Impossibility of detecting quantum nature of gravitational waves

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in collaboration with Y. Miyauchi, H. Omiya, A. Ota, H. Takeda.

See also, Y. Bao, D.Y. Cheong, N.L. Rodd, J. Takach, L.-T. Wang and K. Zhou, ”Suppressed QuantumEffects of Weakly Coupled Waves”, 2607.27313.

Is Gravity quantum?

There is no obvious reason to avoid quantization of gravity as it couples to quantum matter.

Nevertheless, there are several proposals:

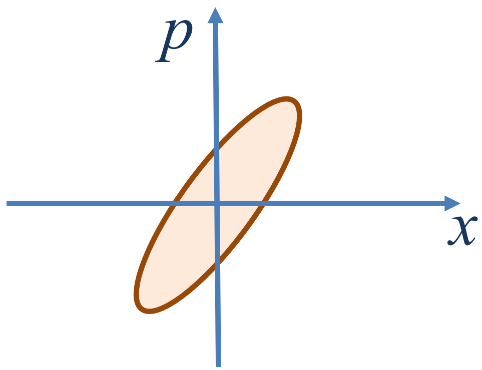
Stochastic gravity
L. Diósi (1984, 1989)-Penrose (1996, 2014)
J. Oppenheim (2018–2023)

There are many proposals to prove the quantumness of gravity:

Tobar et al. (Nature Communications(2024))
"Detecting single gravitons with quantum sensing"
Ikeda et al. (2507.01609)
"Toward graviton detection via photon–graviton quantum conversion"
Kanno–Soda–Tokuda (2103.17053)
"Indirect detection of gravitons through quantum entanglement"

Our main focus is on the question whether one can use a large squeezing of graviton to prove the quantumness.

Squeezing and Witness



$$\begin{pmatrix} M & N \\ N & M^* \end{pmatrix} = \left\langle \begin{pmatrix} a^2 & a^\dagger a \\ a^\dagger a & a^{\dagger 2} \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} x^2 & \frac{xp+px}{2} \\ \frac{xp+px}{2} & p^2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2N+1+M+M^* & i\left(M-M^*\right) \\ i\left(M-M^*\right) & 2N+1-\left(M+M^*\right) \end{pmatrix}$$

Eigenvalues

$$\lambda = \lambda_\pm \equiv N + \frac{1}{2} \pm |M|$$

In the case of squeezed state, the witness defined by

$$W \equiv \frac{1}{2} - \lambda_- = |M| - N$$

is positive → Width in the narrowest direction is less than 1/2.

But the definition changes under the canonical transformation.

Squeezing generated during inflation

Inflation is the accelerated expansion phase assumed to solve the problem of homogeneity, flatness and so on.

Inflation predicts the initial density perturbation as the quantum vacuum fluctuation.

We focus on GWs here, but the mechanism for the generation of density perturbation is the same.

$$h_{ij} = \sum_{\boldsymbol{k},\lambda} \tilde{u}_{\boldsymbol{k}} e_{ij}^{\lambda} \tilde{a}_{\lambda,\boldsymbol{k}} + (h.c.) \qquad \tilde{a}_{\lambda,\boldsymbol{k}} \left|0\right\rangle = 0$$

Solving the equation for the mode function on the inflationary background, at a later epoch, we obtain

$$\tilde{u}_{\boldsymbol{k}} \approx \mathcal{A}\left(e^{-i\omega t} + e^{+i\omega t}\right) e^{ikx} = \cosh r\, u_{\boldsymbol{k}} + e^{i\phi} \sinh r\, u_{\boldsymbol{k}}^{\dagger}$$

dominance of the growing mode

Natural positive frequency function after inflation

We describe the quantum state after inflation by using the natural positive frequency function after inflation.

$$h \approx \int d^3k\, e^{-i\omega t} e^{i\boldsymbol{k}\boldsymbol{x}} a_{\boldsymbol{k}} + h.c. = \int d^3k\, \tilde{u}_{\boldsymbol{k}} \tilde{a}_{\boldsymbol{k}} + h.c.$$

$$a_{\boldsymbol{k}} |0\rangle = 0 \qquad \tilde{a}_{\boldsymbol{k}} = \cosh r\, a_{\boldsymbol{k}} - e^{i\phi} \sinh r\, a^{\dagger}_{-\boldsymbol{k}}$$

The squeezed state after inflation

$$|\tilde{0}\rangle = \prod_{\boldsymbol{k}} \exp\left[\zeta_{\boldsymbol{k}} a^{\dagger}_{\boldsymbol{k}} a^{\dagger}_{-\boldsymbol{k}} - h.c.\right] |0\rangle \qquad \zeta_{\boldsymbol{k}} = e^{i\phi} r$$

$$a_{\boldsymbol{k}} \exp\left(\zeta_{\boldsymbol{k}} a^{\dagger}_{\boldsymbol{k}} a^{\dagger}_{-\boldsymbol{k}} - h.c.\right) = \exp\left(\zeta_{\boldsymbol{k}} a^{\dagger}_{\boldsymbol{k}} a^{\dagger}_{-\boldsymbol{k}} - h.c.\right)\left(\cosh r\, a_{\boldsymbol{k}} + e^{i\phi} \sinh r\, a^{\dagger}_{-\boldsymbol{k}}\right)$$

$$\tilde{u}_{\boldsymbol{p},\boldsymbol{y}} = \int d^3k\, f_{\boldsymbol{p},\boldsymbol{y}}(\boldsymbol{k}) \tilde{u}_{\boldsymbol{k}}$$

Wave packet of gravitational waves.

Positive frequency function prepared by the inflation

$$\tilde{u}_{\boldsymbol{k}} \approx \mathcal{A}\left(e^{-i\omega t} + e^{+i\omega t}\right) e^{i\boldsymbol{k}\boldsymbol{x}}$$

$$|\tilde{0}\rangle \approx \prod_{\boldsymbol{p},\boldsymbol{y}} \exp\left[\zeta_{\boldsymbol{p}} a^{\dagger}_{\boldsymbol{p},\boldsymbol{y}} a^{\dagger}_{-\boldsymbol{p},\boldsymbol{y}} - h.c.\right] |0\rangle$$

Two-mode squeezing between a left-moving mode and a right-moving mode.

$$\left|\tilde{0}\right\rangle = \prod_{p,y} \exp\left[\zeta_p a^\dagger_{p,y} a^\dagger_{-p,y} - h.c.\right]\left|0\right\rangle \approx \prod_{p,y} \exp\left[\zeta_p a^\dagger_{p,y} a^\dagger_{-p,y} - h.c.\right]\left|0\right\rangle$$

Just a unitary transformation

$$a_{p,y} \exp\left(\zeta_p a^\dagger_{p,y} a^\dagger_{-p,y} - h.c.\right) = \exp\left(\zeta_p a^\dagger_{p,y} a^\dagger_{-p,y} - h.c.\right)\left(\cosh r\, a_{p,y} + e^{i\phi} \sinh r\, a^\dagger_{-p,y}\right) \oplus$$

$$M = \left\langle \Psi \right| a^2_{p,y} \left| \Psi \right\rangle = 0 \qquad N = \left\langle \Psi \right| a^\dagger_{p,y} a_{p,y} \left| \Psi \right\rangle = \sinh^2 r$$

⇨ $W = \left|M\right| - N = -\sinh^2 r$ Witness is largely negative

⇨ Squeezing nature is lost before the detection.

There is a partner wave packet that propagates in the opposite direction.

⇨ That partner cannot be detected by the observer.

The situation is similar to the Hawking particles, which are thermal due to entanglement with modes inside the horizon.

Stochastic background with Hypothetical Single-mode Squeezing

Single-mode squeezed state of many graviton modes:

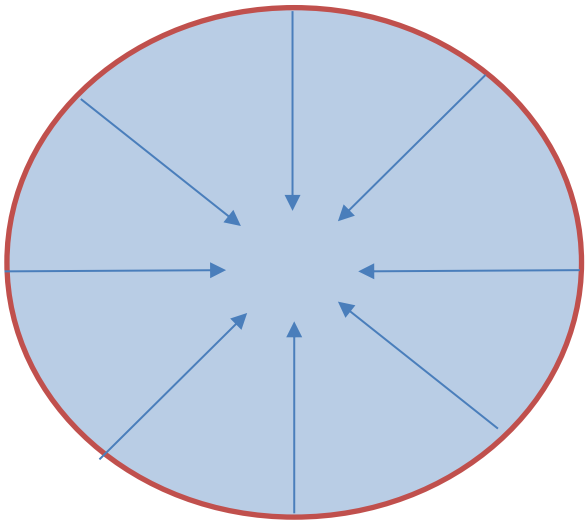
$$|\Psi\rangle = \prod_j \exp\left[\frac{\zeta_j}{2}\left(a_j^\dagger\right)^2 - h.c.\right]|0\rangle$$

$$\tilde{a} := \sum_j c_j a_j \Big/ \sqrt{\sum_j |c_j|^2} \qquad \zeta_j = e^{i\phi_j} r_j$$

is the operator coupled to the detector.

$$M = \langle\Psi|\tilde{a}^2|\Psi\rangle = \left[2\sum_j |c_j|^2\right]^{-1} \sum_j c_j^2 e^{i\phi_j} \sinh 2r_j \approx 0$$

$$N = \langle\Psi|\tilde{a}^\dagger\tilde{a}|\Psi\rangle = \left[\sum_j |c_j|^2\right]^{-1} \sum_j |c_j|^2 \sinh^2 r_j$$



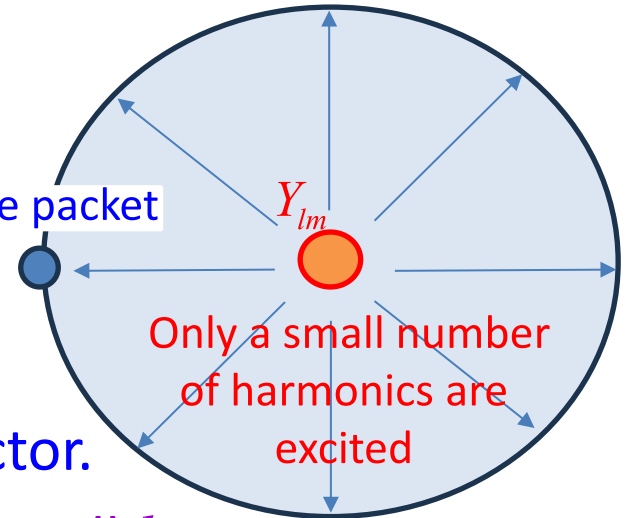
Observed mode is a superposition of waves coming from various directions

$\Rightarrow$ $W = |M| - N = -\sinh^2 \bar{r}$ Witness is largely negative

Isolated source of Single-mode Squeezing

The source of GWs is very far in general.

Detectors are sensitive only to wave packet passing through them.



$$\tilde{a} := \sum_{lm} c_{lm} a_{lm}$$

is the operator coupled with the detector.

Signal will be present only in modes with small lm.

$$M = \langle \Psi | \tilde{a}^2 | \Psi \rangle = \sum_{lm} c_{lm}^2 e^{i\phi_{lm}} \frac{\sinh 2r_{lm}}{2} \approx \frac{e^{2r}}{\mathcal{N}}$$

$$N = \langle \Psi | \tilde{a}^\dagger \tilde{a} | \Psi \rangle = \sum_{lm} |c_{lm}|^2 \sinh^2 r_{lm}$$

$$\mathcal{N} \equiv (\text{Number of modes to descrive the wave packets})$$

$$\approx (\max l)^2 \approx [(\text{source distance}) / (\text{detector size})]^2$$

$$W = |M| - N < \sum |c_{lm}|^2 \frac{1 - e^{-2r_{lm}}}{2} \approx O\left(\frac{1}{\mathcal{N}}\right)$$

Witness is almost 0.

Squeezing is necessary or not?

a is the annihilation operator of the graviton and
b is that of the detector photon.

$$a = \cosh r_a \, \tilde{a} + e^{i\phi_a} \sinh r_a \, \tilde{a}^\dagger \qquad b = \cosh r_b \, \tilde{b} + e^{i\phi_b} \sinh r_b \, \tilde{b}^\dagger$$

$$\tilde{a} \left| \Psi_{init} \right\rangle = \tilde{b} \left| \Psi_{init} \right\rangle = 0$$

Squeezed state both for the graviton and the photon

$$i \int H_{\text{int}} dt = \varepsilon a b^\dagger - h.c. = x \tilde{a} \tilde{b}^\dagger + y \tilde{a}^\dagger \tilde{b}^\dagger - h.c.$$

$$\Rightarrow \quad \left| \Psi_{fin} \right\rangle = \exp\left[i \int H_{\text{int}} dt \right] \left| \Psi_{init} \right\rangle$$

$$x = \varepsilon \left(\cosh r_a \cosh r_b + e^{i\Delta\phi} \sinh r_a \sinh r_b \right)$$

$$y = \varepsilon e^{i\phi_a} \left(\sinh r_a \cosh r_b + e^{i\Delta\phi} \cosh r_a \sinh r_b \right)$$

$$\Delta\phi = \phi_b - \phi_a$$

Squeezing of graviton state is equivalent to the squeezing of the detector photon state.

The photon witness $W_{\tilde{b}} > 0$ is impossible, if the graviton is classical.

How does the quantum nature of the graviton is transferred to the photon state.

$$M_{\tilde{b}} := \langle \Psi | \tilde{b}^2 | \Psi \rangle = xy \left(\frac{\sin \varepsilon}{\varepsilon} \right)^2 \qquad \varepsilon^2 := |x|^2 - |y|^2$$

$$x = \varepsilon \left(\cosh r_a \cosh r_b + e^{i\Delta\phi} \sinh r_a \sinh r_b \right)$$

$$y = \varepsilon e^{i\phi_a} \left(\sinh r_a \cosh r_b + e^{i\Delta\phi} \cosh r_a \sinh r_b \right)$$

$$N_{\tilde{b}} := \langle \Psi | \tilde{b}^\dagger \tilde{b} | \Psi \rangle = |y|^2 \left(\frac{\sin \varepsilon}{\varepsilon} \right)$$

$$\Longrightarrow \quad W_{\tilde{b}} = |M_{\tilde{b}}| - N_{\tilde{b}} = \frac{\sin^2 \varepsilon}{\varepsilon^2} |y| \left(|x| - |y| \right)$$

$$= \sin^2 \varepsilon \times \underline{\frac{1}{2} \left\{ 1 - \left(q - \sqrt{q^2 - 1} \right) \right\}} < \frac{1}{2} \sin^2 \varepsilon$$

the factor controlled by the squeezing

$$q = \left(\cosh 2r_a \cosh 2r_b - \cos \Delta\phi \sinh 2r_a \sinh 2r_b \right)$$

And the variance of W is also calculated as

$$\left(\Delta W_{\tilde{b}} \right)^2 = 2 \left(\frac{1}{2} - W_{\tilde{b}} \right)^2 \Longrightarrow$$

To prove $W_{\tilde{b}} > 0$, we need $W_{\tilde{b}} \gtrsim 0.3$.

To make $W_{\tilde{b}}$ large, we need a large bare original coupling ε of $O(1)$.

An alternative way is repeated measurements $> W_{\tilde{b}}^{-2} = \varepsilon^{-2}$.

Required sensitivity is ridiculously high.

GW frequency	maximum sensitivity	Threshold for $T=\omega^{-1}$ (h_{th})	Single graviton amplitude for $T=\omega^{-1}$ (h_{QG})	$\frac{h_{QG}}{h_{th}}$
nHz	$10^{-17}\ \mathrm{Hz}^{-1/2}$	$10^{-21.5}$	10^{-52}	$10^{-30.5}$
mHz	$3\times 10^{-21}\ \mathrm{Hz}^{-1/2}$	10^{-22}	10^{-46}	10^{-24}
0.1–1 Hz	$10^{-24}\ \mathrm{Hz}^{-1/2}$	10^{-24}	$10^{-43.5}$	$10^{-19.5}$
30–300 Hz	$10^{-25}\ \mathrm{Hz}^{-1/2}$	10^{-24}	10^{-41}	10^{-17}
1–10 kHz	A few $\times\, 10^{-25}\ \mathrm{Hz}^{-1/2}$	10^{-23}	$10^{-39.5}$	$10^{-16.5}$
100 kHz	$10^{-22}\ \mathrm{Hz}^{-1/2}$	$10^{-19.5}$	10^{-38}	$10^{-18.5}$
MHz	$10^{-20}\ \mathrm{Hz}^{-1/2}$	10^{-17}	10^{-37}	10^{-20}
GHz	$10^{-20}\ \mathrm{Hz}^{-1/2}$	$10^{-15.5}$	10^{-34}	$10^{-18.5}$
THz	$10^{-19.5}\ \mathrm{Hz}^{-1/2}$	$10^{-13.5}$	10^{-31}	$10^{-17.5}$

$i\int H_{\mathrm{int}}dt = \varepsilon a b^{\dagger} - h.c. \propto \sqrt{T\omega_{GW}}$ if the detection is not noise-bounded.

If we consider Gertsenshtein effect, assuming the $10^{20}\mathrm{J/m^3}$ laser（target strength of the pulse laser）fills a large volume cavity of $\mathrm{O(1km)}$, and the coherence of the system is maintained for 1 year long, the effective coupling becomes as large as $\mathrm{O}(10^{-3})$.

$$\int H_{\mathrm{int}} dt \approx \int dv \int dt\, hFF$$

$$\approx a^\dagger b \frac{VT}{m_{pl}} \sqrt{\frac{\rho_{EM}\omega_\gamma}{VT\sigma\omega_{GW}}} + h.c.$$

$$\leq 10^{-3} a^\dagger b \sqrt{\frac{L}{1\mathrm{km}} \frac{T}{1\mathrm{yr}} \frac{\rho_{EM}}{10^{20}\,\mathrm{J/m^3}} \frac{\omega_\gamma}{\omega_{GW}}} + h.c.$$

$$h \propto \frac{1}{m_{pl}\sqrt{T\sigma}}$$

$$F_{probe} \propto \sqrt{\frac{\omega_\gamma}{V}} \approx \sqrt{\frac{\omega_\gamma}{L\sigma}}$$

$$F_{source} \approx \sqrt{\rho_{EM}}$$

If we assume a laser confined in a cavity, the strength will scale down to

$10^{9}\mathrm{J/m^3}$ for static magnetic field,

$10^{7}\mathrm{J/m^3}$ for laser in a cavity.

In the case of an ordinary interferometer, $F_{source} \approx \sqrt{\omega_\gamma N_\gamma / L\sigma}$

but we cannot coherently accumulate the signal because of phase cancelation.

$$\int H_{\mathrm{int}} dt \approx a^\dagger b \frac{\omega_\gamma}{m_{pl}} \sqrt{N_\gamma} + h.c. \approx 10^{-18} a^\dagger b \left(\frac{\lambda_\gamma}{10^{-5}\,\mathrm{cm}}\right)^{-1} \sqrt{\frac{N_\gamma}{10^{20}}} + h.c.$$

Even with the factor $\sqrt{T\omega} \approx 10^{12} \left(\frac{T}{1\mathrm{year}}\right)^{1/2} \left(\frac{\omega}{10^{15}\,Hz}\right)^{1/2}$, still $O(1)$ coupling is far.

Summary

Three no-go arguments for the detection of the squeezing of GWs:

1) Two-mode squeezing of primordial GWs cannot be seen, because the partner mode does not travel toward us.

2) Squeezing of the stochastic GW background, even if its state is in a single-mode squeezing, cannot be detected, because what we observe as stochastic GWB is an incoherent superposition of many modes.

3) Even if there exists an ideal single source of a squeezed GW, the squeezing parameter is reduced to almost 0 under the propagation

To see the graviton is quantum or not, the squeezing of GWs is not is not important.

Discrimination between quantum and classical GWs requires the detector sensitivity to be higher than that needed for the detection of a single graviton.