Holographic Lattices

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Holographic Lattices

CFT with a deformation by an operator that breaks translation invariance

Why?

- Translation invariance $\Rightarrow$ momentum is conserved $\Rightarrow$ no dissipation $\Rightarrow$ DC response are infinite. To model more realistic behaviour we can use a lattice

- The lattice deformation can lead to novel ground states at T=0. Can realise novel metals and insulators

- Can model metal-insulator or metal-metal transitions

- General holographic results: thermo-electric DC conductivities in terms of black hole horizon data

c.f $\eta = \frac{s}{4\pi}$ [Policastro,Kovtun,Son,Starinets]

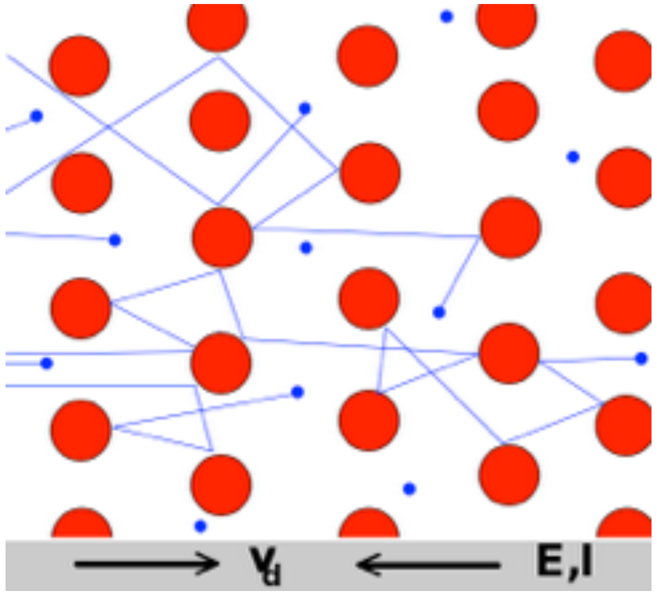
Plan

- Drude physics and coherent metals
- Some examples of holographic lattices and some results
- DC conductivities and Navier-Stokes on the horizon

A new connection between fluids and gravity differing from e.g [Bhattacharya,Hubeny,Minwalla,Rangamani]
[Bhattacharya,Minwalla,Wadia]
[Fouxon,Oz]
[Bredberg,Keller,Lysov,Strominger]

with Aristomenis Donos
Elliot Banks, Christiana Pantelidou

Drude Model of transport in a metal



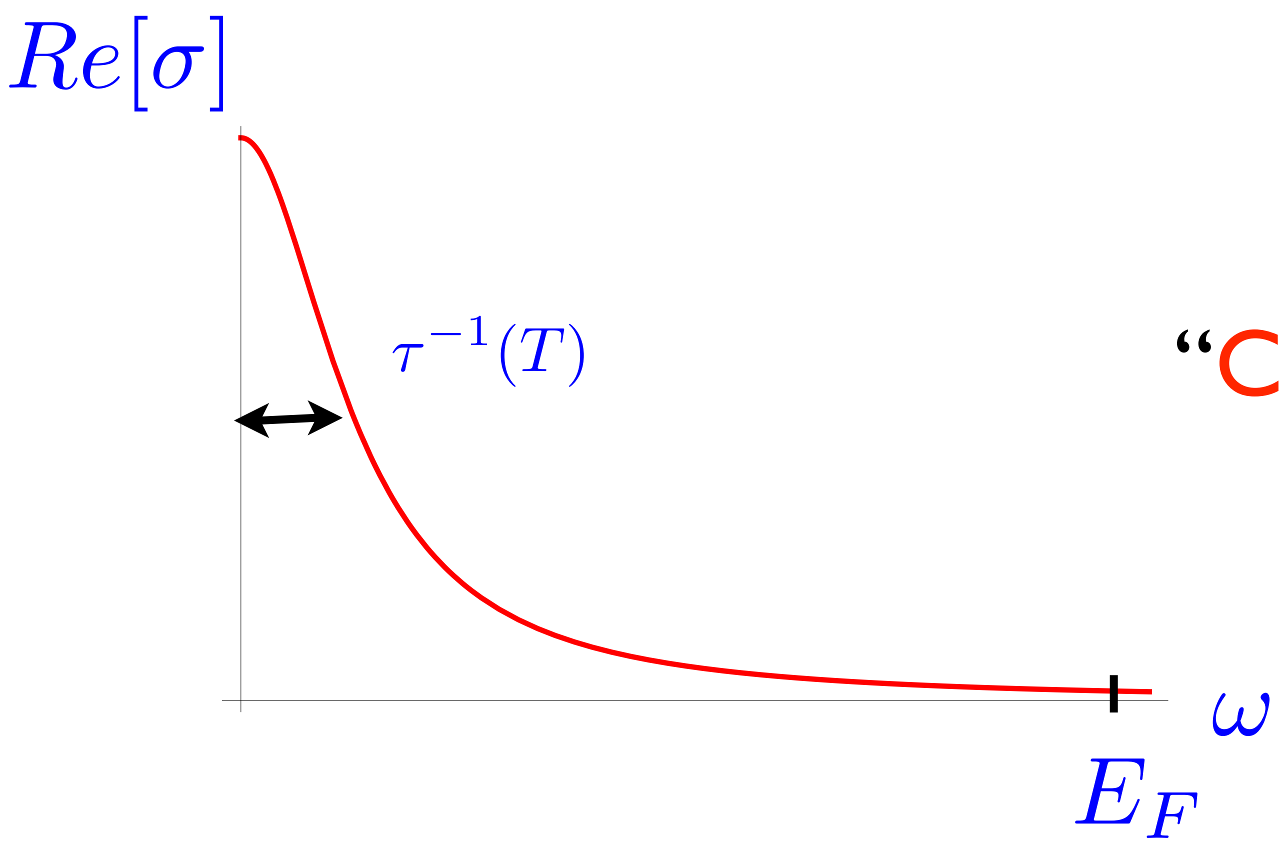
$$J = \sigma_{DC} E \qquad \text{with} \qquad \sigma_{DC} = \frac{q^2 \tau}{m}$$

$$E = E(\omega)e^{-i\omega t}$$

$$J = J(\omega)e^{-i\omega t}$$

$$\Rightarrow \boxed{J(\omega) = \sigma(\omega)E(\omega)}$$

$$\boxed{\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}} \qquad \sigma_{DC} = \frac{q^2\tau}{m}$$



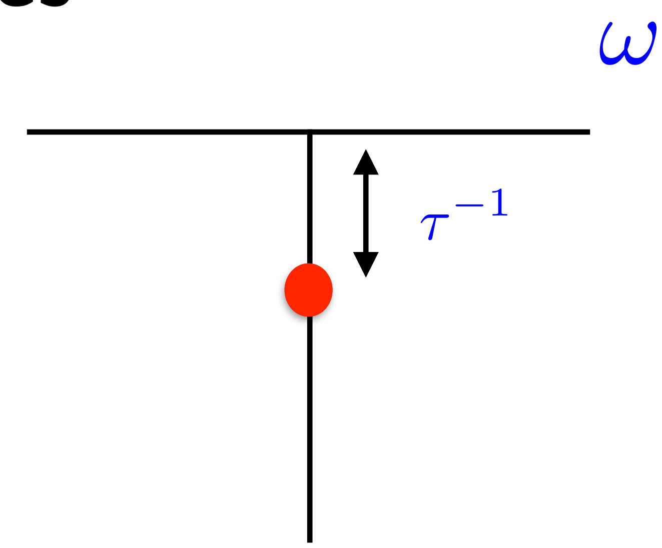
"Coherent" or "good" metal

When $\tau \to \infty$ $\sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$

- Drude physics doesn't require quasi-particles

Coherent metals arise when momentum is nearly conserved with dominant pole on imaginary axis

[Hartnoll,Hofman]



- Similar comments apply to thermal conductivity $Q = -\bar{\kappa} \nabla T$

- In nature there are also "incoherent" metals without Drude peaks

- Insulators with $\sigma_{DC} = \bar{\kappa}_{DC} = 0$ at T=0

Of particular interest to realise these in holography

Holographic CFTs at finite charge density

Focus on d=3 CFT and consider D=4 Einstein-Maxwell theory:

$$S = \int d^4x \sqrt{-g}\Big[R + 6 - \frac{1}{4}F^2 + \ldots\Big]$$

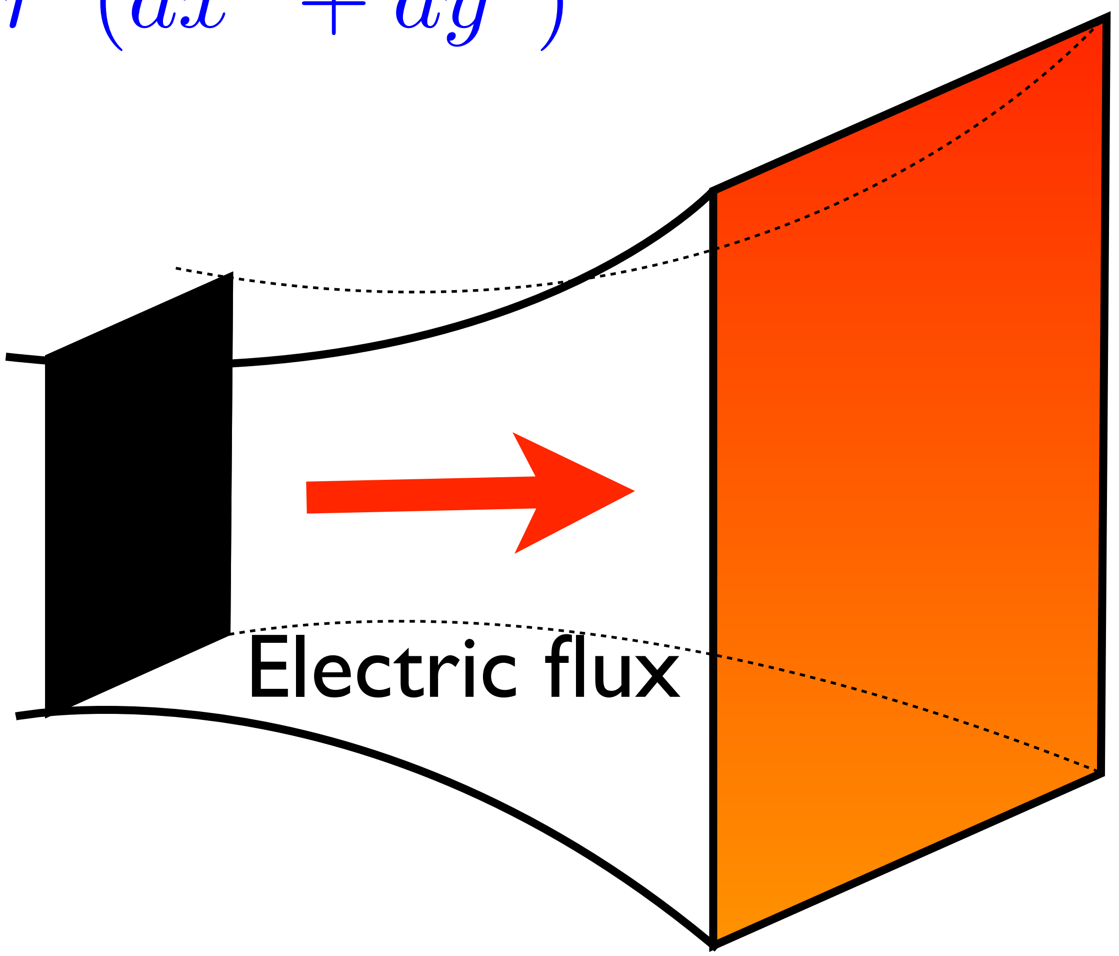
Admits AdS_4 vacuum $\leftrightarrow$ d=3 CFT with global U(1)

Electrically charged AdS-RN black hole (brane)

Describes holographic matter at finite charge density that is translationally invariant

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + r^2(dx^2 + dy^2)$$

$$A_t = \mu(1 - \frac{r_+}{r})$$



d=3 CFT

$\mu \quad T$

T=0 limit: $AdS_2 \times \mathbb{R}^2 \longleftarrow AdS_4$

IR UV

By perturbing the black hole and using holographic tools we can calculate the electric conductivity and find a delta function at $\omega = 0$ [Hartnoll]

Construct lattice black holes dual to CFT with $\mu(x)$

$$A_t(x,r) \sim \mu(x) + \mathcal{O}(\frac{1}{r}) \qquad r \to \infty$$

$$g_{\mu\nu}(x,r)$$

Need to solve PDEs in two (or more) variables

e.g. Monochromatic lattice: [Horowitz, Santos, Tong]

[Ling, Niu, Wu, Xian]

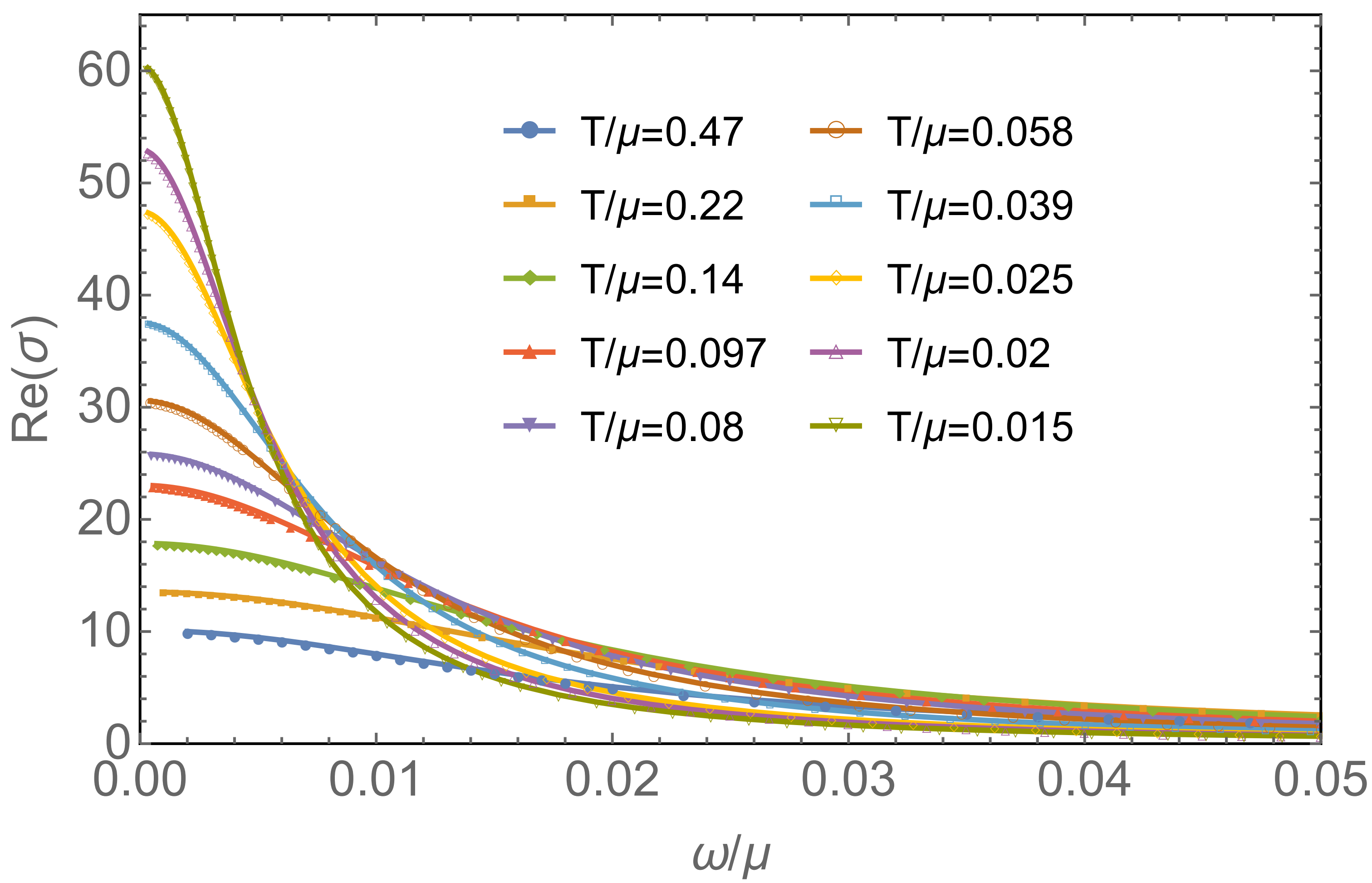
$$\mu(x) = \mu + A\cos kx$$

[Chesler, Lucas, Sachdev]

[Donos, JPG]

After constructing black holes, one can perturb, again solving PDEs, to extract thermo-electric conductivities

Find sharp Drude peaks at low T

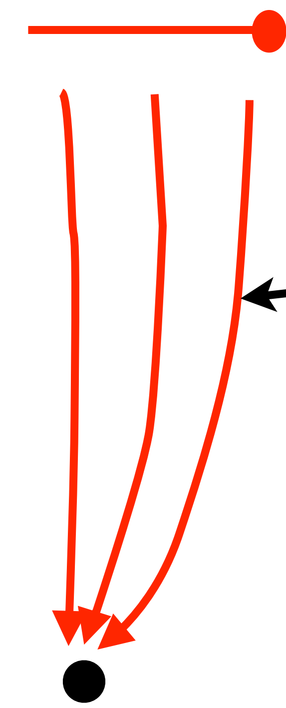


Coherent metal phases

Can be understood by analysing T=0 solutions:

UV data

$k/\mu \quad A/\mu$



AdS-RN

IR fixed point $AdS_2 \times \mathbb{R}^2$

At T=0 the black holes approach $AdS_2 \times \mathbb{R}^2$ in the IR

perturbed by irrelevant operator with $\Delta(k_{IR}) \geq 1$

[Hartnoll,Hofman]

Don't find exceptions to this behaviour, for these lattices, even for "dirty lattices" e.g.

$$\mu(x) = 1 + A \sum_{n=1}^{10} \cos(n\,k\,x + \theta_n)\,,$$

Holographic Q-lattices

[Donos,JPG]

- Illustrative D=4 model

$$\mathcal{L} = R - \frac{1}{2}|\partial\varphi|^2 + V(|\varphi|) - \frac{Z(|\varphi|)}{4}F^2$$

- Choose V, Z so that AdS-RN is a solution at $\varphi = 0$

- Now $\varphi \leftrightarrow \mathcal{O}$ in CFT. Want to build a holographic lattice by deforming with the operator $\mathcal{O}$

- The model has a gauge $U(1)$ and a global $U(1)$ symmetry

  Exploit the global bulk symmetry to break translations so that we only have to solve ODEs

Ansatz for fields

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx^2 + e^{2V_2} dy^2$$

$$A_t = a(r)$$

$$\varphi(r, x) = \phi(r) e^{ikx}$$

UV expansion: approaches AdS with deformation

$$U = r^2 + \ldots, \qquad e^{2V_1} = r^2 + \ldots \qquad e^{2V_2} = r^2 + \ldots$$

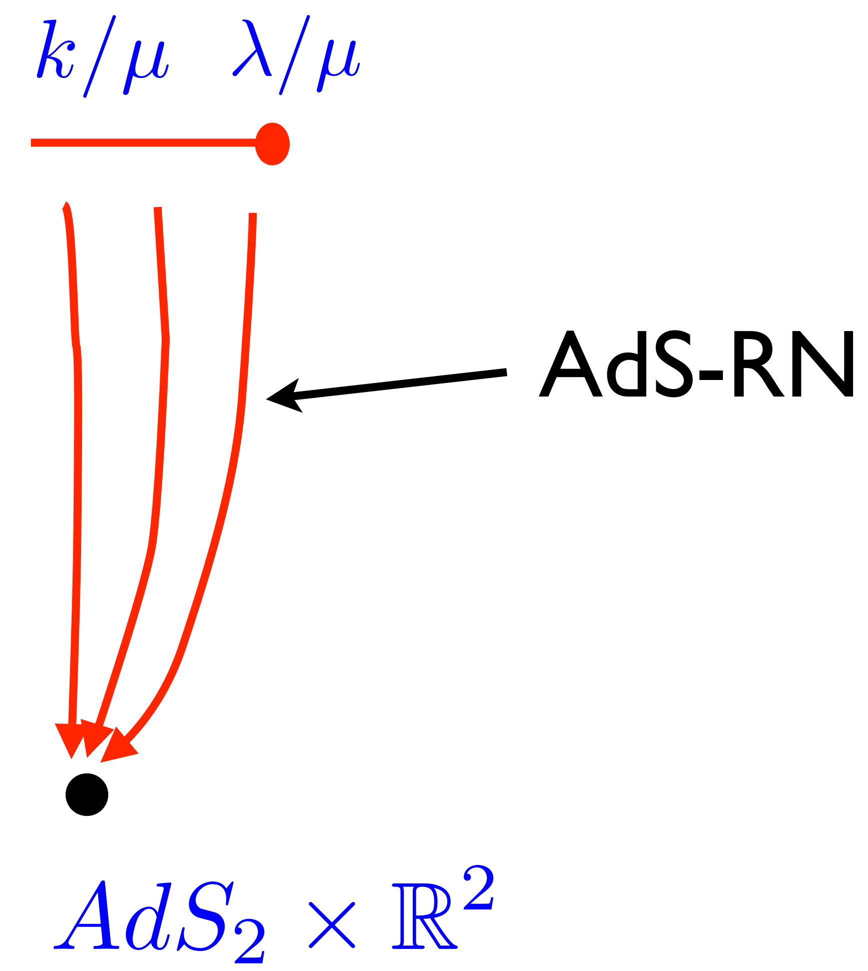
$$a = \mu + \frac{q}{r} \ldots, \qquad \phi = \frac{\lambda}{r^{3-\Delta}} + \ldots$$

Homogeneous and anisotropic and periodic holographic lattices

UV data: $T/\mu \qquad \lambda/\mu^{3-\Delta} \qquad k/\mu$

For small deformations from AdS-RN we find Drude peaks for small T corresponding to coherent metals.

This can be understood by examining T=0 behaviour of solutions:



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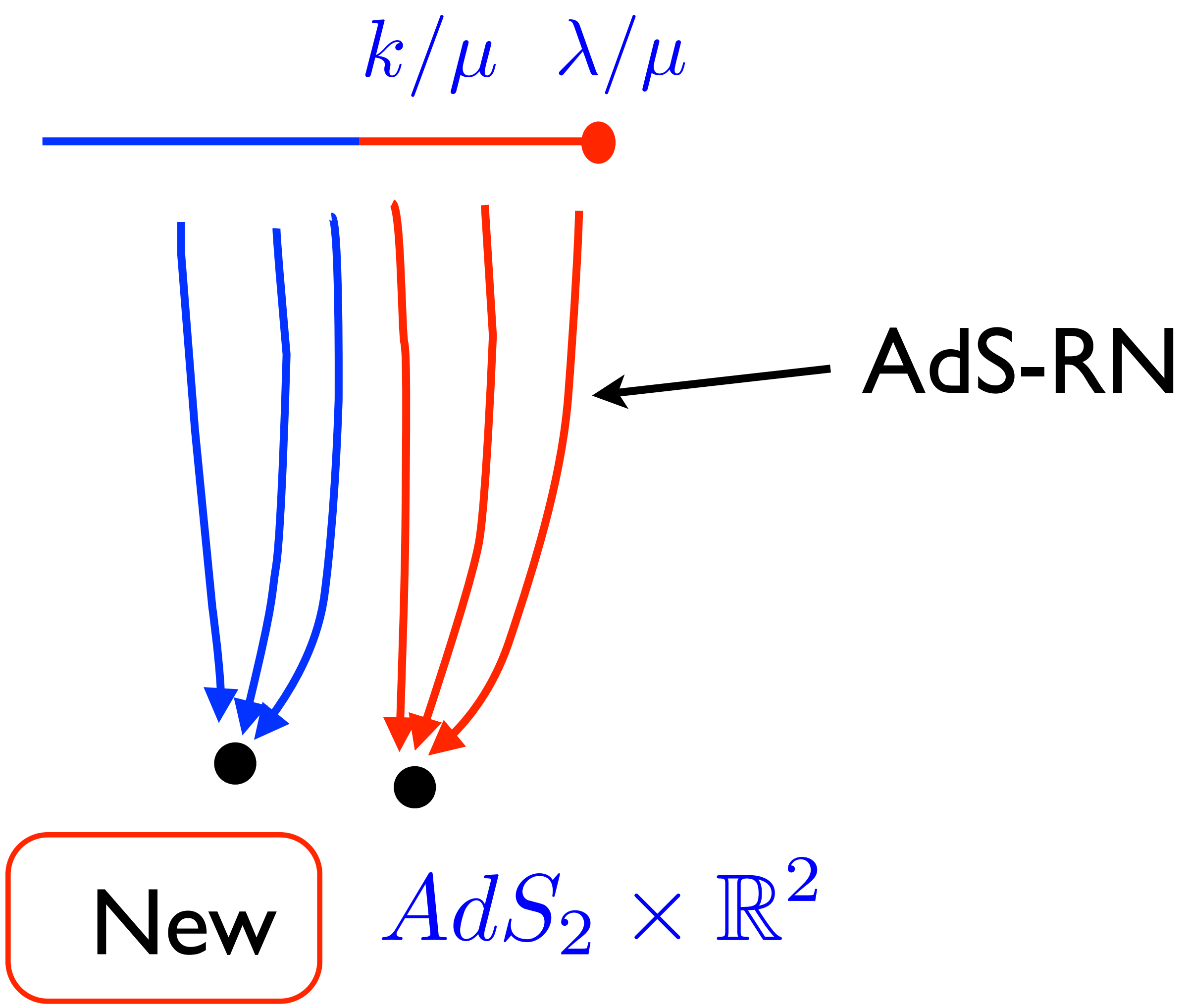


For larger deformations, for specific models, we find a transition to new behaviour. The new ground states - which break transaltions - can be both insulators and also incoherent metals!

See also: [Gouteraux][Andrade,Withers]

DC conductivity of Q-lattices

Generalised Ohm/Fourier Law:

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

J^a	Electric current
$Q^a = T^{ta} - \mu J^a$	Heat current

For Q-lattice black holes the DC matrices $\sigma, \alpha, \bar{\alpha}, \bar{\kappa}$ diagonal

Can obtain from the behaviour of the Q-lattice solutions at the black hole horizon [Donos,JPG] (more later!)

$$\alpha = \bar{\alpha} = -\left[\frac{4\pi\rho}{k^2\Phi(\phi)}\right]_{r=r_+} \qquad \bar{\kappa} = \left[\frac{4\pi s T}{k^2\Phi(\phi)}\right]_{r=r_+}$$

$$\sigma = -\left[e^{-V_1+V_2}Z(\phi) + \frac{4\pi\rho^2 s^{-1}}{k^2\Phi(\phi)}\right]_{r=r_+}$$

First term in σ is actually $(J/E)_{Q=0} \equiv \sigma - \alpha^2\bar{\kappa}^{-1}T$
"Pair evolution" term

Second term "Dissipation" term

Different ground states can be dominated by first or second term

Notice that the first term is finite when $k \to 0$

- For dissipation dominated T=0 ground states κ and $\bar{\kappa}$ can have different low temperature scaling (n.b. $\kappa = \bar{\kappa}$ for FL)

- Wiedemann-Franz law violated: $\kappa/\sigma T$ not constant

- Metals $\sigma \to \infty$ both at small and large T

  Hence there is a minimum conductivity $\Leftrightarrow$ a maximum resistivity at some finite T c.f Mott-Ioffe-Regel bound

- Figure of merit: $ZT \equiv \dfrac{\alpha^2 T}{\kappa\sigma}$

  Measure of the efficiency of thermoelectric engines.
  Maximum known value is about 3

  Holography: ZT diverges at high temperature and also diverges at low temperature for coherent metal ground states

Other lattices

- Scalar lattice - one-dimensional. Solve PDEs

  [Horowitz, Santos, Tong]

  Similar to Q-lattice results

  [Rangamani, Rozali, Smyth]

- Helical

  [Donos, Hartnoll] [Donos, JPG, Pantelidou][Donos, Gouteraux, Kiritsis]
  [Erdmenger, Herweth, Klug, Meyer, Shalm] c.f. [Nakamura, Ooguri, Park]

  Replace spatial $\mathbb{R}^3$ with Bianchi VII_0

  There is a universal helical deformation of all d=4 CFTs.
  In holography this leads to a conformally invariant ground state at T=0 [Donos, JPG, Pantelidou]

- Others: [Balasubramanian, Herozg] [Jain, Kundu, Sen, Sinha, Trivedi] [Vegh]

Axion lattices and N=4 SYM

[Banks,JPG]

- Consistent KK truncation of IIB on S^5

$$S = \int d^5x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial\chi)^2 + 12\right)$$

- Axionic lattice black holes

[Azeyanagi,Li,Takayanagi]

[Mateos,Trancanelli]

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + e^{2V_1}(dx^2 + dy^2) + e^{2V_3} dz^2$$

$$\phi = \phi(r) \qquad \boxed{\chi = az}$$

Describe anisotropic phase of N=4 SYM

- Zero temperature limit approach in the IR

$$ds^2 = L^2\left(\frac{dr^2}{r^2} + r^2(-d\bar{t}^2 + d\bar{x}^2 + d\bar{y}^2) + r^{4/3} d\bar{z}^2\right)$$

Thermal insulator with $\kappa \sim T^{7/3}$

These black holes are unstable

Kaluza-Klein reduction on S^5 gives rise to 20 scalars with masses that saturate the AdS_5 BF bound but violate an analogous bound for the IR Lifshitz solution

Study via a consistent KK truncation that keeps one of the 20 scalars [Banks,JPG]

$$S = \int d^5x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial\chi)^2 - 3X^{-2}(\partial X)^2 + 4(X^2 + 2X^{-1})\right)$$

Find that [Mateos,Trancanelli] black holes undergo a phase transition, with a condensation of the X-field

Curiously the transition has critical exponents NOT mean field type

Ground state is a kind of hyper-scaling type solution $\kappa \sim T^{10/3}$

DC Conductivity and Navier Stokes [Donos,JPG]

Consider D=4 Einstein-Maxwell Theory

- Background lattice black holes

$$ds^2 = -UG\,dt^2 + \frac{F}{U}\,dr^2 + g_{ij}dx^i dx^j$$

$$A = a_t\,dt$$

with $U = U(r)$ and everything else depends on (r, x^i)

- Behaviour at AdS boundary

$$U \to r^2 \qquad F \to 1$$

$$G \to \bar{G}(x) \qquad g_{ij}(r,x) \to r^2 \bar{g}_{ij}(x) \qquad a_t(r,x) \to \mu(x)$$

Very general class of UV lattice data

- Demand that they have regular black hole horizon at $r = 0$

- Perturb the black hole

$$\delta(ds^2) = \delta g_{\mu\nu} dx^\mu dx^\nu + 2tGU\zeta_i dt dx^i$$

$$\delta A = \delta a_\mu dx^\mu - tE_i dx^i + ta_t \zeta_i dx^i$$

- Behaviour at AdS boundary

The only sources are provided by the closed one-forms

$E(x), \zeta(x)$ - they source the electric and heat currents

- Regular behaviour at the horizon

Kruskal coordinate $v = t + \frac{\ln r}{4\pi T} + \ldots$

e.g. $\delta a_i = \frac{\ln r}{4\pi T}(-E_i + a_t \zeta_i) + \ldots$

- Electric current $J^i = \sqrt{-g} F^{ir}$

At the AdS boundary, $J^i|_\infty$, this is the electric current

$$\partial_i J^i = 0\,, \qquad \partial_r J^i = \partial_j \left(\sqrt{-g} F^{ji}\right)$$

- Heat current

Want a suitable two-form. Let $k = \partial_t$ and define

$$G_{\mu\nu} = \nabla_{[\mu} k_{\nu]} + \tfrac{1}{2} k_{[\mu} F_{\nu]\sigma} A^\sigma + \theta\, F^{\mu\nu}$$

(think of First Law or Kaluza-Klein reduction)

$$Q^i = -2\sqrt{-g} G^{ir}$$

$$\partial_i Q^i = 0\,, \qquad \partial_r Q^i = -\partial_j \left(2\sqrt{-g} G^{ji}\right)$$

• Evaluate currents on the horizon and examine the linear equations of motion satisfied by the perturbation

Define $v_i \equiv \delta g_{it}^{(0)}$ $\qquad w \equiv \delta a_t^{(0)}$

$$p \equiv 4\pi T \frac{\delta g_{rt}^{(0)}}{G^{(0)}} + \delta g_{it}^{(0)} g_{(0)}^{ij} \nabla_j \ln G^{(0)}$$

Find

$$\nabla_i v^i = 0$$

$$\nabla^2 w - v^i \nabla_i (a_t^{(0)}) = -\nabla_i E^i$$

$$\nabla^2 v_j + R_{ji} v^i - a_t^{(0)} \nabla_j w - \nabla_j p = 4\pi T \zeta_j + a_t^{(0)} E_j$$

Linear, time-independent, forced Navier-Stokes equations for a charged, incompressible fluid on the curved black hole horizon!

Note: no hydrodynamic limit has been taken.

- Solutions are uniquely fixed by the sources $E(x), \zeta(x)$ (unless there is Killing vectors on the horizon)

- Given J^i, Q^i on the horizon, what do we know about $J^i|_\infty, Q^i|_\infty$ (the holographic electric currents)?

Need to integrate:

$$\partial_r J^i = -\partial_j \left(2\sqrt{-g}F^{ji}\right) \qquad \partial_r Q^i = -\partial_j \left(2\sqrt{-g}G^{ji}\right)$$

Can always define constant averaged currents e.g. if the lattice deformation is periodic in x^i with period $2\pi L_i$

$$\bar{J}^1 \equiv \frac{1}{2\pi L_2}\int J^1 dx^2 \qquad \bar{J}^2 \equiv \frac{1}{2\pi L_1}\int J^2 dx^1$$

and also for Q, and we can obtain the averaged DC conductivity

Examples

- Can recover earlier results for Q-lattices and one-dim lattices

- Perturbative, periodic lattice about AdS-RN black brane

Let λ be the expansion parameter

The black hole horizon is a small expansion about flat space

$$g_{(0)ij} = g\,\delta_{ij} + \lambda\,h^{(1)}_{ij} + \cdots$$

$$G^{(0)} = f_{(0)} + \lambda\,f_{(1)} + \cdots$$

$$a^{(0)}_t = a + \lambda\,a_{(1)} + \cdots$$

Solve Navier-Stokes perturbatively in λ

- At leading order in λ we find

$$\alpha_{ij} = \bar{\alpha}_{ij} = \frac{L_{ij}^{-1}}{\lambda^2} 4\pi\rho + \ldots \qquad \bar{\kappa}_{ij} = \frac{L_{ij}^{-1}}{\lambda^2} 4\pi s T + \ldots$$

$$\sigma ij = \frac{L_{ij}^{-1}}{\lambda^2} \frac{4\pi\rho^2}{s} + \ldots$$

where $$L_{ij} = \int_H l_{ij}(h_{kl}^{(1)}, a^{(1)})$$

Consistent with memory matrix formalism
[Barkeshli,Hartnol,Mahajan]

Summary/Final Comments

- Holographic lattices are interesting

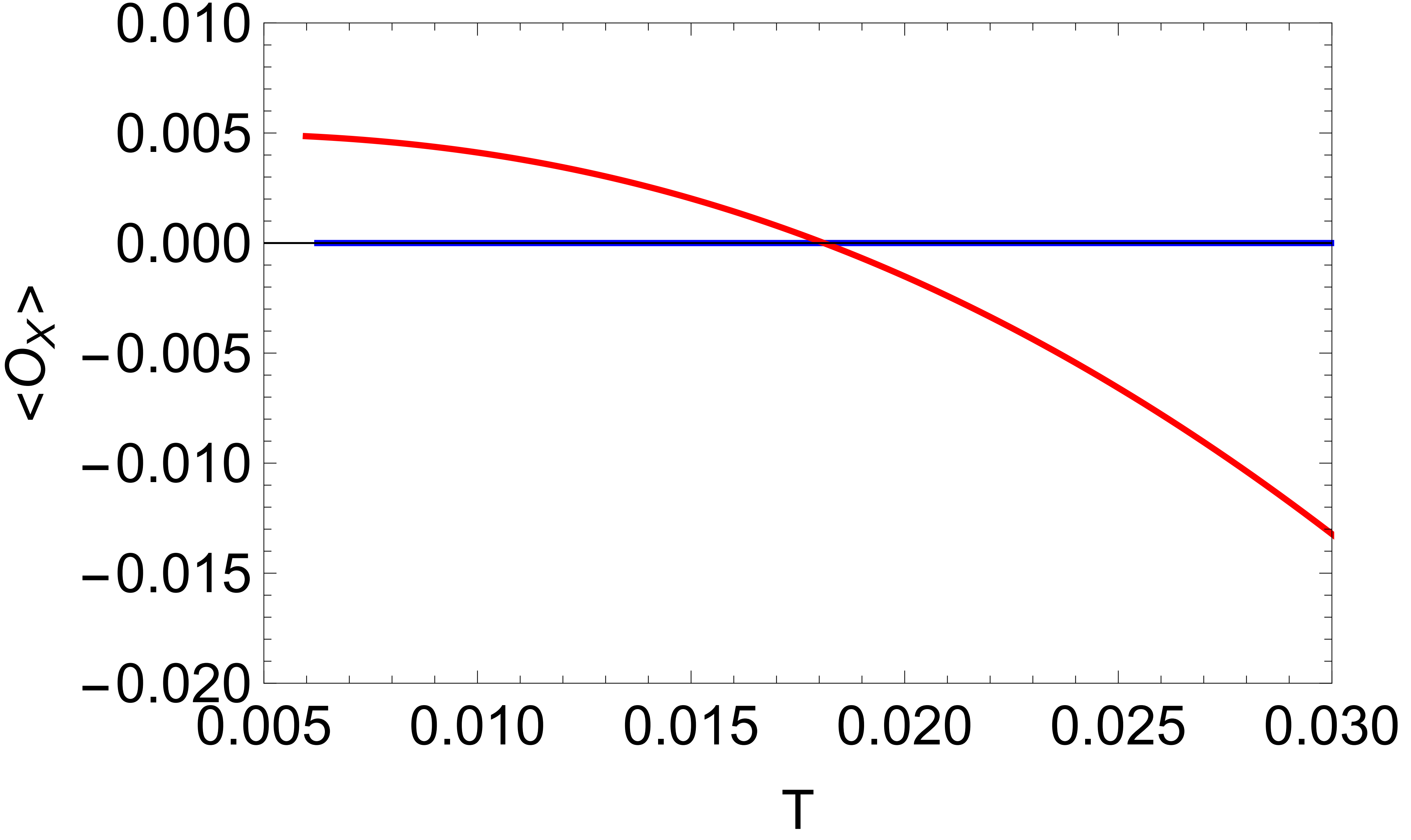
- Coherent metals/Drude physics can be understood by the appearance of translationally invariant ground states in the far IR e.g. $AdS_2 \times \mathbb{R}^2$ perturbed by irrelevant operators

- For larger deformations the Q-lattices (and others) can realise incoherent metallic and insulating phases and transitions between them

The new T=0 ground states break translation invariance and have novel thermoelectric transport properties.
What is the landscape? How can they be realised?

- New appearance of Navier-Stokes on black hole horizons

Can be used to obtain DC thermoelectric conductivities



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T

Phase transition is not mean field!

$$\langle \mathcal{O} \rangle \sim (T_c - T)^{\beta} \qquad \beta = 1$$

$$C \sim (T_c - T)^{-\alpha} \qquad \alpha = 1$$

Mean field: $\beta = 1/2 \quad \alpha = 0$

T=0 ground states in the far IR:

$$ds^2 \sim \rho^{\frac{-2(3-\theta)}{3}} \left(d\rho^2 - d\bar{t}^2 + d\bar{x}^2 + d\bar{y}^2 + \rho^{-2(z-1)} d\bar{z}^2 \right)$$

$$e^{\phi} \sim \rho^{-2/3}, \quad X \sim \rho^{-2/3}, \quad \chi = a\bar{z}$$

$$\theta = -1, \quad z = 2/3$$