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Tales in the realm of Holographic Conductivity

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Bibliography

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. arXiv:1005.4690 [hep-th]

Introduction

• Conductivity is one of the most important observables in condensed matter systems

• It is relatively easy to define and measure.

• It tells us a lot about the dynamics of charge carriers in a medium

• For low enough voltages it is controlled by the retarded current two-point function.

Strongly correlated electrons

• In metals, although bare electrons are strongly coupled, there are "dressed" (fermionic) quasiparticles that are weakly-coupled and behave as almost free electrons. They are responsible for transporting charge.

• In most materials that are on the border with magnetism (like the cuprates) there are no weakly-coupled quasiparticles for most of their phase diagram.

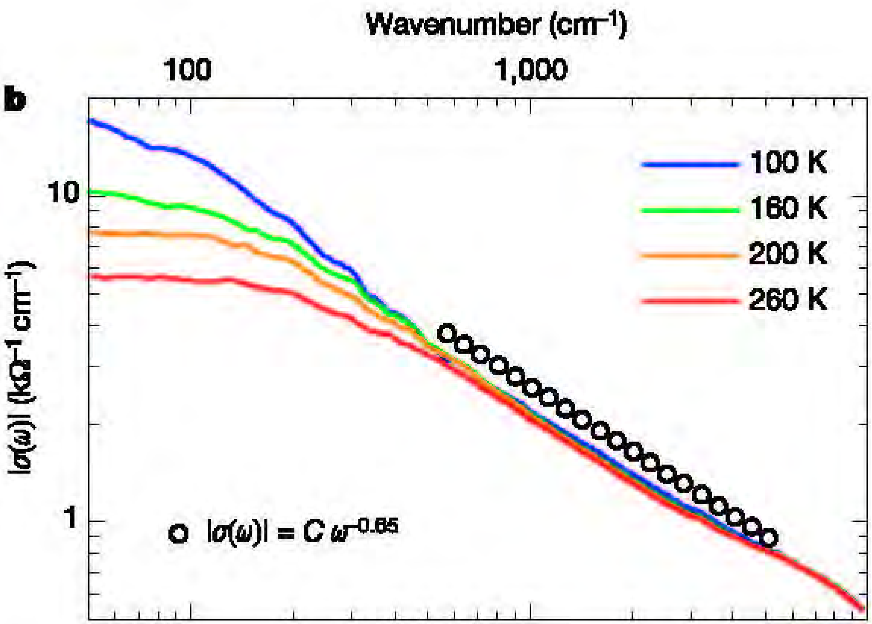
• Such systems are Mott insulators in some part of the their phase diagram.

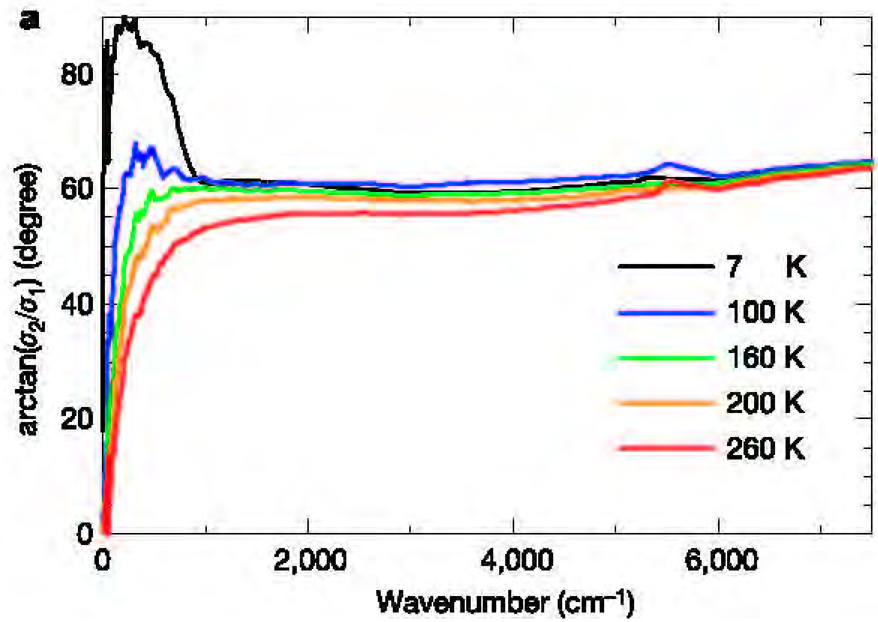
• They have a benchmark linear conductivity in the area above their superconducting dome.

• In some of them the AC Conductivity shows a very simple scaling law,

$$\sigma(\omega) \quad \sim \quad \omega^{-\frac{2}{3}}$$

Scaling in AC conductivity





Van Der Marel et al.

Insulators

• There are several mechanisms for insulating behavior in condensed matter:

1) Band gap insulators, where the conduction band is empty, and there is therefore a gap that prevents current transport.

2) Anderson localization that is effective usually in two dimensions and where strong disorder inhibits conduction.

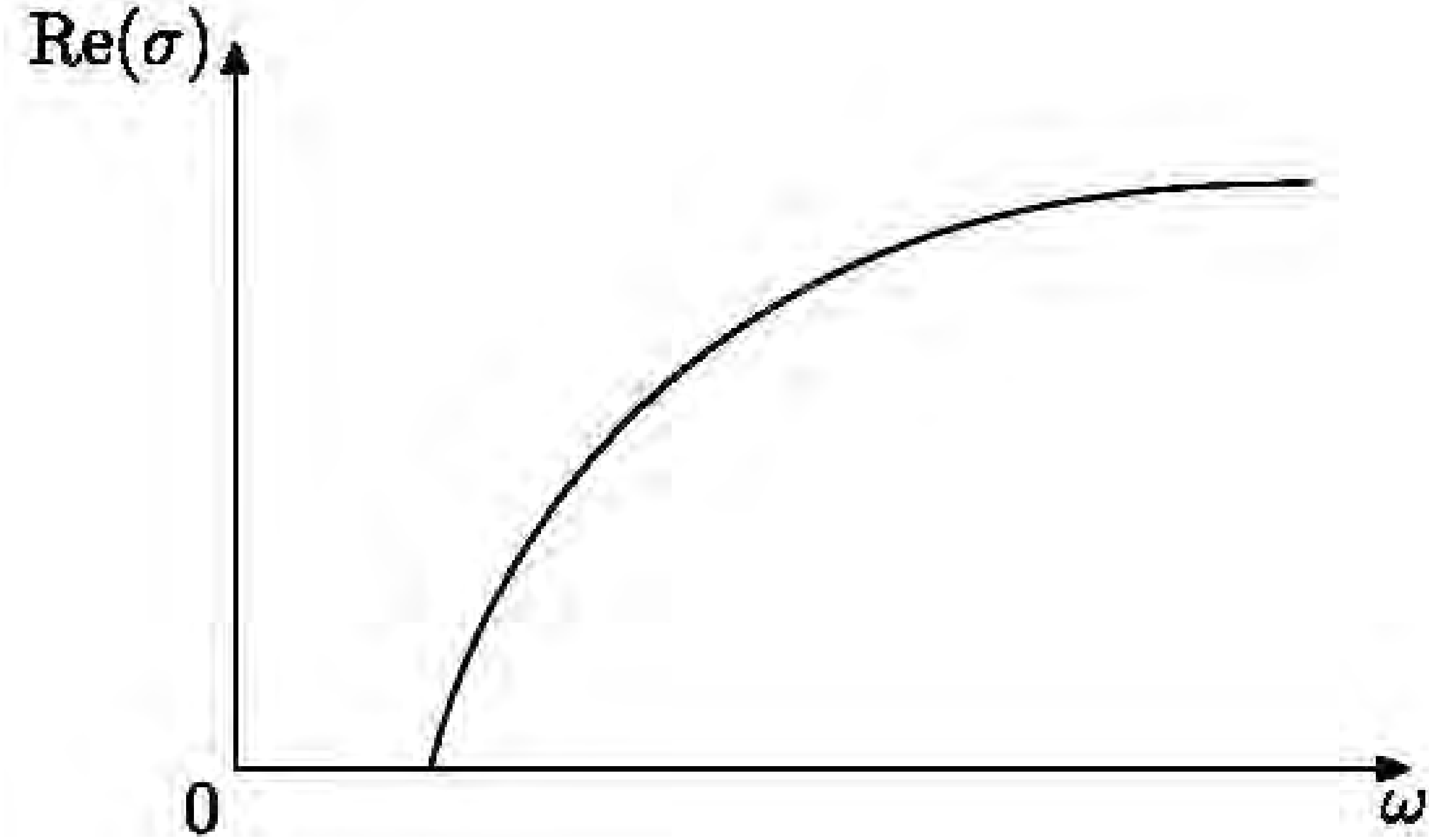
3) Mott localization, where strong onsite interactions localize electrons. This has been argued to be at work in insulating anti-ferromagnets.

4) A new mechanism at strong coupling: Momentum-dissipating interactions become relevant (strong) in the IR, and they inhibit conduction

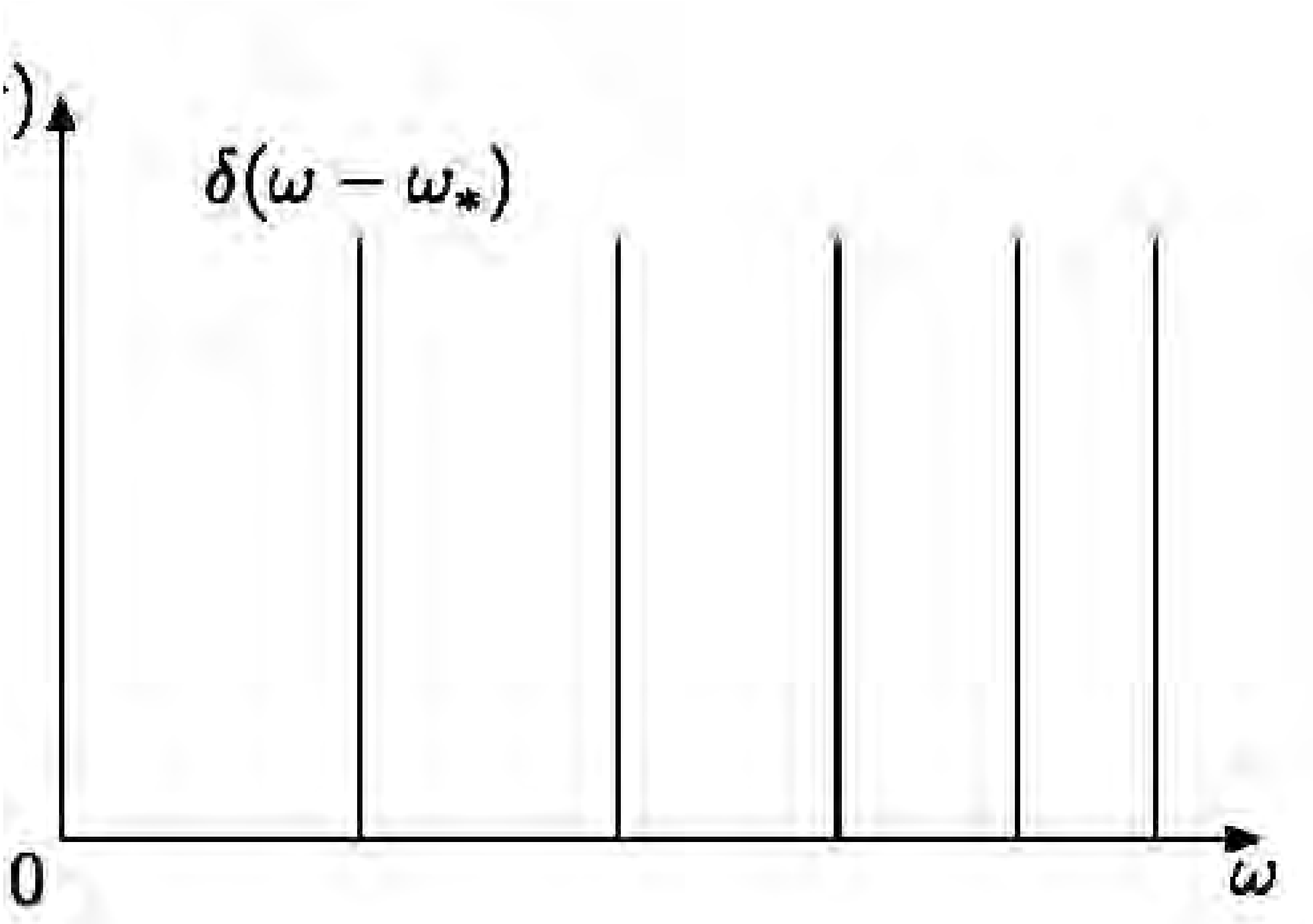
Donos+Hartnoll

• This mechanism was generalized to Einstein-Maxwell-Dilaton (EMD) theories with many saddle points found corresponding to insulators, bad metals and conventional metals.

Donos+Gouteraux+Kiritsis, Donos+Gauntlett,Gouteraux



The insulators above may or may not have a gap, and a continuum above it.



I will discuss insulators whose conductivity looks more like the above.

Supersolids

• A supersolid is a generalization of a superfluid state. It is characterized by a spontaneously broken U(1) symmetry which guarantees a superfluid component (a zero frequency δ-function).

• It has (spontaneously) broken translational invariance. and is therefore a solid.

• Therefore the appropriate two-point function has a discretely localized spectral density.

• If probed at a generic non-zero frequency it is non-responsive. If it is probed at zero frequency it behaves as a superfluid.

• They have been theoretically anticipated and studied, especially in the last 2-3 years.

Legget, Fisher+Nelson, Anderson, Nicolis+Penco+Rosen

• There are proposed realizations with cold atoms.

Keilmann+Cirac+Roscilde

• There have been claims for presence in solid Helium^4 as well as a recent refutation.

Kim+Chan

The Plan of the rest

- Conductivity in Holographic Theories

- Gapped ground states in EMD theories

- Adding momentum dissipation.

- Power tails and scaling of the AC conductivity

The Wilsonian setup

- Holographic theories are generically RG flows between fixed points.

- The first step in mapping the lanscape of theories: Classification of Scale Invariant/Fixed-point theories (The Wilsonian approach in holography).

- The strategy is to use Effective Holographic Theories (in order to explore all possible QC holographic scale invariant theories with given symmetries.

Charmousis+Gouteraux+Kim+Kiritsis+Meyer (2010)

- Once fixed points are classified for a given bulk action and symmetry ansatz, the conductivity can be studied first "locally" (in the IR fixed point) and then globally (along the whole flow).

Generic Scaling Geometries: A classification of all QC points in holography

Charmousis+Gouteraux+Kim+Kiritsis+Meyer (2010), Gouteraux+Kiritsis (2011),
Huisje+Sachdev+Swingle, (2011), Iizuka+Kachru+Kundu+Narayan+Sircar+trivedi+Wang (2012)

- Assume translation and rotational invariance in space and time.

- The QC points generically break hyperscaling invariance and are characterized by several exponents. Two appear in the metric (z, θ).

- $z \to$ dynamical exponent

- $\theta \to$ hyperscaling violation exponent

$$ds^2 = r^{\frac{2\theta}{d}} \left[\frac{dt^2}{r^{2z}} + \frac{dr^2 + dx_1^2 + dx_2^2 + \cdots + dx_d^2}{r^2} \right]$$

- There is invariance under:

$$x_i \to \lambda\ x_i \quad , \quad t \to \lambda^z\ t \quad , \quad r \to \lambda\ r \quad , \quad ds \to \lambda^{\frac{\theta}{d}}\ ds$$

- The entropy scales as

$$S \sim T^{\frac{d-\theta}{z}}$$

which gives an interpretation to the hyperscaling violation exponent.

- There is a third exponent, associated with the charge density, the conduction exponent ζ:

Gouteraux+Kiritsis, Gouteraux

$$A_t = Q\ r^{\zeta - z}$$

- If non-zero it also violates hyperscaling.

Conductivity:basics

• It can be calculated in the linear regime from the correlators of the currents

$$\sigma_{ij}(\omega,\vec{k}) = \frac{1}{i\omega}\int d^p x dt\ e^{-i\omega t - i\vec{k}\cdot\vec{x}}\langle J_i(t,\vec{x})J_j(0,0)\rangle$$

or more generally as the (non-linear) response to an external electric field

$$J_i = \sigma_{ij}\ E_j$$

• Translational invariance and finite density imply a pole at zero frequency for the conductivity.

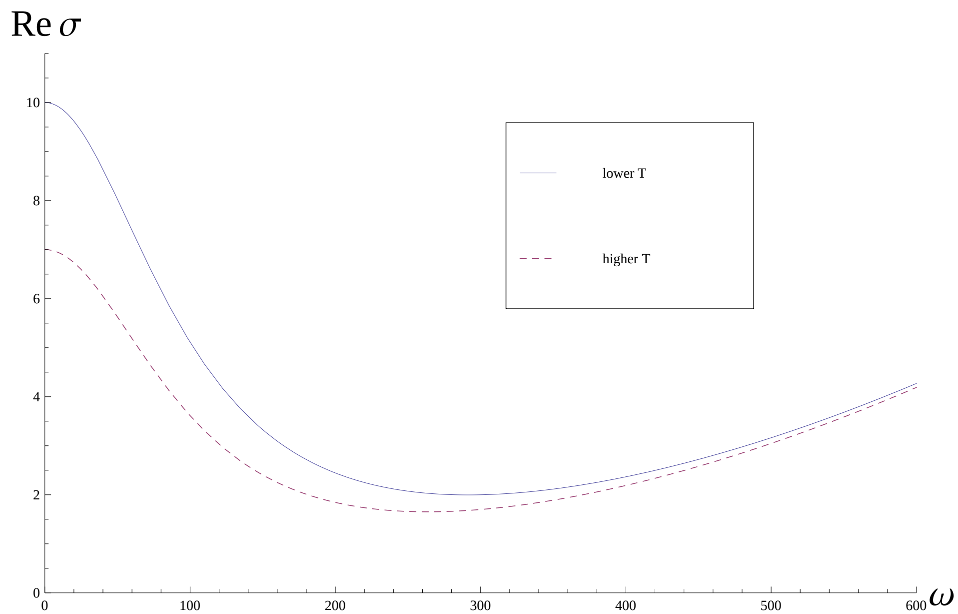
$$\sigma(\omega) \simeq K\left(\delta(\omega) + \frac{i}{\omega}\right) + \cdots \quad , \quad K = \frac{4\rho^2}{\varepsilon + p}$$

• K is the Drude weight.

• Weak scattering over ions or impurities, resolves the zero frequency pole

$$\sigma(\omega) \simeq \frac{K\tau}{1 - i\omega\tau} + \cdots \quad , \quad \frac{1}{\tau} = \text{scattering rate}$$

• This is the so-called Drude peak and defines the response of a metal at low frequencies.



• As $T \to 0$, $\tau \to \infty$. In this limit $\tau \to \infty$ we obtain back the zero frequency pole and δ-function.

• When the U(1) symmetry is spontaneously broken there is a δ-function at zero frequency marking the superfluid. The Drude weight is then the superfluid density.

General features of holographic conductivities

• When the bulk action of the U(1) gauge field is linear in F^2

$$S_F \sim \int d^4x\sqrt{g}\ Z(\phi)\ F^2$$

the DC conductivity has the schematic form

$$\sigma_{DC} \sim \sigma_{pair} + \sigma_{drag}$$

Blake+Tong, Donos+Gauntlett

• The first term $\sigma_{DC} \sim Z$ was interpreted as a term coming from the pair-production of charge.

Karch+O'Bannon

• It is non-zero even when $Q = 0$.

• It does not contribute to thermal transport.

Donos+Gauntlett

• It is finite, even when there is no momentum dissipation (does not contribute to the Drude weight).

• $\sigma_{drag} \sim \tau$ is due to momentum-dissipating interactions.

• The above are suggestive but the true story appears to be more complicated.

Gouteraux+Richardson

• If more than one mechanisms of momentum dissipation are at work,

$$\sigma_{drag} = \sum_I \sigma^I_{drag} \quad , \quad \tau = \sum_I \tau_I$$

(inverse Mathiessen law)

Hints from Donos+Gouteraux+Kiritsis

• In the limit of vanishing dissipation, $\tau \to \infty$, the Drude δ-function is regenerated.

• When charge dynamics is described by the DBI action then

$$\sigma_{DC} = \sqrt{\sigma_{pp}^2 + \sigma_{drag}^2}$$

Holographic systems at finite density

• There are two ways that a system can be insulating at T=0:

♠ Charged excitations are gapped. In that case the conductivity is non-zero only above the gap.

♠ There is no gap but the limit $\omega \to 0$ gives a vanishing conductivity. In both cases the operator/mechanism that breaks translation invariance, is relevant in the IR.

• Gapped holographic systems are known at zero charge density, and they are in use as models of YM.

Witten, '98, Gursoy+Kiritsis+Nitti, '07, Nishioka+Ryu+Takatanagi, '10

• At finite density, many saddle-points with discrete spectrum were found, in the classification of QC points in EMD Theories.

Charmousis+Gouteraux+Kiritsis+Kim+Meyer, '10, McGreevy+Balasubramanian, '10

We parametrize the effective holographic action as

$$S=\int d^4x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-V(\phi)-\frac{Z(\phi)}{4}F^2\right]\quad,\quad V\sim e^{-\delta\phi}\quad,\quad Z\sim e^{\gamma\phi}$$

- The generic IR geometry is hyperscaling violating with (z,θ,ζ) being functions of (γ,δ).

- The IR charge density is also fixed as a function of (γ,δ).

- Such extremal geometries have naked singularities. One should impose the Gubser constraints that put restrictions on (z,θ,ζ). Such singularities are expected to be resolvable by effects neglected (KK states, stringy corrections, etc.)

- As we shall see there may be more constraints when we start considering correlation functions.

The conductivity

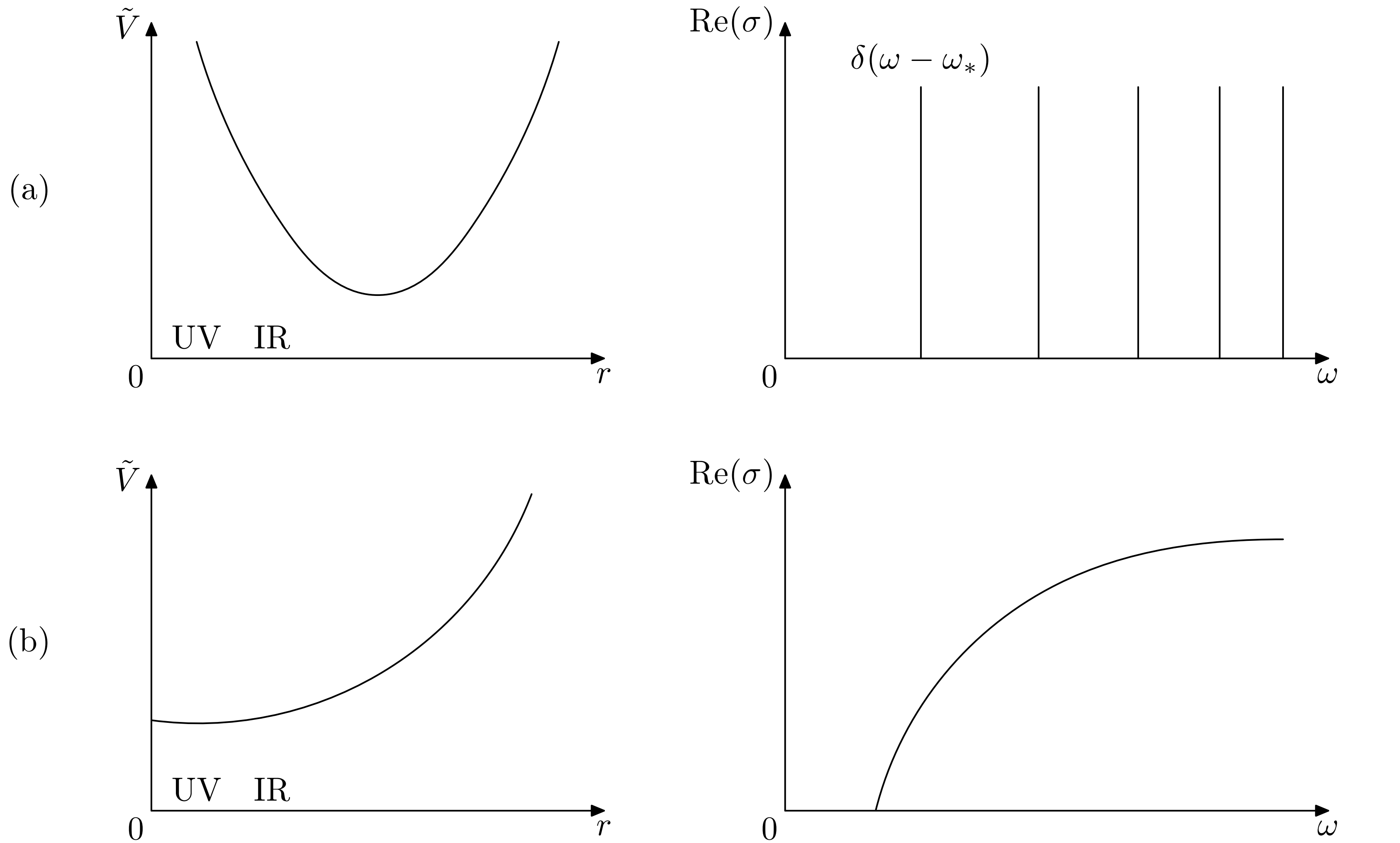
• The conductivity is obtained by solving for the fluctuations of the gauge field, $\delta A_i = a_i(r)e^{i\omega t}$.

$$\frac{1}{Z}\sqrt{\frac{g_{rr}}{g_{tt}}}\partial_r\left(Z\sqrt{\frac{g_{tt}}{g_{rr}}}a_i'\right) + \left[\frac{g_{rr}}{g_{tt}}\omega^2 - \frac{Q^2\ g_{rr}}{Zg_{xx}^2}\right]a_i = 0$$

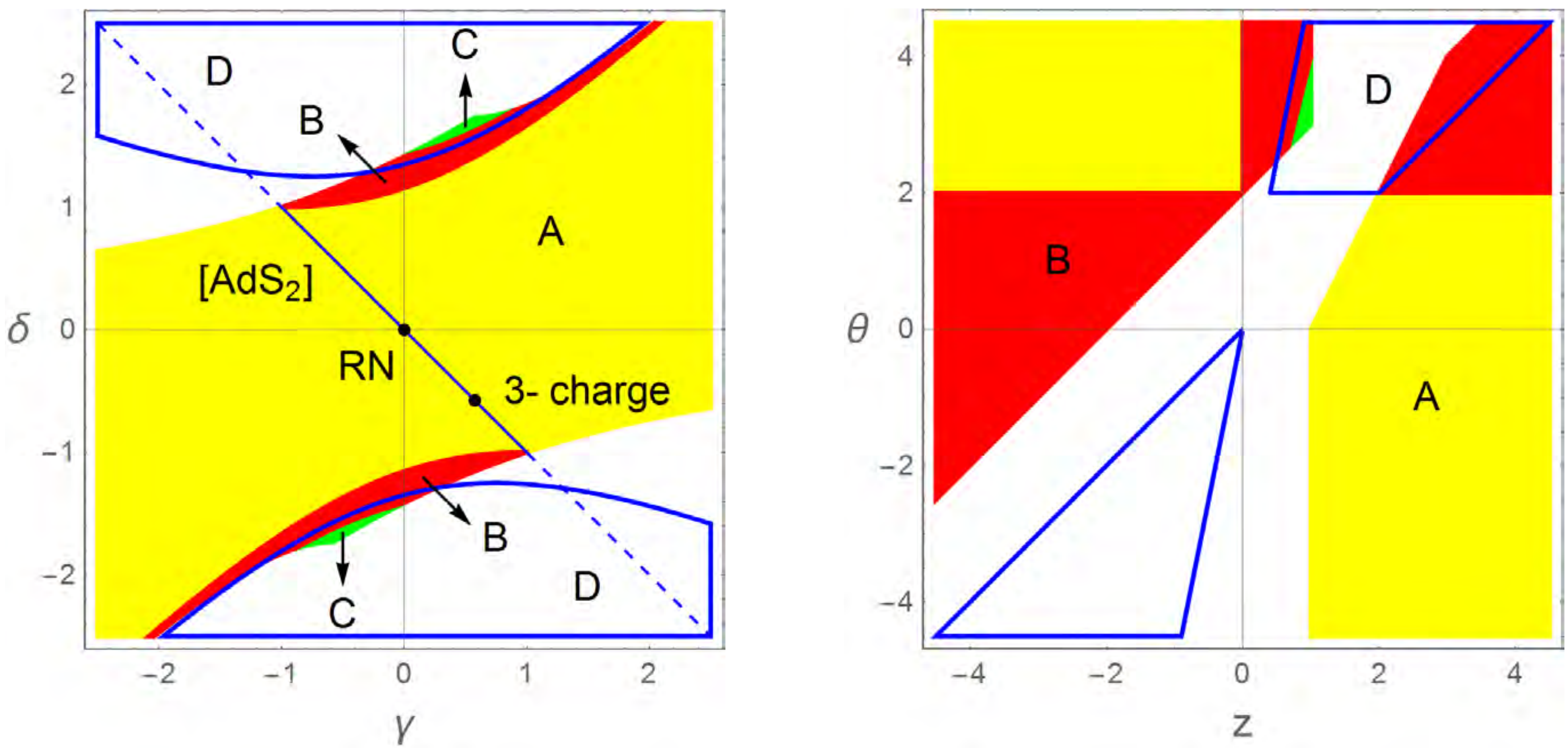
• By a field and coordinate redefinitions it can be mapped into a Schrödinger problem

$$-\psi'' + V_{eff}\ \psi = \omega^2\psi$$

$$V_{eff} = V_1 + Q^2\ V_2$$


(a)
$\tilde{V}$
UV
IR
0
r
$\mathrm{Re}(\sigma)$
$\delta(\omega - \omega_*)$
0
ω
(b)
$\tilde{V}$
UV
IR
0
r
$\mathrm{Re}(\sigma)$
0
ω

The parameter space



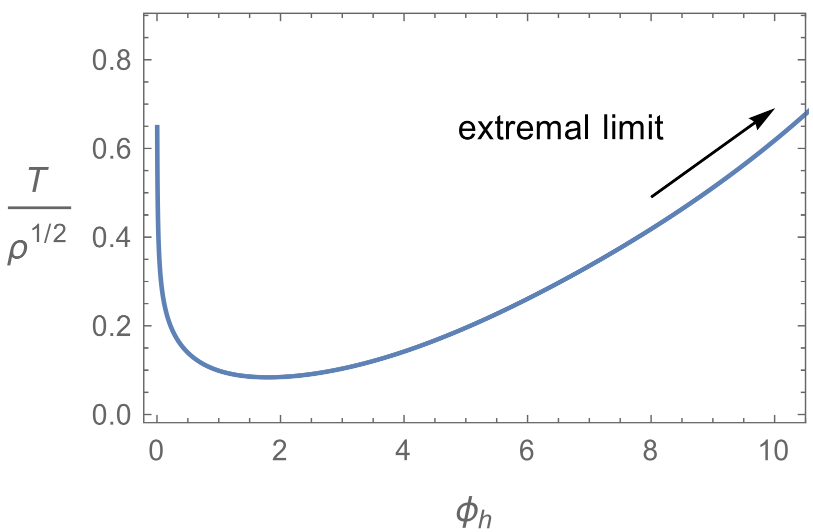
The white part is not allowed by the Gubser criterion. The regions A, B, and C are the parameter space allowed by the Gubser criterion. In region A (yellow), the extremal limit is at $T \to 0$, and the current-current correlator is gapless. In region B (red), the extremal limit is at $T \to \infty$, and the current-current correlator is gapped. In region C (green), the extremal limit is at $T \to \infty$, and the current-current correlator is gapless. Region D (enclosed by blue boundaries) is holographically unreliable.

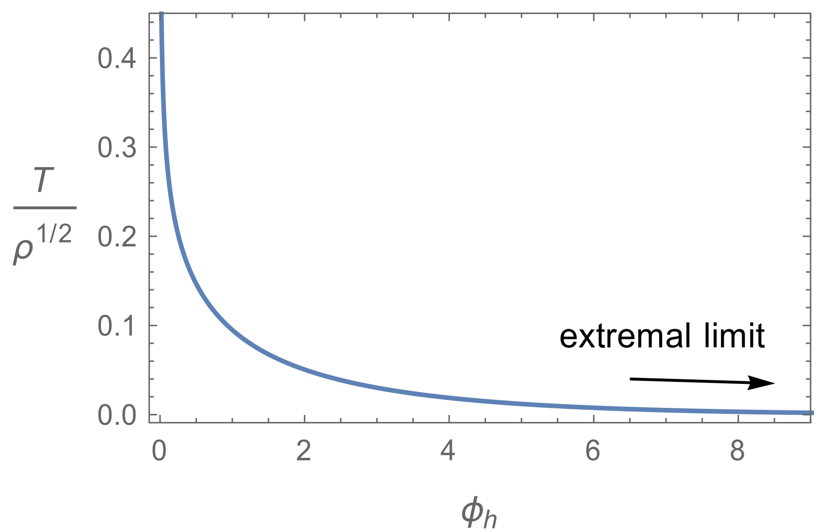
The finite temperature picture

• The gapped systems above are at finite density, and have therefore a zero-frequency δ-function in the AC conductivity: They are perfect conductors.

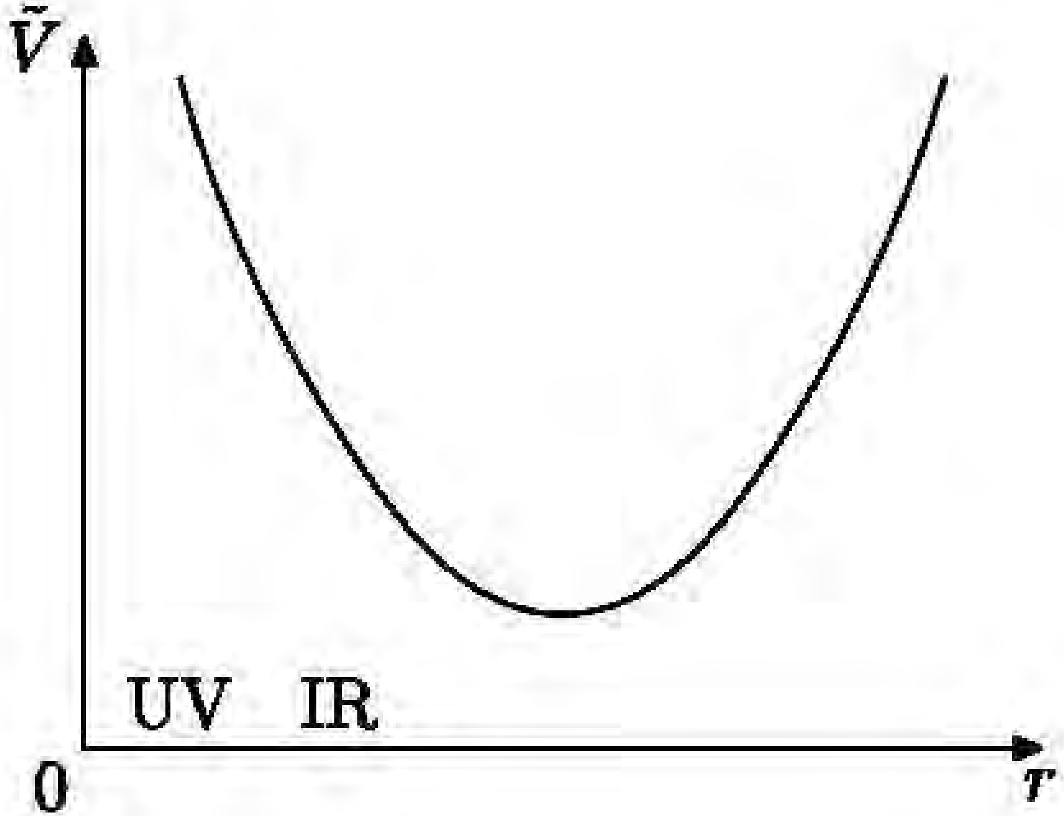
• Note also that we are at $T = 0$. In this respect as far as the δ-function is concerned they resemble real metals at $T = 0$.

• Up to $T = T_{min}$ this is the only saddle-point for the system. For $T > T_{min}$ there are also two black holes that are competing at the same temperature.





• At $T = T_c > T_{min}$ there is a first order phase transition to the large black hole phase that is a gapless plasma phase. This is very similar to the confinement-deconfinement phase transition in gauge theories.



• In gapped saddle points, the intuition that the interior geometry is controlling the IR properties of the theory is incorrect.

• The far-interior (naked singularity) is controlling the UV asymptotics of the spectrum.

Adding Momentum dissipation

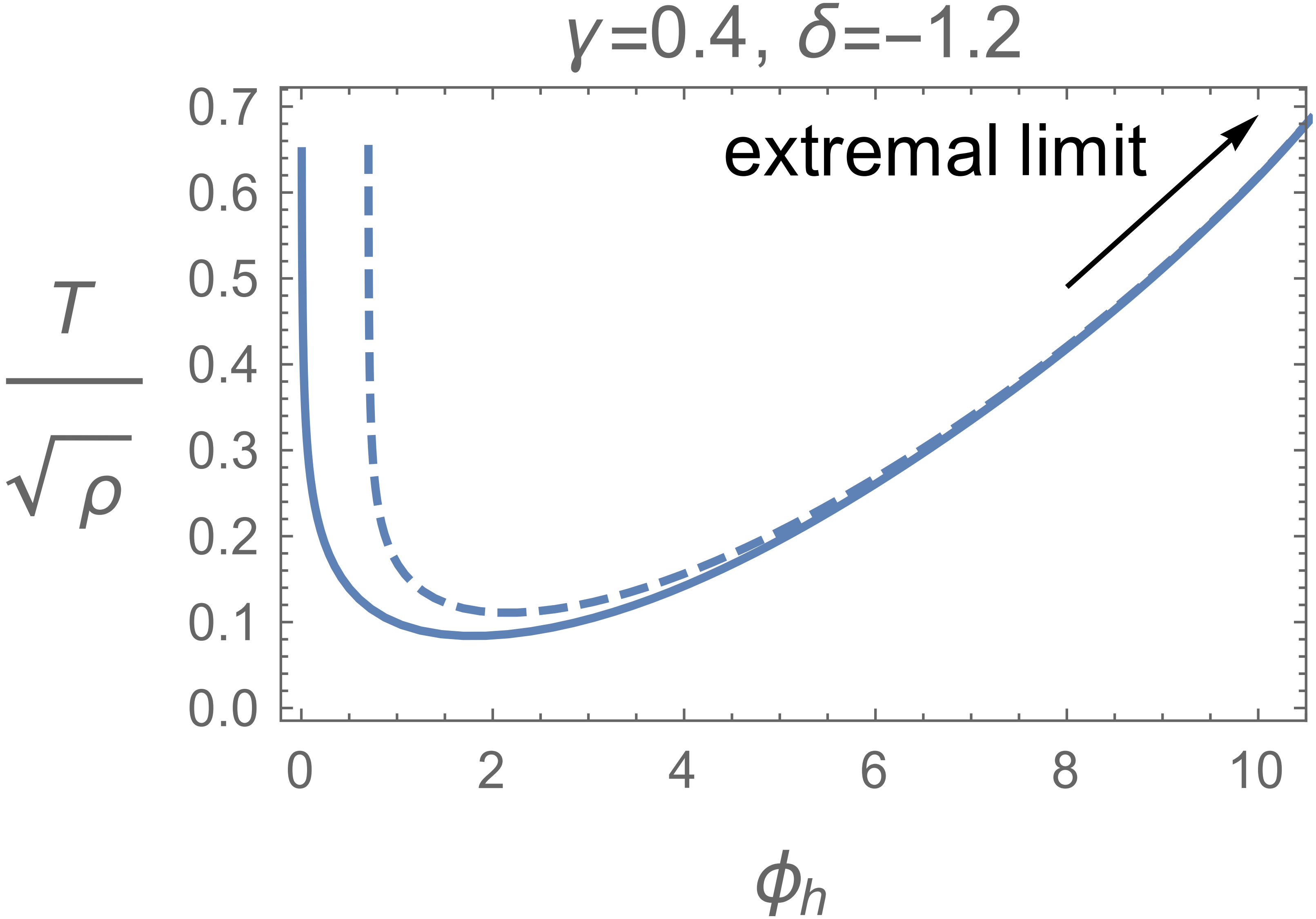
• We use axions (goldstone-bosons for broken translational invariance) as a source of momentum dissipation.

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\psi_i)^2\right],$$

$$V(\phi) \sim e^{-\delta\phi}, \qquad Z(\phi) \sim e^{\gamma\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}.$$

$$\psi_1 = kx \quad , \quad \psi_2 = k\ y$$

• We also choose λ in the region where the axions are "irrelevant" in the IR. This means that leading IR solution is unaffected by the axions.


γ=0.4, δ=−1.2
extremal limit
T/√ρ
φ_h
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
2
4
6
8
10

• Momentum dissipation could remove the zero-frequency δ-function but:

(a) We are in the T=0 geometry as long as $T < T_c$

(b) The axions/translation symmetry breaking are irrelevant in the deep IR.

• A detailed analysis of the Drude weight is needed.

• The fluctuation equations for the conductivity involve not only δA_i but also the axions.

• There is a direct formula for the DC conductivity in the presence of a regular (non-extremal) horizon.

$$\sigma_{\rm DC} = H(r = r_h) \equiv Z_h + \frac{q^2}{k^2 (g_{xx})_h Y_h} \equiv Z_1 + \frac{q^2}{k^2} Z_2$$

Gouteraux, Donos+Gauntlett

• There is also a direct formula for the Drude weight:

$$\Pi(\omega) = f H \lambda_1' - \frac{q}{k^2} f Z_2 \left(\frac{Z_1}{Z_2}\right)' \lambda_2, \quad , \quad \partial_r \Pi = \mathcal{O}(\omega^2)$$

$$\lambda_1 = \frac{Z_1}{H}\left(a_x - \frac{q}{k^2}\frac{Z_2}{Z_1} b_x\right), \qquad \lambda_2 = \frac{Z_2}{H}\left(q a_x + b_x\right).$$

• It can be shown that $\Pi(0)$ is the Drude weight. We can then establish that:

(a) when $\sigma_{DC} \to \infty$ at extremality then there is a zero-frequency δ-function, and this happens in the gapless geometries.

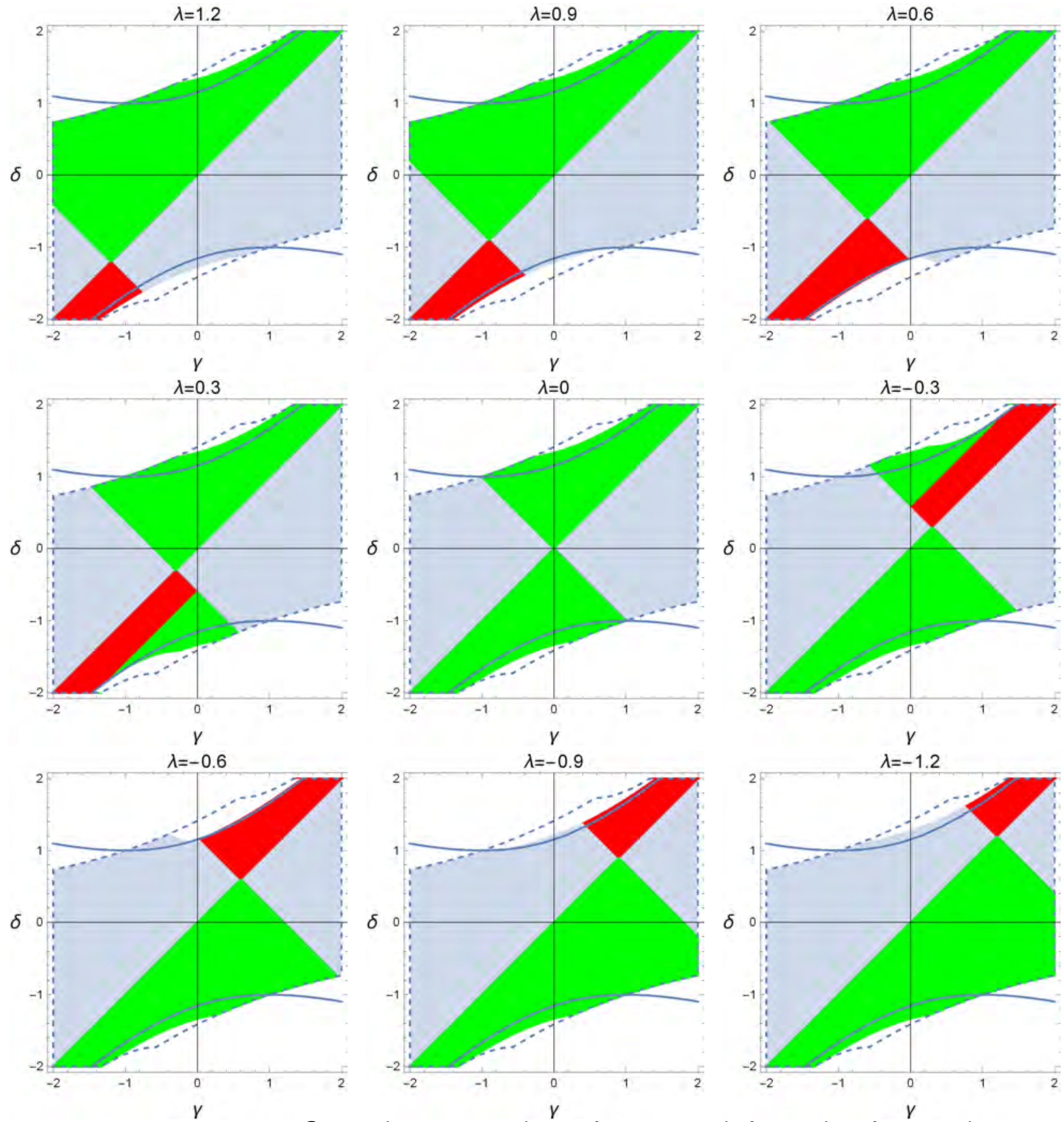
(b) when $\sigma_{DC} \to 0$ at extremality, then there is no zero-frequency δ-functions and this happens for the gapped geometries.

• We can also show in general that: The Gubser criterion, and irrelevance of axions in the IR = the conductivity of small near-extremal black-holes is dominated by the momentum dissipation term.

• It is crucial for all of the above, to exclude holographically unreliable cases.

• We have therefore found holographic systems with a charged spectrum that is gapped and discrete. The stress-energy correlators are also gapped.

• These are insulators that share properties of both band-gap insulators and Mott insulators. We believe this is a novel mechanism of insulating behavior.



Parameter space for the conducting and insulating phases.

An interesting borderline case

• The two-charge black hole (in 4d) is an analytic solution, obtained from the general STU four-charge BH.

Cvetic+Du+Hoxha+Liu+Lu+Lu+Martinez-Acosta+Pope (1999)

• It has effective EMD functions:

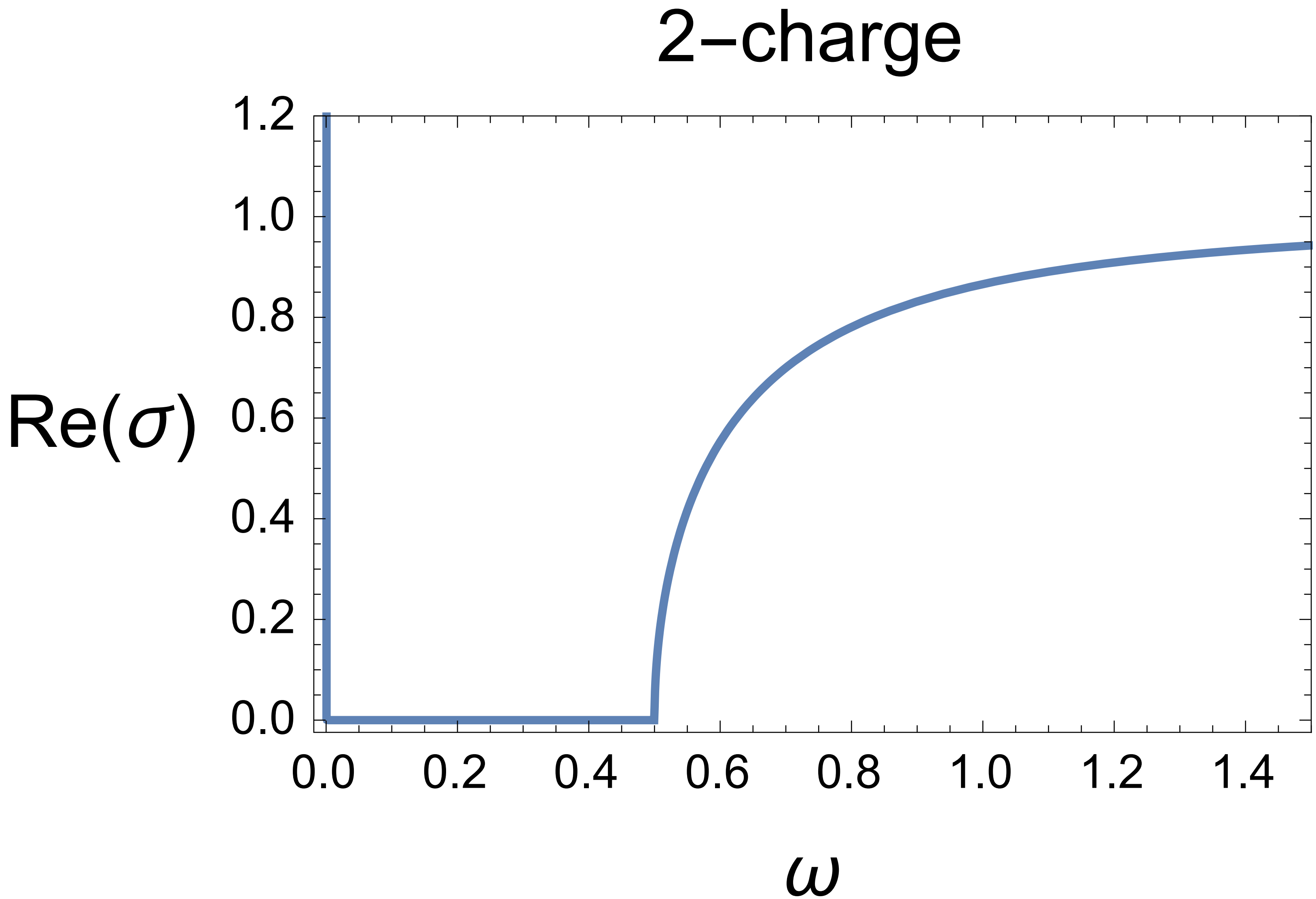
$$V(\phi) = 2(\cosh\phi + 2) \quad , \quad Z(\phi) = e^{\phi}$$

$(\gamma = 1, \delta = -1) \rightarrow (z = 1, \theta \rightarrow \infty)$.

• The $T = 0$ AC conductivity can be calculated analytically

$$\sigma(\omega) = i\frac{\left(Q - i\sqrt{4\omega^2 - Q^2}\right)}{2\omega}\Big|_{\omega\rightarrow\omega+i\epsilon}$$

• There is a non-zero Drude weight and a mass gap, with continuous spectrum above.



• The real part of the AC conductivity calculated from the 2-charge black hole in AdS_4 at extremality. There is a δ-function at $\omega = 0$.

• The solution has well defined IR boundary conditions above the gap only.

The AC conductivity in a holographic strange metal

Kim+Kiritsis+Panagopoulos

- The bulk is the AdS Schwarzschild black hole.

- The charge is described by a standard DBI action coupled to the metric

$$S_{DBI} = \mathcal{N} \int d^5x \sqrt{\text{det}(g+F)}$$

- The ansatz for the ground state is

$$A = (Ey + h_+(u))dx^+ + (b^2 Ey + h_-(u))dx^- + (b^2 Ex^- + h_y(u))dy,$$

- It is a stationary solution in lightcone coordinates with a nontrivial charge density and a light-cone electric field $F_{+y} = E$

- The DBI equations can be solved exactly and the conductivity computed à la Karch-O'Bannon.

The DC conductivity

- The parameters are E, J^+, T. E is similar to the doping parameter of cuprates.

- They can be combined in one scaling variable t and parameter J.

$$t = \frac{\pi \ell T}{2\sqrt{E}} \quad , \quad J^2 = \frac{J_+^2}{(2\mathcal{N})^2 \sqrt{2} (E)^3} \, ,$$

and the DC conductivity becomes

$$\sigma_{DC} = \sigma_0 \sqrt{\sigma_{DR}^2 + \sigma_{QC}^2} \, ,$$

$$\sigma_{DR}^2 = \frac{J^2}{t^2 A(t)} \quad , \quad A(t) = t^2 + \sqrt{1 + t^4} \quad , \quad \sigma_{QC}^2 = \frac{t^3}{\sqrt{A(t)}} \, .$$

- We will also define the ratio

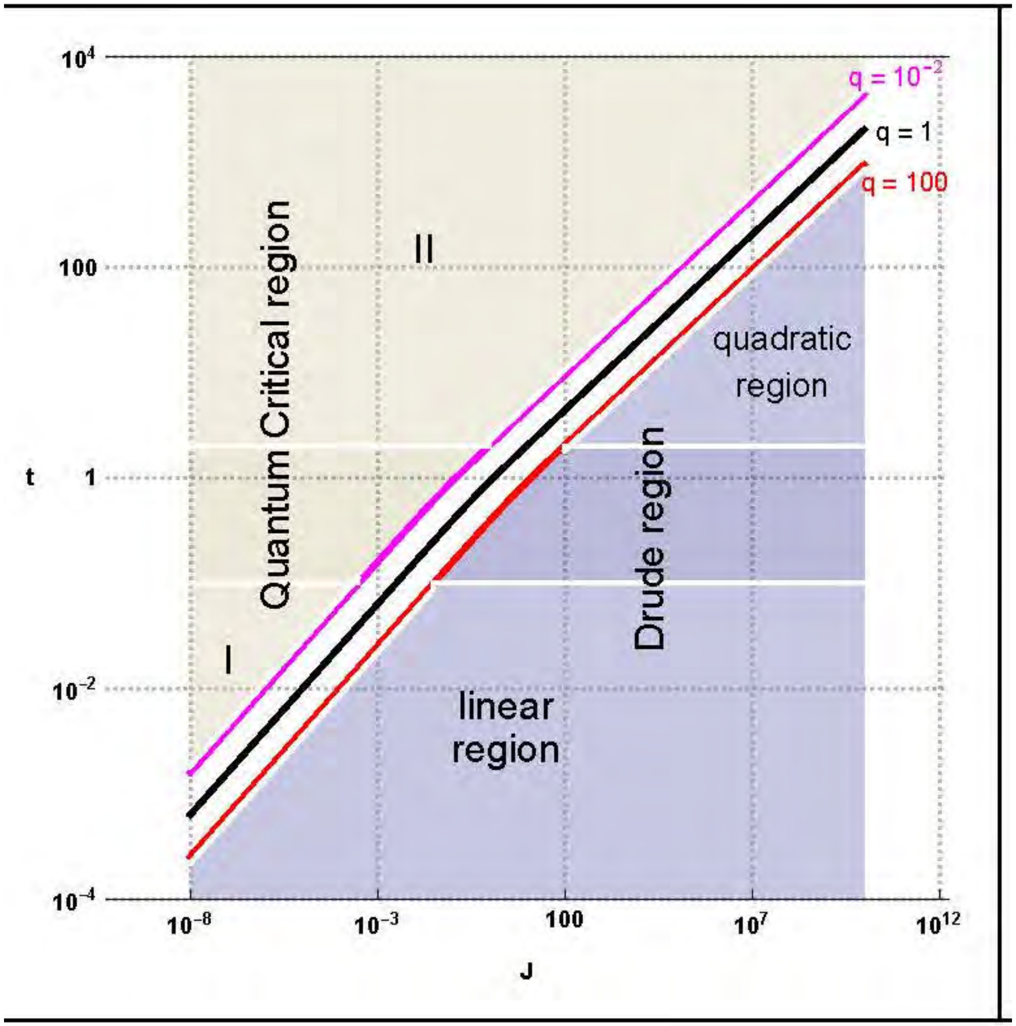
$$q \equiv \frac{\sigma_{DR}^2}{\sigma_{QC}^2}$$

that determines in which regime we are.

- The conductivity is as follows:

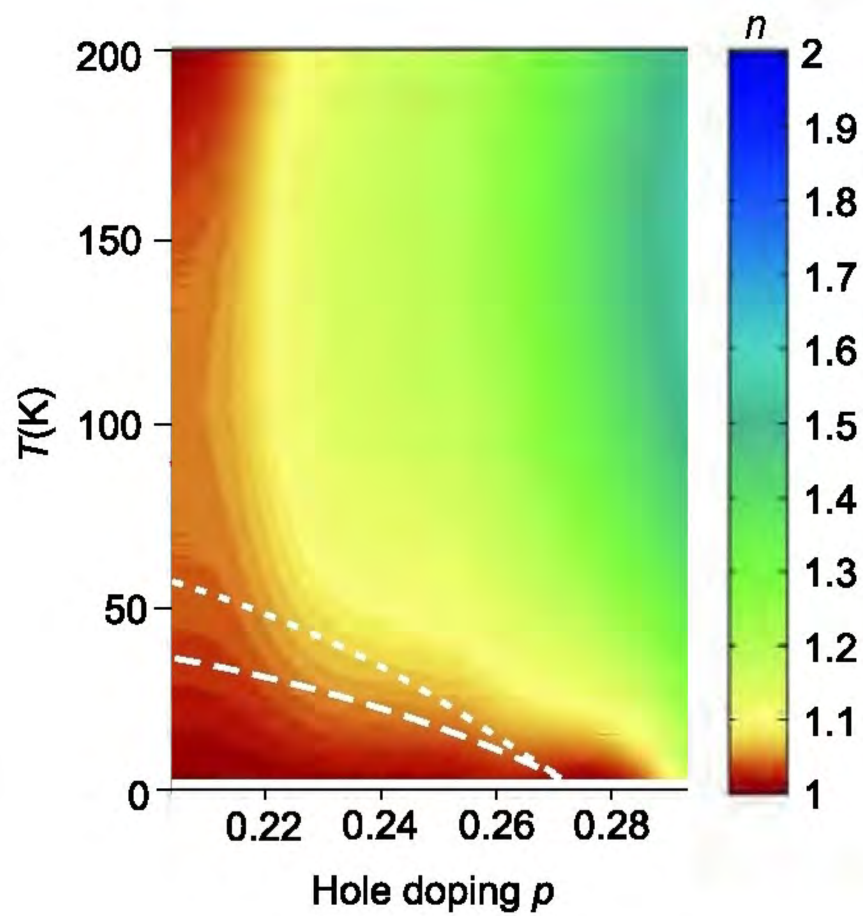
$$\rho = \begin{cases} (\sigma_0 J)^{-1}\, t & q \gg 1, t \ll 1 \quad \text{linear} \\ \sqrt{2}\sigma_0^{-1} J\, t^2 & q \gg 1, t \gg 1 \quad \text{quadratic} \end{cases} \Big| \text{Drude regime (DR)}$$

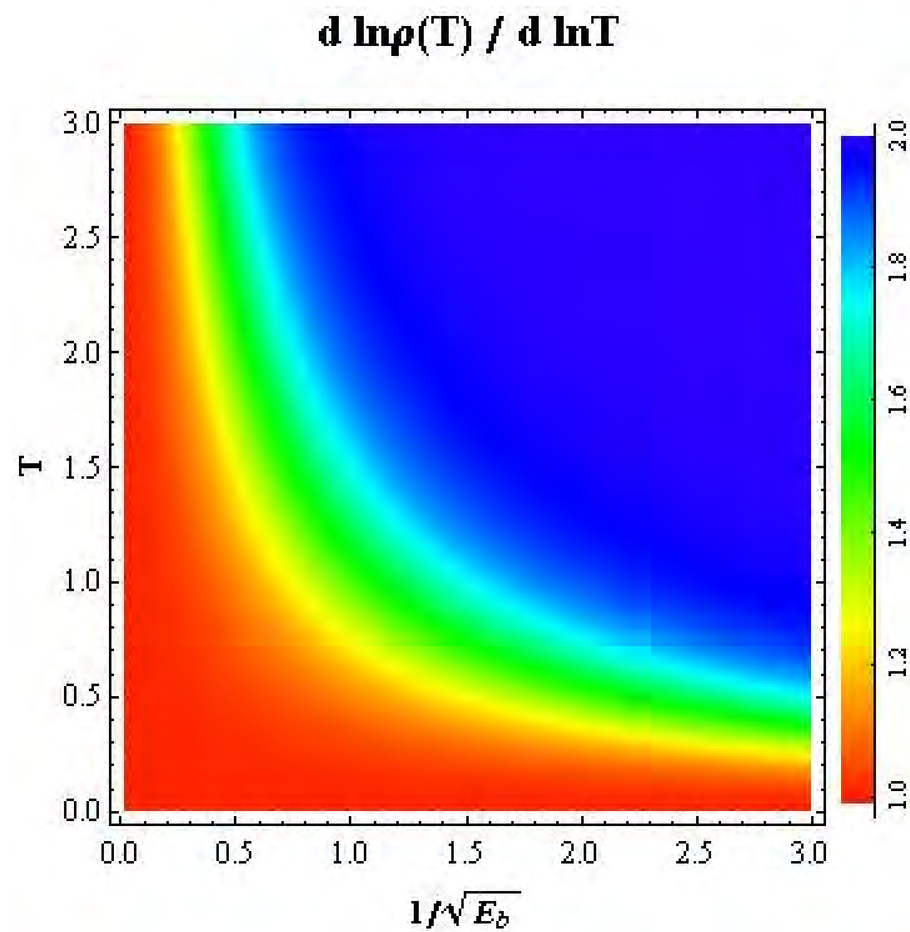
$$\rho = \begin{cases} \sigma_0^{-1}\, t^{-3/2} & q \ll 1, t \ll 1 \quad \text{regime I} \\ \sqrt{2}\sigma_0^{-1}\, t^{-1/2} & q \ll 1, t \gg 1 \quad \text{regime II} \end{cases} \Big| \text{pair production regime (QC)} \tag{1}$$



Center: Location of the four regimes in the space of parameters (J, t) log-log scale. The black line $q = 1$, separates the DR respect to the QC regime. The magenta line represents the region with $q = 10^{-2}$ and the red one $q = 100$.

$La_{2-x}Sr_xCuO_4$





Cooper et al. Science (2009)

Kim+Kiritsis+Panagopoulos

The AC conductivity

- From the AC conductivity equations one can compute the asymptotics

- The large ω behavior

$$\left(b^2 \ell \mathcal{N} T\right)^{-1} \sigma(\omega) = i \left(\frac{2\pi}{3}\right)^{7/3} \frac{2}{\Gamma\left(1/3\right) \Gamma\left(7/3\right)} \, \omega^{-\frac{1}{3}} \, e^{i\pi/6}$$

- The generalized relaxation time defined as

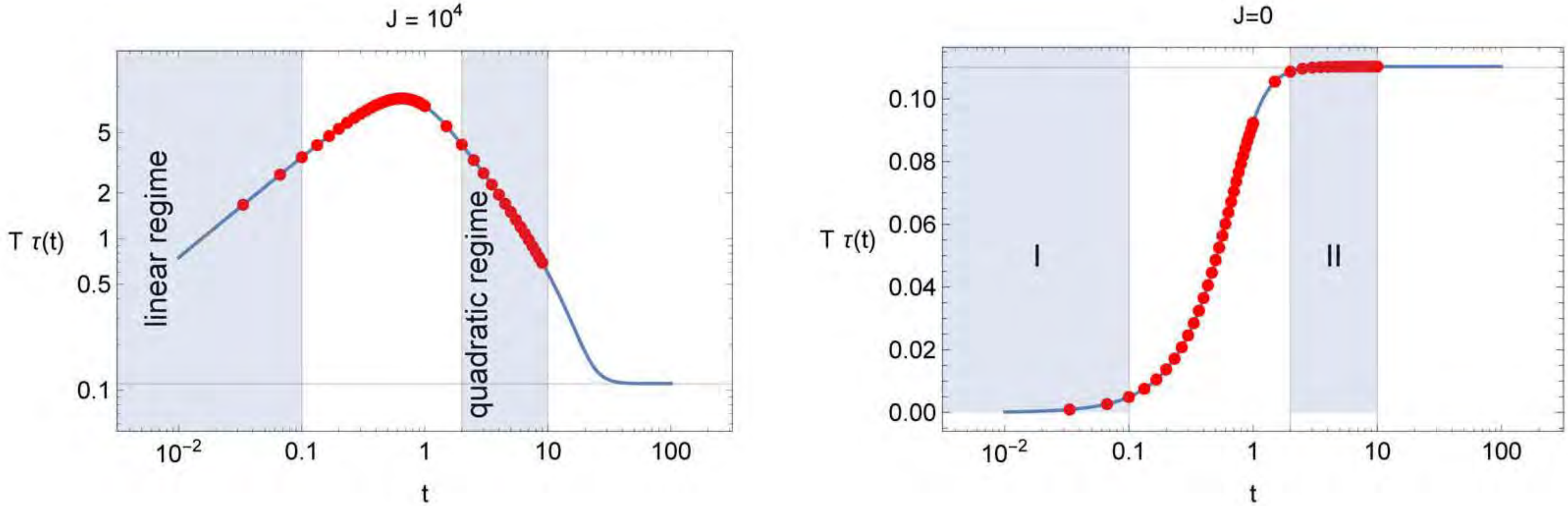
$$\sigma(\omega) \approx \sigma_{DC} \left(1 + i\tau\omega + \mathcal{O}(\omega^2)\right) \, ,$$

can be computed analytically but has a complicated formula.

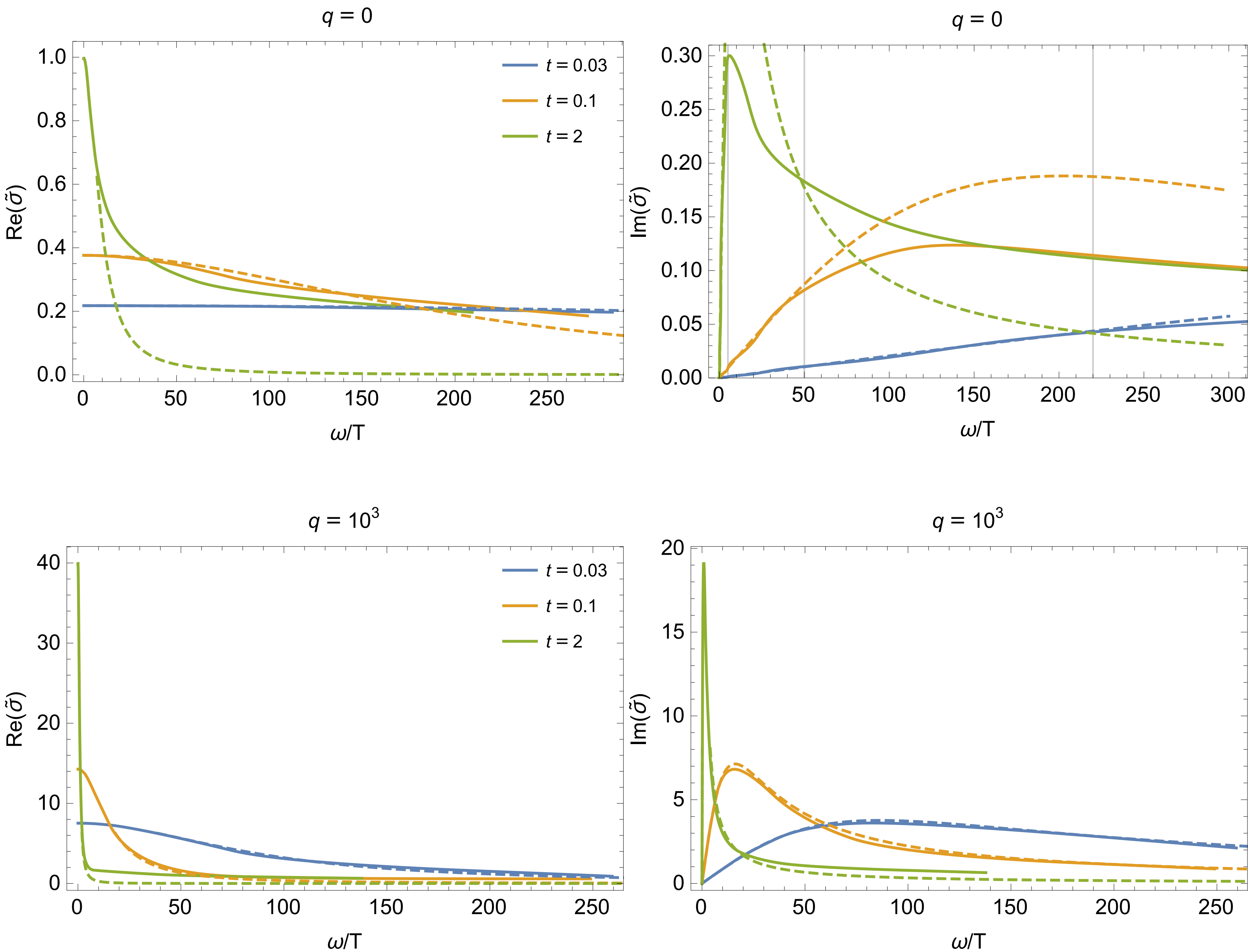
- There is another intermediate asymptotic scaling regime, in which

$$\sigma \sim \omega^{-\frac{1}{2}}$$

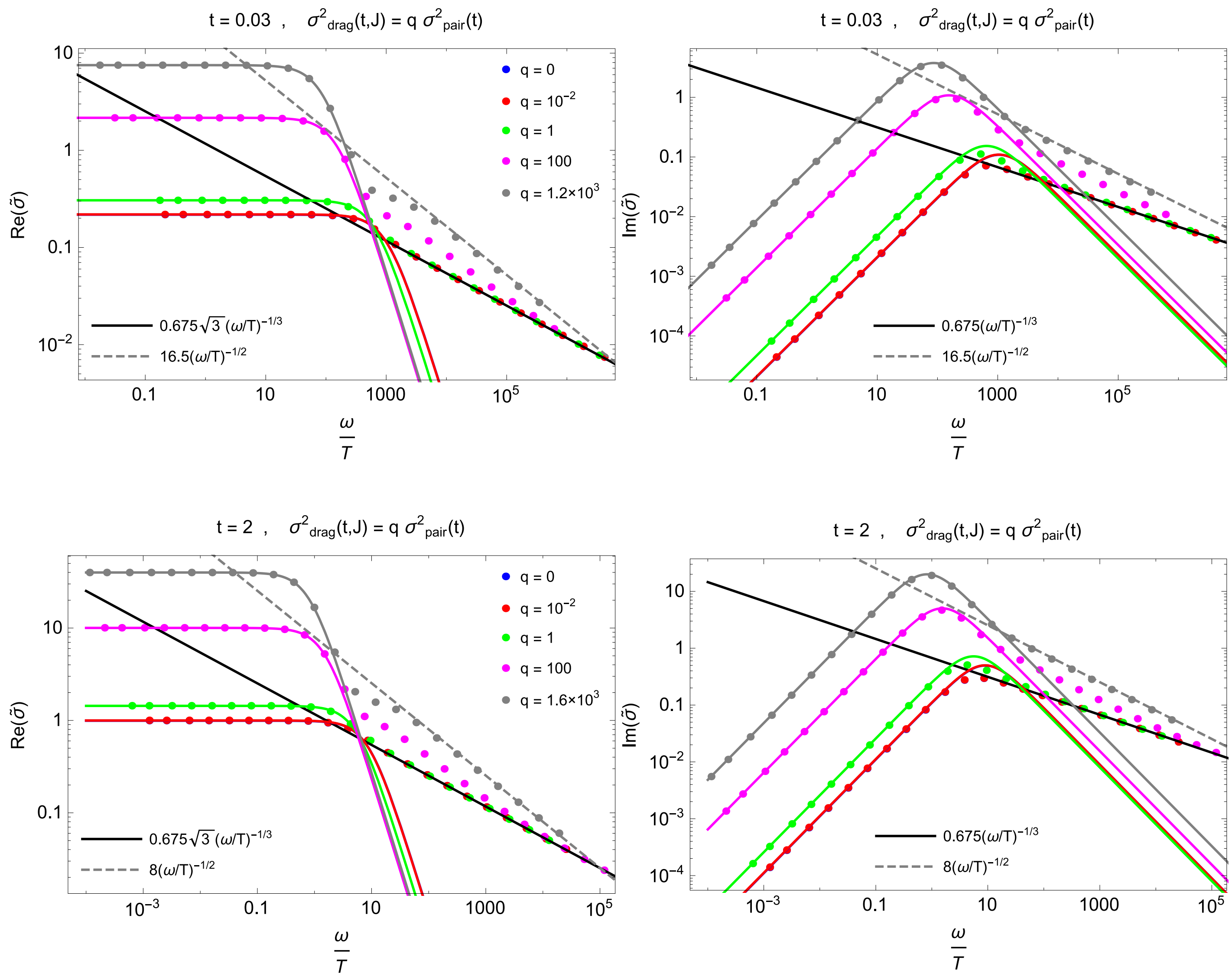
. It was found numerically, and it coincides with a feature in the effective Schrödinger potential



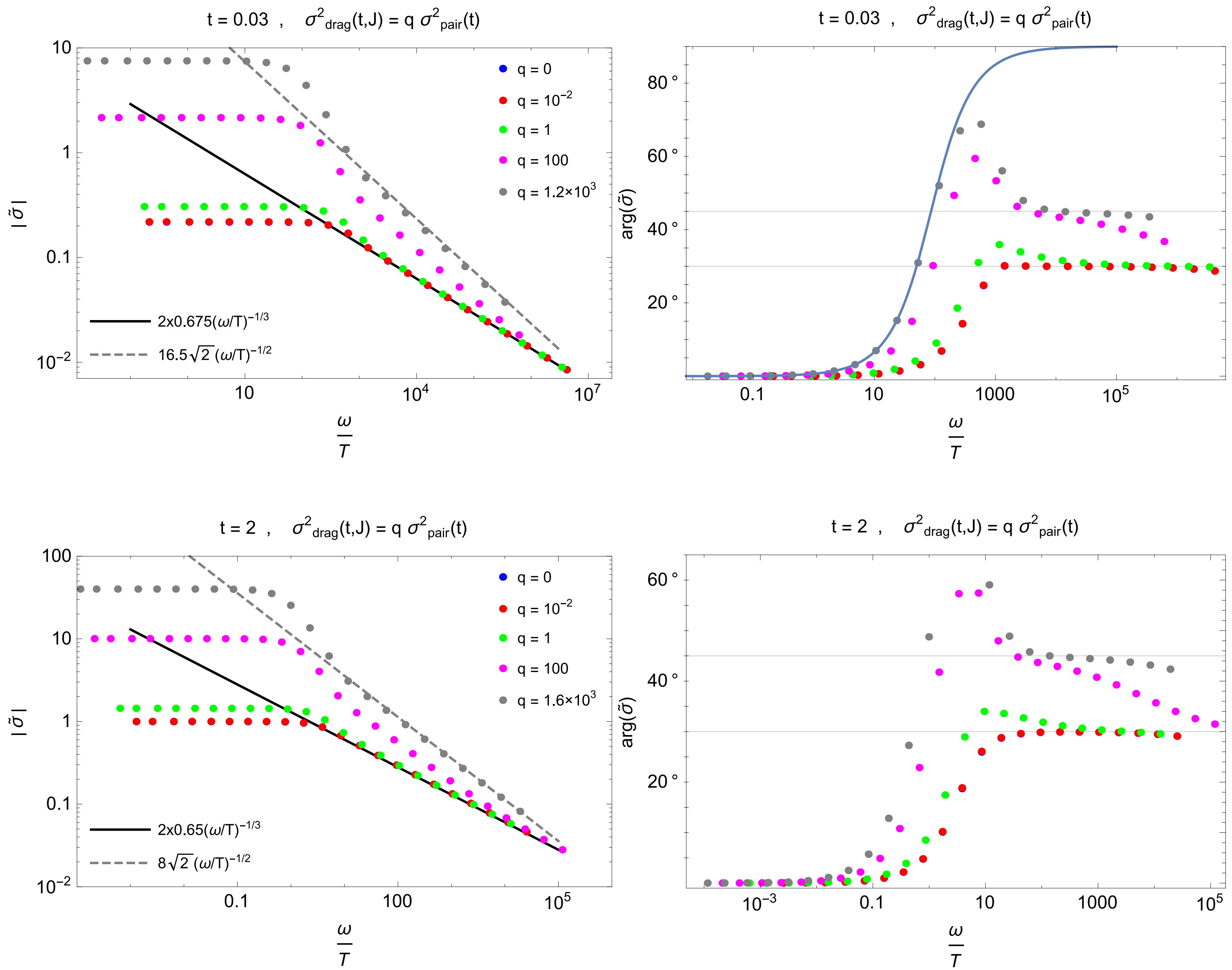
The generalized relaxation time τ as a function of the scaling temperature variable t.The left plot shows the behaviour of the dimensionless quantity $T\tau$ (T is the temperature) deep in the Drude regime (DR). The right plot shows the behavior in the Quantum Critical regime (QCR) at zero density. Dots show numerical data and the continuous line is the analytic formula obtained in the text using perturbation theory .



Conductivity for three different temperatures in the QC (up) and in the DR (down) regimes. The left figures show the real part of the conductivity while the right figures the imaginary part of the conductivity. Continues lines represent the numerical data, dashed lines correspond to a fit to the Drude peak formula.



The colored continuous lines show the Drude fitting using the analytic computation of the generalized relaxation time. Straight black and dashed gray lines show the UV and the intermediate power law behavior of the AC conductivity.



Absolute value (left) and argument (right) part of the conductivity. The blue continuous line shows the Drude fit using the analytic computation of the generalized relaxation time. Straight black and dashed gray lines show the UV and the intermediate power law behavior of the AC conductivity.

Lessons learned

• When the conductivity is dominated by the drag mechanism (momentum dissipation) there is a clear Drude peak.

• When the conductivity is dominated by the pair-production mechanism there is no Drude peak.

• There is an associated scaling tail in the conductivity (here $\sigma \sim \omega^{-\frac{1}{3}}$). This always survives beyond the Drude peak as it falls off slower than ω^{-1}.

• It is controlled by the pair-production contribution and is there even if $Q = 0$.

• It has the features seen in experiment by Van der Marel et al.:

a) Constant phase matching the power falloff.
b) Temperature independence.
c) It implies that the QC point is hyperscaling-violating.

• These properties seem to hold more generally and beyond the system under study.

General Scaling

Charmousis+Gouteraux+Kiritsis+Kim+Meyer
Gouteraux, Kiritsis+Pena-Benitez

Consider EMD solutions with general scaling exponents z, θ, ζ. (unbroken U(1)) and gapless charge excitations).

• The conductivity is obtained by solving for the fluctuations of the gauge field, $\delta A_i = a_i(r)e^{i\omega t}$.

$$\frac{1}{Z}\sqrt{\frac{g_{rr}}{g_{tt}}}\partial_r\left(Z\sqrt{\frac{g_{tt}}{g_{rr}}}a_i'\right) + \left[\frac{g_{rr}}{g_{tt}}\omega^2 - \frac{Q^2\ g_{rr}}{Zg_{xx}^2}\right]a_i = 0$$

• By a field and coordinate redefinitions it can be mapped into a Schrödinger problem

$$-\psi'' + V_{eff}\ \psi = \omega^2\psi \quad , \quad V_{eff}(x) = V_1(x) + Q^2V_2(x)$$

$$V_1(x) \sim \frac{1}{x^2} \quad , \quad V_2 \sim \frac{1}{x^{2a}} \quad , \quad x \to \infty$$

$$a \geq 1$$

• It is the coefficient of $1/x^2$ that controls the scaling power of the AC conductivity.

Goldstein+Kachru+Prakash+Trivedi (2010)

• The Charge density is "supporting" the IR geometry. Q^2 is fixed in terms of z, θ. Both terms in the potential contribute at the same order.

$$|\sigma| \sim \omega^m \quad , \quad Arg(\sigma) \simeq -\frac{\pi \; m}{2}$$

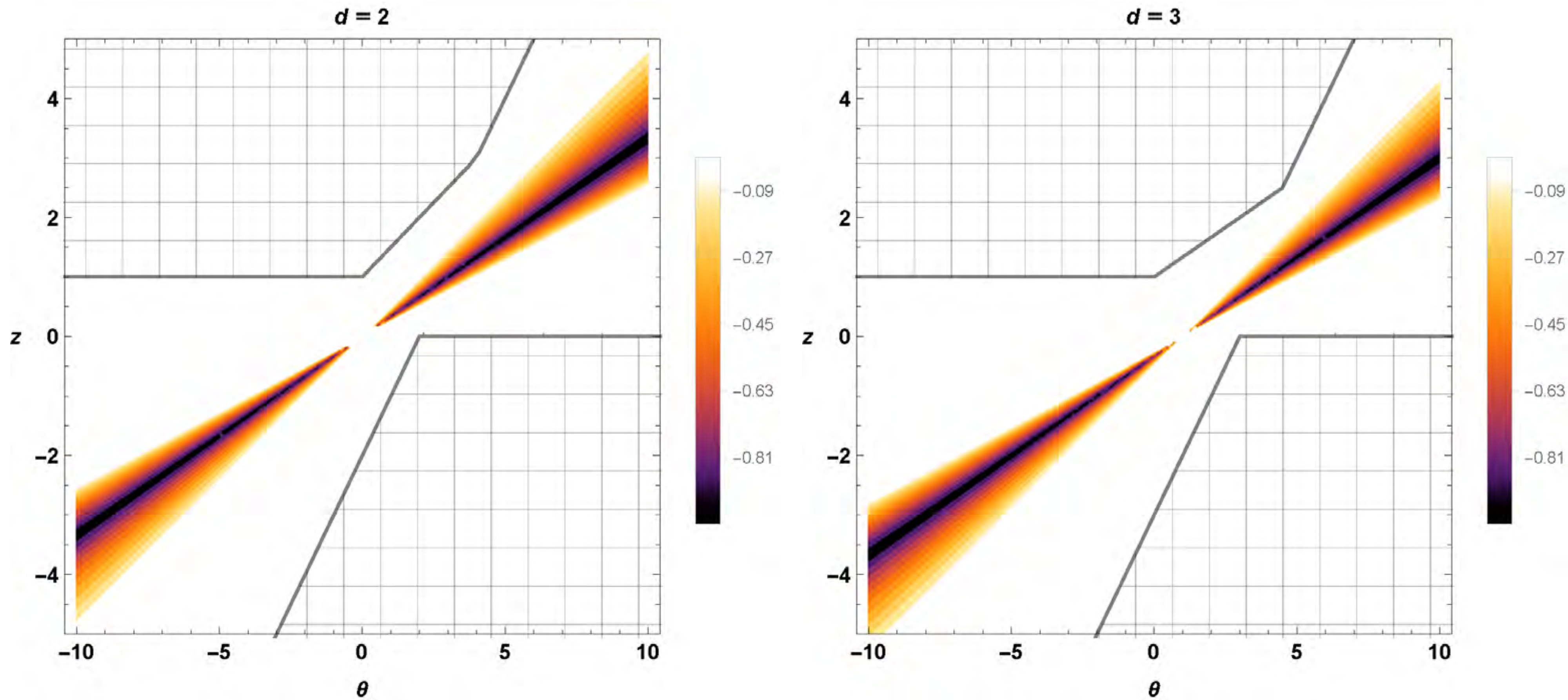
$$m = \frac{2(z-1) + d - \theta}{z}$$

• In his case m is always positive.

• The case where the IR geometry is AdS_2 can be obtained for $z \to \infty$ giving $m = 2$.

• For hyperscaling violating semilocal geometries, we must take, $\theta \to \infty$, $z \to \infty$ with $\frac{\theta}{z} = -\eta$ fixed.

$$m \;\; = \;\; 2 + \eta \;\; > \;\; 0$$



Contour plots to illustrate the region in the parameter space where the exponent m takes negative values for the single charged model. Left: Conductivity in the charged case for $d = 2$. Right: Conductivity in the charged regime for $d = 3$. The allowed values for the parameters are bounded by the gray mesh. The negative values for m are outside the permitted region.

• The Charge density is a probe in the IR geometry. The charge term is subleading or absent.

$$|\sigma| \sim \omega^m \quad , \quad Arg(\sigma) \simeq -\frac{\pi\ m}{2}$$

$$m = \left|\frac{z + \zeta - 2}{z}\right| - 1\,,$$

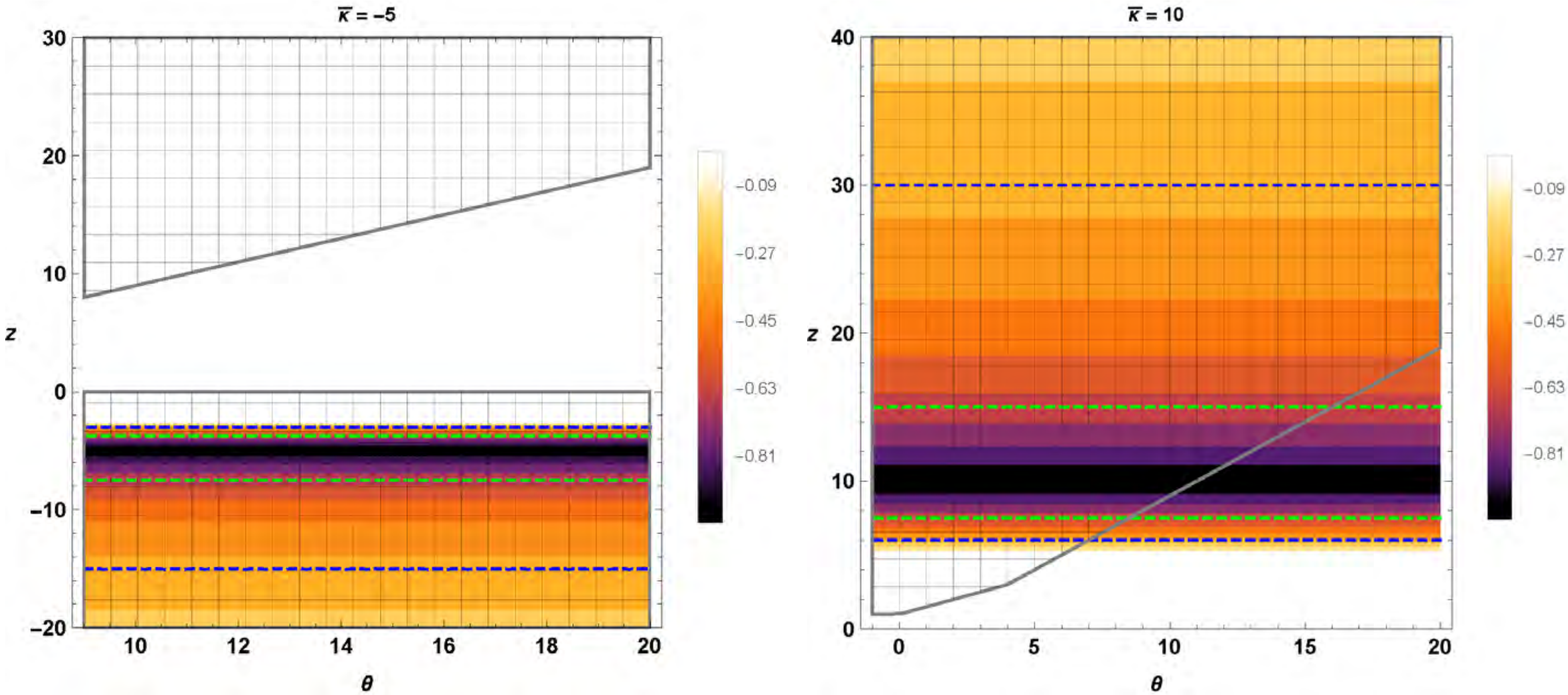
• It allows for negative values in m but always $m \geq -1$ (unitarity bound)

• For an AdS_2 IR geometry the exponent can be obtained by an $z \to \infty$ giving $m = 0$.

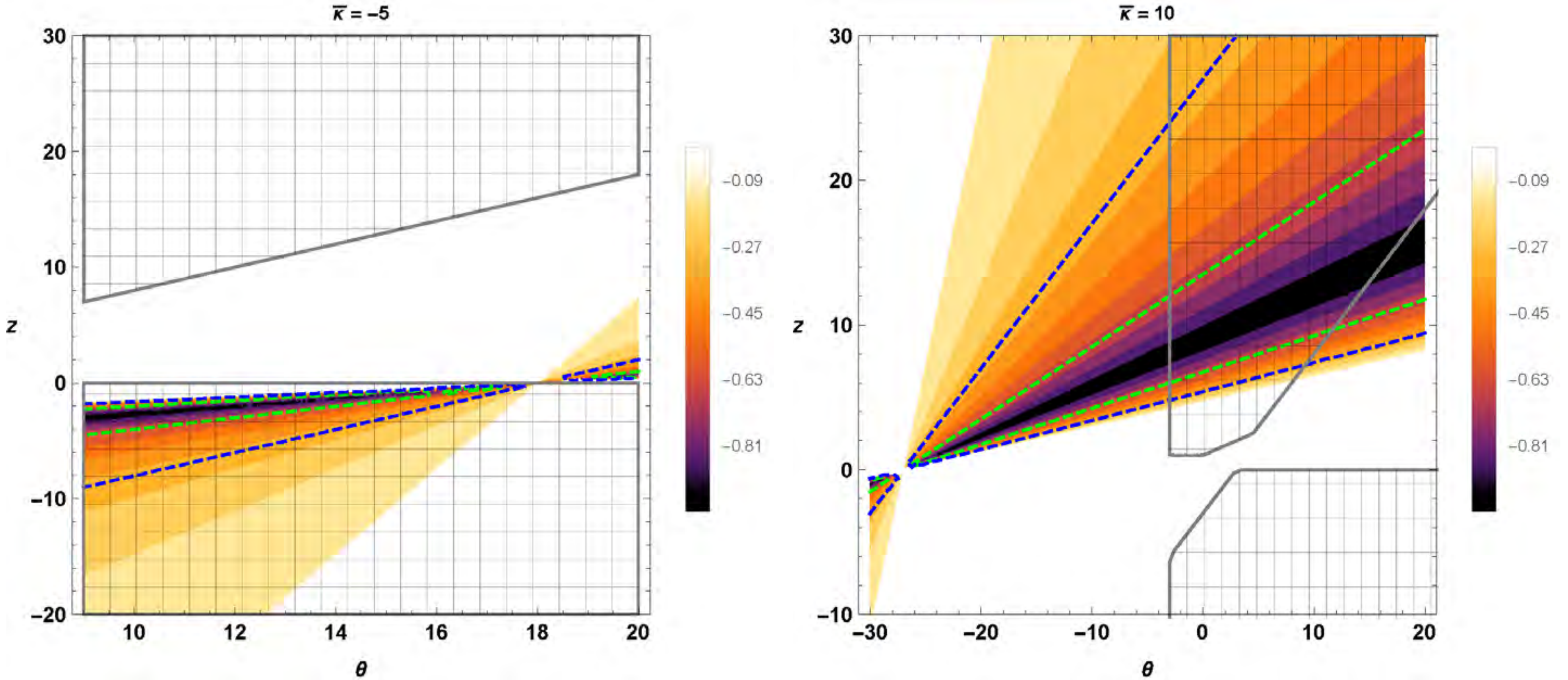
• For hyperscaling violating semilocal geometries we must take $\theta \to \infty$, $z \to \infty$ with $\frac{\theta}{z} = -\eta$ fixed and obtain

$$m \;\; = \;\; \frac{d-2}{d}\,\eta \quad > 0$$

• Finally, for the gauge field conformal case we obtain $m = 0$ when $d = 2$.



Contour plots to illustrate the region in the parameter space where the exponent m takes negative values for the probe charge density case. The plots above correspond to $d = 2$. Left: m for $\bar{\kappa} = -5$. Right: m for $\bar{\kappa} = 10$. The allowed values for the parameters are bounded inside the gray mesh. Green dashed lines are the contour levels where $m = -2/3$ and blue dashed lines $m = -1/3$.



Contour plots to illustrate the region in the parameter space where the exponent m takes negative values for the probe charge density case. The plots above correspond to $d = 3$ Left: m for $\bar{\kappa} = -5$. Right: m for $\bar{\kappa} = 10$. The allowed values for the parameters are bounded inside the gray mesh. Green dashed lines are the contour levels where $m = -2/3$ and blue dashed lines $m = -1/3$.

Outlook

- Holography provides a rich landscape of possibilities for conductivity.

- It realizes novel types of insulators and supersolid-like. behavior.

- More work is needed to formulate the general theory of conductivity in holography. We have just scratched the surface.

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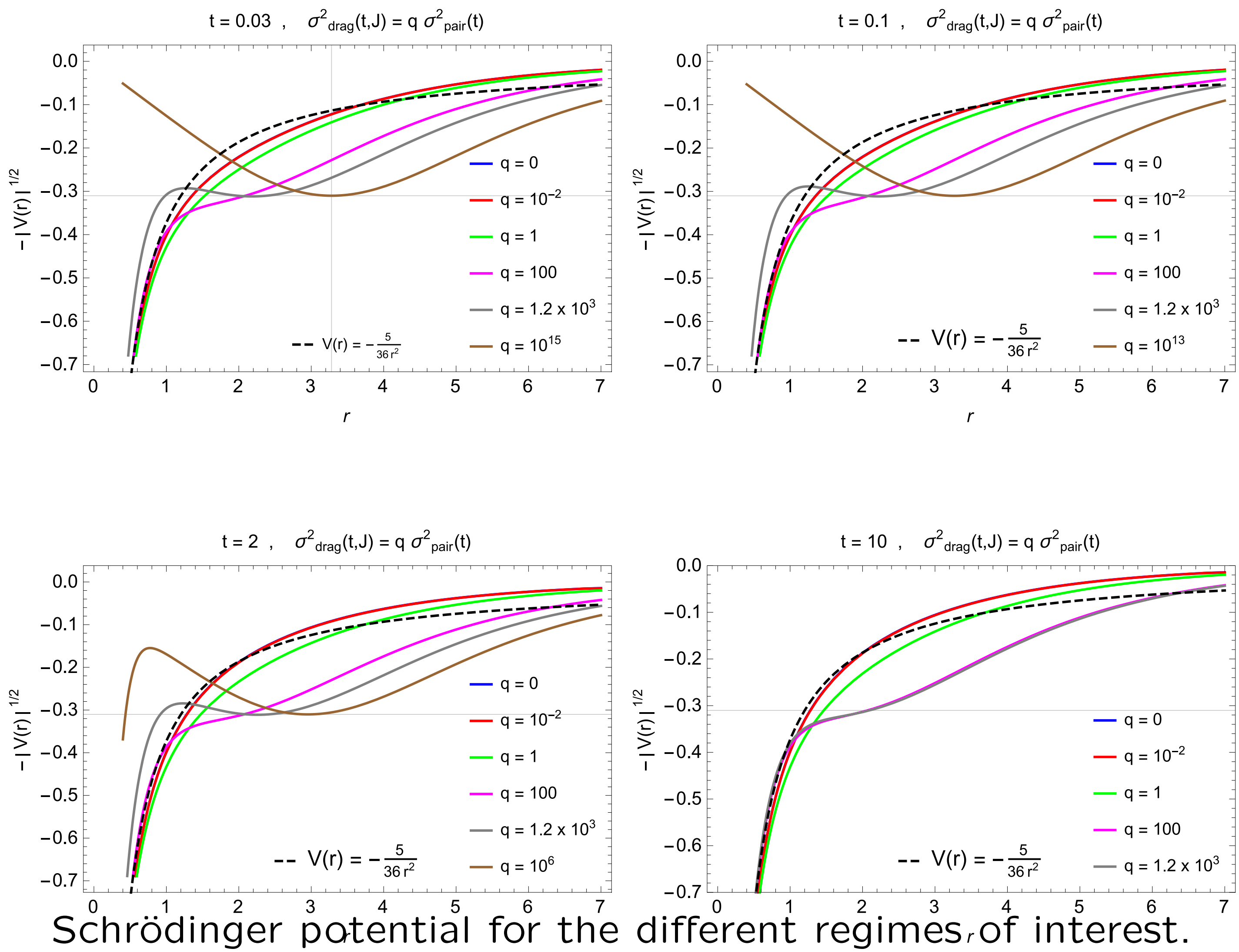
THANK YOU

Effective Schrödinger potentials



Schrödinger potential for the different regimes of interest.

RETURN

Overview of the solutions

- The gravitational action:

$$S = M^3 \int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z_1(\phi)}{4}F_1^2 - \frac{Z_2(\phi)}{4}F_2^2\right]$$

$$\text{with} \qquad F^i_{\mu\nu} \equiv \partial_\mu A^i_\nu - \partial_\nu A^i_\mu$$

and

$$V(\phi) \to_{IR} \sim V_0\, e^{-\delta\phi}, \quad Z_1(\phi) \underset{IR}{\sim} Z_{10}\, e^{\gamma_1\phi}, \quad Z_2(\phi) \underset{IR}{\sim} Z_{20}\, e^{\gamma_2\phi}.$$

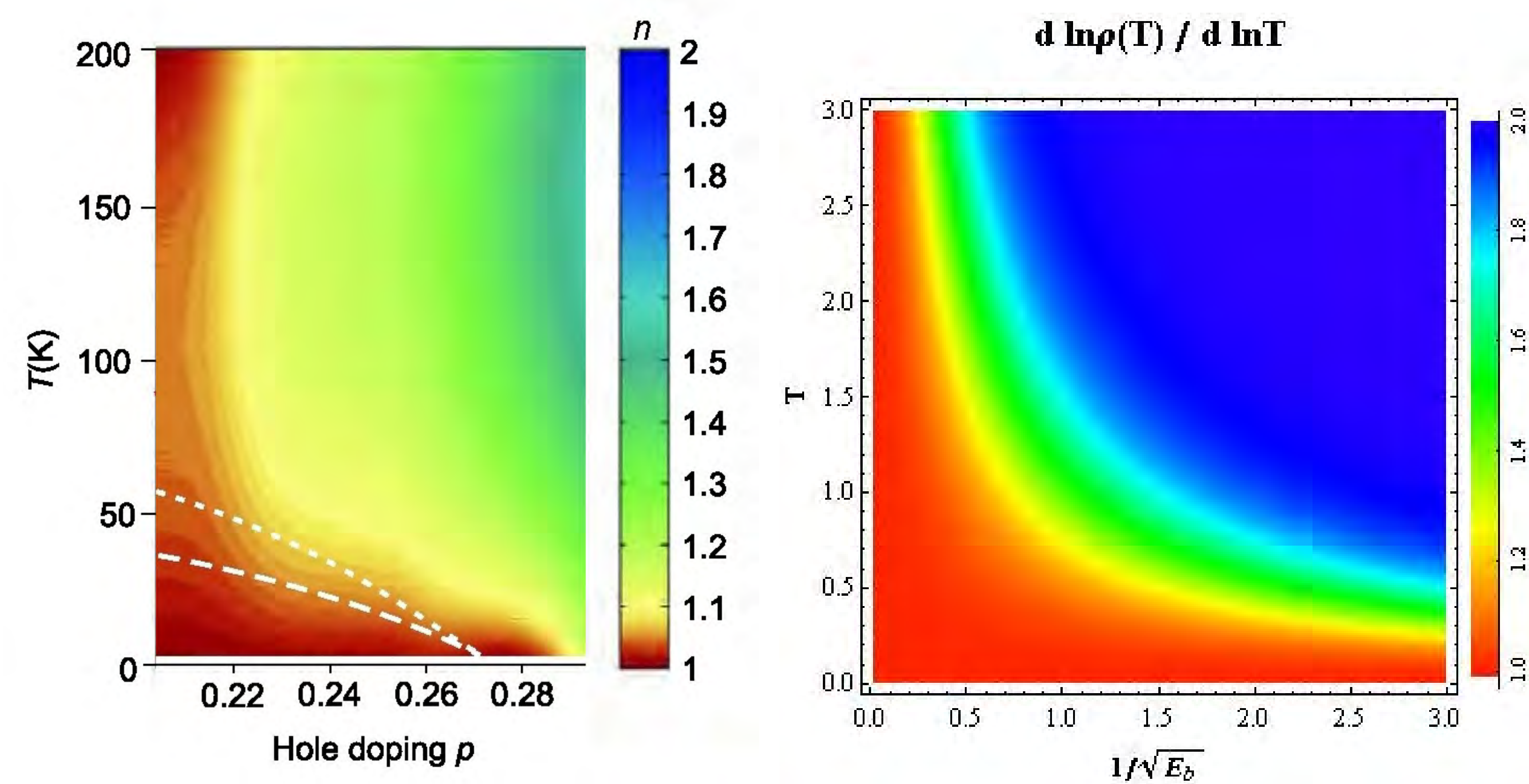
- The metric ansatz has helical symmetry and we parametrize the scaling solutions as :

$$ds^2 = r^{\frac{2\theta}{3}}\left[-\frac{\mathrm{d}t^2}{z^{2z}} + \frac{L^2\mathrm{d}r^2 + {\omega_1}^2}{r^2} + \frac{1}{r^{2z_2}}\left({\omega_2}^2 + \frac{\lambda}{r^2}{\omega_3}^2\right)\right] \quad , \quad A_1 \sim r^{\zeta - z}$$

where

$$\omega_1 = \mathrm{d}x_1\,, \omega_2 = \cos(kx_1)\mathrm{d}x_2 + \sin(kx_1)\mathrm{d}x_3\,, \quad \omega_3 = \sin(kx_1)\mathrm{d}x_2 - \cos(kx_1)\mathrm{d}x_3$$

Critical lines vs critical points in Cuprates



Kim+Kiritsis+Panagopoulos

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$.

R.A. Copper et. al. 2009

Breaking Translation Invariance

• Charge lattices lead to non-linear (gravitational) PDEs that are in general very difficult to solve.

Horowitz+Santos+Tong (2012), Lin+Liu+Wu+Xiang (2013), Donos+Kiritsis (2013)

• Some other string mechanisms of translation invariance breaking (momentum dissipation) are simpler to analyze:

♠ An effective "phenomenological" treatment of momentum dissipation using massive gravity.

Vegh (2013), Davison (2013), Blake+Tong (2013), Davison+Schalm+Zannen (2013)

♠ Interactions of charge with a bulk sector that carries most of the energy (DBI probe approximation)

Karch+O'Bannon (2007), Charmousis+Gouteraux+Kiritsis+Kim+Meyer (2010)

♠ Interactions with string axions that model a kind of homogeneous disorder.

Andrade+Withers (2013), Gouteraux (2013), Donos+Gauntlett (2013)

♠ The use of random field disorder in holography.

Hartnoll+Herzog (2008), Davison+Schalm+Zaanen (2013), Lucas+Sachdev+Schalm (2014)

♠ Study of saddle points with helical symmetry.

Kachru et al. (2012), Donos+Gauntlett (2012), Donos+Hartnoll (2012)

DC conductivity

• The general structure of the holographic DC conductivity at strong coupling:

$$\sigma_{\mathrm{DC}} = \sqrt{(\sigma_{DC}^{pp})^2 + (\sigma_{DC}^{drag})^2} \quad , \quad \sigma_{\mathrm{DC}} = \sigma_{DC}^{pp} + \sigma_{DC}^{drag}$$

• σ^{pp} is the pair-creation contribution: it exists also when $Q = 0$.

• σ^{drag} is the "drag" contribution. It originates from the force and dissipation a fractionalized charge (a string) feels as it is moving in the strongly coupled medium.

Karch+O'Bannon, (2007)

Ground-states with helical symmetry

Work with A. Donos and B. Gouteraux, to appear

• The gravitational action:

$$S = M^3 \int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z_1(\phi)}{4}F_1^2 - \frac{Z_2(\phi)}{4}F_2^2\right]$$

with $\quad F^i_{\mu\nu} \equiv \partial_\mu A^i_\nu - \partial_\nu A^i_\mu$

and

$$V(\phi) \to_{IR} \sim V_0\, e^{-\delta\phi}, \quad Z_1(\phi) \underset{IR}{\sim} Z_{10}\, e^{\gamma_1\phi}, \quad Z_2(\phi) \underset{IR}{\sim} Z_{20}\, e^{\gamma_2\phi}.$$

• The metric ansatz has helical symmetry and we parametrize the scaling solutions as :

$$ds^2 = r^{\frac{2\theta}{3}}\left[-\frac{dt^2}{z^{2z}} + \frac{L^2 dr^2 + \omega_1{}^2}{r^2} + \frac{1}{r^{2z_2}}\left(\omega_2{}^2 + \frac{\lambda}{r^2}\omega_3{}^2\right)\right] \quad , \quad A_1 \sim r^{\zeta - z}$$

where

$$\omega_1 = dx_1\,, \omega_2 = \cos(kx_1)dx_2 + \sin(kx_1)dx_3\,, \quad \omega_3 = \sin(kx_1)dx_2 - \cos(kx_1)dx_3$$

Anisotropic saddle points

(a) IR marginal current ($\zeta = \theta - 2 - 2z_2$)

$$S_a \sim T^{\frac{2z_2+2-\theta}{z}}$$

vanishes at $T \to 0$ • For the DC conductivity

$$\sigma^a_{DC} \sim T^{\frac{\zeta-2}{z}}$$

• σ_{DC} along the helical axis vanishes always at $T = 0$ (power insulators)

• For the IR limit of AC conductivity we find

$$\sigma^a_{AC}(T=0) \sim \omega^{\frac{\theta-2z_2-4}{z}}$$

• This is the same power as the DC conductivity and $\sigma^a_{AC}(T=0) \to 0$ as $\omega \to 0$.

• (b) IR Irrelevant current ($z = \frac{3}{2} z_2$).

$$S_b \sim T^{\frac{(2z_2+2-\theta)}{z}} \quad ,$$

• For the DC conductivity

$$\sigma^b_{DC} \sim T^{\frac{\zeta-2}{z}}$$

• It is sometimes diverging (metal) and sometimes vanishing (insulator).

• σ_{DC} comes always for the pair creation term. Therefore in the metallic case we expect no Drude peak (incoherent metals) .

• For the AC conductivity at zero T:

$$\sigma^a_{AC}(T=0) \sim \omega^{\min\{n_1,n_2\}}$$

$$n_1 = -1 + \left|\frac{-4+2\zeta+3z_2}{3z_2}\right| \quad , \quad n_2 = -1 + \frac{1}{3}\sqrt{\frac{4\theta^2+96z_2-76\theta z_2+217z_2^2}{z_2^2}}$$

• Whenever the system is an insulator, n_1 dominates and the the AC and DC exponents are equal.

• When n_2 dominates, the system is metallic with a decaying low-frequency AC tail. A $\delta(\omega)$ may bridge this behavior.

• When the system is metallic and n_1 dominates, it can have:

1. Diverging AC frequency power tail and match the DC scaling

2. Diverging AC frequency power tail without matching the DC scaling

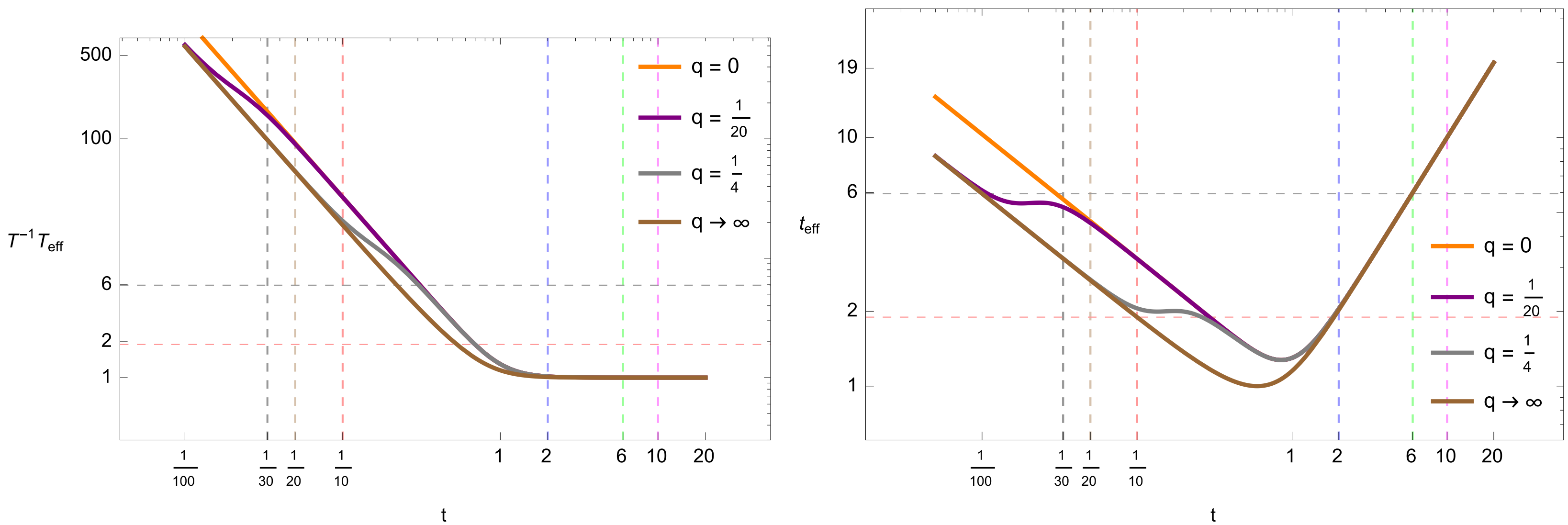
3. Decaying AC frequency power tail.

• Whenever $\sigma_{AC} \sim \omega^0$ then $\sigma_{DC} \sim T^0 \quad or \quad \sigma_{DC} \sim T^{-2}$

The effective temperature

The effective temperature on the brane is not t for non-trivial E:

$$t_{eff} = t\sqrt{\frac{J^2 A(t)^2\sqrt{A(t)-2t^2} + t^5\left(t^2A(t)+2t^4+3\right)}{2\sqrt{2}t^3\left(t^5\sqrt{A(t)}+J^2\right)}}$$



Dependence of the world-volume temperature T_{eff} with the parameters J and t, the effective temperature is always bigger than the background temperature T.

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 2 minutes
- Strongly Correlated Electrons 6 minutes
- Insulators 8 minutes
- Supersolids 10 minutes
- The plan for the rest 11 minutes
- The Wilsonian Program 12 minutes
- Generic Scaling Geometries: A classification of all QC points in holography 14 minutes

- Conductivity 18 minutes
- General features of holographic conductivity 20 minutes
- Holographic systems at finite density 22 minutes
- The conductivity 27 minutes
- The finite temperature picture 30 minutes
- Adding Momentum dissipation 38 minutes
- An interesting borderline case 40 minutes
- The AC conductivity in a holographic strange metal 42 minutes
- The DC conductivity 46 minutes
- The AC conductivity 55 minutes
- General Scaling 61 minutes
- Outlook 62 minutes