Quantum Crossroads@ IPMU

Many-body states under measurements: from quantum entanglement to magic

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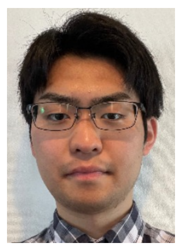
Masahiro Hoshino (UT)



Ryota Matsuda (UT)

Masaki Oshikawa (Ohio State)

Shunsuke Furukawa (Keio)

Yohei Fuji (Ritsumeikan)

First part:

YA, Gong, Ueda, (review)	Adv. Phys. (2021)
Fuji and YA,	PRB (2020)
YA, Furukawa, Oshikawa,	PRB (2024)

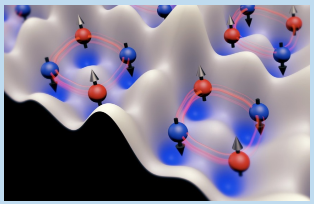
Second part:

Hoshino, Oshikawa, YA,	PRX (2026); PRB (2025)
Hoshino and YA,	PRL (2026)
Matsuda, Hoshino, YA,	arXiv:2607.05343

Overview: Quantum many-body physics × Quantum optics/Info

Quantum Many-Body Physics

Quantum Optics/Info



- **Measurement-induced phenomena**
- **Entanglement transitions**
- **Many-body quantum resources**

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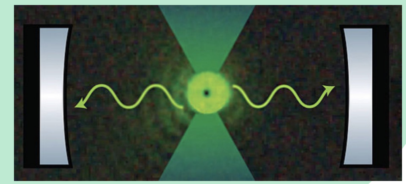


macro

micro

- Quantum criticality
- Quantum phase transitions
- Field theory

- Quantum measurement
- Dissipation, open systems
- Quantum resource theory

Motivation: many-body states under measurements

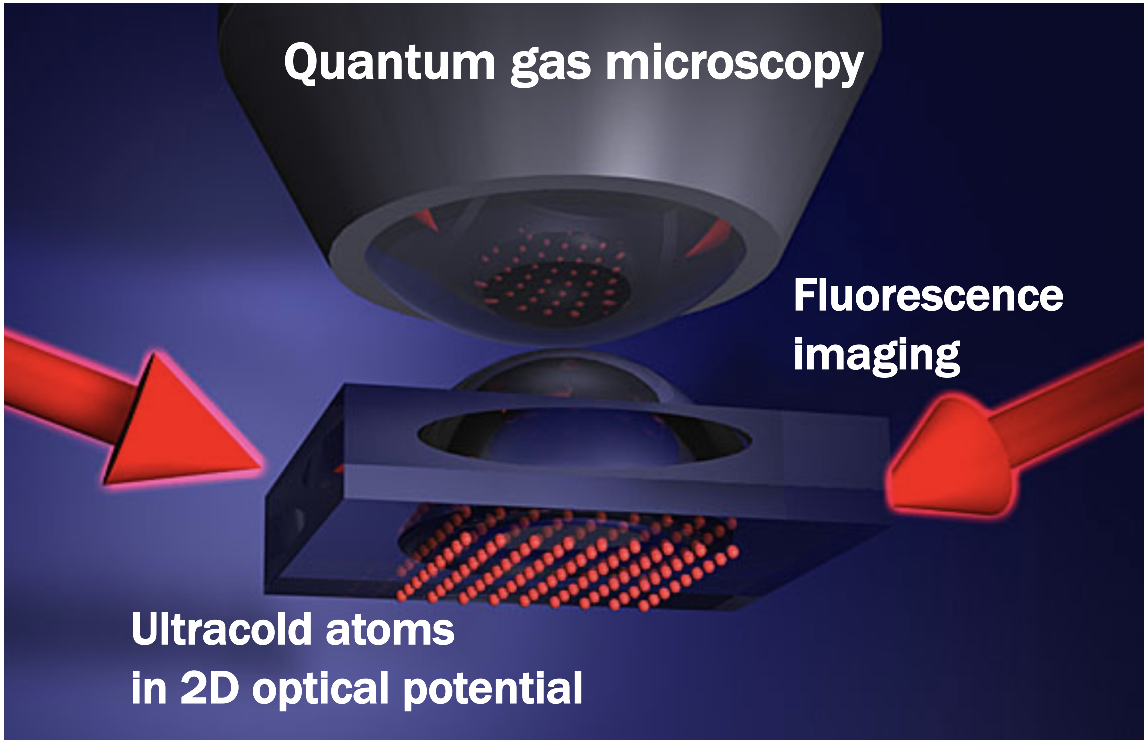
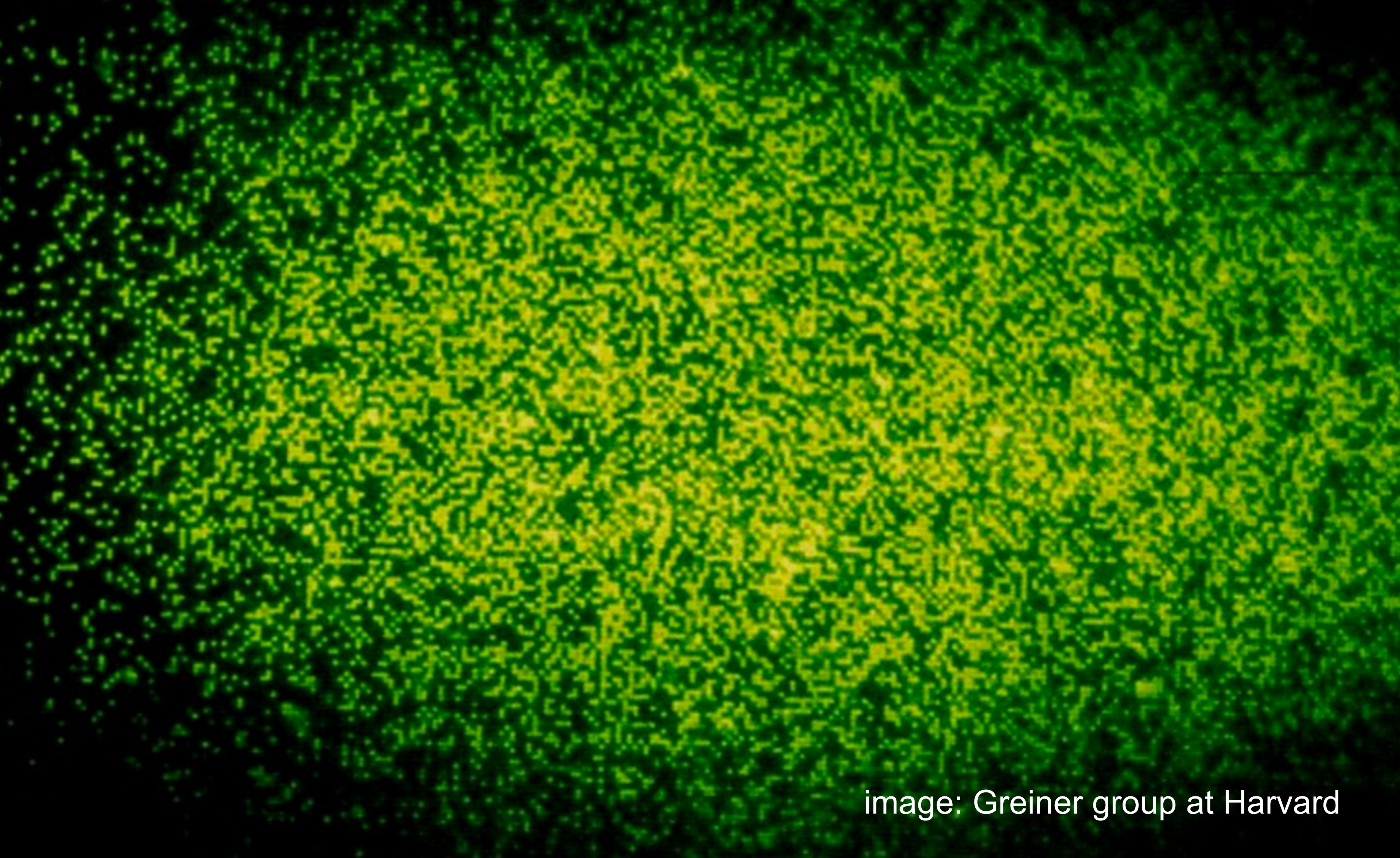


image: Greiner group at Harvard

Measuring and controlling *macroscopic* many-body wavefunction at *microscopic* single-atom resolution



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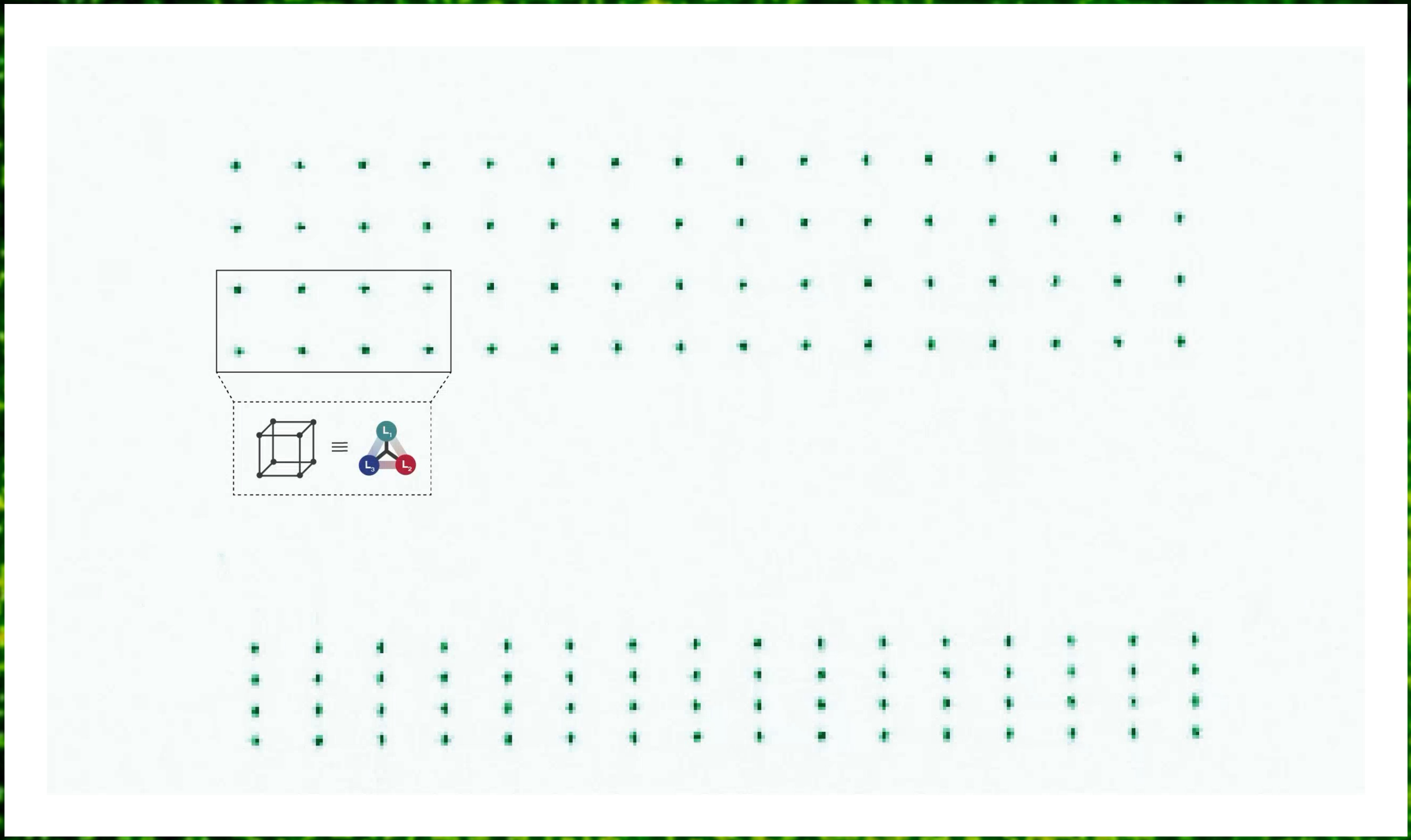
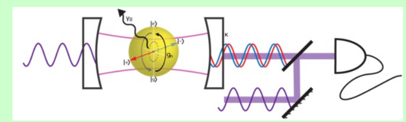


image: Greiner group and Lukin group at Harvard

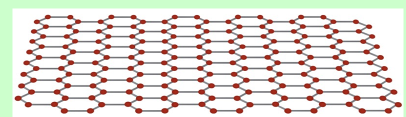
Measuring and controlling *macroscopic* many-body wavefunction at *microscopic* single-atom resolution

Integrate

- *Open* quantum systems (Quantum measurement theory)



- Quantum *many-body* systems (Condensed matter physics)



→ **How does measurement affect many-body states?**

image: Greiner group and Lukin group at Harvard

Continuous Measurement of Quantum Systems: Quantum Trajectory

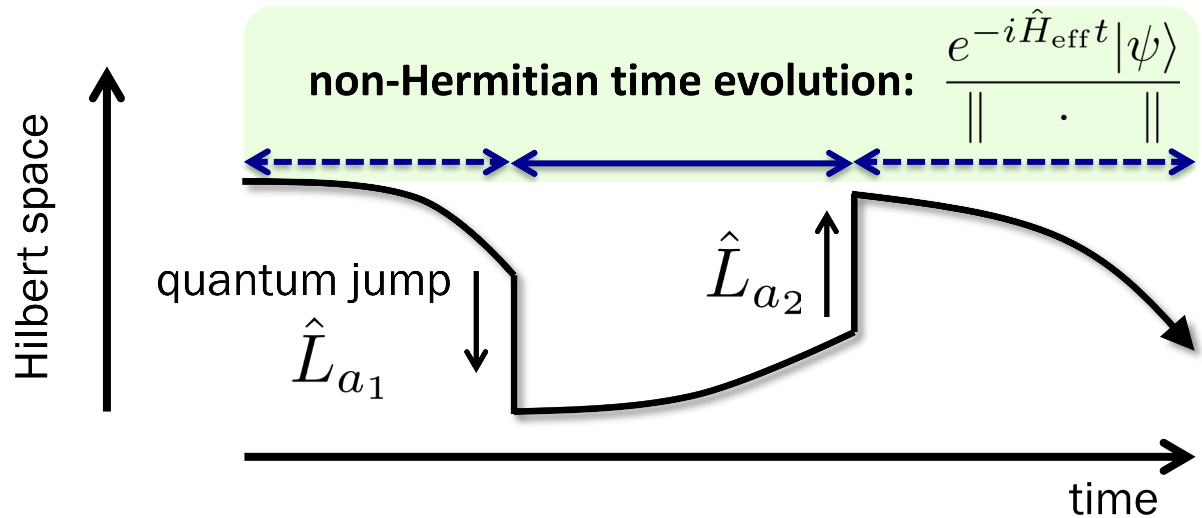
Stochastic time-evolution under continuous measurement

(norm preservation)

$$d|\psi\rangle = \underbrace{-i\hat{H}|\psi\rangle dt}_{\text{unitary evolution}} - \underbrace{\sum_a \frac{1}{2}\left(\hat{L}_a^\dagger \hat{L}_a - \langle \hat{L}_a^\dagger \hat{L}_a\rangle\right)|\psi\rangle dt}_{\text{non-unitary evolution without quantum jumps}} + \underbrace{\sum_a \left(\frac{\hat{L}_a|\psi\rangle}{\sqrt{\langle \hat{L}_a^\dagger \hat{L}_a\rangle}} - |\psi\rangle\right) dN_a}_{\text{stochastic quantum jumps}}$$

*Taking ensemble average E over dN_a, $\hat{\rho} = E[|\psi\rangle\langle\psi|]$ obeys Lindblad master eq.

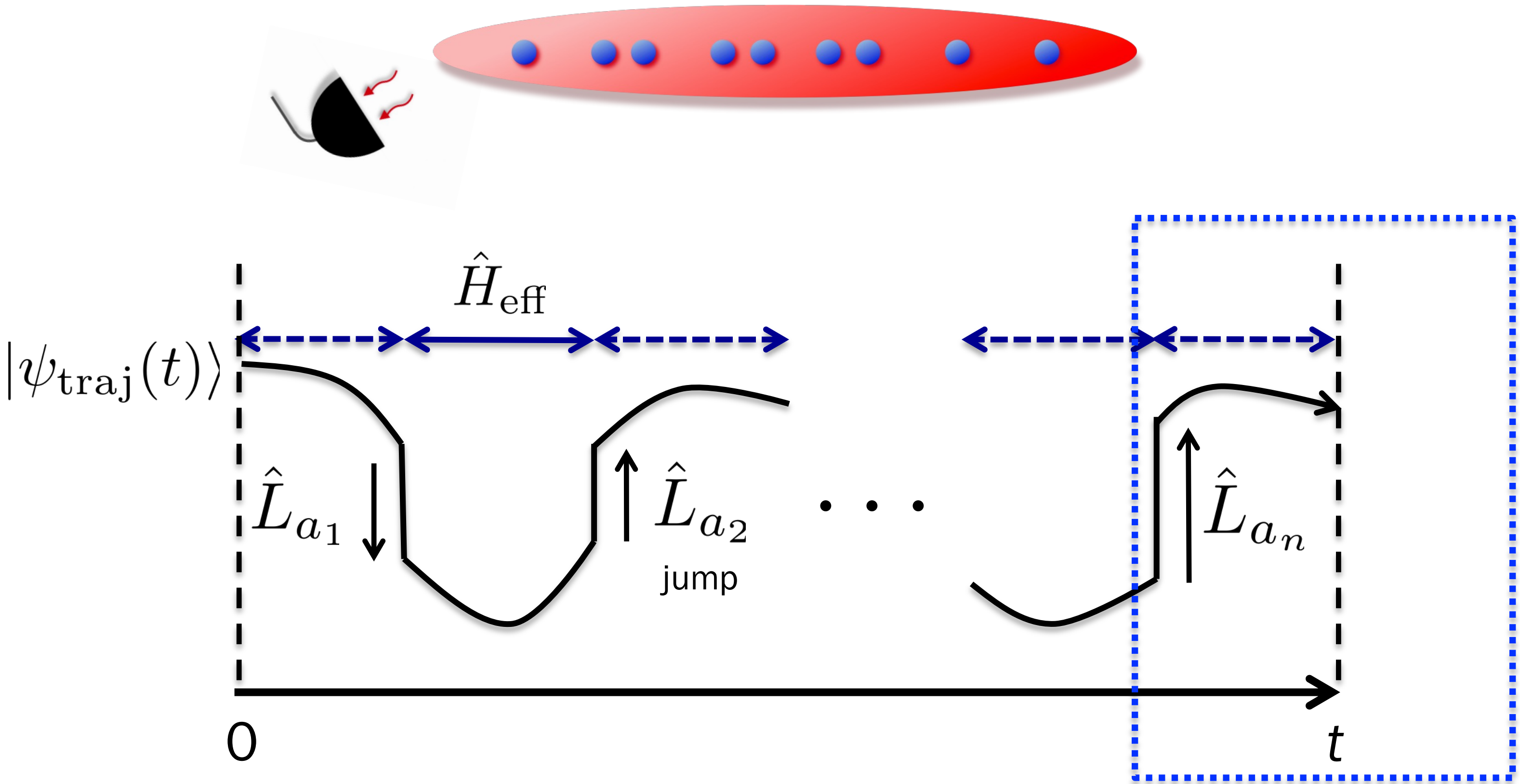
Reviews: Daley (2014); YA, Gong, Ueda (2020)



(effective non-Hermitian Hamiltonian)

$$\hat{H}_{\text{eff}} = \hat{H} - (i\gamma/2)\sum_m \hat{L}_m^\dagger \hat{L}_m$$

Entanglement transition induced by measurement



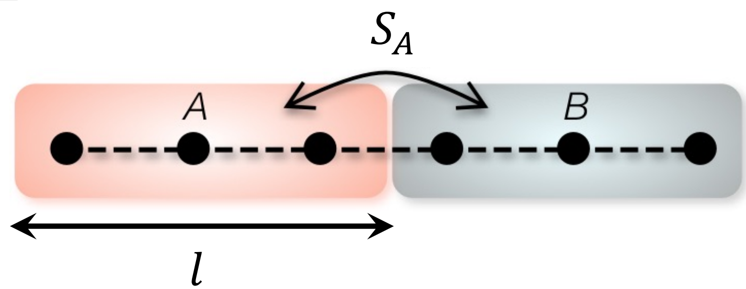
After transient dynamics, the system is expected to reach a pure *steady* state $|\psi_{\text{traj}}\rangle$.

How does the **entanglement** behave under continuous **measurements**?

Entanglement in quantum many-body systems

Entanglement: $\rho \neq \sum_i p_i \, \rho_i^A \otimes \rho_i^B$

1D pure state $|\psi\rangle$



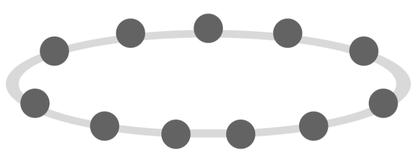
quantified by entanglement entropy $S_A = -\mathrm{tr}[\rho_A \ln \rho_A]$ $\rho_A = \mathrm{tr}_B[|\psi\rangle\langle\psi|]$

Example: universal scaling of many-body entanglement at criticality

Holzhey, Larsen, Wilczek 1994; Calabrese & Cardy 2004

1D critical state

periodic

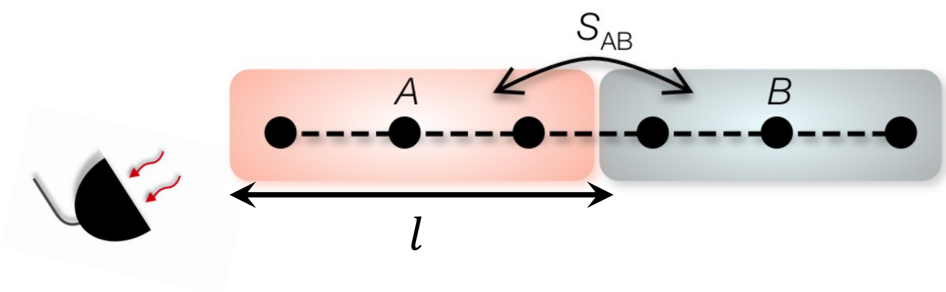


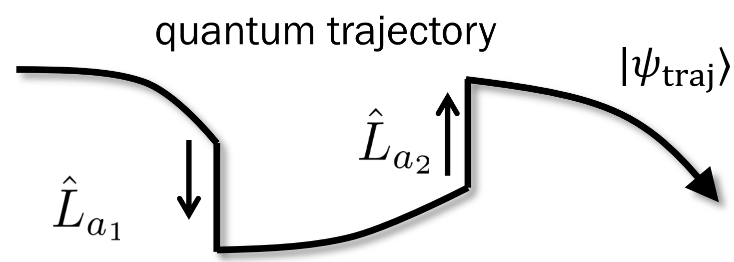
$$S_A = \frac{c}{3} \ln l + O(1)$$

l : size of region A

c : universal (central charge)

Entanglement transition induced by measurement



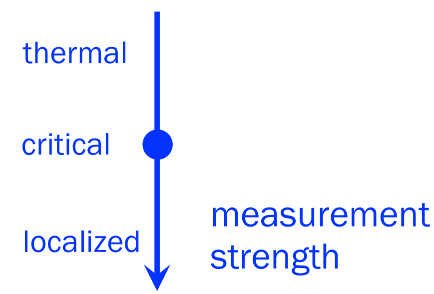


Entanglement entropy of $|\psi_{\mathrm{traj}}\rangle$: $S_{AB} = \mathrm{E}[-\mathrm{tr}[\rho_A \ln \rho_A]]$

averaged over trajectories

Measurement-induced transition and criticality

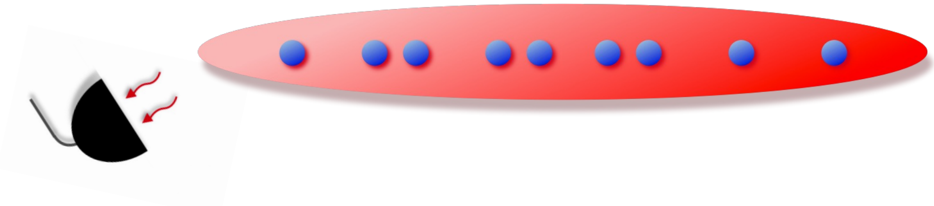
- Volume law $S_A \propto l$ thermal states
- Logarithmic scaling $S_A \propto \ln l$ gapless critical states
- Area law $S_A \propto \mathrm{const.}$ gapped states



Fuji and YA, PRB 102, 054302 (2020); review: Fisher et al., Ann. Rev. Cond. Matt. 14, 335 (2023)

More recent developments…

Euclidean path-integral formulation of **critical** states under **measurements**:



Weak measurements:
Garratt, Weinstein, Altman, PRX 13, 021026 (2023)
Murciano, Sala, Liu, Mong, Alicea, PRX 13, 041042 (2023)
Lee, Jian, Xu, PRXQ 4, 030317 (2023)
Yang, Mao, Jain, PRB 108, 165120 (2023)
Sala, Murciano, Liu, Alicea, PRXQ 5, 030307 (2024)
Patil and Ludwig, arXiv:2409.02107
…

Strong / Projective measurements:
Weinstein, Sajith, Altman, Garratt, PRB 107, 245132 (2023)
Zou, Sang, Hsieh, PRL 130, 250403 (2023)
YA, Furukawa, Oshikawa, PRB 110, 094404 (2024)
Liu, Murciano, Mross, Alicea, PRR 7, 023293 (2025)
Hoshino, Oshikawa, YA, PRB (2025)
Popov and Wang, arXiv:2504.06203
…

local measurement ≃ imaginary-time evolution ≃ Euclid boundary action

Ex) z-axis meas at site i with outcome m_i

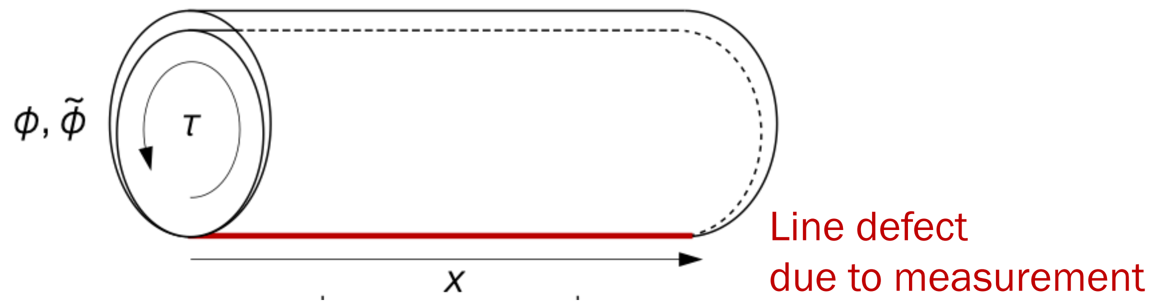
$$|\psi\rangle \to \frac{\prod_i K_i^{m_i}|\psi\rangle}{\|\prod_i K_i^{m_i}|\psi\rangle\|} \qquad K_i^{m_i} = \frac{\exp(\tau\, m_i Z_i)}{\sqrt{\cosh 2\tau}}$$

Kraus operator

($\tau \to \infty$ projective meas)

More recent developments…

Euclidean path-integral formulation of critical states under measurements:



local measurement $\simeq$ imaginary-time evolution $\simeq$ **Euclid boundary action**

Ex) z-axis meas at site i with outcome m_i

$$|\psi\rangle \to \frac{\prod_i K_i^{m_i} |\psi\rangle}{\| \prod_i K_i^{m_i} |\psi\rangle \|} \qquad K_i^{m_i} = \frac{\exp(\tau\, m_i Z_i)}{\sqrt{\cosh 2\tau}} \qquad \prod_i K_i^{m_i} \propto e^{\tau \sum_i m_i Z_i} \sim e^{\mathcal{S}[\phi]}$$

Kraus operator

($\tau \to \infty$ projective meas)

Measurement $\simeq$ Line defect

System-environment entanglement in open many-body systems

Open-system dynamics $\Leftrightarrow$ CPTP map $\mathcal{E}$

$$\hat{\rho}_{\mathcal{E}} = \mathcal{E}(\hat{\rho}_S)$$

YA, Furukawa, Oshikawa, PRB 110, 094404 (2024)

System-environment entanglement in open many-body systems

Open-system dynamics ⇔ CPTP map $\mathcal{E}$ ⇔ Stinespring representation

$$\hat{\rho}_{\mathcal{E}} = \mathcal{E}(\hat{\rho}_S) = \mathrm{tr}_E[\hat{U}(\hat{\rho}_S \otimes \underbrace{\hat{\rho}_E}_{\text{pure}})\hat{U}^\dagger]$$

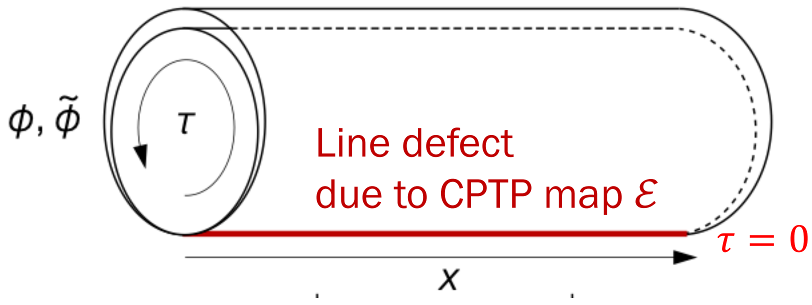
$\rightarrow \rho_{\mathcal{E}}$ is a reduced density matrix on S obtained from a pure state on $S + E$.

Entanglement *between* S and *E* can be quantified by Renyi entropy of $\rho_{\mathcal{E}}$:

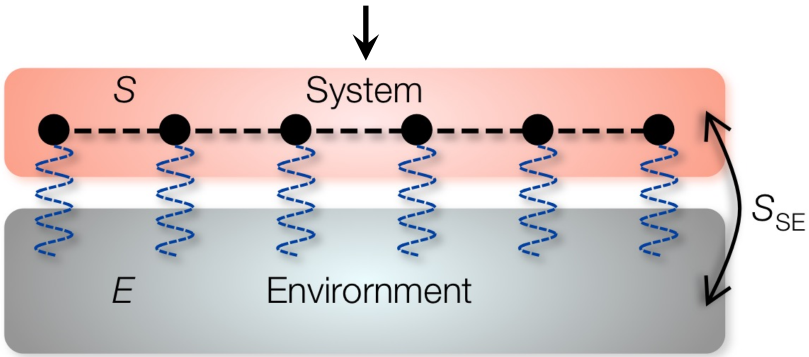
$$S_{SE}^{(n)} = \frac{1}{1-n} \ln \mathrm{tr}_S\,[\rho_{\mathcal{E}}^n]$$

Universal behavior in S_{SE} ?

Path-integral rep of the replica theory



Tomonaga-Luttinger liquid (TLL) = free-boson CFT

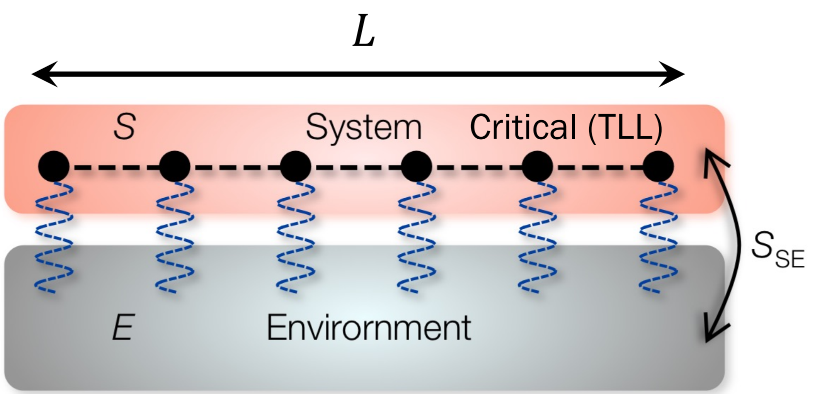


YA, Furukawa, Oshikawa, PRB 110, 094404 (2024)

Universality and transition in system-environment entanglement

Scaling of system-environment entanglement S_{SE} :

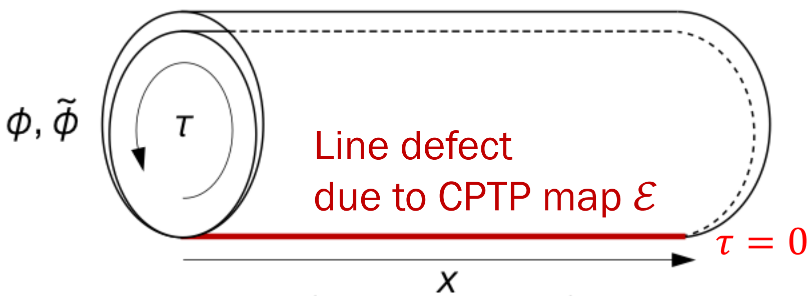
$$S_{SE} = s_1 L - \boxed{s_0} + \mathrm{o}(1)$$



s_1 : nonuniversal coefficient (depending on UV cutoff)

s_0 : universal term (Affleck-Ludwig boundary entropy)

$s_0 \neq 0 \Leftrightarrow$ CPTP map $\mathcal{E}$ is relevant to low-energy properties.



YA, Furukawa, Oshikawa, PRB 110, 094404 (2024)

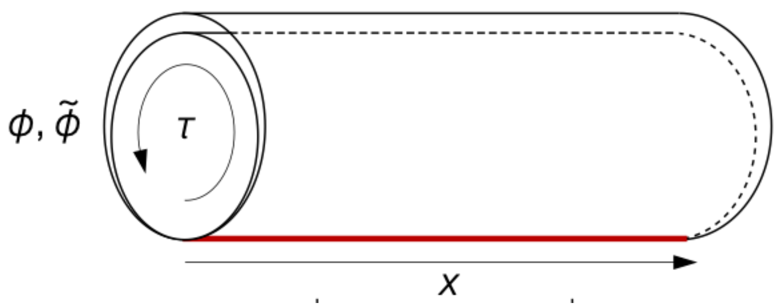
Quantum magic in many-body systems

✓ these theoretical methods are directly relevant to the analysis of quantum computational resources (a.k.a. magic).

✓ the common universal behavior:

$$S_{SE} = s_1 L - \boxed{s_0} + \mathrm{o}(1) \quad \text{sys-env entanglement}$$

$$M_\alpha = m_\alpha L - \boxed{c_\alpha} + o(1) \quad \text{magic}$$



Hoshino, Oshikawa, YA, PRX (2026)

Hoshino and YA, PRL (2026)

Matsuda, Hoshino, YA, arXiv 2607.05343 (2026)

Quantum magic : resource for universal quantum computation

Universal gate set = Clifford gates + a non-Clifford gate (e.g., T gate)

Quantum magic : resource for universal quantum computation

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Clifford group

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

- transversal fault-tolerant implementation
- classical simulable[1] $\rightarrow$ no quantum advantage

Stabilizer state:

$$|\mathrm{STAB}\rangle = \mathrm{Clifford}\ |0\rangle^{\otimes L}$$

[1]Gottesman and Knill 1998

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T gate

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

- non-transversal[2]

→ difficult to implement in a fault-tolerant manner

Nonstabilizer (magic) state:

$$|\mathrm{magic}\rangle \neq \mathrm{Clifford}\ |0\rangle^{\otimes L}$$

Clifford operations + Magic state = Universal quantum computation[3]

Nonstabilizerness (magic)
= resource for universal quantum computation

[1]Gottesman and Knill 1998 [2]Eastin and Knill 2009 [3]Bravyi and Kitaev 2005

Stabilizer Rényi entropy : computatable measure of nonstabilizerness (magic)

Stabilizer Rényi entropy (SRE) :

$$M_\alpha(\psi) = \frac{1}{1-\alpha} \ln \sum_{\vec{m}} \frac{\mathrm{Tr}^{2\alpha}[\sigma^{\vec{m}}\psi]}{2^L}$$

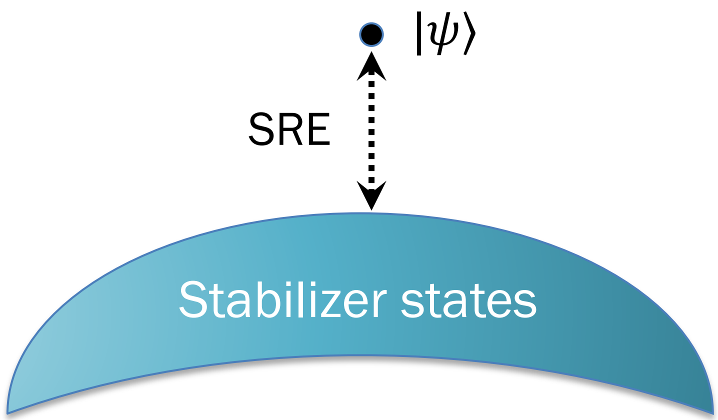
$\psi = |\psi\rangle\langle\psi|$: L-qubit pure state

$\sigma^{\vec{m}} = \sigma^{m_1}\sigma^{m_2}\cdots\sigma^{m_L}$: Pauli string

$$\sigma^{00} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma^{01} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^{10} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^{11} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Leone, Oliviero, Hamma, PRL 2022



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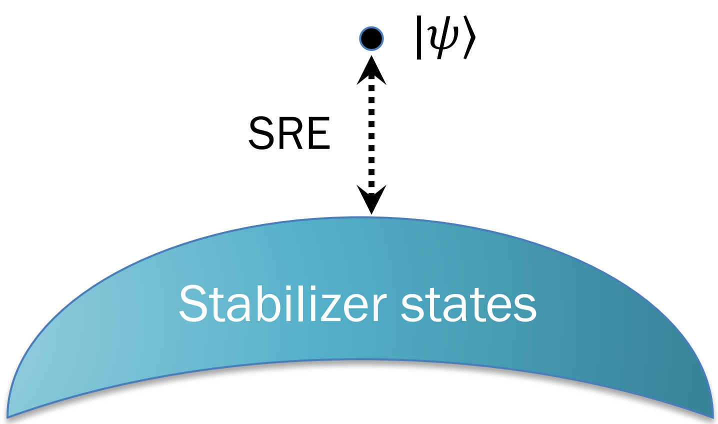
1. Faithful : $M_\alpha(\psi) = 0 \iff$ stabilizer state $|\psi\rangle =$ **Clifford** $|0\rangle^{\otimes L}$
2. Additive : $M_\alpha(\psi \otimes \phi) = M_\alpha(\psi) + M_\alpha(\phi)$
3. Monotonic : nonincreasing under stabilizer protocol ($\alpha \geq 2$)
4. Efficiently computable by MPS

Leone, Oliviero, Hamma, PRL 2022

Haug, Lee, Kim PRL 2024



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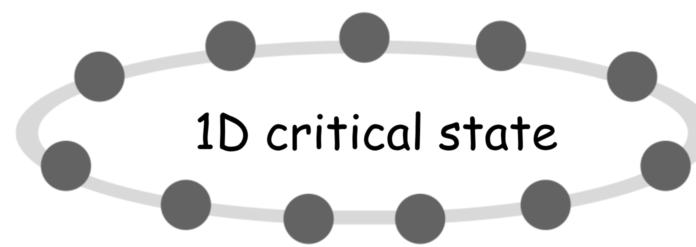
Leone, Oliviero, Hamma, PRL 2022

Haug, Lee, Kim PRL 2024

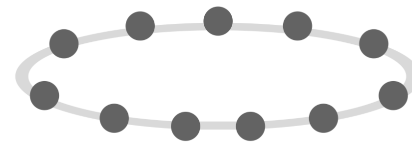
Universality in many-body quantum magic?

So far, many numerical studies exist,
but no theoretical understanding

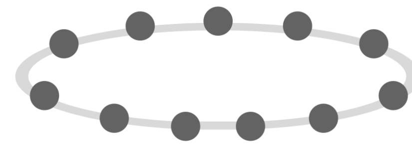


Tarabunga, Tirrito, Chanda, Dalmonte, PRXQ 4, 040317 (2023)
Oliviero, Leone, Hamma, PRA 106, 042426 (2022)
Piemontese, Roscilde, Hamma, PRB 107, 134202 (2023)
Tarabunga and Castelnovo, Quantum 8, 1347 (2024)
Sierant, Stornati, Turkeshi, arXiv:2506.00116
…

Universal behavior of entanglement and magic in 1D critical states

state / resource	1D critical state on periodic chain 
Entanglement (entanglement entropy) S_A	$S_A = \frac{c}{3}\ln l + O(1)$ l : size of region A c : universal (central charge) Holzhey, Larsen, Wilczek PRL 1994 Calabrese & Cardy J. Stat. 2004
Magic (stabilizer entropy) M	?

Universal behavior of entanglement and magic in 1D critical states

resource \ state	**1D critical state on periodic chain** 
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Magic (stabilizer entropy) M	$$M = mL - s + o(1)$$ L : total system size m : nonuniversal $s = \ln g$: universal (g function) Hoshino, Oshikawa, YA PRX 2026 Hoshino & YA PRL 2026 Matsuda, Hoshino, YA arXiv2607.05343

Link to measurements: SRE as participation entropy in the doubled Hilbert space

SRE = participation entropy of a doubled state in the Bell basis

projection onto the product of Bell states

$$M_\alpha(\psi) = \frac{1}{1-\alpha} \ln \sum_{\vec{m}} \mathrm{Tr}^\alpha [P^{\vec{m}} (\psi \otimes \psi^*)] - (\ln 2) L \qquad \psi = |\psi\rangle\langle\psi|$$

See also: Haug and Kim, PRXQ 4, 010301 (2023)

Link to measurements: SRE as participation entropy in the doubled Hilbert space

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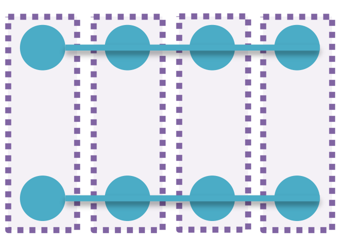
$p_{\vec{m}} = |\langle \vec{m} | \Psi \rangle|^2$: Born probability of observing the Bell-meas outcomes $\vec{m}$

$|\Psi\rangle = |\psi\rangle|\psi^*\rangle$



$|\psi\rangle$

$\otimes$

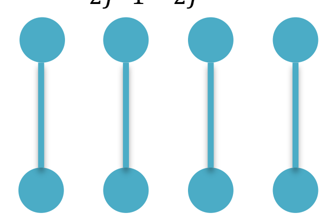
$|\psi^*\rangle$

Bell measurements

$P^{\vec{m}}$

$|\vec{m}\rangle = \otimes_{j=1}^{L} |\mathrm{Bell}^{m_{2j-1} m_{2j}}\rangle$

$m_{2j-1} m_{2j}$



$m_{2j-1} m_{2j} \in \{00, 01, 10, 11\}$

$$|\mathrm{Bell}^{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \quad |\mathrm{Bell}^{01}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\mathrm{Bell}^{10}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, \quad |\mathrm{Bell}^{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

See also: Haug and Kim, PRXQ 4, 010301 (2023)

Path-integral representation of the SRE under PBC

Many-body magic ~ many-body states under Bell measurements

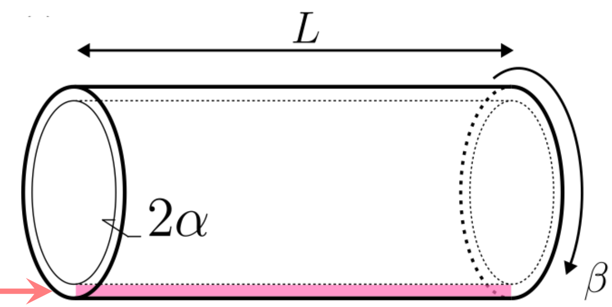
Rewrite M_α by the replica method:

$$M_\alpha(\psi) = \frac{1}{1-\alpha} \ln \sum_{\vec{m}} \mathrm{Tr}^\alpha [P^{\vec{m}}(\psi \otimes \psi^*)] - (\ln 2)L$$

$$= \frac{1}{1-\alpha} \ln \mathrm{Tr} \left[\sum_{\vec{m}} (P^{\vec{m}})^{\otimes \alpha} (\psi \otimes \psi^*)^{\otimes \alpha} \right] - (\ln 2)L$$

projection measurement
= boundary action $e^{\mathcal{S}[\phi]}$ ⟶ line defect

critical state under PBC
= (1+1)d CFT on torus



Folding

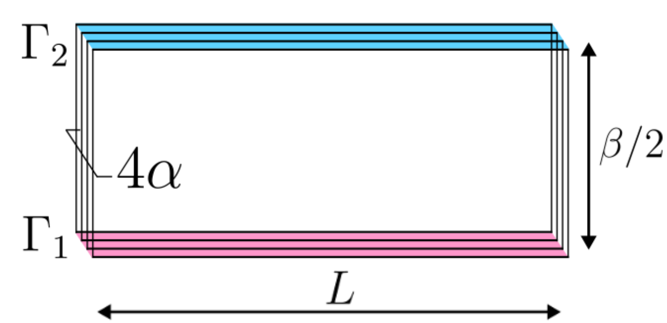
After folding, partition function can be described as the transition amplitude between boundary states $\Gamma_{1,2}$:

$$Z_{2\alpha} = \langle \Gamma_2 | e^{-\frac{\beta}{2} H} | \Gamma_1 \rangle$$

boundary state at the artificial boundary

boundary state at the line defect

cylinder with boundaries $\Gamma_{1,2}$



Boundary CFT analysis of the SRE under PBC

zero temp. limit
$(\beta \gg L)$

$$Z_{2\alpha} = \langle \Gamma_2 | e^{-\frac{\beta}{2}H} | \Gamma_1 \rangle \simeq \underbrace{\langle \Gamma_1 | \mathrm{GS} \rangle \langle \mathrm{GS} | \Gamma_2 \rangle}_{g \text{ function (universal quantity)}} e^{-\frac{\beta}{2} E_{\mathrm{GS}}(L)}$$

$g_2 = \langle \mathrm{GS} | \Gamma_2 \rangle = 1$ ∵) artificial boundary

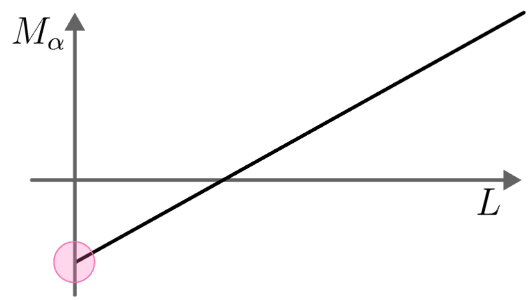
$g_1 = \langle \mathrm{GS} | \Gamma_1 \rangle$ → (in general) noninteger value

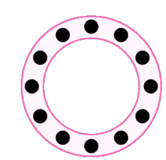
Long-distance behavior of the SRE :

$$M_\alpha(\psi) = \frac{1}{1-\alpha} \ln \frac{Z_{2\alpha}}{Z^{2\alpha}} - (\ln 2)L = m_\alpha L - c_\alpha + o(1)$$

$$c_\alpha = \frac{\ln g_1}{\alpha - 1}$$

In magic, the size-independent term is universal and governed by the g-function





<u>Fermionic/Bosonic Magic</u> in many-body systems

Non-Gaussianity

Matsuda, Hoshino, YA, arXiv 2607.05343 (2026)

Matsuda, Hoshino, YA, in prep.

Non-Gaussianity / quantum magic : resource for universal quantum computation

Universal gate set = Gaussian gates + a non-Gaussian gate (e.g., SWAP gate)

Non-Gaussianity / quantum magic : resource for universal quantum computation

Universal gate set = Gaussian gates + a non-Gaussian gate (e.g., SWAP gate)

Gaussian unitary

$$U = \exp\left(\sum_{nm} A_{nm}\gamma_n\gamma_m\right) \qquad A_{nm} = -A_{mn} \qquad \{\gamma_n, \gamma_m\} = 2\delta_{nm}$$

linear $\quad U^\dagger \boldsymbol{\gamma} U = e^A \boldsymbol{\gamma} \qquad e^A \in SO(2L)$

Gaussian state:

$$|\mathrm{Gauss}\rangle = U\,|0\rangle^{\otimes L}$$

- classical simulable[1] → no quantum advantage

[1] Jozsa and Miyake 2008

Non-Gaussianity / quantum magic : resource for universal quantum computation

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SWAP gate

$$\mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- non-Gaussian operation

Non-Gaussian (magic) state:

$$|\mathrm{magic}\rangle \neq U\,|0\rangle^{\otimes L}$$

J-W transformation :

$$\gamma_{2i-1} = c_i + c_i^\dagger = Z_1 Z_2 \cdots Z_{i-1} X_i$$
$$\gamma_{2i} = i(c_i^\dagger - c_i) = Z_1 Z_2 \cdots Z_{i-1} Y_i$$

nonlinear (=non-Gaussian)

$$\mathrm{SWAP}^\dagger \gamma_3 \mathrm{SWAP} = \mathrm{SWAP}^\dagger Z_1 X_2 \mathrm{SWAP} = X_1 Z_2 = -i\gamma_1\gamma_3\gamma_4$$

[1]Jozsa and Miyake 2008

Non-Gaussianity / quantum magic : resource for universal quantum computation

Universal gate set = Gaussian gates + a non-Gaussian gate (e.g., SWAP gate)

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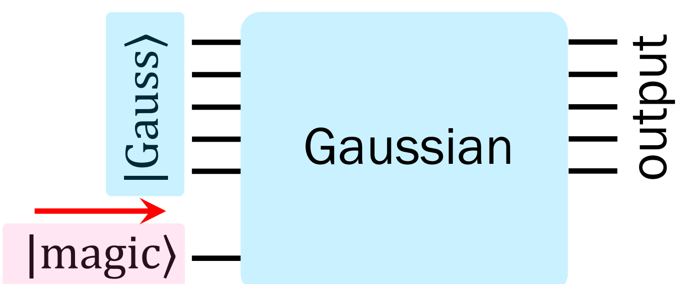
Gaussian operations + Magic state = Universal quantum computation[2]



Non-Gaussianity (a.k.a magic)
= resource for universal quantum computation

[1]Jozsa and Miyake 2008 [2]Hebenstreit et al. 2019

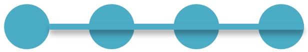
How to quantify non-Gaussianity/magic ?

Convolution

Boson:	Cushen and Hudson, 1971
Fermion:	Hudson, 1973
Extensions:	Bu, Gu, Jaffe, 2025
	Lyu & Bu, 2025
	Coffman et al., 2025
	Hahn & Takagi, 2025
	Matsuda, Hoshino, YA, 2026
	...

How to quantify non-Gaussianity/magic ?

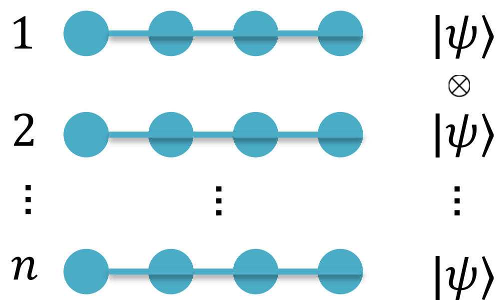
Convolution



$|\psi\rangle$

How to quantify non-Gaussianity/magic ?

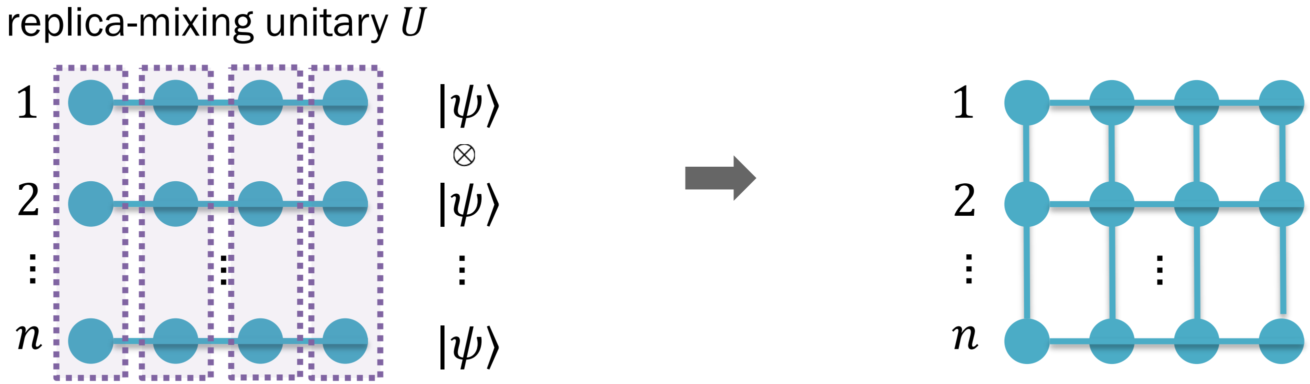
Convolution



1. Prepare n copies $|\psi\rangle^{\otimes n}$

How to quantify non-Gaussianity/magic ?

Convolution



1. Prepare n copies $|\psi\rangle^{\otimes n}$

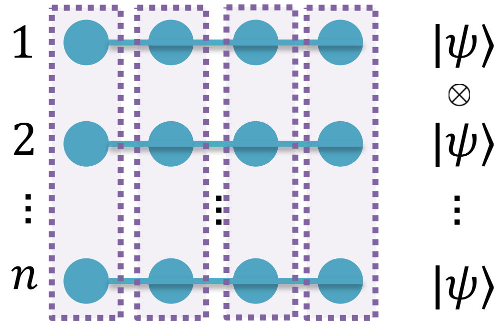
2. Perform the replica-mixing unitary U

Matsuda, Hoshino, YA, 2026.

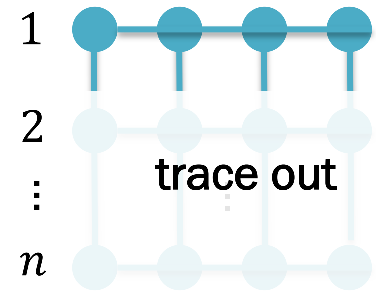
How to quantify non-Gaussianity/magic ?

Convolution

replica-mixing unitary U



$$\mathcal{C}(\rho) \equiv \mathrm{tr}_{2,3,\ldots,n}[U\rho^{\otimes n}U^{\dagger}]$$



$\rho = |\psi\rangle\langle\psi|$

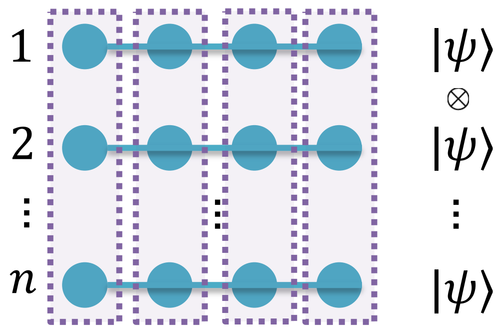
1. Prepare n copies $|\psi\rangle^{\otimes n}$
2. Perform the replica-mixing unitary U
3. Trace out the replicas 2,3, … , n.

Matsuda, Hoshino, YA, 2026.

How to quantify non-Gaussianity/magic ?

Convolution as a <u>non-Gaussianity measure</u>

replica-mixing unitary U



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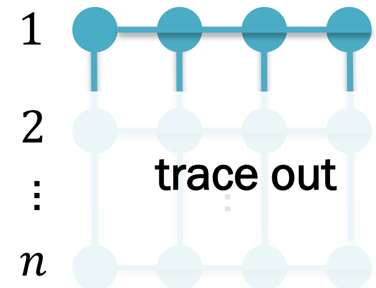


$$\rho = |\psi\rangle\langle\psi|$$

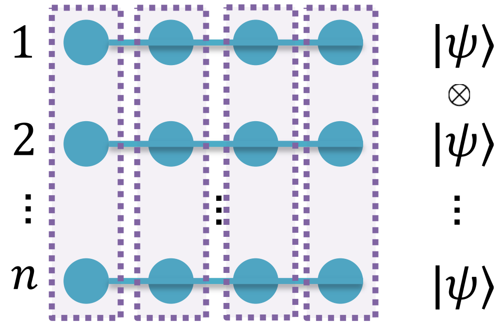
$\mathcal{C}(\rho)$ is pure $\leftrightarrow$ $|\psi\rangle$ is Gaussian

Matsuda, Hoshino, YA, 2026.

How to quantify non-Gaussianity/magic ?

Convolution as a non-Gaussianity measure

replica-mixing unitary U



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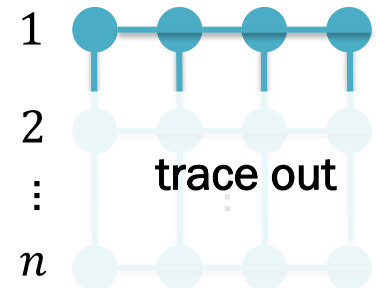


$$\rho = |\psi\rangle\langle\psi|$$

$\mathcal{C}(\rho)$ is mixed $\leftrightarrow$ $|\psi\rangle$ is non-Gaussian

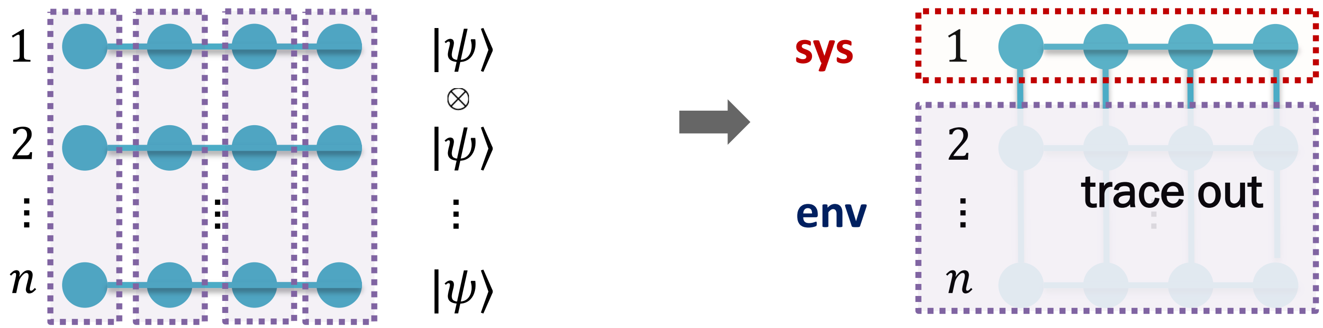
Matsuda, Hoshino, YA, 2026.

How to quantify non-Gaussianity/magic ?

Convolution as a <u>non-Gaussianity measure</u>

replica-mixing unitary U

$$\mathcal{C}(\rho) \equiv \mathrm{tr}_{2,3,\ldots,n}[U\rho^{\otimes n}U^\dagger]$$



$$\rho = |\psi\rangle\langle\psi|$$

$\mathcal{C}(\rho)$ is mixed $\leftrightarrow$ $|\psi\rangle$ is non-Gaussian

Convolution-based magic measure

$$M_n(\psi) = -\ln \mathrm{tr}_1[\mathcal{C}(\rho)^2]$$

1. Faithful : $M_n(\psi) = 0$ iff $|\psi\rangle$ is Gaussian
2. Additive : $M_n(\psi \otimes \phi) = M_n(\psi) + M_n(\phi)$
3. **Monotone** (nonincreasing under Gaussian protocols)
4. **Computable** (no optimization needed, efficient MPS sampling method)

Useful measure to quantify non-Gaussianity in many-body states !

Matsuda, Hoshino, YA, 2026.

Doubled-Hilbert space expression of non-Gaussian entropy

$$\mathcal{C}_n(\rho) = \mathrm{tr}_{2,3,\ldots,n}\left[U\,\rho^{\otimes n} U^\dagger\right]$$

Convolution of n replicas
= replica mixing by U + trace out $2,3,\ldots,n$

$$M_n(\rho) = \frac{1}{1-n}\ln \mathrm{tr}\left[\mathcal{C}_n(\rho)^2\right]$$

Non-Gaussian entropy
= purity of the output state $\mathcal{C}(\rho)$

Vectorize

$A \to |A\rangle\rangle$
$\mathrm{tr}[A] \to \langle\langle I|A\rangle\rangle$

Unnormalized enlarged "pure" state

$$|\mathcal{C}_n(\rho)\rangle\rangle = \langle\langle I|_{1^c}\, U \otimes \tilde{U}\, |\rho\rangle\rangle^{\otimes n}$$

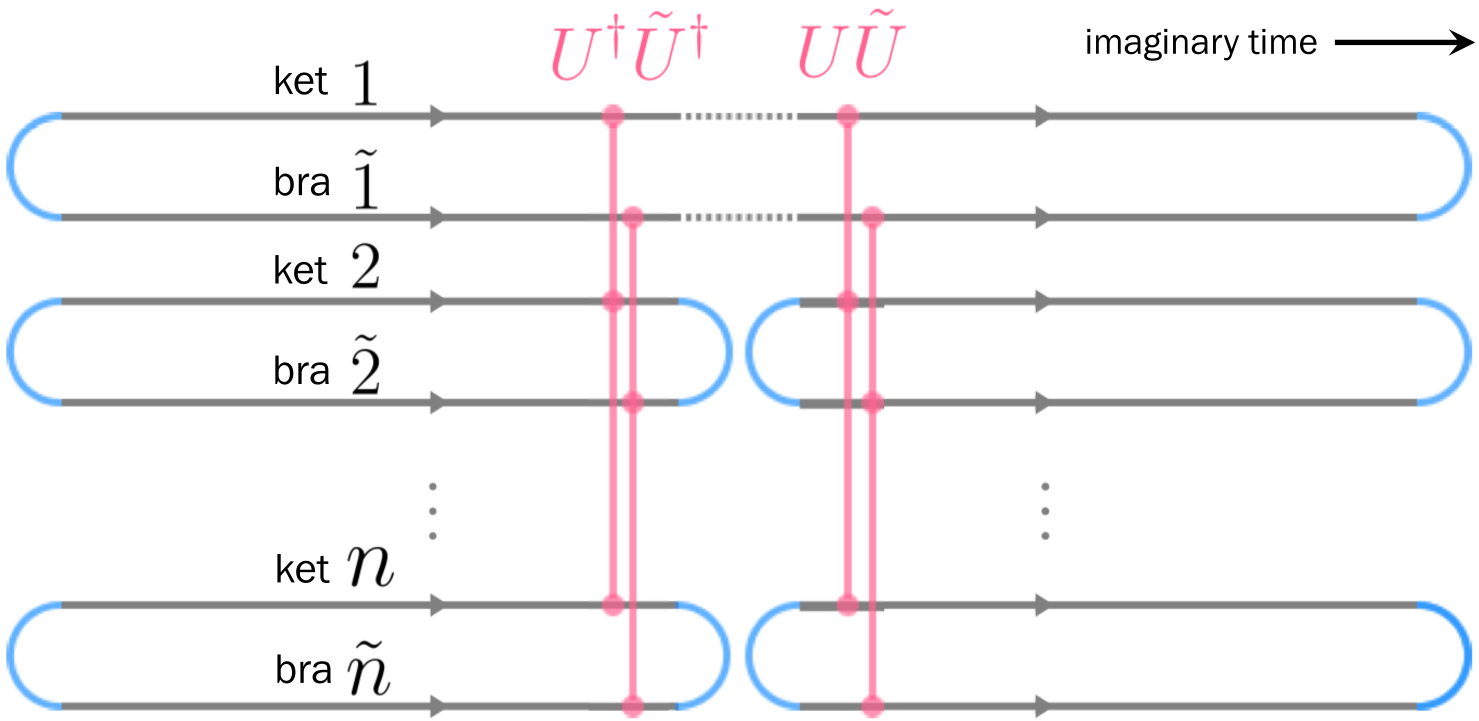
Maximally entangled states

$$\langle\langle I|_{1^c} \equiv \bigotimes_{r=2}^{n} \langle\langle I|_{r\tilde{r}}$$

$\rightarrow$ trace out the replicas $r = 2,3,\ldots,n$

$$M_n(\rho) = \frac{1}{1-n}\ln\langle\langle \mathcal{C}_n(\rho)|\mathcal{C}_n(\rho)\rangle\rangle$$

Path-integral representation of non-Gaussian entropy

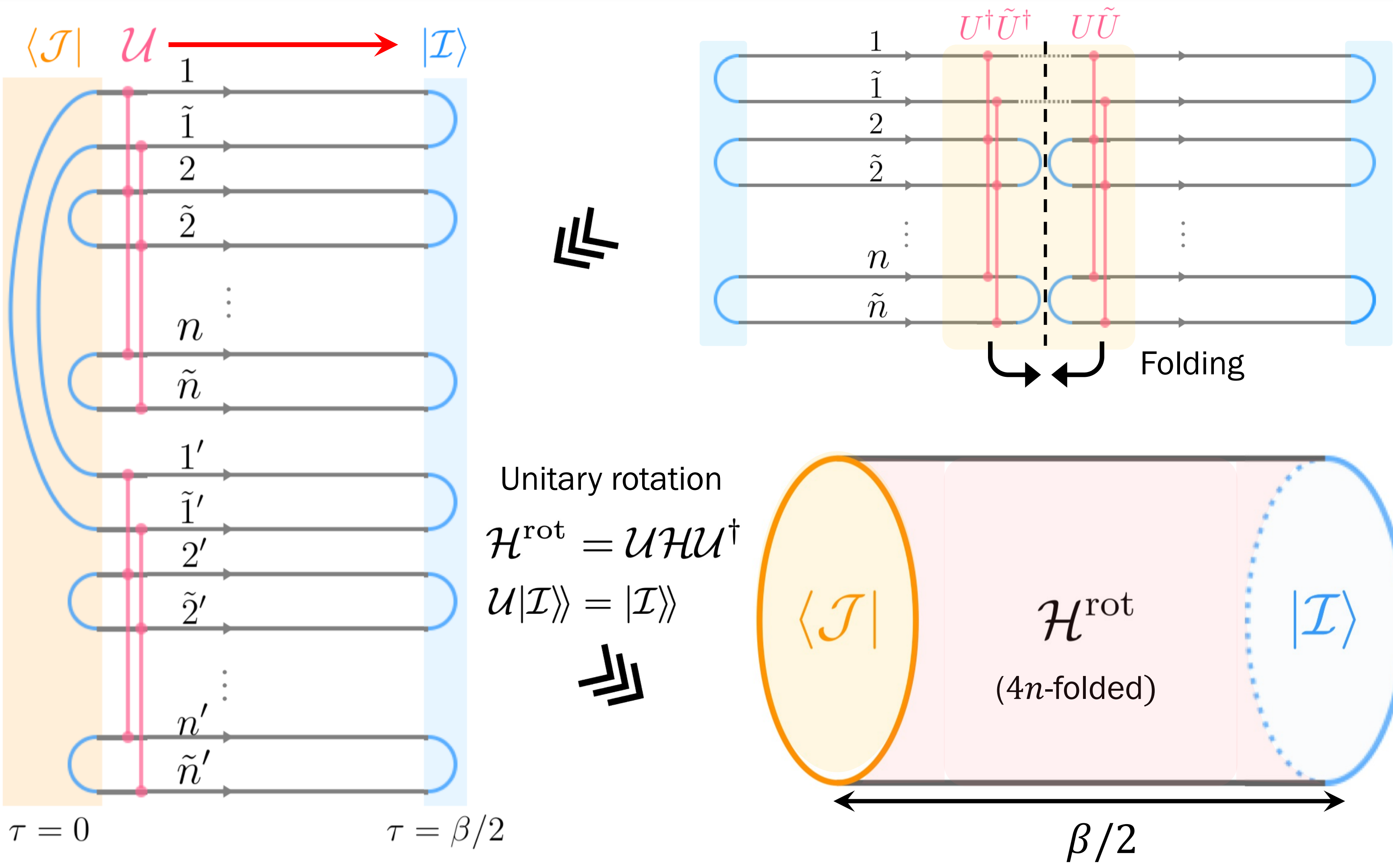


$$Z_n = \langle\!\langle \rho|^{\otimes n} (U \otimes \tilde{U})^\dagger \left(I_{1\tilde{1}} \otimes |I\rangle\!\rangle_{1^c} \langle\!\langle I_{1^c}| \right) (U \otimes \tilde{U}) |\rho\rangle\!\rangle^{\otimes n}$$

$$M_n(\rho) = \frac{1}{1-n} \ln \boxed{\langle\!\langle \mathcal{C}_n(\rho) | \mathcal{C}_n(\rho) \rangle\!\rangle}$$

$$|\mathcal{C}_n(\rho)\rangle\!\rangle = \langle\!\langle I|_{1^c}\, U \otimes \tilde{U}\, |\rho\rangle\!\rangle^{\otimes n}$$

Path-integral representation of non-Gaussian entropy



$$Z_n = \langle\langle\mathcal{J}|\mathcal{U}e^{-\frac{\beta}{2}\mathcal{H}}|\mathcal{I}\rangle\rangle$$

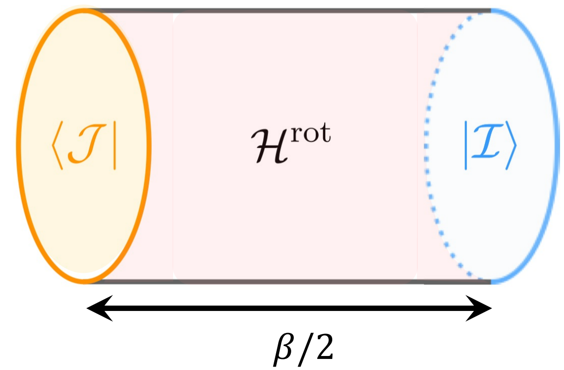
$$Z_n = \langle\langle\mathcal{J}|\, e^{-(\beta/2)\mathcal{H}^{\text{rot}}}\, |\mathcal{I}\rangle\rangle$$

Universal behavior of magic in 1D critical states

$$Z_n = \langle\langle \mathcal{J} | \, e^{-(\beta/2)\mathcal{H}^{\mathrm{rot}}} \, | \mathcal{I} \rangle\rangle$$

Trivial boundary (identity defect)

Nontrivial boundary



$\beta \gg L$

$$\simeq \underbrace{\langle\langle \mathcal{J} | \mathrm{CFT}^{\mathrm{rot}} \rangle\rangle \langle\langle \mathrm{CFT}^{\mathrm{rot}} | \mathcal{I} \rangle\rangle}_{g \text{ function (universal quantity)}} e^{-\frac{\beta}{2} E_0(L)}$$

$$g_{\mathcal{I}} = \langle\langle \mathcal{I} | \mathrm{CFT}^{\mathrm{rot}} \rangle\rangle = 1$$

$$g_{\mathcal{J}} = \langle\langle \mathcal{J} | \mathrm{CFT}^{\mathrm{rot}} \rangle\rangle$$

$$M_n = \frac{1}{1-n} \ln \frac{Z_{2n}}{Z^{2n}} = m_n L - \boxed{s_n} + o(1) \qquad s_n = \frac{\ln \boxed{g_{\mathcal{J}}}}{n-1}$$

In non-Gaussianity, the size-independent term is universal and governed by the g-function.

Universal behavior of non-Gaussianity in the 1D integrable model

1D critical fermion chain with nearest-neighbor interactions

$$H = -t\sum_i (a_i^\dagger a_{i+1} + \mathrm{H.c.}) + V\sum_i n_i n_{i+1}$$

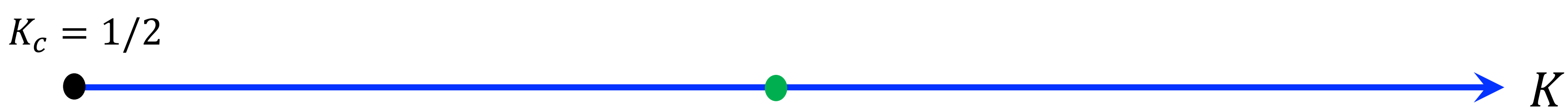
gapless

$|V| < 2t$

bosonization $a_{R/L}(x) \sim e^{i\theta(x) \mp i\phi(x)}$

Low-energy theory: Tomonaga-Luttinger liquid (TLL, $c = 1$ CFT)

$$H \simeq \frac{v}{2\pi}\int dx \left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right]$$



repulsive ($V > 0$)
$\frac{1}{2} < K < 1$

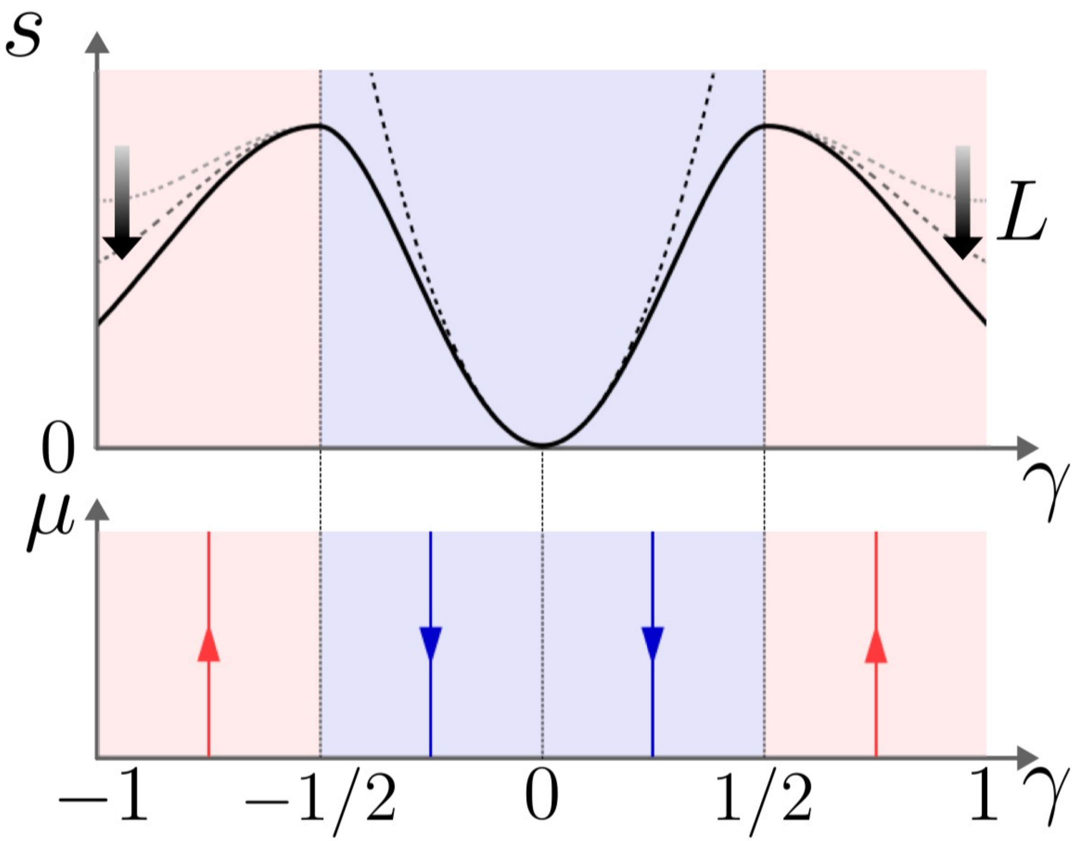
free fermion ($V = 0$)
$K = 1$

attractive ($V < 0$)
$K > 1$

Universal behavior of fermionic magic in the 1D interacting model

Non-Gaussian entropy

$$M = mL - \boxed{s}$$



$$\gamma \equiv \frac{K-1}{K+1}$$

Non-Gaussianity

Summary of CFT results

- ✓ Duality $s(\gamma) = s(-\gamma)$
- ✓ Perturbative formula

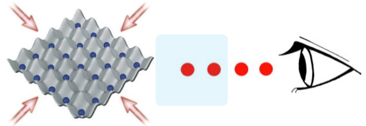
$$s = \frac{3}{2}\gamma^2 + O(\gamma^4)$$

- ✓ Transitions at $K_c = \frac{1}{3}, 3$
- ✓ Monotonic decrease of s as L increases above the thresholds.
  (g theorem)

Matsuda, Hoshino, YA, arXiv 2607.05343

Summary: Many-body states under measurements

- **Measurement-induced phenomena**



Entanglement phase transitions induced by measurements
System-environment entanglement transitions

$$S_{SE} = s_1 L - s_0 + \mathrm{o}(1)$$

Review: YA, Gong, Ueda, Adv. Phys. 69, 3 (2020)

Refs: Fuji & YA PRB (2020); YA, Furukawa, Oshikawa PRB (2024) …

- **Many-body magic: resources for quantum computation**

$$M_\alpha = m_\alpha L - c_\alpha + o(1)$$

Universal behavior and CFT calculations

Refs: Hoshino, Oshikawa, YA PRX (2026); Hoshino, YA PRL (2026)
Matsuda, Hoshino, YA, arXiv 2607.05343 (2026)