Holographic Approaches to Non-Equilibrium Steady States

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"International Workshop on Condensed Matter Physics AdS/CFT"

Kavli IPMU, The University of Tokyo, Japan

29$^{\text{th}}$ May 2015

Outline

- Motivation from condensed matter
- Gauge-gravity duality
- Far from equilibrium dynamics
- Recent work on energy flow
- Einstein equations, hydrodynamics, transport
- Current status and future developments

MJB, Benjamin Doyon, Andrew Lucas, Koenraad Schalm
"Energy flow in quantum critical systems far from equilibrium"
Nature Physics (2015)

Progress in AdS/CMT

Transport Coefficients

Viscosity, Conductivity, Hydrodynamics, Bose–Hubbard, Graphene

Strange Metals

Non-Fermi liquids, instabilities, cuprates

Holographic Duals

Superfluids, Fermi Liquid, $O(N)$, Luttinger Liquid

Equilibrium or close to equilibrium

Utility of Gauge-Gravity Duality

Quantum dynamics **Classical Einstein equations**

Finite temperature **Black holes**

Real time approach to finite temperature quantum dynamics in interacting systems, with the possibility of anchoring to $1+1$ and generalizing to higher dimensions

Non-Equilibrium **Beyond linear response**

Organizing principles out of equilibrium

Quantum Quenches

Simple protocal

Parameter in H abruptly changed

$$H(g) \to H(g')$$

System prepared in state $|\Psi_g\rangle$ but time evolves under $H(g')$

Quantum quench to a CFT

Calabrese & Cardy, PRL **96**, 136801 (2006)

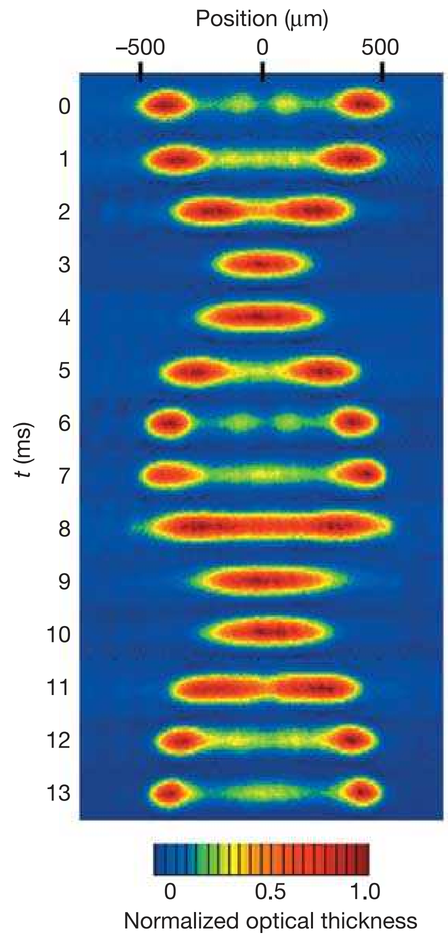
Spin chains, BCS, AdS/CFT ...

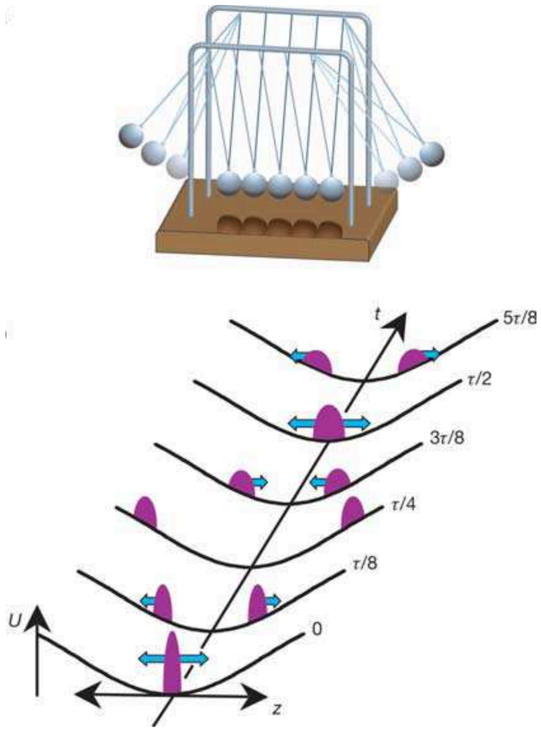
Thermalization

Experiment

Weiss *et al* “A quantum Newton’s cradle”, Nature **440**, 900 (2006)

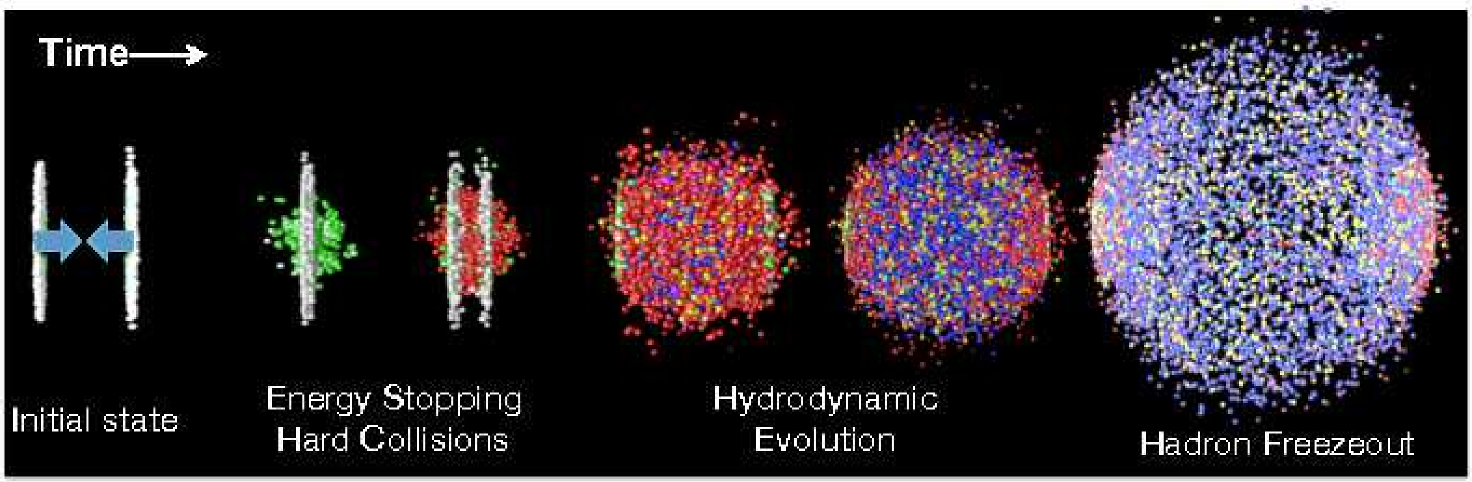
Non-Equilibrium 1D Bose Gas





Integrability and Conservation Laws

Heavy Ion Collisions



Heavy Ions: Results from the Large Hadron Collider, arXiv:1201.4264

Non-Equilibrium High Energy Physics

Thermalization in Strongly Coupled Gauge Theories

Chesler & Yaffe (2009), de Boer & Keski Vakkuri (2011),

Buchel, Lehner, & Myers (2012),

Craps, Lindgren, Taliotis, Vanhoof, & Zhang (2014) ...

Quantum Quenches

Aparício & López (2011), Albash & Johnson (2011), Basu & Das (2012),

Das, Galante, & Myers (2014) ...

Hydrodynamics

Minwalla, Bhattacharyya, Hubeny, Rangamani ...

Non-Equilibrium AdS/CMT

Current Noise

Sonner and Green, *"Hawking Radiation and Nonequilibrium Quantum Critical Current Noise"*, PRL **109**, 091601 (2013)

Hawking Radiation

Quenches in Holographic Superfluids

MJB, Gauntlett, Simons, Sonner & Wiseman, *"Holographic Superfluids and the Dynamics of Symmetry Breaking"* PRL (2013)

Quasi-Normal-Modes

Amado, Kaminski, Landsteiner (09); Murata, Kinoshita, Tanahashi (10); Witczak-Krempa, Sørensen, Sachdev (13)

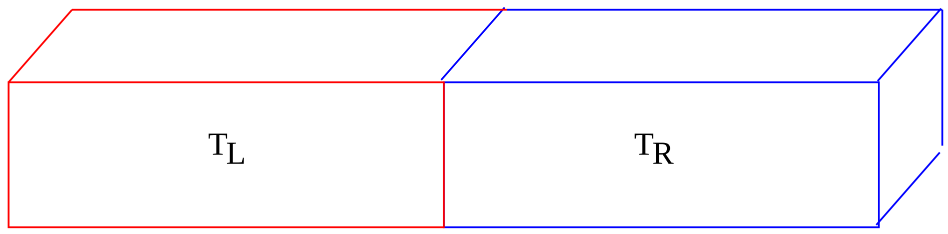
Superfluid Turbulence

Chesler, Liu and Adams, *"Holographic Vortex Liquids and Superfluid Turbulence*, Science **341**, 368 (2013); also arXiv:1307.7267

Fractal Horizons

Thermalization

Condensed matter and high energy physics

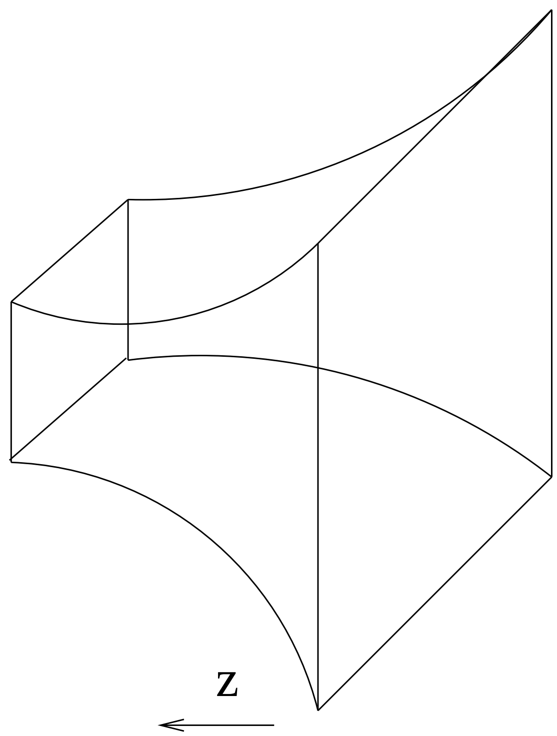


Why not connect two strongly correlated systems together and see what happens?

AdS/CFT

Energy flow may be studied within pure Einstein gravity

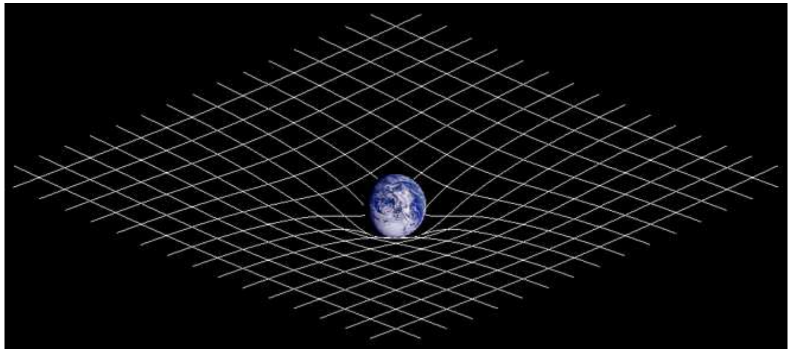
$$S = \frac{1}{16\pi G_{\rm N}} \int d^{d+2}x\, \sqrt{-g}(R - 2\Lambda)$$



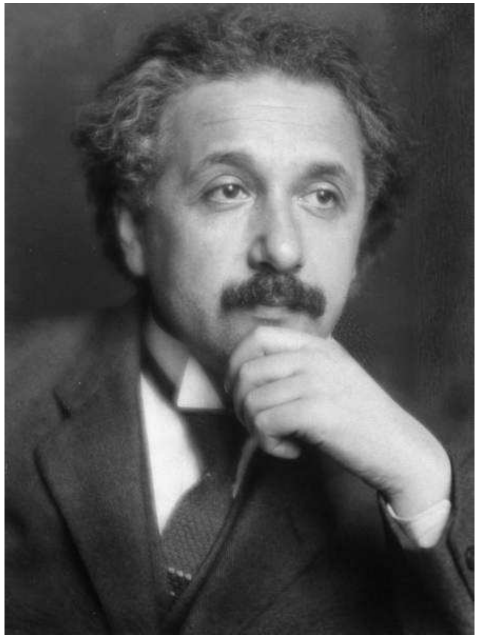
$$g_{\mu\nu} \leftrightarrow T_{\mu\nu}$$

Einstein Centenary

The Field Equations of General Relativity (1915)



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$



http://en.wikipedia.org/wiki/Einstein_field_equations

$g_{\mu\nu}$ metric $R_{\mu\nu}$ Ricci curvature R scalar curvature

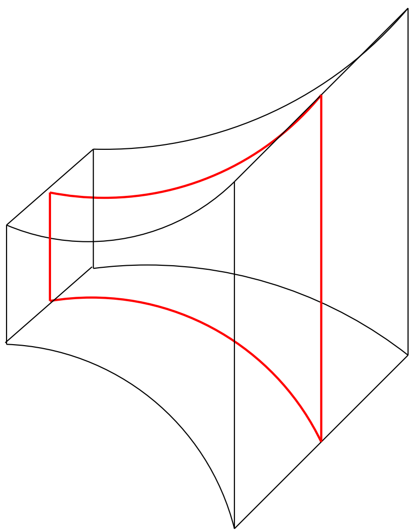
Λ cosmological constant $T_{\mu\nu}$ energy-momentum tensor

Coupled Nonlinear PDEs

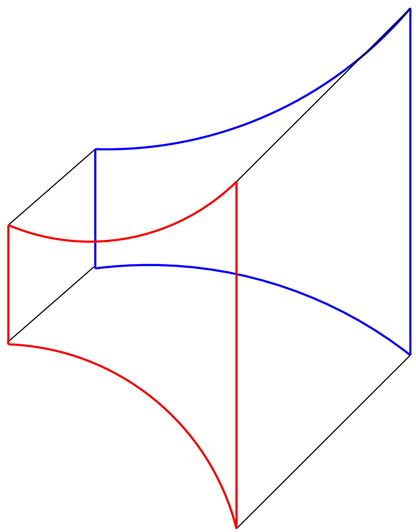
Schwarzchild Solution (1916) $R_{\mathrm{S}} = \frac{2MG}{c^2}$ **Black Holes**

Possible Setups

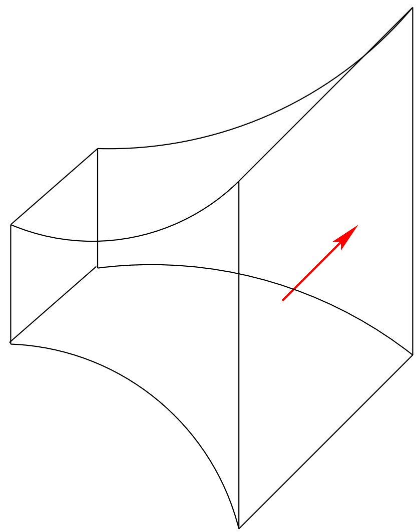
Local Quench



Driven Steady State



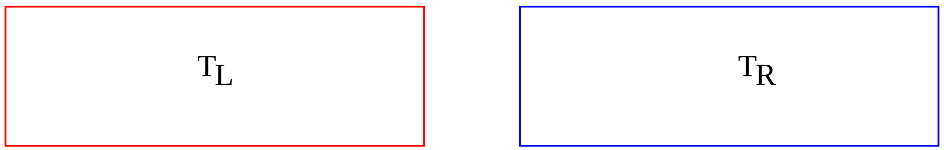
Spontaneous



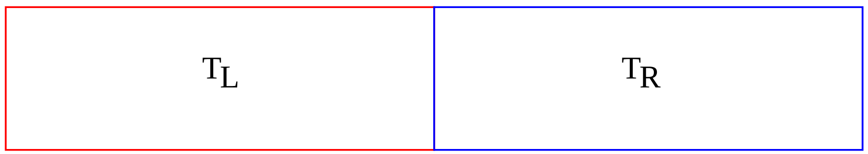
Non-Equilibrium CFT

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

Two critical 1D systems (central charge c)
at temperatures T_L & T_R



Join the two systems together



Alternatively, take one critical system and impose a step profile

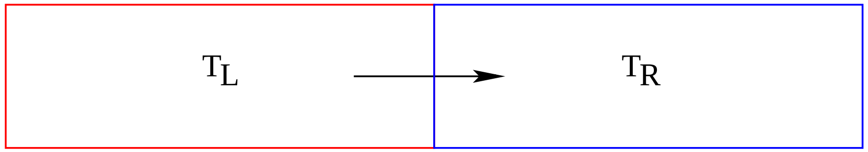
Local Quench

Steady State Energy Flow

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45** 362001 (2012)

If systems are very large ($L \gg vt$) they act like heat baths

For times $t \ll L/v$ a steady heat current flows



Non-equilibrium steady state

$$J = \frac{c\pi^2 k_B^2}{6h}(T_{\mathrm{L}}^2 - T_{\mathrm{R}}^2)$$

Universal result out of equilibrium

Direct way to measure central charge; velocity doesn't enter

Sotiriadis and Cardy. J. Stat. Mech. (2008) P11003

Stefan–Boltzmann

Linear Response

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

$$J = \frac{c\pi^2 k_B^2}{6h}(T_{\mathrm{L}}^2 - T_{\mathrm{R}}^2)$$

$$T_{\mathrm{L}} = T + \Delta T/2 \qquad T_{\mathrm{R}} = T - \Delta T/2 \qquad \Delta T \equiv T_{\mathrm{L}} - T_{\mathrm{R}}$$

$$J = \frac{c\pi^2 k_B^2}{3h} T\Delta T \equiv g\Delta T \qquad g = cg_0 \qquad g_0 = \frac{\pi^2 k_B^2 T}{3h}$$

Quantum of Thermal Conductance

$$g_0 = \frac{\pi^2 k_B^2 T}{3h} \approx (9.456 \times 10^{-13}\,\mathrm{WK}^{-2})\,T$$

Free Fermions

Fazio, Hekking and Khmelnitskii, PRL **80**, 5611 (1998)

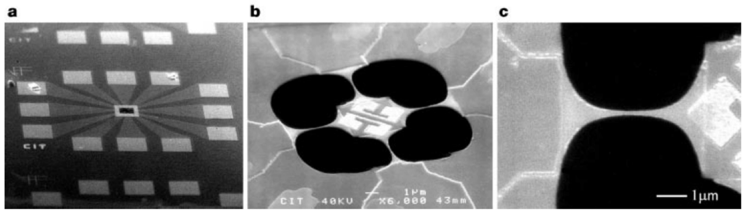
Wiedemann–Franz $\quad \frac{\kappa}{\sigma T} = \frac{\pi^2}{3e^2} \qquad \sigma_0 = \frac{e^2}{h} \qquad \kappa_0 = \frac{\pi^2 k_B^2 T}{3h}$

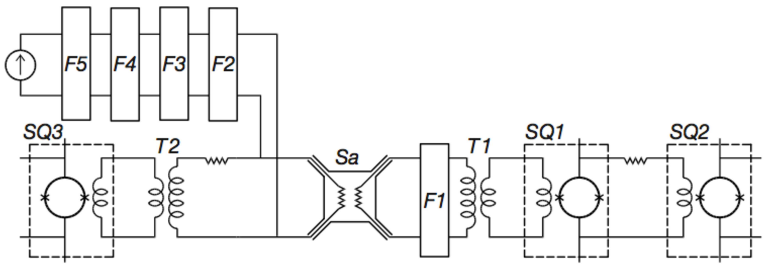
Conformal Anomaly

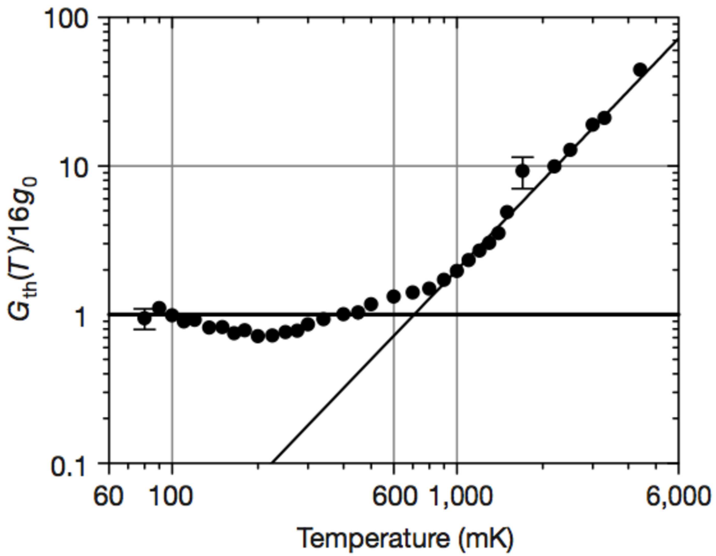
Cappelli, Huerta and Zemba, Nucl. Phys. B **636**, 568 (2002)

Experiment

Schwab, Henriksen, Worlock and Roukes, *Measurement of the quantum of thermal conductance*, Nature **404**, 974 (2000)



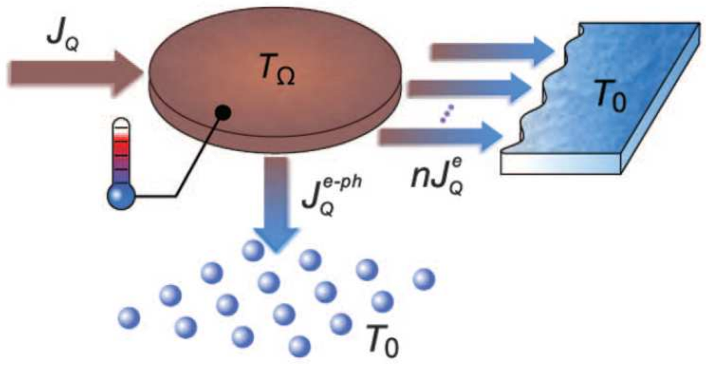


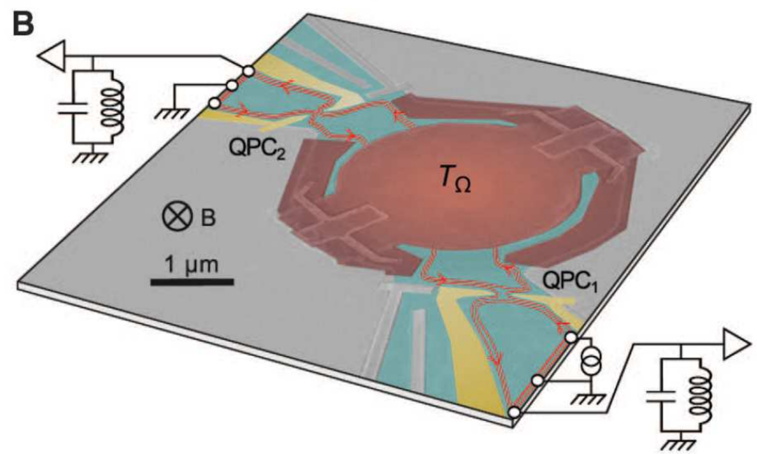


Quantum of Thermal Conductance

Experiment

S. Jezouin *et al*, *"Quantum Limit of Heat Flow Across a Single Electronic Channel"*, Science **342**, 601 (2013)





Electrons heated up by a known Joule power J_Q

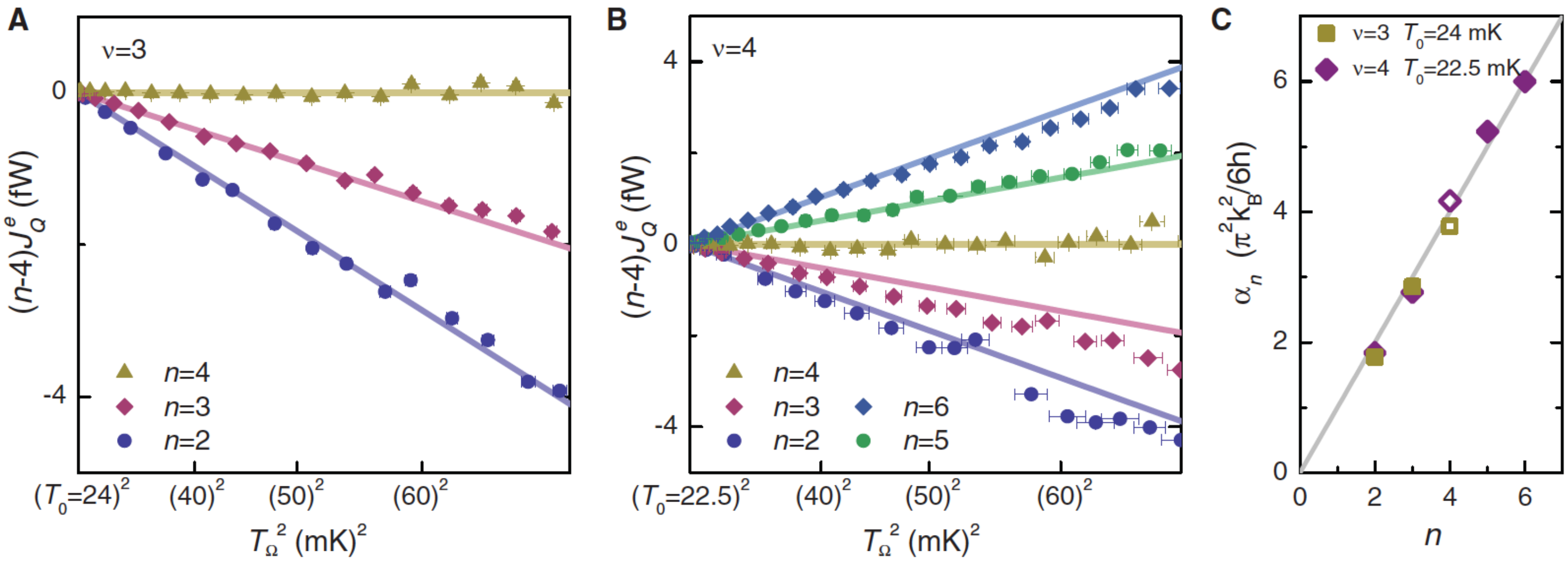
$$J_Q = nJ_Q^e(T_\Omega, T_0) + J_Q^{e-ph}(T_\Omega, T_0)$$

J_Q^e: increase of power to keep T_Ω constant when a channel is opened

$$J_Q^e(T_\Omega, T_0) = \frac{\pi^2 k_B^2}{6h}(T_\Omega^2 - T_0^2)$$

Experiment

S. Jezouin *et al*, *"Quantum Limit of Heat Flow Across a Single Electronic Channel"*, Science **342**, 601 (2013)



$$\frac{J_Q^e(T_\Omega,T_0)}{T_\Omega^2-T_0^2}=(1.06\pm0.07)\times\frac{\pi^2k_B^2}{6h}$$

Energy Current Fluctuations

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

Generating function for all moments

$$F(z) \equiv \lim_{t\to\infty} t^{-1} \ln\langle e^{z\Delta_t Q}\rangle$$

Exact Result

$$F(z) = \frac{c\pi^2}{6h}\left(\frac{z}{\beta_l(\beta_l - z)} - \frac{z}{\beta_r(\beta_r + z)}\right)$$

$$F(z) = \frac{c\pi^2}{6h}\left[z\left(\frac{1}{\beta_l^2} - \frac{1}{\beta_r^2}\right) + z^2\left(\frac{1}{\beta_l^3} + \frac{1}{\beta_r^3}\right) + \ldots\right]$$

$$\langle J\rangle = \frac{c\pi^2}{6h} k_B^2 (T_{\mathrm{L}}^2 - T_{\mathrm{R}}^2) \qquad \langle \delta J^2\rangle \propto \frac{c\pi^2}{6h} k_B^3 (T_{\mathrm{L}}^3 + T_{\mathrm{R}}^3)$$

Poisson Process $\int_0^\infty e^{-\beta\epsilon}(e^{z\epsilon} - 1)d\epsilon = \frac{z}{\beta(\beta - z)}$

Non-Equilibrium Fluctuation Relation

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

$$F(z) \equiv \lim_{t\to\infty} t^{-1} \ln\langle e^{z\Delta_t Q}\rangle = \frac{c\pi^2}{6h}\left(\frac{z}{\beta_l(\beta_l - z)} - \frac{z}{\beta_r(\beta_r + z)}\right)$$

$$F(-z) = F(z + \beta_l - \beta_r)$$

Irreversible work fluctuations in isolated driven systems

Crooks relation $$\frac{P(W)}{\tilde{P}(-W)} = e^{\beta(W-\Delta F)}$$

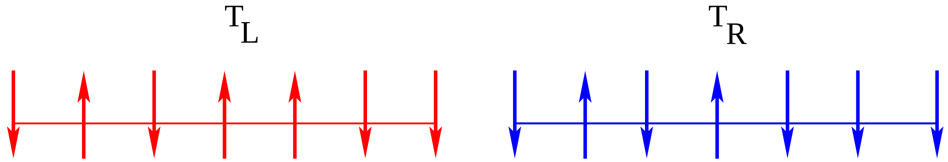
Jarzynski relation $$\langle e^{-\beta W}\rangle = e^{-\beta\Delta F}$$

Entropy production in non-equilibrium steady states

$$\frac{P(S)}{P(-S)} = e^{S}$$

Esposito *et al*, *"Nonequilibrium fluctuations, fluctuation theorems, and counting statistics in quantum systems"*, RMP **81**, 1665 (2009)

Lattice Models



Quantum Ising Model

$$H = J \sum_{\langle ij \rangle} S_i^z S_j^z + \Gamma \sum_i S_i^x$$

$\Gamma = J/2$ Critical $c = 1/2$

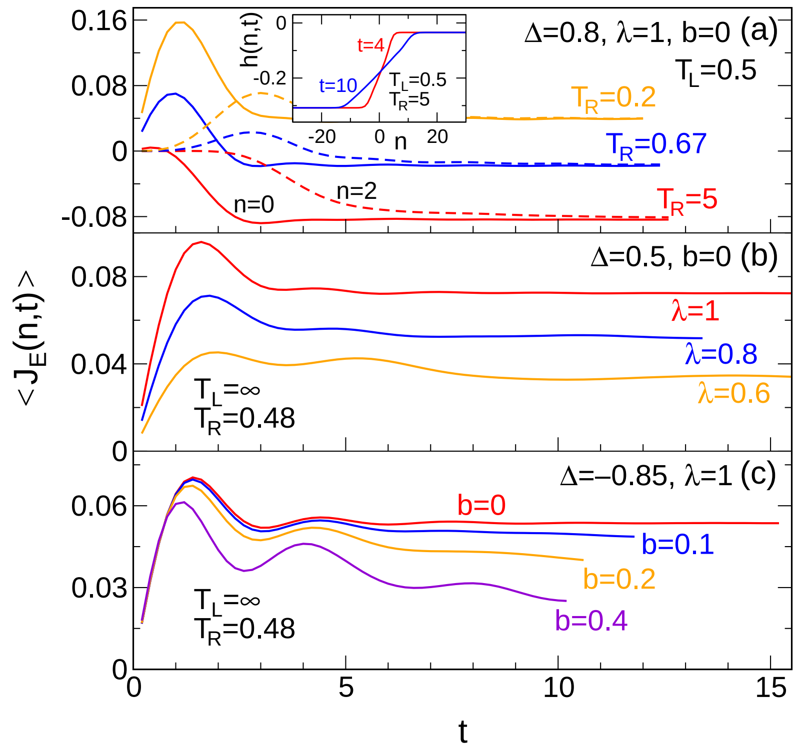
Anisotropic Heisenberg Model (XXZ)

$$H = J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right)$$

$-1 < \Delta < 1$ Critical $c = 1$

Time-Dependent DMRG

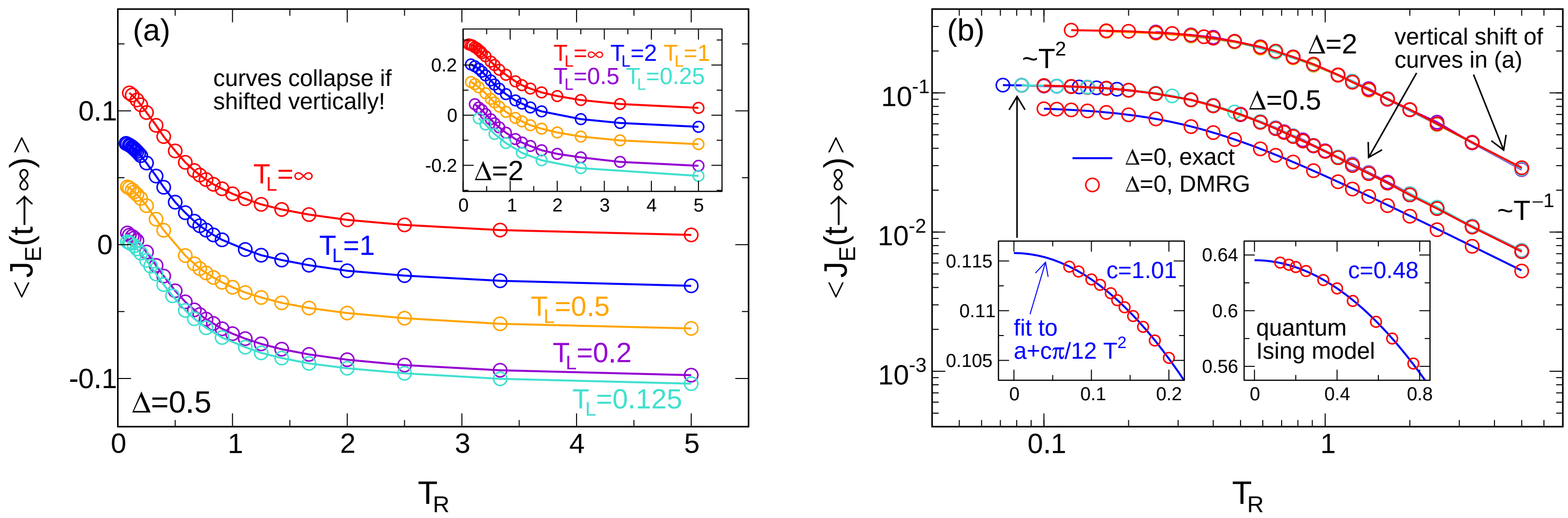
Karrasch, Ilan and Moore, *Non-equilibrium thermal transport and its relation to linear response*, Phys. Rev. B **88**, 195129 (2013)



$$\text{Dimerization } J_n = \begin{cases} 1 & n\,\text{odd} \\ \lambda & n\,\text{even} \end{cases} \quad \Delta_n = \Delta \quad \text{Staggered } b_n = \frac{(-1)^n b}{2}$$

Time-Dependent DMRG

Karrasch, Ilan and Moore, *Non-equilibrium thermal transport and its relation to linear response*, Phys. Rev. B **88**, 195129 (2013)

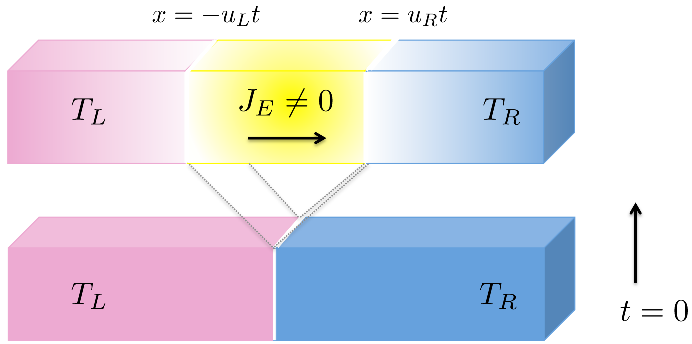


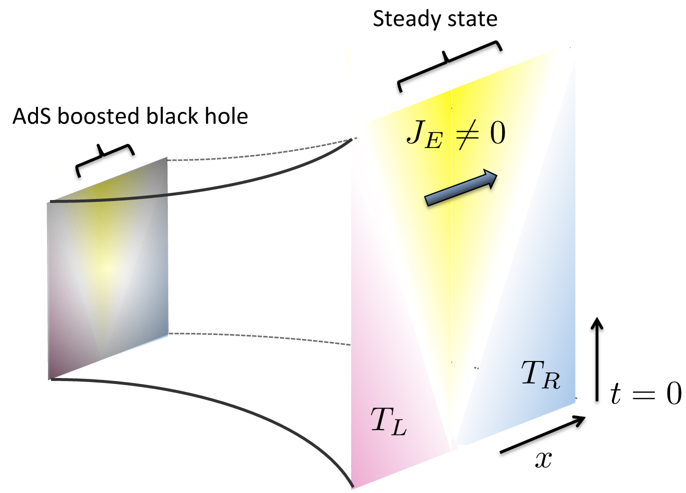
$$\lim_{t\to\infty}\langle J_E(n,t)\rangle = f(T_L) - f(T_R)$$

$$f(T) \sim \begin{cases} T^2 & T \ll 1 \\ T^{-1} & T \gg 1 \end{cases}$$

Beyond CFT to massive integrable models (Doyon)

AdS/CFT





Steady State Region

Spatially Homogeneous

Solutions of Einstein Equations

$$S = \frac{1}{16\pi G_N} \int d^{d+2}x \sqrt{-g}(R - 2\Lambda)$$

$$\Lambda = -d(d+1)/2L^2$$

Unique homogeneous solution = boosted black hole

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[\frac{\mathrm{d}z^2}{f(z)} - f(z)(\mathrm{d}t\cosh\theta - \mathrm{d}x\sinh\theta)^2 + (\mathrm{d}x\cosh\theta - \mathrm{d}t\sinh\theta)^2 + \mathrm{d}y_\perp^2\right]$$

$$f(z) = 1 - \left(\frac{z}{z_0}\right)^{d+1} \qquad z_0 = \frac{d+1}{4\pi T}$$

Fefferman–Graham Coordinates

$$\langle T_{\mu\nu}\rangle_{\mathrm{s}} = \frac{L^d}{16\pi G_{\mathrm{N}}} \lim_{Z\to 0}\left(\frac{d}{dZ}\right)^{d+1} \frac{Z^2}{L^2}\, g_{\mu\nu}(z(Z))$$

$$z(Z) = Z/R - (Z/R)^{d+2}/[2(d+1)z_0^{d+1}] \qquad R = (d!)^{1/(d-1)}$$

Boost Solution

Lorentz boosted stress tensor of a finite temperature CFT

Perfect fluid

$$\langle T^{\mu\nu}\rangle_{\rm s} = a_d\, T^{d+1}\left(\eta^{\mu\nu} + (d+1)u^\mu u^\nu\right)$$

$$\eta^{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1), \qquad u^\mu = (\cosh\theta, \sinh\theta, 0, \ldots, 0)$$

$$\langle T^{tx}\rangle_{\rm s} = \tfrac{1}{2} a_d\, T^{d+1}(d+1)\sinh 2\theta$$

$$a_d = (4\pi/(d+1))^{d+1} L^d / 16\pi G_{\rm N}$$

One spatial dimension

$$a_1 = \frac{L\pi}{4G_{\rm N}} \qquad c = \frac{3L}{2G_{\rm N}}$$

$$T_{\rm L} = T e^{\theta} \qquad T_{\rm R} = T e^{-\theta} \qquad \langle T_{tx}\rangle = \frac{c\pi^2 k_B^2}{6h}(T_{\rm L}^2 - T_{\rm R}^2)$$

Run past a thermal state with temperature $T = \sqrt{T_L T_R}$

Steady State Density Matrix

$$\langle \mathcal{O} \ldots \rangle = \frac{\mathrm{Tr}(\rho_s \mathcal{O} \ldots)}{\mathrm{Tr}(\rho_s)}$$

$$\rho_s = e^{-\beta E \cosh\theta + \beta P_x \sinh\theta}$$

$$\beta = \sqrt{\beta_{\mathrm{L}} \beta_{\mathrm{R}}} \qquad e^{2\theta} = \frac{\beta_{\mathrm{R}}}{\beta_{\mathrm{L}}}$$

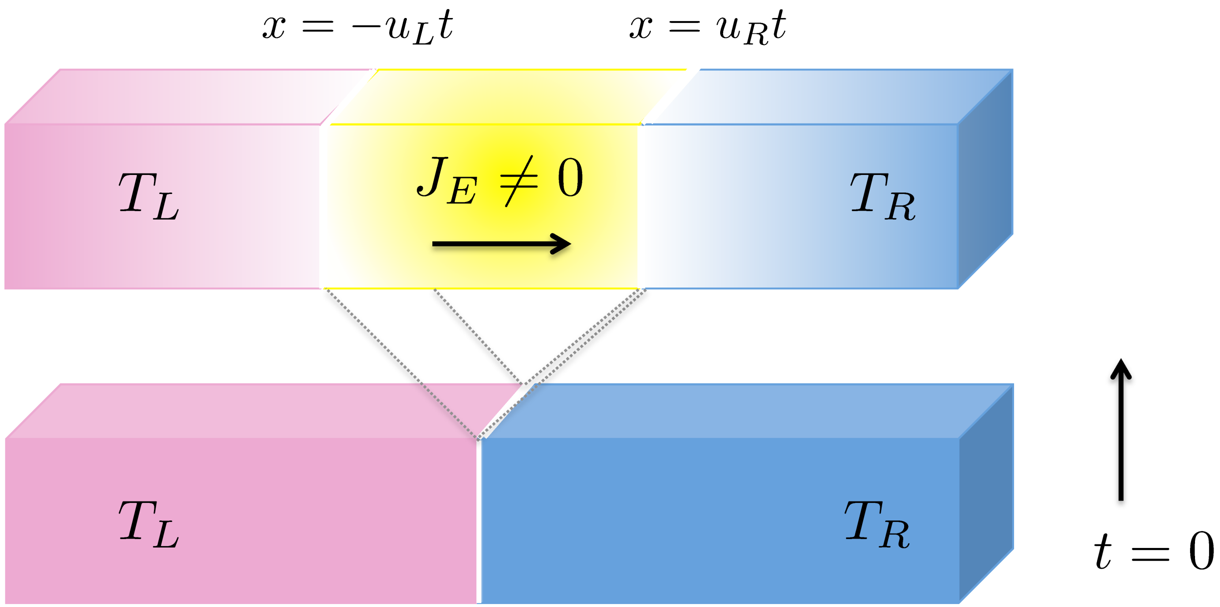
Lorentz boosted thermal density matrix

Describes all the cumulants of the energy transfer process

The non-equilibrium steady state (NESS)
is a Lorentz boosted thermal state

Higher Dimensions

Shock Waves



Energy-Momentum conservation across the shocks

Shock Solutions

Rankine–Hugoniot

Energy-Momentum conservation $\partial_\mu T^{\mu\nu} = 0$ across shock

$$\langle T^{tx} \rangle_{\mathrm{s}} = a_d \left(\frac{T_{\mathrm{L}}^{d+1} - T_{\mathrm{R}}^{d+1}}{u_{\mathrm{L}} + u_{\mathrm{R}}} \right)$$

Invoking boosted steady state gives $u_{\mathrm{L,R}}$ in terms of $T_{\mathrm{L,R}}$:

$$u_{\mathrm{L}} = \frac{1}{d}\sqrt{\frac{\chi+d}{\chi+d^{-1}}} \qquad u_{\mathrm{R}} = \sqrt{\frac{\chi+d^{-1}}{\chi+d}} \qquad \chi \equiv (T_{\mathrm{L}}/T_{\mathrm{R}})^{(d+1)/2}$$

Steady state region is a boosted thermal state with $T = \sqrt{T_{\mathrm{L}} T_{\mathrm{R}}}$

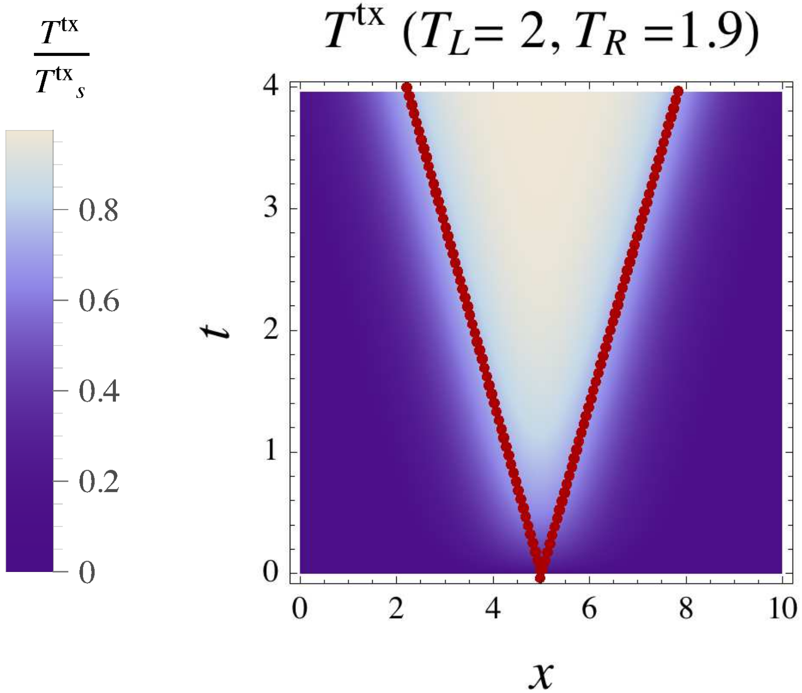
Boost velocity $(\chi - 1)/\sqrt{(\chi + d)(\chi + d^{-1})}$ Agrees with $d = 1$

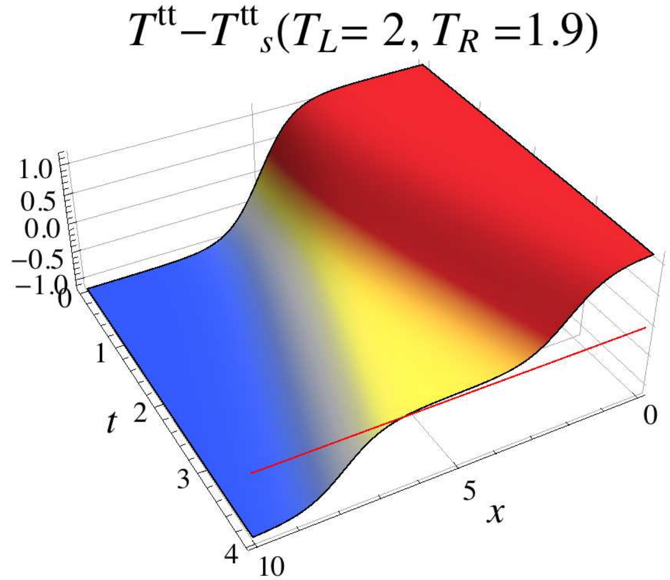
Shock waves are non-linear generalizations of sound waves

EM conservation: $u_{\mathrm{L}} u_{\mathrm{R}} = c_{\mathrm{s}}^2$, where $c_{\mathrm{s}} = v/\sqrt{d}$ is speed of sound

$c_s < u_{\mathrm{R}} < v \qquad c_s < u_{\mathrm{L}} < c_s^2/v$ reinstated microscopic velocity v

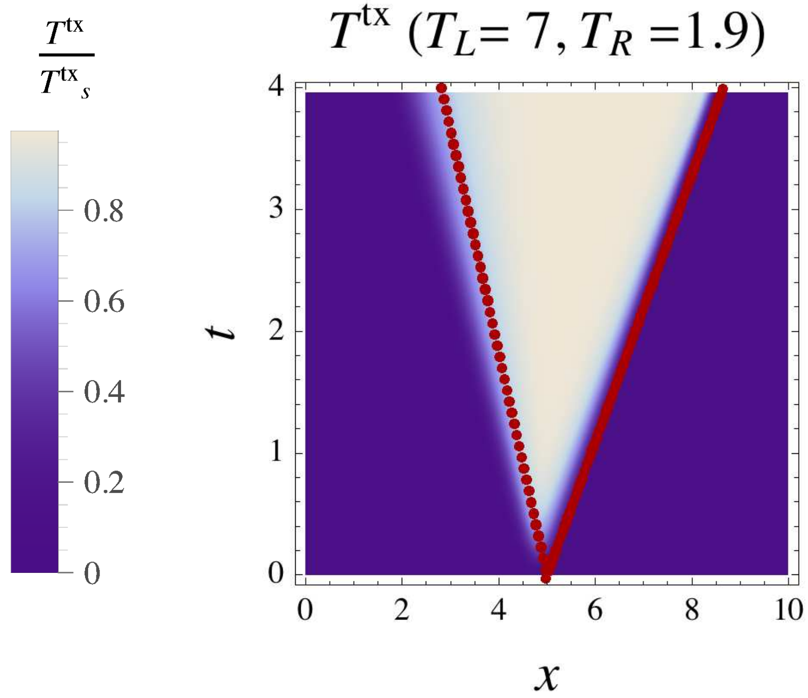
Numerics I

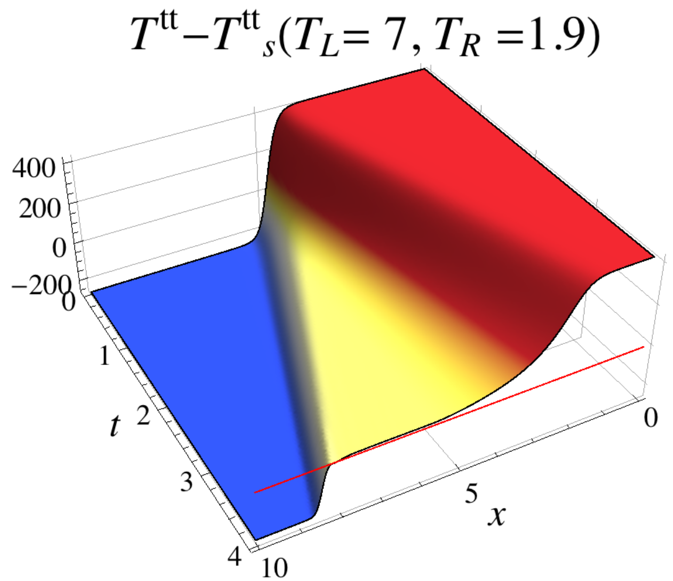




Excellent agreement with predictions

Numerics II





Excellent agreement far from equilibrium

Asymmetry in propagation speeds

Conclusions

Average energy flow in arbitrary dimension

Lorentz boosted thermal state

Energy current fluctuations

Exact generating function of fluctuations

Generalisations

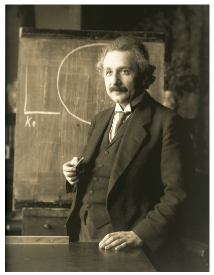
Other types of charge noise Non-Lorentz invariant situations

Different central charges Fluctuation theorems Numerical GR

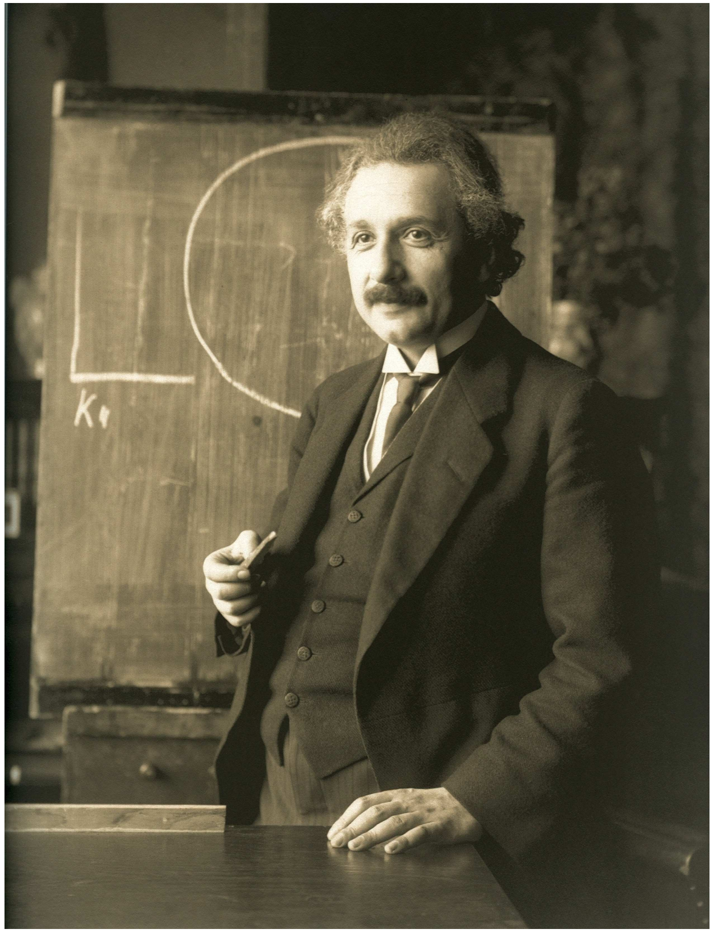
Acknowledgements

B. Benenowski, D. Bernard, P. Chesler, A. Green
D. Haldane C. Herzog, D. Marolf, B. Najian, C.-A. Pillet
S. Sachdev, A. Starinets

100 years after the advent of General Relativity
Einstein still has an enormous amount to teach us



100 years after the advent of General Relativity



Einstein still has an enormous amount to teach us

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