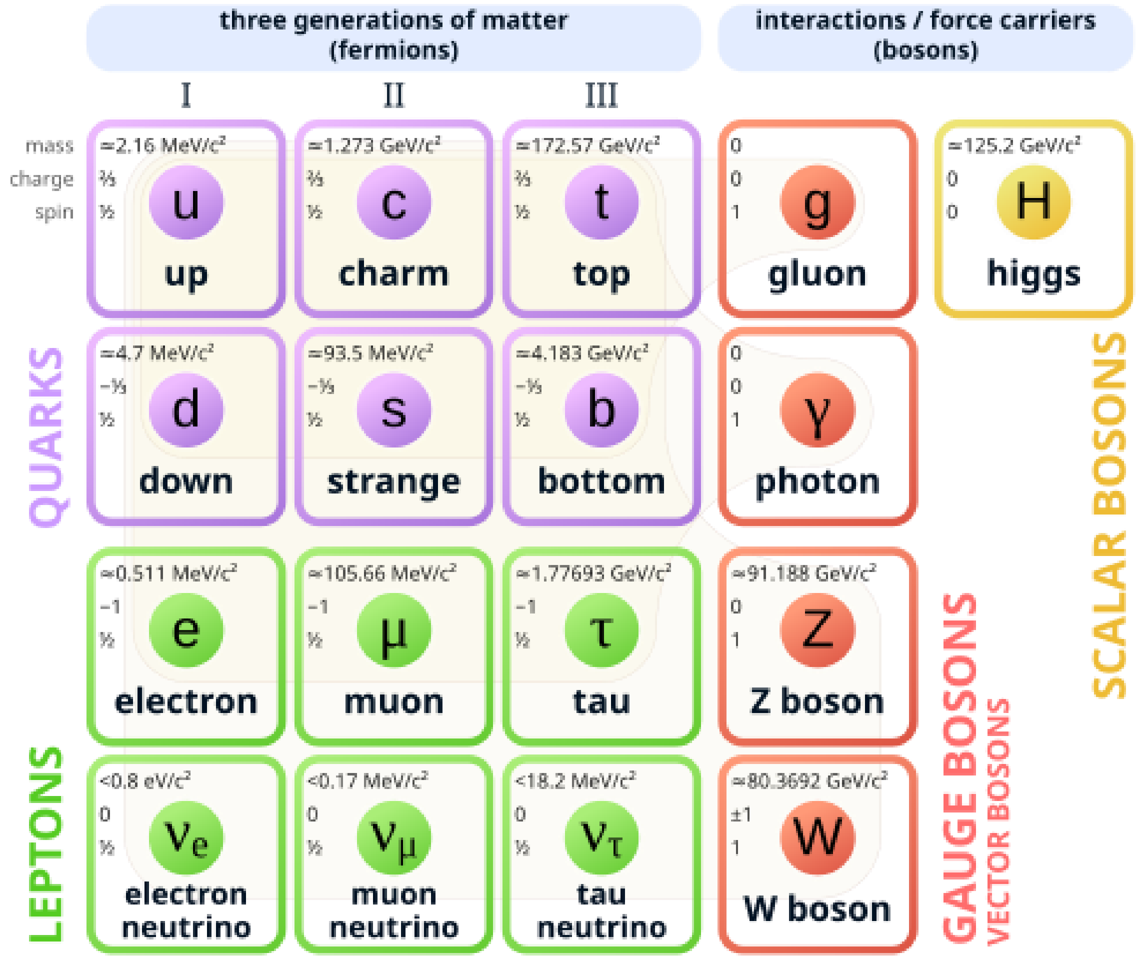
An Efficient Framework for Lattice Gauge Theories with Fermions in 3+1d

Arnab Pradhan
York U.

Quantum Crossroads, August 7

Based on work with R. Lewis, S. Pourbakhsh, and L. Siquioco 2606.21563

Standard Model of Particle Physics

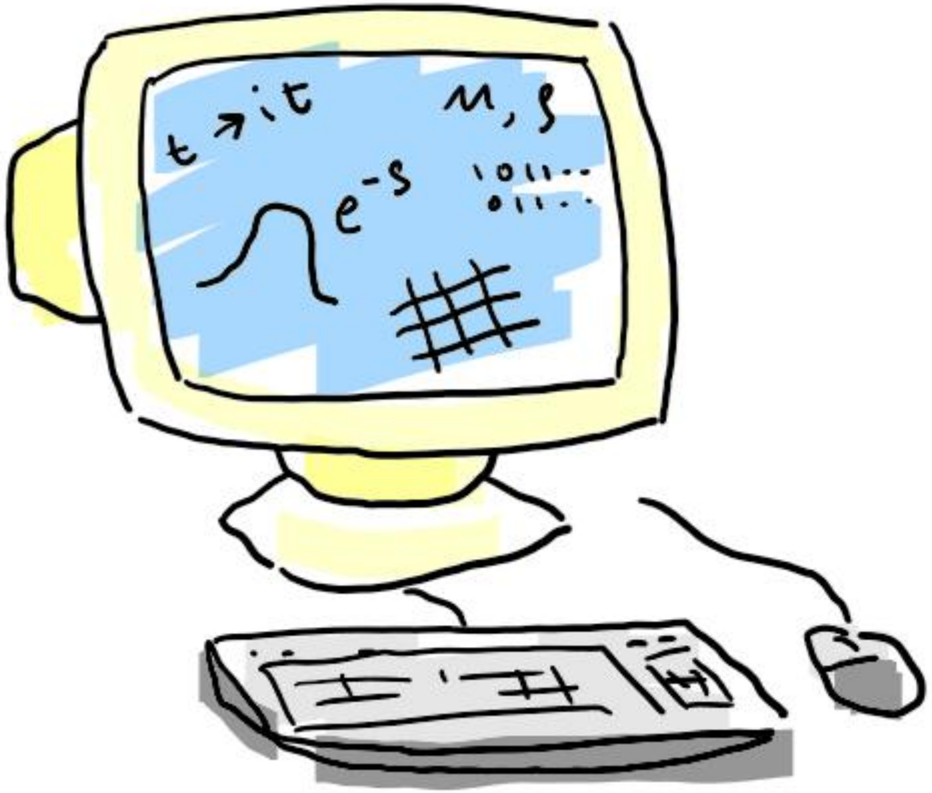


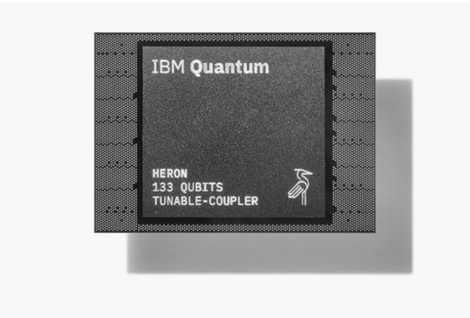
Fermions vs Bosons

Matter vs interaction mediator

FD vs BE statistics

QC for QCD



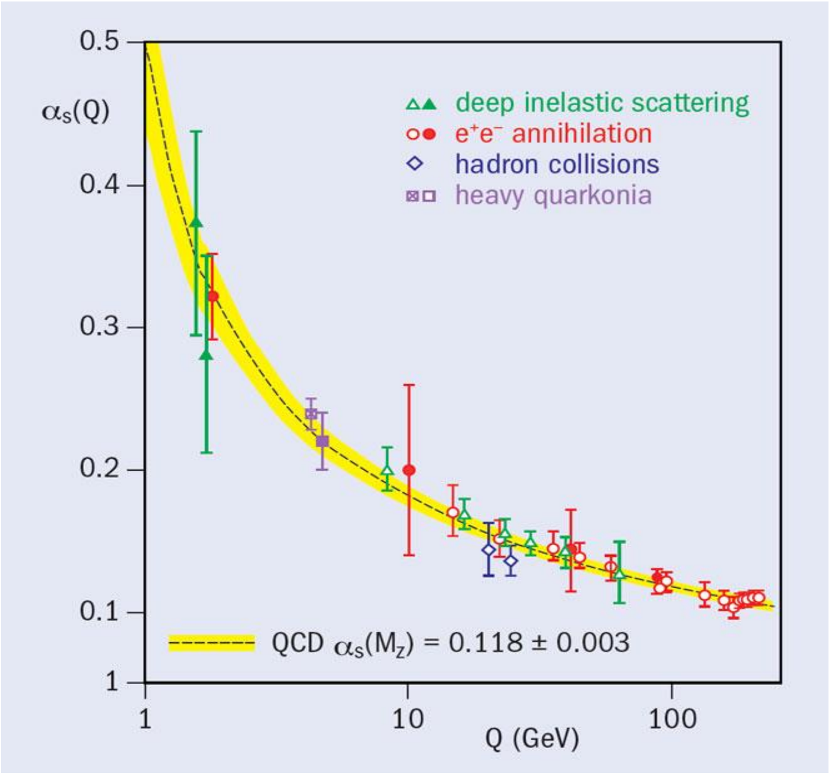


QCD : quarks + gluons : non-perturbative at low E

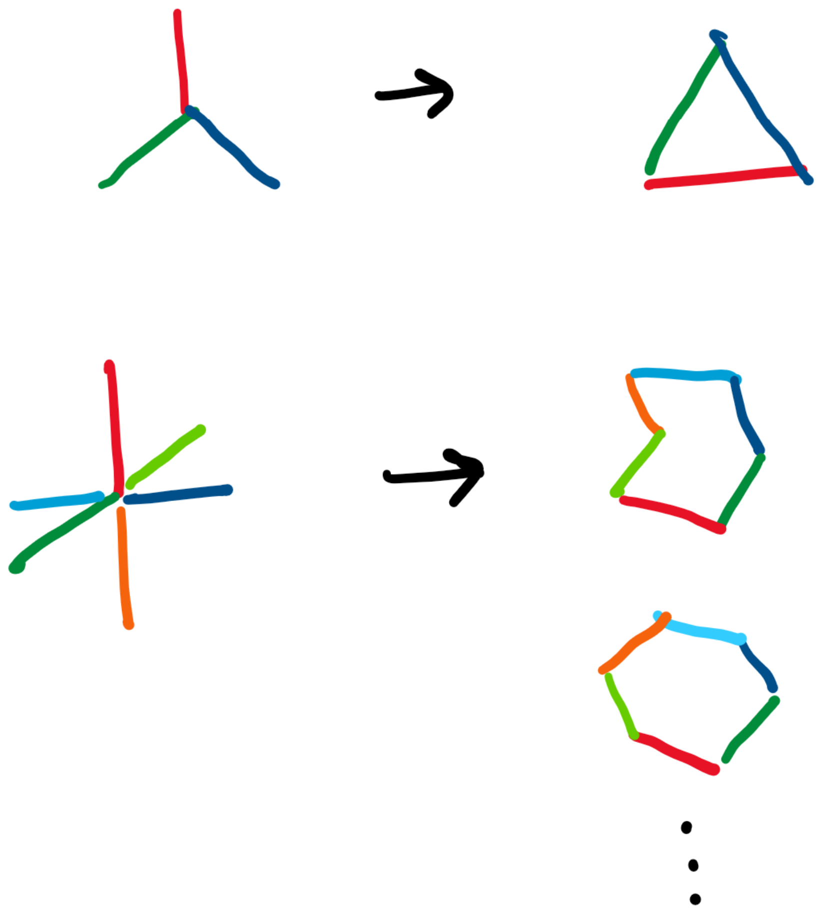
Discretize + imag. time + Monte Carlo

Fails for real time, finite density, osc. phase

QC : Hamiltonian, real time, no sign problem



Gauss Law



Gauging SU(2) Color – non-Abelian, easier than SU(3)

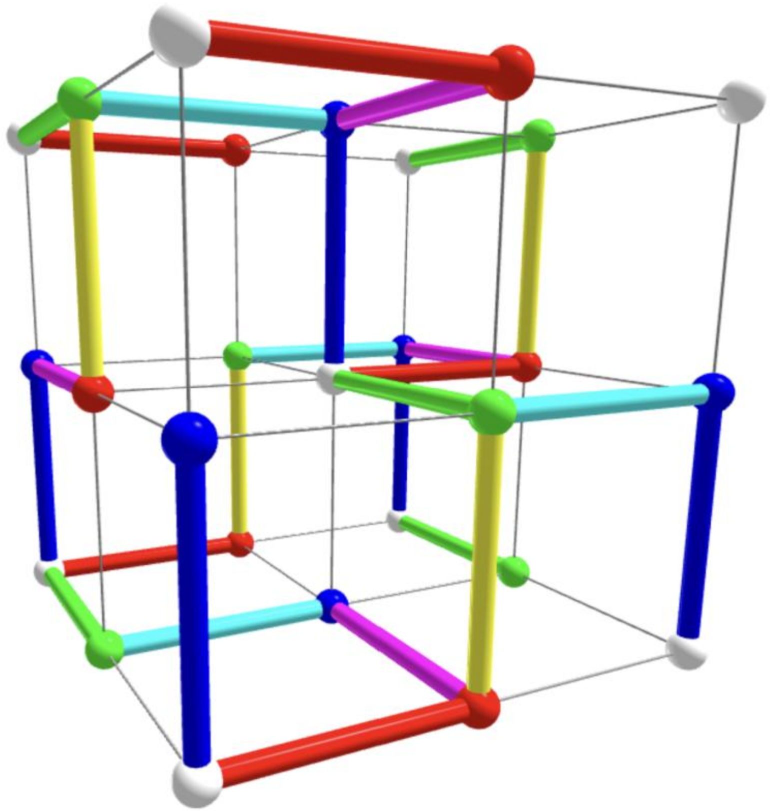
Problem : #qubits, connectivity

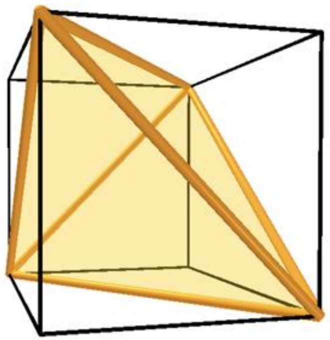
j = 0, ½, 1, ... imposing maximal truncation

Trivalent vertex : unique singlet, minimal #qubits

A.H.Z. Kavaki, R. Lewis 2401.14570

Semi-simple cubic lattice





Delete half of the links in simple cubic

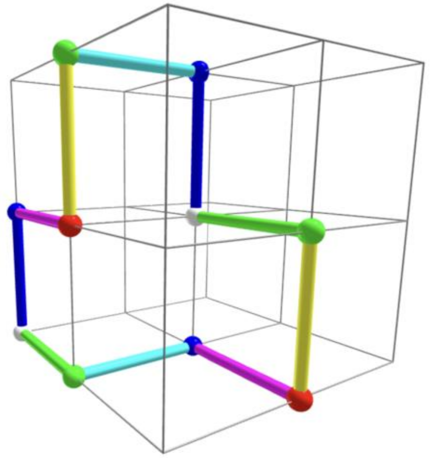
Topologically equivalent to K_4 lattice

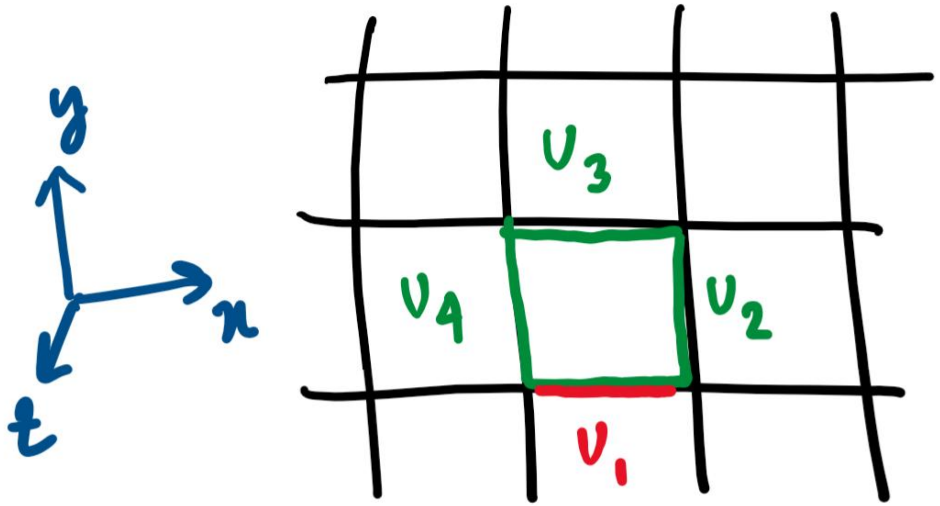
Screw symmetry, 3-fold symm about body diagonal : $I2_1 3$

Enough symmetry to classify states by angular momentum quantum #

ℓ	0	1	2	3	4	5	6	7
A	1	0	0	1	1	0	2	1
E_+	0	0	1	0	1	1	1	1
E_-	0	0	1	0	1	1	1	1
T	0	1	1	2	2	3	3	4

Pure Gauge Hamiltonian





$$U_1 \sim e^{iaA_x}$$

Electric + Magnetic contribution

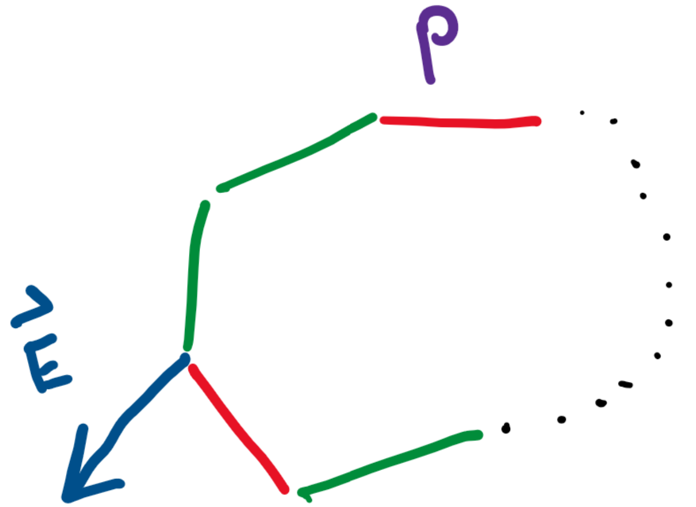
Gauge fields live on links

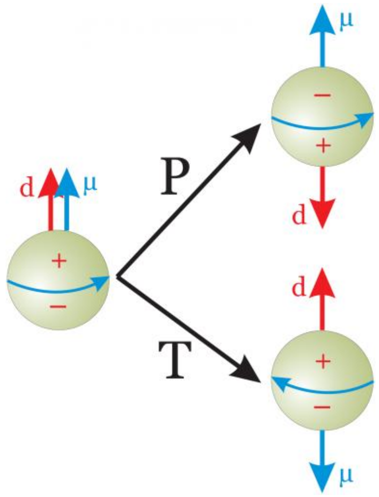
Magnetic contribution in closed loops

$$H \sim E^2 + B^2$$

$$P_{xy} = Tr(U_1 U_2 U_3^\dagger U_4^\dagger) \sim Tr(I) - cB_z^2$$

Θ term





Strong CP problem of QCD

Topological : total derivative, EOMs unchanged

Conventional LGT has a sign problem : $e^{iQ\Theta}$

Tool to study TQFT : SPT, anomalies, ...

$$\mathcal{L}(\vec{n}) = \frac{g^2\theta}{8\pi^2 a^2}\epsilon^{ijk}\mathrm{Tr}\left(E_i(\vec{n})F_{jk}(\vec{n})\right)$$

$$\sim \theta \vec{E}\cdot\vec{B}$$

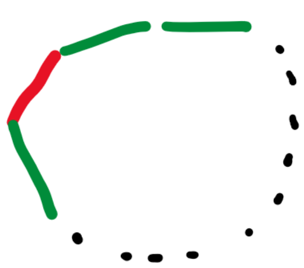
2105.06019 Kan et. al.

Qubit Encoding

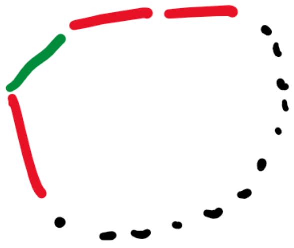
$$H_E = \frac{3g^2}{8} \sum_{n=\text{link}} \left(\frac{1 - Z_n}{2} \right)$$

$H_E \sim j(j+1)$

H_B



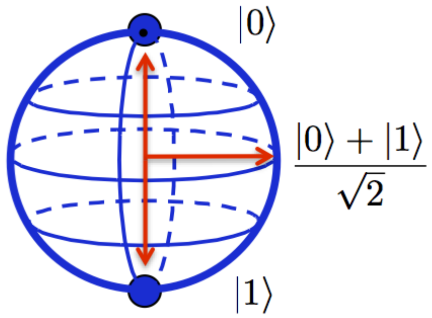
$\sim$



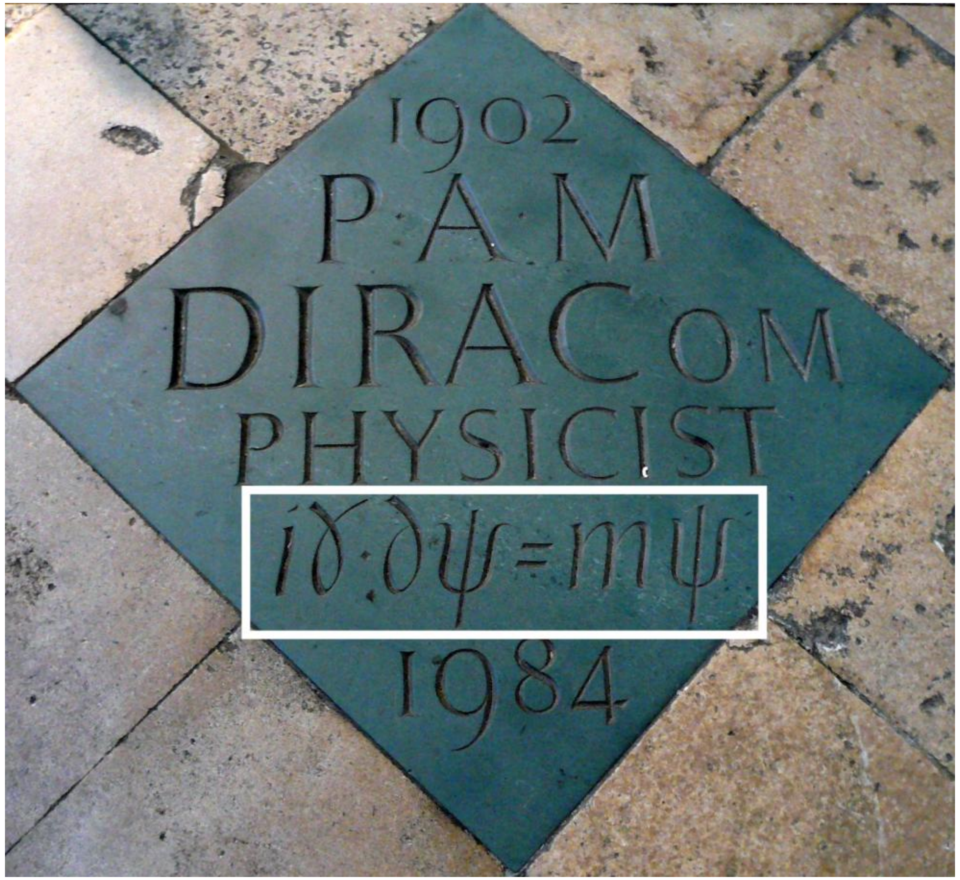
j=0 : |0>
j=1/2 : |1>

Use Pauli gates (X, Z) to construct the Hamiltonian

e^{-iHt} : time evolution



The Dirac Equation



$$\Psi \sim \begin{pmatrix} p_\uparrow \\ p_\downarrow \\ \bar{p}_\uparrow \\ \bar{p}_\downarrow \end{pmatrix}$$

Relativity + Quantum Mechanics

Describes matter

4 component spinor : particle/anti-particle and spin up/down

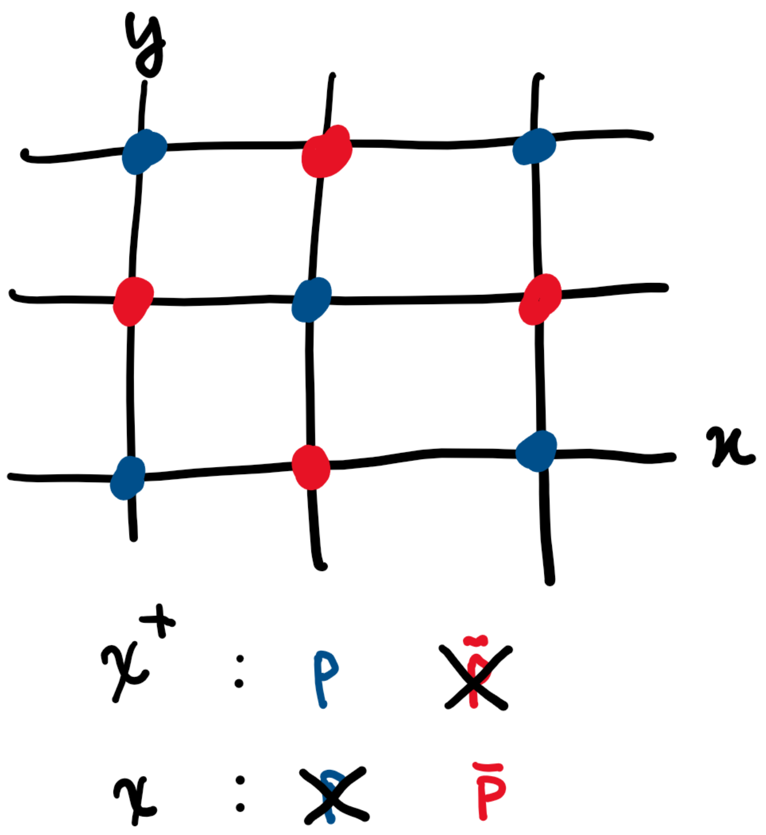
Dirac Gamma matrices

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu} I_4$$

Hamiltonian Description

$$\Psi(x) = (\gamma_0\gamma_i)^x \chi(x)$$

$$H = \underbrace{\sum_{x,i} i\eta_i(\chi^\dagger(x)\chi(x+i) + h.c.)}_{\text{Kinetic}} + \underbrace{m\sum_{x} \epsilon(x)\chi^\dagger(x)\chi(x)}_{\text{Mass}}$$

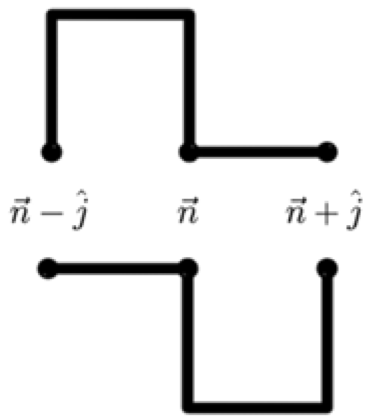


Staggering : easier handling on a QC

Particle vs anti-particle : even vs odd sites

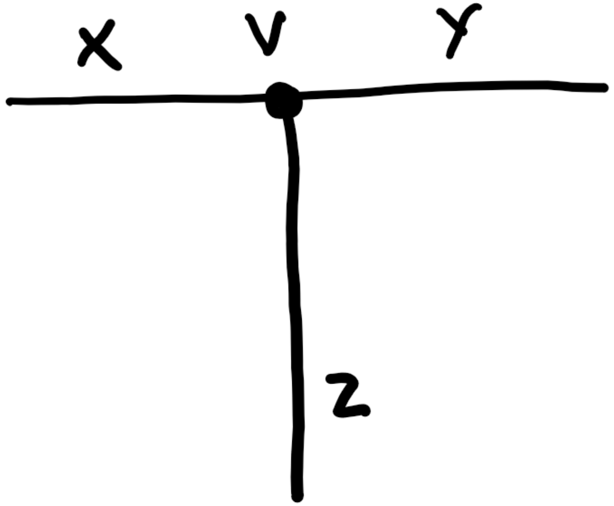
Spin structure in phases

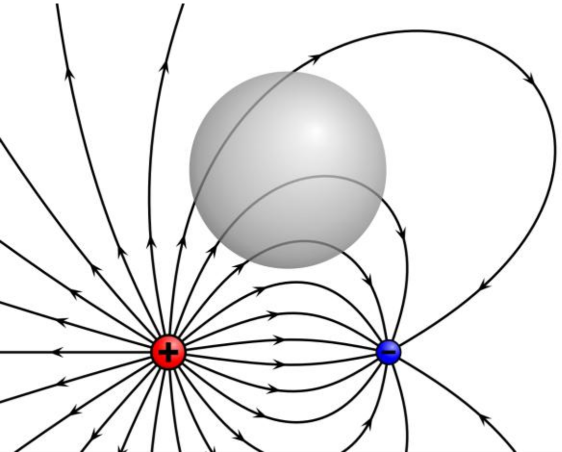
Pair creation/destruction



S. Catterall, **AP**, A. Samlodia 2501.10862
N. Modi, A. Ciavarella, J. Halimeh, C. Bauer 2602.02344

Gauss Law





Links On	Vertex Fermions
0 or 2	0 or 2
1 or 3	1

Truncated SU(2) : j = 0, ½

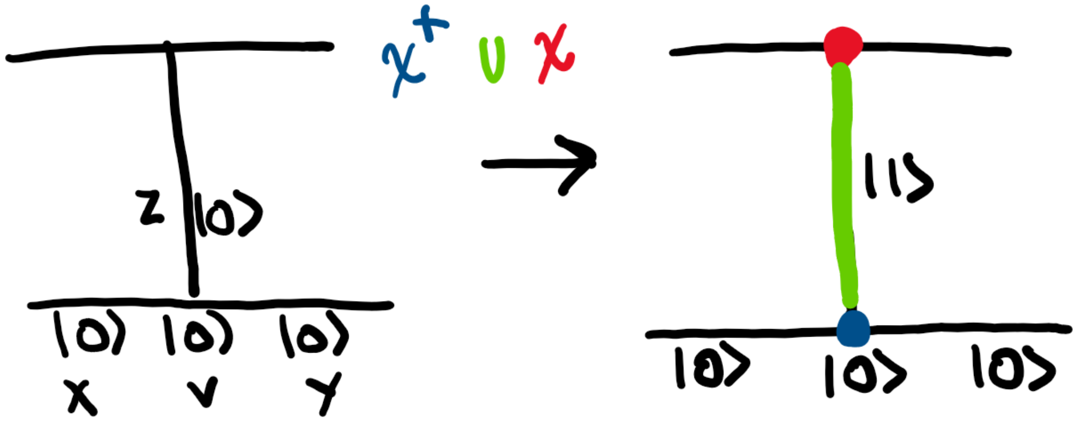
Links : gauge field interactions

Sites : fermion/anti-fermion

Trivalent vertex + staggering : qubits ↓

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

Qubit mapping



Projectors : links and vertex

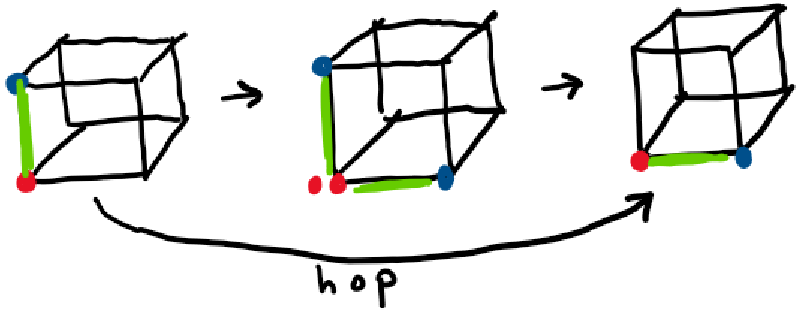
Allowed and forbidden transitions : Gauss law

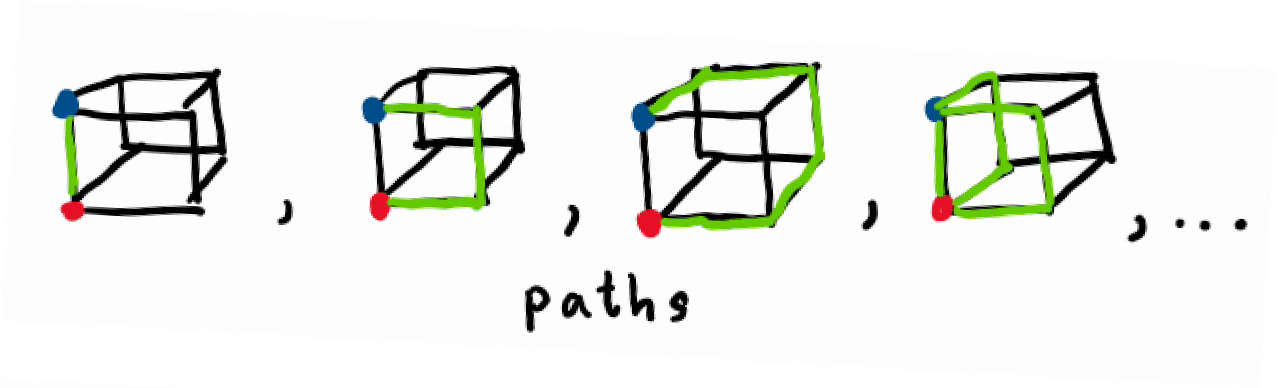
Paulis : X |0> = |1>, Z |1> = -|1>

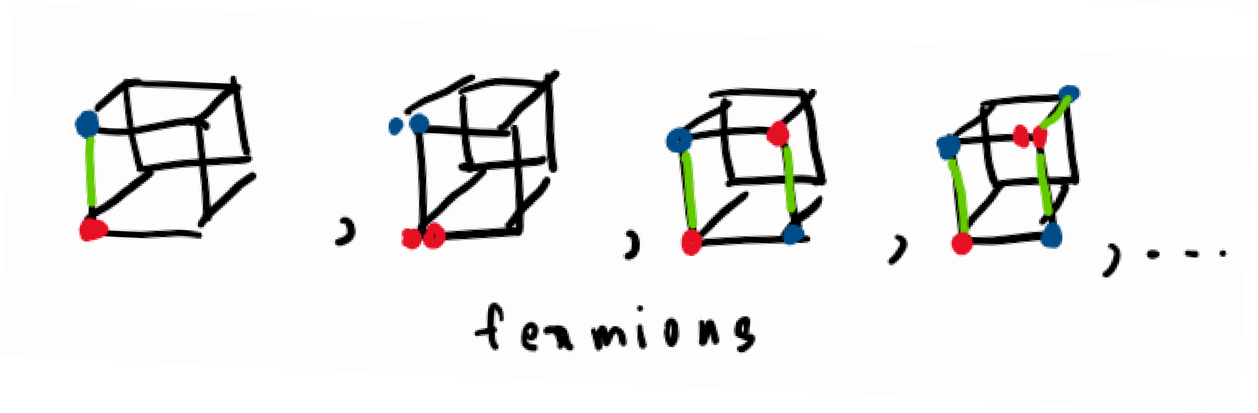
No need for Jordan-Wigner

$$
\begin{aligned}
(H_k)_z &= \sum_{z\ \text{links}} (-1)^{n_x+n_y} D^\dagger_{\vec{n}} X_z D_{\vec{n}+\hat{z}}\,, \\
D &= \frac{1}{4}(1+Z_v)(1+X_v-(1-X_v)Z_xZ_yZ_z)
\end{aligned}
$$

Examples





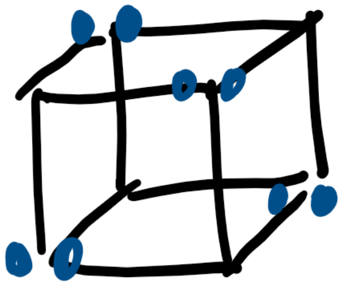


Basis states : N_f, N_g, N_v

Time evolution : $e^{-iHt}|\Psi>$

Particles identified by their symmetries

Resource Requirements

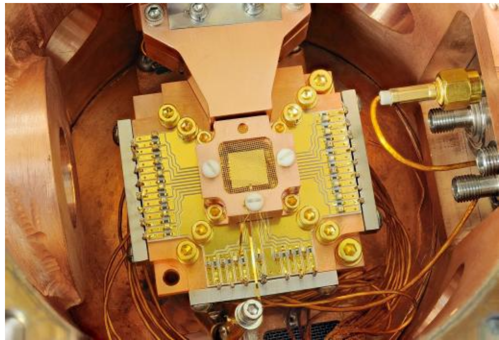


B	N_f	N_g	N_v	N_B
4	1	32	0	32
3	36	32	528	1680
2	266	32	7026	15538
1	784	32	27120	52208
0	1107	32	41637	77061
−1	784	32	27120	52208
−2	266	32	7026	15538
−3	36	32	528	1680
−4	1	32	0	32
total	3281	32	110915	215977

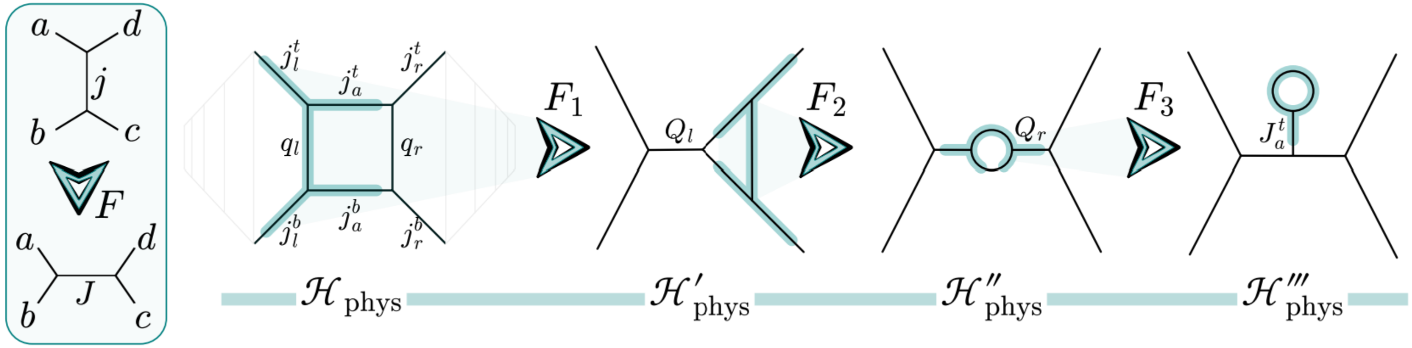
Baryon number classification

Pure gauge vs fermions

20 (12 + 8) : 18 qubits



What's next?



Putting it on a quantum hardware: connectivity

Higher j : q-deformations ($SU(2)_q$), consistently transforming plaquette variables

Noise : use unphysical states for error correction

2511.13721 Yao
2605.15076 Webb-Mack, Klco
2304.02527 Zache, Gonzalez-Cuadra, Zoller

Takeaway

Thank you!

LGT on classical computer : limited phenomenology

QC : real time dynamics, finite number density, no sign problem

Non-Abelian gauge th. with fermions : hard

Staggering + trivalent geometry : qubits ↓

Arnab Pradhan
York U.