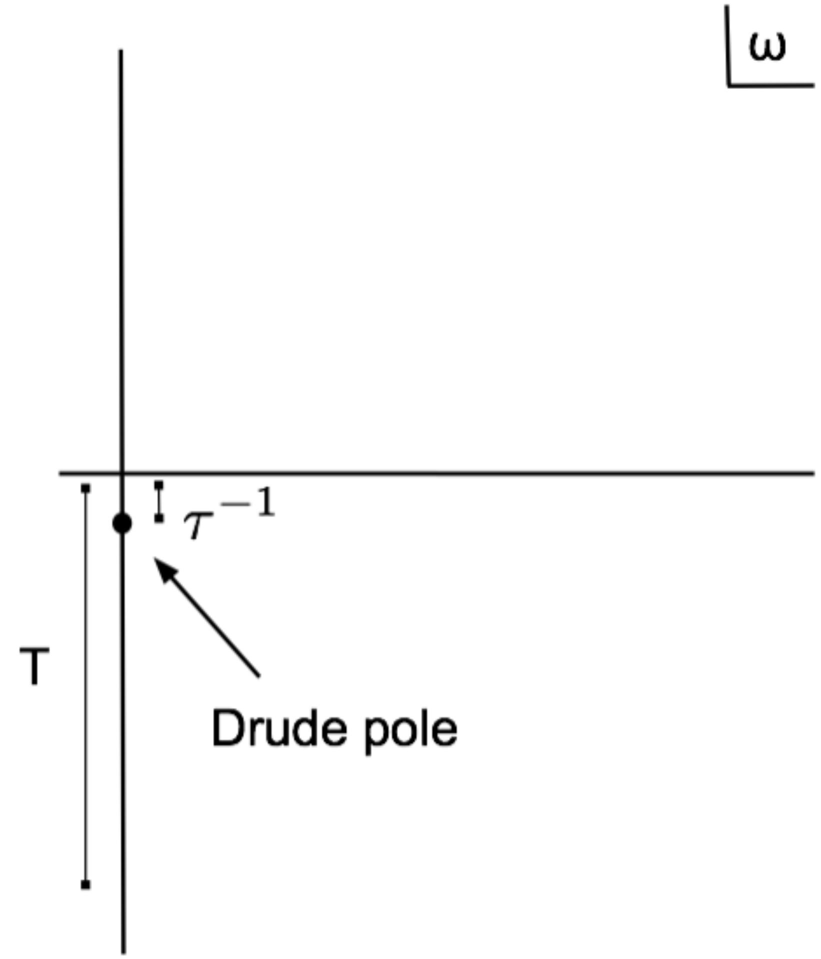
$$\sigma_{AB}(\omega) = \frac{\chi_{AP}\chi_{PB}}{\chi_{PP}} \frac{1}{-i\omega + \Gamma}$$

**Thermodynamic susceptibilities**

**P relaxation rate. ∃ a formula for it.**

# “Coherent” metals

- **An essential aspect of a coherent metal is that the conductivity is parametrically controlled by a single pole in the complex frequency plane.**



# "Incoherent" metals

(SAH '14)

- <u>Nothing</u> is long-lived that overlaps with the currents J and $J^Q$.

- Heat and charge will diffuse:

$$\sigma^L(\omega, k) = \frac{-i\omega D\chi}{i\omega - Dk^2}$$

- The Einstein relations (neglecting 'thermoelectric' effects):

$$\sigma = \chi D_{\text{charge}}$$
$$\kappa = c\, D_{\text{heat}}$$

# Coherent vs Incoherent

- Holographic models with momentum relaxation explicitly show crossovers from coherent to incoherent behavior.

(Davison and Gouteraux: 1411.1062 + 1505.05092)

# IV. The Lorenz ratio

| Metalle. | Für den lufterfüllten Raum. | | Für den luftverdünnten Raum. | |
|---|---|---|---|---|
| | *q.* | *l.* | *q.* | *l.* |
| Silber | 2,057 | 100 | 2,020 | 100 |
| Kupfer | 2,072 | 77,4 | 2,025 | 80,2 |
| Gold | 2,093 | 60,1 | 2,0315 | 63,7 |
| Messing I. | 2,202 | 27,9 | 2,0665 | 30,2 |
| Messing II (dicker) | 2,179 | 25,8 | 2,063 | 26,0 |
| Zinn | 2,297 | 15,4 | 2,099 | 16,1 |
| Eisen | 2,441 | 13,1 | 2,172 | 11,8 |
| Stahl | 2,4485 | 12,8 | 2,176 | 11,5 |
| Blei | 2,502 | 9,3 | 2,176 | 9,3 |
| Platin | 2,670 | 9,2 | 2,182 | 11,7 |
| Neusilber | 2,869 | 6,8 | 2,246 | 8,3 |
| Rose'sches Metall | 3,529 | 3,2 | 2,502 | 3,3 |
| Wismuth | 5,104 | 1,8 | — | — |

# The Lorenz ratio

- **Matrix of conductivities:**

$$\begin{pmatrix} j \\ j^Q \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\alpha & T\overline{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- **Thermal conductivity at j = 0.**

$$\kappa = \overline{\kappa} - \frac{\alpha^2 T}{\sigma}$$

- **Lorenz ratio:**

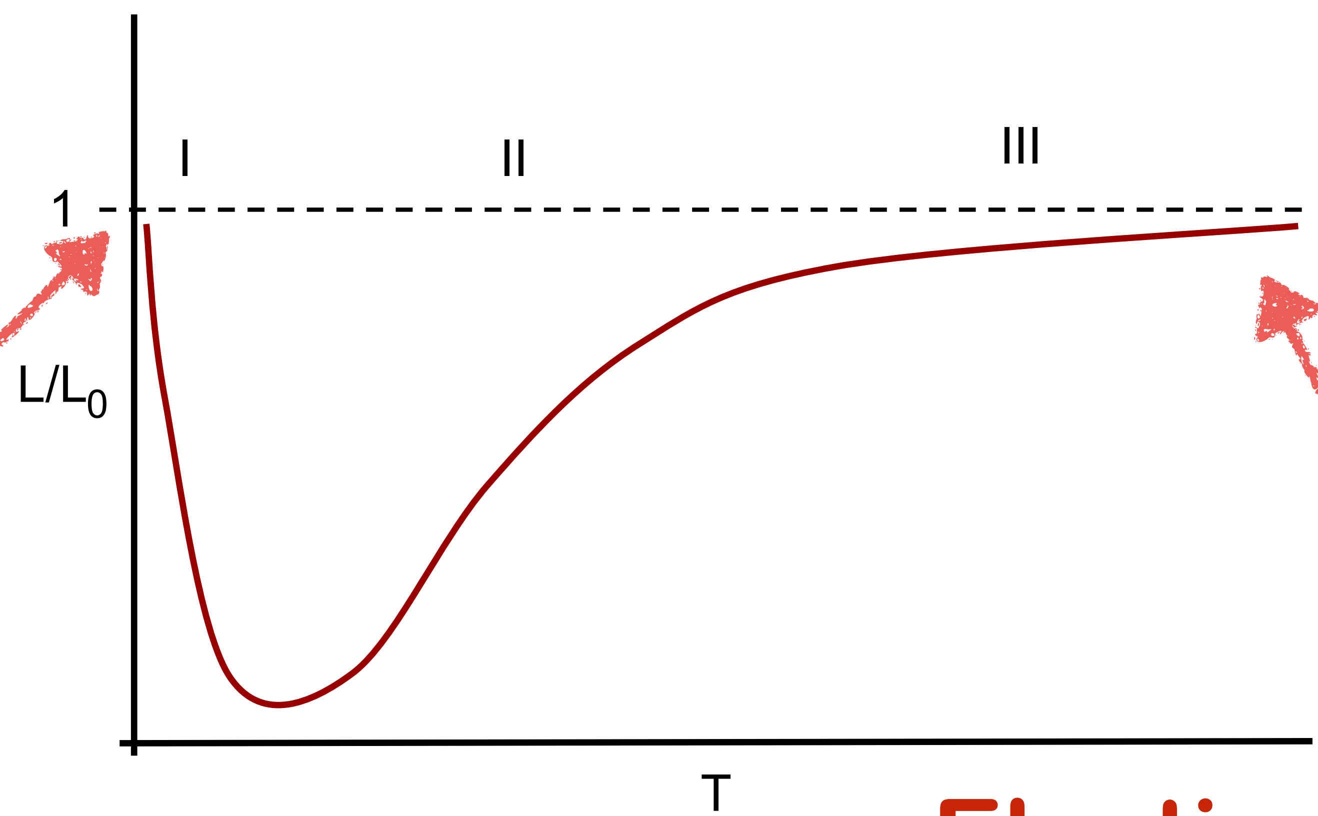
$$L = \frac{\kappa}{\sigma T}$$

# Wiedemann-Franz law

- **In a conventional metal (if you can subtract the phonon contribution to thermal transport):**

**Fermi Dirac**

$$L_0 = \frac{\pi^2}{3}$$

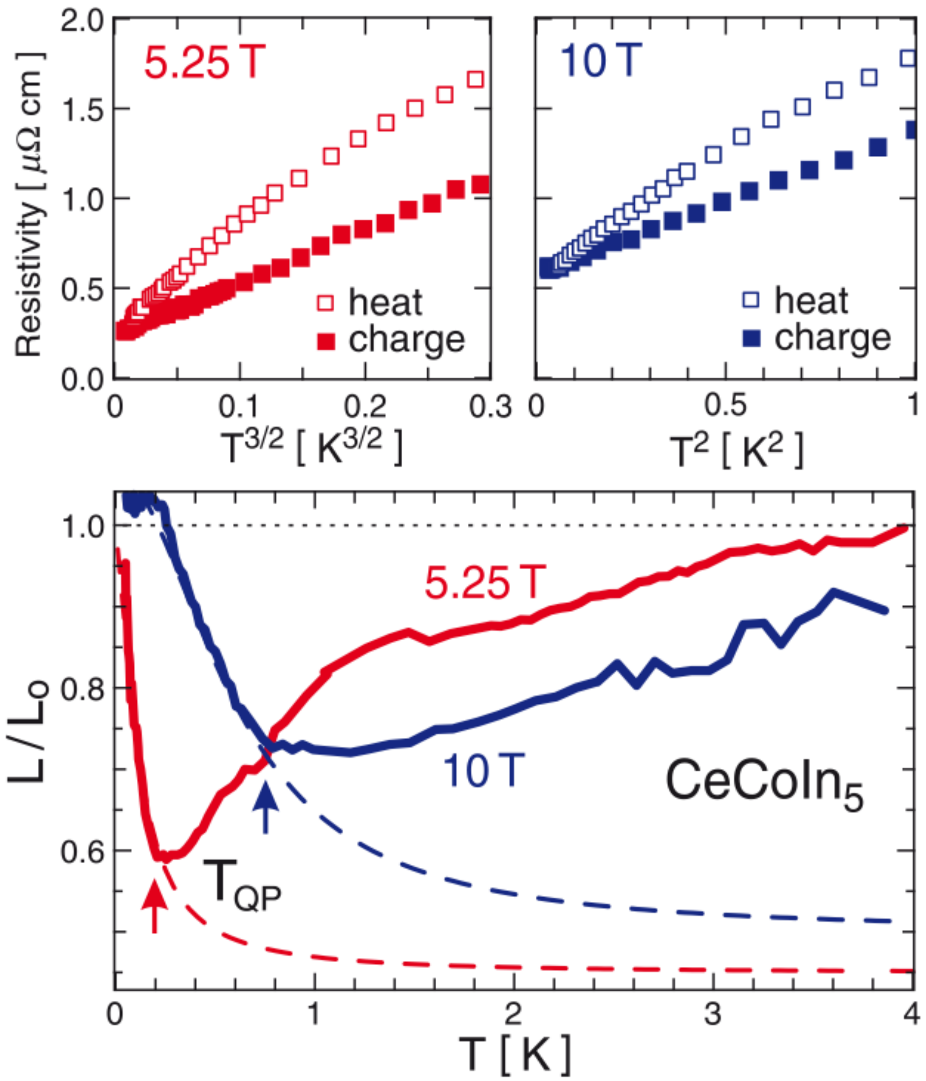


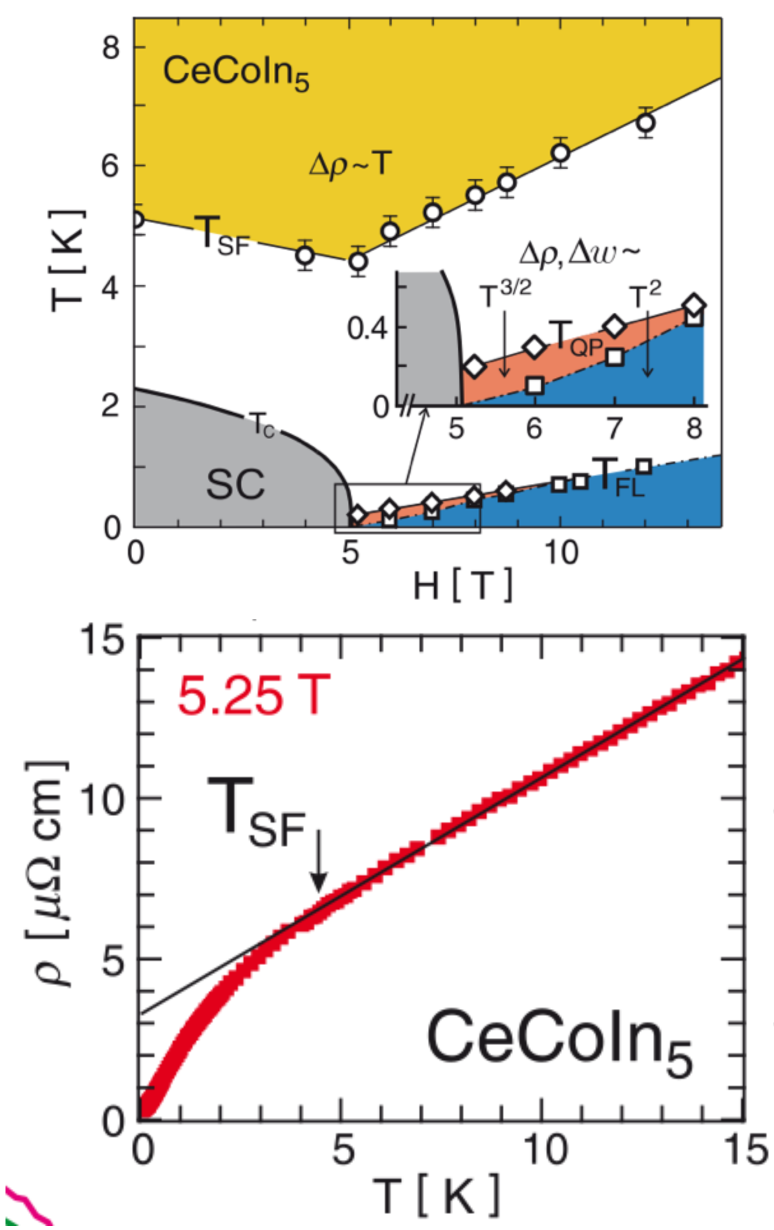

**Elastic disorder scattering**

**Elastic phonon scattering**

# Quantum criticality with WF law I

- **CeCoIn$_5$: critical and non-critical (a-axis)**

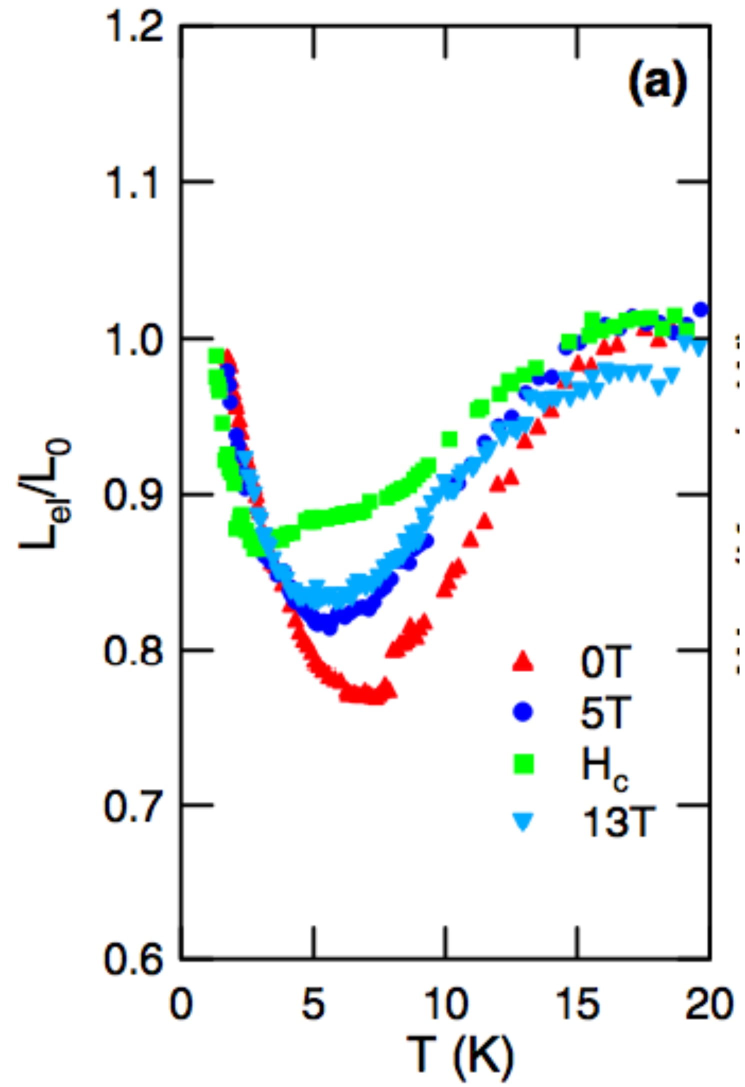


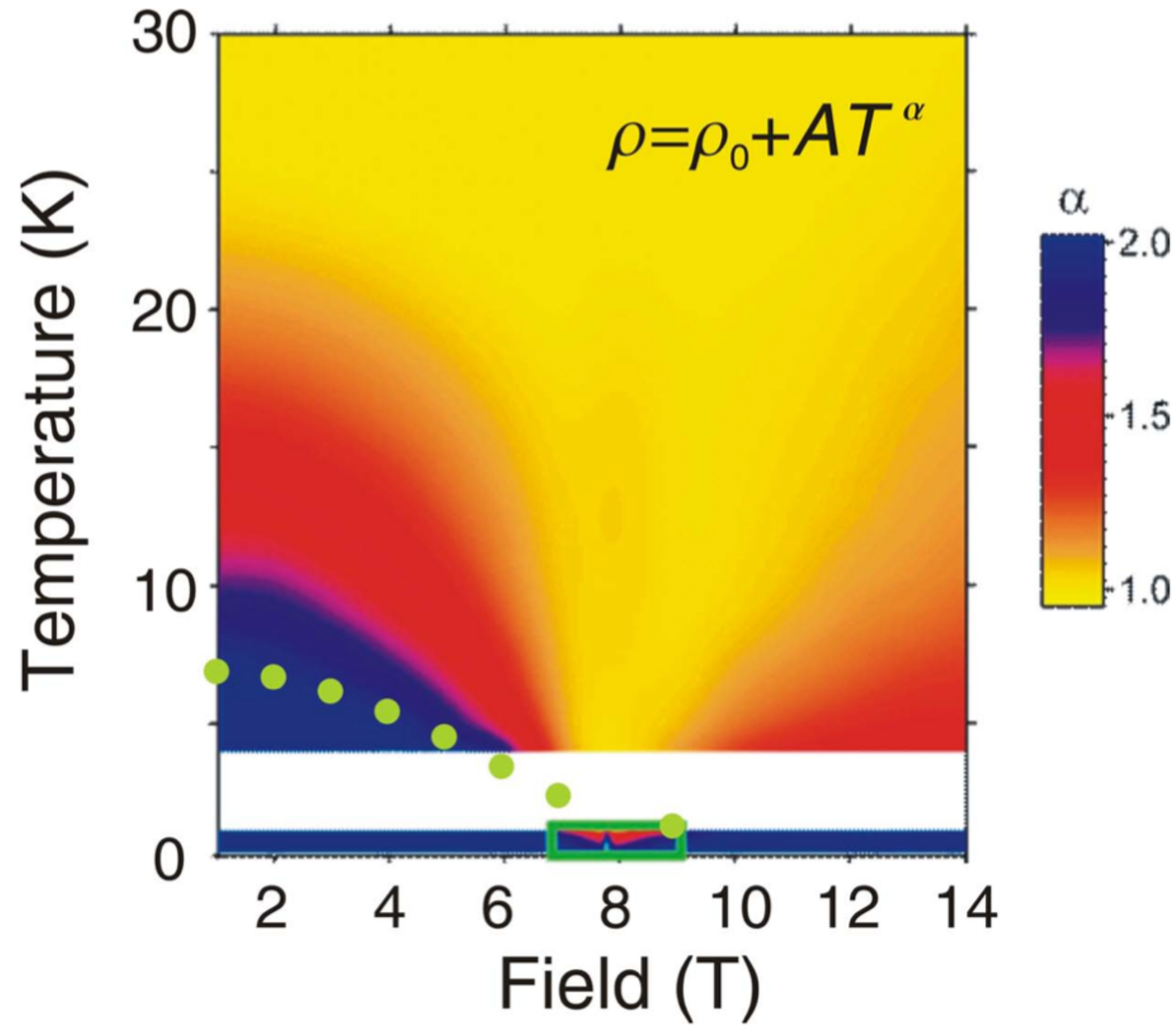




- **Paglione et al. PRL '06**

# Quantum criticality with WF law I

- $Sr_3Ru_2O_7$: critical and non-critical

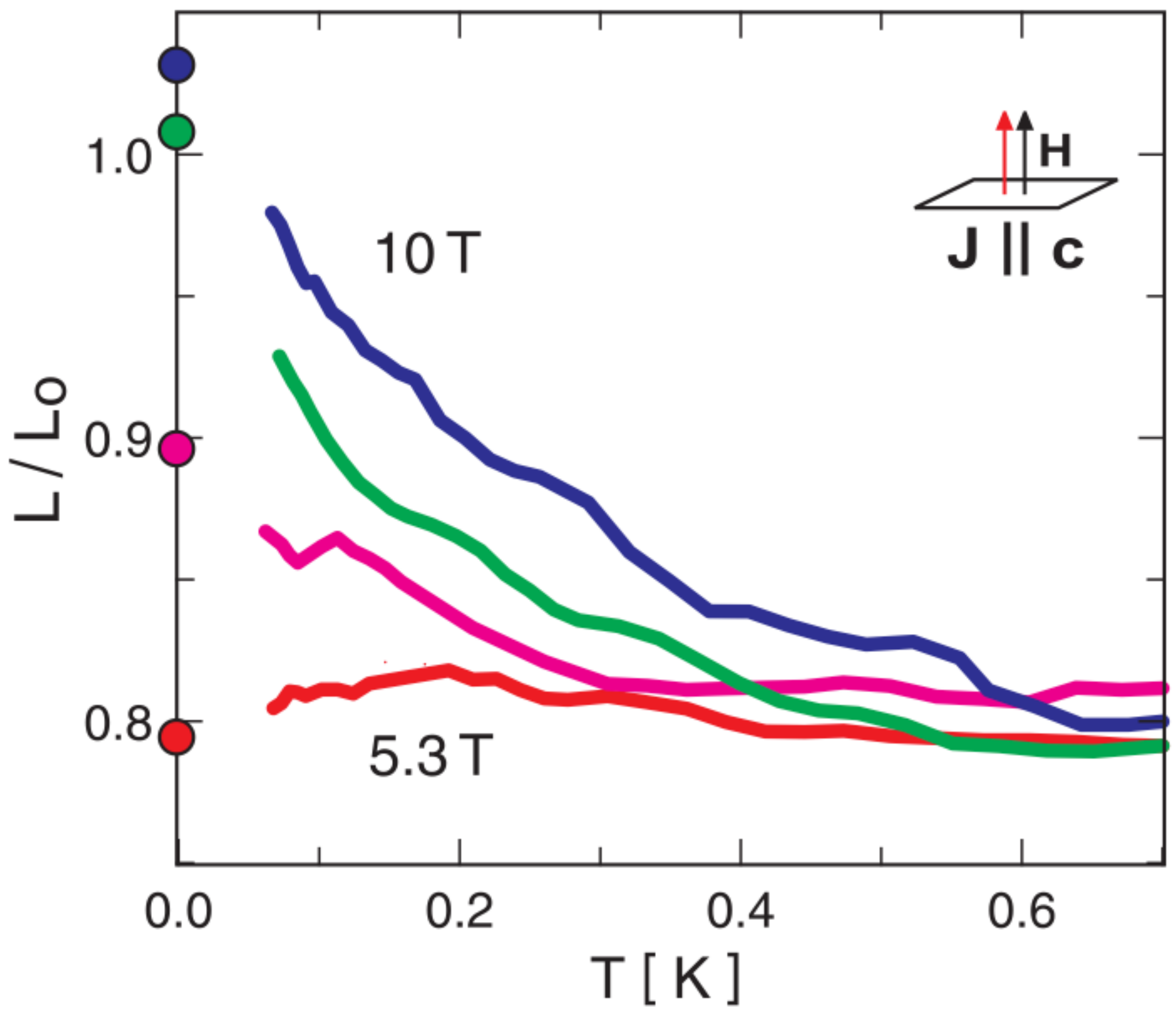


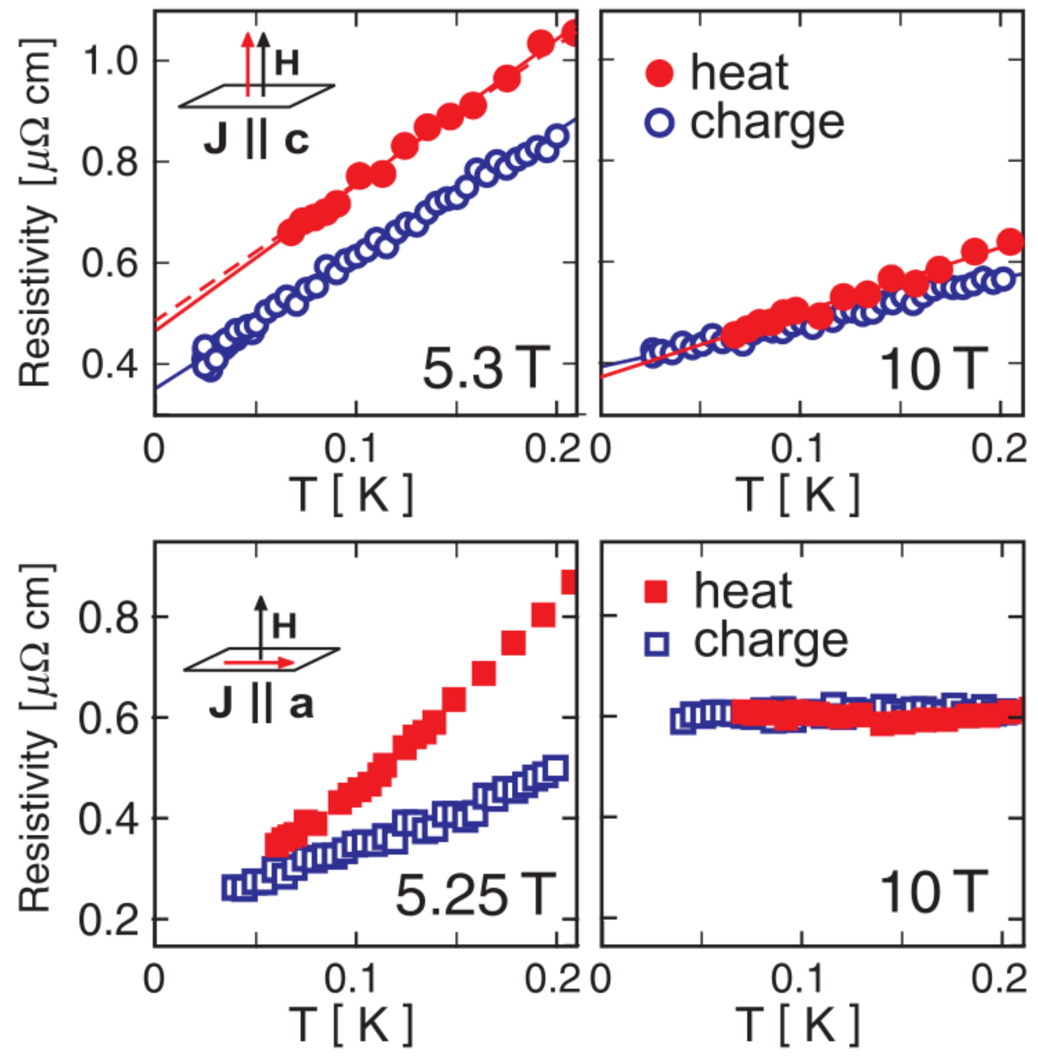




- Ronning et al. PRL '06; Rost et al. PNAS '11

# Linear T without WF

- $CeCoIn_5$: c-axis conductivity

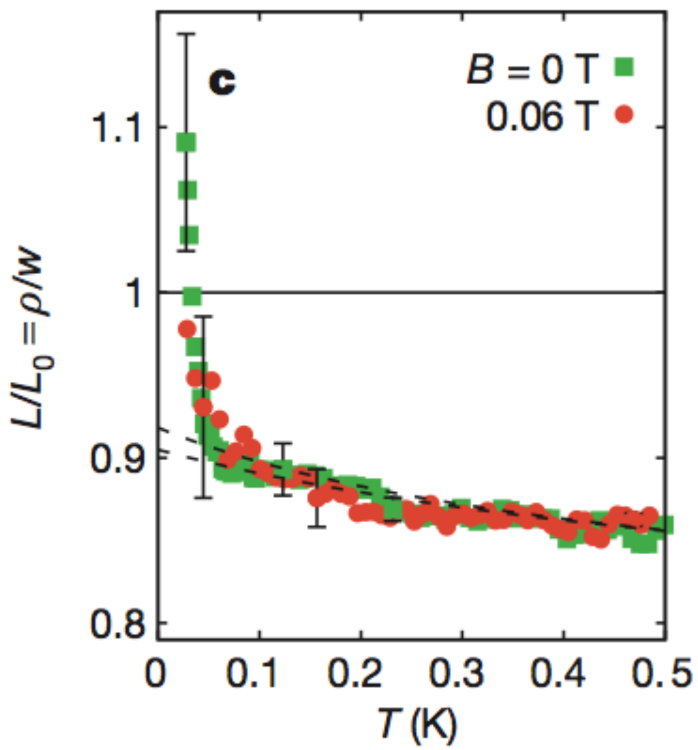


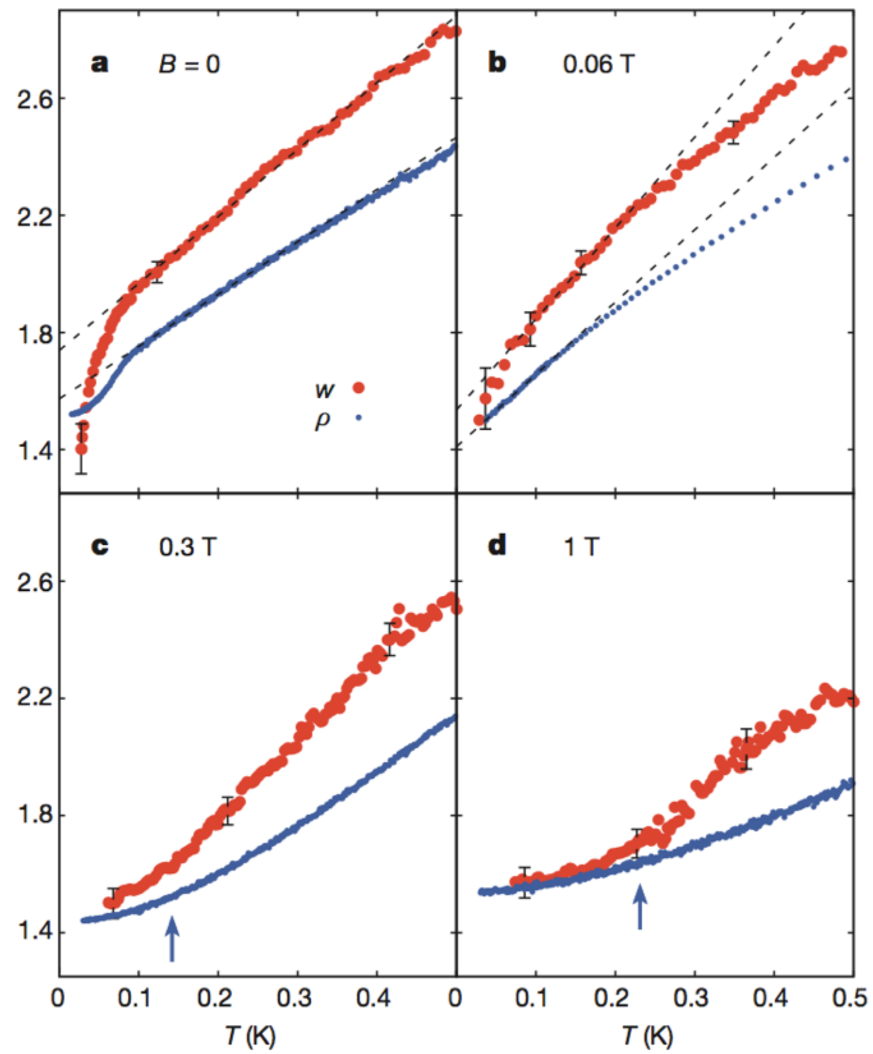



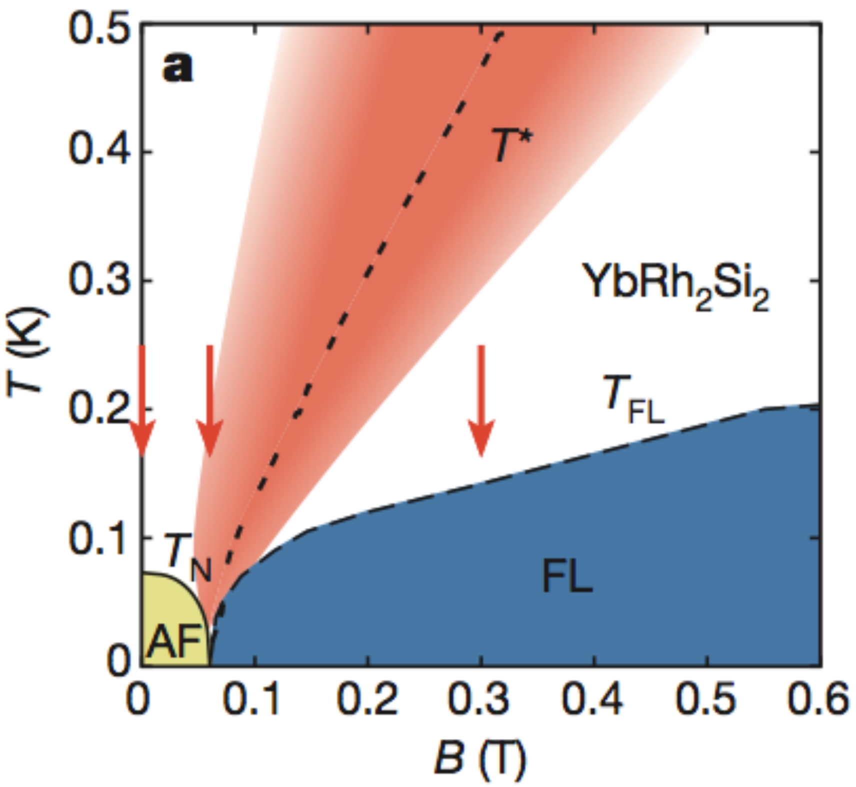

- Tanatar et al. Science '07

# Linear T without WF

- $YbRh_2Si_2$ tuned to criticality



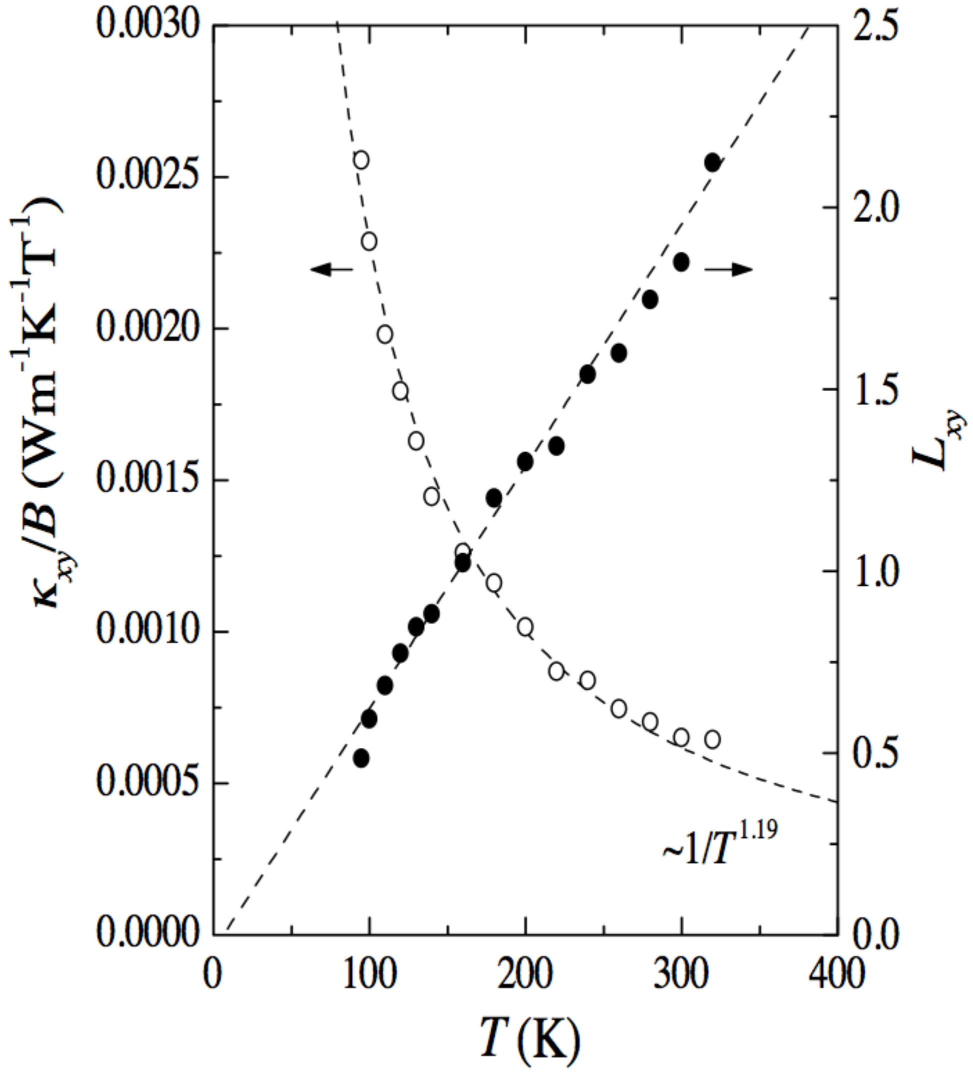




- **Pfau et al. Nature ’12.**
  **See also Machida et al.; J-Ph. Reid et al.**
  **Disagreement of interpretation T→0, higher T already interesting!**

# Linear T without WF

- Hall Lorentz ratio in optimally doped YBCO




- Hall ratio avoids phonons
- $L/L_0 < 1$ and linear.
- Down to $L/L_0 \sim 0.15$.

- Zhang et al. PRL ’00

# Coherent metals

(Barkeshli-Hartnoll-Mahajan '13)

- **More conventional thermal conductivity**

$$\kappa = \overline{\kappa} - \frac{\alpha^2 T}{\sigma}$$

Cancellation!

- **Two ratios of conductivities**

$$\frac{\overline{\kappa}}{\sigma T} = \frac{1}{T^2}\frac{\chi_{QP}^2}{\chi_{JP}^2} \qquad \frac{\kappa}{\sigma T} \ll 1$$

Universal ratio of thermodynamic susceptilibites

- $\kappa \ll \overline{\kappa}$ **distinctive signature (equal in FL)**

# Incoherent metals

- An incoherent quantum critical metal, the conductivities are finite and computable within the low energy scaling theory.

- This scaling has been done many times. Previously, several exponents were set to zero.

- Explicit holographic examples indicated extra exponents were necessary.

(Gouteraux, Kiritsis, Karch,...)

# The three exponents

**(Hartnoll-Karch '15)**

- **z: dynamical critical exponent**

$$\xi \sim \frac{1}{T^{1/z}}$$

- **θ: hyperscaling violation exponent**

$$f \sim T \cdot T^{(2-\theta)/z}$$

- **Φ: anomalous dimension for charge**

$$n \sim T^{(2-\theta+\Phi)/z}$$

# Scaling in cuprates

(Hartnoll-Karch '15)

- We found that the exponents:

$$z = \frac{4}{3}\,, \qquad \theta = 0\,, \qquad \Phi = -\frac{2}{3}\,.$$

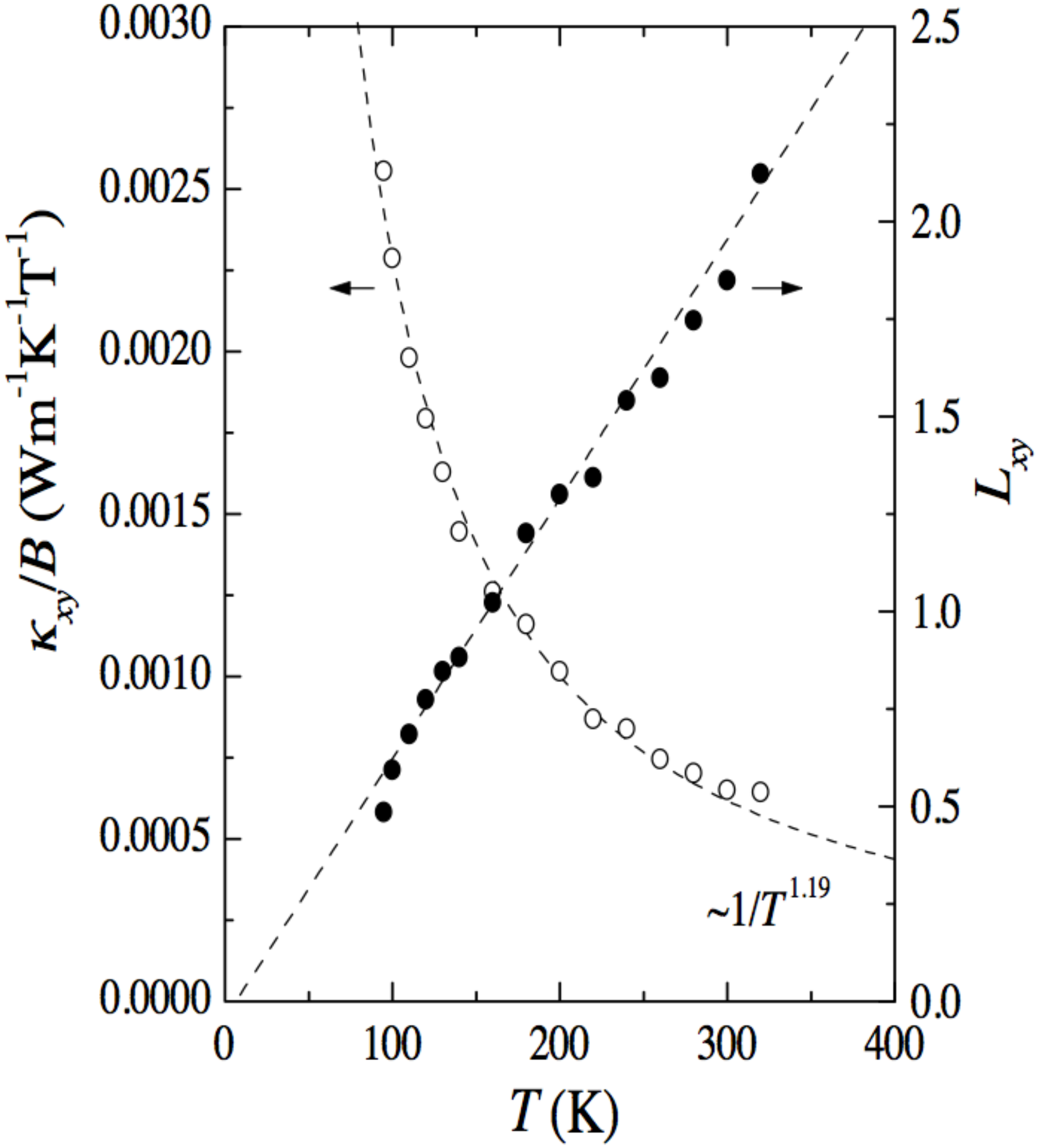
**Matched multiple scalings in transport quantities in the cuprates. The Lorenz ratio is particularly interesting:**

$$L \sim T^{-2\Phi/z}$$

**<u>Need a nonzero Φ</u> to get anything other than a constant!**

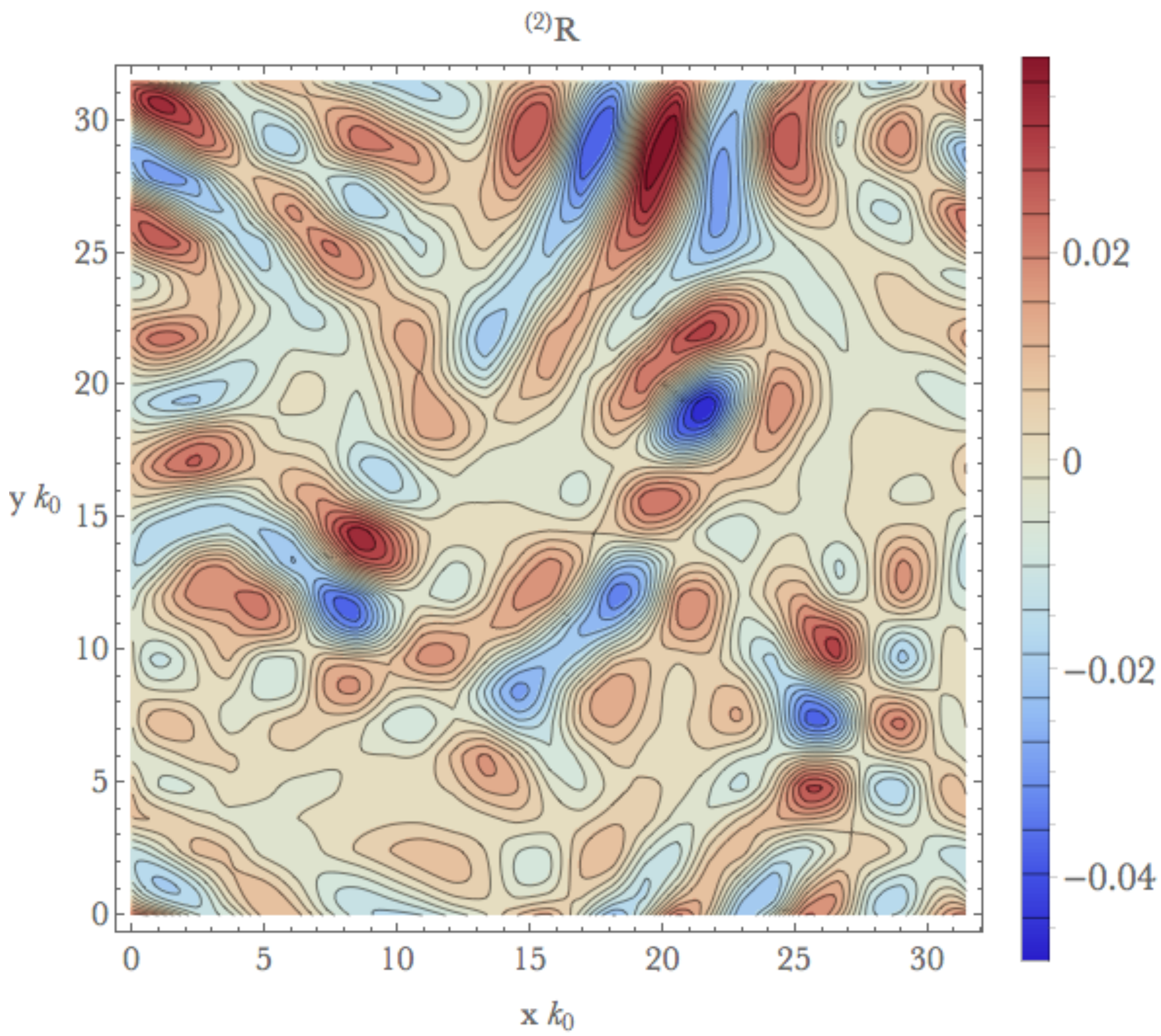
(cf. Khveshchenko)

# Some questions




- **Does this behavior occur in more than one cuprate?**
- **Does it continue up to higher temperatures?**
- **Is it scaling or an intermediate inelastic scattering regime?**

# V. Disordered fixed points in holography



# Disorder physics

- The simplest models of incoherent metals may come from disordered fixed points.
- Because disorder breaks translation invariance at all scales (unlike a lattice). It can easily have strong effects on the far IR.
- In QFT the way the continuum description and non-conservation of momentum are married is traditionally through the "replica trick".

# Disorder physics

- However, attempts to find controlled interacting disordered fixed points in general dimension have not been successful.
- Eg. in attempts à la Wilson-Fisher, the second term in the beta function has the wrong sign:

$$\mu \frac{dV}{d\mu} = -\epsilon V - \# V^2 + \cdots$$

(e.g. Sachdev book)

# Disorder physics

- Holography allows study of disorder physics without the replica trick.

# Disordered fixed points in holography

(Hartnoll-Santos '14)

- The relevance of a disordered coupling

$$\int dt d^d x \, h(x) \mathcal{O}(t,x)$$

- Is determined by the 'Harris criterion':

$$\Delta > \frac{d+2}{2}$$

- If relevant, need to follow the flow to IR.

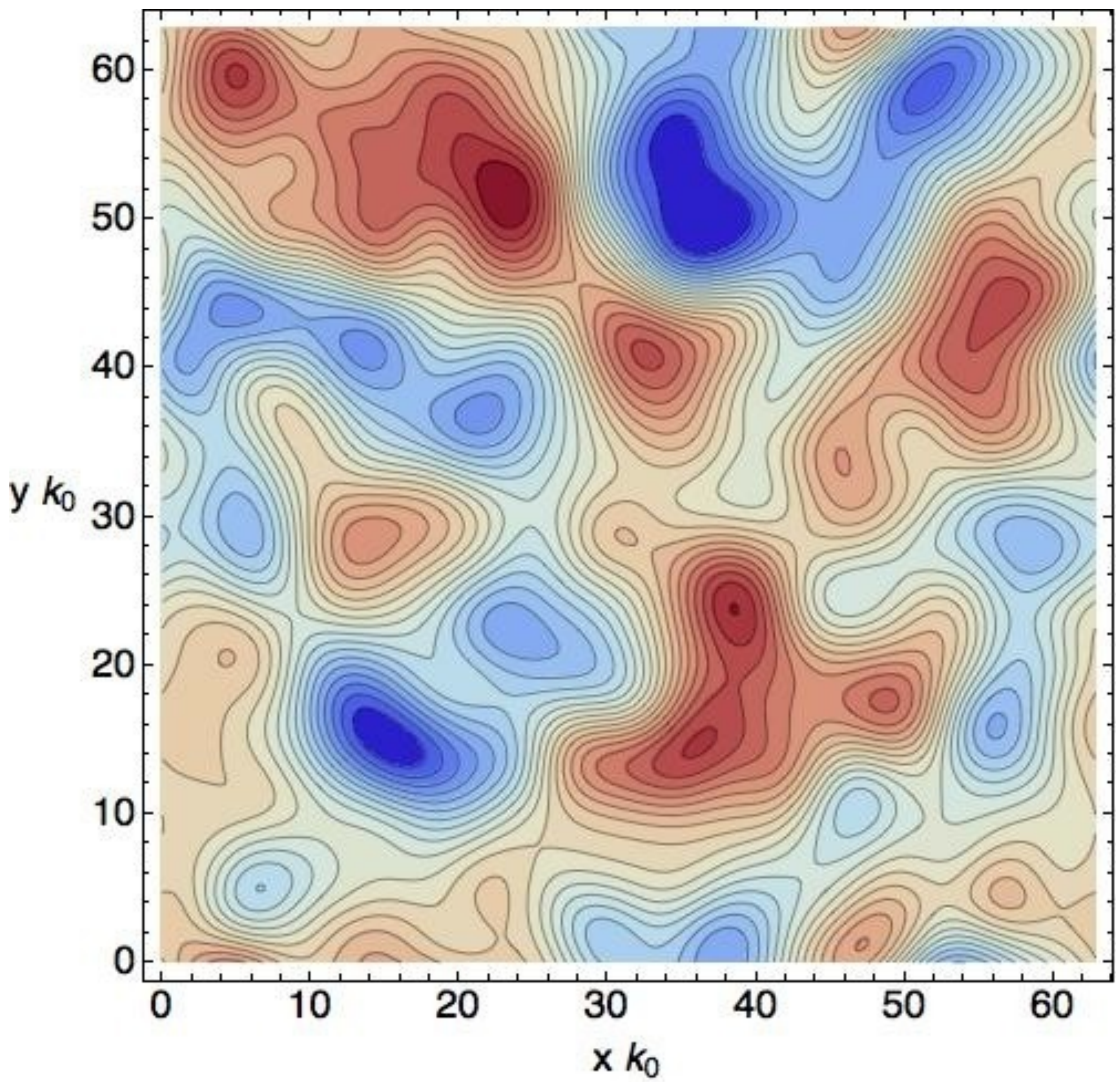
# Disordered fixed points in holography

- A random coupling is generated by

$$h(x) = \bar{V} \sum_{n=1}^{N-1} 2\sqrt{\Delta k}\, \cos(n\Delta k\, x + \gamma_n)$$

- Solved Einstein-scalar bulk theory with a marginally relevant random source.
- Resummed the logarithmic growth to find stable disordered IR fixed points.
- Confirmed and extended results via numerical simulation of full disorder.

# Disordered fixed points in holography




- **Zero temperature <u>averaged</u> IR geometry:**

$$\langle ds^2_{\rm IR} \rangle = -\frac{dt^2}{r^{2\bar{z}}} + \frac{dr^2 + dx^2 + dy^2}{r^2}$$

$$\bar{z} = 1 + \frac{\pi^{(d-1)/2}}{2}\Gamma\left(\frac{d+1}{2}\right)\bar{V}^2 + \cdots$$

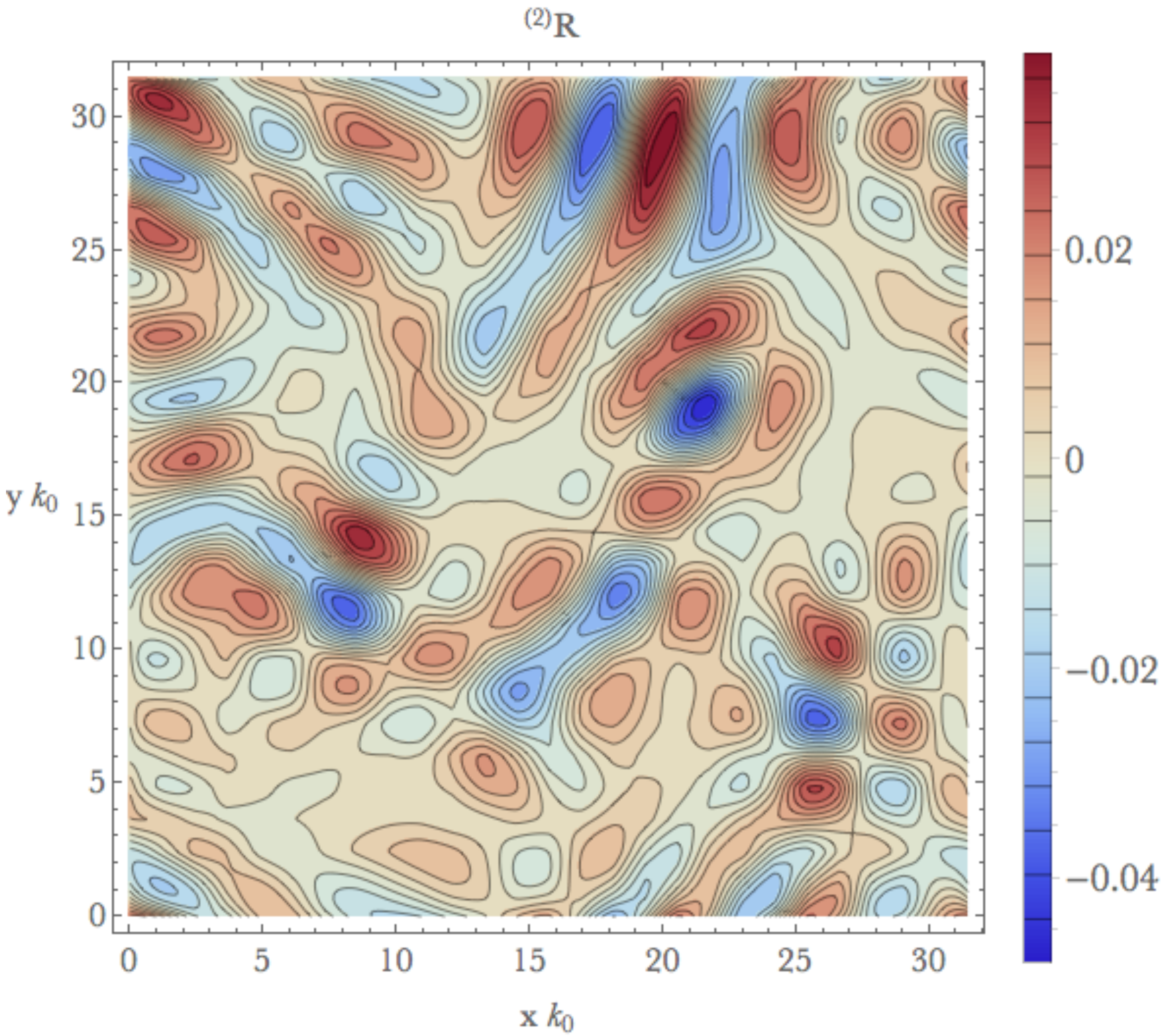
# Disordered fixed points in holography

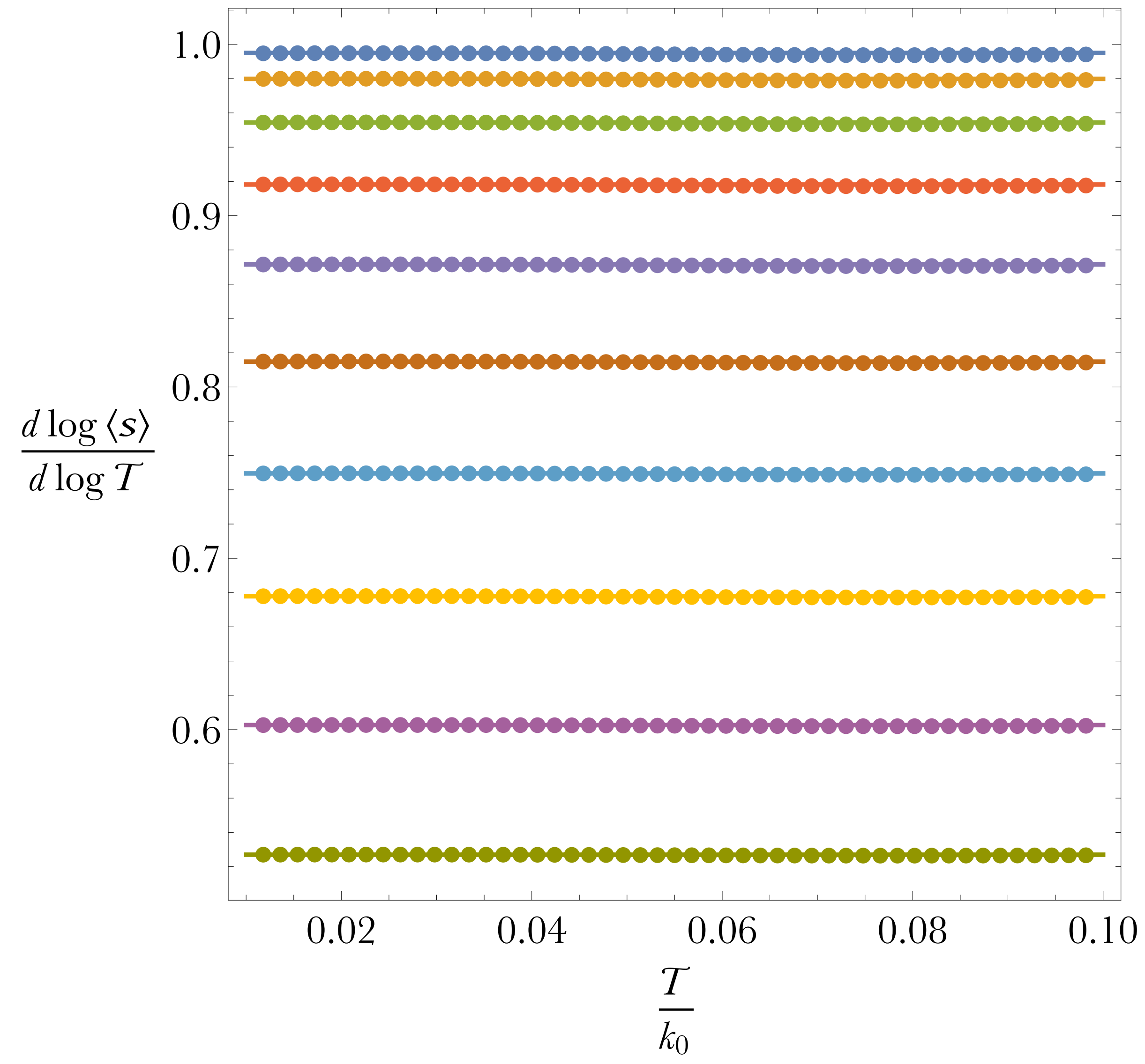
(Hartnoll-Ramirez-Santos ’15)

- Scale invariance of the disorder averaged metric suggested an IR disordered fixed point.
- Not clear what quantities controlled by z.
- By constructing finite T solutions, have shown analytically and numerically:

$$s \sim T^{2/z}$$

# Disordered fixed points in holography



# Disordered fixed points in holography

- Compelling evidence for a disordered fixed point, of the type that is elusive in weakly interacting QFT.

- Transport calculations underway. Natural thing to look at is heat transport, model for incoherent 'metal'.

# Summary

- Unconventional metals cannot be described using formulae from textbooks.

- The coherence vs incoherence question is a useful way to organize ones thoughts.

- The Lorenz ratio:
  (i) Direct diagnostic of coherent metal.
  (ii) Directly sees exponent $\Phi$ incoherent case.

- Holography gives explicit instances of disordered quantum critical points.