Metal-Insulator Transition by Holography

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References:

- Building a doped Mott system by holography.
  Y. Ling, P. Liu, C. Niu, J. Wu, **To appear**.

- Holographic entanglement entropy close to quantum phase transitions
  Y. Ling, P. Liu, C. Niu, J. Wu, Z. Xian, **arXiv:1502.03661.**

- Metal-insulator transition by holographic charge density waves.
  Y. Ling, C. Niu J. Wu, Z. Xian, H. Zhang, **Phys.Rev.Lett.** 113 (2014) 091602.

Outlines

PART I Brief introduction

- Metal-insulator transition by holography

PART II MIT as a thermal phase transition

- Holographic charge density waves

PART III MIT as a quantum critical phenomenon

- Q-lattice and novel MIT
- HEE close to quantum phase transitions
- Towards a holographic Mott-like insulator

Part I

Brief introduction: Metal-insulator transition by holography

Metal-Insulator Transition (MIT) by Holography

- Classification of Insulators:

Insulators

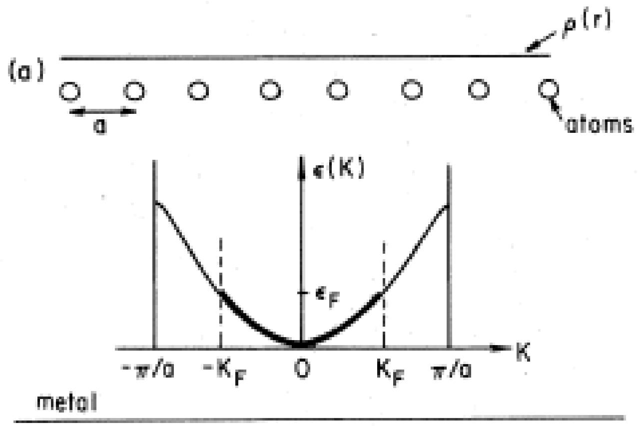
- **I. Electron-ion interaction**
  - Band Insulator
  - Peierls Insulator
  - Anderson Insulator
- **II. Electron-electron interaction**
  - Mott Insulator
    - Mott-Heisenberg Insulator
    - Mott-Hubbard Insulator

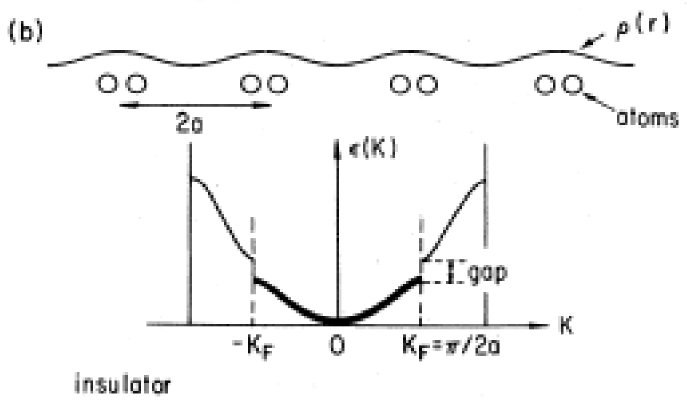
Metal-Insulator Transition (MIT) by Holography

- MIT as a thermal phase transition:

Charge density waves (CDW):

$$\rho(x) = \rho_0 + \rho_1 \cos(2k_F x + \varphi)$$





$$\left.\begin{aligned} N &= \frac{L}{a} \\ N &= 2\frac{L \bullet 2k_F}{2\pi} \end{aligned}\right\} \Rightarrow k_F = \frac{N\pi}{L2} = \frac{\pi}{2a}$$

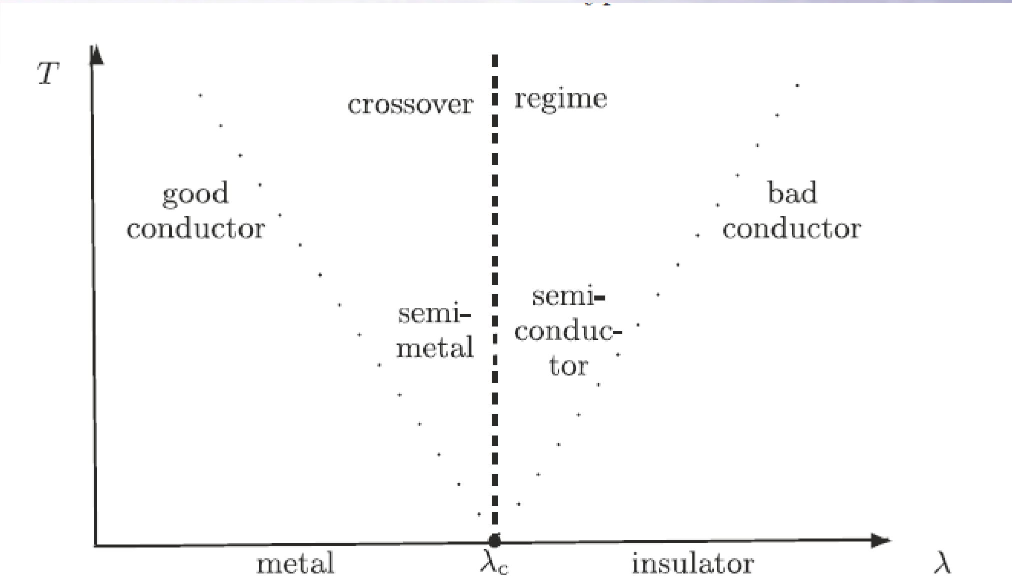
Peierls phase transition

$$\vec{F} = \vec{E}q = \frac{d\vec{p}}{dt} = \frac{d\vec{k}}{dt}$$

Gruner, Rev.Mod.Phys.Vol.60(1988),No.4

Metal-Insulator Transition (MIT) by Holography

- MIT as a quantum critical phenomenon:



Gebhard, The Mott metal-insulator transition, Springer press,1997.

- Mott-Hubbard model:

$$H_{BH} = -J \sum_{\langle i,j \rangle} b_i^{\dagger} b_j + \frac{U}{2} \sum_i b_i^{\dagger} b_i^{\dagger} b_i b_i \qquad \lambda_c \propto U / J$$

Towards a Holographic Bose-Hubbard Model
Fujita, Harrison , Karch , Meyer and Paquette. JHEP 1504 (2015) 068

Metal-Insulator Transition (MIT) by Holography

Many MITs occur due to strong correlations !

Metal-Insulator Transition (MIT) by Holography

- **General method for optical conductivity in holographic approach:**

1、Setup for asymptotical AdS $dS^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2)$ $z = \frac{1}{r} \to 0$ Infinity

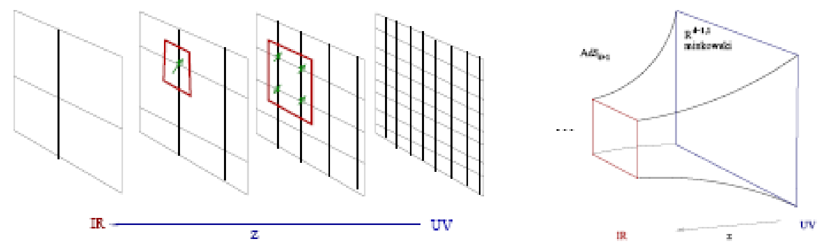


Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory.

McGreevy, Adv.High Energy Phys. 2010 (2010) 723105

2、Solving perturbation equations

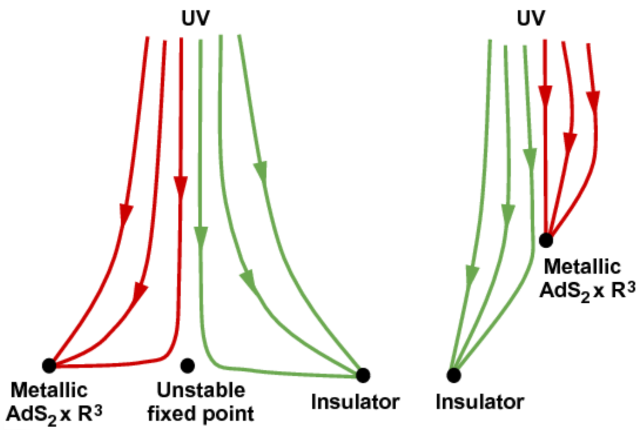
$$a_x(x,z) = a_x^{(0)}(x) + a_x^{(1)}(x) z + \dots$$

$$\sigma = \frac{1}{i\omega} G^R(\omega) = -i \frac{a_x^{(1)}(x)}{\omega a_x^{(0)}(x)} = -i \frac{a_x^{(1)}(x)}{\omega}$$

Metal-Insulator Transition (MIT) by Holography

- **How can one implement MIT by holography?**

  - IR fixed point plays a crucial role in understanding the holographic quantum phase structure.
  - The low energy behavior of a system in long wave- length limit such as the DC conductivity is completely controlled by the near horizon geometry.



- Two renormalization group flow scenarios that arise in theories, mediating quantum phase transitions between metallic and insulating phases.

Donos and Hartnoll, Nature Phys.9, 649 (2013).

Building a lattice background in holographic gravity

Holographic gravity over an inhomogeneous or anisotropic background

↓

The breaking of spatially translational invariance

- Impurities
  - Massive gravity
  - Spatially dependent axions
- Lattice
  - Scalar lattice
  - Ionic lattice
  - Q-lattice，Bianchi models
- Spontaneous breaking instability
  - Topological terms
  - Non-topological terms

↓

CDW

Horowitz, Santos and Tong, JHEP 1207 (2012) 168.

Part II

MIT as a thermal phase transition: charge density waves

Holographic Charge Density Waves (HCDW)

- Motivations and recent progress

1. It is essential to introduce some mechanism inducing the instability of the bulk geometry which is usually of spatial homogeneity.

   H. Ooguri and C. -S. Park, Phys. Rev. Lett. 106, 061601 (2011)
   A. Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)

2. Striped black hole solutions as the examples of spatially modulated unstable modes have been presented.

   M. Rozali, D. Smyth, E. Sorkin and J. B. Stang, Phys. Rev. Lett. 110, 201603 (2013)
   A. Donos, JHEP 1305, 059 (2013)

3. The dynamics of CDW in the holographic approach.

   Y. Ling, C. Niu, J. Wu, Z. Xian and H. Zhang, Phys.Rev.Lett. 113 (2014) 091602.
   Y. Ling, C. Niu, J. Wu, Z. Xian and H. Zhang, To appear.

Holographic Charge Density Waves (HCDW)

- Spontaneous breaking of translational invariance

1、Topological terms

$$S_{top} \sim \int d^4x \frac{c_1 \Phi}{16\sqrt{3}} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

2、Non-topological terms

$$S_{non-top} \sim \int d^4x \sqrt{-g}[-\frac{1}{4} t(\Phi) F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} - \frac{1}{2} u(\Phi) F^{\mu\nu} G_{\mu\nu}]$$

Striped black holes

$$ds^2 = \frac{1}{z^2}[-f(z)Qdt^2 + \frac{S}{f(z)}dz^2 + T[dx + z^2 U dz]^2 + V dy^2]$$

$$A = \mu(1-z)\psi dt \qquad B = (1-z)\chi dt \qquad \Phi = z\phi$$

$$F = dA, G = dB,$$

$$t(\Phi) = 1 - \frac{\beta}{2} L^2 \Phi^2$$

$$u(\Phi) = \frac{\gamma}{\sqrt{2}} L\Phi$$

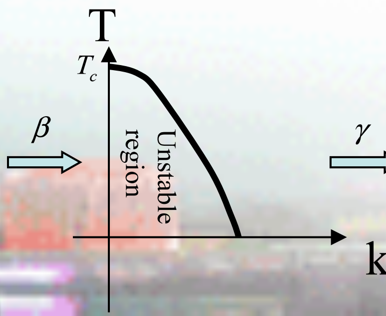
RN black brane

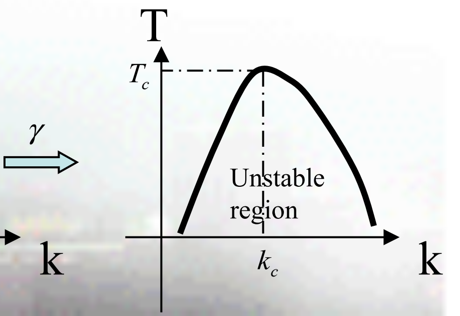
$$Q(x,z) = S(x,z) = T(x,z) = V(x,z) = 1, U(x,z) = 0 \qquad f(z) = 4(1 + z + z^2 - z^3\mu^2/16)$$

$$\psi(x,z) = 1, \qquad \chi(x,z) = 0, \qquad \phi(x,z) = 0$$

$$\frac{T}{\mu} = \frac{48 - \mu^2}{16\pi\mu}$$

RN black brane → Striped black holes





Holographic Charge Density Waves (HCDW)

- Instability of near horizon geometry which is AdS2

AdS-RN black holes:

A. Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)

$$T=0 \qquad AdS_2\times R^2$$

$$ds^2=-12r^2dt^2+\frac{dr^2}{12r^2}+(dx^2+dy^2)$$

$$F=2\sqrt{3}dr\wedge dt$$

$$S_{top}\sim\int d^4x\frac{c_1\varphi}{16\sqrt{3}}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$\delta g_{ty}=2\sqrt{3}h_{ty}(t,r)\sin(kx)$$

$$\delta g_{xy}=h_{xy}(t,r)\cos(kx)$$

$$\delta A_y=a(t,r)\sin(kx)$$

$$\delta\varphi=w(t,r)\cos(kx)$$

$$\Rightarrow\quad \Box_{AdS2}\vec{V}-M^2\vec{V}=0$$

$$\vec{V}=(\phi_{xy},a,w)$$

$$M^2=\begin{pmatrix} k^2 & \frac{1}{\sqrt{3}}k & 0\\ 24\sqrt{3} & 24+k^2 & -c_1k\\ 0 & -c_1k & k^2+m^2\end{pmatrix}$$

AdS2 BF bound

$$m^2\geq -3$$

Could be violated !

Holographic Charge Density Waves (HCDW)

- The holographic charge density waves

$$L^2 = \frac{1}{24}, m^2 = -8$$

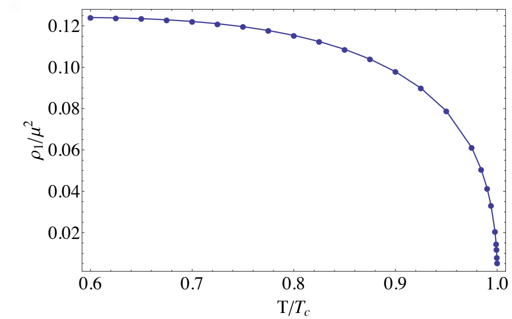
$$B = (1 - z)\chi dt$$

$$T_c = 0.078\mu, k_c = 0.325\mu \quad \Leftarrow \quad \beta = -138, \gamma = 17.1$$

$$\frac{T}{\mu} = \frac{48 - \mu^2}{16\pi\mu}$$

Charge density

$$\rho(x) = \left(1 - \frac{\partial \chi}{\partial z}\Big|_{z=0}\right) \quad \Leftarrow \quad \chi(x,z) = 0 + z\chi_1(x) + z^2\chi_2(x) + \ldots$$



$$\rho(x) = \rho_1 \cos[k_c x] + \rho_3 \cos[3k_c x] + \ldots$$

$$\rho_0 \cong \rho_2 \cong 0\ldots, \quad \rho_1, \quad \rho_3 \quad \rightarrow CDW$$

$\Downarrow$

No free electrons

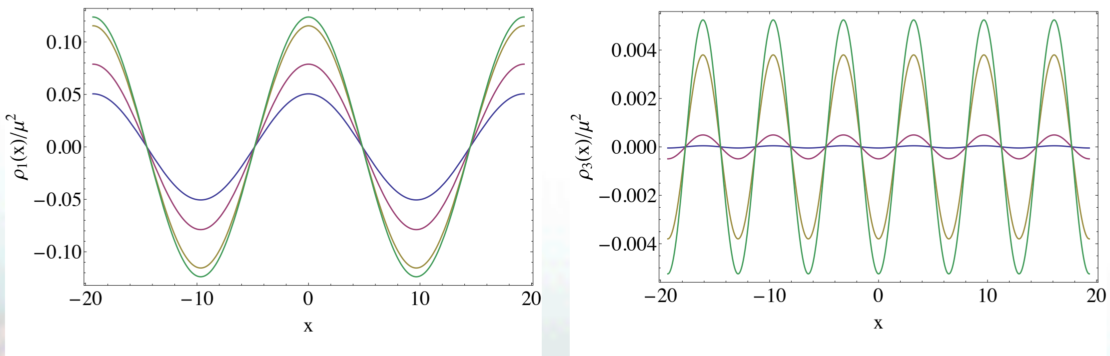
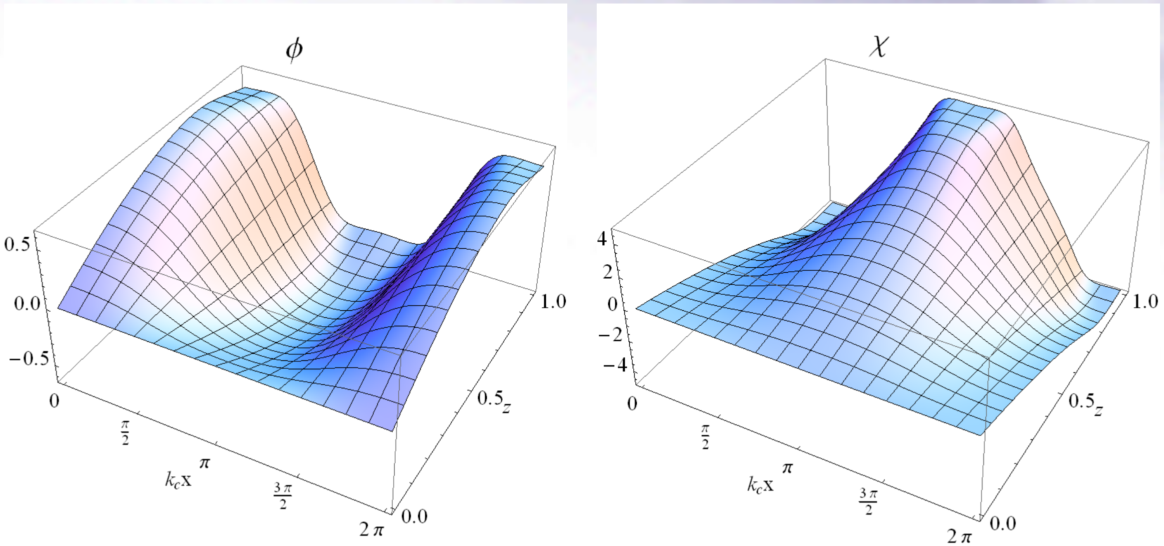


FIG. 2: The first and third modes of CDW for $T/T_c = 0.6, 0.8, 0.95, 0.98$ from top to down.

Holographic Charge Density Waves (HCDW)

- The examples of background solutions



The solutions of scalar field and the time component of the gauge field at T=0.8Tc

- The striped profile increases when one goes deeper into the horizon.
- Such a striped phase is triggered by the instability of near horizon geometry.

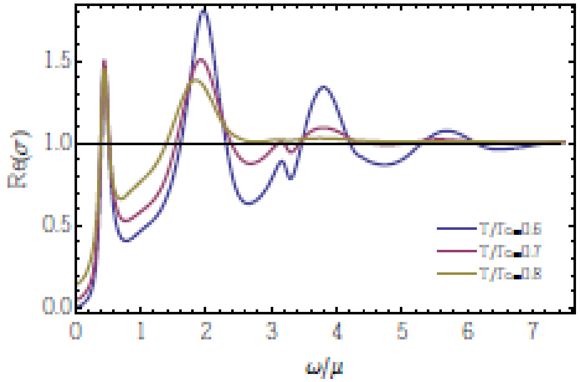
- The optical conductivity

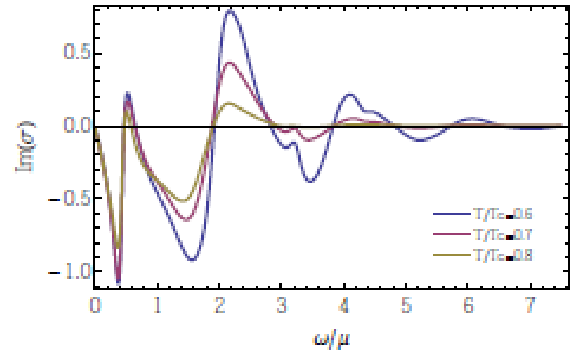
$$b_x = (1 + j_x(x) z + \dots) e^{-i\omega t}$$

$$\sigma(\omega/\mu) = \frac{4 j_x^{(0)}}{i\omega}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, A_\mu = \bar{A}_\mu + a_\mu, B_\mu = \bar{B}_\mu + b_\mu, \Phi = \bar{\Phi} + \varphi.$$

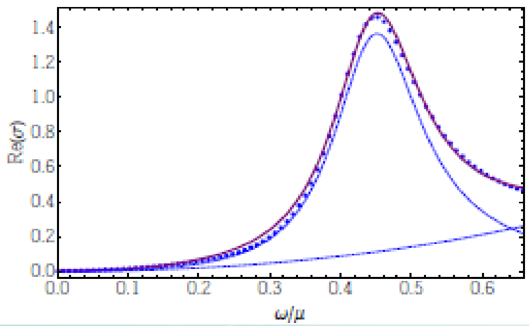
$$\nabla^\mu \hat{h}_{\mu\nu} = 0, \nabla^\mu a_\mu = 0, \nabla^\mu b_\mu = 0. \qquad \hat{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2.$$

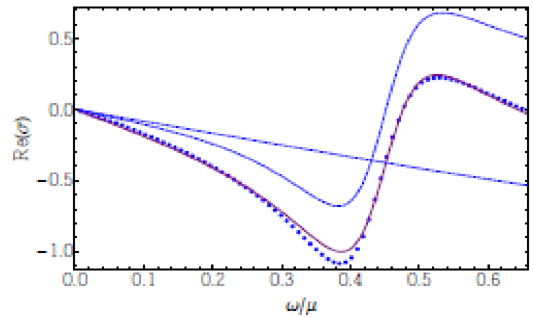




- Two Lorentz formulae $\sigma_{tot}(\omega) = \sigma_{CDW1}(\omega) + \sigma_{CDW2}(\omega)$

$$\sigma_{CDW}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2/\omega^2)}$$





- Single-particle gap:

$$\frac{2\Delta}{T_c} \cong 20.51$$

Remark: metal to insulator phase transition!

15.80 for Single crystalline TbTe3

Y. Ling *et.al.*, Phys.Rev.Lett. 113 (2014) 091602.

Holographic Charge Density Waves (HCDW)

- **Summary：**

1、Superconductivity

$$\sigma(\omega) \propto K\left(\delta(\omega) - \frac{1}{i\omega^{\alpha}}\right)$$

Theory： The breaking of U(1) gauge symmetry
Strategy： Introducing a complex scalar field

2、Drude law for metals

$$\sigma = \frac{\sigma_{DC}}{1 - i\omega\tau}$$

Theory： The breaking of translational symmetry
Strategy： Introducing lattice structure

3、CDW

$$\sigma_{CDW}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2 / \omega^2)}$$

Theory： Spontaneously breaking of translational symmetry
Strategy： Introducing a topological term

Part III

MIT as a quantum critical phenomenon

Q-lattice and novel MIT

- 4D Setup

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R+6-\frac{1}{2}F_{ab}F^{ab}-|\partial\Phi|^2-m^2|\Phi|^2]$$

Φ : Complex scalar field

Equations of motion:

$$G_{ab}=R_{ab}+3g_{ab}-....=0$$

$$\nabla_a F^a{}_b=0$$

$$(\Box-m^2)\Phi=0$$

A. Donos and J. P. Gauntlett, JHEP 1404, 040 (2014).

Q-lattice and novel MIT

- **Ansatz of variables**

$$\Phi = e^{ikx} z^{3-\Delta} \phi(z)$$

$$ds^2 = \frac{1}{z^2}[-(1-z)P(z)U(z)dt^2 + \frac{1}{P(z)(1-z)U(z)}dz^2 + V_1(z)dx^2 + V_2(z)dy^2]$$

$$A = \mu(1-z)\psi(z)dt \qquad \Delta = 3/2 \pm \left(9/4 + m^2\right)^{1/2} \qquad P(z) = 1 + z + z^2 - \frac{\mu^2 z^3}{2}$$

$$m^2 = -2$$

- **A family of three-parameter black brane solutions:** (T, λ, k)

Boundary condition $\phi(0) = \lambda$

$$T/\mu, \qquad \lambda/\mu^{3-\Delta}, \qquad k/\mu$$

Temperature (T/μ)

Lattice Amplitude ($\lambda/\mu^{3-\Delta}$)

Wave number (k/μ)

Q-lattice and novel MIT

- Linear perturbations and optical conductivity:

$$\delta g_{tx} = h_{tx}(t,z), \quad \delta A_{\mu} = a_x(t,z), \quad \delta\Phi = i e^{ikx} z^{3-\Delta} \varphi(t,z).$$

Boundary condition:

$$a_x(0) = 1, \quad \varphi(0) = (ik\lambda/\omega) h_{tx}^{-2}, \qquad h_{tx} = \frac{h_{tx}^{(-2)}}{z^2} + \ldots$$

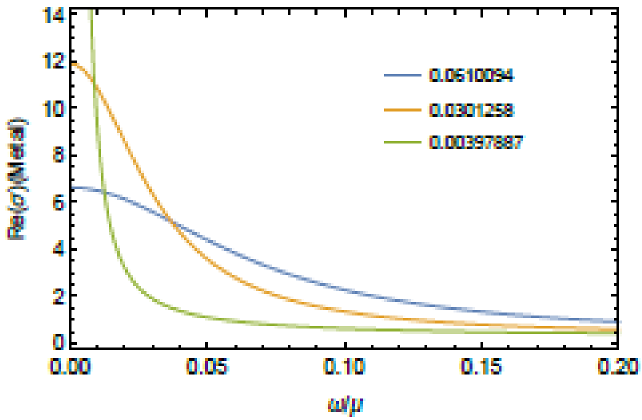
Optical conductivity:

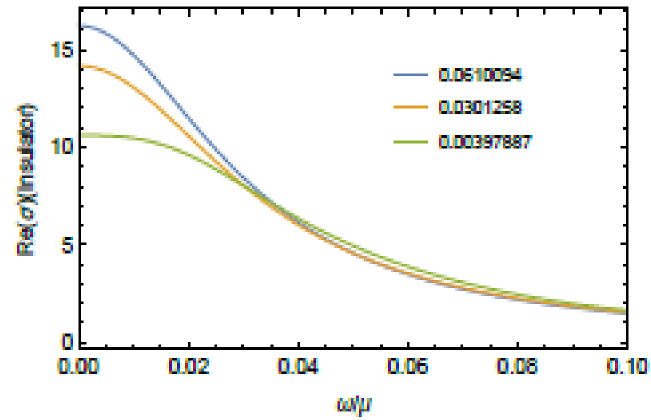
$$a_x = (1 + j_x z + \ldots) e^{-i\omega t}$$

$$\sigma(\omega/\mu) = \frac{j_x}{i\omega}$$

Q-lattice and novel MIT

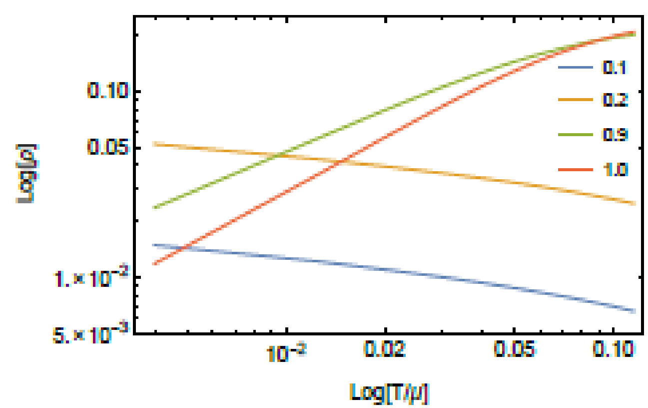
- **Novel MIT:**





Metal $\frac{d\rho_{DC}}{dT} > 0$

Insulator $\frac{d\rho_{DC}}{dT} < 0$



Q-lattice and novel MIT

- **Phase Diagram:** $T/\mu = 0.001$

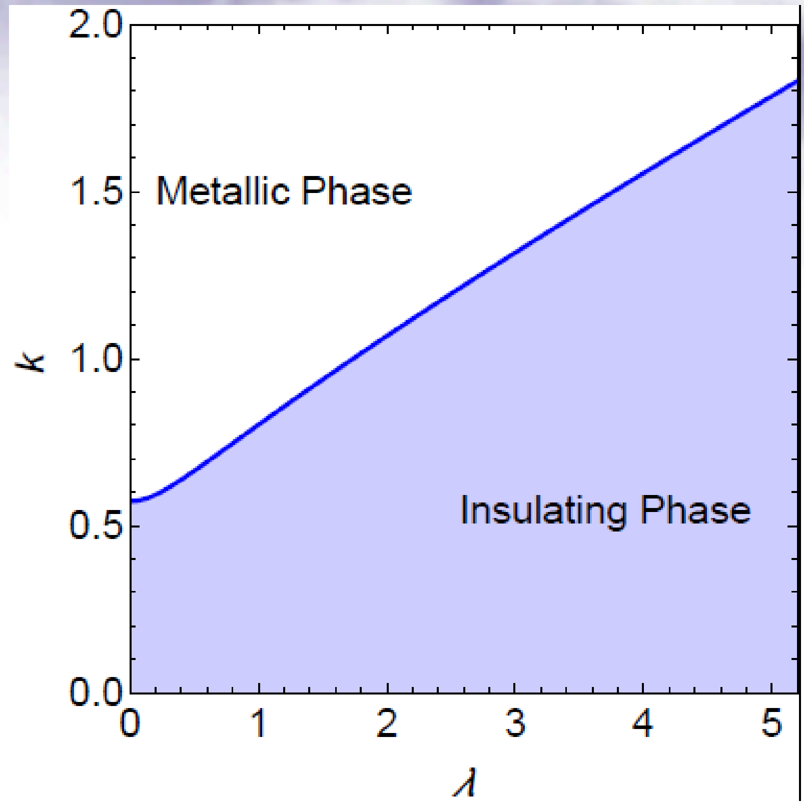
Critical points: $\partial_T \sigma_{DC}(k,\lambda) = 0$

i) The zero-frequency limit of AC conducutivity

$$\sigma_{DC} = \lim_{\omega \to 0} \sigma_{AC}(\omega)$$

ii) It is completely determined by the near horizon geometry

$$\sigma_{DC} = \left(\sqrt{\frac{V_2}{V_1}} + \frac{\mu^2 a^2 \sqrt{V_1 V_2}}{k^2 \phi^2} \right) \Bigg|_{z=1}$$



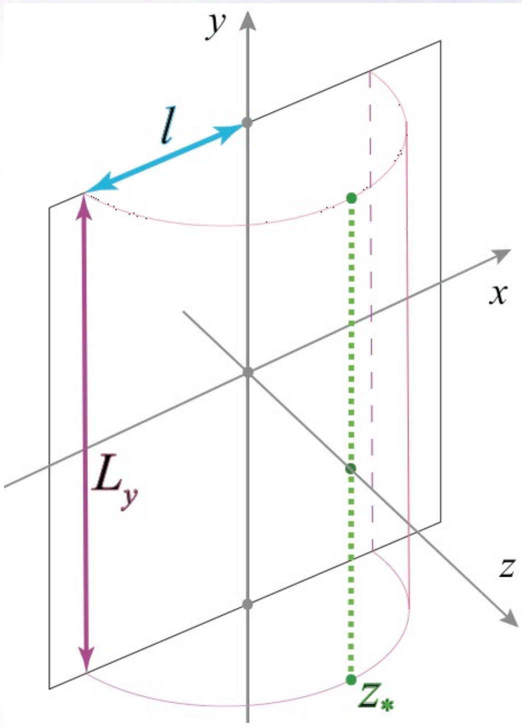
Question:
What role can the holographic entanglement entropy play in quantum phase transition?

Y.Ling, et.al.,arXiv:1502.03661

Holographic Entanglement Entropy (HEE) close to QPT

- **Holographic description of entanglement entropy: RT formula**

$$S_{full} = \frac{\mathrm{Area}(\gamma_A)}{4G_N} \quad \longrightarrow \quad 4G_N S_{full} = 2L_y (S + 1/\varepsilon)$$



  - It is suggested that the reduced HEE measures the degrees of freedom in CFT on the boundary.

  T. Nishioka, S. Ryu and T. Takayanagi, J.Phys.A42:504008,2009

- **The reduced HEE:**

$$l = \mu \int_0^{z_*} dz z^2 \sqrt{\frac{V_1(z_*)V_2(z_*)}{P(z)V_1(z)W(z_*,z)}}$$

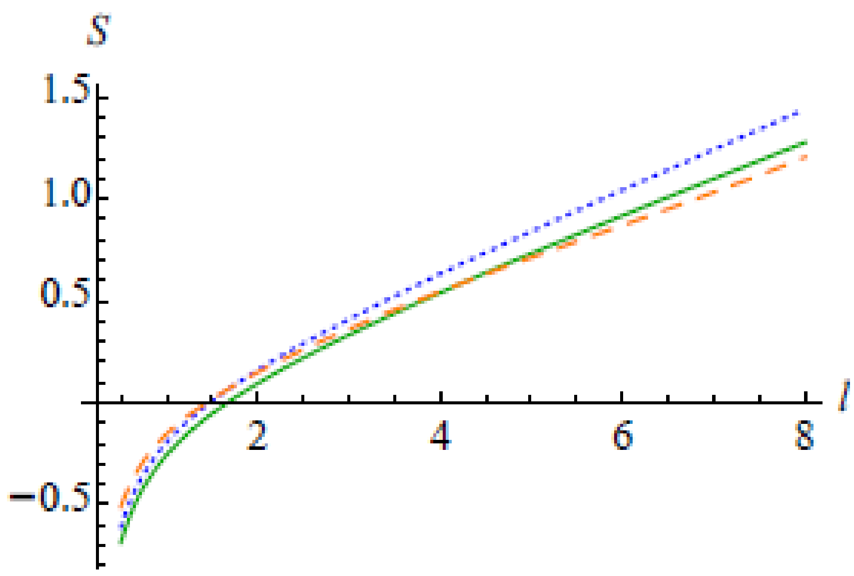
$$S = \frac{1}{\mu}\left\{ -\frac{1}{z_*} + \int_0^{z_*} \frac{dz}{z^2}\left[\frac{z_*^2 V_1(z)V_2(z)}{\sqrt{P(z)V_1(z)W(z_*,z)}} - 1 \right]\right\}$$

$$W(z_*,z) = z_*^4 V_1(z)V_2(z) - z^4 V_1(z_*)V_2(z_*)$$

Y.Ling, et.al.,arXiv:1502.03661

Holographic Entanglement Entropy (HEE) close to QPT

- **Numerical results:**



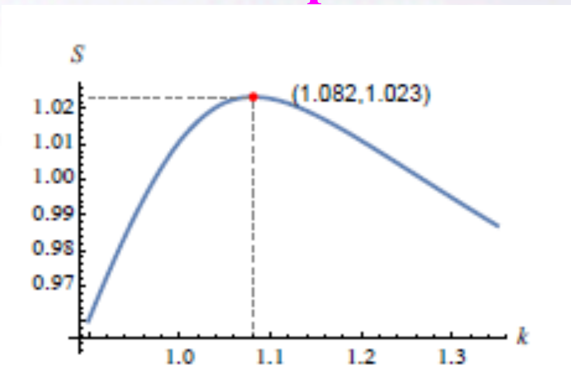
- Q-lattice dual to metallic phase
- AdS-RN black brane
- Q-lattice dual to insulating phase

- The reduced HEE increases in a linear fashion with the width of the strip when l is relatively large

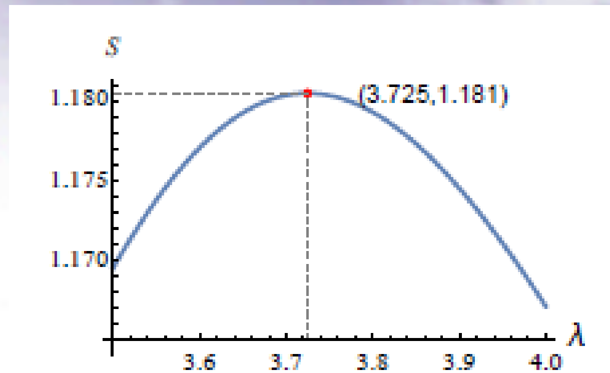
Y.Ling, et.al.,arXiv:1502.03661

Holographic Entanglement Entropy (HEE) close to QPT

- **Pronounced peaks :**



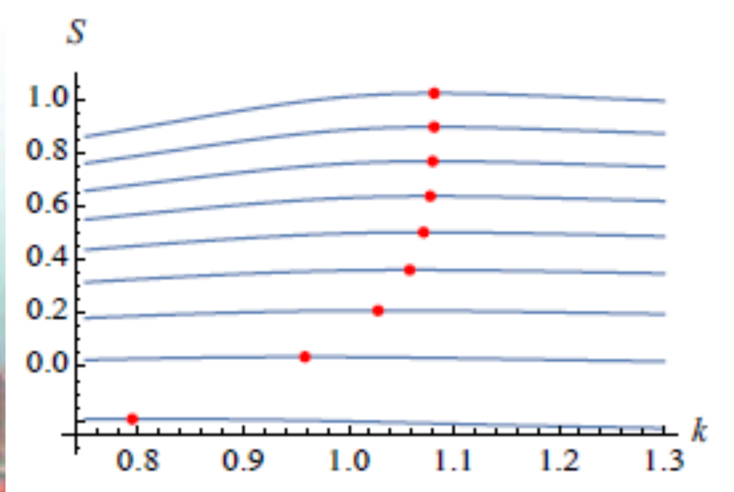
S vs k with λ fixed



S vs λ with k fixed

- All the strips have the same width $2l$.

- **Independence of the width of the strip :**



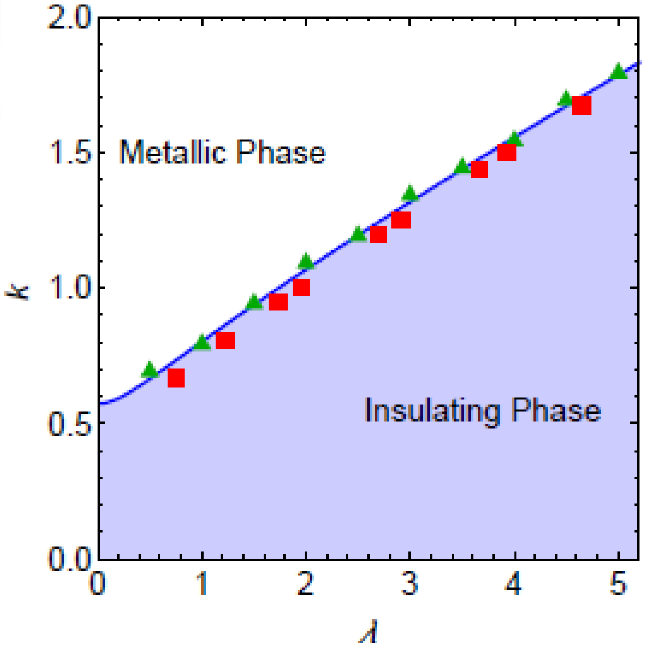
- The half-width of the strip l is increased from 0.91 to 5.56 uniformly.

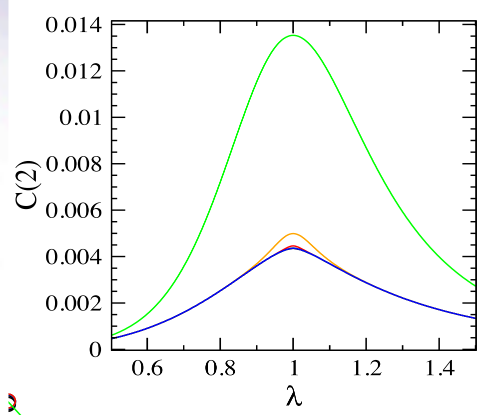
- The location of turning points becomes independent of the width of the strip.

Y.Ling, et.al.,arXiv:1502.03661

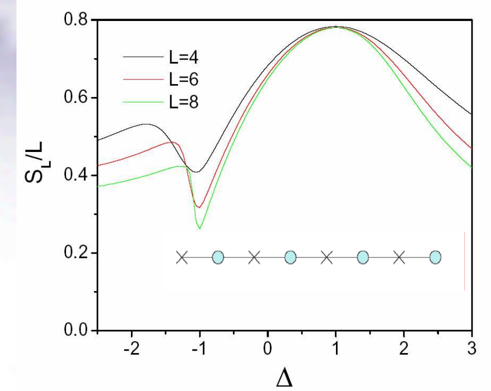
Holographic Entanglement Entropy (HEE) close to QPT

- **HEE close to QPTs :**





Osterloh, *et.al.*
Nature 416,608(2002).



Chen, *et.al.*
New J. Phys. 8, 97(2006).

- All the turning points of the reduced HEE are distributed in the vicinity of quantum critical points.
- It indicates that HEE can be viewed as a signature of the occurrence of QPTs.
- This feature coincides with the behavior of EE in CMT.

Y.Ling, et.al.,arXiv:1502.03661

Holographic Entanglement Entropy (HEE) close to QPT

- **Near horizon analysis:** $z_* \to 1$

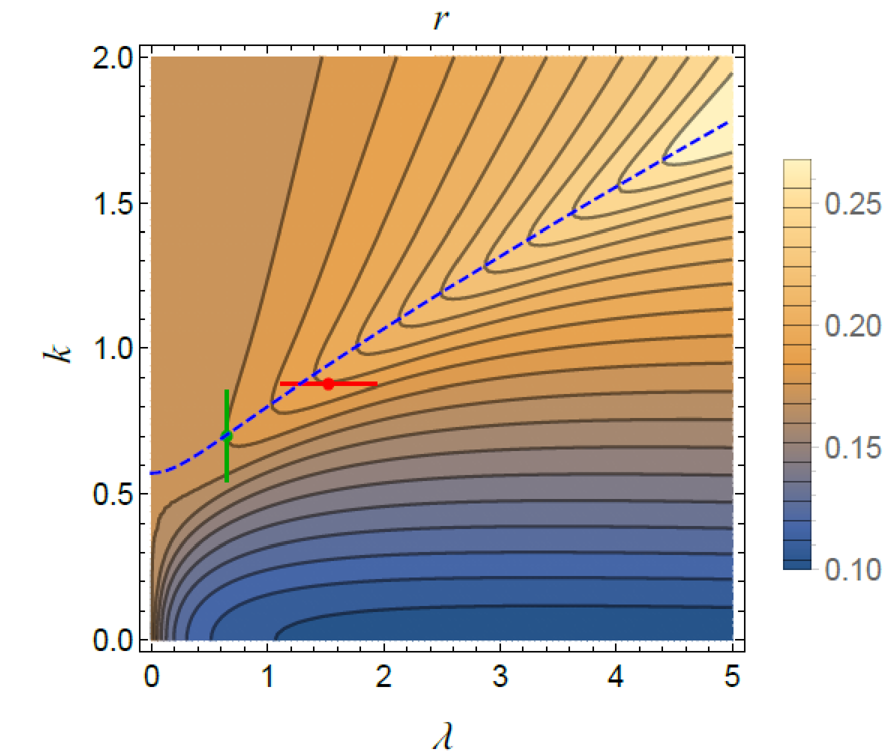
$$S \sim \frac{V_2}{\mu}\sqrt{\frac{V_1}{pUB}}\bigg|_{z=1} \log\left(\frac{1}{1-z_*}\right) + \ldots,$$

$$l \sim \mu\sqrt{\frac{V_2}{pUB}}\bigg|_{z=1} \log\left(\frac{1}{1-z_*}\right) + \ldots,$$

$$B = 4V_1V_2 - V_1{}'V_2 - V_1V_2{}'$$

$$p = 1 + z + z^2 - \mu^2 z^3/2$$

$$r = \lim_{z_*\to 1}\frac{S}{l} = \frac{\sqrt{V_1V_2}}{\mu^2}\bigg|_{z=1}$$



A contour plot of the quantity r

- For large l, the reduced HEE receives the dominant contribution from the near horizon regime!

Y.Ling, et.al.,arXiv:1502.03661

Towards a holographic Mott-like insulator

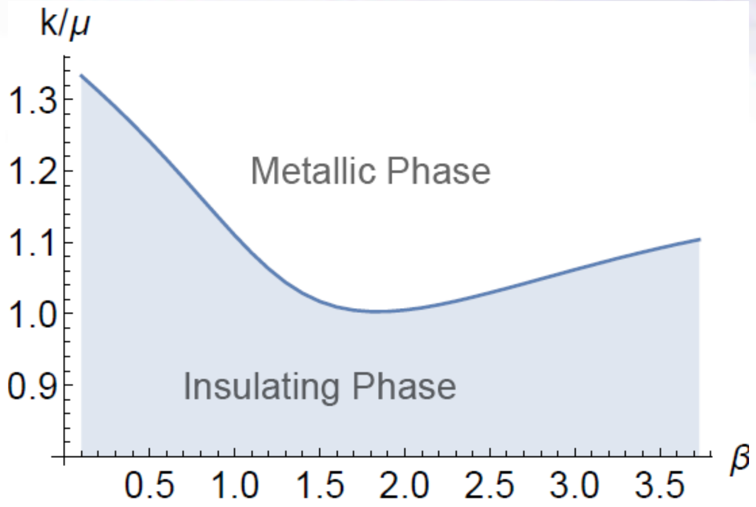
- 4D Setup:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R+6-\frac{V(\Phi)}{2}F_{ab}F^{ab}-|\partial\Phi|^2-m^2|\Phi|^2]$$

$$V(\Phi)=1-\beta|\Phi|^2$$

$$(\beta, T, \lambda, k)$$

- Periodic structure: $a=2\pi/k$



$T=0.2,\quad \lambda=2$

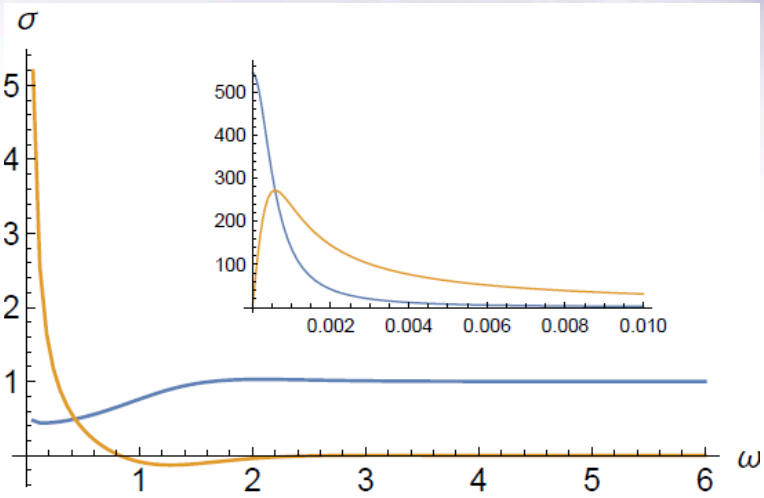
- Finite temperature region

i) The transition from a metallic phase to an insulating phase occurs when the lattice constant becomes larger, for a given (β, T, λ)

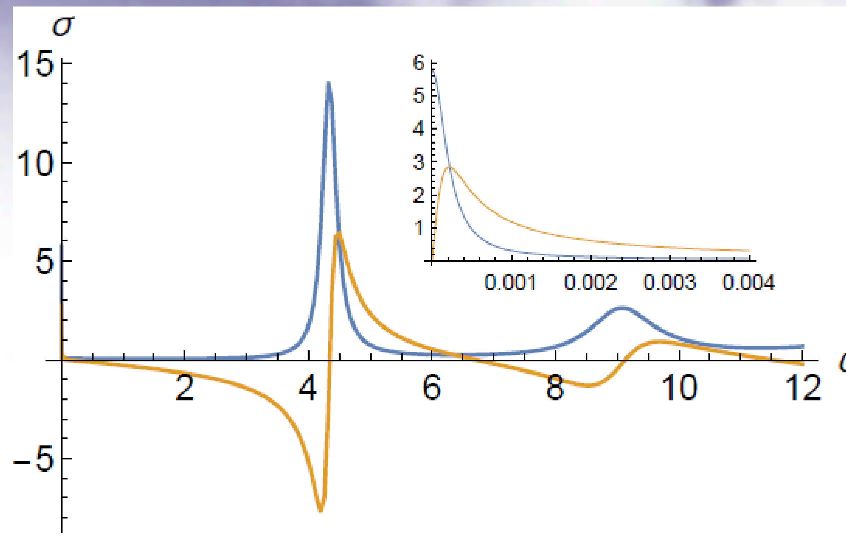
Dynamically generating a Mott gap in the probe limit by holography has been proposed in Phys. Rev. Lett. 106 (2011) 091602, by Edalati, Leigh and Phillips.

Y.Ling, et.al.,to appear

Towards a holographic Mott-like insulator



$T = 0.2, \quad \lambda = 2, \quad k=0.03, \quad \beta=0$



$T = 0.2, \quad \lambda = 2, \quad k=0.03, \quad \beta=3.0$

ii) A gap in insulating phase can be manifestly observed from the optical conductivity when the parameter β is increased to an appropriate value.

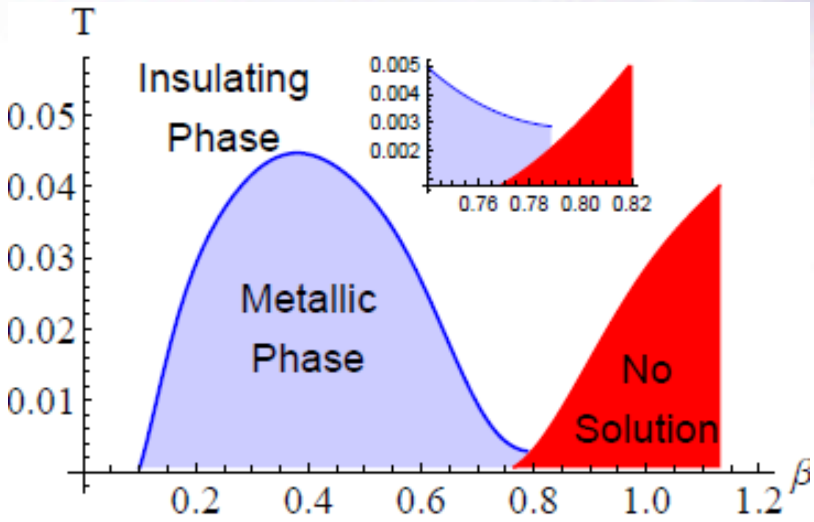
Similar results appeared in e-Print: ***arXiv:1503.03481*** *by Kiritsis and Ren*

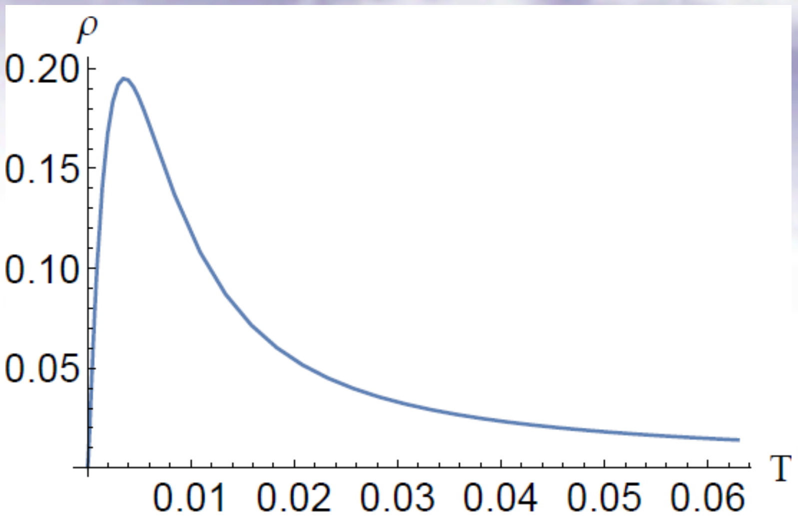
Y.Ling, et.al.,to appear

Towards a holographic Mott-like insulator

- In zero temperature limit

$\lambda=2$, k=0.03, β=0.76





- This model exhibits a novel metallic behavior again at the extremely low temperature featured by a gap as well as a zero-frequency mode with tiny spectral weight.
- It implies that it is a doped (or incommensurate) system where umklapp scattering is frozen in zero temperature limit.
  (*T. Giamarchi, Quantum physics in one dimension,2004*)
- β plays a double role in this model, namely, generating a gap and doping the system.
- Its behavior is analogous to some organic linear chain conductors observed in experiments. (*Vescoli et. al., Science 281 (1998) 1181.*)

Y.Ling, et.al.,to appear

Summary:

- We investigate the metal-insulator transition in a holographic approach.

- As a thermal phase transition, a holographic model for charge density waves has been constructed with a spontaneous breaking of translational symmetry.

- In Q-lattice backgrounds, the reduced HEE always displays a peak in the vicinity of quantum critical points, indicating that the reduced HEE can be used to characterize the quantum phase transition.

- Building a holographic background dual to Mott insulator is on the road.

Thank you !