Bottom-up open EFT for non-Abelian gauge theory with dynamical color environment

Kanji Nishii

(Keio Univ., KQCC)

Based on work with

Yoshihiko Abe (Keio Univ., KQCC, SQAI)

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Motivation

- Many realistic systems are open, meaning that they interact with the surrounding environment.

→ In gauge theory, an example is quarkonium in a quark-gluon plasma.

- Consistency with gauge symmetry in an open system is nontrivial.

→ Non-locality generally appears.

- I will present a bottom-up construction in Schwinger-Keldysh EFT.

(The previous talk presented a top-down construction.)

Contents

Setup: Schwinger-Keldysh EFT

$$Z[J_+, J_-] = \int \mathscr{D}A_\pm \mathscr{D}\psi_{i\pm}\, \rho_0[A_\pm, \psi_{i\pm}] \exp\big(\underline{\mathrm{i}S[A_+, \psi_{i+}] - \mathrm{i}S[A_-, \psi_{i-}] + \mathrm{i}J_+A_+ - \mathrm{i}J_-A_-}\big)$$

$$\left.\begin{aligned} &= \int \mathscr{D}A_\pm \mathscr{D}X_\pm\, \rho_0'[A_\pm, X_\pm] \exp\big(\mathrm{i}S\underline{[A_+, X_+] - \mathrm{i}S[A_-, X_-] + \mathrm{i}I_{\mathrm{IF}}[A_\pm, X_\pm}]\big) \\ &= \int \mathscr{D}A_\pm\, \rho_0''[A_\pm] \exp\big(\underline{\mathrm{i}S[A_+] - \mathrm{i}S[A_-] + \mathrm{i}I_{\mathrm{IF}}[A_\pm}]\big) \end{aligned}\right\} \mathrm{i}I_{\mathrm{eff}}[A_\pm; X_\pm]$$

- The first line is the full SK action; ψ_i $(i = 1, \ldots, n)$ denotes matter fields of the many-body system.

- In the second line, the microscopic degrees of freedom are integrated out. At a suitable EFT scale, the long-wavelength modes, that survive coarse-graining, remain as X.

- In the third line, X is also integrated out; if long-wavelength modes are relevant, nonlocal terms appear.

Setup: Assumptions

- Focus on the **semiclassical** regime.

→ Higher-order terms in a-variables are ignored.

- Consider a **bottom-up** construction, not a top-down ([Kaplanek-Mylova-Tolley 2604.26941] and the previous talk).

- Consider the non-Abelian gauge field $A_\mu \in su(N)$.

- Consider the color-charge dissipation, not the energy dissipation ([Lau-**KN**-Noumi 2412.21136]).

- Below the cutoff scale Λ_{EFT}, the gauge field and the long-wavelength modes exist.

Doubled gauge fields and r-a basis

- The gauge field is doubled in the SK formalism

$$A_{\pm\mu} = A_{r\mu} \pm \frac{1}{2} A_{a\mu} \qquad \begin{aligned} A_{r\mu} &\coloneqq \frac{1}{2}\left(A_{+\mu} + A_{-\mu}\right) \\ A_{a\mu} &\coloneqq A_{+\mu} - A_{-\mu} \end{aligned}$$

and the gauge transformation $U_{\pm} \in SU(N)$ is

$$A_{\pm\mu} \mapsto A_{\pm\mu}^{U_\pm} = U_\pm A_{\pm\mu} U_\pm^{-1} - \frac{\mathrm{i}}{\mathrm{g}} U_\pm \partial_\mu U_\pm^{-1}$$

- Under a diagonal transformation, $U_+ = U_- = U_r$,

$$A_{r\mu} \mapsto A_{r\mu}^{U_r} = U_r A_{r\mu} U_r^{-1} - \frac{\mathrm{i}}{\mathrm{g}} U_r \partial_\mu U_r^{-1}\,, \qquad A_{a\mu} \mapsto U_r A_{a\mu} U_r^{-1}\,.$$

Non-diagonal gauge transformation

- The non-diagonal part can be parametrized as

$$U_{\pm} = U_r \exp\left(\pm\frac{\mathrm{i}}{2}\alpha_a\right)\,, \qquad \alpha_a \in \mathfrak{su}(N)$$

which leads to

$$U_-^{-1} U_+ = \exp(\mathrm{i}\alpha_a)$$

- Under the non-diagonal transformation ($X \mapsto X + \delta_a X$), the a-type variations δ_a are given by

$$\delta_a A_{r\mu} = -\frac{\mathrm{i}}{4}[A_{a\mu}, \alpha_a] = \mathcal{O}(a^2)\,,$$

$$\delta_a A_{a\mu} = -\frac{1}{\mathrm{g}} D_\mu[A_r]\alpha_a\,, \qquad D_\mu[A] \coloneqq \partial_\mu + \mathrm{i}\mathrm{g}A_\mu$$

Why a naive Ohmic term fails: Naïve lesson

- As a naive dissipative model for the Yang-Mills theory, consider

$$I_{\mathrm{YM,Ohm}} = 2\int \mathrm{d}^d x\, \mathrm{Tr}\, A_{a\mu}\Big(\underbrace{D_\nu[A_r]F_r^{\nu\mu} - J^\mu_{\mathrm{Ohm}}}_{=\,0 \text{ is the EoM}}\Big) + \mathcal{O}(a^2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{ig}[A_\mu, A_\nu] \qquad J^\mu_{\mathrm{Ohm}} = \sigma_h E_r^\mu = \sigma_h u_\nu F_r^{\mu\nu}$$

σ_h: color conductivity

u^μ: time-like vector

- The action is invariant under diagonal gauge transformations, but non-diagonal gauge invariance requires

$$D_\mu[A_r]J^\mu_{\mathrm{Ohm}} = 0$$

This implies that there is no sink for dissipation-driven outflow, and that the Ohmic term represents only apparent dissipation.

Why it failed: intuition

- Gauge constraints involve the total color charge.

→ A system-only friction term cannot represent an untracked sink of color charge.

→ The environmental DoFs are needed to capture the total color charge.

- Additional DoFs are required if we want to construct a bottom-up EFT using only local terms.

- If the environmental slow mode is integrated out, the gauge-only theory includes generally nonlocal and non-Markov terms.

How to construct the environment

- We need a time-like vector u^μ from the environmental sector to specify the direction of dissipation.

- **HydroEFT** is a good candidate of the environment for a dissipative gauge-invariant theory. [Glorioso-Liu 1805.09331][Lau-**KN**-Noumi 2412.21136]

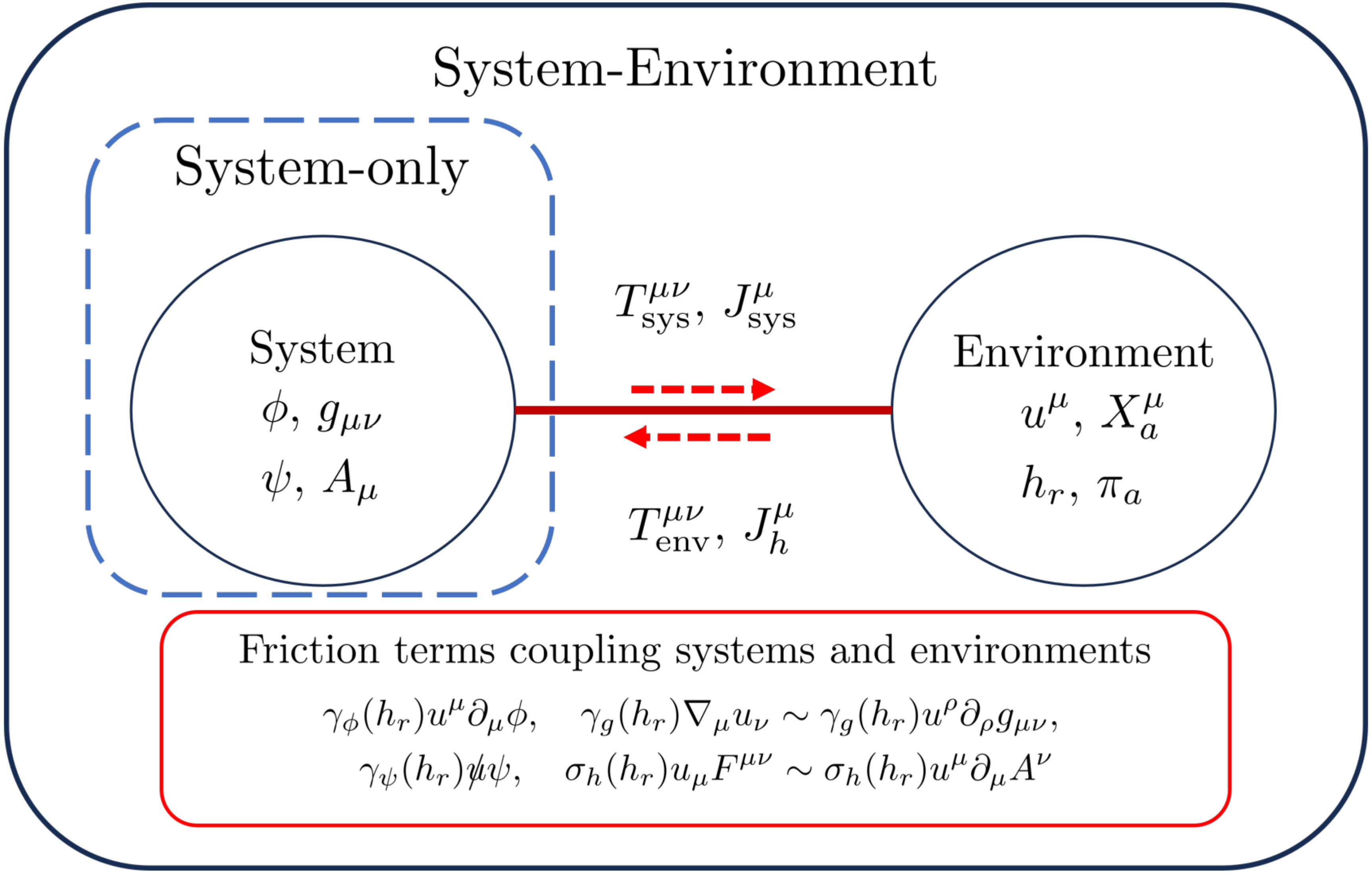
→ The Stückelberg field restores the a-type gauge covariance, but it does not by itself provide dissipation.

→ Make **the Stückelberg field dynamical as the environment**.

- The environmental current can carry and store the color charge.

→ The result is a **Markovian embedding** rather than a genuinely non-Markovian theory.

Our setup


System-Environment
System-only
System
$\phi,\ g_{\mu\nu}$
$\psi,\ A_\mu$
$T^{\mu\nu}_{\text{sys}},\ J^\mu_{\text{sys}}$
$T^{\mu\nu}_{\text{env}},\ J^\mu_h$
Environment
$u^\mu,\ X^\mu_a$
$h_r,\ \pi_a$
Friction terms coupling systems and environments
$\gamma_\phi(h_r)u^\mu\partial_\mu\phi, \quad \gamma_g(h_r)\nabla_\mu u_\nu \sim \gamma_g(h_r)u^\rho\partial_\rho g_{\mu\nu},$
$\gamma_\psi(h_r)\not{u}\psi, \quad \sigma_h(h_r)u_\mu F^{\mu\nu} \sim \sigma_h(h_r)u^\mu\partial_\mu A^\nu$

Gauge-covariant environmental color frame

- A naive dissipative term violates the a-type gauge symmetry.

→ Compensate using the Stückelberg trick.

- Introduce a color frame $h_{\pm}(x) \in SU(N)$ which transforms as

$$h_{\pm} \mapsto U_{\pm} h_{\pm}$$

We can dress the gauge field using the Stückelberg field $h_{\pm}$ as

$$\mathcal{A}_{\pm\mu} := h_{\pm}^{-1} A_{\pm\mu} h_{\pm} - \frac{\mathrm{i}}{\mathrm{g}} h_{\pm}^{-1} \partial_{\mu} h_{\pm}$$

- This is trivially gauge-invariant.

Stückelberg field in the r-a basis

- r-a fields can be constructed in the same way as in the previous slides

$$h_\pm := h_r \exp\left(\pm\frac{\mathrm{i}}{2}\pi_a\right) \approx h_r\left(\mathbf{1} \pm \frac{\mathrm{i}}{2}\pi_a\right) , \qquad \pi_a \in \mathfrak{g} ,$$

$$h_-^{-1} h_+ = \exp(\mathrm{i}\pi_a)$$

- Under the diagonal gauge transformation,

$$h_r \mapsto U_r h_r \qquad \pi_a \mapsto \pi_a$$

- The a-type gauge transformations are

$$\delta_a h_r = \mathcal{O}(a^2) , \qquad \delta_a \pi_a = h_r^{-1} \alpha_a h_r + \mathcal{O}(a^2) .$$

Dressed fields in the r-a basis

- Gauge-invariant r-a dressed fields can be constructed as

$$\mathcal{A}_{r\mu} := \frac{1}{2}(\mathcal{A}_{+\mu} + \mathcal{A}_{-\mu}) = h_r^{-1} A_{r\mu} h_r - \frac{\mathrm{i}}{\mathrm{g}} h_r^{-1} \partial_\mu h_r + \mathcal{O}(a^2) \,,$$

$$\mathcal{A}_{a\mu} := \mathcal{A}_{+\mu} - \mathcal{A}_{-\mu} = h_r^{-1} A_{a\mu} h_r + \frac{1}{\mathrm{g}} \mathcal{D}_\mu[\mathcal{A}_r] \pi_a + \mathcal{O}(a^2)$$

and the dressed field strength and the covariant derivative are

$$\mathcal{F}_{r\mu\nu} := \partial_\mu \mathcal{A}_{r\nu} - \partial_\nu \mathcal{A}_{r\mu} + \mathrm{ig}[\mathcal{A}_{r\mu}, \mathcal{A}_{r\nu}] = h_r^{-1} F_{r\mu\nu} h_r \,,$$

$$\mathcal{D}_\mu[\mathcal{A}_r] := \partial_\mu + \mathrm{ig}\mathcal{A}_{r\mu} = h_r^{-1} D_\mu[A_r] h_r \,,$$

- These are trivially gauge-invariant.

Gauge-invariant SK effctive action

- The gauge-invariant SK-YM effective action is

$$
\begin{aligned}
I_{\mathrm{YM}} &= 2\int \mathrm{d}^d x\, \mathrm{Tr}\, \mathcal{A}_{a\mu}\Big(\mathcal{D}_\nu[\mathcal{A}_r]\mathcal{F}_r^{\nu\mu} - \mathcal{J}^\mu_{\mathrm{Ohm}}\Big) + \mathcal{O}(a^2) \\
&= 2\int \mathrm{d}^d x\, \mathrm{Tr}\, A_{a\mu}\Big(D_\nu[A_r]F_r^{\nu\mu} - J^\mu_{\mathrm{Ohm}}\Big) \qquad\qquad \mathcal{J}^\mu := h_r^{-1} J^\mu h_r\,, \\
&\quad - \frac{2}{\mathrm{g}}\int \mathrm{d}^d x\, \mathrm{Tr}\, \pi_a \mathcal{D}_\mu[\mathcal{A}_r]\Big(\mathcal{D}_\nu[\mathcal{A}_r]\mathcal{F}_r^{\nu\mu} - \mathcal{J}^\mu_{\mathrm{Ohm}}\Big) + \mathcal{O}(a^2)
\end{aligned}
$$

- This is not yet dissipative.

→ The non-dynamical color frame does not by itself carry the exchanged color charge. A dynamical environmental current sector is still required.

Environmental color current

- Naïvely, the environmental color current is given by

$$\mathcal{J}_h^\mu = \underbrace{\chi_h \mu_h u^\mu}_{\mathcal{J}^\mu_{\text{fluid}}} + \underbrace{\sigma_h \left[u_\nu \mathcal{F}_r^{\mu\nu} - T \Delta_u^{\mu\nu} \mathcal{D}_\nu[\mathcal{A}_r] \left(\frac{\mu_h}{T} \right) \right]}_{\mathcal{J}^\mu_{\text{YM,fric}}} + \cdots ,$$

$$\mu_h := u^\mu \mathcal{A}_{r\mu} = u^\mu \left(h_r^{-1} A_{r\mu} h_r - \frac{\mathrm{i}}{\mathrm{g}} h_r^{-1} \partial_\mu h_r \right) \qquad \Delta_u^{\mu\nu} := \eta^{\mu\nu} + u^\mu u^\nu$$

μ_h: chemical potential χ_h: color susceptibility T: local temperature

- The first term describes color-charge storage. The remaining terms form the dissipative electrochemical response.

→ Dynamical KMS symmetry fixes the associated local noise kernel.

Local system–environment SK action

- The gauge-invariant SK effective action is

$$I_{\mathrm{YM,fluid}} = 2\int \mathrm{d}^d x\, \mathrm{Tr}\, \mathcal{A}_{a\mu}\left(\mathcal{D}_\nu[\mathcal{A}_r]\mathcal{F}_r^{\nu\mu} - \mathcal{J}_h^{\mu}\right)$$
$$+\,\mathrm{i}\int \mathrm{d}^d x\, \mathrm{Tr}\, \mathcal{A}_{a\mu}(x) N_{h,\mathrm{loc}}^{\mu\nu}(x;\mathcal{A}_r,u,T)\mathcal{A}_{a\nu}(x) + I_{\mathrm{hydro,env}}[u,T] + \cdots \,.$$

$$\text{Ward identity}: \quad \mathcal{D}_\mu[\mathcal{A}_\mathrm{r}]\mathcal{J}^{\mu}_{\mathrm{YM,fric}} = -\mathcal{D}_\mu[\mathcal{A}_\mathrm{r}]\mathcal{J}^{\mu}_{\mathrm{fluid}}$$

- The noise term is fixed by dynamical KMS invariance, and SK unitarity requires positivity.

$$N_{h,\mathrm{loc}}^{\mu\nu} = 2T\sigma_h \Delta_u^{\mu\nu} + \cdots\,, \qquad \sigma_h \geq 0$$

- Matter fields (fermions and bosons) can be added (see our paper).

Summary and limitations

- We gave a **bottom-up** and gauge-consistent recipe for an open-system EFT.

→ Introducing **the environment is important**.

- Our primary construction is an enlarged local EFT of the gauge sector (**Markovian embedding**) and retained environmental sector. The gauge-only open EFT is its reduced description. Depends on the meaning of “open”.

- A fluid environment is natural in EFT but not essential. The construction is sufficient, but necessary-and-sufficient conditions remain open.

- The construction is semiclassical and sufficient only; it does not establish consistency with a quantum open-system description (see [Kaplanek-Mylova-Tolley 2512.17089, 2604.26941]).

Future directions

- BRST-completion matching

- Analyze a certain phenomenon

→Open EFT of quarkonium in QGP including non-Markovian effects

- Dissipation with anomaly

- Dissipative lattice construction with the color-frame $h_r \in SU(N)$.

- Dissipative charged black hole

Backup slides

Relation to the previous talk

Previous: top-down [Kaplanek-Mylova-Tolley 2026]

- Start from a gauge-fixed quantum SK path integral
- Treat physical initial states and finite-time boundaries carefully
- Integrate out charged matter or hard gauge modes
- Derive BRST constraints on the reduced influence functional

This talk: bottom-up

- Work in the semiclassical r-a expansion
- Retain slow environmental color response explicitly
- Introduce a color frame and a charge-carrying current sector
- Construct a local gauge-invariant system–environment EFT with dissipation and noise

The previous talk constrains the reduced influence functional; this talk constructs an enlarged local theory that can dynamically generate such a response.

Gravitational SKEFT

- A diffeo-invariant action is

$$I_{\text{naive}} = \int \mathrm{d}^d x \sqrt{-g_r} \left[\frac{1}{2} \left(-M_P^2 G_r^{\mu\nu} + T_{\phi_r}^{\mu\nu} \right) \mathcal{G}_{a\mu\nu} + \left(\Box\phi_r - \gamma_\phi u^\mu \partial_\mu \phi_r - V'(\phi_r) \right) \varphi_a \right] + \mathcal{O}(a^2) \,,$$

$$\mathcal{G}_{a\mu\nu} \coloneqq g_{a\mu\nu} - \nabla_\mu X_{a\nu} - \nabla_\nu X_{a\mu} = g_{a\mu\nu} - \pounds_{X_a} g_{r\mu\nu} \,,$$
$$\varphi_a \coloneqq \phi_a - X_a^\mu \partial_\mu \phi_r = \phi_a - \pounds_{X_a} \phi_r \,,$$

It is completed by adding the hydro sector $T_{env}^{\mu\nu}(X_r)$

Full SKEFT

- A diffeo- and gauge-invariant full action is

$$I_{h,\text{fluid}} = \frac{1}{2}\int \mathrm{d}^d x\, T^{\mu\nu}_{\text{env}}(u, T, \ldots)\mathcal{G}_{a\mu\nu} - 2\int \mathrm{d}^d x\, \mathrm{Tr}\, \mathbb{A}_{a\mu}\mathcal{J}^{\mu}_{h,\text{fluid}}(\mathcal{A}_r, \mu_h, u, T)$$
$$+\, \mathrm{i}\int \mathrm{d}^d x\, \mathrm{Tr}\, \mathbb{A}_{a\mu}(x) N^{\mu\nu}_{h,\text{loc}}(x; \mathcal{A}_r, u, T)\mathbb{A}_{a\nu}(x) + I_{\text{hydro,KMS}}[u, T, \mathcal{G}_a] + \cdots\,.$$

where

$$\mathbb{A}_{a\mu} := \mathcal{A}_{a\mu} - \pounds_{X_a}\mathcal{A}_{r\mu}$$

Integration of environmental variables

- The EoM is

$$\mathcal{D}_\mu[\mathcal{A}_r]\left(\mathcal{J}^\mu_{\text{sys}}+\mathcal{J}^\mu_{h,\text{fluid}}\right)=0\,.$$

where

$$\mathcal{J}^\mu_{h,\text{fluid}}=\chi_h\mu_h u^\mu+\sigma_h\mathfrak{F}^\mu_h$$

$$\mathfrak{F}^\mu_h:=\mathcal{E}^\mu_r-T\Delta^{\mu\nu}_u\mathcal{D}_\nu[\mathcal{A}_r]\left(\frac{\mu_h}{T}\right)\,,\qquad u_\mu\mathfrak{F}^\mu_h=0\,,\qquad \mathcal{E}^\mu_r:=\mathcal{F}^{\mu\nu}_r u_\nu$$

- It can be solved formally as

$$\bar{\mu}_h=-[\hat{\mathfrak{L}}_h]^{-1}_{\text{ret}}\mathcal{D}_\mu[\mathcal{A}_r]\left[\mathcal{J}^\mu_{\text{sys}}+\sigma_h\mathcal{E}^\mu_r\right]+\cdots\,.$$

$$\hat{\mathfrak{L}}_h\mu_h:=\mathcal{D}_\mu[\mathcal{A}_r]\left[\chi_h\mu_h u^\mu-\sigma_h T\Delta^{\mu\nu}_u\mathcal{D}_\nu[\mathcal{A}_r]\left(\frac{\mu_h}{T}\right)\right]$$

This causes nonlocality.

Fermionic SKEFT

- A gauge-invariant action with a fermion is

$$I_{\psi,0} = -\int \mathrm{d}^d x \left[\bar{\Psi}_a \left(\not{D}[\mathcal{A}_r] + m\right) \Psi_r + \bar{\Psi}_r \left(-\overleftarrow{\not{D}}[\mathcal{A}_r] + m\right) \Psi_a\right] - 2 \int \mathrm{d}^d x \, \mathrm{Tr}\, \mathcal{A}_{a\mu} \mathcal{J}_{\Psi}^{\mu},$$

$$I_{h\psi} = -\int \mathrm{d}^d x \left(\bar{\Psi}_a \mathcal{M}_\psi[\mathcal{A}_r, h_r] \Psi_r + \bar{\Psi}_r \mathcal{M}_{\psi,A}[\mathcal{A}_r, h_r] \Psi_a\right)$$

where

$$\mathcal{M}_\psi[\mathcal{A}_r, h_r] = -\gamma_\psi(\mu_h, T, \ldots)\not{u} + \mathcal{O}(\mathcal{D}[\mathcal{A}_r], \mathcal{F}_r)\,,$$

$$\mathcal{M}_{\psi,A} = -\gamma^0 \mathcal{M}_\psi^\dagger \gamma^0$$