Magnetic impurities and universal relations in AdS/CMT

Johanna Erdmenger

Max–Planck–Institut für Physik, München

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

1. Kondo models from holography

- Model — J.E., Hoyos, O'Bannon, Wu 1310.3271
- Screening, resistivity
- Quantum quenches — J.E., Flory, Newrzella, Wu in progress
- Entanglement entropy — J.E., Flory, Newrzella 1410.7811

J.E., Hoyos, Newrzella, O'Bannon, Wu in progress

2. S-Wave Superconductivity in Anisotropic Holographic Insulators

J.E., Herwerth, Klug, Meyer, Schalm 1501.07615

- Scalar condenses in helical Bianchi VII background
- Homes' Law

Kondo models from gauge/gravity duality

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

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Motivation for study within gauge/gravity duality:

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1. Kondo model: Simple model for a RG flow with dynamical scale generation

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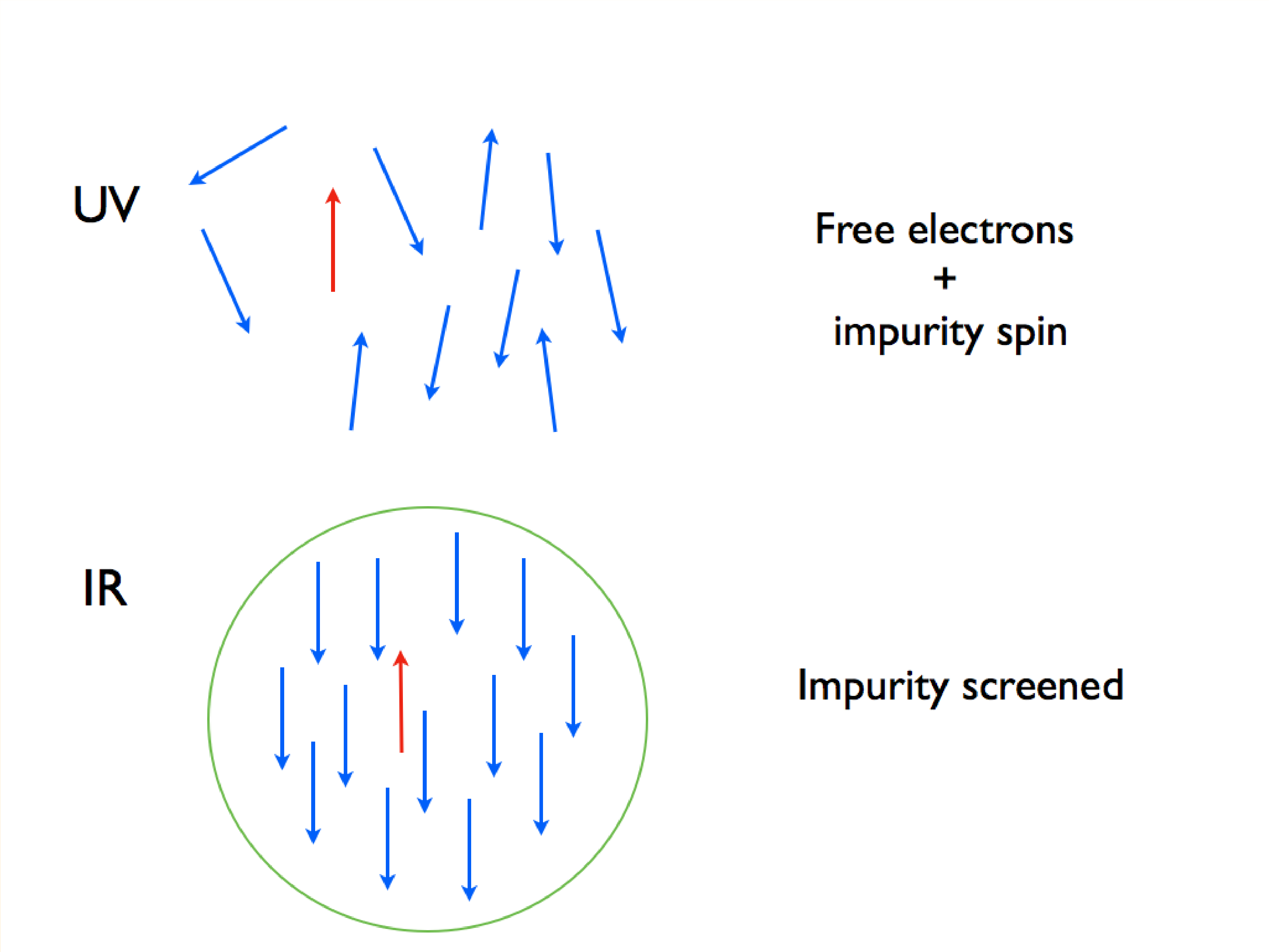
2. New applications of gauge/gravity duality to condensed matter physics:

- Magnetic impurity coupled to strongly correlated electron system
- Entanglement entropy
- Quantum quench
- Kondo lattice

Kondo effect



Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

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Impurity screened at low temperatures:
Logarithmic rise of resistivity at low temperatures

Dynamical scale generation

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Due to symmetries: Model effectively $(1+1)$-dimensional

Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}\,, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Gauge/gravity requires large N: Spin group $SU(N)$

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In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = Q$

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Screened phase has condensate $\langle \mathcal{O} \rangle$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192
Senthil, Sachdev, Vojta cond-mat/0209144

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation
- Holographic superconductor: Condensate forms in AdS_2
- Power-law scaling of conductivity in IR with real exponent
- Screening, phase shift

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \to$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \to \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $Q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

Bottom-up gravity dual for Kondo model

Action:

$$S = S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi} \int_{AdS_3} \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right),$$

$$S_{AdS_2} = -N \int d^3x \, \delta(x) \sqrt{-g} \left[\frac{1}{4} \mathrm{Tr} f^{mn} f_{mn} + g^{mn} \left(D_m \Phi \right)^\dagger D_n \Phi + V(\Phi^\dagger \Phi) \right]$$

$$V(\Phi) = M^2 \Phi^\dagger \Phi$$

Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} \left(\frac{dz^2}{h(z)} - h(z) \, dt^2 + dx^2 \right),$$

$$h(z) = 1 - z^2 / z_H^2 \,, \qquad T = 1/(2\pi z_H)$$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

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Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

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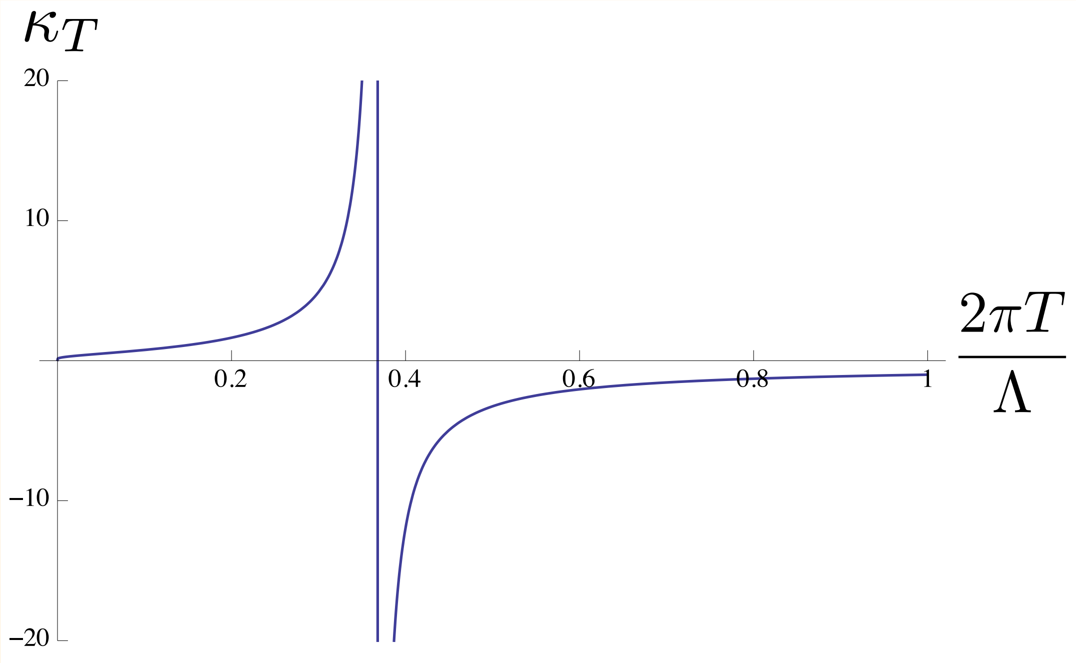
Dynamical scale generation

Scale generation

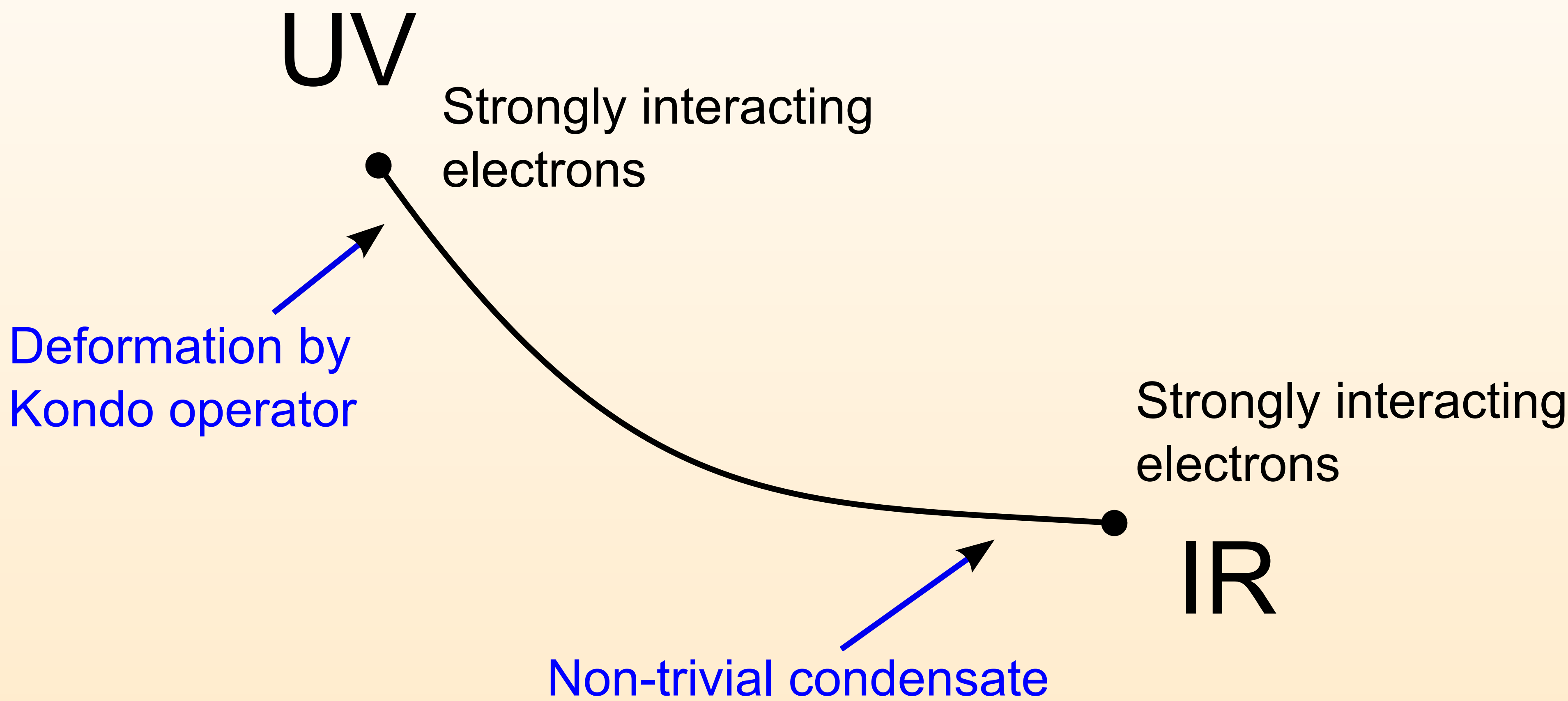


Divergence of Kondo coupling determines Kondo temperature T_K

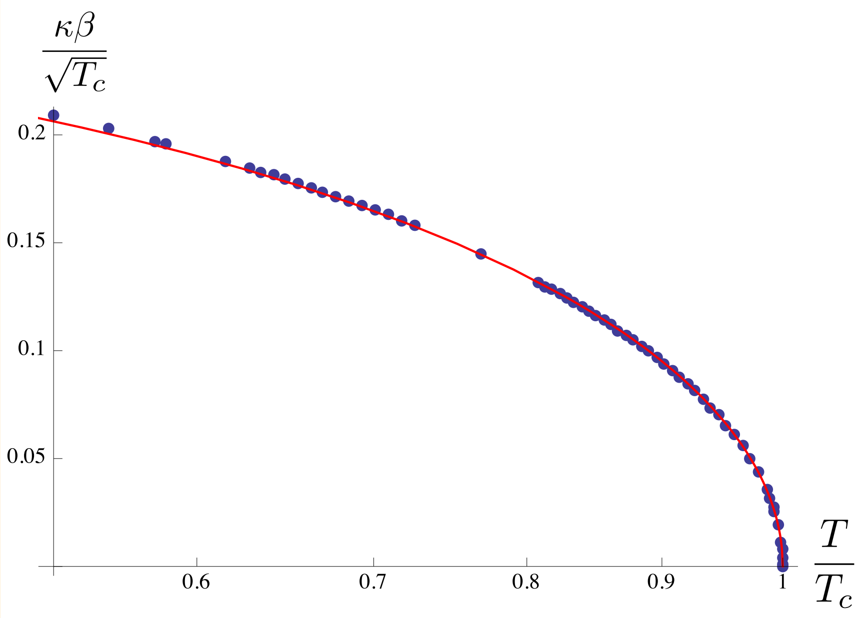
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

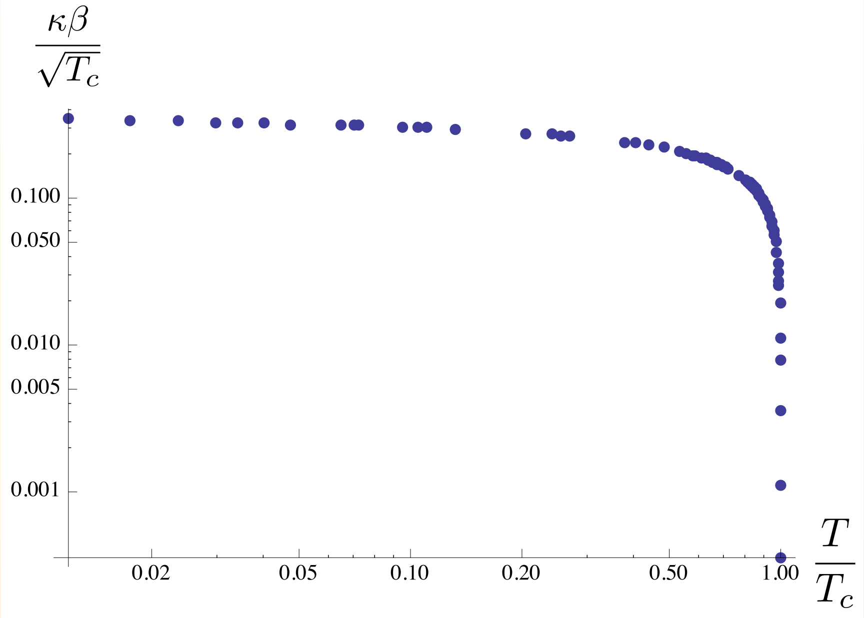
RG flow


UV
Strongly interacting
electrons
Deformation by
Kondo operator
Strongly interacting
electrons
IR
Non-trivial condensate

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa\beta$ as function of the temperature



(a)

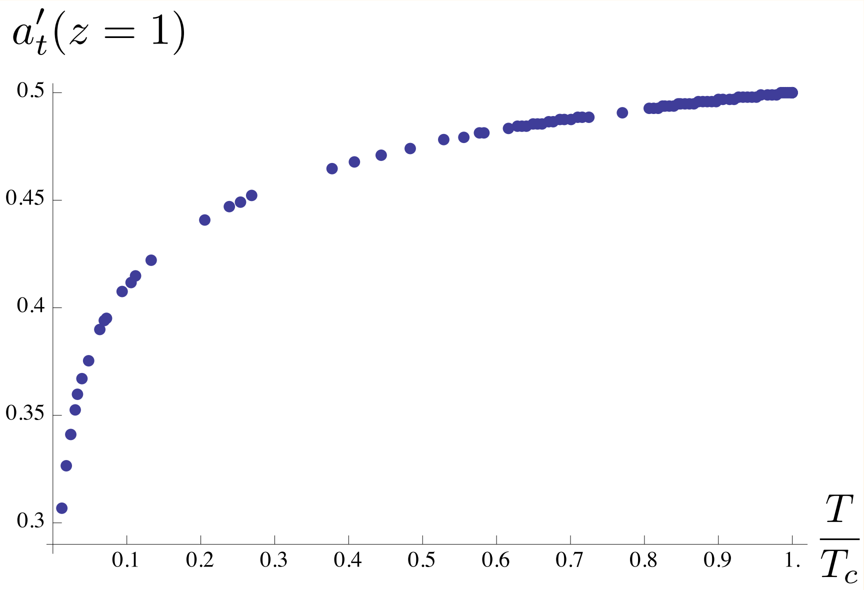


(b)

Mean field transition

$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

Electric flux at horizon



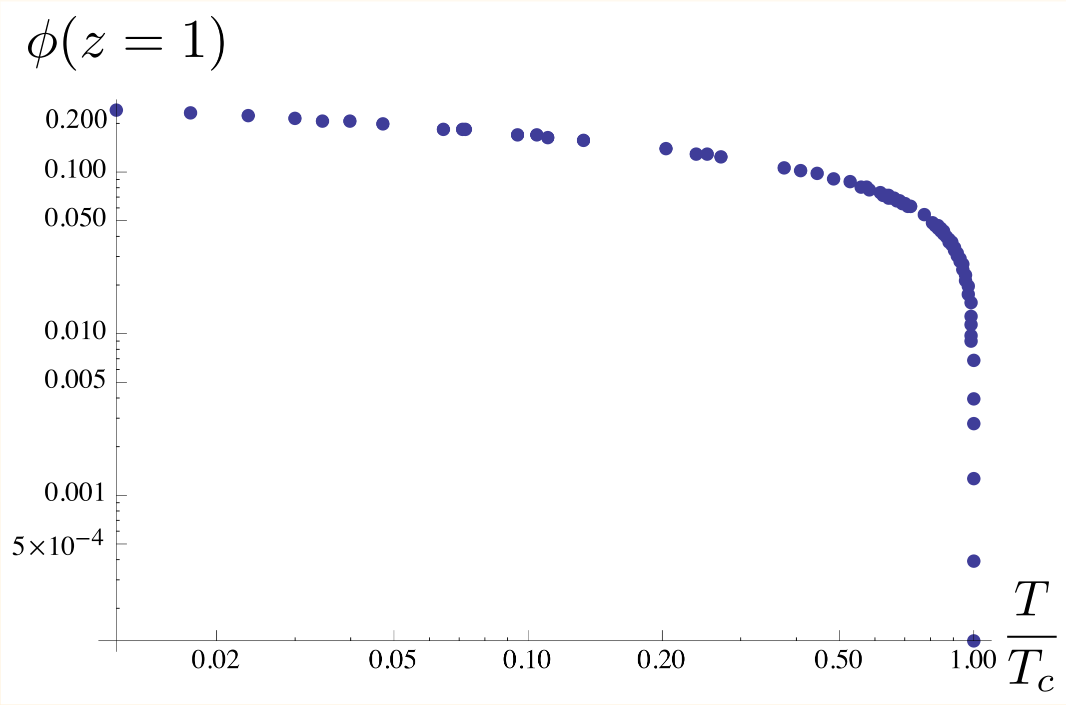
(a)

$$\sqrt{-g} f^{tr} \Big|_{\partial AdS_2} = q,$$ charge density $q = Q/N$

Impurity is screened

Resistivity obtained from leading irrelevant operator

(No logarithmic behaviour due to the large N limit)



Dimension: $\Delta = 1/2 + \sqrt{1/4 + 2\phi_\infty^2} = 1.07$

Entropy density: $s = s_0 + c_s \lambda_{\mathcal{O}} T^{\Delta - 1}$

Resistivity: $\rho = \rho_0 + c_\rho \lambda_{\mathcal{O}} T^{\Delta}$

Time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

(see poster by Jackson Wu)

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Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

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(see poster by Jackson Wu)

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- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

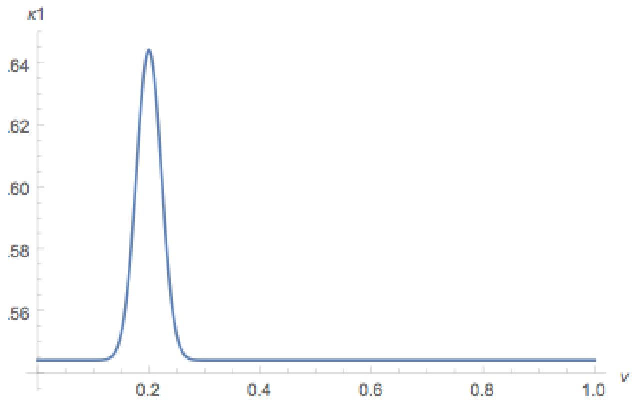
Observations:

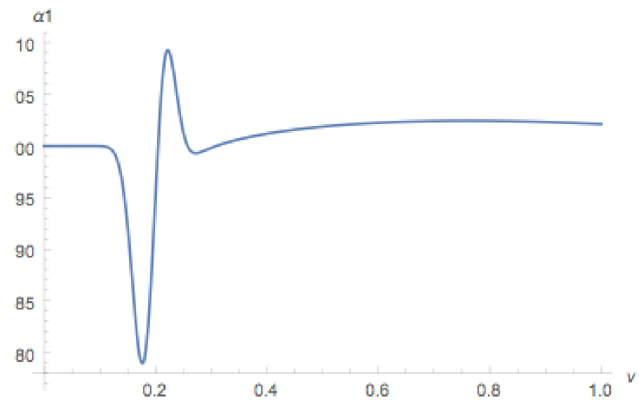
Different timescales depending on whether the condensate is asymptotically small or large

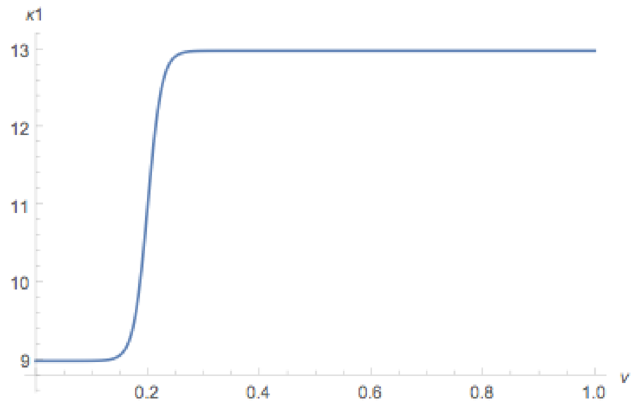
Anderson orthogonality catastrophe? $\tau \sim 1/\langle \text{initial}|\text{final}\rangle$

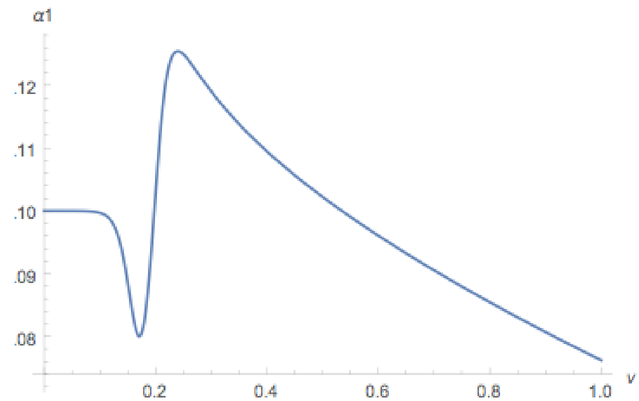
Time dependence

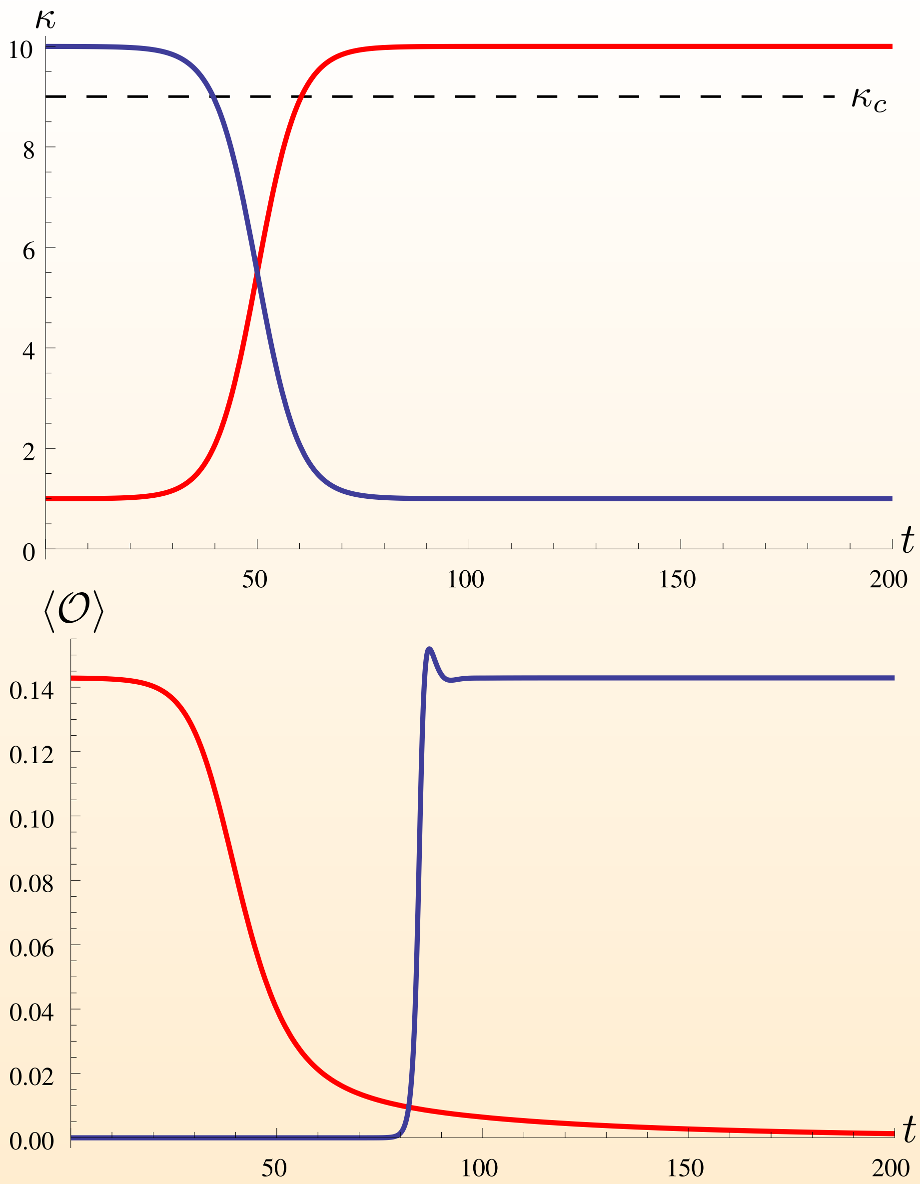
Kondo coupling → Condensate











Quantum quenches in holographic Kondo model

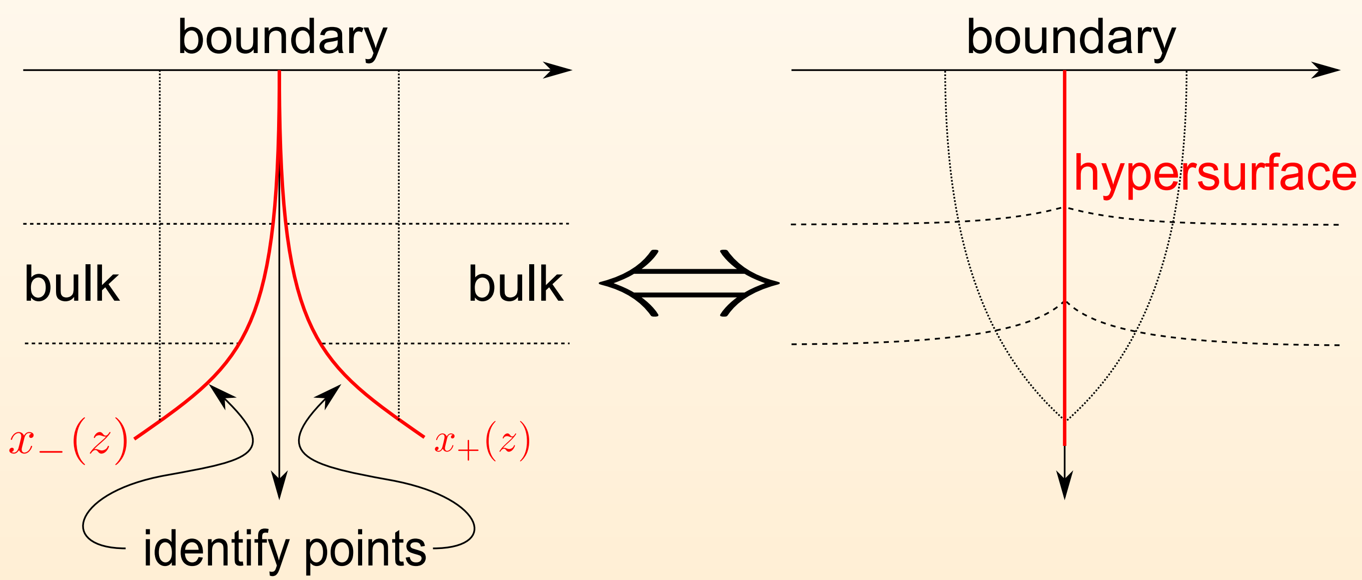
To and from condensed phase

Timescales determined by quasinormal modes

J.E., Flory, Newrzella, Strydom, Wu

Including the backreaction using a thin brane and Israel junction conditions

Israel junction conditions $K_{\mu\nu} - \gamma_{\mu\nu} K = -\frac{\kappa}{2} T_{\mu\nu} \Leftrightarrow$ Energy conditions

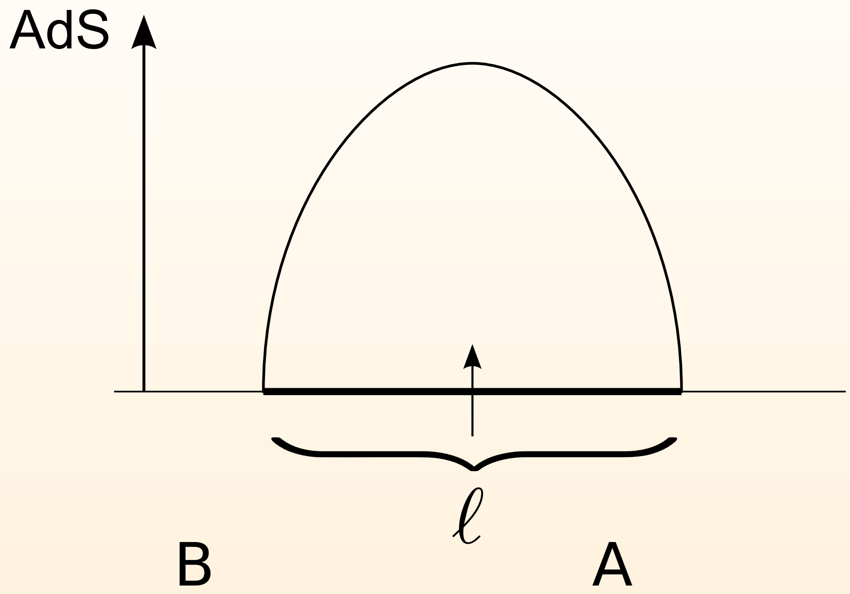


J.E., Flory, Newrzella 1410.7811

In extension of previous work on holographic BCFT

Takayanagi; Fujita, Takayanagi, Tonni 2011; Nozaki, Takayanagi, Ugajin 2012

Entanglement entropy for magnetic impurity



Impurity entropy:

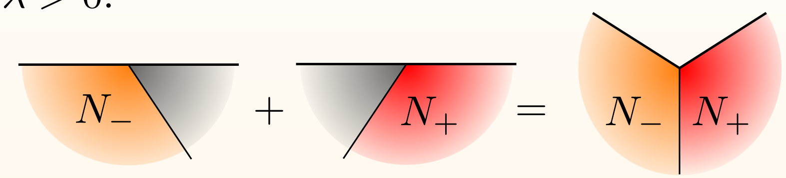
$$S_{\text{imp}} = S_{\text{condensed phase}} - S_{\text{normal phase}}$$

Subtraction also guarantees UV regularity

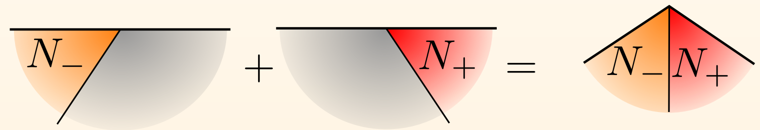
Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811

$\lambda > 0$:



$\lambda < 0$:



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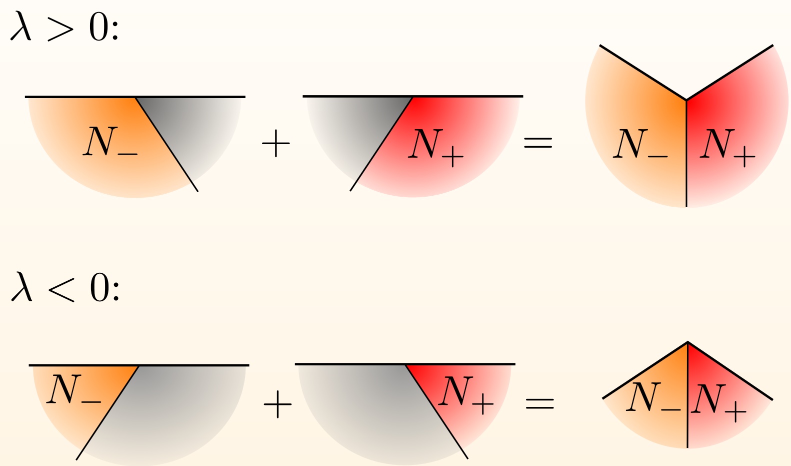
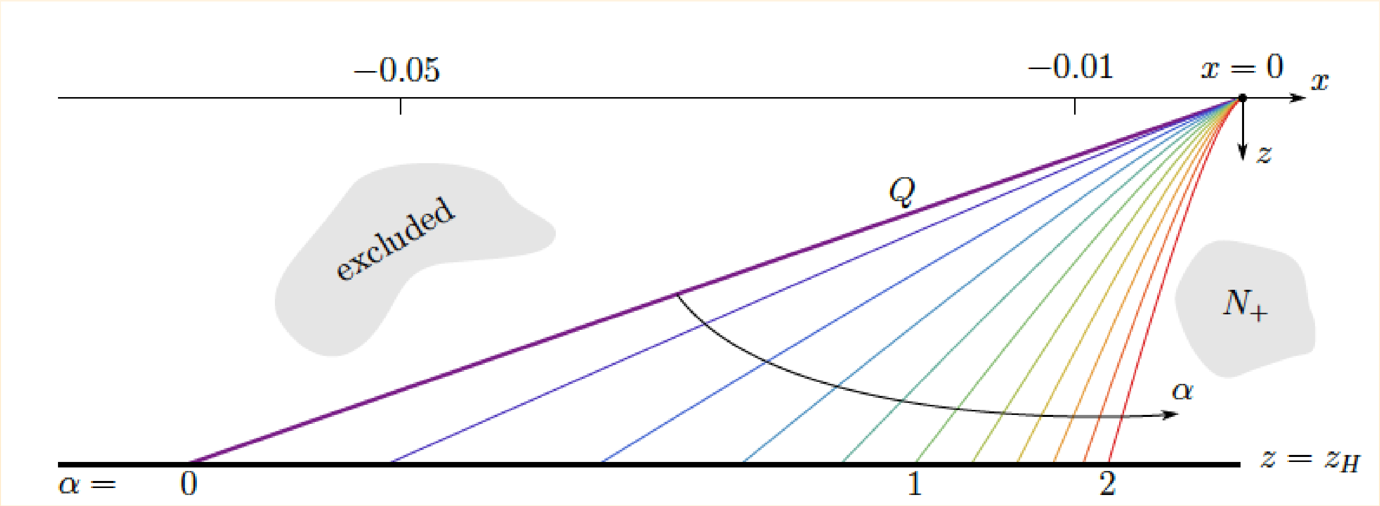
J.E., Flory, Newrzella 1410.7811

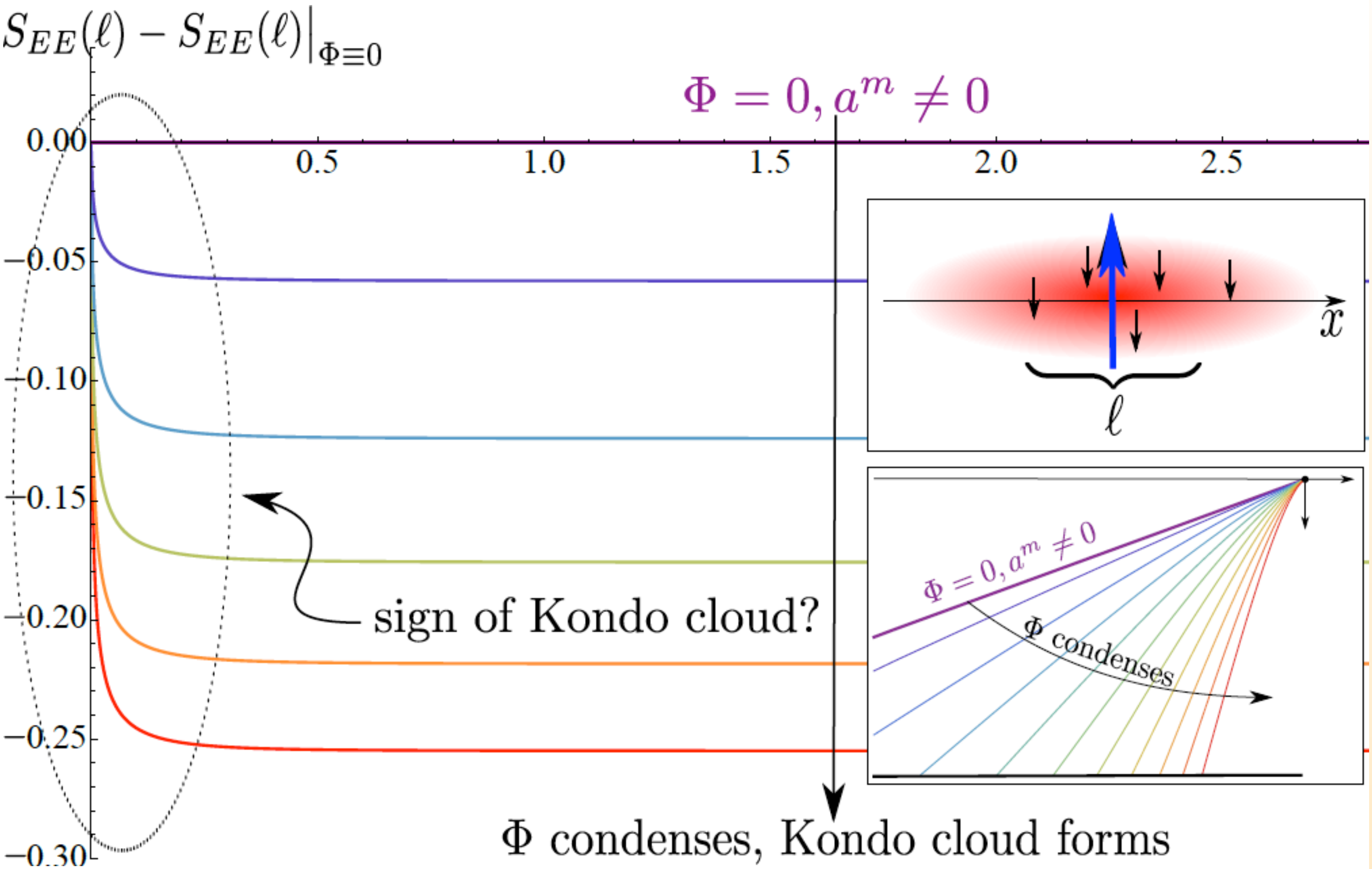




The larger the condensate, the shorter the geodesic

Entanglement entropy

J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu in progress



Entanglement entropy for magnetic impurity: Comparison to field theory

Field theory result

Sorensen, Chang, Laflorencie, Affleck 2007

$$\Delta S_{EE}(\ell) = \tilde{c}_0 + \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T)$$

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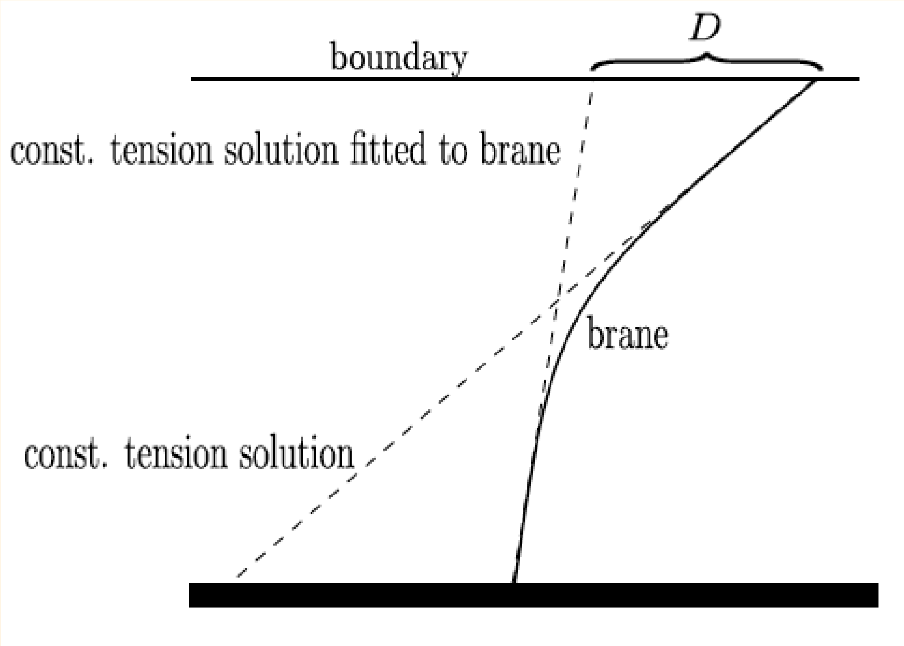
In our gravity approach:



Entanglement entropy for magnetic impurity: Comparison to field theory

On gravity side:

Impurity entropy from difference of entanglement entropies for constant tension branes

$$\Delta S_{EE}(\ell) = c_0 + S_{BH}(\ell + D) - S_{BH}(\ell)$$

$$S_{BH}(\ell) = \frac{c}{3} \ln \left(\frac{1}{\pi \epsilon T} \sinh(2\pi \ell T) \right)$$

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For $D \ll \ell$:

$$\Delta S_{EE}(\ell) \sim c_0 + D \cdot \partial_\ell S_{BH}(\ell) = c_0 + \frac{2\pi c D T}{3} \coth(2\pi \ell T)$$

Agrees with field theory result subject to identification $cD \sim \xi_K$

Universal properties of superconductors

Universality: IR fixed point determines physical properties
Macroscopic properties do not depend on microscopic degrees of freedom

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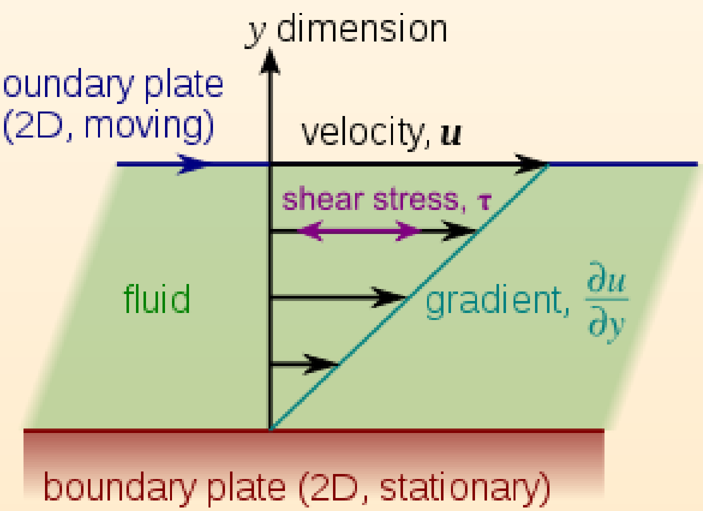
Macroscopic properties do not depend on microscopic degrees of freedom

Example: Universal result from gauge/gravity duality:

Shear viscosity over entropy density:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$



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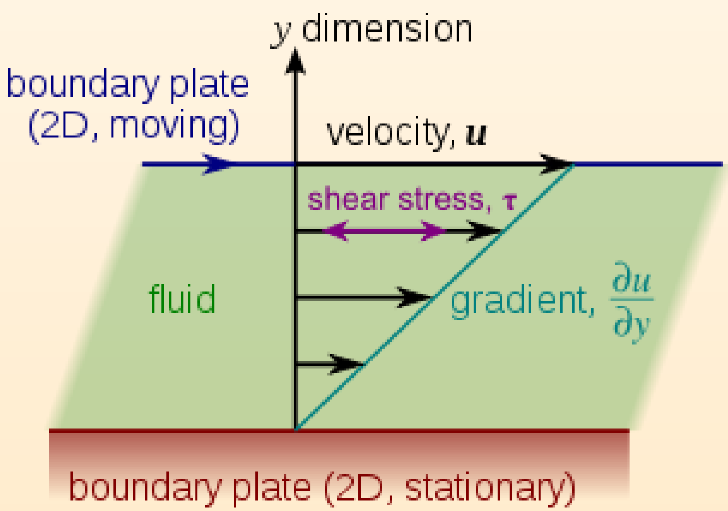


Planckian dissipator: relaxation time $\tau = \frac{\hbar}{k_B T}$

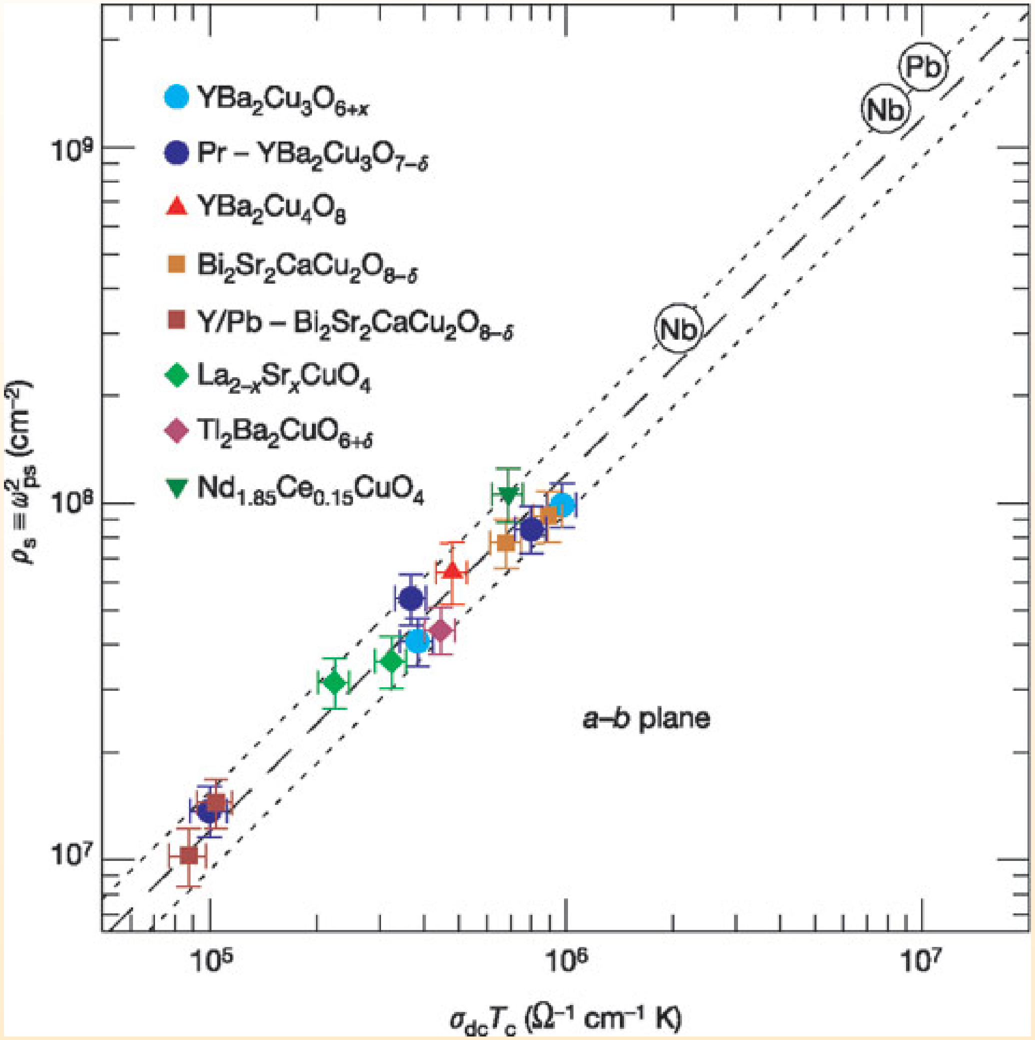
Damle, Sachdev 1997

Universal properties of superconductors

Is there a similiar universal result for applications of the duality within condensed matter physics?

Universal properties of superconductors

Candidate: Homes' relation
$\rho_s(T=0) = C\,\sigma_{\rm DC}(T_c)\,T_c$



C. Homes et al, Nature 2004

Universal properties of superconductors

Homes' relation $\rho_s(T=0) \, = \, C \, \sigma_{\rm DC} \, T_c$

general form may be deduced from Planckian dissipation Zaanen 2004

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J.E., Herwerth, Klug, Meyer, Schalm arXiv:1501.07615:

Investigation of C in a family of gauge/gravity duality models

In particular region of parameter space:

$$C \approx 6.2$$

BCS superconductor in 'dirty limit': $C = 8.1$,
High-T_c superconductors: $C = 4.4$

Holography:

J.E., Kerner Müller 2012

Conditions for identifying ρ_s:

Universal properties of superconductors

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Horowitz, Santos 2013

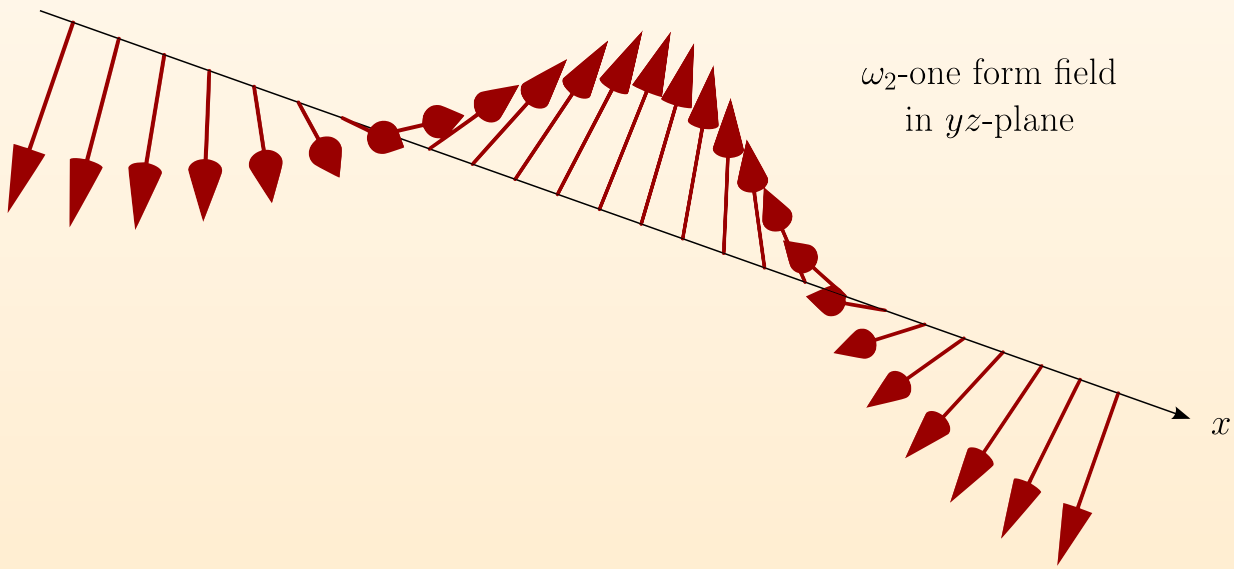
Use background with helical symmetry

(see talk by Koenraad Schalm on Friday)

Background: Helical Bianchi VII symmetry

Donos, Gauntlett 2011; Donos, Hartnoll 2012

Model with broken translation symmetry:



Gauge/gravity duality with helical symmetry

Background: (Hartnoll, Donos)

$$S_{\text{helix}} = \int \mathrm{d}^{4+1}x \sqrt{-g}\left[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2 B_\mu B^\mu\right] - \frac{\kappa}{2}\int B \wedge F \wedge W.$$

$$B = w(r)\omega_2, \qquad w(\infty) = \lambda$$

$$\omega_1 = \mathrm{d}x\,,$$
$$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$$
$$\omega_3 = \sin(px)\,\mathrm{d}y + \cos(px)\,\mathrm{d}z$$

S-wave superconductivity in helical symmetry background

S-wave superconductivity in helical symmetry background

Add charged scalar:

$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[-\left|\partial\rho - \mathrm{i}qA\rho\right|^2 - m_\rho^2|\rho|^2\right]$$

S-wave superconductivity in helical symmetry background

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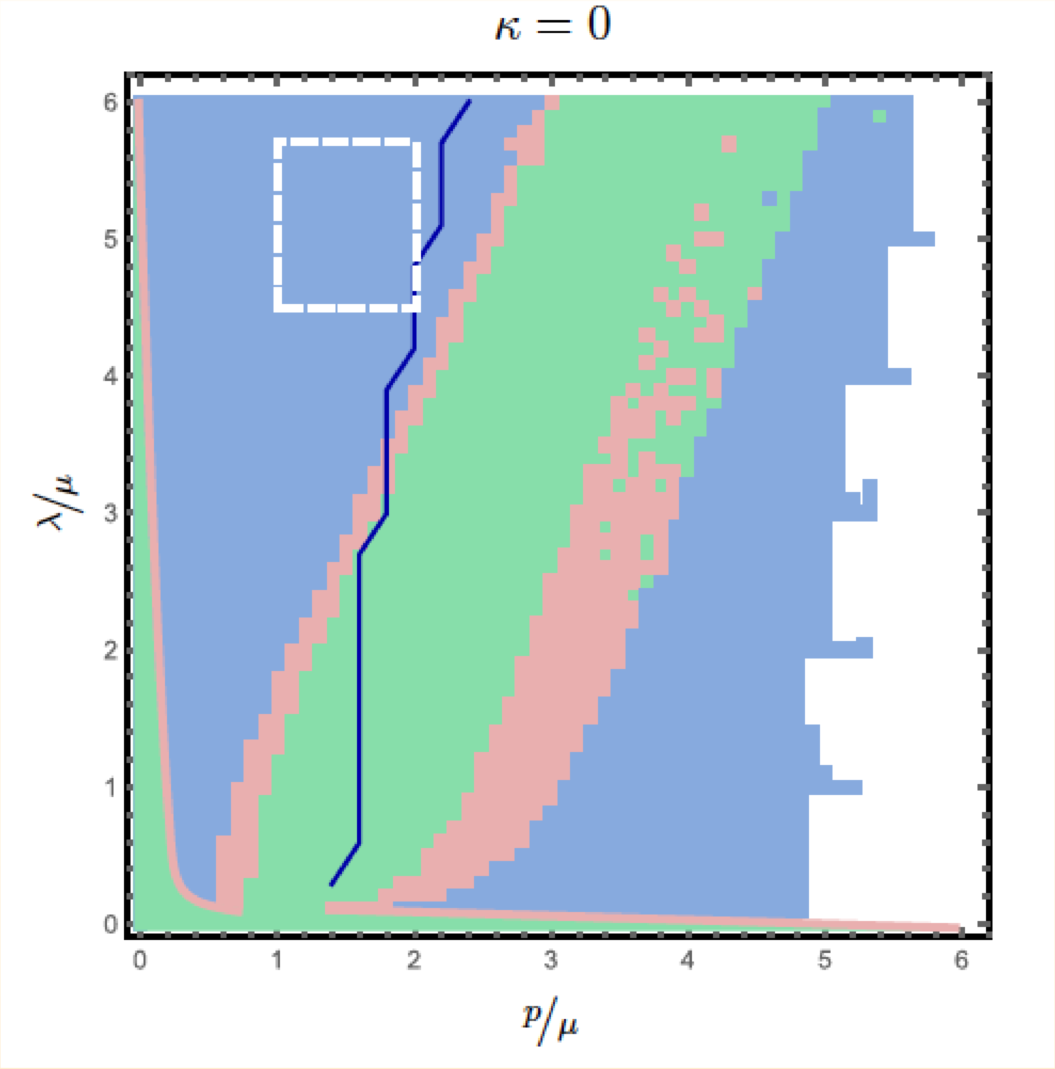
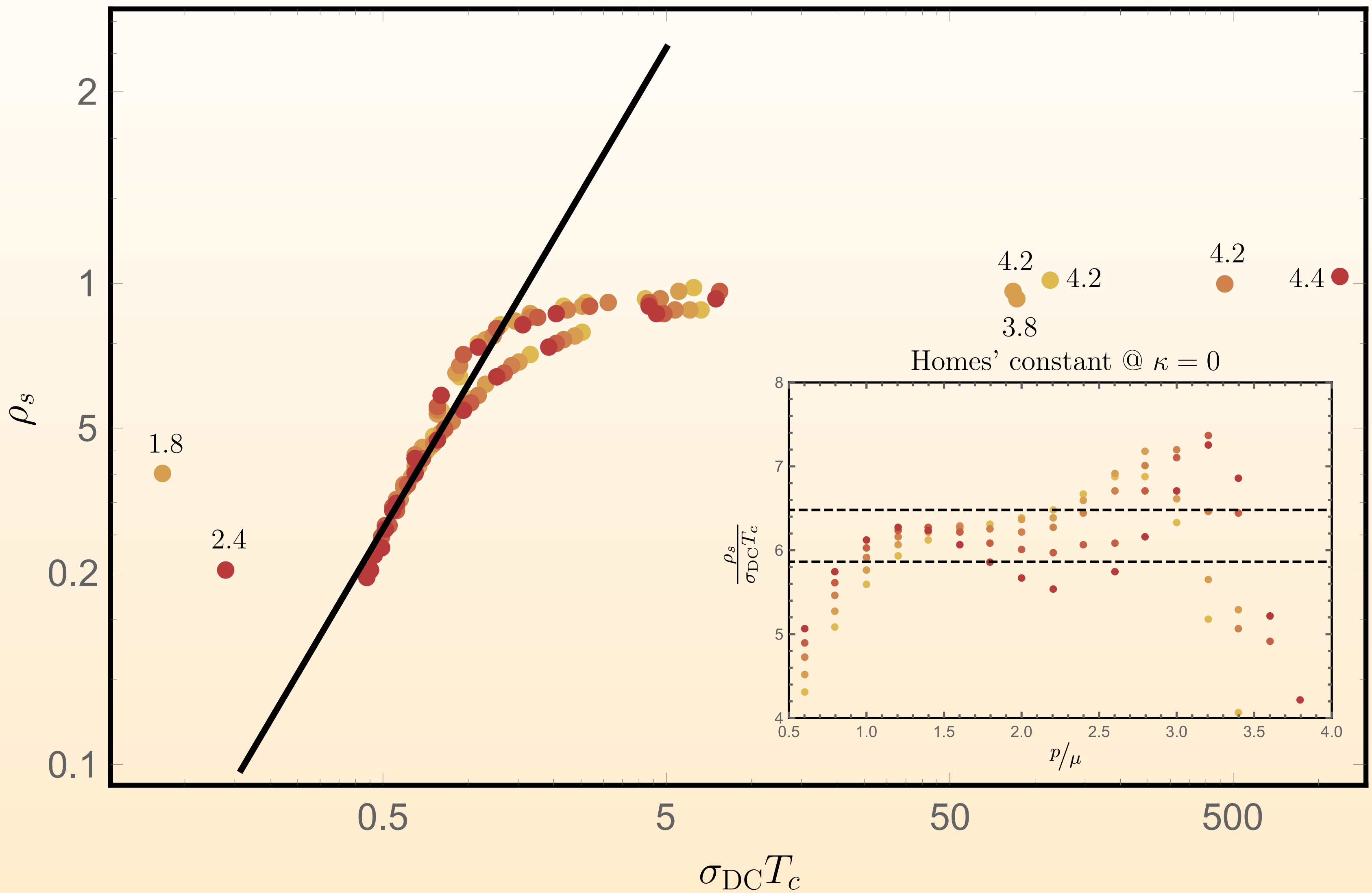
$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x \sqrt{-g} \Big[- |\partial\rho - \mathrm{i}qA\rho|^2 - m_\rho^2 |\rho|^2 \Big]$$

All charged degrees of freedom condense at $T = 0$

Phase diagram



Homes' relation for $q = 6$ & $\kappa = 0$



J.E., Herwerth, Klug, Meyer, Schalm 1501.07615

Conclusions and outlook

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Quantum quench
- Entanglement entropy
- S-wave superconductor in Bianchi VII background:
- Homes' Law

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