

Quantum correlations in interacting inflationary perturbations

J. Martin, **A. Micheli**, and V. Vennin, EPL 142, 18001

J. Martin, **A. Micheli**, and V. Vennin, JCAP. 2022, 051

A. Micheli and P. Peter, « Quantum Cosmological Gravitational Waves? » in Handbook of Quantum Gravity

A. Micheli, Y. Oshima and T. Takahashi, PRD, 113, 123554

Amaury Micheli

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Motivation: probe quantum gravity

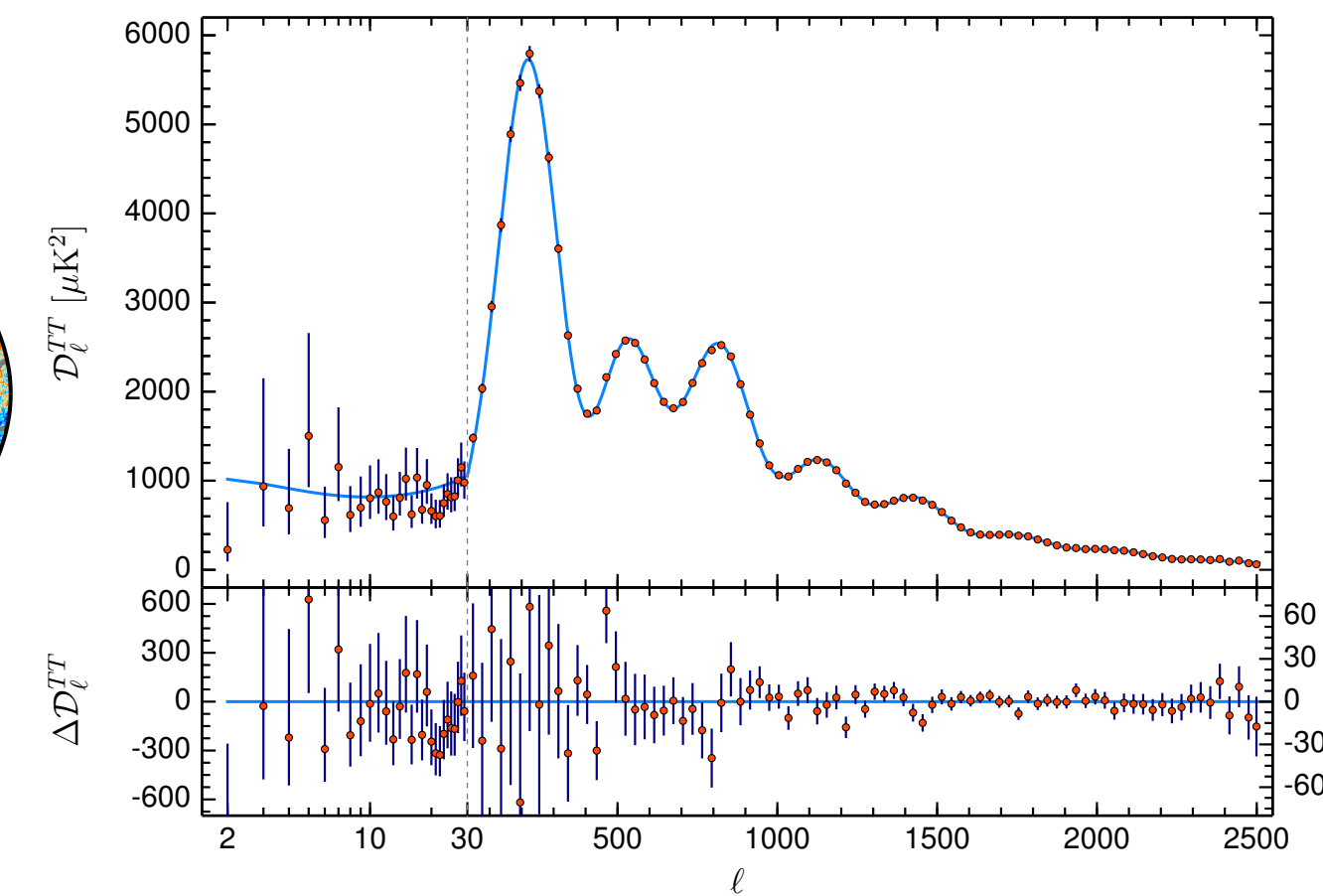
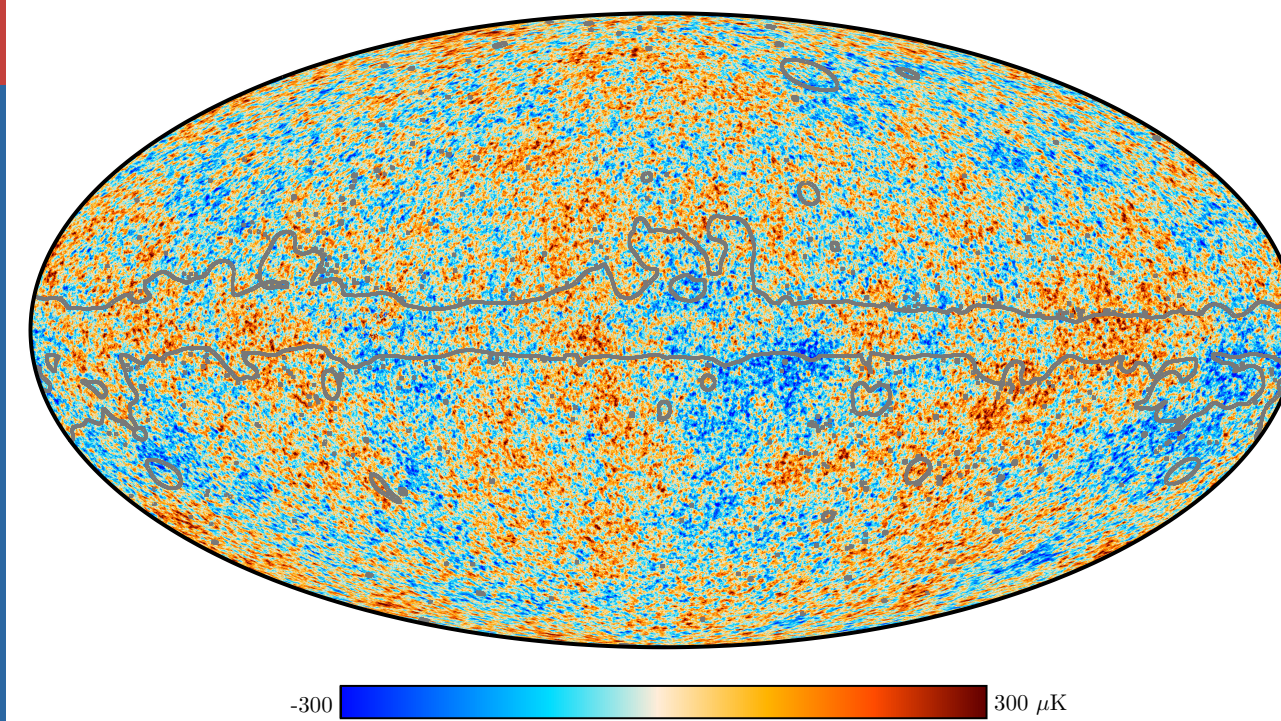
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Leading Scenario

Initial quantum vacuum fluctuations of matter and gravity during inflation generates inhomogeneities in very good agreement with observation¹.



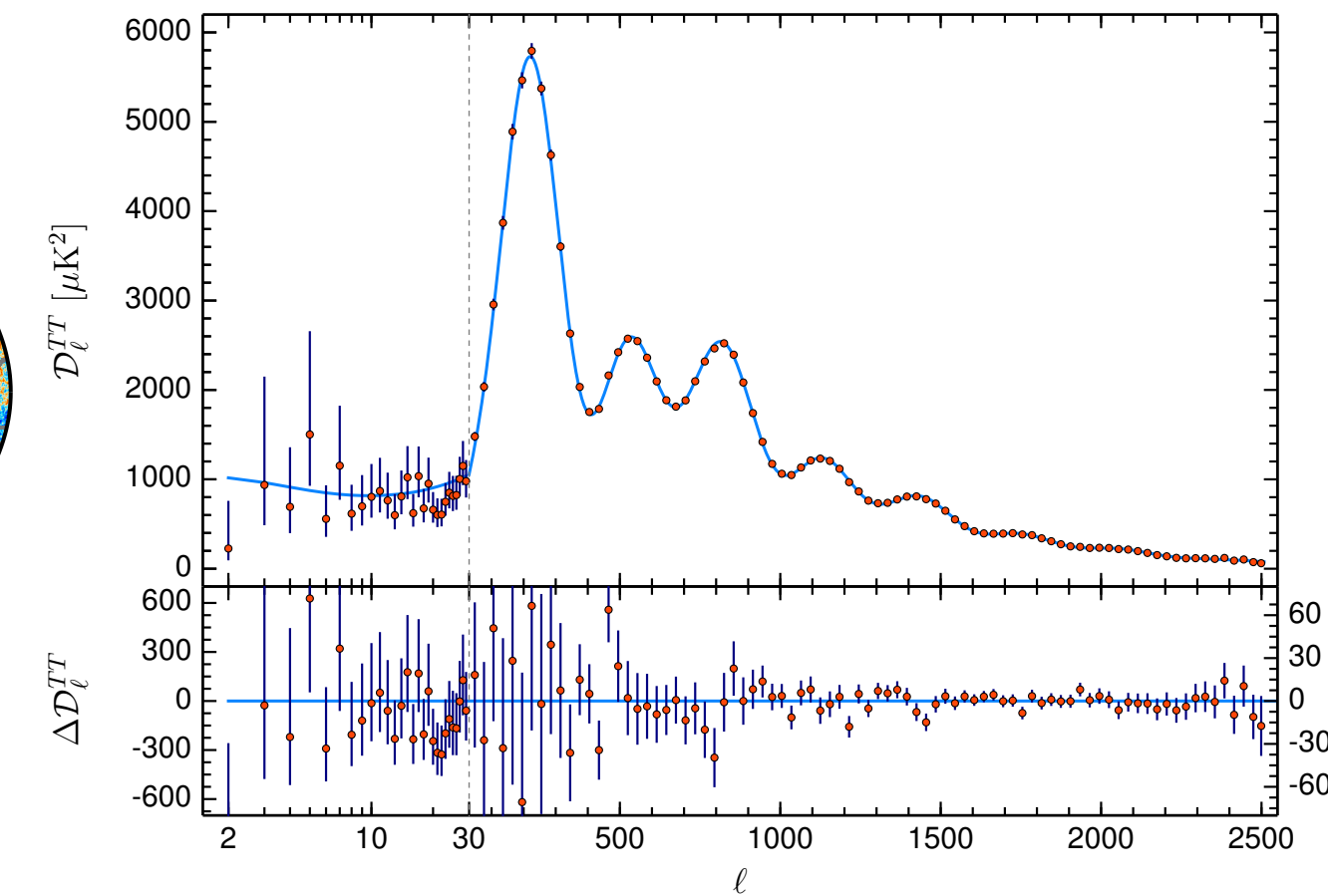
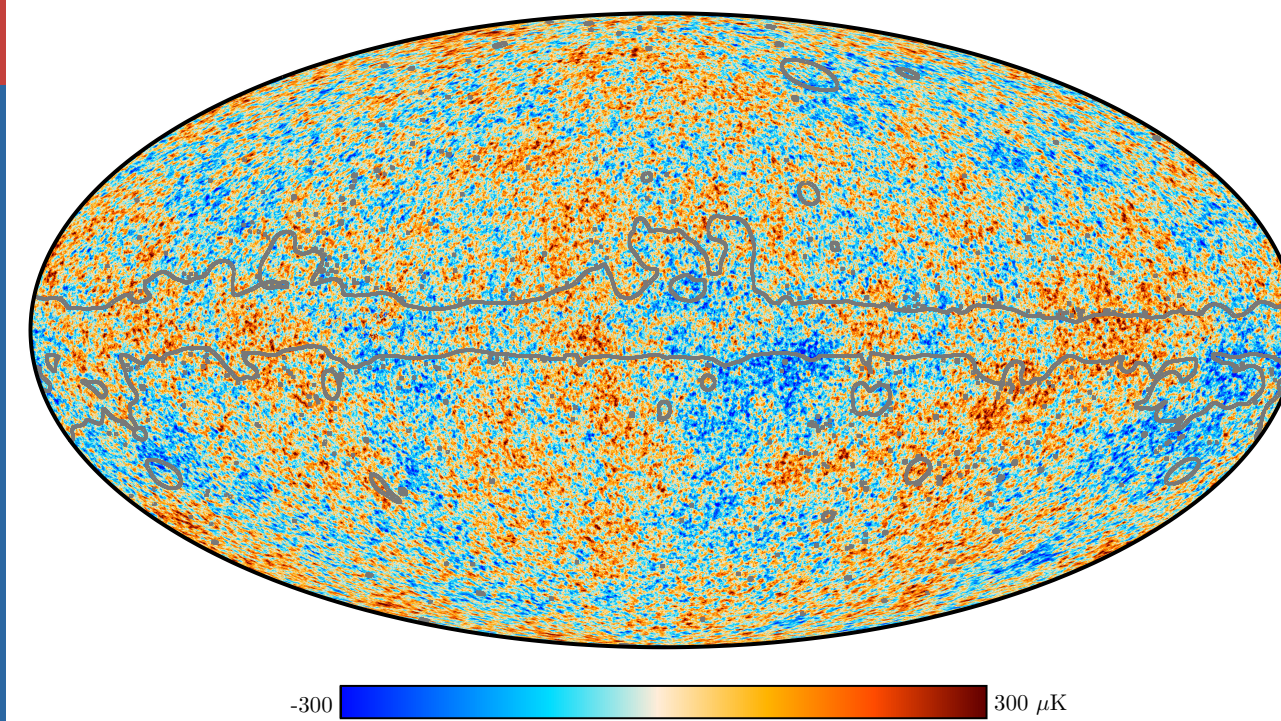
1. [arXiv:1807.06209 Planck Collaboration]

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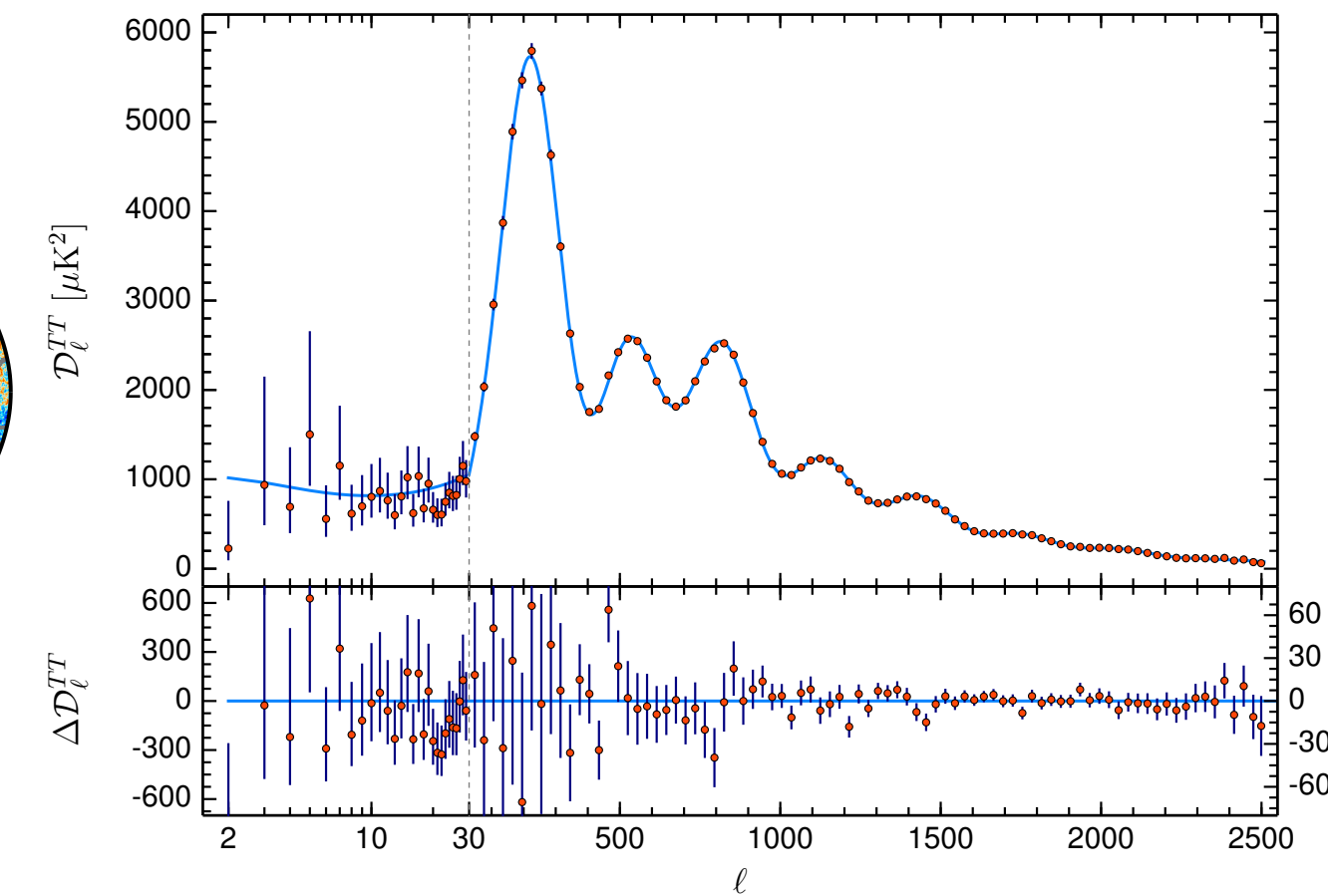
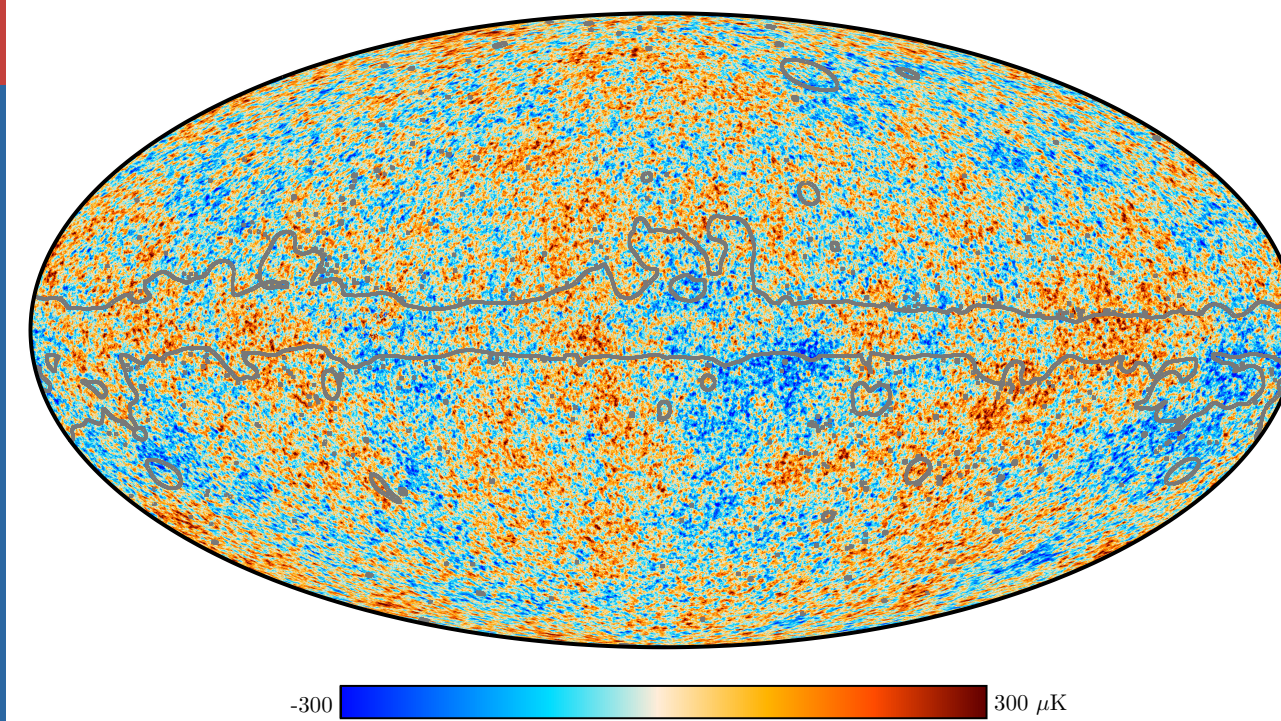
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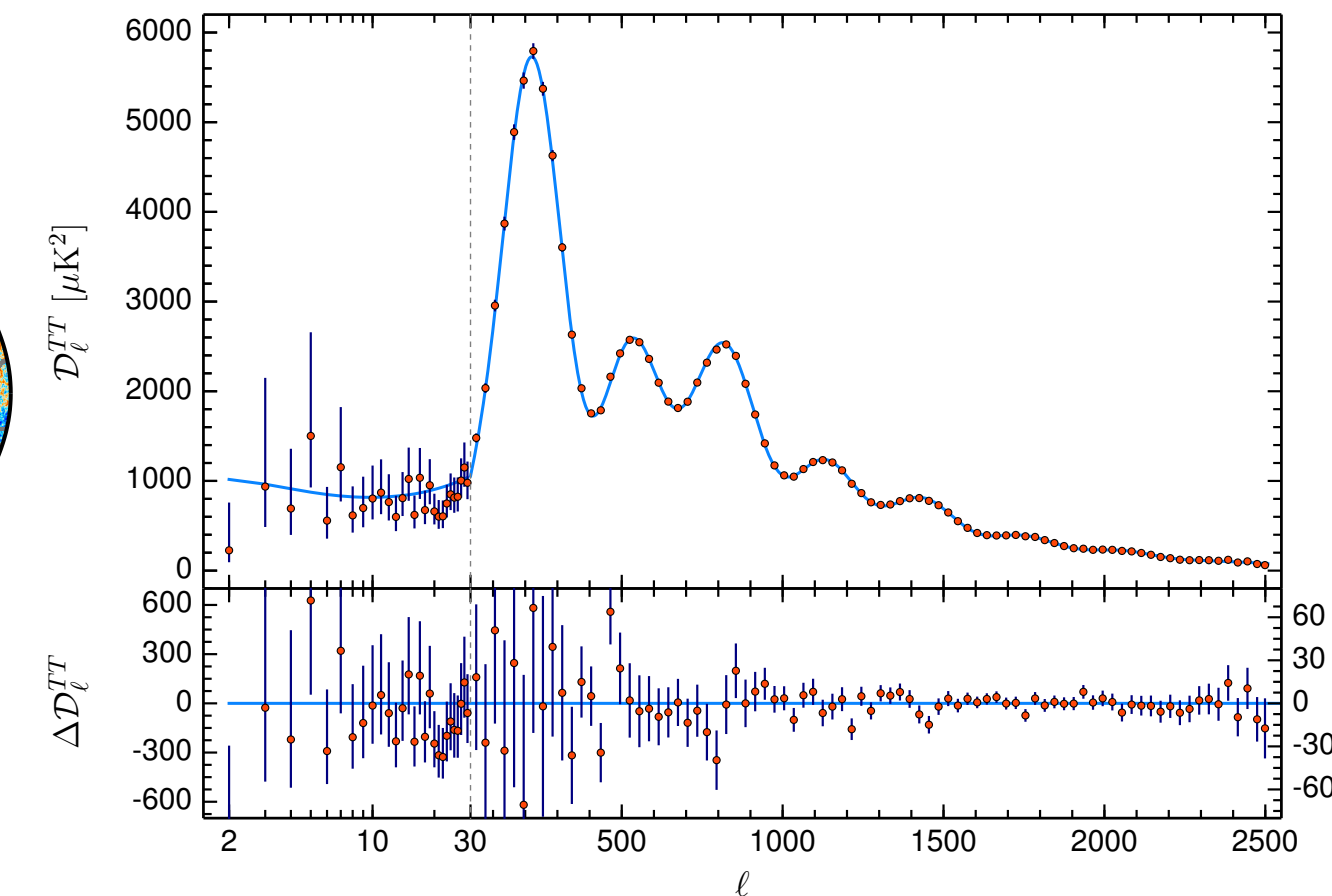
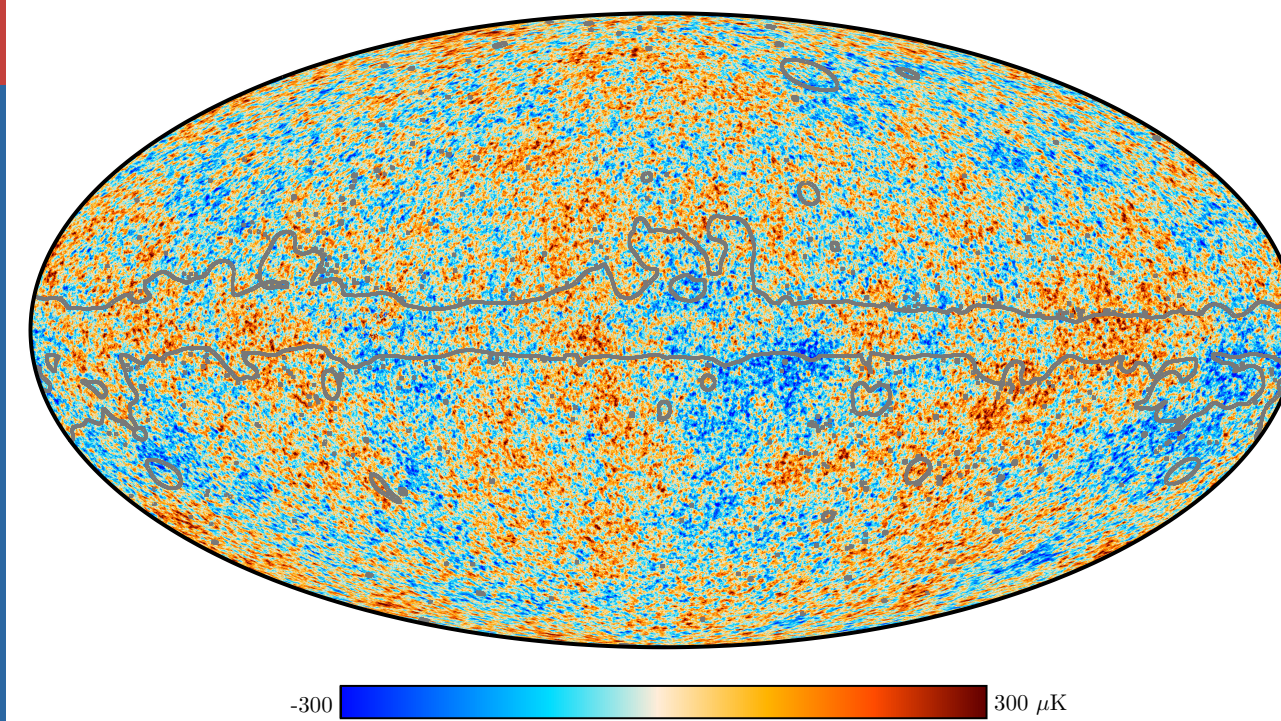
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- In particular, a **direct proof** that inhomogeneities cannot have a classical origin would show that **gravity** can and should be **quantised**.

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Conclusions & Perspectives

- A. In the simplest models, **inflationary perturbations** enjoy at least some **quantum correlations** that **cannot** be explained **classically**.
 - B. Even a slight amount of **interaction** of the perturbations, will **degrade** the **correlations...** but **not necessarily classicalise them!**
 - C. However, we **cannot reveal these correlations** with typical **cosmological observations**.
-
- D. Have to seek **other** type of **correlations** that may appear in **richer scenarios**, for **example** featuring **non-Gaussian** perturbations.

Plan

- I. Context and motivations
- II. Evolution of perturbations during inflation
- III. Quantum features in the perturbations?
- IV. Apparent classicality and detectability
- V. Conclusions and Perspectives

II - Evolution of perturbations during inflation

Classical cosmological perturbations in inflation - I

- **GR** $g_{\mu\nu}$ with a **single scalar field** ϕ :
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right]$$

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- Inflationary background: **Homogeneous universe** in accelerated **expansion** due to a **slowly rolling** $\phi_0(t)$

$$ds^2 = -c^2 dt^2 + a^2(t) d\mathbf{r}^2 \quad \text{with} \quad H = \frac{\dot{a}}{a} \approx \text{cst.} > 0 \quad \text{Hubble parameter}$$

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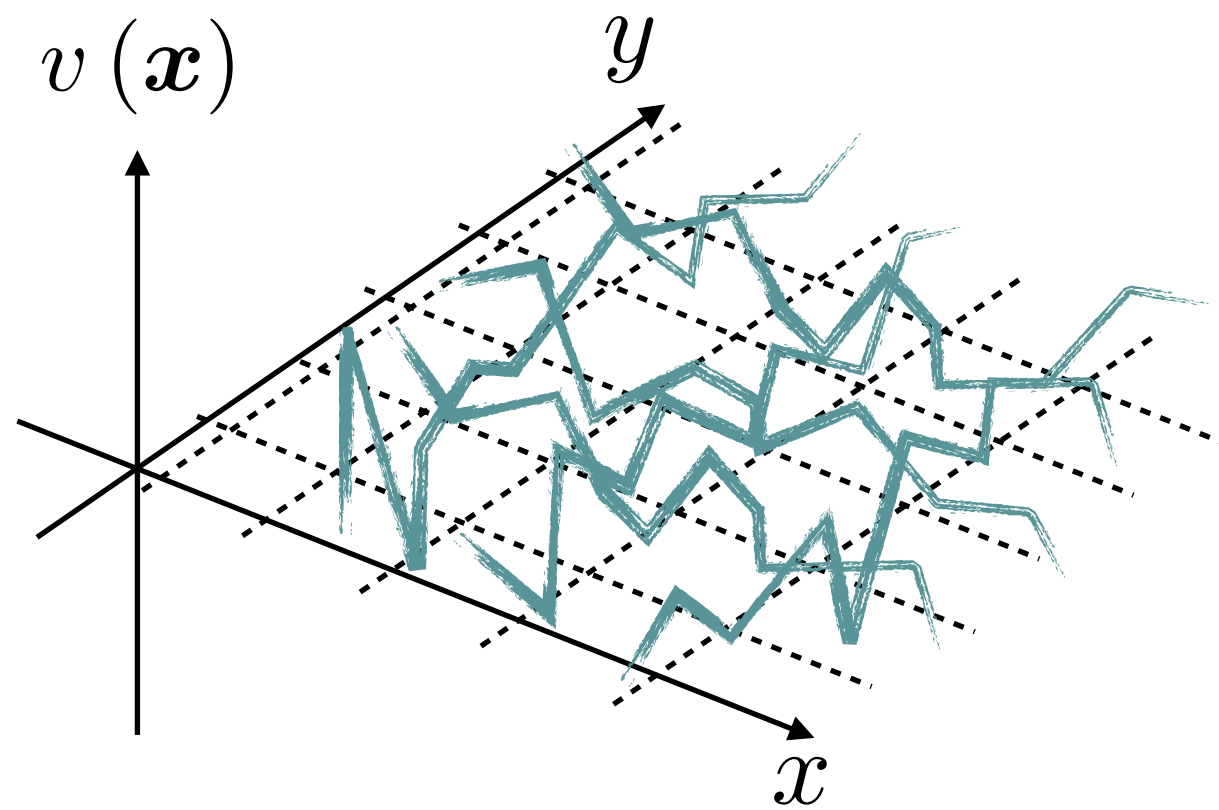
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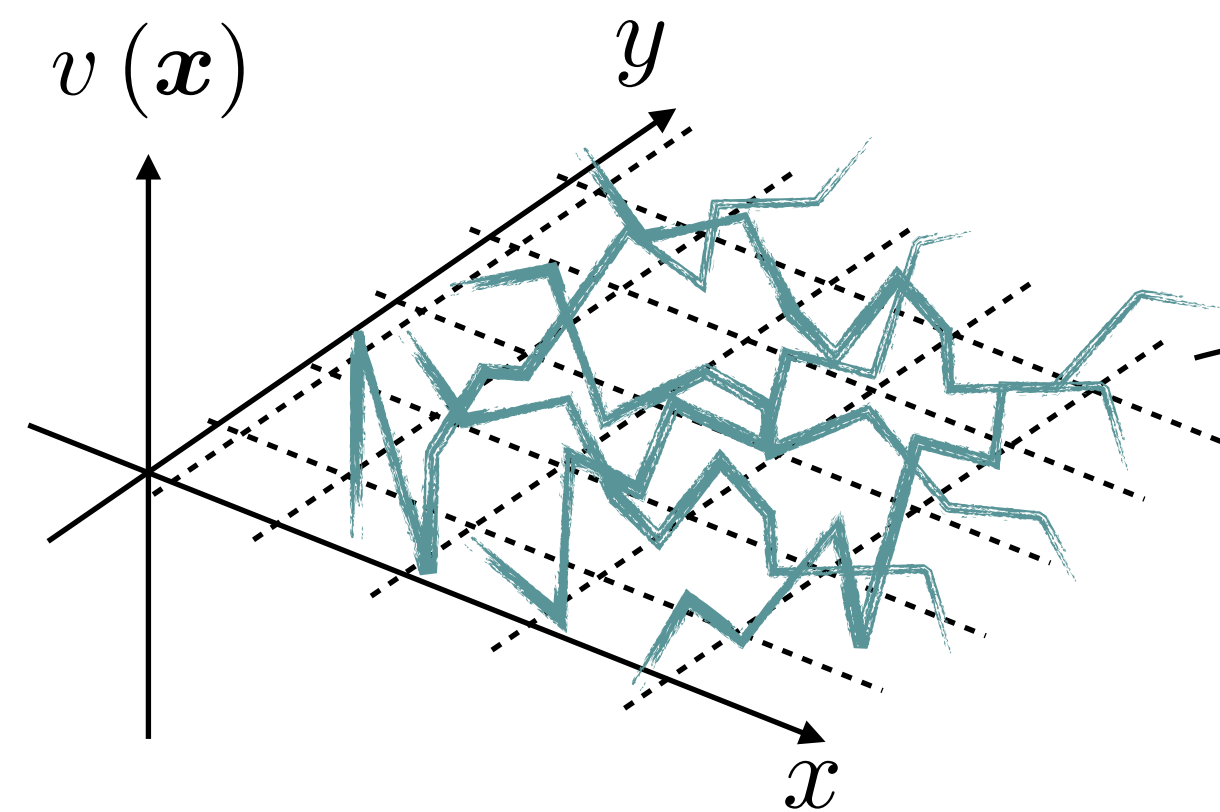
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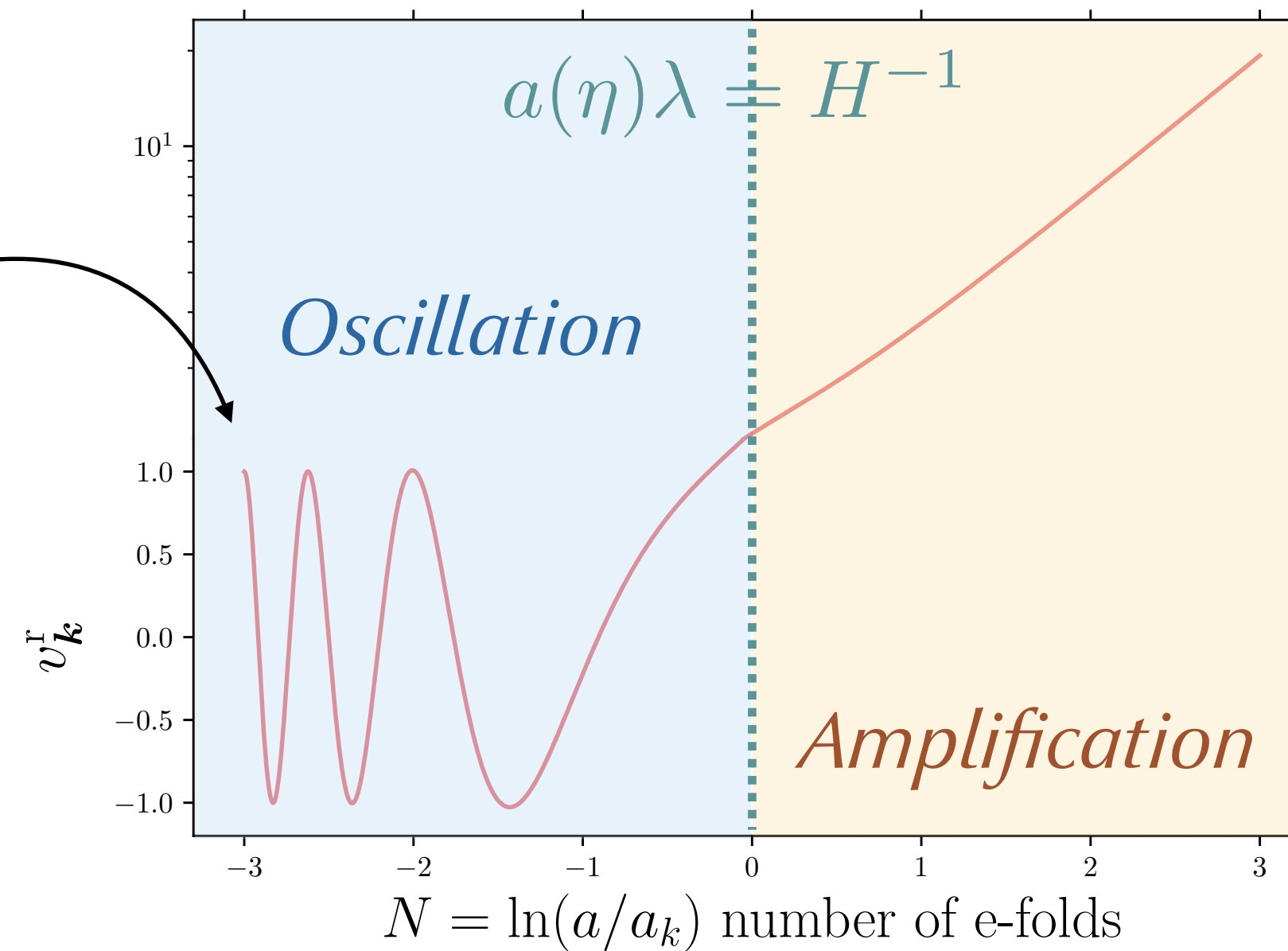
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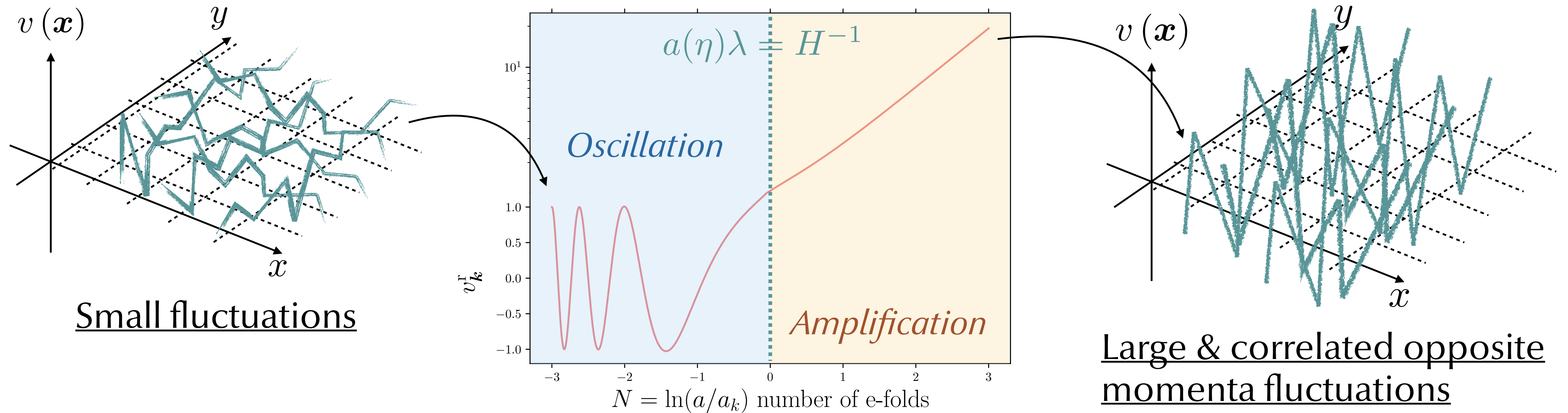


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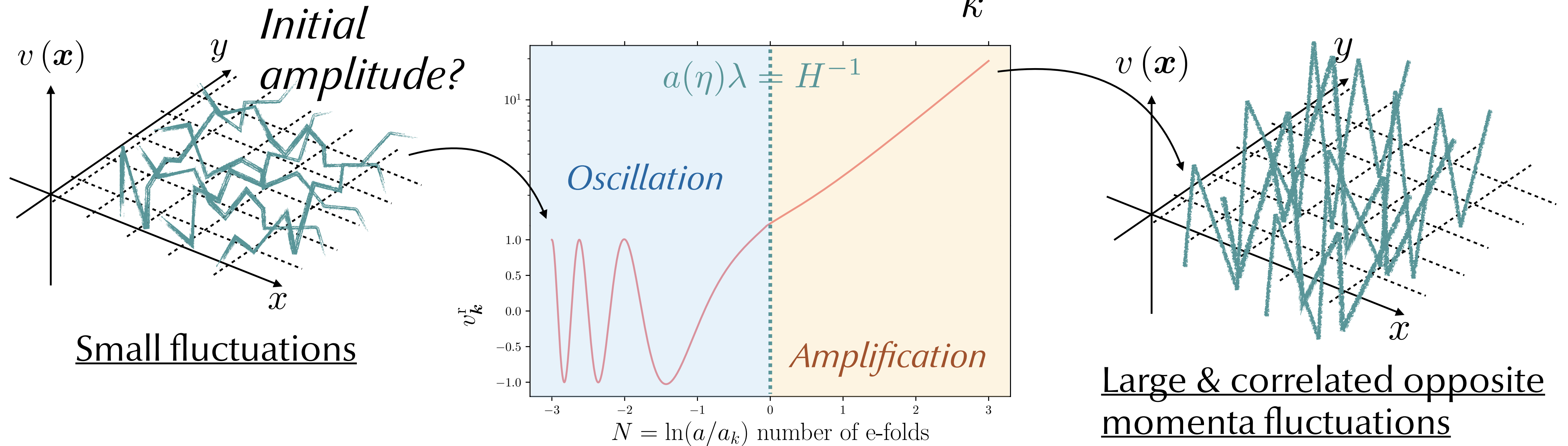
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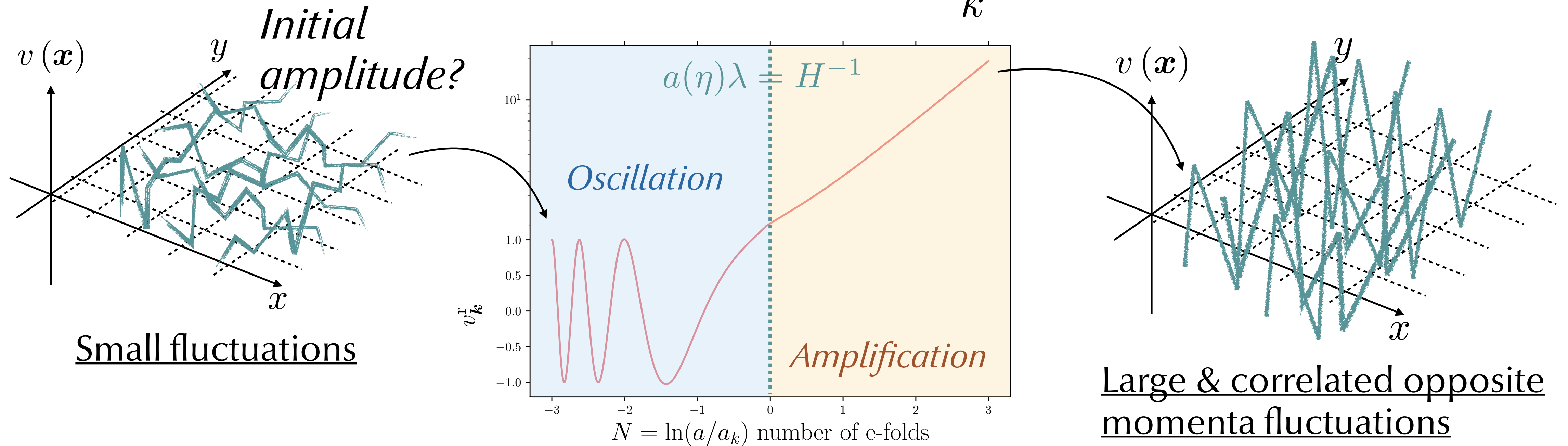
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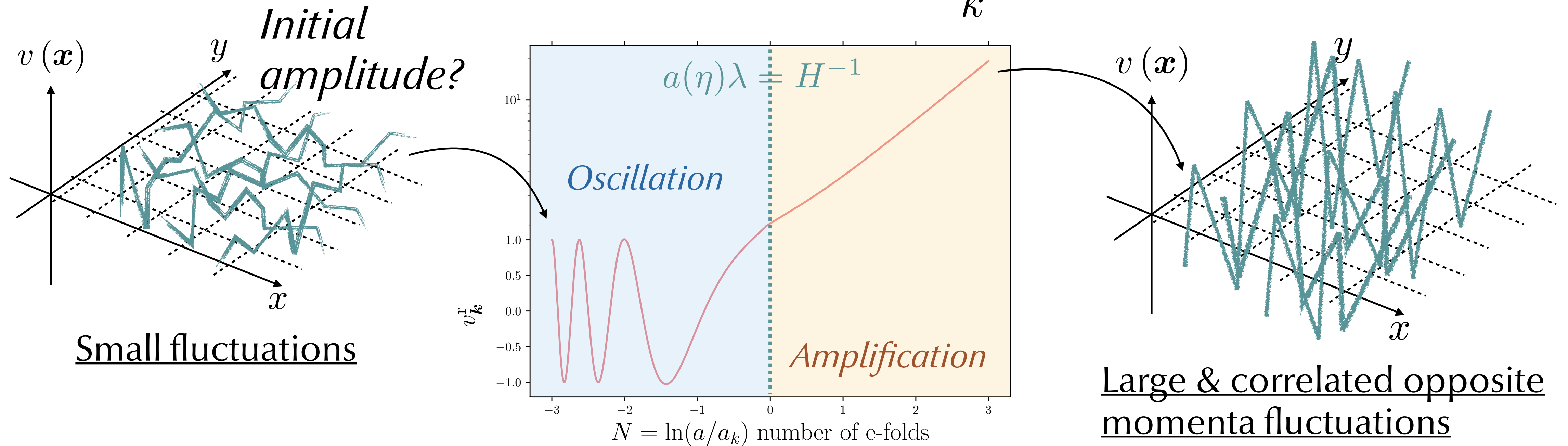
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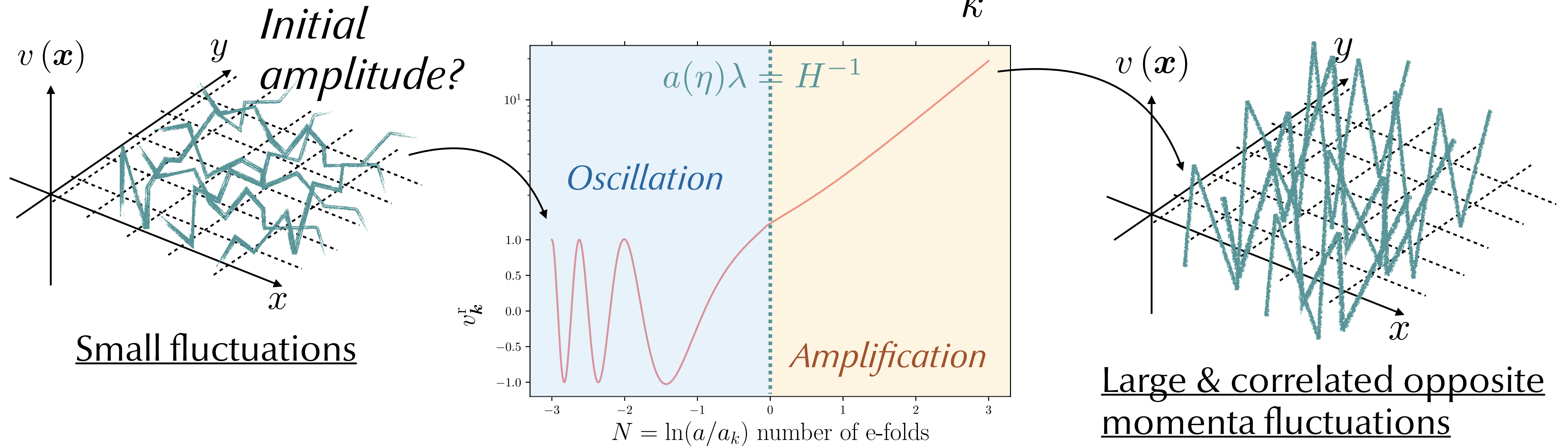
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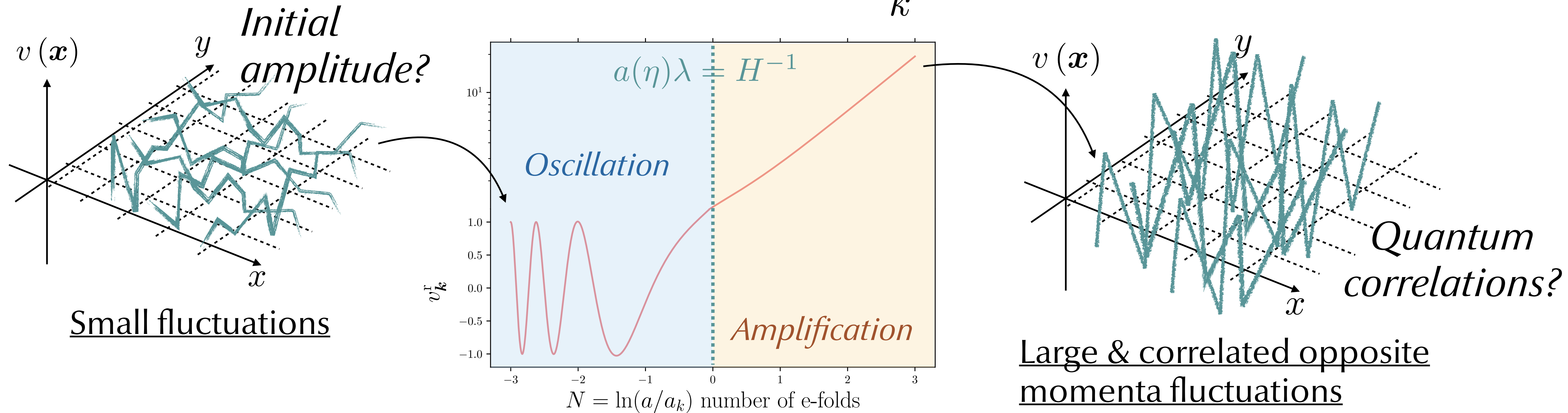
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- Introduce Hermitian operators $(\hat{v}_{\mathbf{k}}^r, \hat{\pi}_{\mathbf{k}}^r, \hat{v}_{\mathbf{k}}^i, \hat{\pi}_{\mathbf{k}}^i) = \sqrt{2} [\text{Re}(\hat{v}_{\mathbf{k}}), \text{Re}(\hat{\pi}_{\mathbf{k}}), \text{Im}(\hat{v}_{\mathbf{k}}), \text{Im}(\hat{\pi}_{\mathbf{k}})]$

$$\Psi_{\mathbf{k}, -\mathbf{k}} = \sqrt{\frac{k}{\pi\hbar\gamma_{11}}} e^{-\frac{k}{2\hbar} \frac{(1-i\gamma_{12})}{\gamma_{11}} [(v^r)^2 + (v^i)^2]} = \Psi(v^r) \Psi(v^i)$$

Phase-space representation of TMSV - I

- Density matrix $\hat{\rho} = |\Psi\rangle \langle \Psi| = |\Psi_r\rangle \langle \Psi_r| \otimes |\Psi_i\rangle \langle \Psi_i| = \hat{\rho}_r \otimes \hat{\rho}_i$ focus on $\hat{\rho}_r$

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- Wigner transform maps **operators** $\hat{O}$ to **phase-space** functions $O(v^r, \pi^r)$

$$\mathcal{W}(\hat{O})(v^r, \pi^r) = \frac{1}{\hbar} \int dx e^{-i \frac{\pi^r \cdot x}{\hbar}} \left\langle v^r + \frac{x}{2} \left| \hat{O} \right| v^r - \frac{x}{2} \right\rangle$$

Phase-space representation of TMSV - I

- Density matrix $\hat{\rho} = |\Psi\rangle \langle\Psi| = |\Psi_r\rangle \langle\Psi_r| \otimes |\Psi_i\rangle \langle\Psi_i| = \hat{\rho}_r \otimes \hat{\rho}_i$ focus on $\hat{\rho}_r$
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Wigner of a TMSV?

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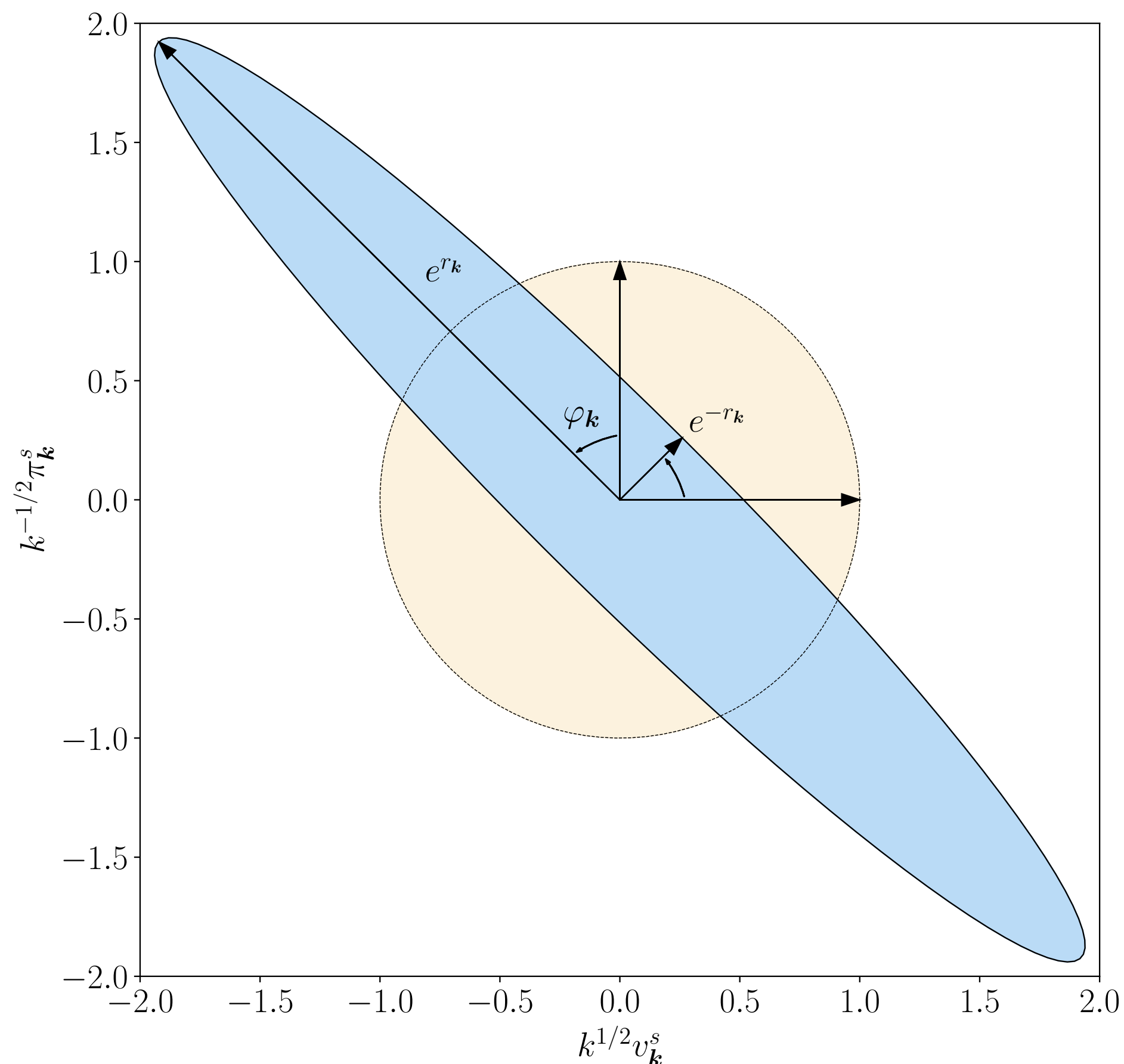
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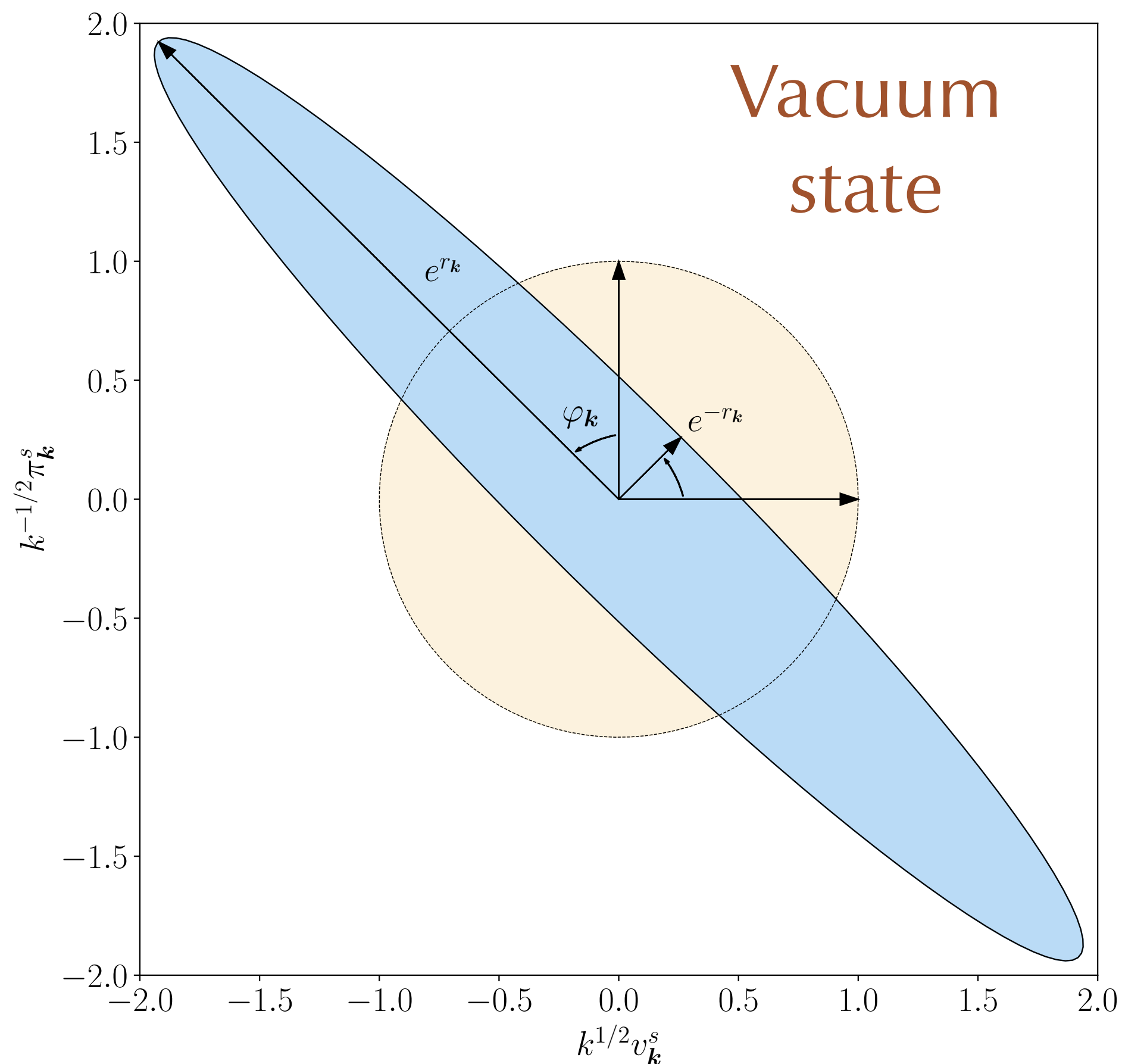


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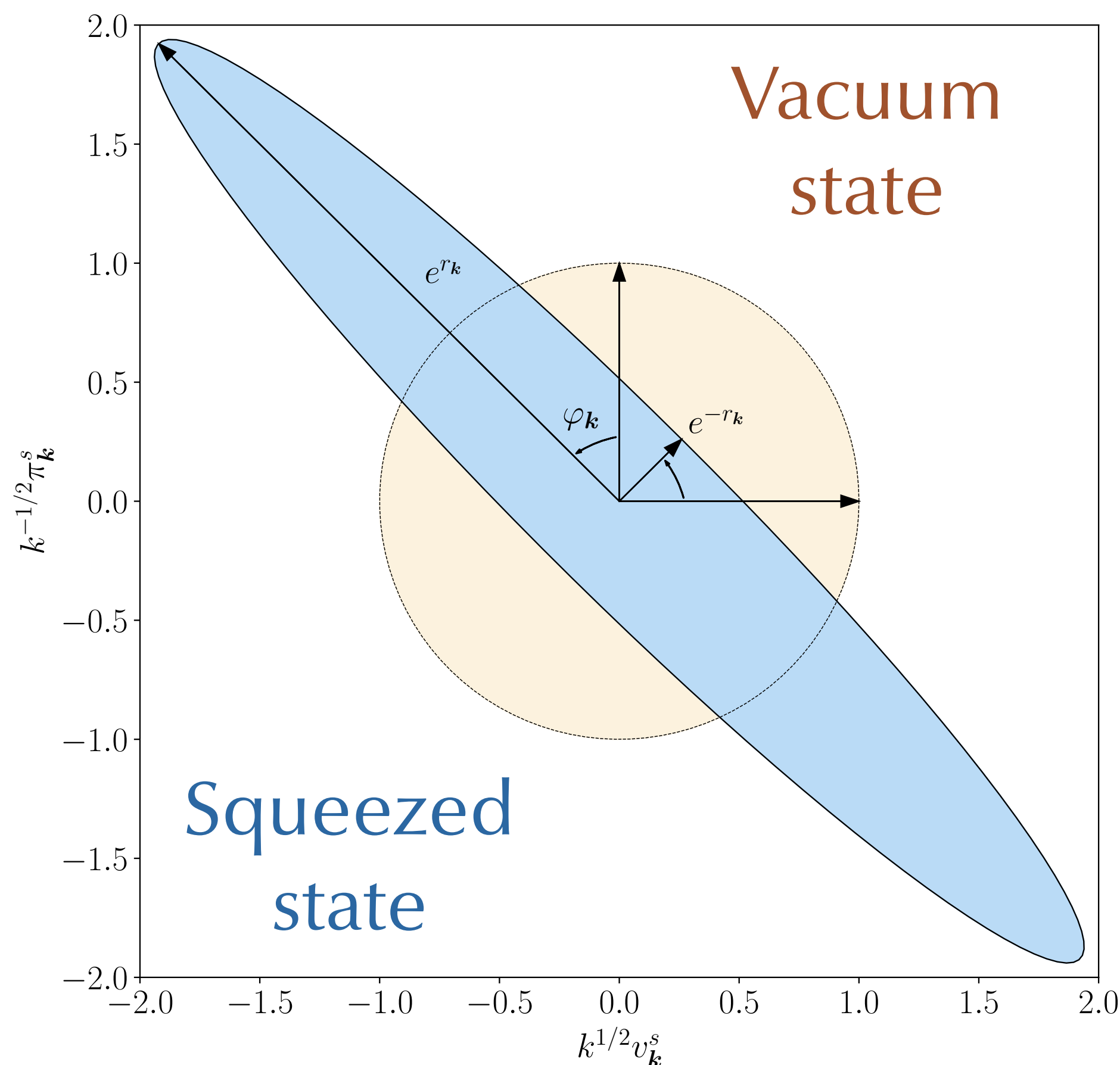
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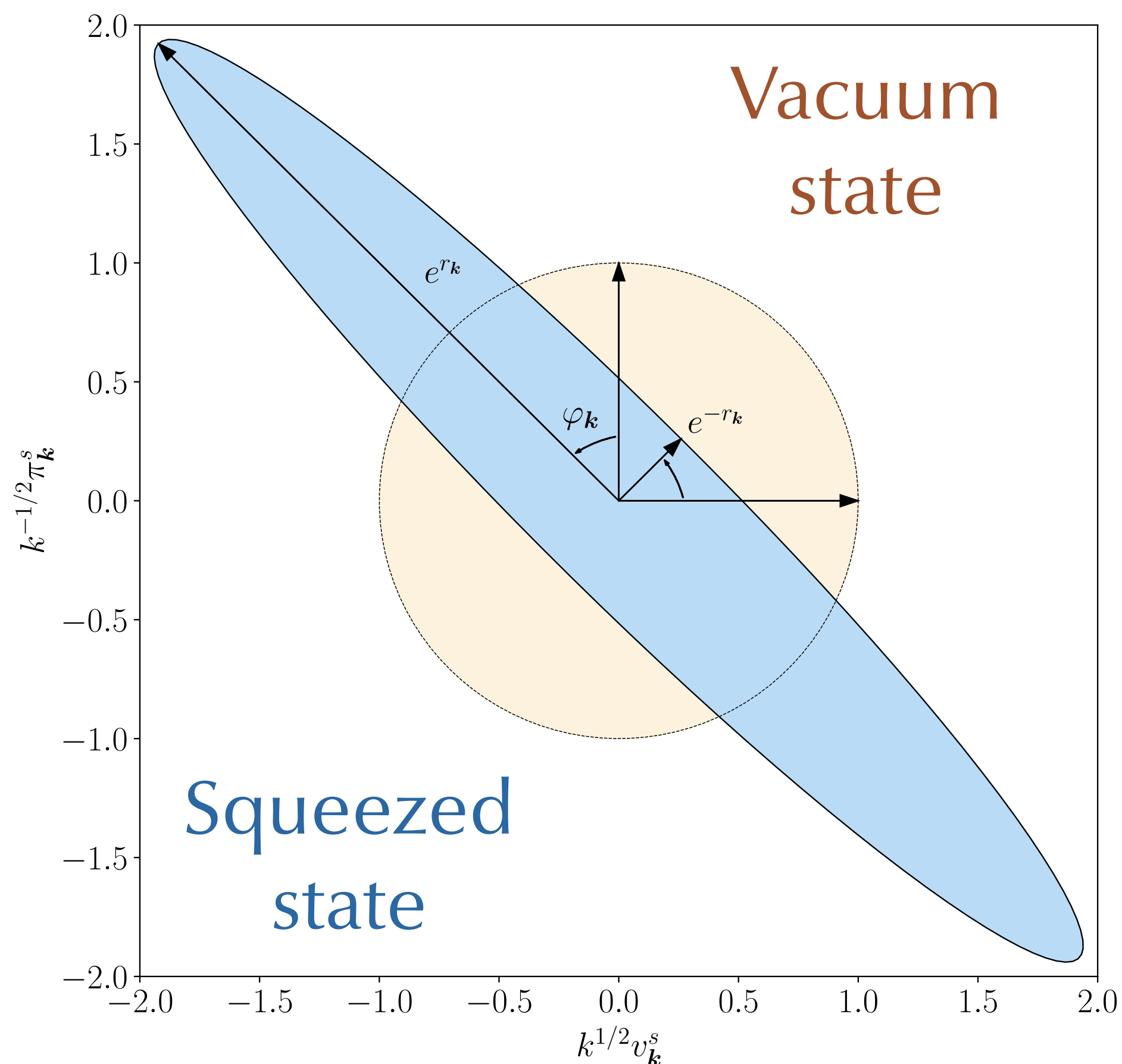
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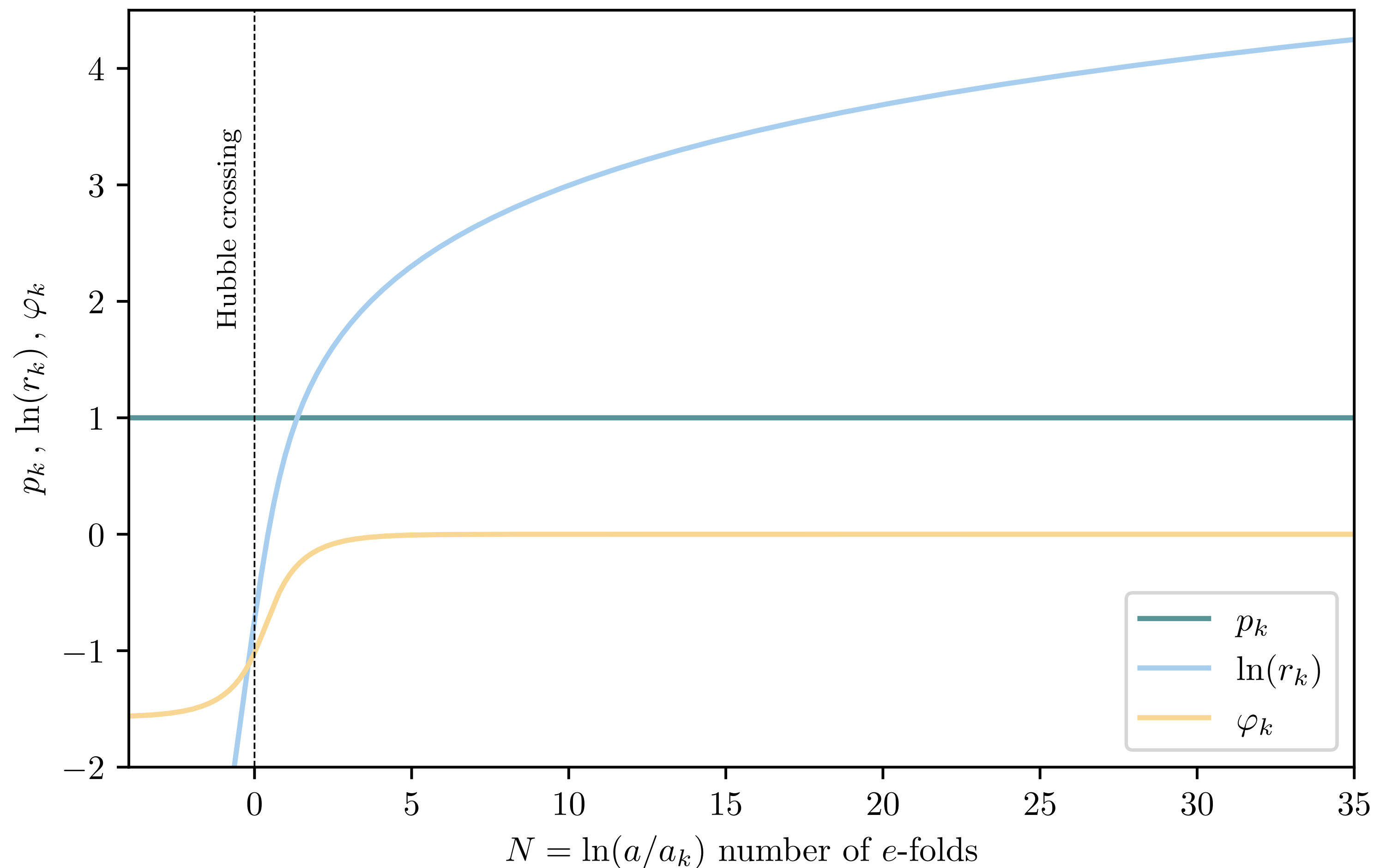
Alternative parameterisation of the state

Inhomogeneities in the early Universe

- Cosmological perturbations: Follow a pair $\pm \mathbf{k}$ during a phase of exponential expansion
 $N = \ln(a/a_k)$ with $a_k/k = H^{-1}$ Hubble crossing.

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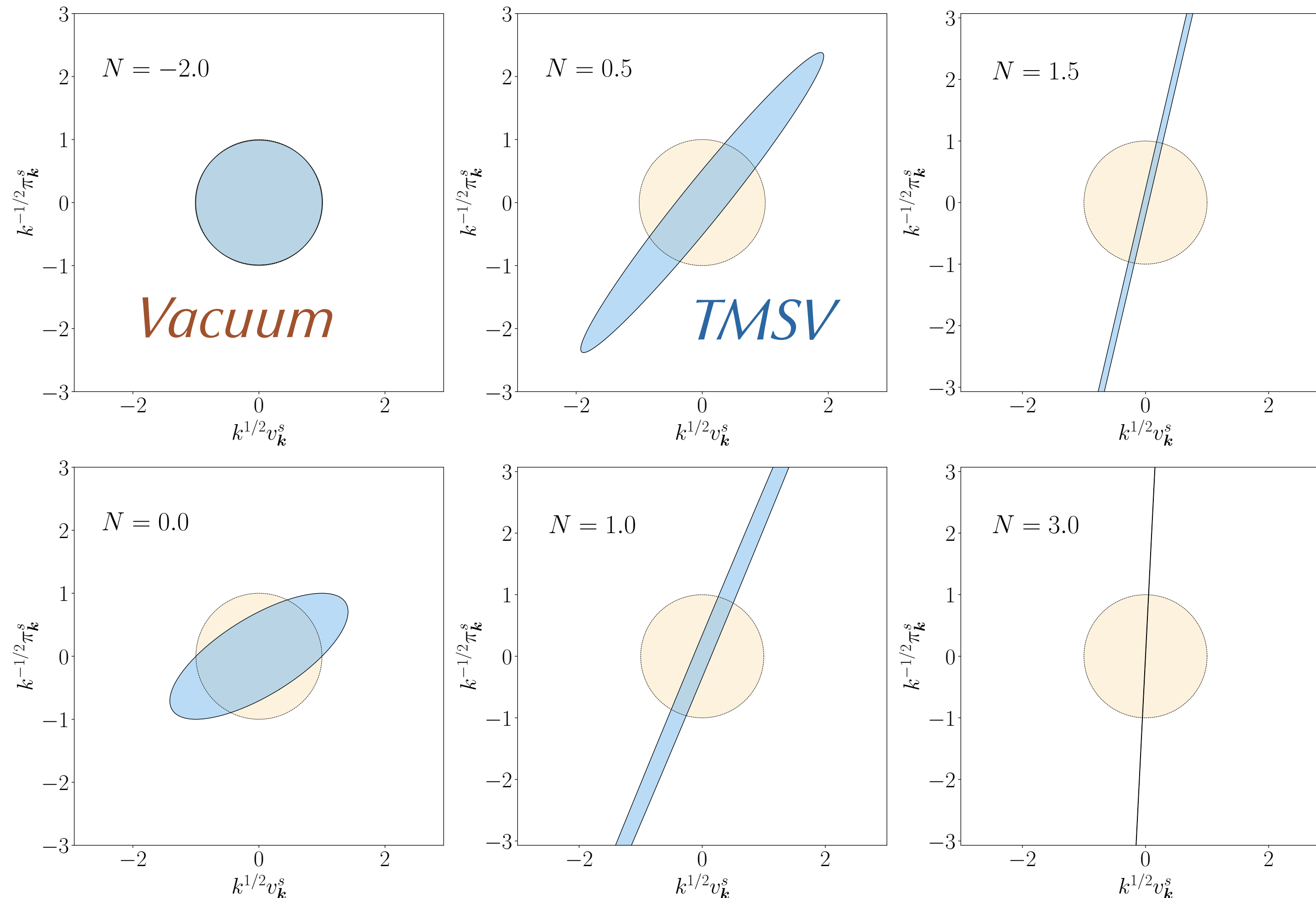
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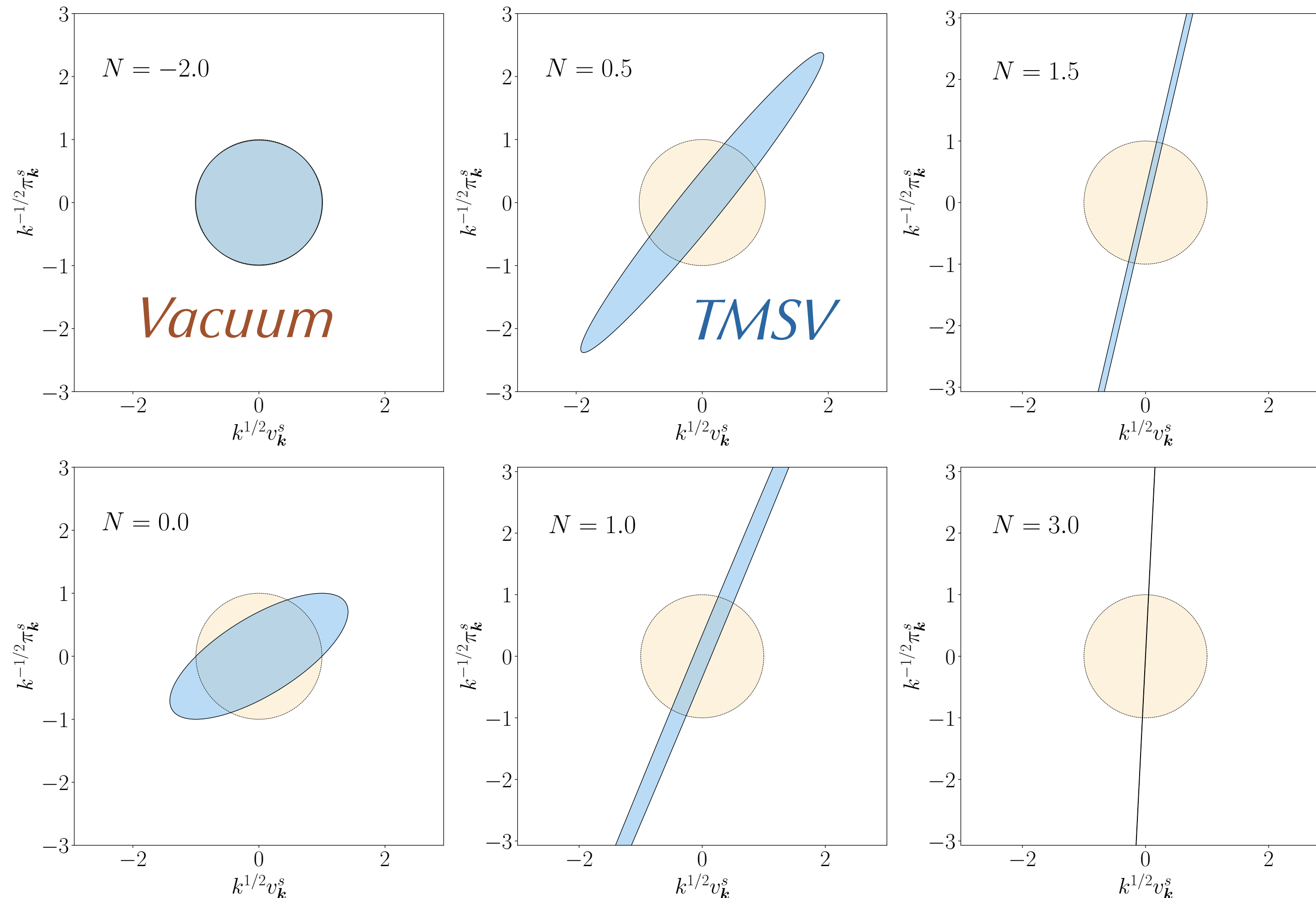
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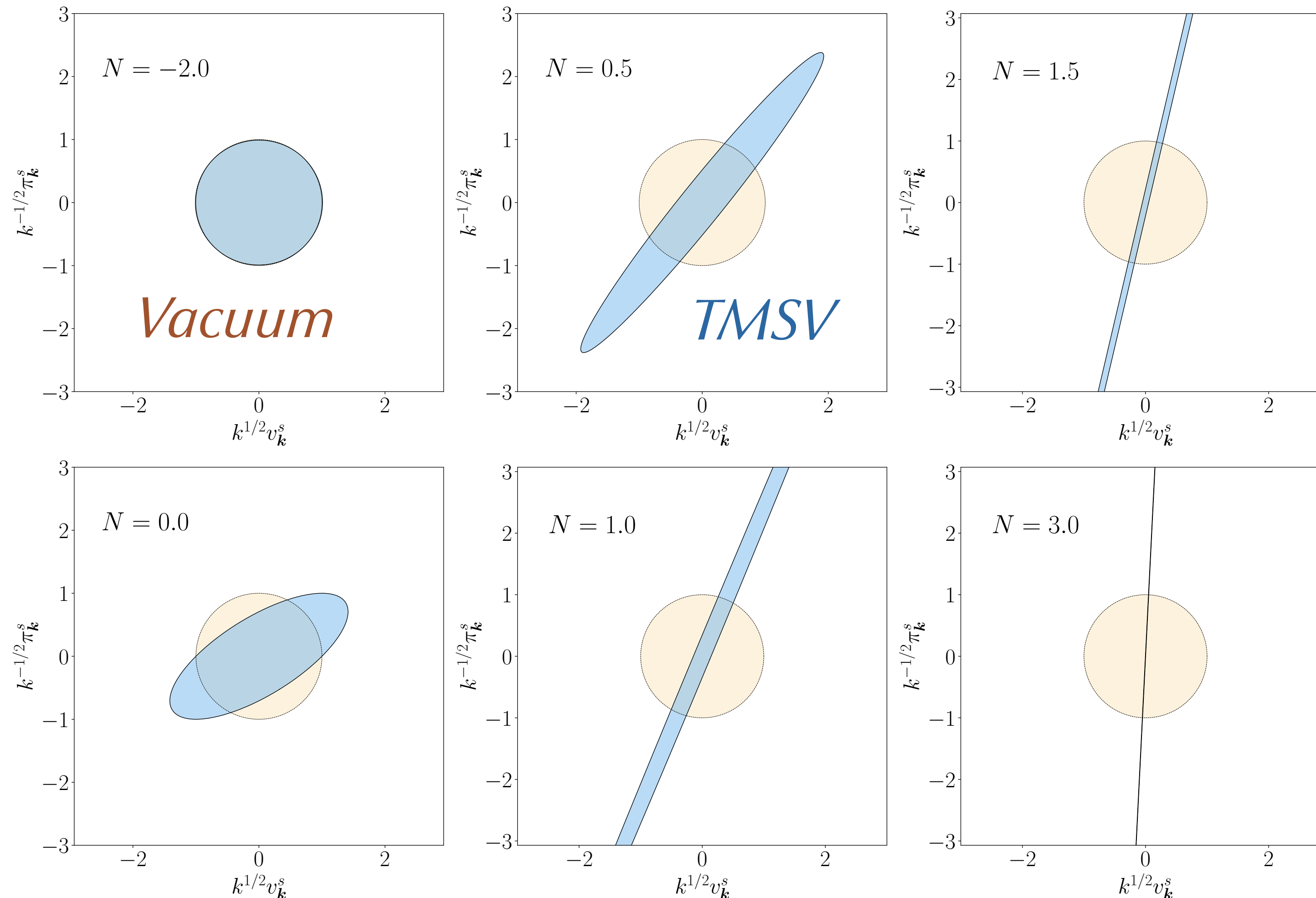
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How quantum is this state?

III - Quantum features in the perturbations?

Quantum correlations - I

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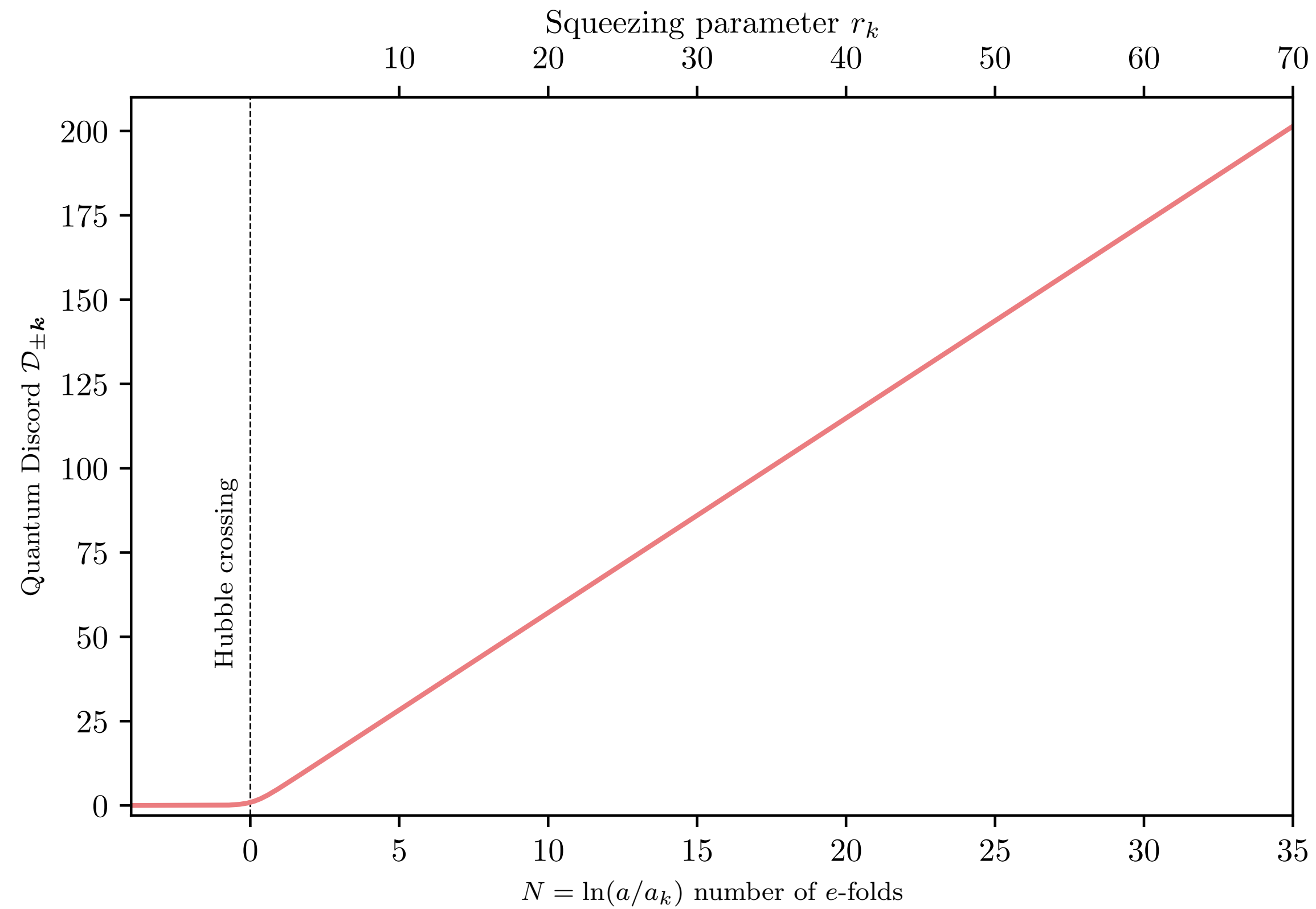
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Discord in a TMSV?

Quantum correlations - II

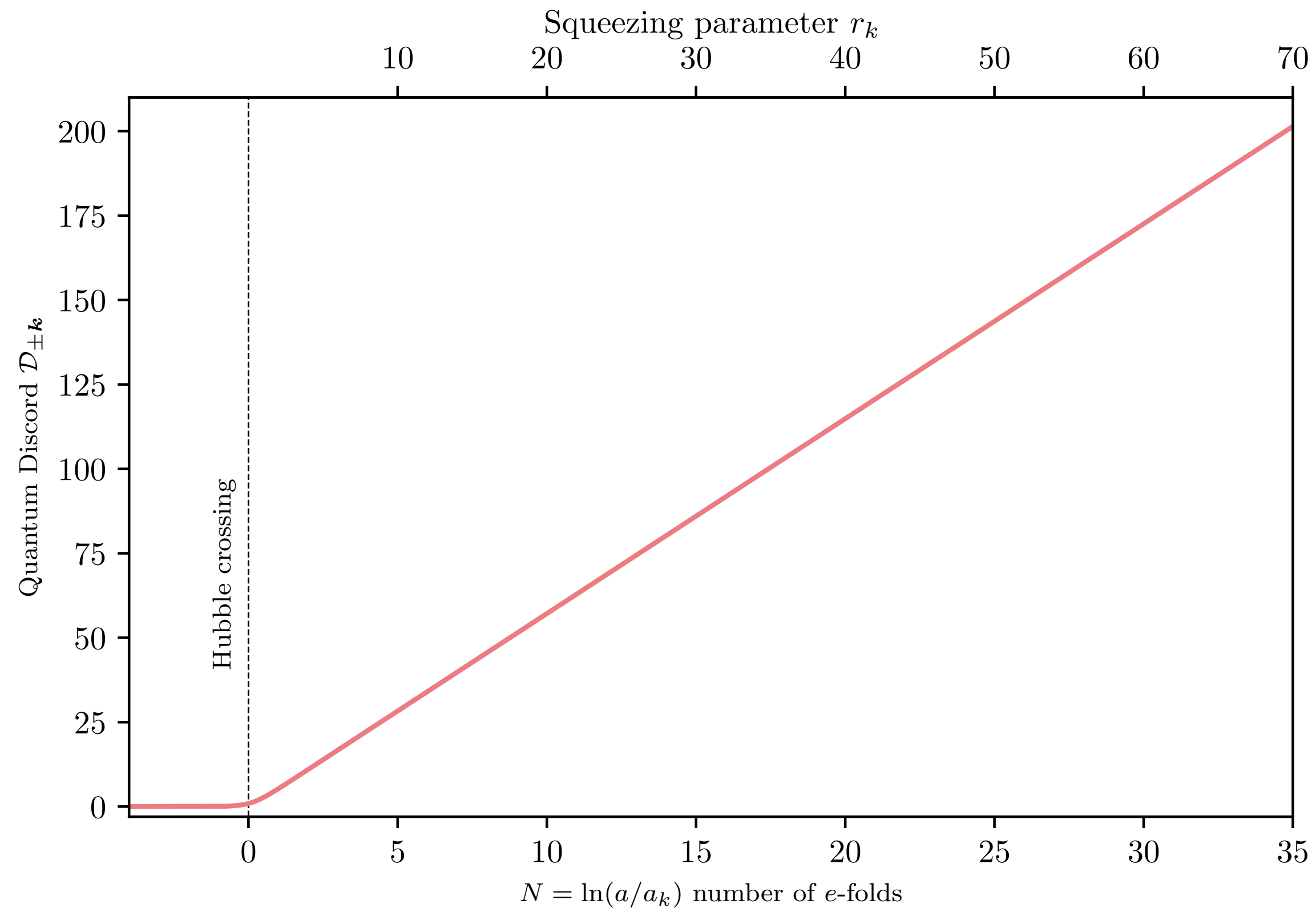
- Discord of $\pm k$ pairs



$$\mathcal{D}_{\pm k} \approx 2r_k / \ln 2 \approx 4N / \ln 2$$

Quantum correlations - II

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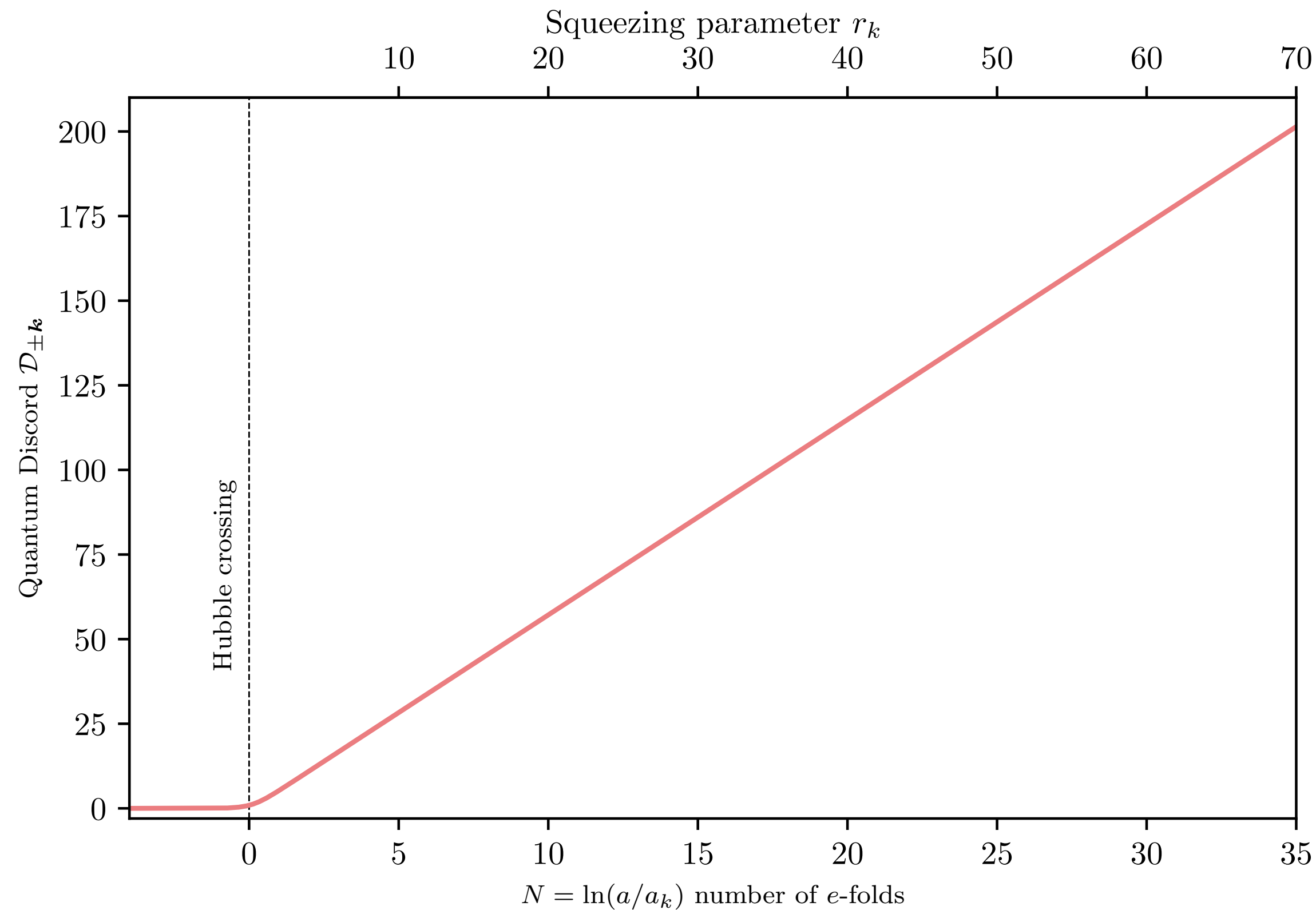
Take Home Message 1

Squeezing generates strong quantum correlations between $\pm k$ modes in the sense of several non-classicality criteria³, e.g., Bell inequality violation.

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Quantum correlations - II

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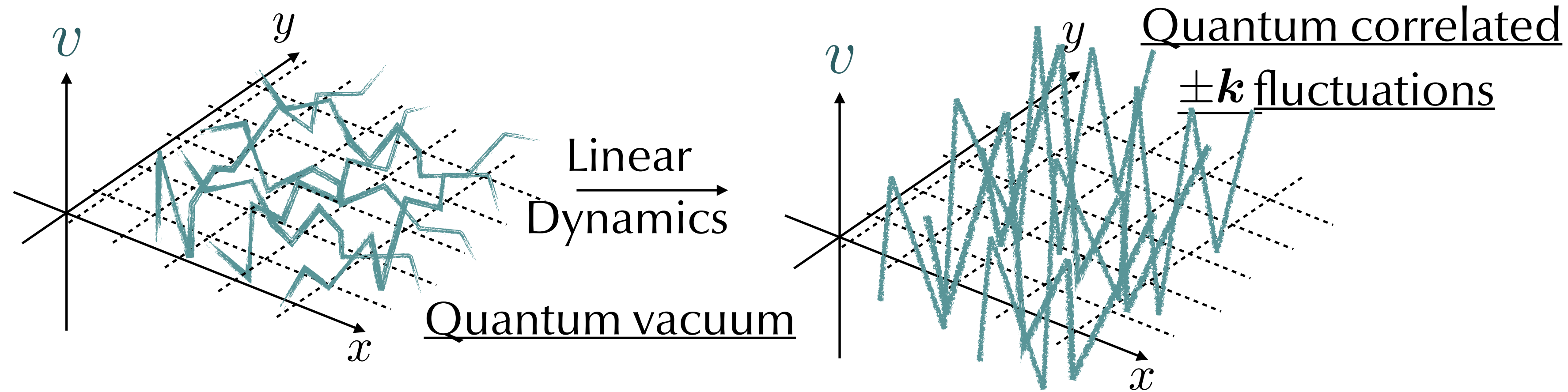
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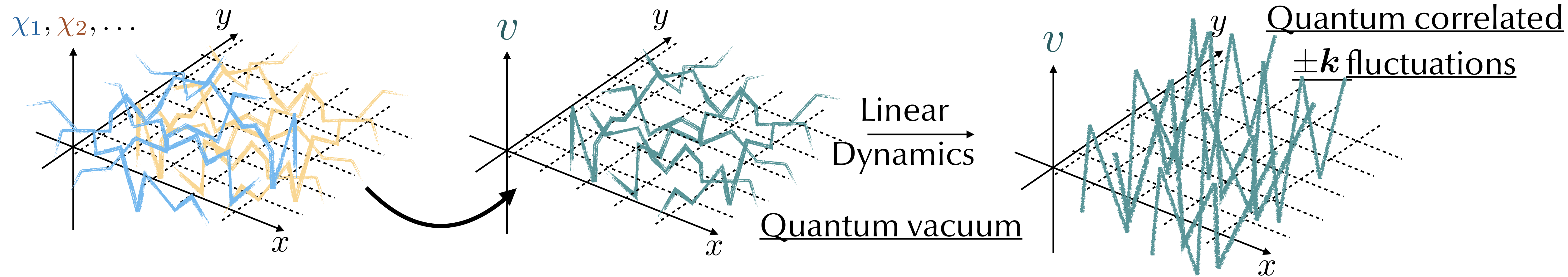
*Is this description too naive e.g. are quantum correlations **robust against interactions**?*

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A simple model for interactions

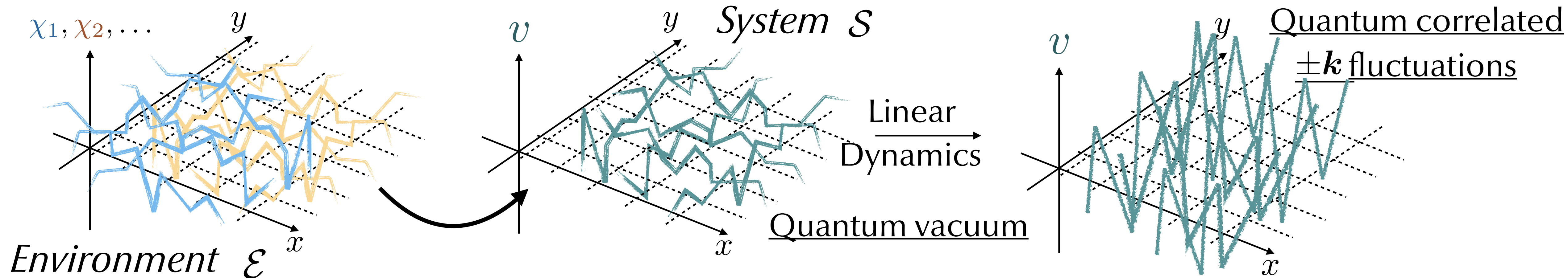


A simple model for interactions



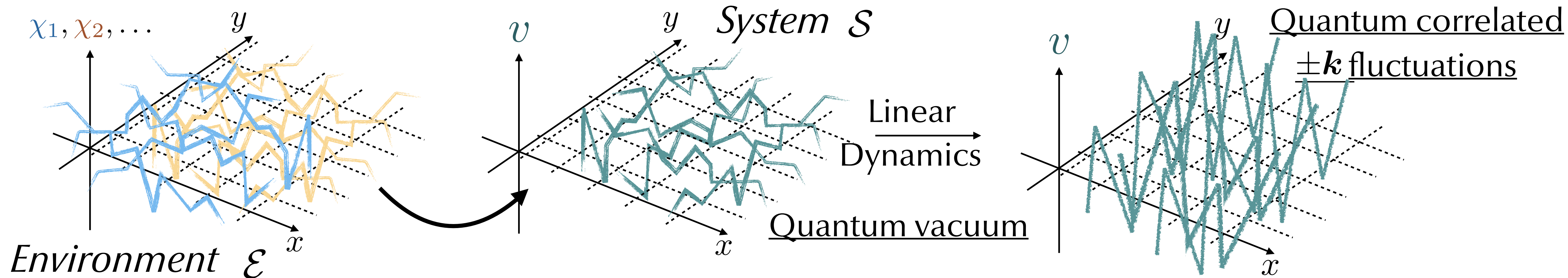
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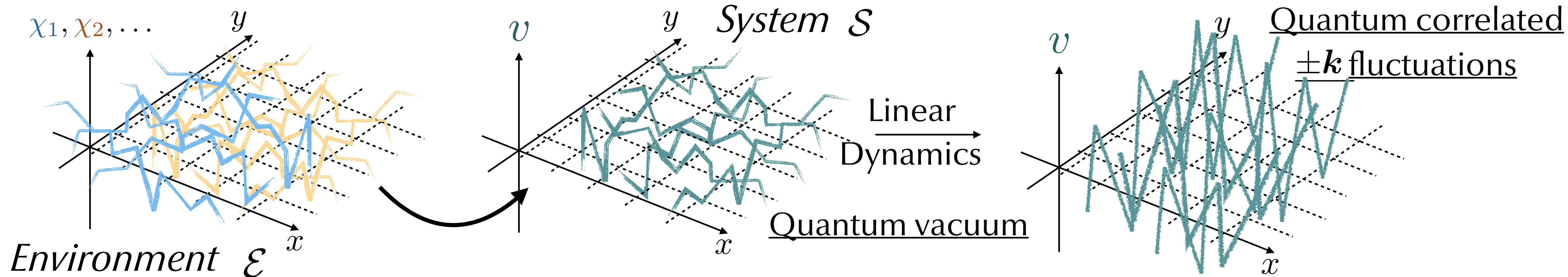
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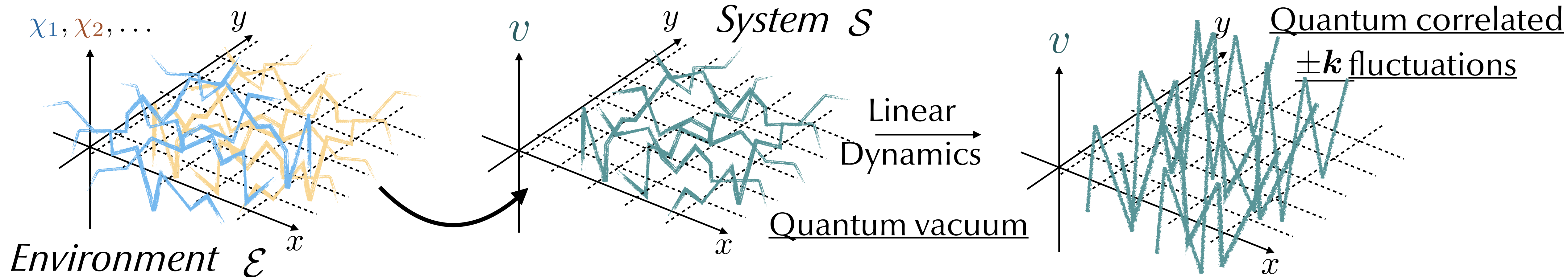
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Lindblad eq. $\frac{d\hat{\rho}_S}{d\eta} = -i \underbrace{[\hat{H}, \hat{\rho}_S]}_{\text{Unitary}} - \int d^3x d^3y \underbrace{C_E(\eta; \mathbf{x}, \mathbf{y})}_{\text{Info on coupling \& } \mathcal{E}} \underbrace{[\hat{v}(\mathbf{x}), [\hat{v}(\mathbf{y}), \hat{\rho}_S]]}_{\text{Non-unitary}}$ with $C_E \propto \underbrace{k_\gamma^2}_{\text{Time-dep.}} \underbrace{a^{p-3}}_{\text{Magnitude}}(\eta)$

A simple model for interactions



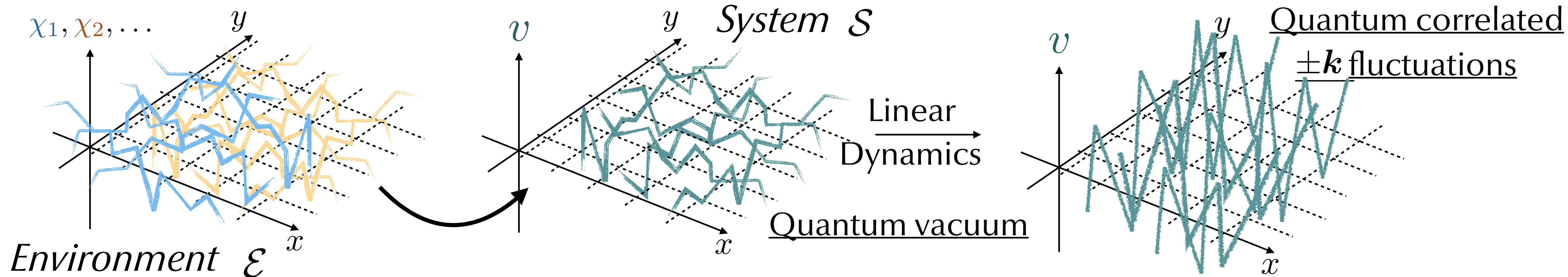
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Lindblad eq. $\frac{d\hat{\rho}_S}{d\eta} = -i \underbrace{[\hat{H}, \hat{\rho}_S]}_{\text{Unitary}} - \int d^3x d^3y \underbrace{C_E(\eta; \mathbf{x}, \mathbf{y})}_{\text{Info on coupling \& } \mathcal{E}} \underbrace{[\hat{v}(\mathbf{x}), [\hat{v}(\mathbf{y}), \hat{\rho}_S]]}_{\text{Non-unitary}} \quad \text{Time-dep. Magnitude}$

with $C_E \propto \underbrace{k_\gamma^2}_{\text{Time-dep.}} \widehat{a^{p-3}}(\eta)$

- Linear coupling: $\hat{\rho}_S = \bigotimes_{\mathbf{k} \in \mathbb{R}^3, +} \hat{\rho}_{\pm \mathbf{k}}$ and parametrised by $r_k, \varphi_k, 0 \leq p_k \leq 1$ 15/24

A simple model for interactions



- Many d.o.f. in early Universe $\chi_1, \chi_2, \dots$ coupled, even very weakly, to v
 $\longrightarrow$ **Decoherence**: large environment weakens quantum correlations of system
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Lindblad eq. $\frac{d\hat{\rho}_{\mathcal{S}}}{d\eta} = -i \underbrace{[\hat{H}, \hat{\rho}_{\mathcal{S}}]}_{\text{Unitary}} - \int d^3x d^3y \underbrace{C_E(\eta; \mathbf{x}, \mathbf{y})}_{\text{Info on coupling \& } \mathcal{E}} \underbrace{[\hat{v}(\mathbf{x}), [\hat{v}(\mathbf{y}), \hat{\rho}_{\mathcal{S}}]]}_{\text{Non-unitary}} \quad \text{Time-dep.} \quad \text{Magnitude}$

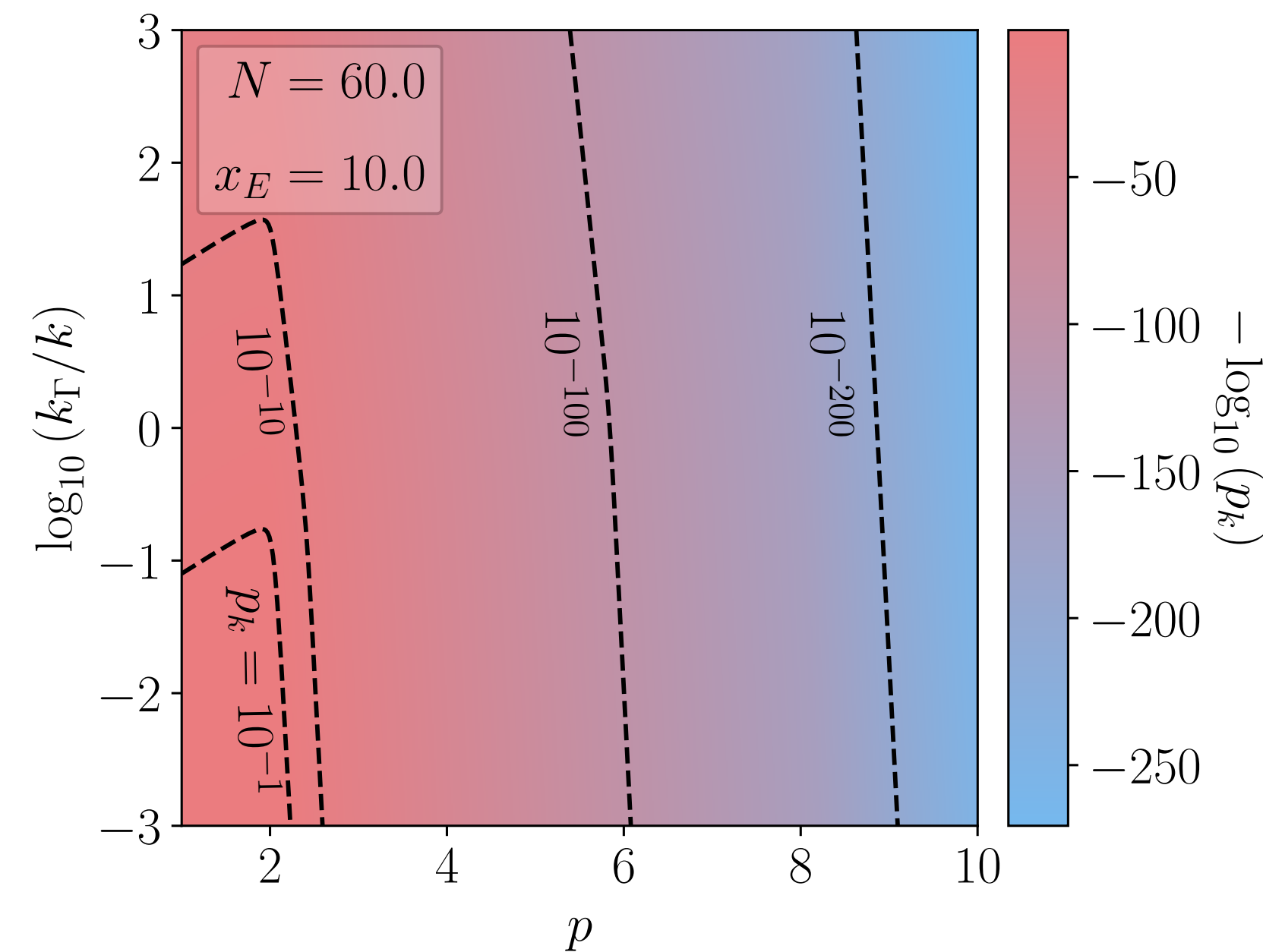
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Effect of decoherence?

Effect of interactions on quantum state

- After 60 e-folds of inflation³ with interaction $C_E \propto k_\gamma^2 a^{p-3}(\eta)$

p_k strongly suppressed

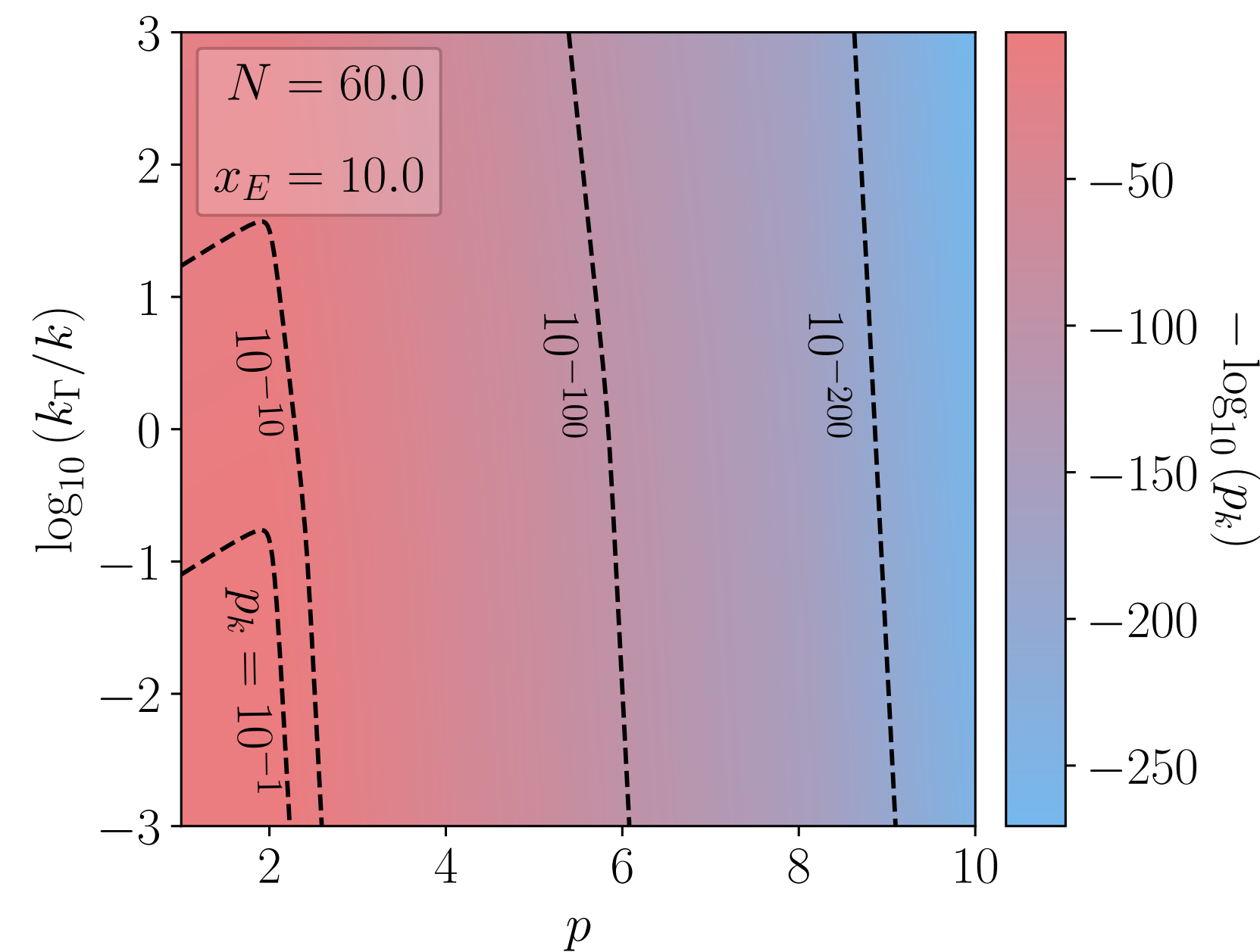


Expected and know

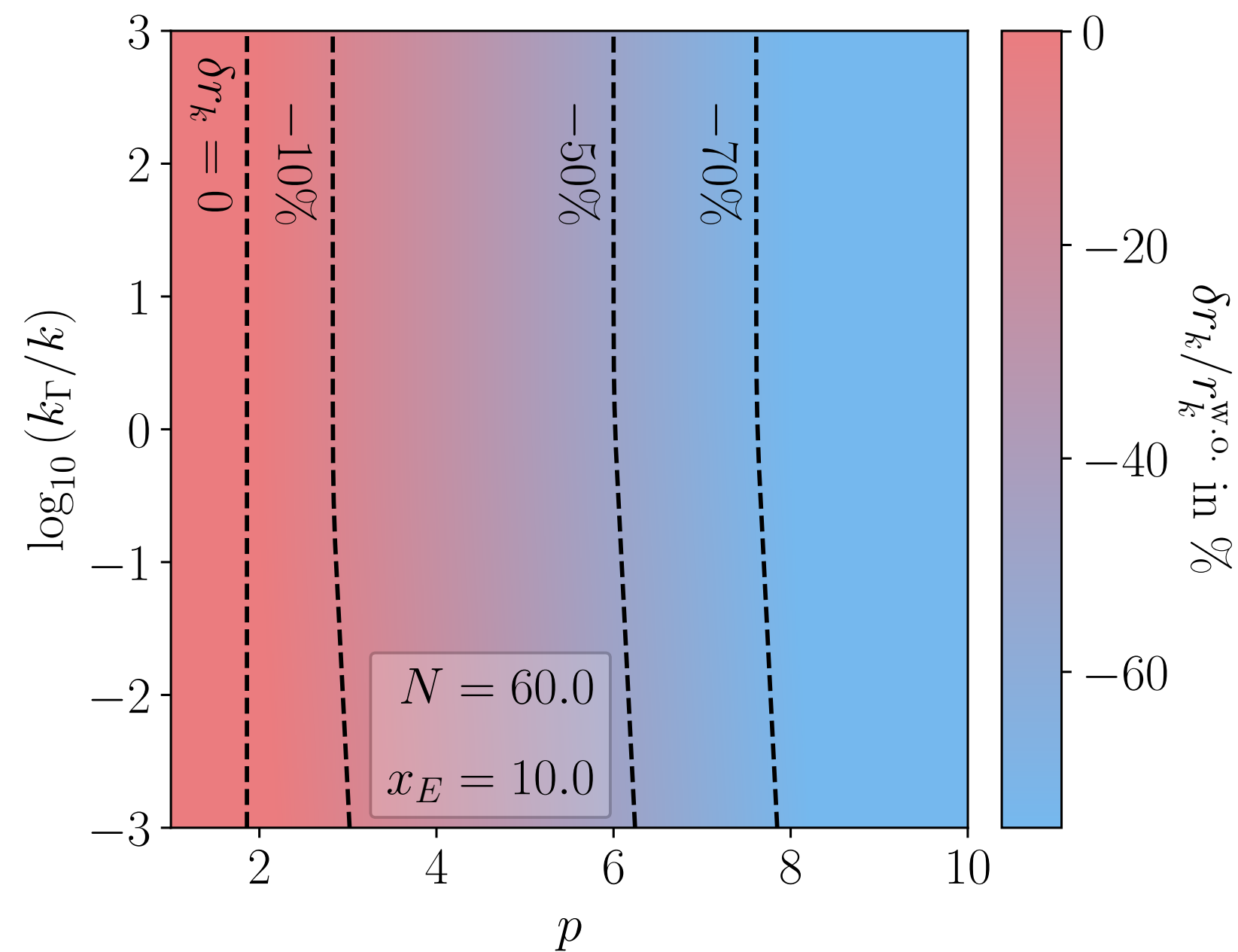
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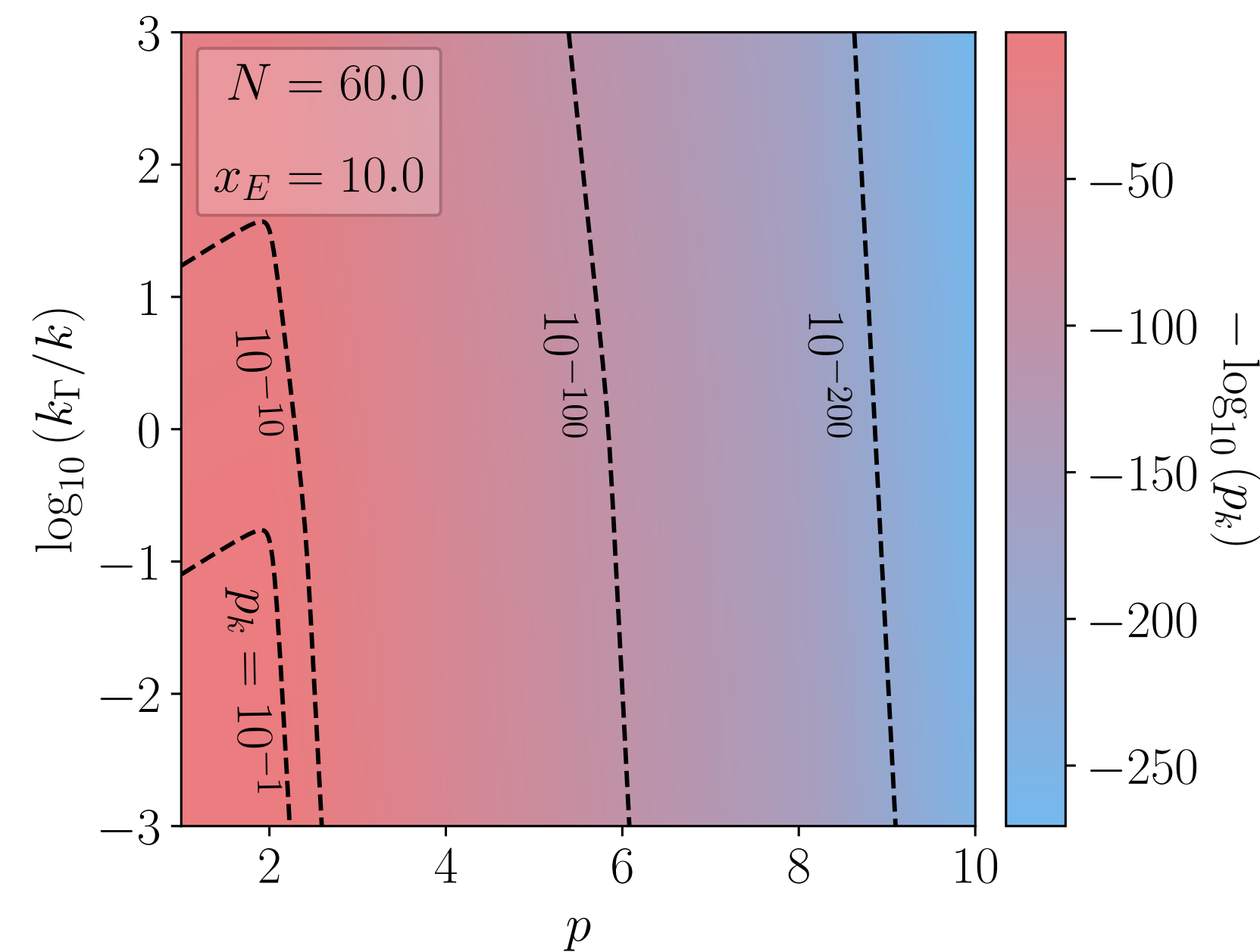
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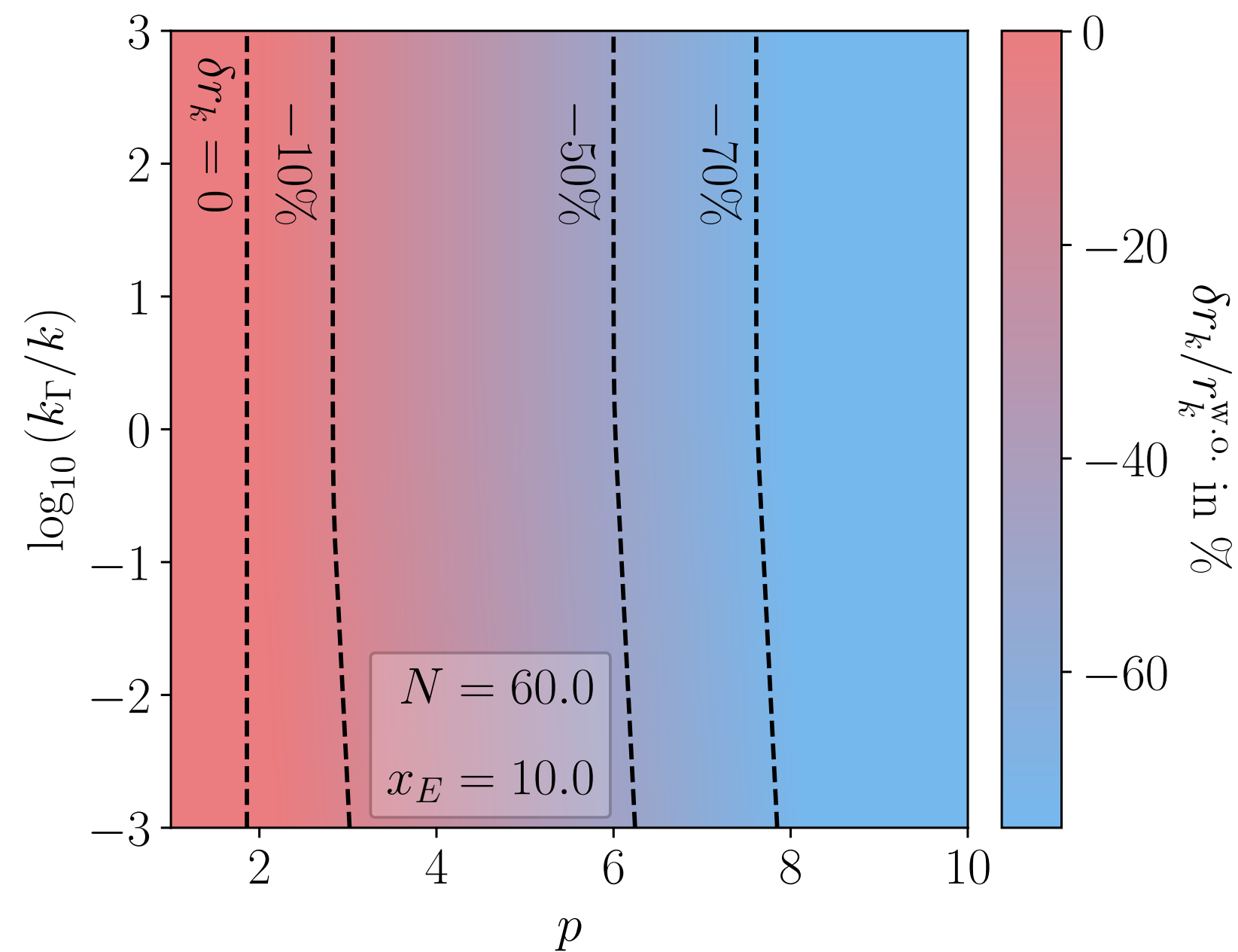
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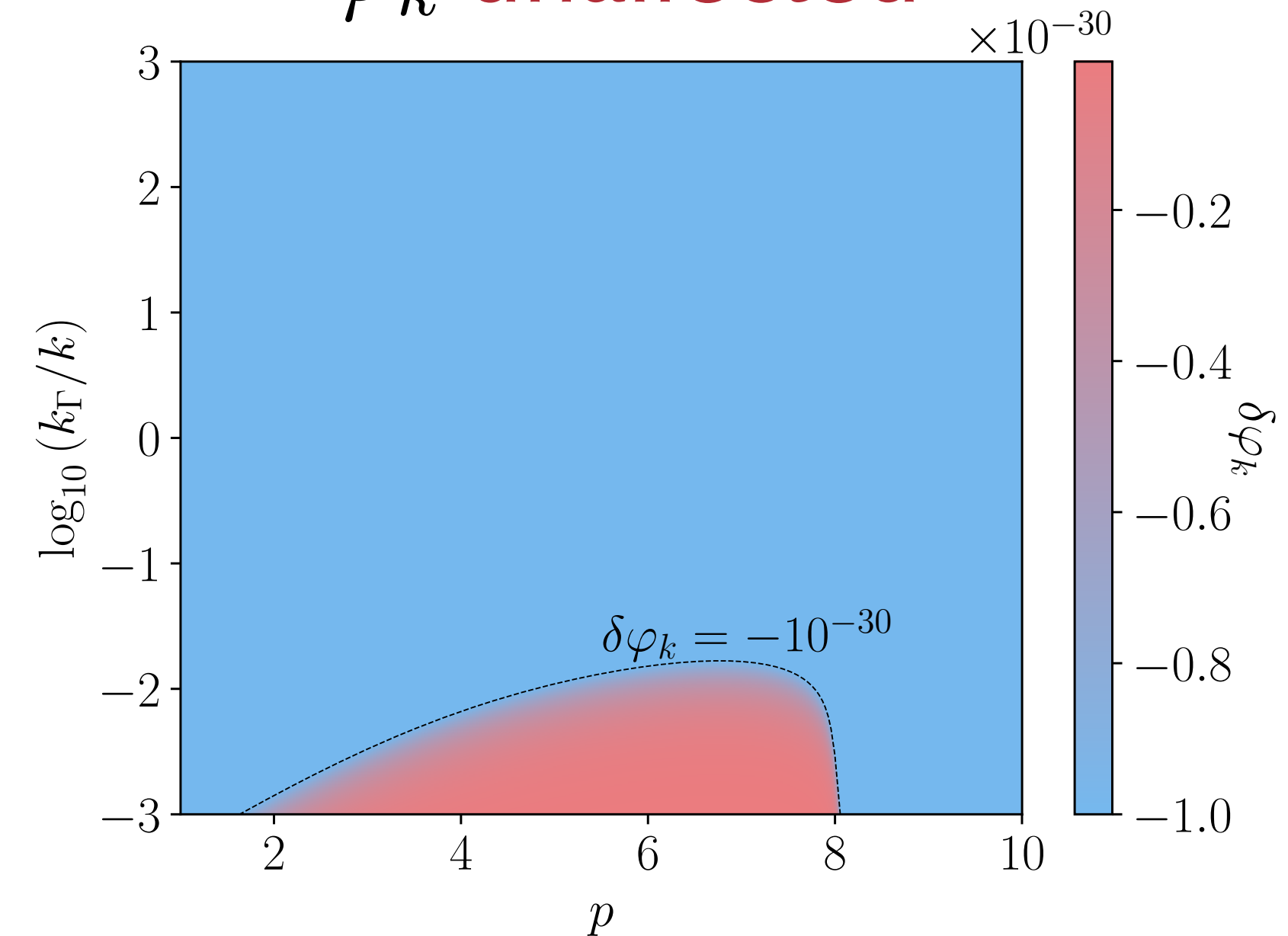
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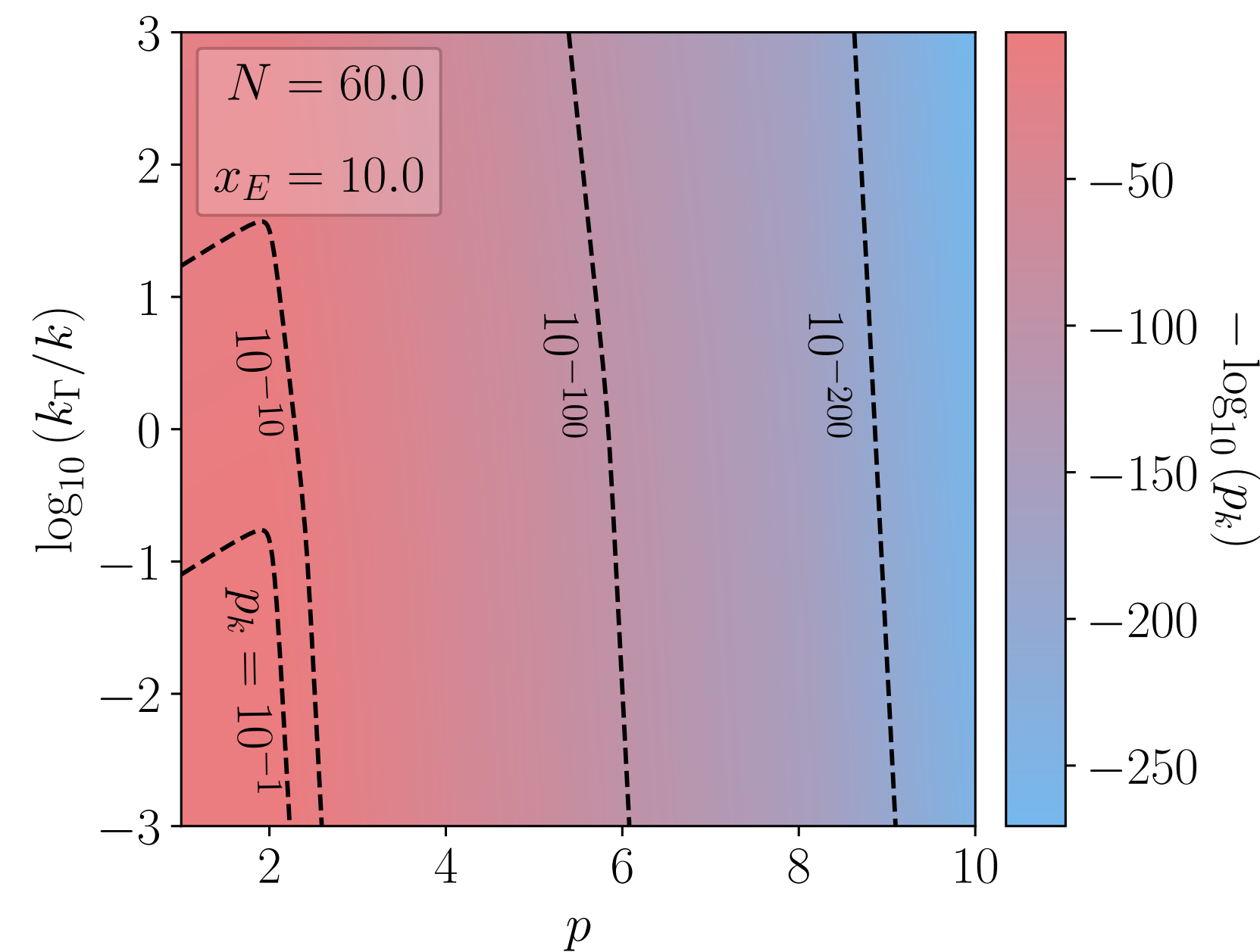
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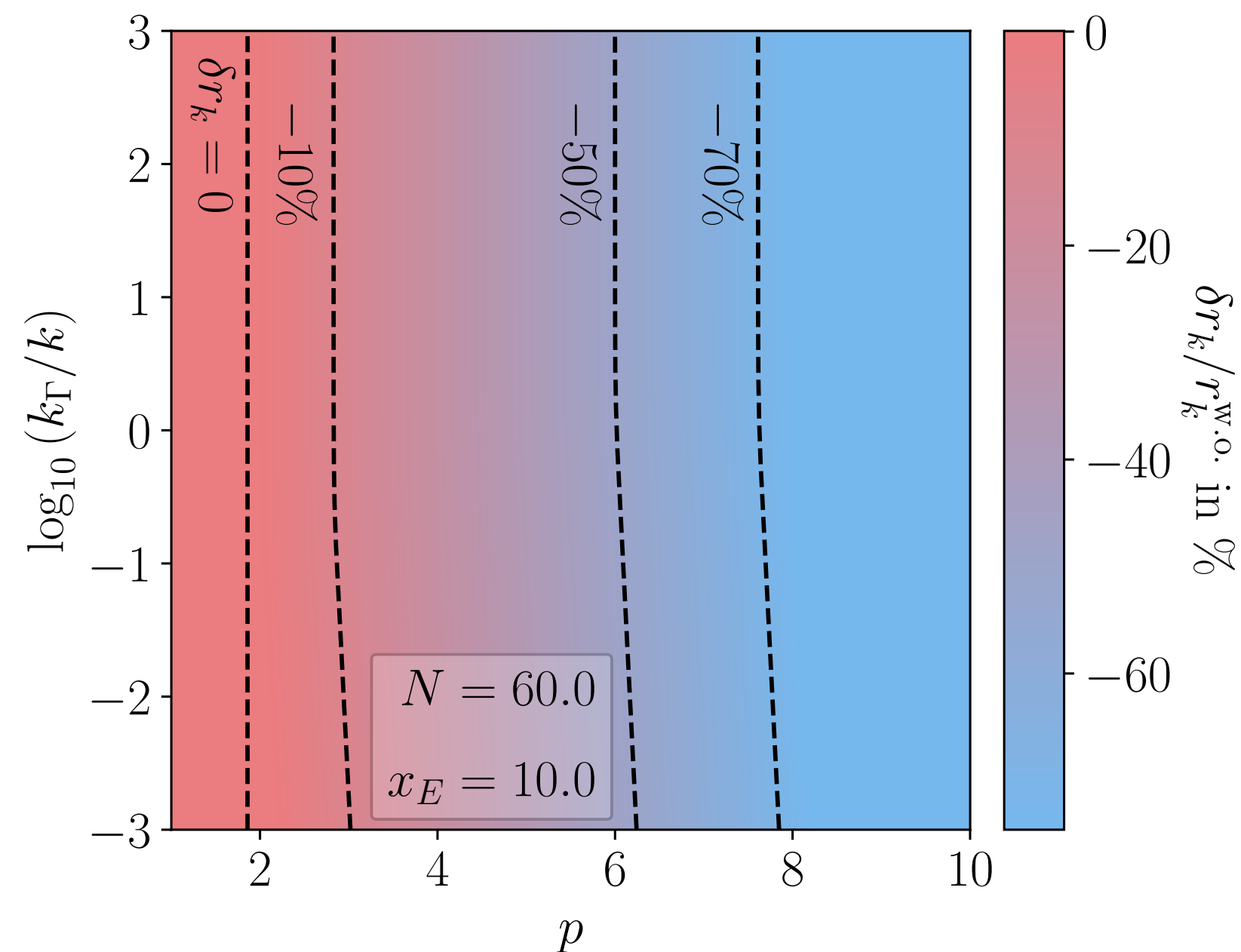
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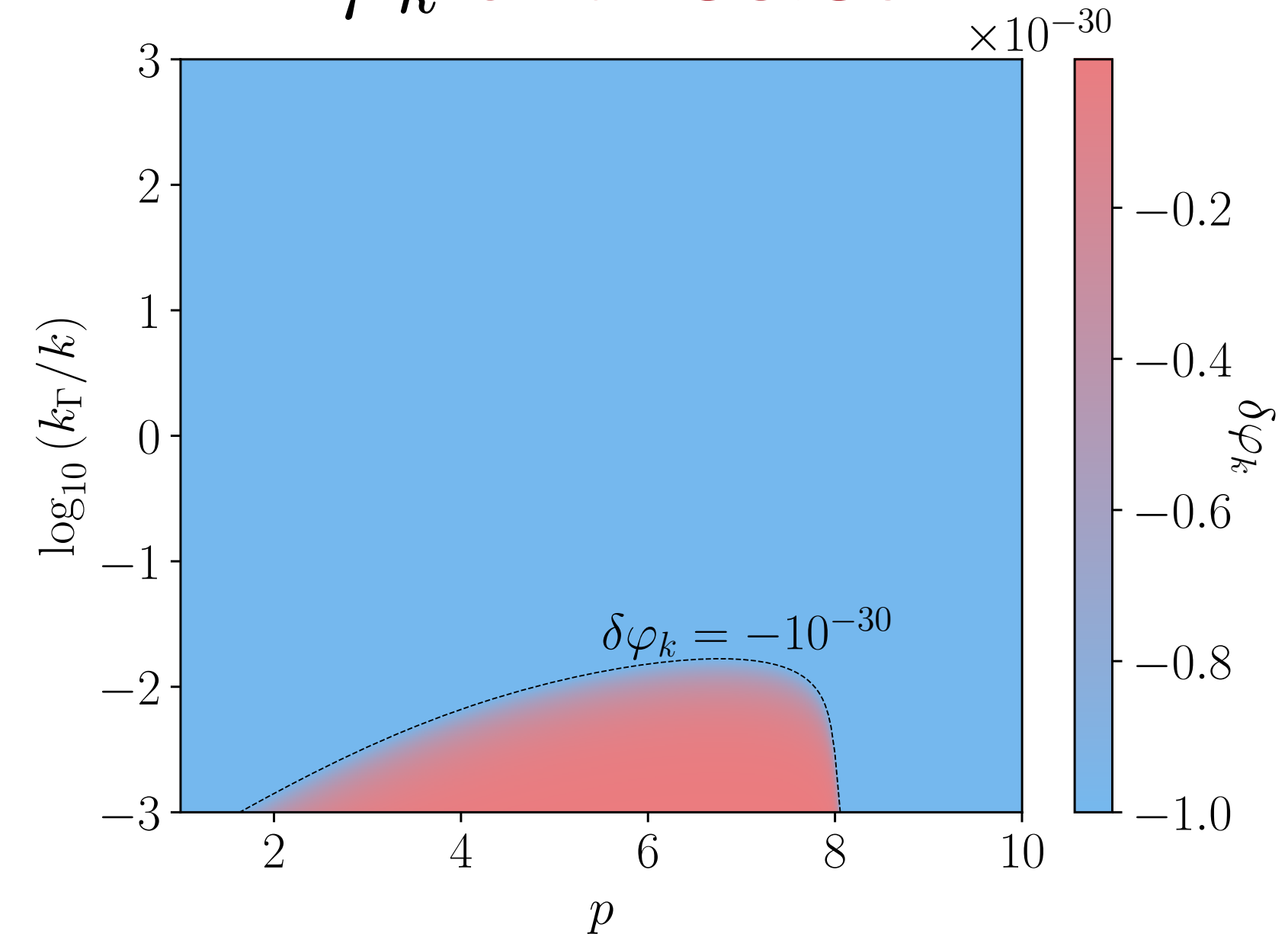
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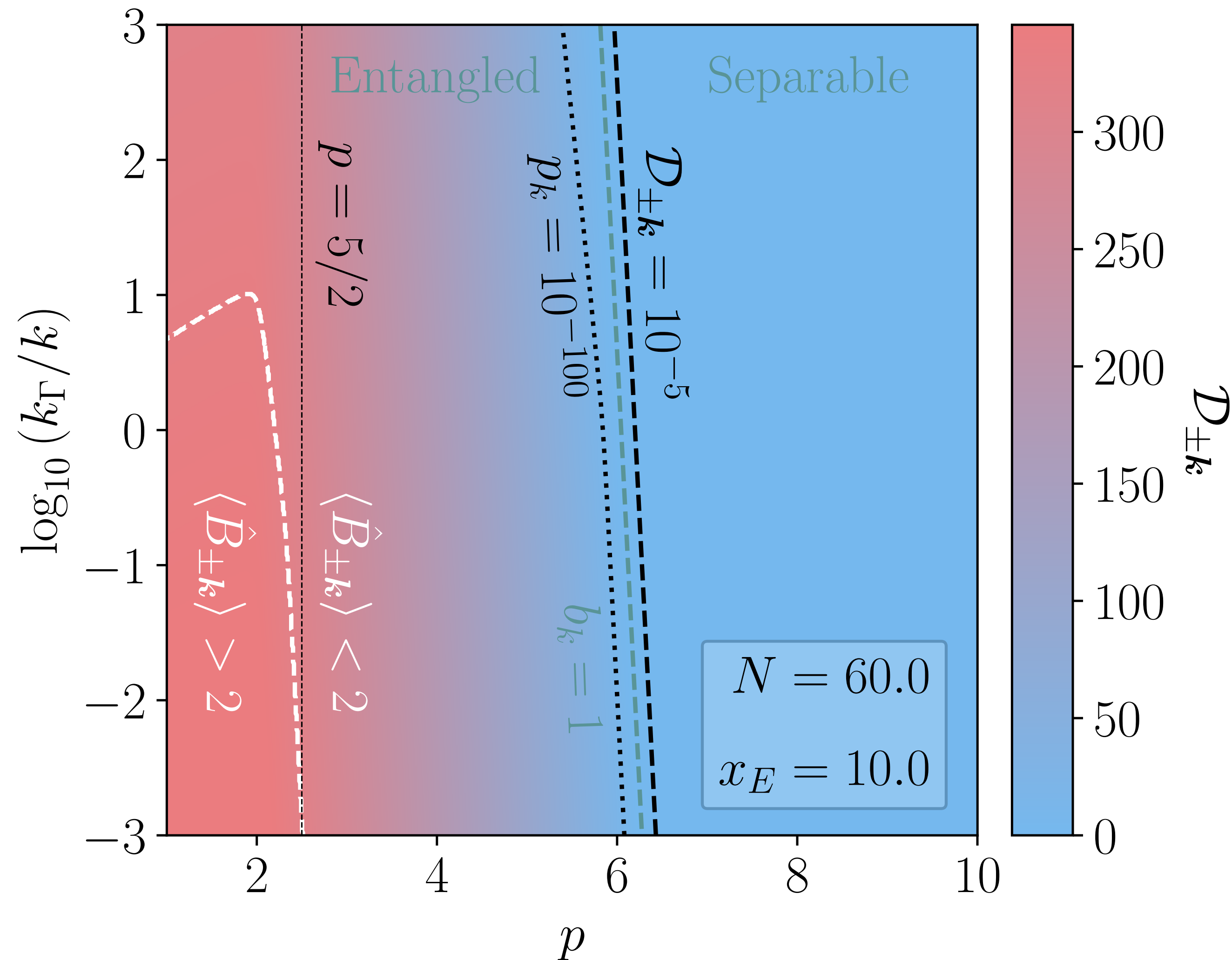
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Effect on correlations?

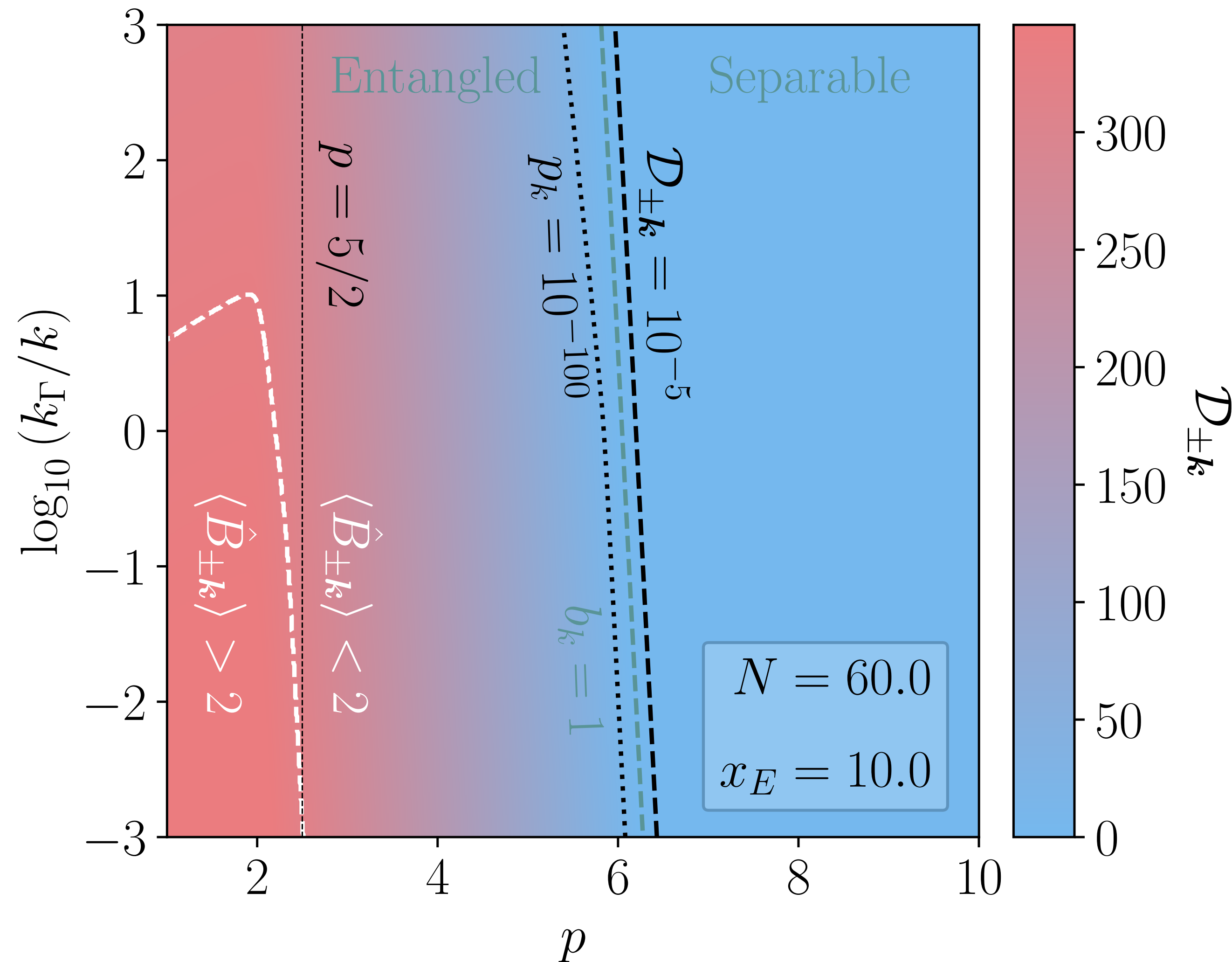
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Take Home Message 2

Purity can be vanishingly small and yet quantum correlations still present, estimating them requires to compute the squeezing parameter as well!

IV - Apparent classicality and detectability

Apparent classicality - I

- Reminder: Perturbations described by positive Wigner function $W(X) \geq 0$

which allows to compute expectation value via $\langle \hat{O} \rangle = \int d\pi^r dv^r \mathcal{W}(\hat{O})(v^r, \pi^r) W(v^r, \pi^r)$

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When does that happen?

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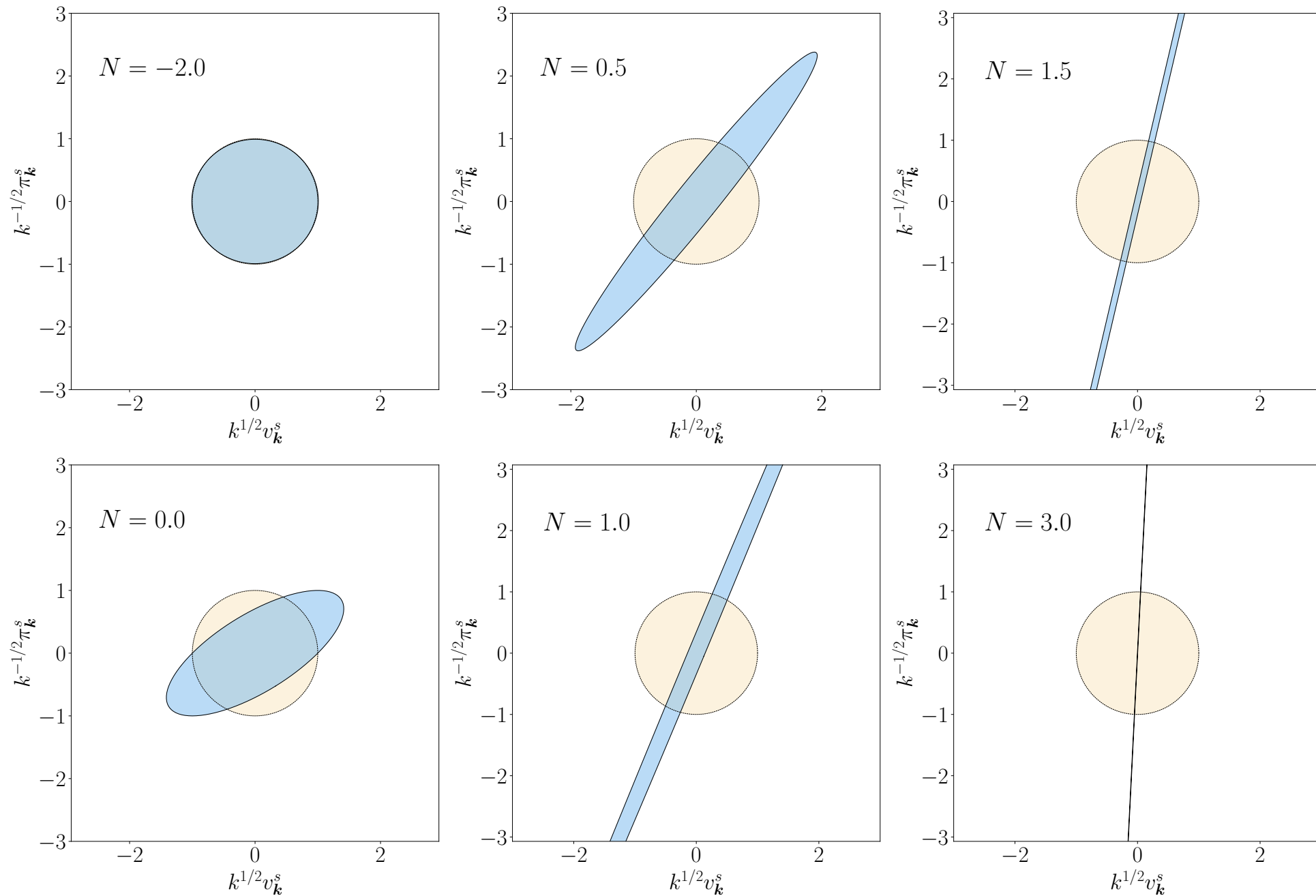
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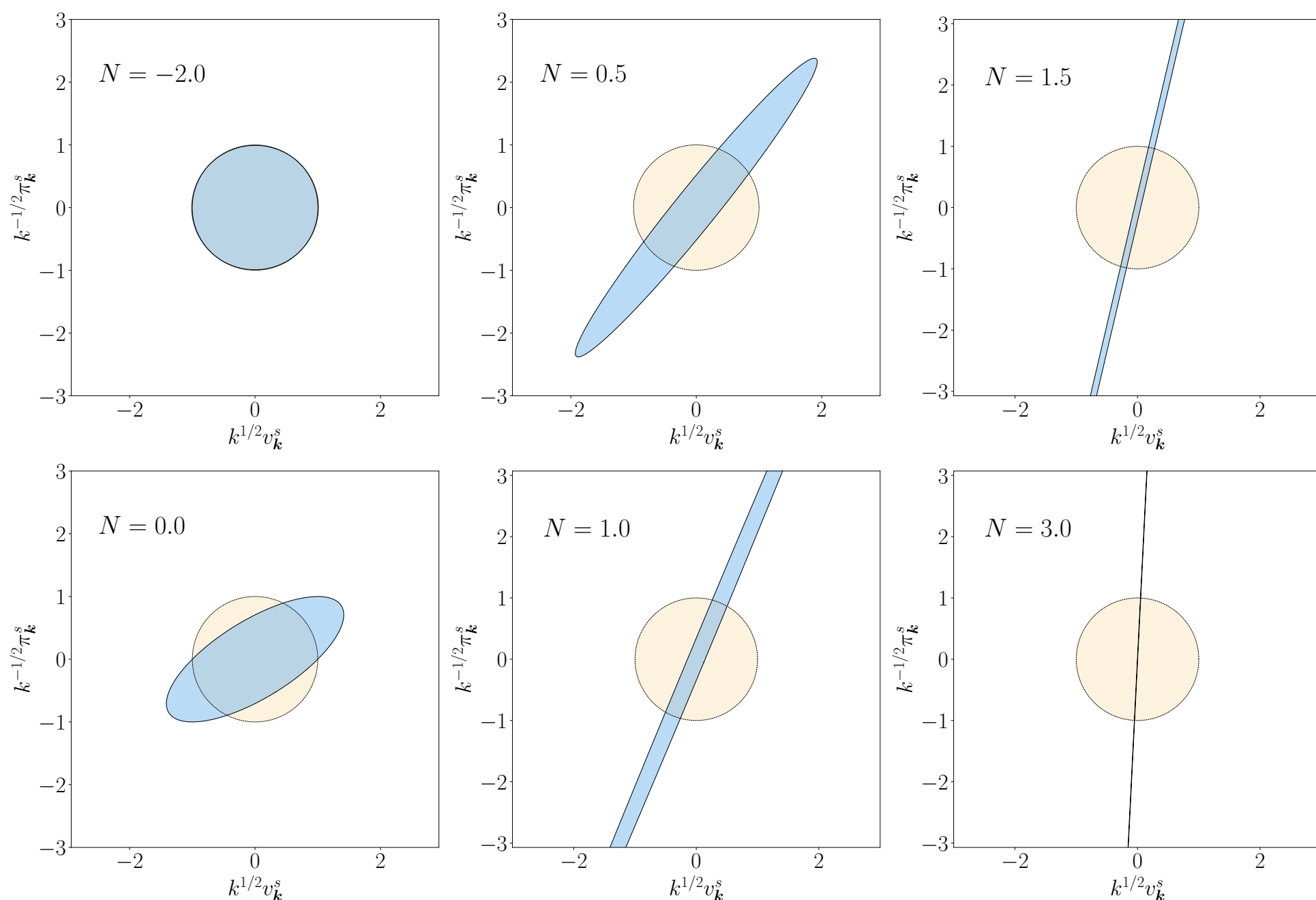
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Take Home Message 3

Because squeezing is very large, for polynomial $f(\hat{v}, \hat{\pi})$, we have $\langle f(\hat{v}, \hat{\pi}) \rangle \approx \langle f(v, \pi) \rangle_{\text{st.}}$

Apparent classicality - III

- NB: No contradiction with previous statements, quantum correlations exist but need $\mathcal{W}(\hat{O})$ **non-polynomial** in (ν, π) to be revealed with a large squeezed state.

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- Example: Pseudo-spin operator $\hat{\sigma}_z^r = - \int_{-\infty}^{\infty} |v_{\mathbf{k}}^r\rangle \langle -v_{\mathbf{k}}^r| dv_{\mathbf{k}}$

————→ Can be used to build a Bell inequality violated by perturbations

Check: its Wigner transform reads $\mathcal{W}(\hat{\sigma}_z^r) = -\pi \delta(v_{\mathbf{k}}^r) \delta(\pi_{\mathbf{k}}^r)$

Non-analytical in (v, π)

Conclusions and perspectives

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-
- D. Have to seek **other** type of **correlations** that may appear in **richer scenarios**^{8,9}, for **example** featuring **non-Gaussian** perturbations.

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5. [arXiv:2512.17622 [Micheli](#), Oshima & Takahashi]

8. [arXiv:2001.09149 Green and Porto]

9. [arXiv:1508.01082 Maldacena]

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