Holographic AME states in black hole interiors

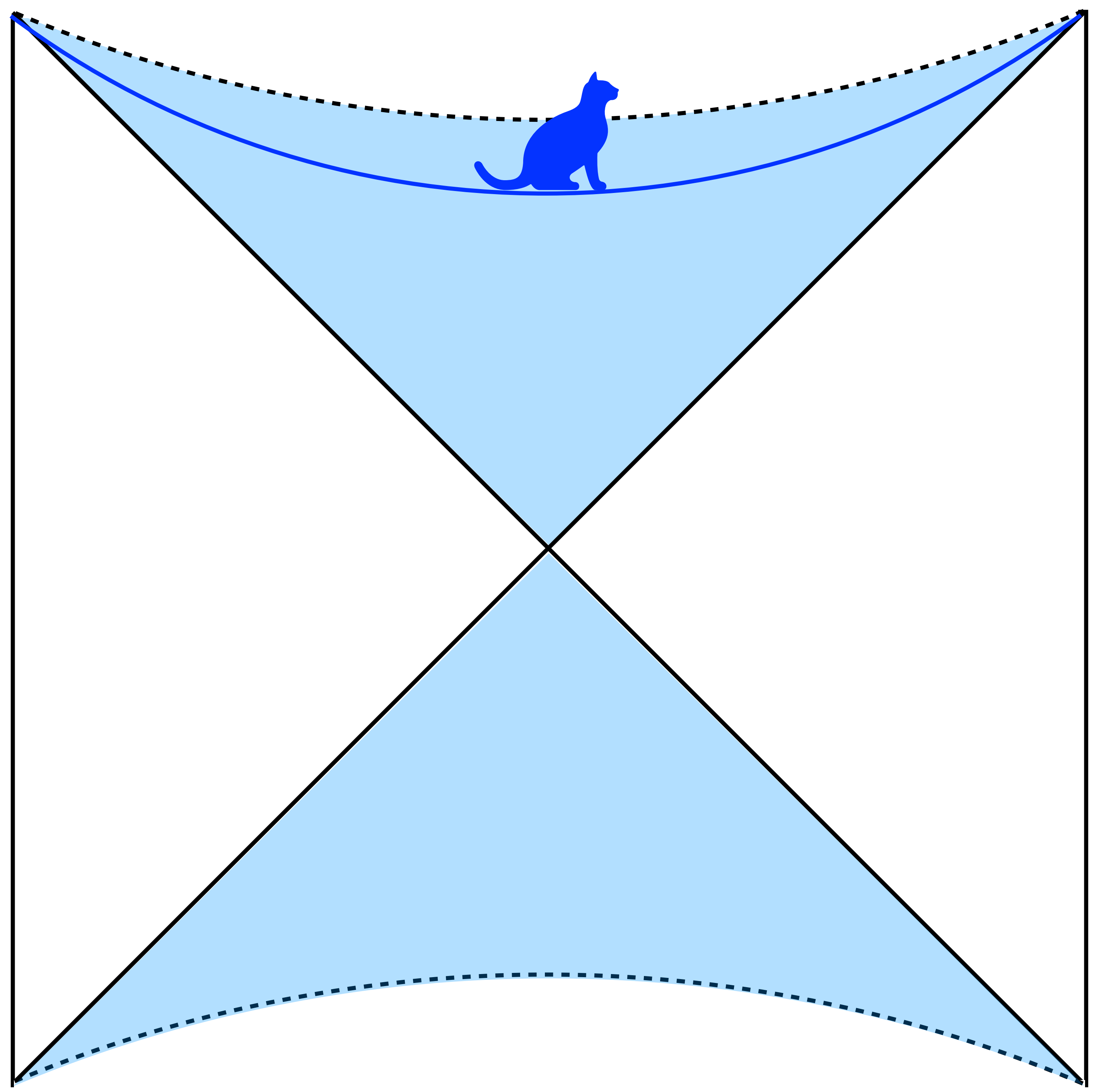
Kotaro Tamaoka (Nihon U.)

Quantum crossroads @ IPMU August 7th, 2026

Can we learn some universal features of the black hole interior?

Some universal features of black hole exteriors can be understood from near-horizon physics

Can we learn some universal features of the interiors as well? (From where?)



This talk: special extremal slice

Mostly based on joint work with

Takanori Anegawa (Yonago N.I.T.) [2508.07634] PRL

See also

Takanori Anegawa and Shota Suzuki (Nihon U.) [2512.21037] PTEP

Main claim:

State dual to a time slice in BH interior (called special extremal slice)

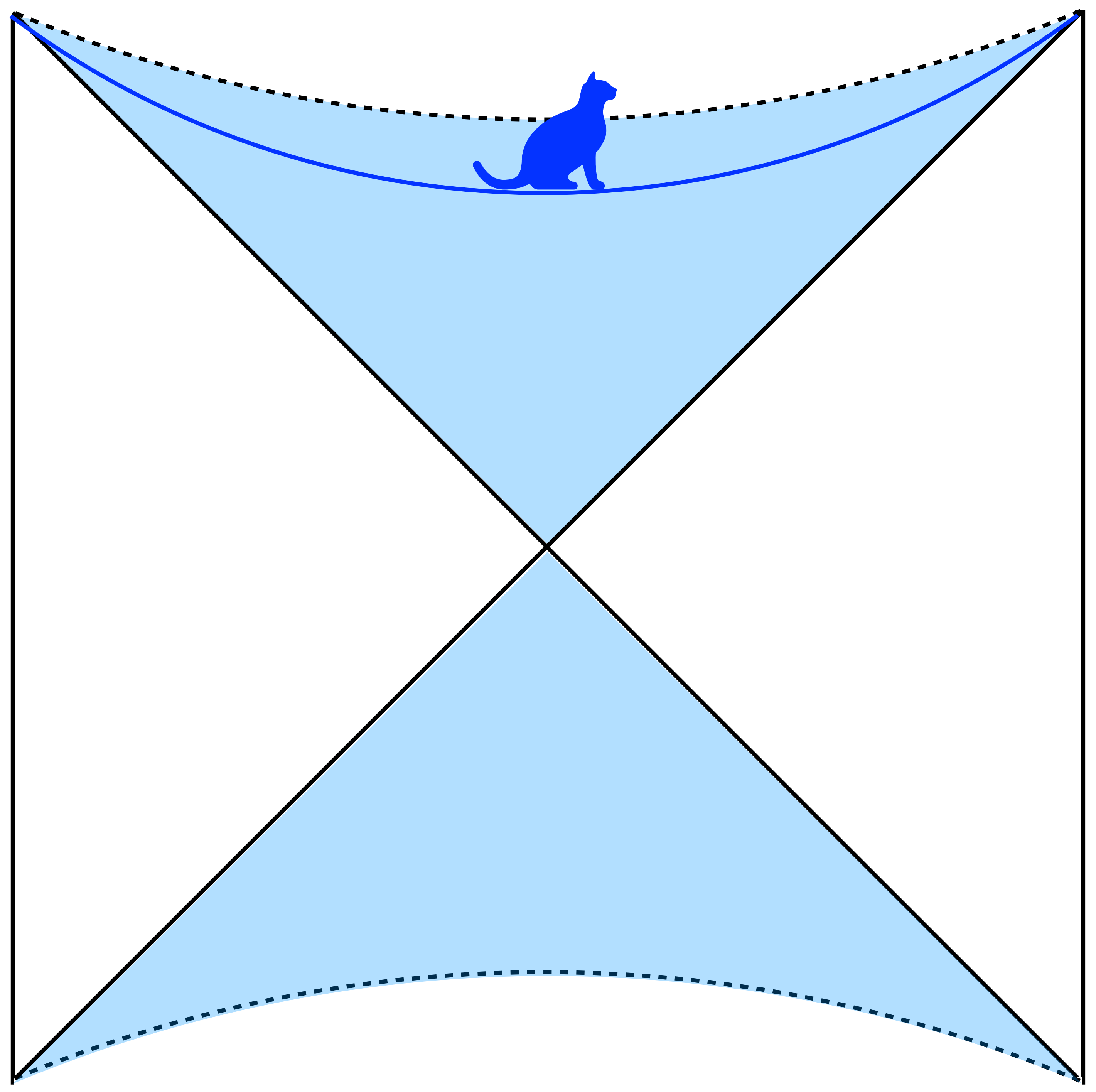
= Absolutely maximally entangled (AME) state

1. Background and brief summary

2. Main argument

3. Connection to non-Hermitian systems

4. Discussion



Background: Black hole (exterior) and Holography

- Area law of black hole entropy Bekenstein, Hawking

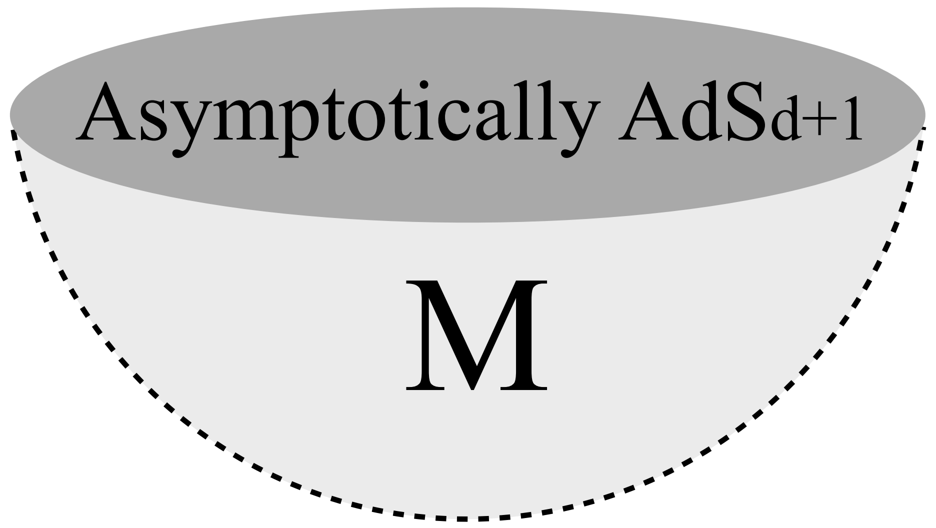
$$S_{BH} = \frac{A}{4G_N}$$

BH does not follow volume law !

- Holography: QG on M = Quantum Many-body System on ∂M

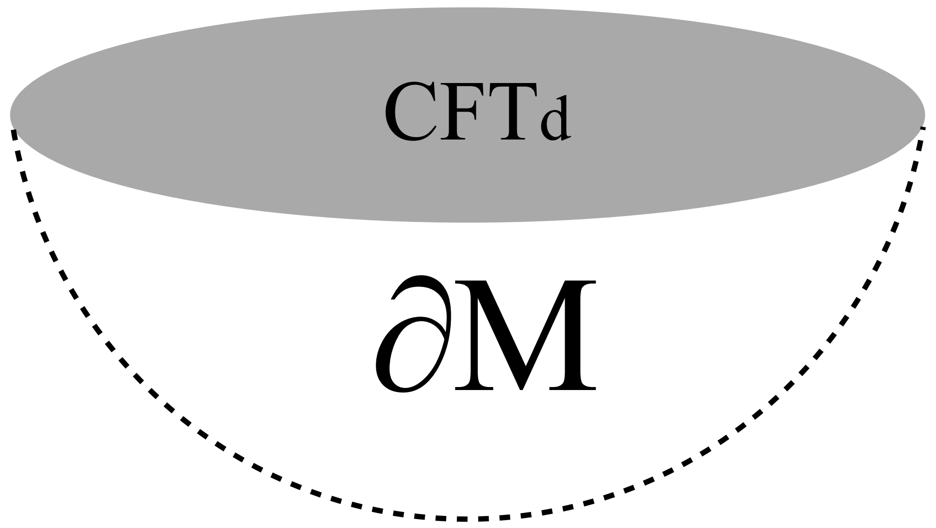
't Hooft, Susskind

Canonical example: AdS/CFT correspondence



=

Maldacena

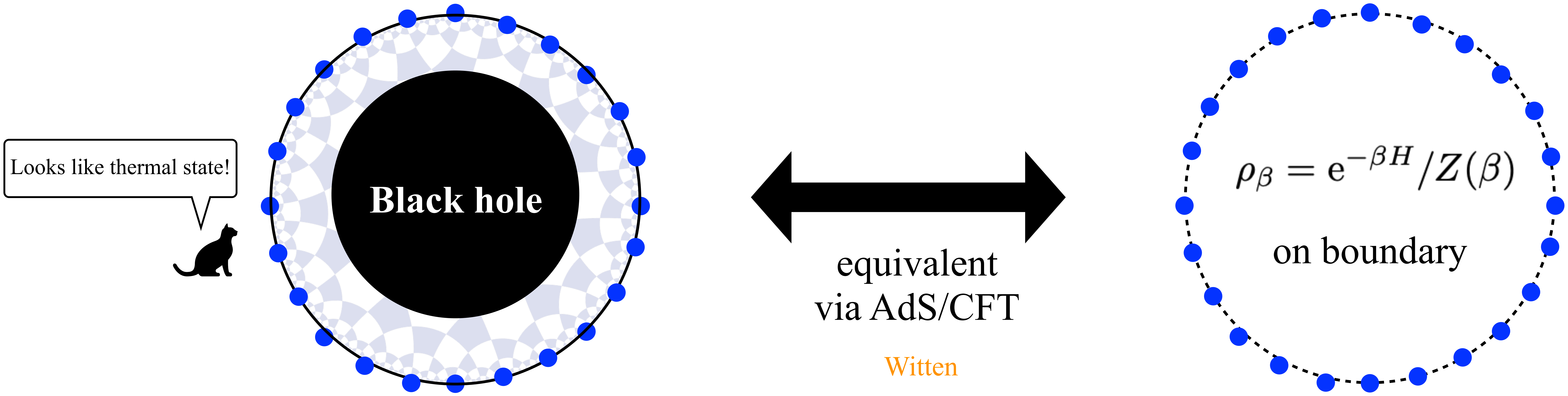


e.g.) partition function

$$Z_{\mathrm{AdS}}[M] = Z_{\mathrm{CFT}}[\partial M]$$

Gbuser-Klebanov-Polyakov, Witten

Background: Black hole (exterior) and Holography



Example:

BH in AdS_{d+1} with inverse temperature β

Thermal state in CFT_d with inv. temp. β

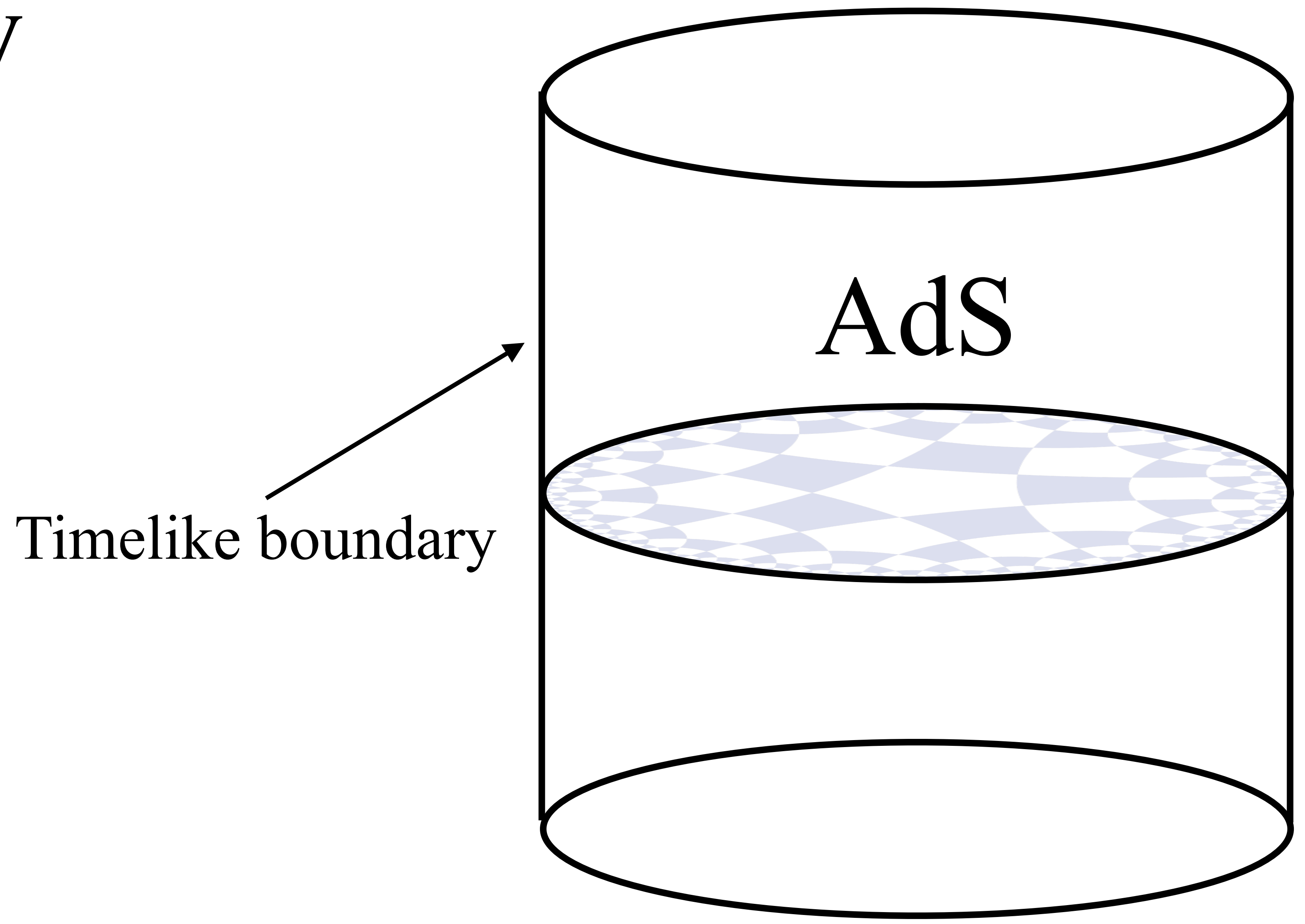
- Microstates of BH ~ Typical states: von Neumann entropy for subsystems follow volume law (in d-dim.)
- Random states nicely explain some physics of black holes *e.g.* Page curve (entanglement entropy),…

These are essentially views from the outside. **How to describe the interior from holography?**

Another motivation: holography *without* timelike boundary

- AdS/CFT [Maldacena] suggests QG with timelike boundary

  can be described by “boundary” QM systems



- Our universe probably has no such boundary!

  c.f. de Sitter spacetime

- Can we formulate similar framework without timelike boundary?

Black hole interiors as relatively tractable examples

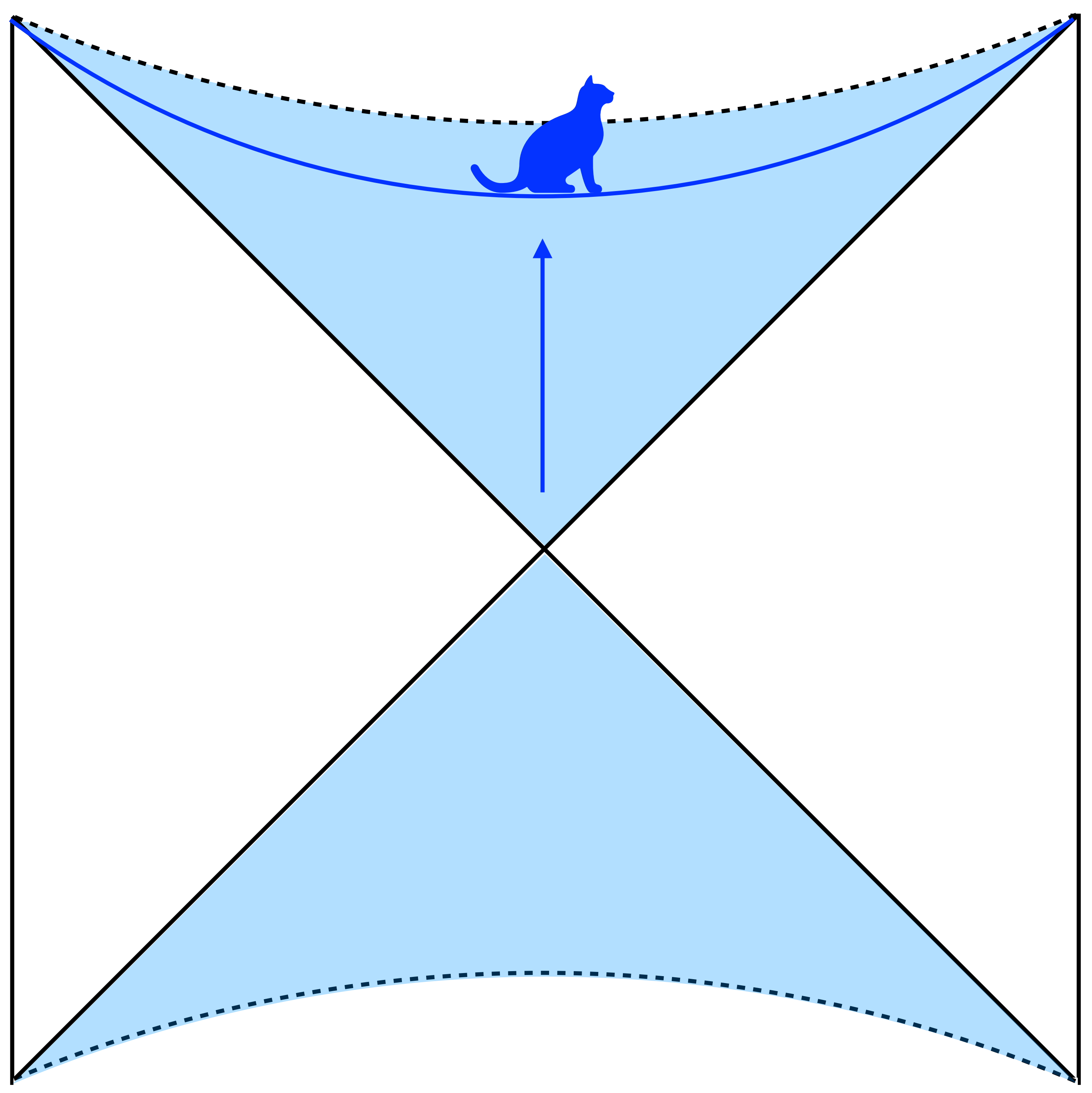
- Similarities with dS holography (dS/CFT)

  Strominger

  - Emergent radial direction = timelike

  c.f. Takayanagi-san's talk

  - Dual theory may be 'non-Hermitian' as well



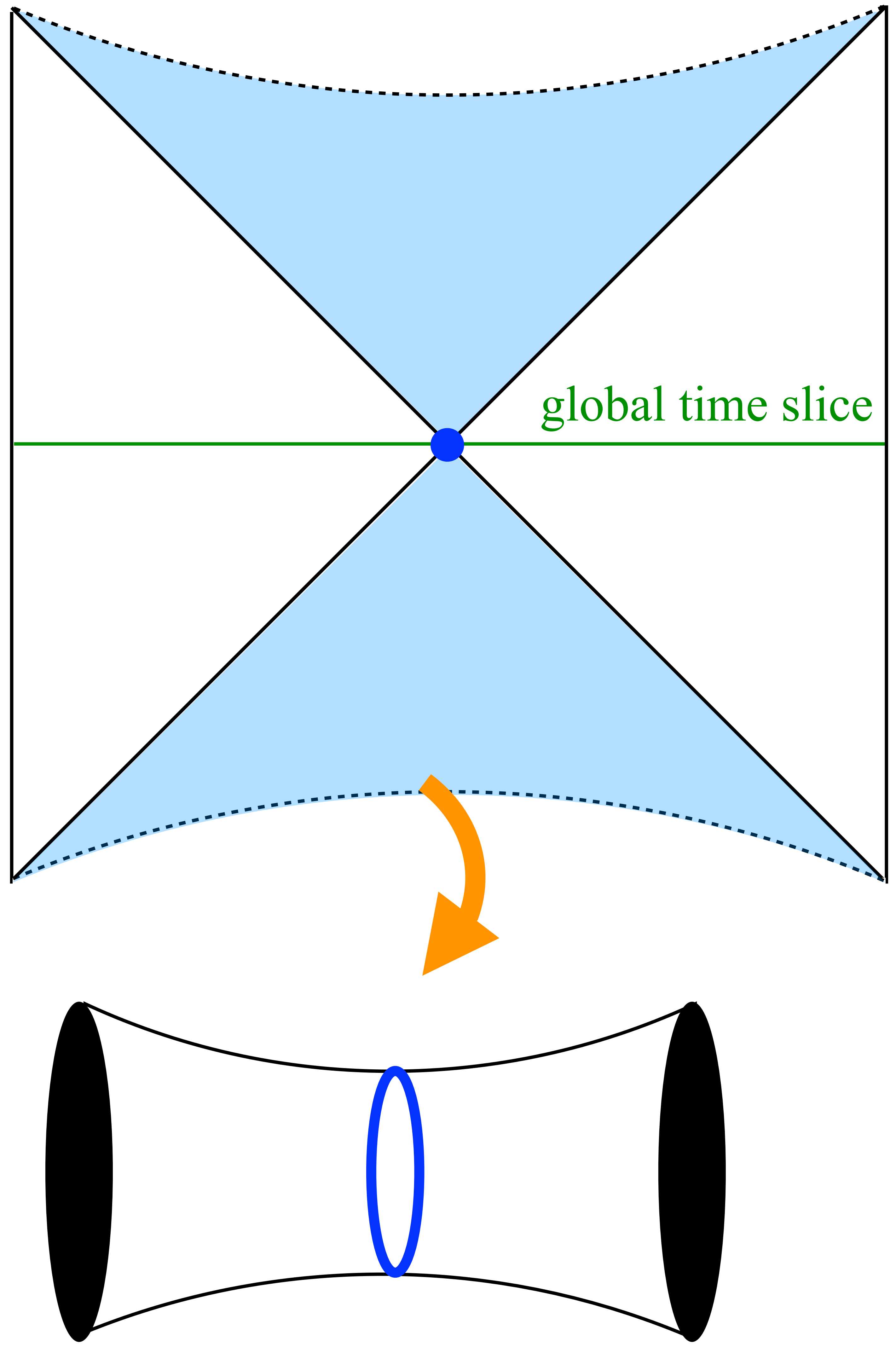
- More controllable than de Sitter

  We are sure about AdS/CFT

  → deformation of standard AdS/CFT (discussed later)
  may give description of interior holography

Better understanding of BH interior may also help the understanding of our universe…

Holographic description of black hole interior using boundary CFTs



- Global time slice

$$\leftrightarrow \quad |\mathrm{TFD}_\beta\rangle = Z^{-\frac{1}{2}}(\beta) e^{-\frac{\beta}{2}H} \sum_n |n_L\rangle\,|n_R\rangle$$

Maldacena (See also Israel, Balasubramanian-Kraus-Lawrence-Trivedi,…)

- A purification of thermal state:

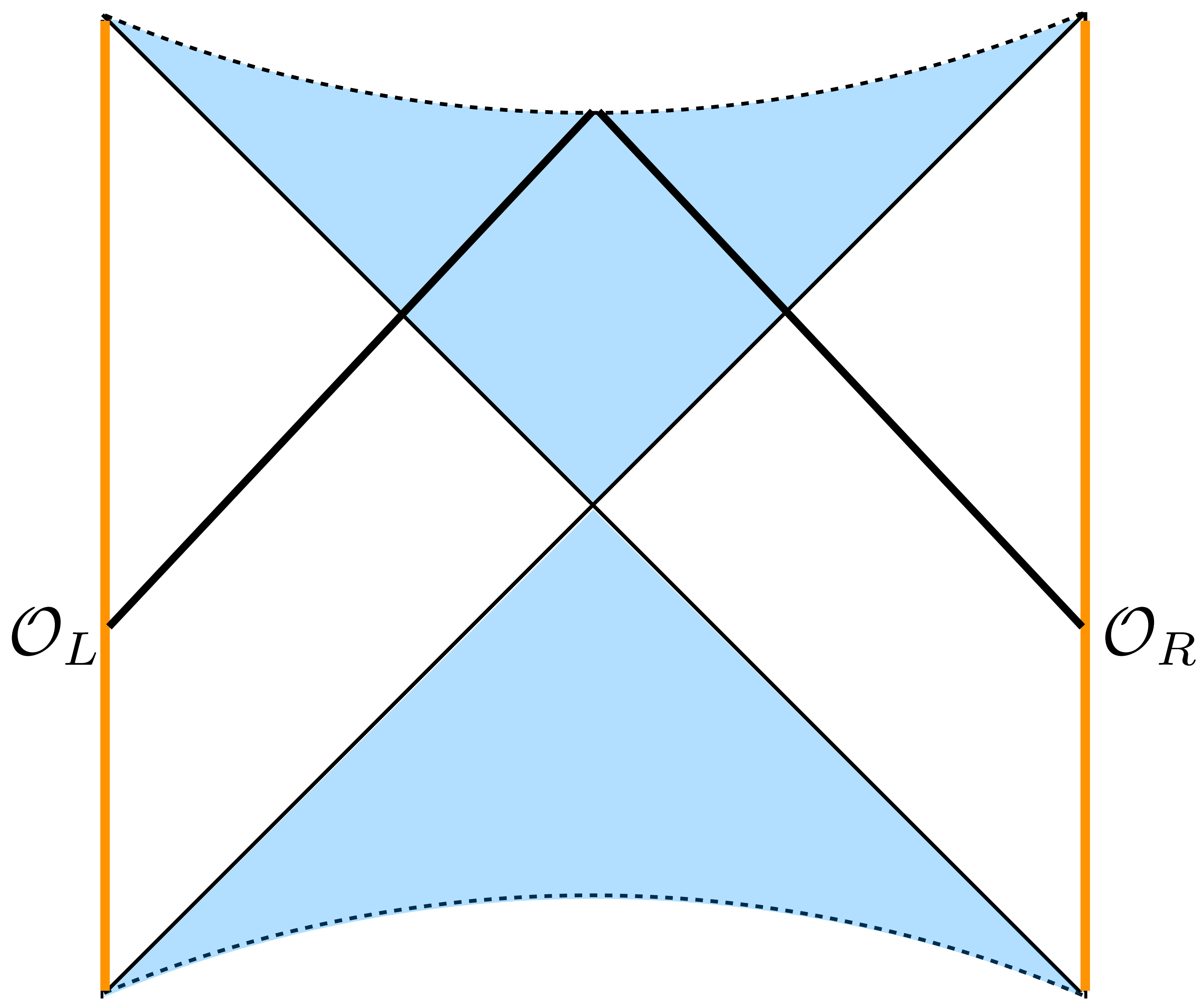
$$\rho_R \equiv \mathrm{Tr}_L |\mathrm{TFD}_\beta\rangle\langle \mathrm{TFD}_\beta| = \frac{1}{Z(\beta)} \sum_n \mathrm{e}^{-\beta E_n} |n_R\rangle\langle n_R| = \rho_\beta$$

- The simplest example of Ryu-Takayanagi formula:

$$S(\rho_R) = -\mathrm{Tr}_R \rho_R \log \rho_R = -\mathrm{Tr}_R \rho_\beta \log \rho_\beta = \underline{S_{\mathrm{BH}}}$$

Holographic Entanglement Entropy = Area of (minimal surface)/$4G_N$

Boundary description of interior is somewhat indirect



e.g.) boundary correlation functions cannot directly probe interior if $\exists$ singularity

Fidkowski-Hubeny-Kleban-Shenker, Festuccia-Liu,…

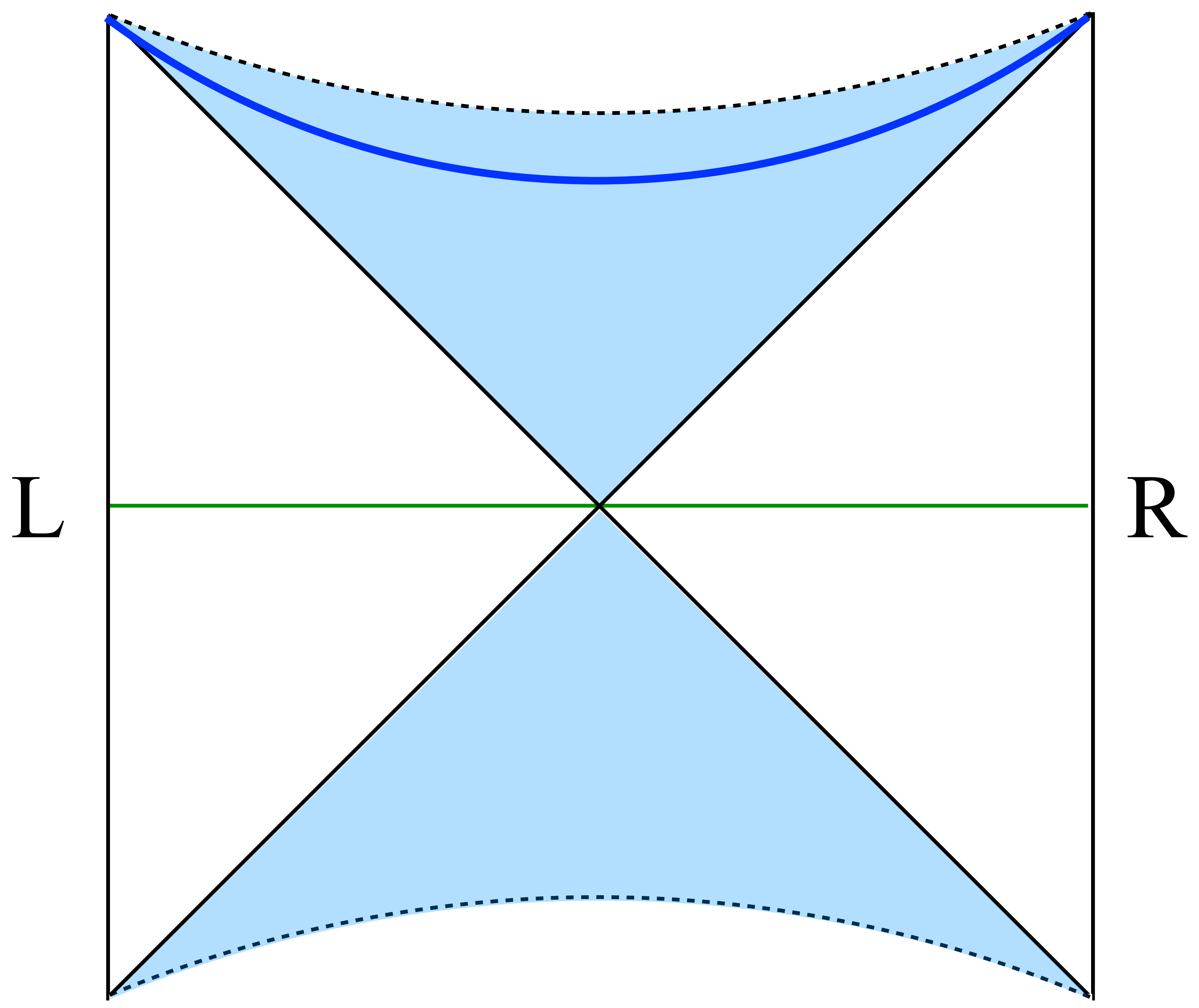
Geodesic approximation does not work (‘bouncing geodesics’)

$$\langle \mathrm{TFD}_\beta | \mathcal{O}_L \mathcal{O}_R | \mathrm{TFD}_\beta \rangle \not\approx e^{-\Delta L_{\mathrm{geodesics}}}$$

Note: Information of the geodesics are hidden in the second sheet of the analytically continued correlator

Q. Simple expression of state in interior holography? (Any “nice slice”?)

A “nice slice”: Special extremal slice

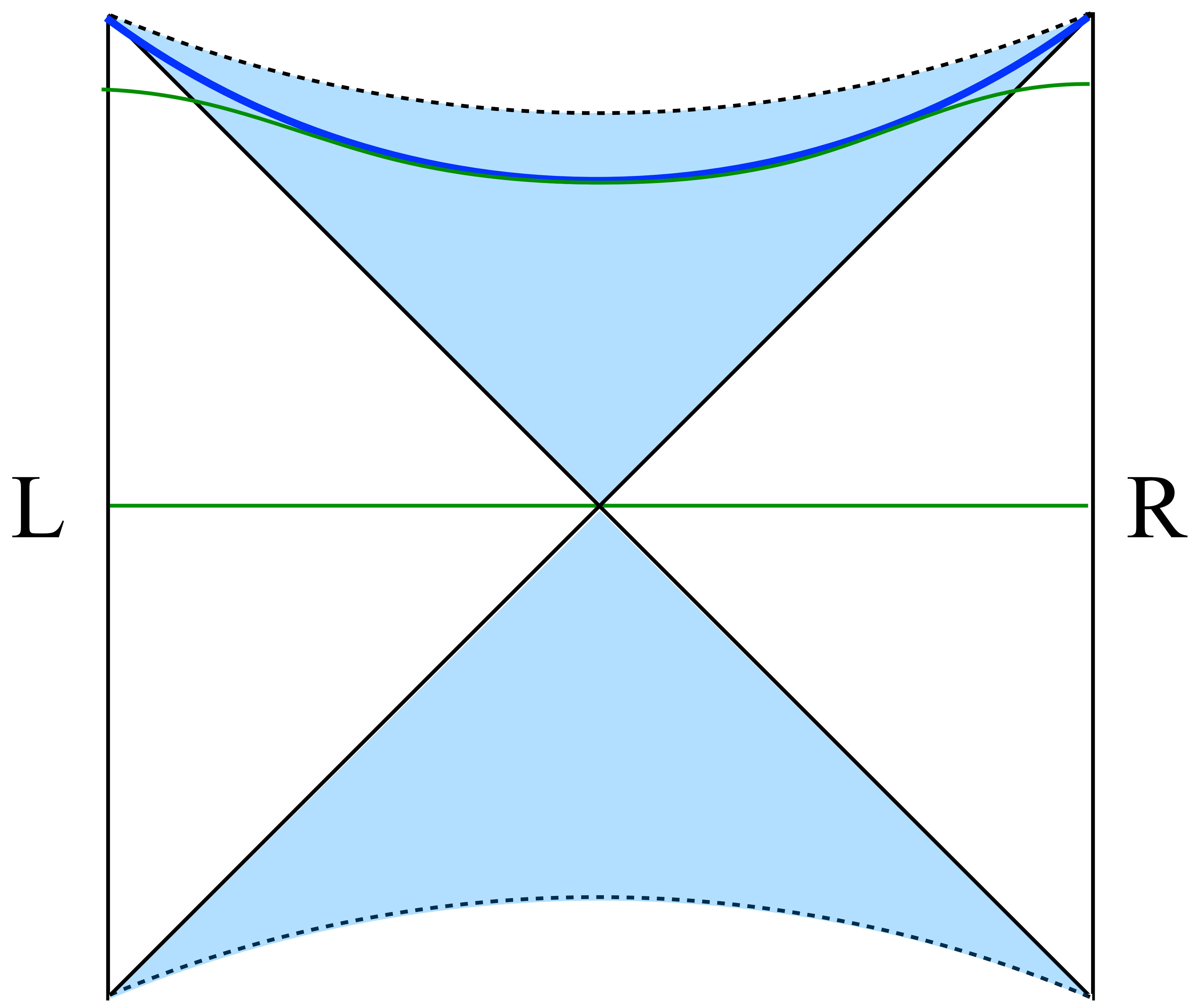


- Constant r slice inside of blackhole
- Defined by maximum of certain area functional A(r)
- Originally appeared when calculating Holographic EE for a global quenched ($-H_R \to +H_R$) TFD state

Hartman-Maldacena

$$|\mathrm{TFD}_\beta(t)\rangle \equiv e^{i(H_L+H_R)t_b/2}\,|\mathrm{TFD}_\beta\rangle$$

A "nice slice": Special extremal slice



- Constant r slice inside of blackhole
- Defined by <u>maximum</u> of certain area functional A(r)
- Originally appeared when calculating Holographic EE

  for a global quenched $(-H_R \to +H_R)$ TFD state

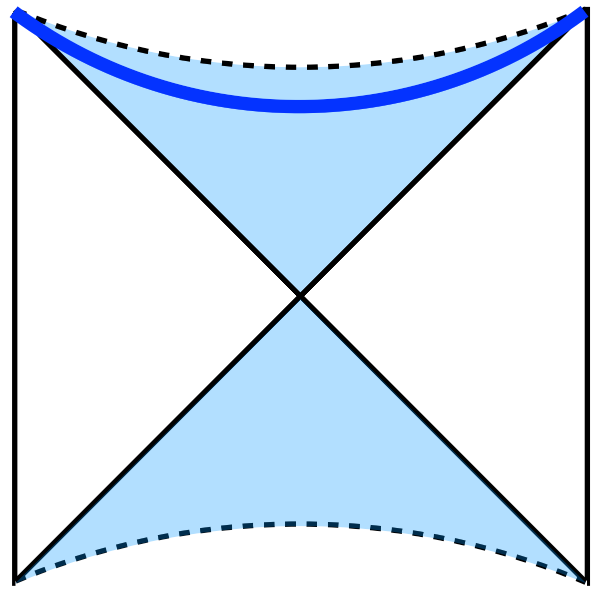
Hartman-Maldacena

$$|\mathrm{TFD}_\beta(t)\rangle \equiv e^{i(H_L+H_R)t_b/2}\,|\mathrm{TFD}_\beta\rangle$$

Summary of Results:

Special extremal slice for eternal black hole ↔ AME state

(c.f. global time slice ↔ TFD state)

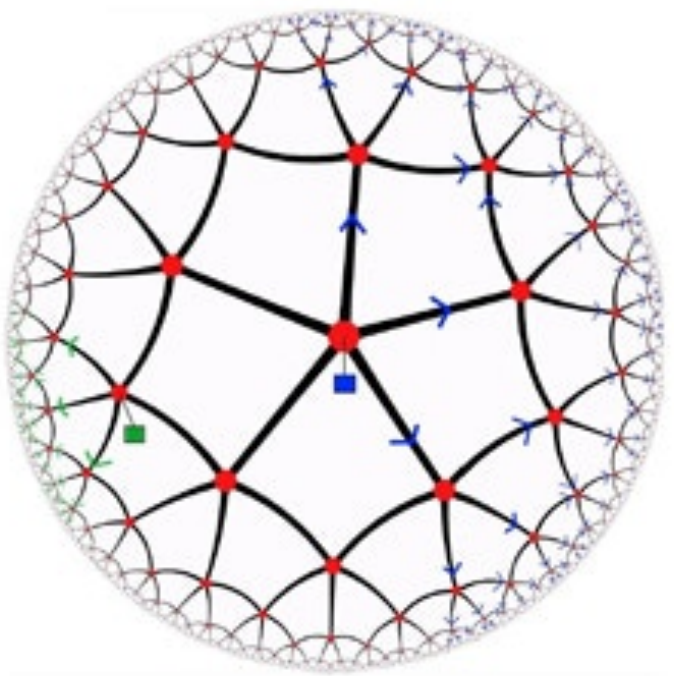


- **Absolutely Maximally Entangled (AME) state** : Maximally entangled state for any bi-partition

→ n-independence of n-th Renyi entropy (**flat spectrum**) $S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n$

- Widely used in tensor network toy model of holography

e.g.) HaPPY code (**Pa**stawski-**Y**oshida-**Ha**rlow-**P**reskill)



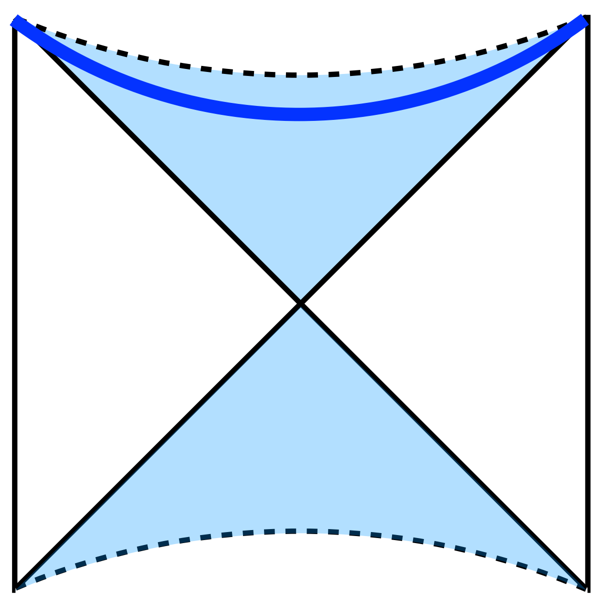
- In the conventional AdS/CFT, states dual to semi-classical spacetimes are *not* AME

c.f. fixed area state Dong-Harlow-Marolf,…

Summary of Results:

Special extremal slice for eternal black hole $\leftrightarrow$ AME state

(c.f. global time slice $\leftrightarrow$ TFD state)

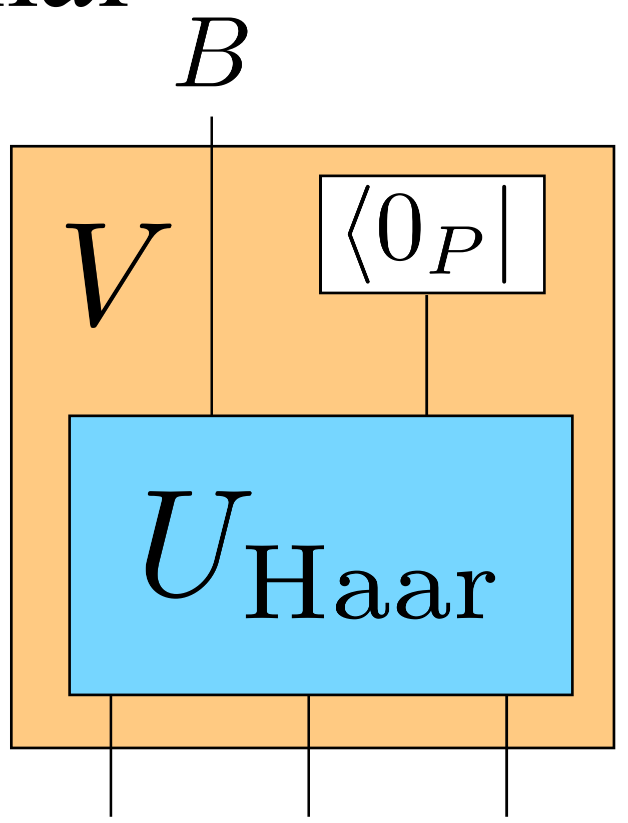


- Our AME states can be approximated as **Haar random states** $|\mathrm{AME}\rangle \simeq U_{\mathrm{Haar}} |\phi_0\rangle$

☑ multi-partite entanglement as well Anegawa-Suzuki-KT

- Black hole interior is indeed highly random as outside, but infinite dimensional

Gives a direct verification for recent non-isometric code model



Main claim:

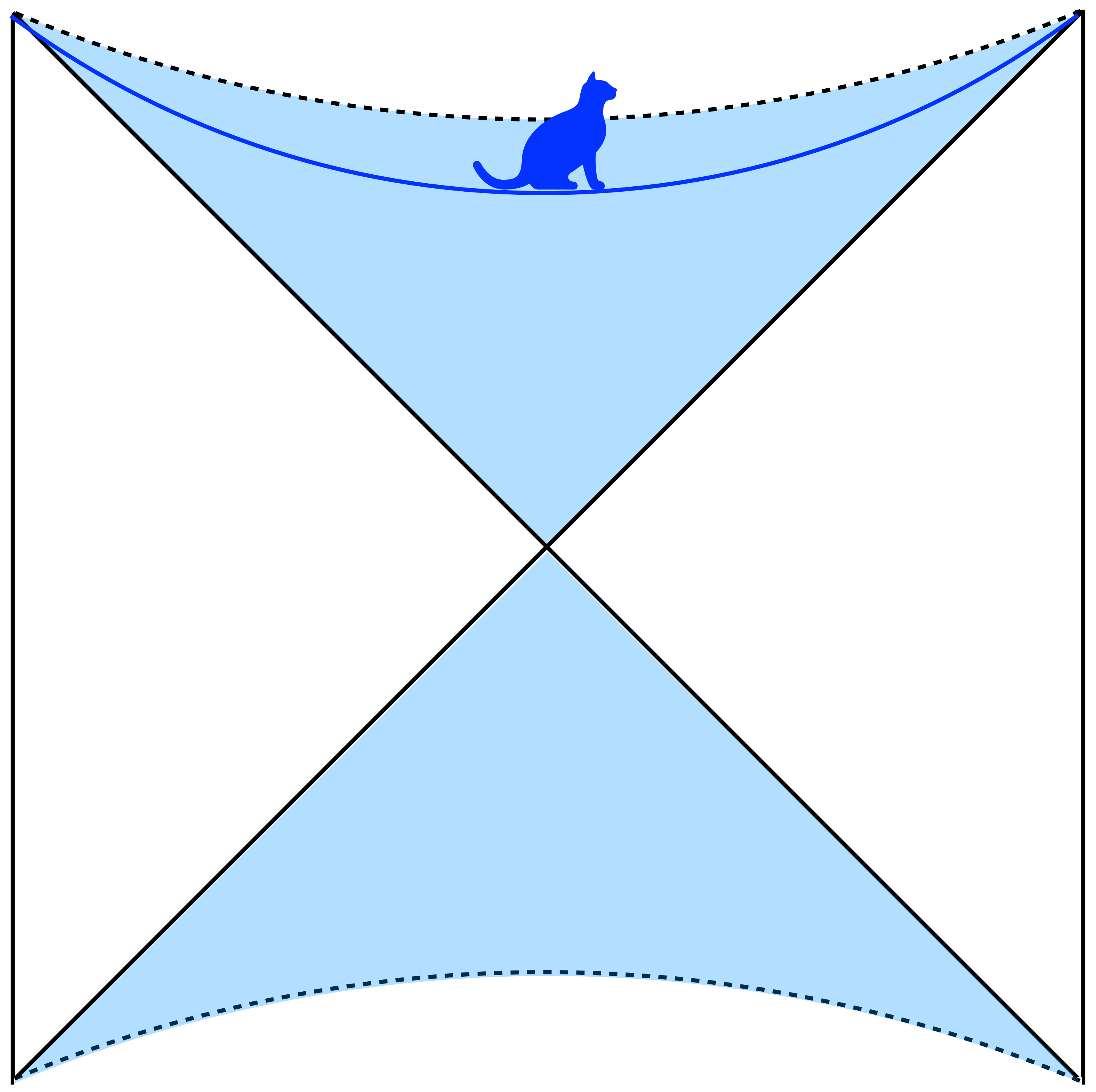
State dual to a time slice in BH interior (called special extremal slice)
= Absolutely maximally entangled (AME) state

1. Background and brief summary

2. Main argument

3. Connection to non-Hermitian systems

4. Discussion



Setup and assumption

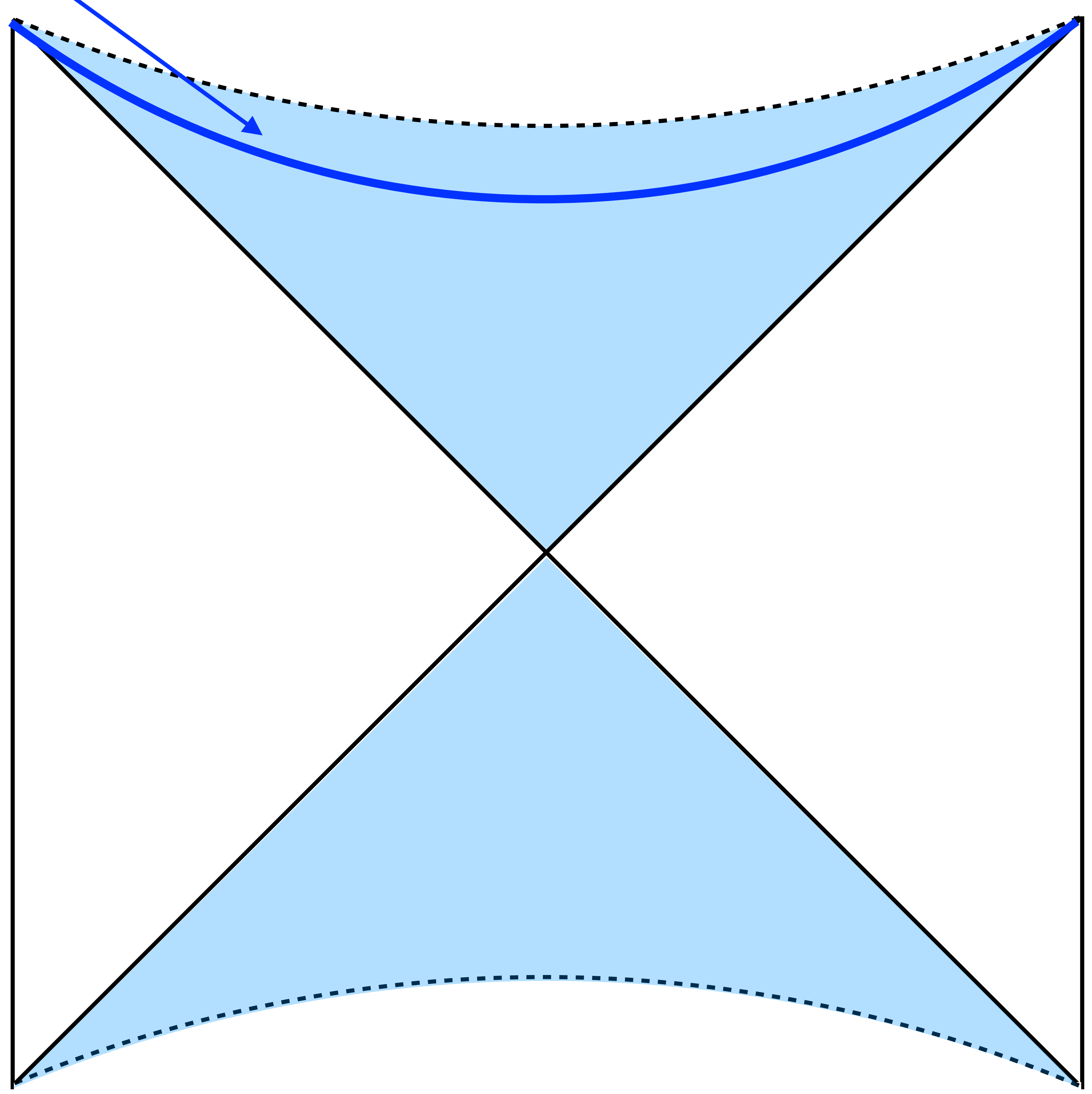
- Pure Einstein gravity in AdS

- Dirichlet boundary condition at special extremal slice

- Existence of dual theory

  ($T\overline{T}$-like deformation in mind)

- Static black hole (for simplicity)



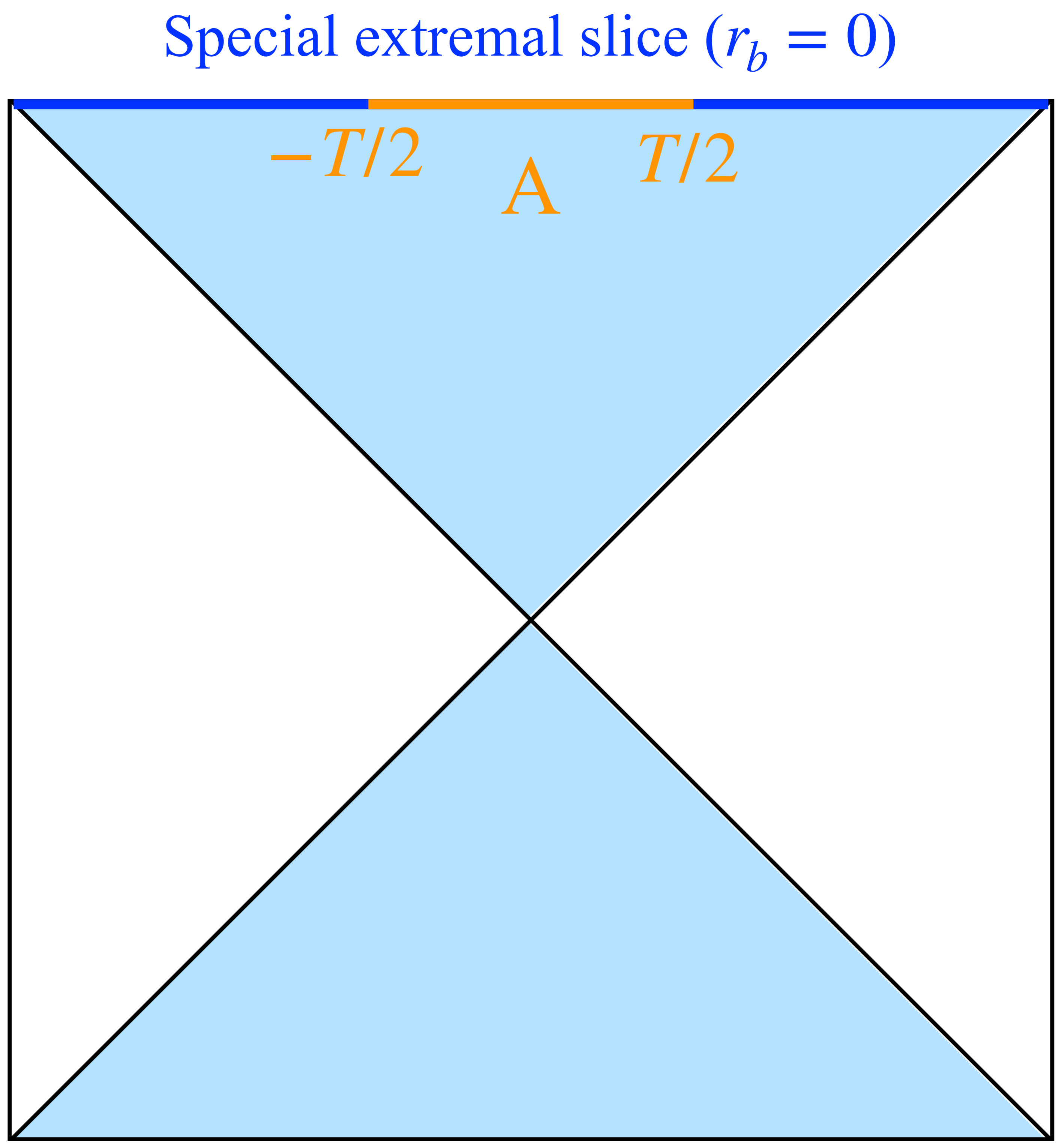
The simplest example: static BTZ black hole

$$ds^2 = (r_+^2 - r^2)\, dt^2 - \frac{dr^2}{r_+^2 - r^2} + r^2\, d\phi^2$$

- t is shifted by $i\frac{\beta}{4}$ compared with asymptotic infinity
- r-direction is timelike as $r < r_+$ (time-dependent b.g.)

Special extremal slice: $r_b = 0$ (Note: $r_b > 0$ for higher dim. BH)

Take a subregion A = [-T/2, T/2] along the t-direction



Holographic EE is computed via
Hubeny-Rangamani-Takayanagi (HRT) prescription (geodesics):

Just "straight line" along the screen! $$S_A = \frac{r_+ T}{4G_N} = \frac{S_{\mathrm{BH}}}{2\pi} T$$

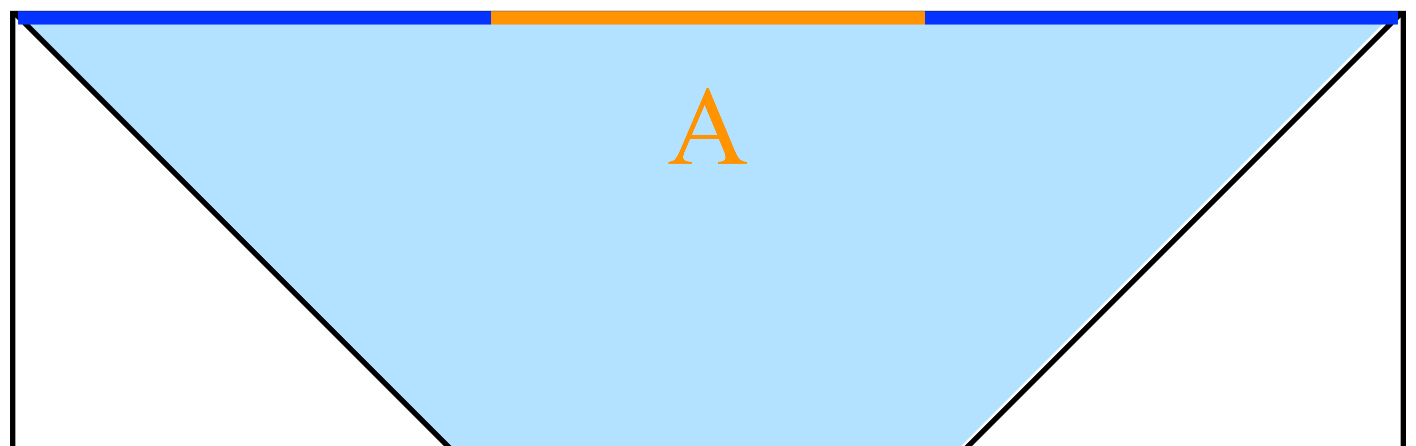
Holographic n-th Renyi entropy:

- It is useful to consider a refined Renyi entropy at first: $\tilde{S}_A^{(n)} = n^2 \partial_n \left(\frac{n-1}{n} S_A^{(n)} \right)$
- Gravity dual: $\tilde{S}_A^{(n)} = \frac{A(C_n)}{4G_N}$ Dong (n>1) , Dong—Kudler-Flam—Rath (0<n<1)

C_n : Area of co-dimension 2 cosmic brane with tension $T_n = \frac{n-1}{4nG_N}$

→ Backreaction onto original spacetime with deficit angle $\Delta\phi = 2\pi \left(1 - \frac{1}{n}\right)$

(1) In 3D gravity, metric is locally AdS_3
(2) By symmetry, we have the same configuration as the extremal surface
→ fixes metric to the original one **up to deficit angle along irrelevant direction**



$$\boxed{\tilde{S}_A^{(n)} = S_A^{(n)} = S_A \quad \text{Maximally entangled state}}$$

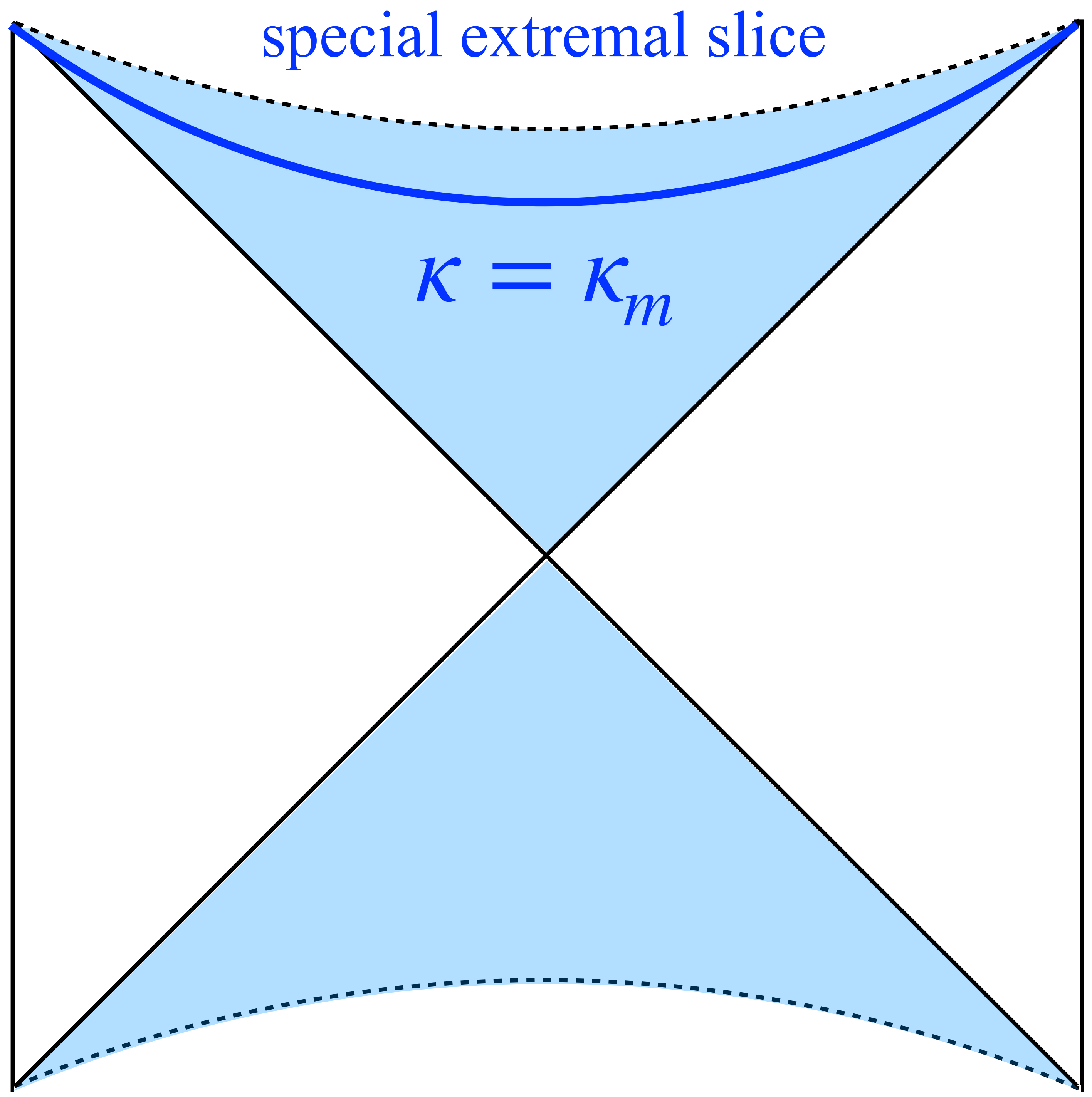
More general intervals have the same properties → AME

Higher dimensional generalization: AdS black brane

$$ds^2 = -g(\rho)^2 dt^2 + d\rho^2 + h(\rho)^2 d\vec{x}^2_{d-1}$$

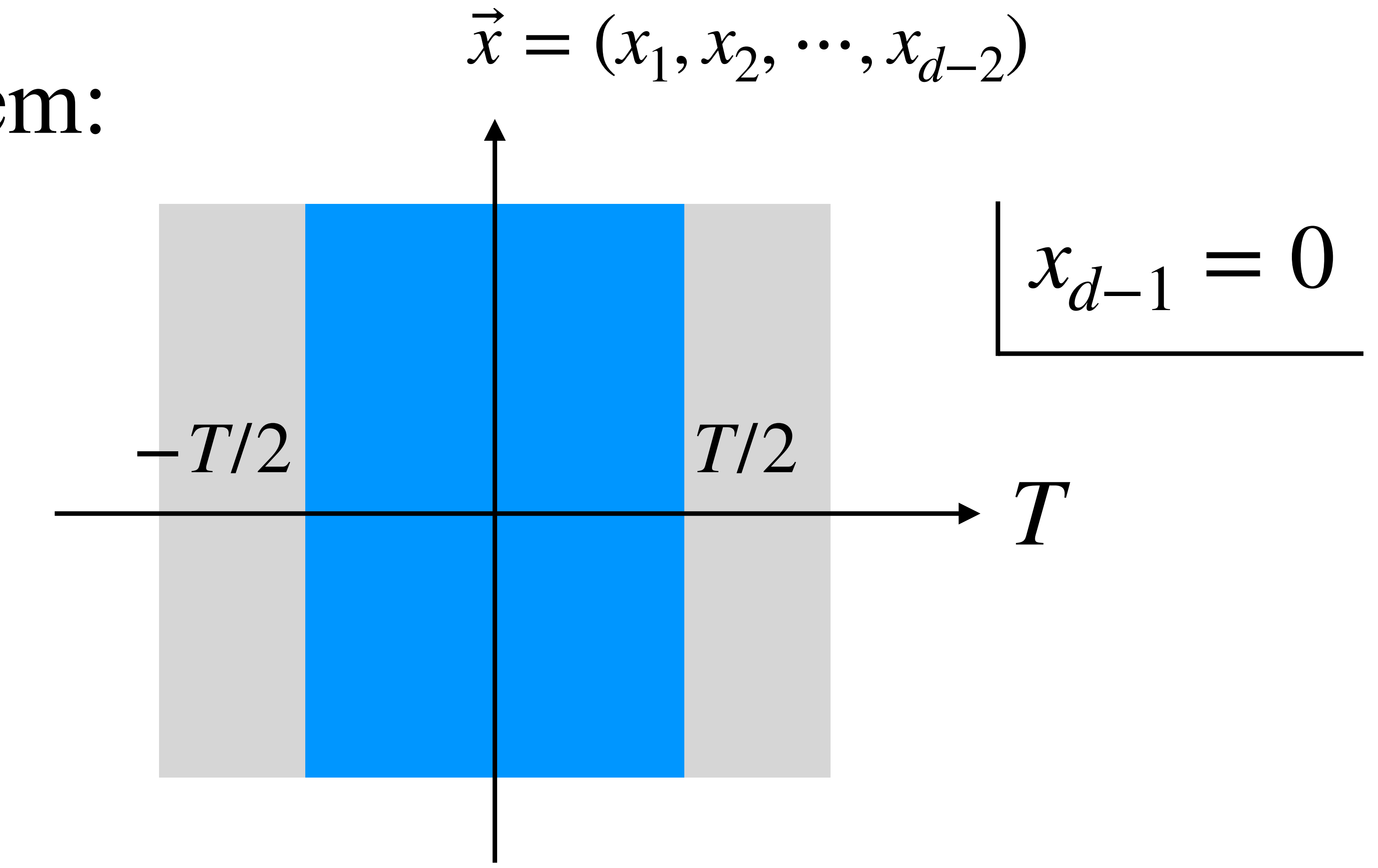
$$h(\rho) = \frac{2}{d}\left(\cosh\frac{d\rho}{2}\right)^{2/d} \qquad g(\rho) = h(\rho)\tanh\frac{d\rho}{2}$$

- Related to Poincare coordinates via $z = z_H\left(\cosh\frac{d\rho}{2}\right)^{-\frac{2}{d}}$
- Interior: $\rho = i\kappa$
- Special extremal slice: $\tan\frac{d\kappa_m}{2} = \sqrt{\frac{d}{d-2}}$



Illustrative example:

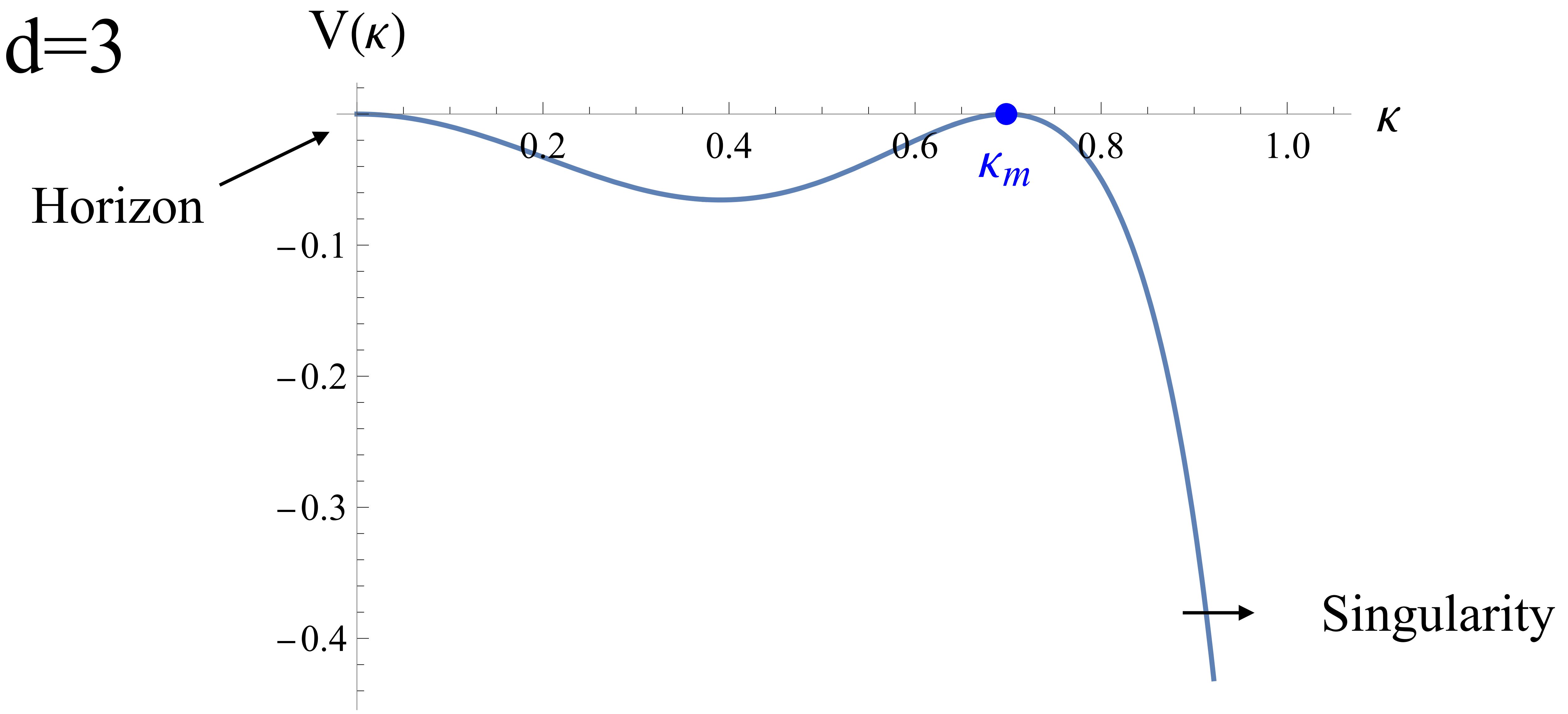
Subsystem:

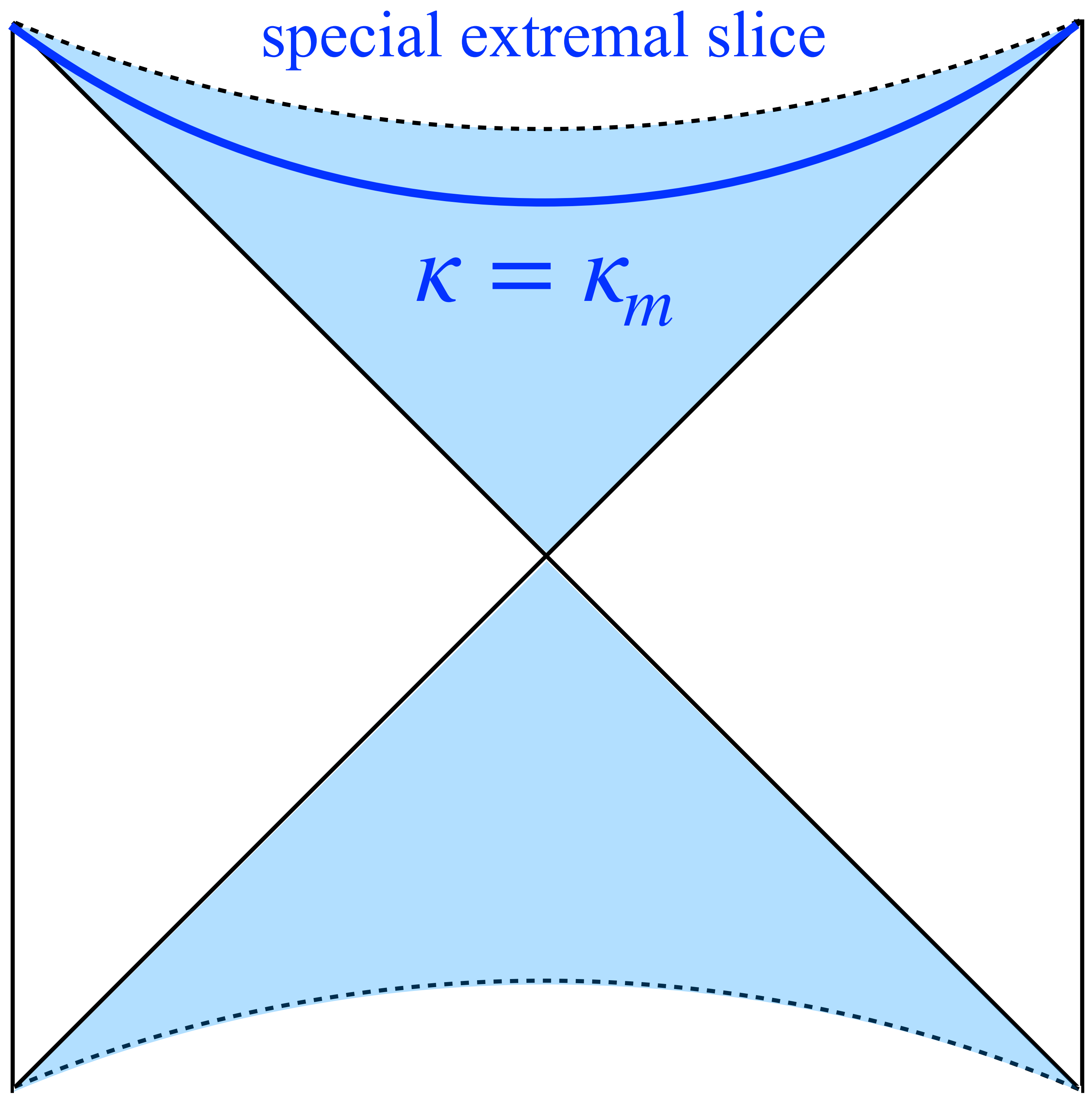


$$A = V_{d-2}^{\perp} \int_{-T/2}^{T/2} dt\, h(\rho)^{d-2} \sqrt{-g(\rho)^2 + \dot{\rho}^2}$$

Reduce to "potential problem": $\dot{\kappa}^2 = -V(\kappa)$ $\rho = i\kappa$

e.g.) d=3





Consistent solution with B.C. ($\kappa(\pm T/2) = \kappa_m$) : $\kappa(t) = \kappa_m$

$$\longrightarrow \quad A = V_{d-2}^{\perp} \sqrt{-g(i\kappa_m)}\, h(i\kappa_m)^{d-2} T$$

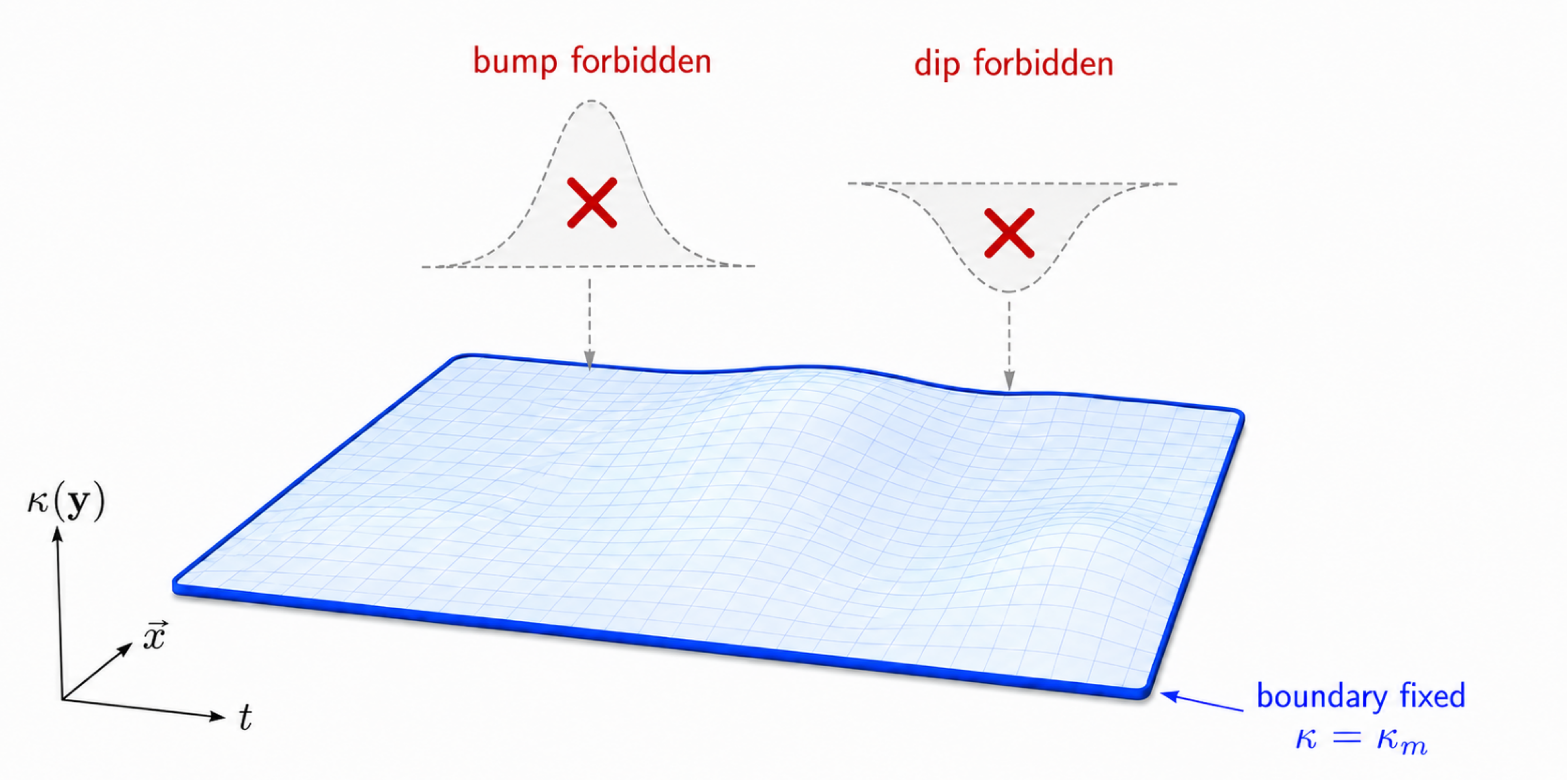
General argument: constant solutions for all n

In the interior, Euler-Lagrange equation becomes (quasilinear) elliptic differential equation

(Note: Hyperbolic for the exterior)

→ maximum principle gives the unique constant solution, $\kappa(\mathbf{y}) = \kappa_m$

c.f. electric potential with no charge: $\nabla^2\phi = 0,\ \phi|_\partial = \phi_0 \Rightarrow \phi = \phi_0$



Comments on multi-partite entanglement

Identifying an AME state does not uniquely determine the structure of multipartite entanglement

e.g.) 3-qutrit AME states

$$|\mathrm{GHZ}_3\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$$

$$|\mathcal{A}\rangle = \frac{1}{\sqrt{6}} \sum_{i,j,k=0}^{2} \epsilon_{ijk} |ijk\rangle$$

Both are AME: $\rho_X = \frac{1}{3} I_X \ (X = A, B, C)$

but two-party density matrix is separable/entangled

We further studied the so-called multi-entropy [Gadde-Krishna-Sharma]
and confirmed that it exhibits the same behavior as Haar random states

[Anegawa-Suzuki-KT] 2512.21037

Main claim:

State dual to a time slice in BH interior (called special extremal slice)

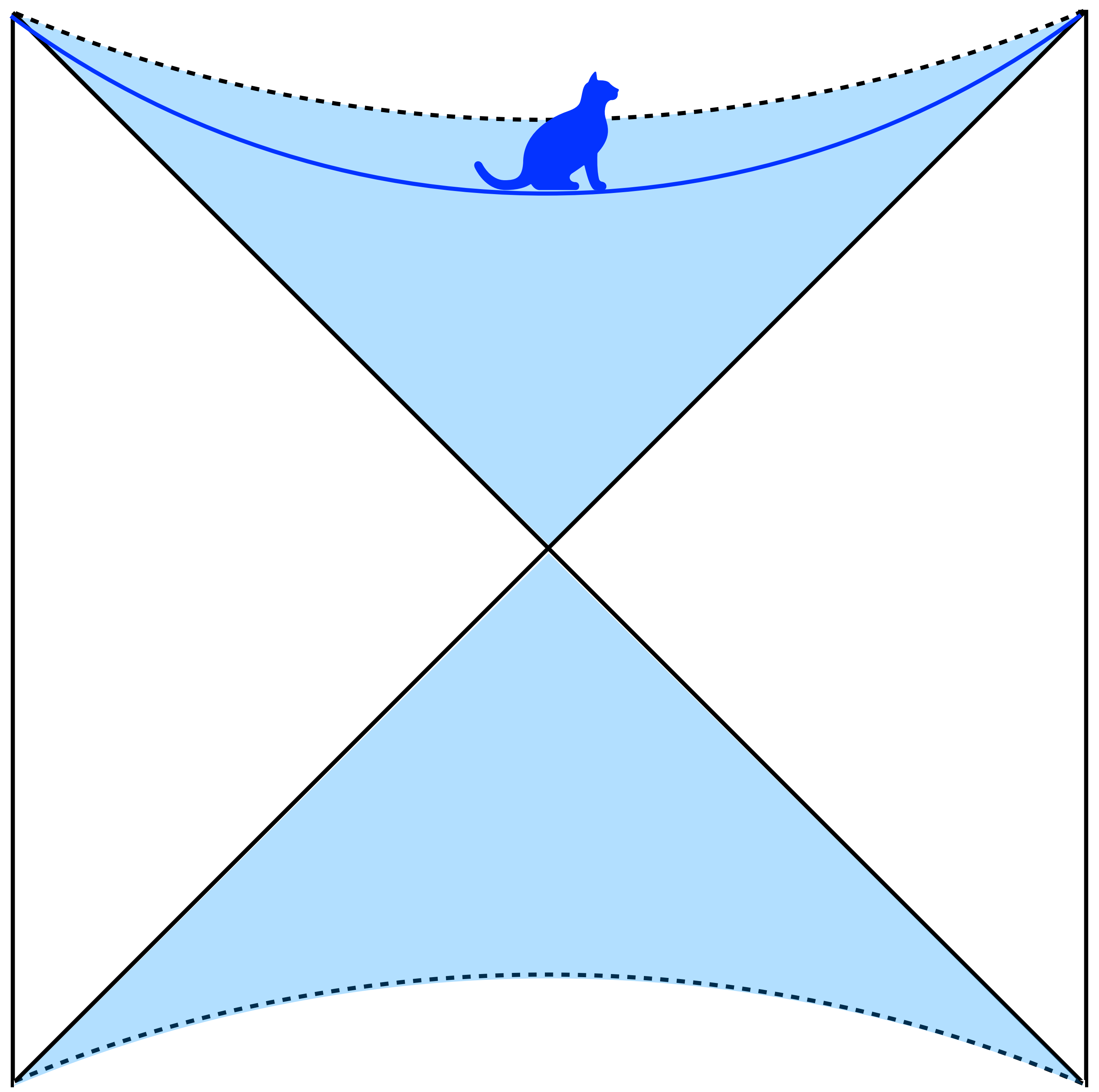
= Absolutely maximally entangled (AME) state

1. Background and brief summary

2. Main argument

3. Connection to non-Hermitian systems

4. Discussion



A known concrete realization of interior holography

- $\mathrm{T\overline{T}}$-deformation: an irrelevant deformation by adding $\mu \int d^2x \mathrm{T\overline{T}}$

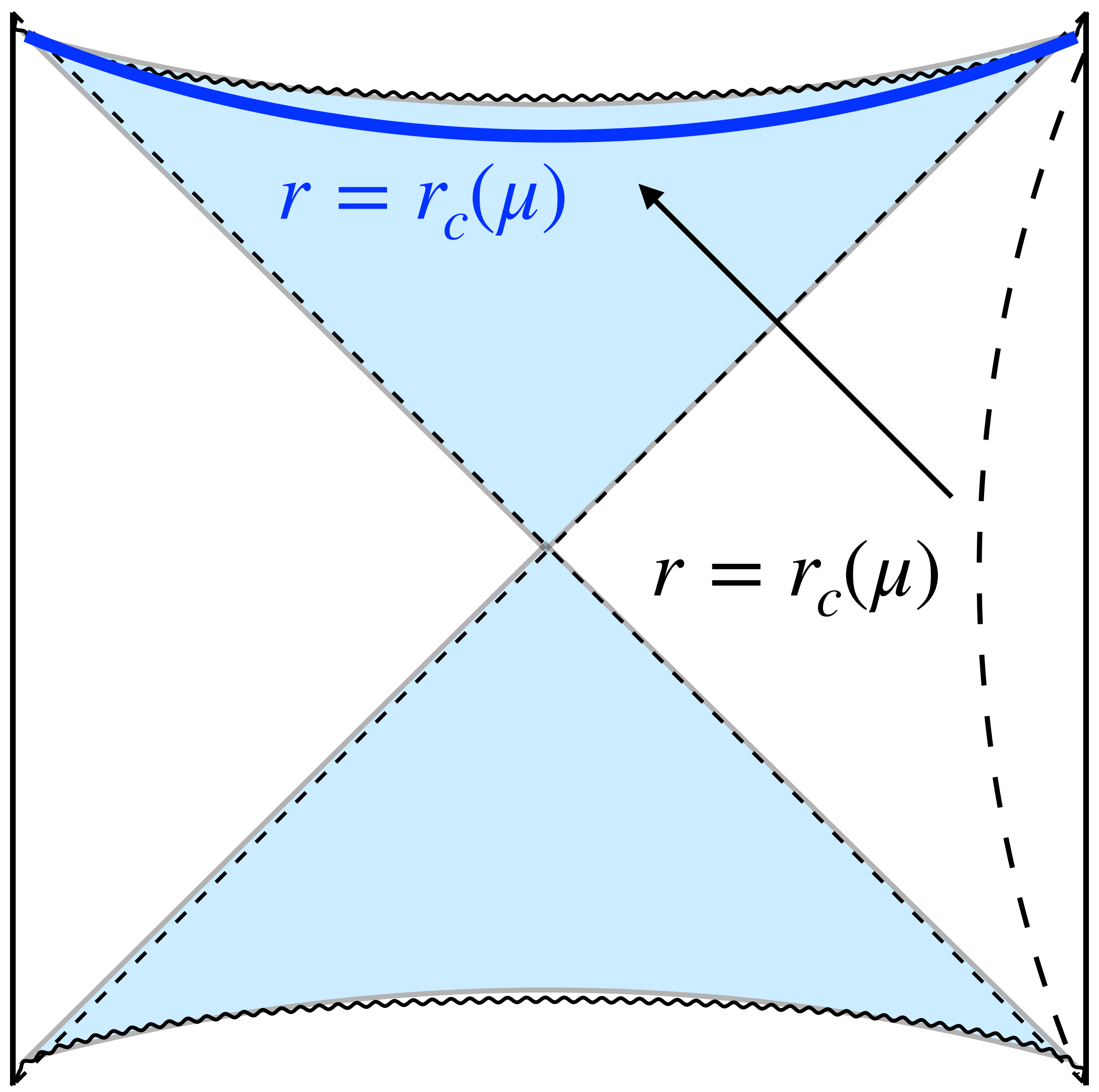
→ Can obtain deformed energy spectrum for 2D CFT Smirnov -Zamolodchikov

- Gravity dual: AdS with Dirichlet boundary condition at $r = r_c(\mu)$

McGough-Mezei-Verlinde

If we require r_c to be smaller than the horizon,
the energy spectrum becomes complex (non-Hermitian system)

→ Cauchy slice holography Araujo-Regado—Khan—Wall



One may view our result as $\mathrm{T\overline{T}}$-deformation
of timelike entanglement

c.f. previous talk by Takayanagi-san

Connection to non-isometric code

- Conventional AdS/CFT (outside of BH) can be understood as an isometric map:

Almheiri-Dong-Harlow,…

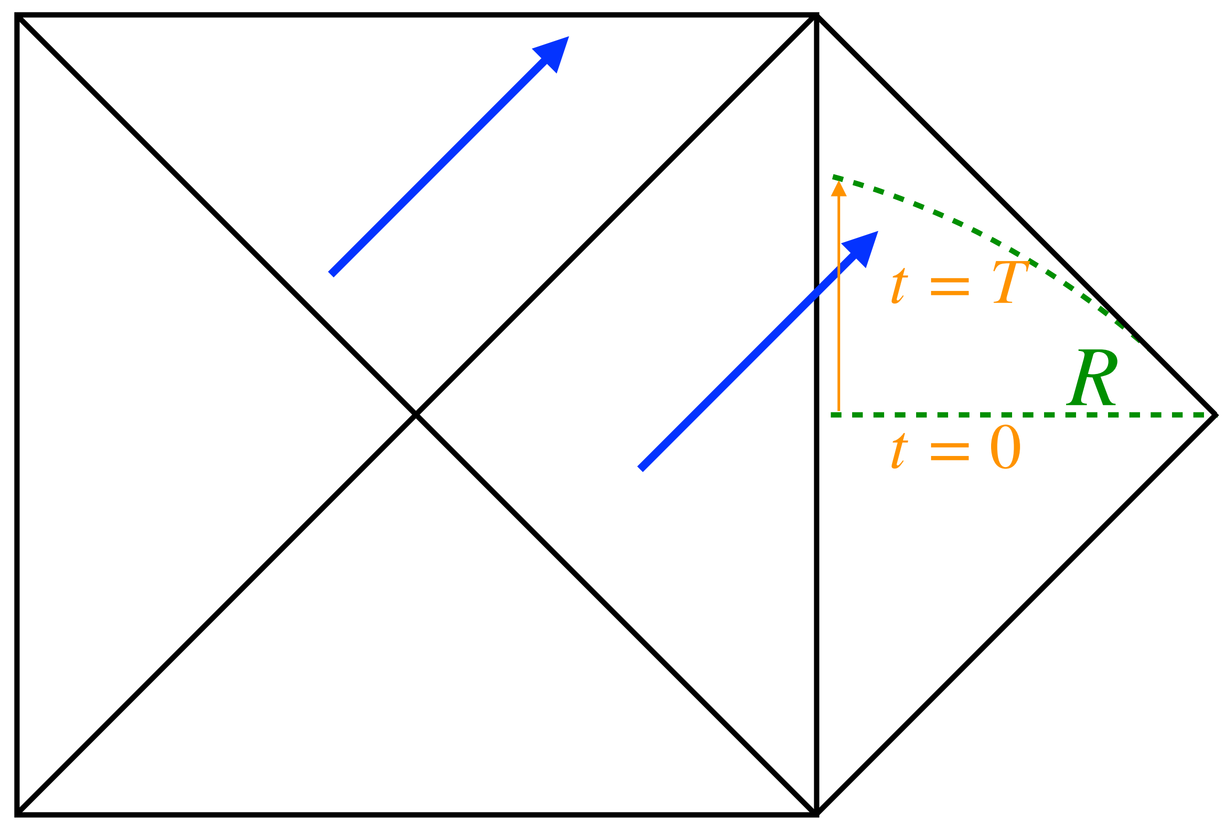
$$\mathcal{H}_{\mathrm{AdS}} \xrightarrow{V} \mathcal{H}_{\mathrm{code}} \subset \mathcal{H}_{\mathrm{CFT}} \quad \text{where} \quad V^\dagger V = 1_{\mathrm{AdS}}$$

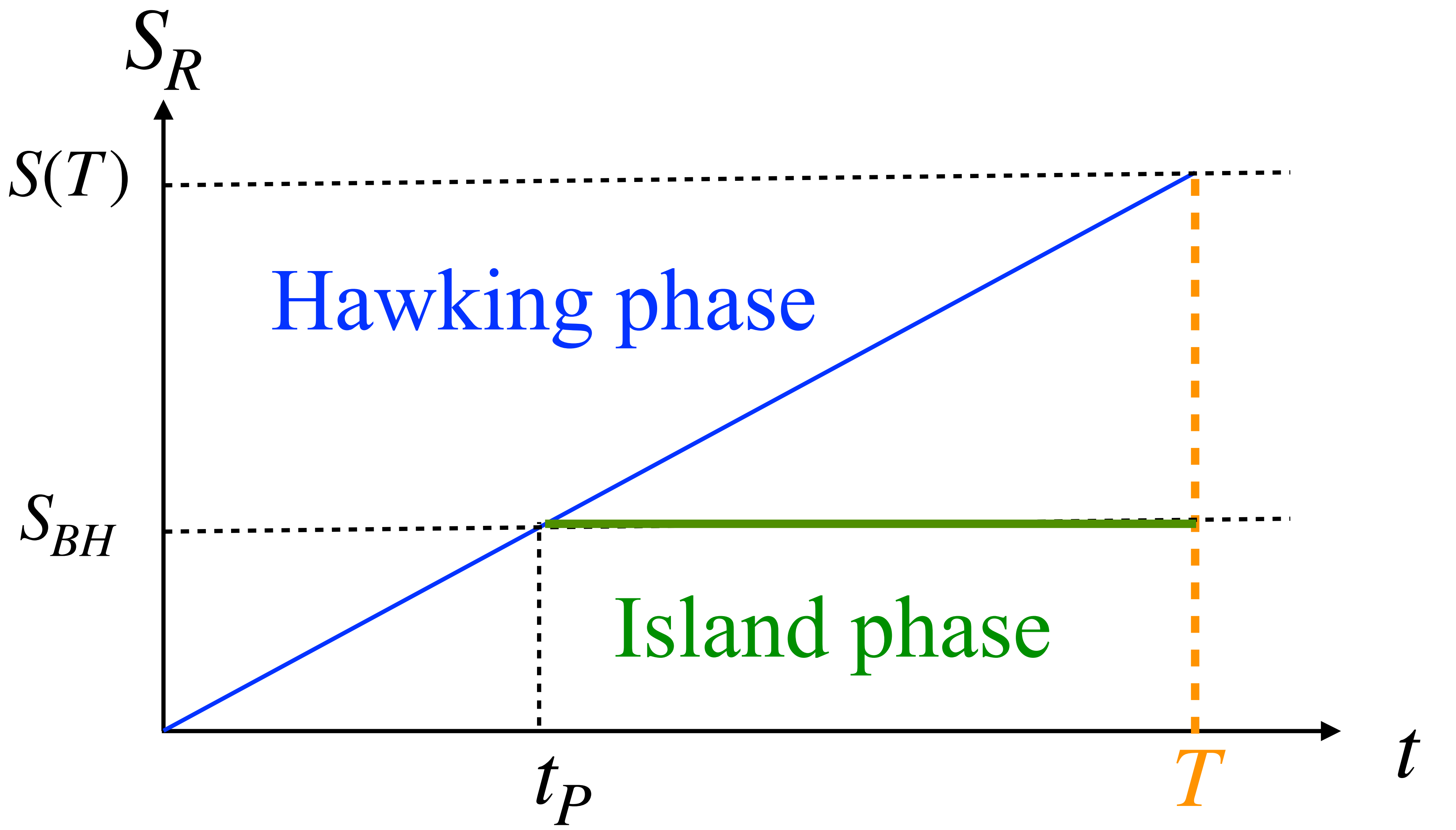
$\mathcal{H}_{\mathrm{AdS}}$: Hilbert space of low-energy effective theory

- Recently, we expect the holography (inside of BH) may be a *non*-isometric map:

Akers-Engelhardt-Harlow-Penington-Vardhan

$$V^\dagger V \neq 1$$





Connection to non-isometric code

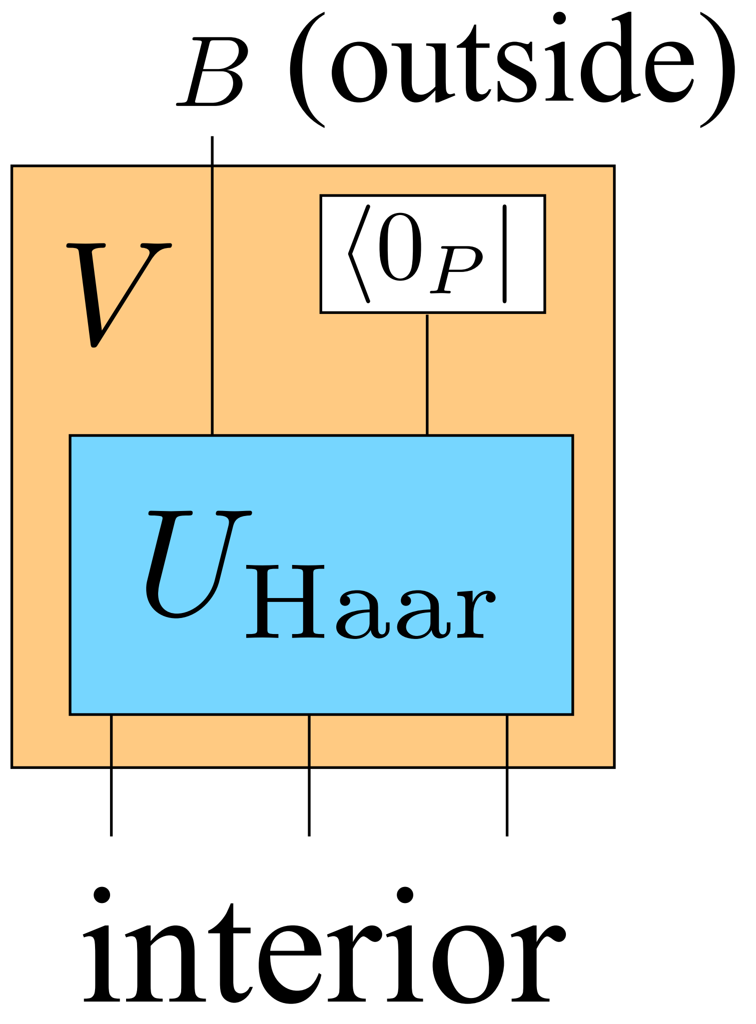
- $\dim \mathcal{H}_{\text{interior}} > \dim \mathcal{H}_{\text{CFT}}$ is a direct consequence of the fact that $\partial_{t_{\text{outside}}}$ becomes spacelike in BH interior

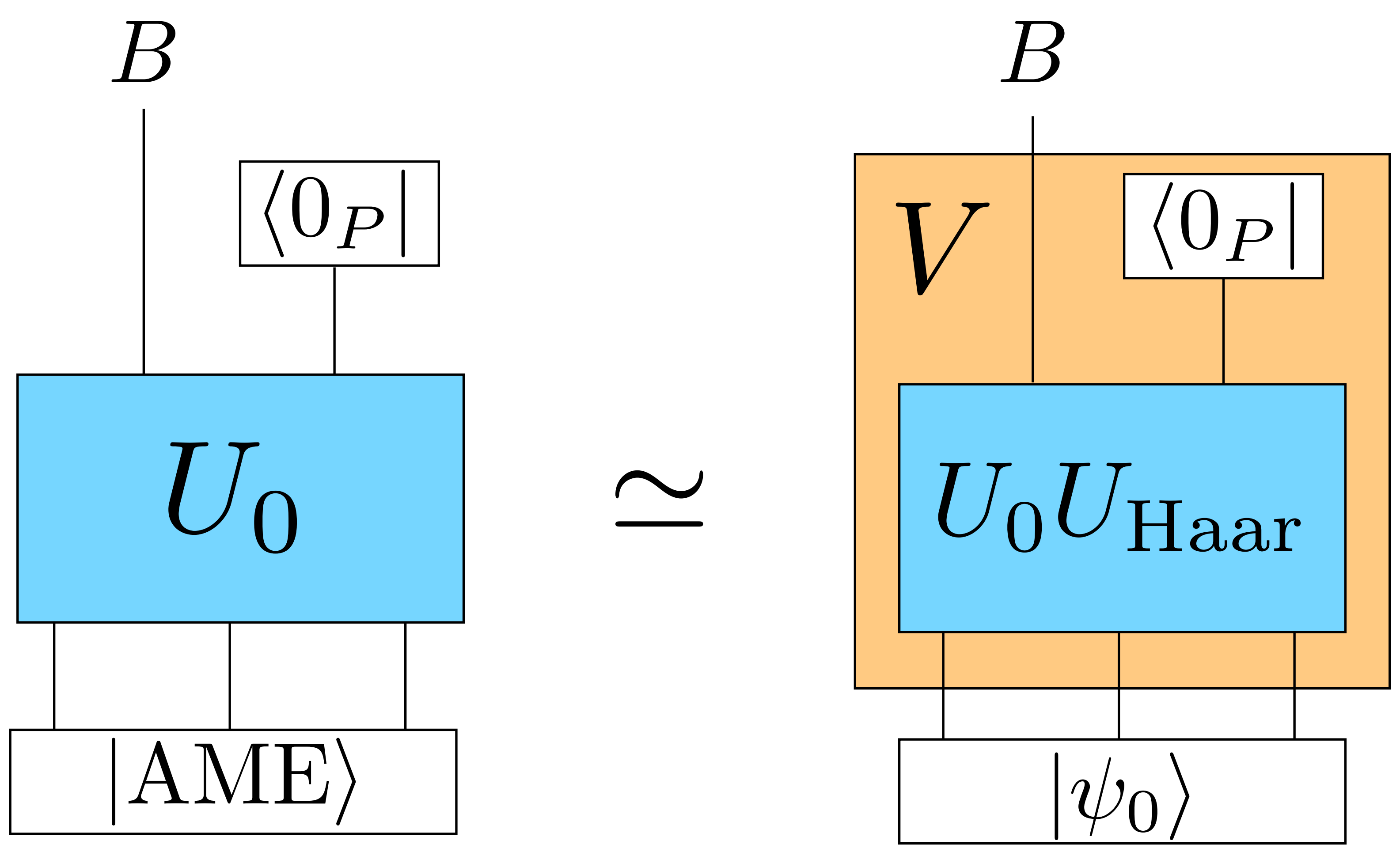
- Furthermore, our results guarantee the appearance of randomness :

  In our setting, $|\text{AME}\rangle \simeq U_{\text{Haar}} |\psi_0\rangle \;\; (d_A \ll d_{\bar{A}})$

e.g.) minimal model discussed in

Akers-Engelhardt-Harlow-Penington-Vardhan



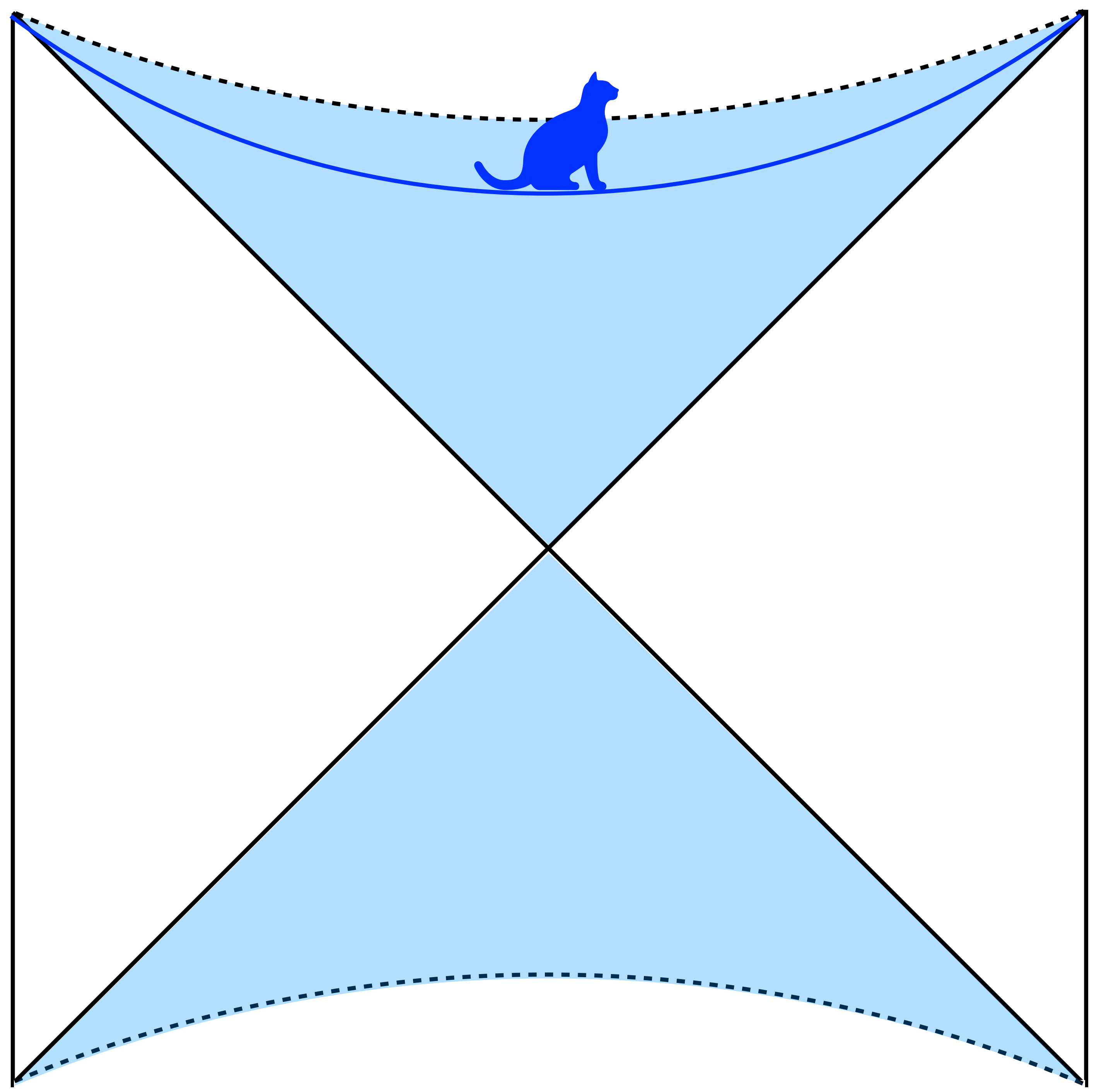


Main claim:

State dual to a time slice in BH interior (called special extremal slice)

= Absolutely maximally entangled (AME) state

1. Background and brief summary
2. Main argument
3. Connection to non-Hermitian systems
4. Discussion



Summary

・Black hole interior has a characteristic slice called special extremal slice

・Special extremal slice in AdS is dual to AME state

・Such AME states can be well approximated as Haar random states

・These findings give a direct consequence of

black hole interior as non-isometric code

Discussion

- Special extremal slices are not unique to static AdS black holes

  → any suggestion to holography for more general spacetimes?

- Going beyond special extremal slice: curvature singularity at the origin ?

- Multipartite entanglement also plays a key role in elongating the ER bridge

Anegawa-Miyata-Suzuki-KT in progress

Summary

- Black hole interior has a characteristic slice called special extremal slice

- Special extremal slice in AdS is dual to AME state

- Such AME states can be well approximated as Haar random states

- These findings give a direct consequence of

  black hole interior as non-isometric code

Thank you!