



THE QUEST FOR THE GRAVITON

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Why gravitons?

In 2015, LIGO directly detected gravitational waves.

➡ Gravity has dynamical degrees of freedom.

Quantum theory is fundamental.

Hence, quantum gravity seems inevitable.

Gravitons are quanta of gravitational waves and the hallmarks of quantum gravity. Therefore, the detection of gravitons is of importance.

Indeed, with the Higgs boson now discovered, the graviton is the only missing particle that everyone believes exists.

Gravity and quantum mechanics become important at the Planck energy scale

$$GE^2 = \left(\frac{E}{M_p}\right)^2 = \left(\frac{E}{10^{19}\text{GeV}}\right)^2$$

It is impossible to create a graviton at an accelerator!!

We may need cosmology.

In fact, primordial gravitational waves are nothing but gravitons created during inflation. Thus, the detection of gravitons is crucial to understanding the early universe.

It is natural to ask if detection of a graviton is possible or not.

If the answer is negative, namely, "Detection of a graviton is in principle impossible" that would have three meanings.

- Meaning (a): We can prove a theorem asserting that detection of a graviton would contradict the laws of physics.

Although no one has succeeded in proving that the graviton does not exist, I will discuss the meaning of this possibility.

- Meaning (b): We have examined a class of possible graviton detectors and demonstrated that they cannot work.

More specifically, we show the interferometry cannot detect a graviton.

- Meaning (c): We have examined a class of graviton detectors and demonstrated that they cannot work in the environment provided by the real world.

Dyson considered photo-electric effect and Gertsenshtein effect as examples. I will explain the photo-electric effect in detail.

Meaning(a): the graviton does not exist

Suppose it is proved that the graviton does not exist in the sense of Meaning (a),
What does that mean?

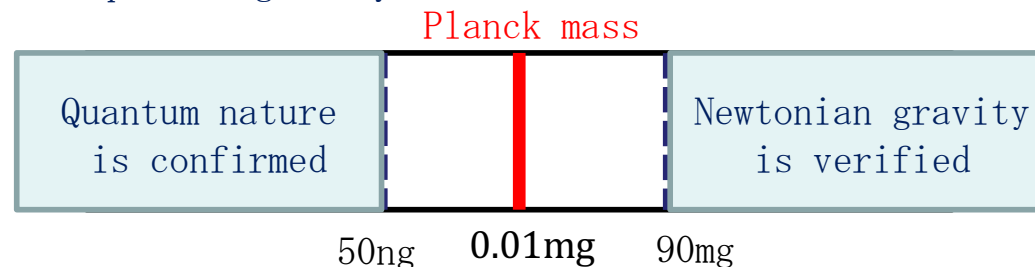
“How can we experimentally verify that these waves are quantized?

Maybe they are not. Maybe gravity is a way that quantum mechanics fails at large.”

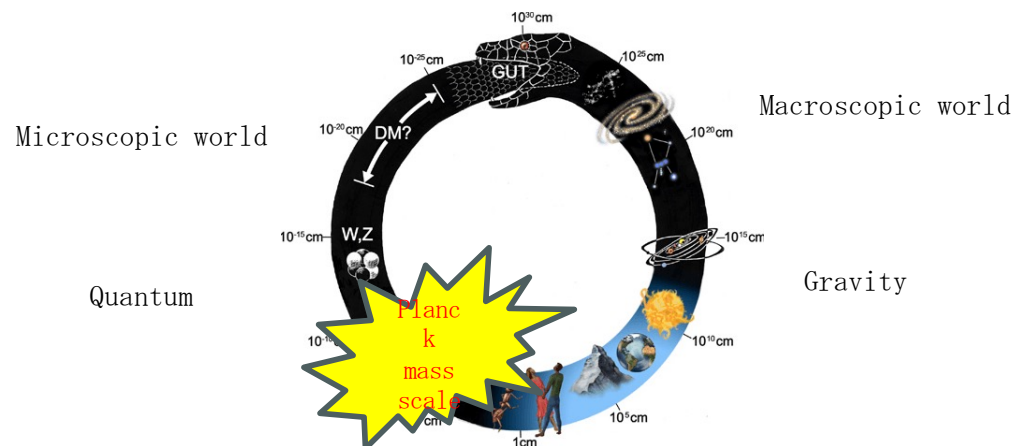
“If this failure of quantum mechanics is connected with gravity, we might speculatively expect this to happen for masses near the Planck mass 0.01 mg, which corresponds to 10^{18} particles.”

--- *Richard Feynman from arXiv:2102.11220*

Experimental status of quantum gravity



Paradigm crisis?



Meaning (b): LIGO cannot detect a graviton

Dyson 2013

The reason is simple. GWs detected at LIGO have

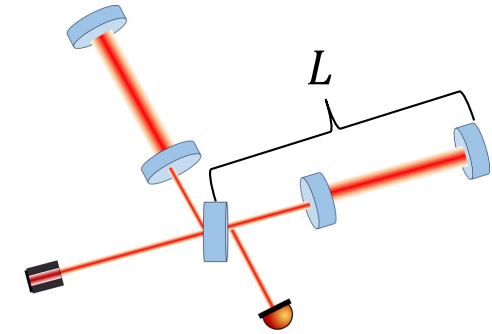
a frequency 1kHz and an amplitude $h \approx 10^{-21}$.

energy density $\rho = \frac{c^2}{32\pi G} \omega^2 h^2$ $\omega = 1\text{kHz}$, $h = 10^{-21} \Rightarrow 10^{-10} \text{erg/cm}^3$

energy density of a single graviton is at most $\rho_s = \frac{\hbar \omega^4}{c^3} \Rightarrow 10^{-47} \text{erg/cm}^3$

number of gravitons detected is at least $n_g = \frac{1}{32\pi} \frac{c^3}{G\hbar} \left(\frac{c}{\omega}\right)^2 h^2 = 10^{37}$

Thus, LIGO cannot resolve individual gravitons.



If we want to make the number to be order unity $n_g \approx 1$, we must have $h \frac{c}{\omega} \approx \ell_p$.

Hence, the variation $\delta \approx hL \approx h \frac{c}{\omega}$ of the arm length L becomes the Planck length ℓ_p .

Using the uncertainty principle $M\delta^2 \geq \hbar T \geq \hbar \frac{L}{c}$,

we can show the detector collapses into a black hole $\frac{GM}{c^2} \geq L$.

Thus, it is impossible to detect a single graviton by any GW interferometers.
This is the impossibility in the Meaning (b).

Meaning (c): gravito-electric effect

To detect a single graviton, we can consider analog of photo-electric effect.

(1) High-frequency gravitational waves

graviton with a frequency $1\text{eV} \approx 10^{15}\text{Hz}$ or more higher frequencies

(2) A novel detection method with a gravito-electric effect

the cross section for interaction of a graviton with an atom is ℓ_P^2 .
the absorption cross section per gram

$$N_A \times \ell_P^2 \approx 10^{-40} \text{cm}^2/\text{g}$$

(3) Possible sources

Let us take thermal gravitons from the sun with a flux $F \approx 10^{-4} / \text{cm}^2 / \text{s}$

If we regard the earth $M \approx 10^{27} \text{g}$ as a detector, we obtain the rate

$$N_A \times \ell_P^2 \times M \times F \approx 10^{-17} / \text{s}$$

which is too small. Thus, it is difficult to detect a graviton.

If we consider white dwarf or neutron star, we have larger rate,
however, the neutrino noise is severe.

Hence, it is impossible to detect a graviton in the Meaning (c).

How to circumvent the difficulty?

As we mentioned, the difficulty stems from the large number of gravitons.

Hence, one way to circumvent it is to reduce the number of gravitons.

To detect a single graviton, we need to consider high-frequency gravitational waves.

We need new tabletop experiments where quantum sensing would be important.

Ito, Ikeda, Miuchi and Soda,
“Probing GHz gravitational waves with graviton-magnon resonance,”
Eur. Phys. J. C80, 179 (2020) [arXiv:1903.04843].

We will not talk about this direction today.

Alternatively, we can utilize the large number of gravitons.

In order to circumvent the difficulty raised by Dyson, we simply ask a different question.

Is the graviton state detectable?

In this talk, we try to answer the following three questions:

Can we observe a squeezed graviton state from inflation through the decoherence?

What is a quantum state of GWs from binary black holes?

Can we see quantum nature of gravitons through gravito-phononic effects?

Plan of my talk



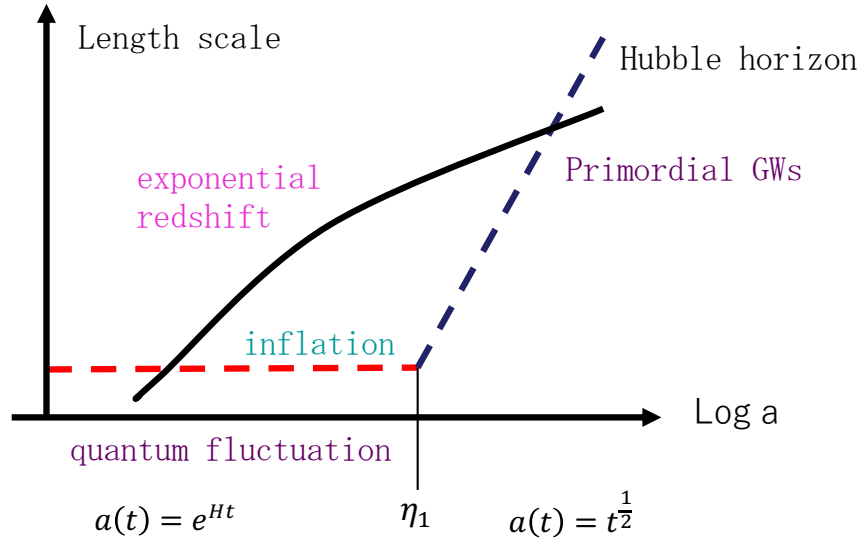
1. Dyson impossibility
2. How can we circumvent it?
3. Inflation creates a squeezed state of gravitons
4. Graviton state detection through decoherence
5. Quantum state description of GWs
6. Graviton detection with gravito-phononic effect
7. Conclusions



INFLATION CREATES A SQUEEZED STATE OF GRAVITONS

PGWs from inflation

GWs in cosmological background: $S = \frac{M_p^2}{4} \int d\eta d^3x a^2(\eta) \left[\frac{1}{2} h'_{ij} h'_{ij} - \frac{1}{2} h_{ij,k} h_{ij,k} \right]$



$$ds^2 = a^2(\eta) [-d\eta^2 + \delta_{ij} dx^i dx^j]$$

$$a(\eta) = \begin{cases} \frac{1}{-H(\eta - 2\eta_1)} & \text{inflation} \\ \frac{\eta}{H\eta_1^2} & \text{radiation} \end{cases}$$

$$k_c = 2\pi f_c = \frac{1}{|\eta_1|}$$

Canonical quantization

$$h_{ij}(x^i, \eta) = \frac{2}{a(\eta)M_p} \int \frac{d^3k}{(2\pi)^{3/2}} \sum_P [e_{ij}^P(k^i) u_k(\eta) a_P(k^i) + e_{ij}^P(-k^i) u_k^*(\eta) a_P^\dagger(-k^i)] e^{i\vec{k} \cdot \vec{x}}$$

$$e_{ij}^P(k^i) e_{ij}^Q(k^i) = \delta^{PQ}$$

$$[a_P(k^i), a_Q^\dagger(p^i)] = \delta_{PQ} \delta(k^i - p^i)$$

$$i u_k^* \partial_\eta u_k - i u_k \partial_\eta u_k^* = 1$$

$$\left(\frac{d^2}{d\eta^2} + k^2 - \frac{a''}{a} \right) u_k(\eta) = 0$$

Bogoliubov transformation

BD vacuum

$$u_k(\eta) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k(\eta - 2\eta_1)} \right) e^{-ik(\eta - 2\eta_1)} \xleftarrow{\eta < \eta_1} U_k(\eta) \xrightarrow{\eta > \eta_1} \alpha_k v_k(\eta) + \beta_k v_k^*(\eta)$$

BD vacuum $a_P(k^i)|BD\rangle = 0$ inflation radiation $b_P(k^i)|0_R\rangle = 0$

$$\xleftarrow{\eta < \eta_1} V_k(\eta) \xrightarrow{\eta > \eta_1} v_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

Since both mode functions are linearly independent and span a complete set, we can define

Bogoliubov transformation $U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^*(\eta)$

Junction conditions at the end of inflation give rise to

$$u_k(\eta_1) = \alpha_k v_k(\eta_1) + \beta_k v_k^*(\eta_1) \quad u'_k(\eta_1) = \alpha_k v'_k(\eta_1) + \beta_k v'^*_k(\eta_1)$$

$$\longrightarrow \alpha_k = \left(1 + \frac{i}{k\eta_1} - \frac{1}{2k^2\eta_1^2} \right) e^{2ik\eta_1} \quad \beta_k = \frac{1}{2k^2\eta_1^2}$$

Squeezed graviton state

Grishchuk 1998

We have two ways to represent the same field. Thus, we have a relation

$$a_P(k^i) = \alpha_k^* b_P(k^i) - \beta_k^* b_P^\dagger(-k^i)$$

The BD vacuum is the solution of the following equation

$$a_P(k^i)|BD\rangle = \left[\alpha_k^* \frac{\partial}{\partial b_P^\dagger(k^i)} - \beta_k^* b_P^\dagger(-k^i) \right] |BD\rangle = 0$$

Thus, we obtain

$$|BD\rangle = \frac{1}{\cosh r_k} \prod_{k,P} e^{\frac{1}{2} \tanh r_k b_P^\dagger(k) b_P^\dagger(-k)} |0_R\rangle \quad \tanh r_k = \frac{\beta_k^*}{\alpha_k^*}$$

$$\propto |0_{\mathbf{k}}\rangle \otimes |0_{-\mathbf{k}}\rangle + \tanh r_k |1_{\mathbf{k}}\rangle \otimes |1_{-\mathbf{k}}\rangle + \tanh^2 r_k |2_{\mathbf{k}}\rangle \otimes |2_{-\mathbf{k}}\rangle + \dots$$

This is a highly entangled two-mode **squeezed state**.

The squeezing parameter r_k can be calculated as

$$k_c = 2\pi f_c = \frac{1}{|\eta_1|}$$

$$\sinh r_k = \frac{1}{2} \left(\frac{f_c}{f} \right)^2$$

$$f_c = 10^9 \sqrt{\frac{H}{10^{-4} M_p}} \text{ Hz}$$

We assume the present graviton state is kept to be squeezed.



GRAVITON STATE DETECTION THROUGH DECOHERENCE

Geodesic motion in GW background

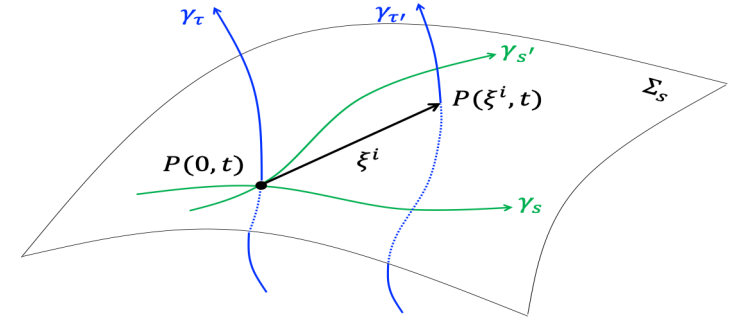
Because of the equivalence principle,
only the relative motion has physical meaning.

Along a geodesic, we can construct the local
Lorentz frame, so-called Fermi-normal coordinate

Action of a geodesic deviation reads

$$S_m = -M \int_{\gamma_\tau} d\tau - m \int_{\gamma_{\tau'}} d\tau' = -m \int d\tau' \sqrt{-g_{\mu\nu}(x^\alpha(\tau')) \frac{dx^\mu(\tau')}{d\tau'} \frac{dx^\nu(\tau')}{d\tau'}}$$

$$= -m \int d\tau' \left[1 + \frac{1}{2} R_{0i0j}(t) \xi^i(t) \xi^j(t) - \frac{1}{2} \left(\frac{d\xi^i(t)}{dt} \right)^2 + \dots \right]$$



$$R_{0i0j} = -\frac{1}{2} \frac{\partial^2 h_{ij}}{\partial t^2} \quad \text{in TT gauge}$$

Interaction between a particle and graviton

$$S_m = \int dt \left[\frac{m}{2} \left(\frac{d\xi^i}{dt} \right)^2 + \frac{m}{4} \ddot{h}_{ij}(x^i=0, t) \xi^i \xi^j \right]$$

Quantum geodesic motion in graviton BG

Equations of motion

$$\ddot{h}^A(\mathbf{k}, t) + k^2 h^A(\mathbf{k}, t) = \frac{m}{2M_p \sqrt{V}} e_{ij}^{*A}(\mathbf{k}) \frac{d^2}{dt^2} \{ \xi^k(t) \xi^l(t) \}$$

$$\ddot{\xi}^i(t) = \frac{1}{M_p \sqrt{V}} \sum_{\mathbf{k}, A} e_{ij}^A(\mathbf{k}) \ddot{h}^A(\mathbf{k}, t) \xi^j(t)$$

Quantum Langevin equation

Parikh, Wilczek, Zahariade 2020, 2021
Kanno, Tokuda, Soda 2021

$$\ddot{\xi}^i(t) + \frac{m}{40\pi M_p^2} \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right) \xi^j \frac{d^5}{dt^5} \{ \xi^k \xi^l \} = -N_{ij} \xi^j$$

reaction force

Noise:
$$N_{ij} = \frac{2}{M_p \sqrt{V}} \sum_{\mathbf{k}, A} k^2 h_I^A(\mathbf{k}, t) e^{i\mathbf{k} \cdot \mathbf{x}} e_{ij}^A(\mathbf{k})$$

$$h_I^A(\mathbf{k}, t) = a_A(\mathbf{k}) u_k(t) + a_A^\dagger(-\mathbf{k}) u_k^*(t)$$

$$[a_A(\mathbf{k}), a_{A'}^\dagger(\mathbf{k}')] = \delta_{AA'} \delta_{\mathbf{k}, \mathbf{k}'}$$

Noise of gravitons in squeezed state

Noise correlation

$$\langle \psi | \{N_{ij}(t), N_{kl}(0)\} | \psi \rangle = \frac{1}{10\pi^2 M_p^2} \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right) F(\Omega_m t) \quad \Omega_m : \text{cutoff}$$

$$F(\Omega_m t) = \int_0^{\Omega_m} dk k^6 \operatorname{Re}(u_k^{sq}(t) u_k^{sq*}(0))$$

$$u_k^{sq}(t) = u_k^M(t) \cosh r_k - u_k^{M*}(t) \sinh r_k$$

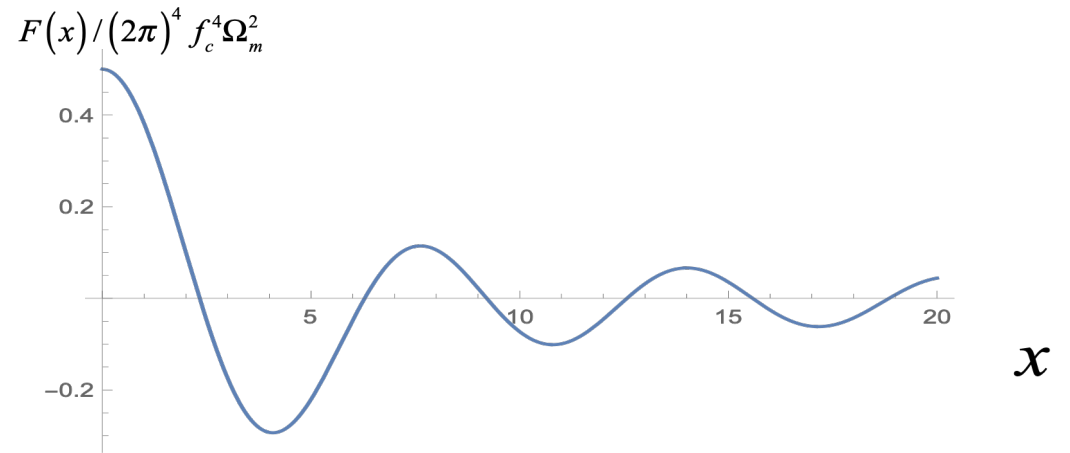
$$u_k^M(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

Recall the formula

$$\sinh r_k = \frac{1}{2} \left(\frac{f_c}{f} \right)^2, \quad k = 2\pi f$$

For $r_k \gg 1$, we obtain

$$F(x) = (2\pi)^4 f_c^4 \Omega_m^2 \frac{x \sin x + \cos x - 1}{x^2}$$



Thus, squeezed state significantly enhances the noise of gravitons.

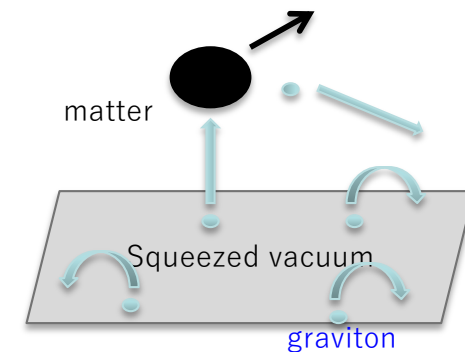
Indirect graviton detection through decoherence

Kanno, Soda and Tokuda, 2021

The cosmological observation of the CMB and the galaxy distribution tells us that the large scale structure of the present universe originates from inflation.

The inflation predicts **primordial gravitational waves**.
The quantum state of gravitons is **squeezed**.
This can be regarded as the condensation of gravitons.

We consider graviton fluctuations in this background.



Suppose a quantum massive object with the mass 40kg is moving.

The object is interacting with gravitons.

This induces decoherence of the quantum entanglement due to the strong coupling

$$GE^2 = \left(\frac{E}{M_p} \right)^2 = \left(\frac{40\text{kg}}{10^{-5}\text{g}} \right)^2 \approx 10^{19} \gg 1$$

If we can observe the decoherence, it would be an evidence of gravitons.



40kg quantum mirrors
can be quantum
mechanically controlled

Entanglement of mirrors

Marshall, Simon, Penrose and Bouwmeester 2003

A single photon goes through a beam splitter or reflected by it, which is described by a superposition state. When the photon hit a mirror, the mirror starts to oscillate. Thus, there arises a superposition state of mirrors.

$$|\psi(t_i)\rangle = \left\{ \frac{1}{\sqrt{2}} |\xi_1\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} |0\rangle \otimes |\xi_2\rangle \right\} \otimes |g\rangle$$

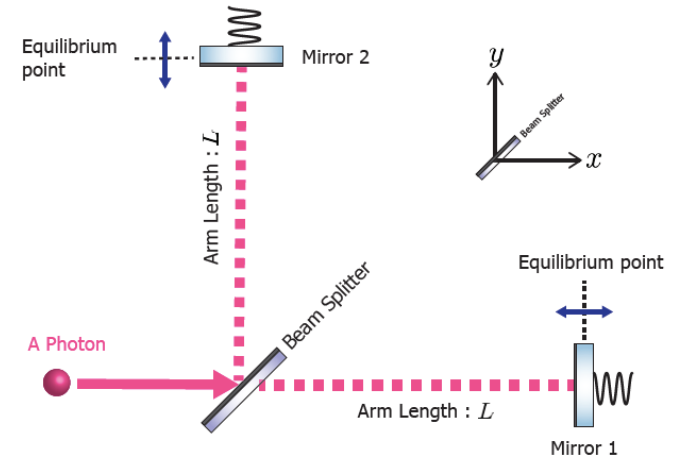
After a while, the mirrors are interacting with gravitons

$$S \simeq \sum_{A=1,2} \int dt \left[\frac{m}{4} \ddot{h}_{11}(0,t) \xi_A^1 \xi_A^1 + \frac{m}{4} \ddot{h}_{22}(0,t) \xi_A^2 \xi_A^2 \right]$$

Consequently, there appears the entanglement between the mirrors and gravitons.

$$|\psi(t_i)\rangle = \frac{1}{\sqrt{2}} |\xi_1\rangle \otimes |0\rangle \otimes |g; \xi_1\rangle + \frac{1}{\sqrt{2}} |0\rangle \otimes |\xi_1\rangle \otimes |g; \xi_2\rangle$$

Thus, mirrors will be decohered due to noises of gravitons.



Decoherence time

Kanno, Soda and Tokuda 2021

By solving non-Markovian master equation, we obtain the decoherence rate

$$\Gamma = \frac{4\pi^3}{5} \underbrace{\left(\frac{m}{M_p}\right)^2}_{\text{effective coupling}} \underbrace{(Lf_c)^4}_{\text{squeezing factor}} \left(\frac{A}{L}\right)^2 \omega \quad M_p \approx 10^{-5} g$$

Substituting

$$m = 40\text{kg}, \quad L = 40\text{ km}, \quad A = 10 \frac{\hbar}{\sqrt{2m\omega}} = 10^{-17}\text{ cm}, \quad \omega = 1\text{kHz}, \quad f_c = 1\text{GHz}$$

we see the decoherence time is 20s.

Note that the decoherence due to air molecule is dominant among other decoherence sources.

$$t_d = 1200 \left(\frac{R}{0.17\text{m}}\right)^2 \left(\frac{T}{10\text{K}}\right)^{3/2} \text{ s} \quad P = 10^{-10} \text{ Pa}$$

This time scale is much larger than the decoherence time due to noise of gravitons. Therefore, there is a chance to detect gravitons indirectly.



QUANTUM STATE DESCRIPTION OF GWS FROM BINARY BLACK HOLES

Quantum theory of GWs

How to describe classical radiation in quantum theory?

It is well known that electromagnetic waves are described by a [coherent state](#) of photon generated by the operator. Glauber 1963

$$T\text{Exp}\left[-i\int_{-\infty}^t H_I(t) dt\right] = T\text{exp}\left[-i\int_{-\infty}^t \int d^3x \mathbf{j}(x^i, t) \cdot \hat{\mathbf{A}}(x^i, t)\right]$$

Here, the current is assumed classical.

How about quantum theory of gravitational waves?

The point is that the gravitational interaction is generically nonlinear

$$T\text{Exp}\left[-i\int_{-\infty}^t H_I(t) dt\right] = T\text{exp}\left[-i\int_{-\infty}^t \int d^3x T_{ij}(x^i, t) \hat{h}_{ij}(x^i, t) + \Lambda_{ijkl} \hat{h}_{ij}(x^i, t) \hat{h}_{kl}(x^i, t) + \dots\right]$$

The first term generates a [coherent state](#).

In contrast to photon, the second term generates a [squeezed state](#).

Thus, we can expect the deviation from the coherent state.

Coherent state of GWs from binary BHs

The binary black holes are described by

$$S = S_{\text{EH}} + S_1 + S_2 = \frac{M_{\text{p}}^2}{2} \int d^4x \sqrt{-g} R - m_1 \int_{\zeta_1} d\tau - m_2 \int_{\zeta_2} d\tau$$

We assume that backreaction is negligible

We treat the system perturbatively. The action of the free part is given by

$$S_{\text{EH}} = \frac{M_{\text{p}}^2}{8} \int d^4x \left(\dot{h}_{ij} \dot{h}_{ij} - h_{ij,k} h_{ij,k} \right) \quad \psi_{ij} = \frac{M_p}{2} h_{ij}$$

The canonical quantization procedure is parallel to the de Sitter case.

$$\begin{aligned} \left[\psi_{ij}(t, \mathbf{x}), \dot{\psi}^{k\ell}(t, \mathbf{x}') \right] &= \frac{i}{2} (P_i^k P_j^\ell + P_i^\ell P_j^k - P_{ij} P^{k\ell}) \delta(\mathbf{x} - \mathbf{x}') & P_{ij} &= \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \\ h_{ij}(t, \mathbf{x}) &= \frac{2}{M_{\text{p}}} \sum_{P=+, \times} \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} \left[\frac{e^{-i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}} e_{ij}^{(P)}(\mathbf{k}) a^{(P)}(\mathbf{k}) + \frac{e^{i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}} e_{ij}^{(P)}(-\mathbf{k}) a^{(P)\dagger}(-\mathbf{k}) \right] e^{i\mathbf{k} \cdot \mathbf{x}} \\ [a^{(P)}(\mathbf{k}), a^{(Q)\dagger}(\mathbf{k}')] &= \delta(\mathbf{k} - \mathbf{k}') \delta^{PQ} \end{aligned}$$

coherent state

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle \quad \hat{D}(\alpha) = \prod_P \exp \left[\int d^3 \mathbf{k} \left(\alpha^{(P)}(\mathbf{k}) a^{(P)\dagger}(\mathbf{k}) - \alpha^{(P)*}(\mathbf{k}) a^{(P)}(\mathbf{k}) \right) \right]$$

Squeezing of GWs from a binary system

Kanno, Soda, Taniguchi 2025

For simplicity, we consider circular motion

$$\begin{aligned} x_1 &= \frac{m_2}{M} a \cos(\Omega t), & y_1 &= \frac{m_2}{M} a \sin(\Omega t), & z_1 &= 0 \\ x_2 &= \frac{m_1}{M} a \cos(\Omega t + \pi), & y_2 &= \frac{m_1}{M} a \sin(\Omega t + \pi), & z_2 &= 0 \end{aligned} \quad M = m_1 + m_2$$

At the second order, we have the squeezing operator

$$\hat{S}(\beta) = \prod_P \exp \left[\int d^3 \mathbf{k} \int d^3 \mathbf{k}' \left(\beta_{\mathbf{k}\mathbf{k}'}^{(P)} a^{(P)\dagger}(\mathbf{k}) a^{(P)\dagger}(\mathbf{k}') - \beta_{\mathbf{k}\mathbf{k}'}^{(P)*} a^{(P)}(\mathbf{k}) a^{(P)}(\mathbf{k}') \right) \right]$$

The order of magnitude is estimated using the formula

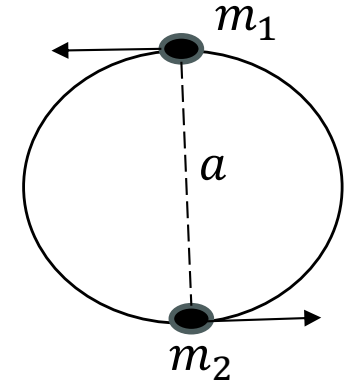
$$\zeta \simeq \frac{4\pi}{3} (2\Omega)^3 |\beta| \simeq \frac{1}{8\pi M_p^2} \mu (a\Omega)^4 f$$

as

$$\zeta \simeq 2 \times 10^{-3} \left(\frac{\mu}{16 M_\odot} \right) \left(\frac{a\Omega}{0.41} \right)^4 \left(\frac{f}{68 \text{ Hz}} \right)$$

A different analysis found the squeezing can be around 0.3.

Das, Parikh, Wilczek, Wutte 2025



PGW Squeezed state detection with Binary BHs

Kanno, Soda, Taniguchi 2025

Our main observation is that
we can utilize binary BHs to gain information of PGWs.

squeezing from binary BHs

$$S(\zeta) = \exp \left[\frac{1}{2} \zeta a^\dagger a^\dagger - \frac{1}{2} \zeta^* a a \right] \quad \zeta = r e^{i\varphi}$$

This is quite small, however,

PGWs have a squeezing $|\xi\rangle = S(\xi) |0\rangle \quad S(\xi) = \exp \left[\frac{1}{2} \xi a^\dagger a^\dagger - \frac{1}{2} \xi^* a a \right]$

Hence, binary black holes emit coherent GWs with the squeezing

$$S(\zeta) S(\xi) |0\rangle \approx S(\xi + \zeta) |0\rangle$$

PGWs around the 100Hz have a squeezing of 280dB corresponding to $r \approx 30$. $\sinh r_k = \frac{1}{2} \left(\frac{f_c}{f} \right)^2$

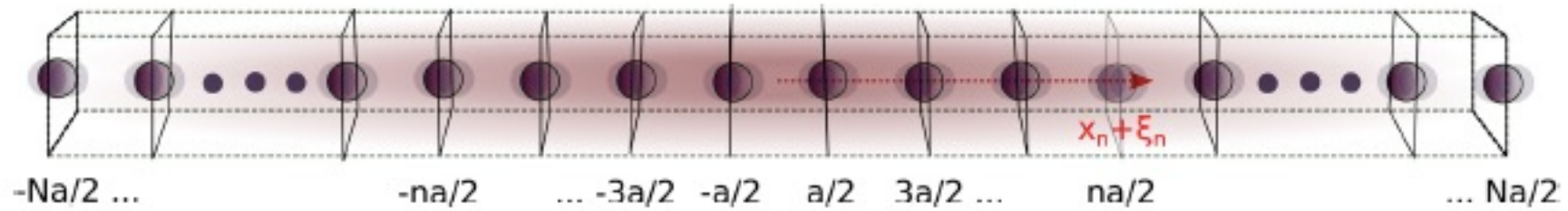
In quantum optics, we can detect this squeezing using the intensity interferometry.

It is interesting to perform intensity interferometry in the context of gravitational waves.



GRAVITON DETECTION WITH GRAVITO-PHONONIC EFFECT

Graviton phonon interaction



$$H = \omega_a a^\dagger a + \omega_b b^\dagger b - g(ab^\dagger + a^\dagger b)$$

$$g = \sqrt{\frac{8\pi G M \omega_a^3 L^2}{\omega_b \pi^4 V}}$$

$$i \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle \quad U(t) = R_a(t) R_b(t) T(t)$$

$$\dot{\theta}_a = \omega_a - \frac{1}{2} \dot{\chi} (\cos 2q - 1)$$

$$\dot{\theta}_b = \omega_b + \frac{1}{2} \dot{\chi} (\cos 2q - 1)$$

$$\dot{q} = -g \sin \chi \cos(\theta_a - \theta_b) + g \cos \chi \sin(\theta_a - \theta_b)$$

$$\dot{\chi} \sin 2q = -2g \cos \chi \cos(\theta_a - \theta_b) - 2g \sin \chi \sin(\theta_a - \theta_b)$$

$$R_a(t) = \exp[-i\theta_a(t)a^\dagger a]$$

$$R_b(t) = \exp[-i\theta_b(t)b^\dagger b]$$

$$T(t) = \exp[q(t)(e^{-i\chi(t)} ab^\dagger - e^{i\chi(t)} a^\dagger b)]$$

Graviton-phonon conversion

Graviton-phonon conversion

$$\begin{aligned} A_{g \rightarrow p}(t) &= \langle 0 | b U(t) a^\dagger | 0 \rangle = \langle 0 | b R_a R_b T a^\dagger | 0 \rangle \\ &= e^{-i(\theta_b + \chi)} \sin q \end{aligned}$$

conversion probability

$$P_{g \rightarrow p}(t) = |\langle 0 | b U(t) a^\dagger | 0 \rangle|^2 = \sin^2 q(t)$$

$q(t)$ is solved numerically

The first order perturbation gives rise to

$$q(t) = gt$$

Thus, we get the conventional result.

$$P_{g \rightarrow p}(t) = \sin^2 gt$$

This probability is quite small.

Graviton-phononic detection of a single graviton

Tober, Manikandan, Beitel, Pikovski 2024

Gravitational waves from binary black holes should be described by a coherent state.

Moreover, number of gravitons is huge.

Namely, the coherent parameter is huge.

Thus, we get the enhancement of the conversion probability

$$P_{\alpha \rightarrow \alpha,1}(t) \approx |\alpha|^2 g^2 t^2$$

Taking realistic parameters

$$M = 10^3 \text{ kg} \quad L = 10 \text{ m} \quad \omega_a = \omega_b = 1 \text{ kHz} \quad V \approx \omega_a^{-3}$$

We obtain the effective coupling $g \approx 10^{-17} \times |\alpha| \text{ Hz}$

For gravitational waves LIGO detected, we have $|\alpha|^2 \approx N \approx 10^{37}$

For observation time 0.1s, we get

$$P_{\alpha \rightarrow \alpha,1}(t) \approx \mathcal{O}(1)$$

Thus, there is a chance to detect a single graviton with gravito-phononic effect.

It should be emphasized that this is not the detection of gravitational waves in the sense that we can reconstruct the waveform.

Unitarity bound

Gouin, Kanno, Soda 2026

The perturbative result grows linearly with the square of the coherent parameter. However, the probability must be less than 1.

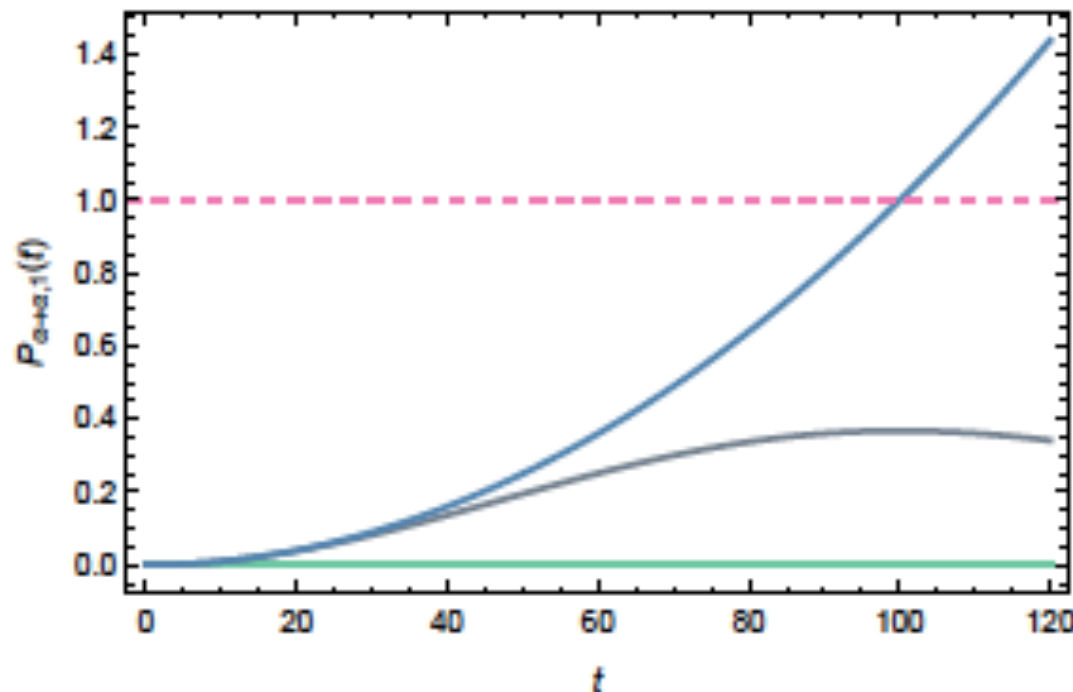
$$A_{\alpha \rightarrow \alpha,1}(t) = \langle \alpha(t), 1 | U(t) | \alpha, 0 \rangle = \langle \alpha(t), 0 | b R_a R_b T | \alpha, 0 \rangle$$

conversion probability

$$P_{\alpha \rightarrow \alpha,1}(t) = |\langle \alpha(t), 1 | U(t) | \alpha, 0 \rangle|^2$$

$$= e^{-i(\theta_b + \chi)} \alpha \sin q \langle \alpha e^{\theta_a - i\omega_a t} | \alpha \cos q \rangle_a \langle 0 | \alpha e^{-i\chi} \sin q \rangle_b$$

$$= |\alpha|^2 \sin^2 q(t) \exp[-2|\alpha|^2 \{1 - \cos q(t) \cos(\theta_a(t) - \omega_a t)\}]$$



$$P_{\alpha \rightarrow \alpha,1}(t) \approx (30 \times t[s])^2$$

$$|\alpha|^2 = 10^{37}$$

$$P_{\alpha \rightarrow \alpha,1}(t) = |\alpha|^2 \sin^2 q(t)$$

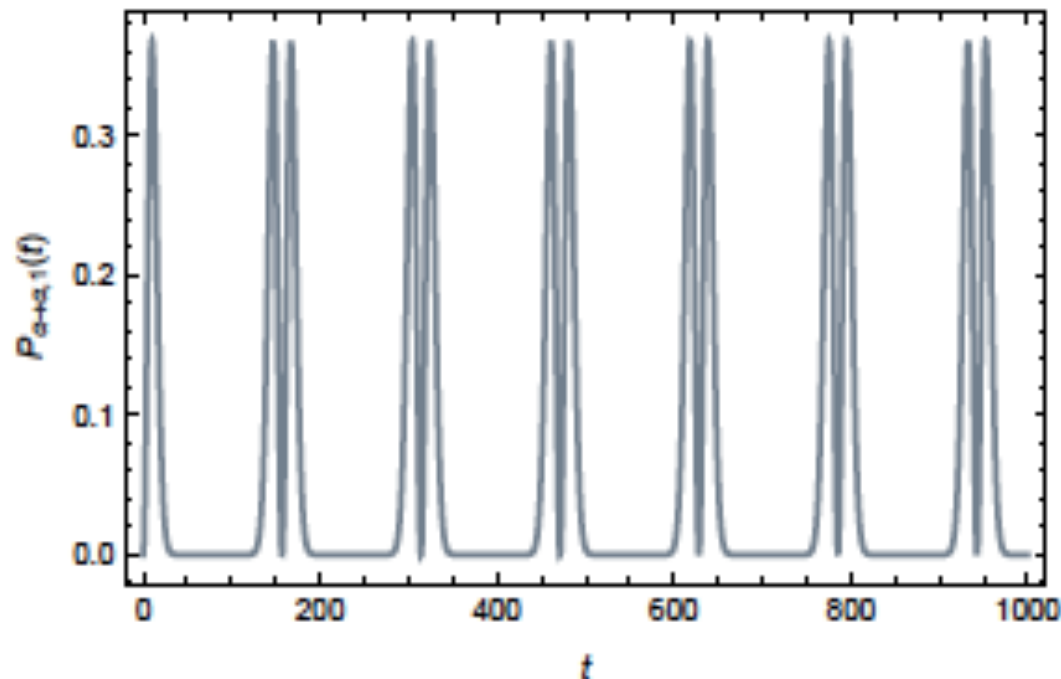
$$\times \exp[-2|\alpha|^2 \{1 - \cos q(t) \cos(\theta_a(t) - \omega_a t)\}]$$

Intermittency in graviton-phonon conversion

Gouin, Kanno, Soda 2026

In the paper by Tober et al., the semiclassical analysis is used.
Hence, even when the phonon is detected,
it cannot be an evidence for quantum nature of gravitons.

When the effective coupling is close to 1, however, the intermittent behavior shows up.
This behavior does not appear in the semi-classical theory.
Thus, we may be able to prove quantum nature of gravitons
through the observations of intermittency.



Summary



➤ Dyson impossibility revisited

We discussed how to circumvent Dyson argument.

➤ Decoherence probes quantum graviton states

➤ Binary black holes may produce coherent and squeezed gravitons.

➤ Gravito-phononic conversion enables quantum detection.

➤ Intermittency is a quantum signature.