Gravitational waves beyond rays: a disordered system approach

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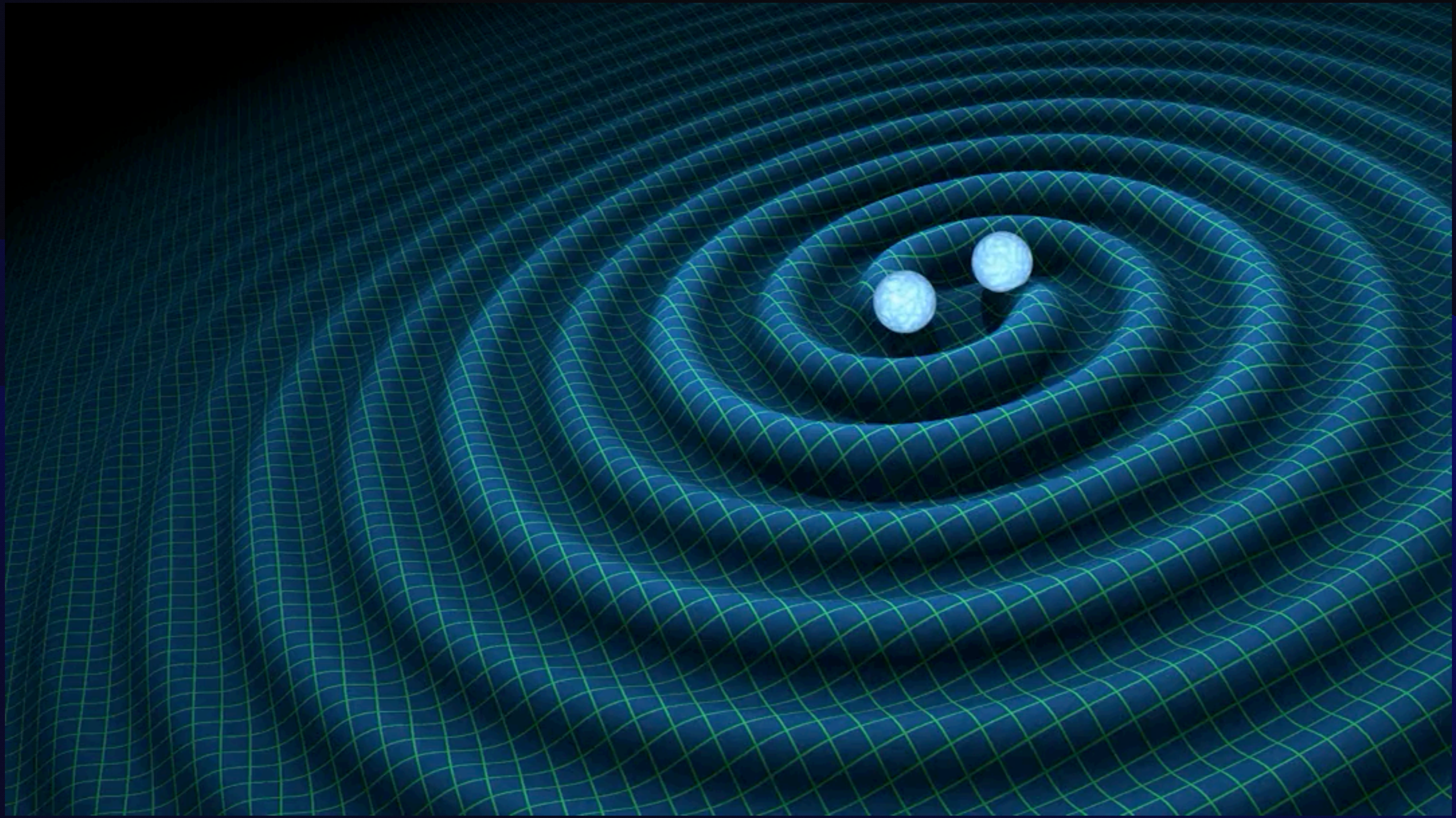
OUTLINE OF THE TALK

- Overview
- What is gravitational wave optics?
- Multiple lenses
- Describing the universe as a disordered system
- Density matrix description: averaging over the disorder
- Computing the evolution of our system
- Decoherence

R.A,
G. Braga,
A. Garoffolo, F. Lopez,
N. Bartolo, S. Matarrese

Slides made with Ginevra

GRAVITATIONAL WAVES

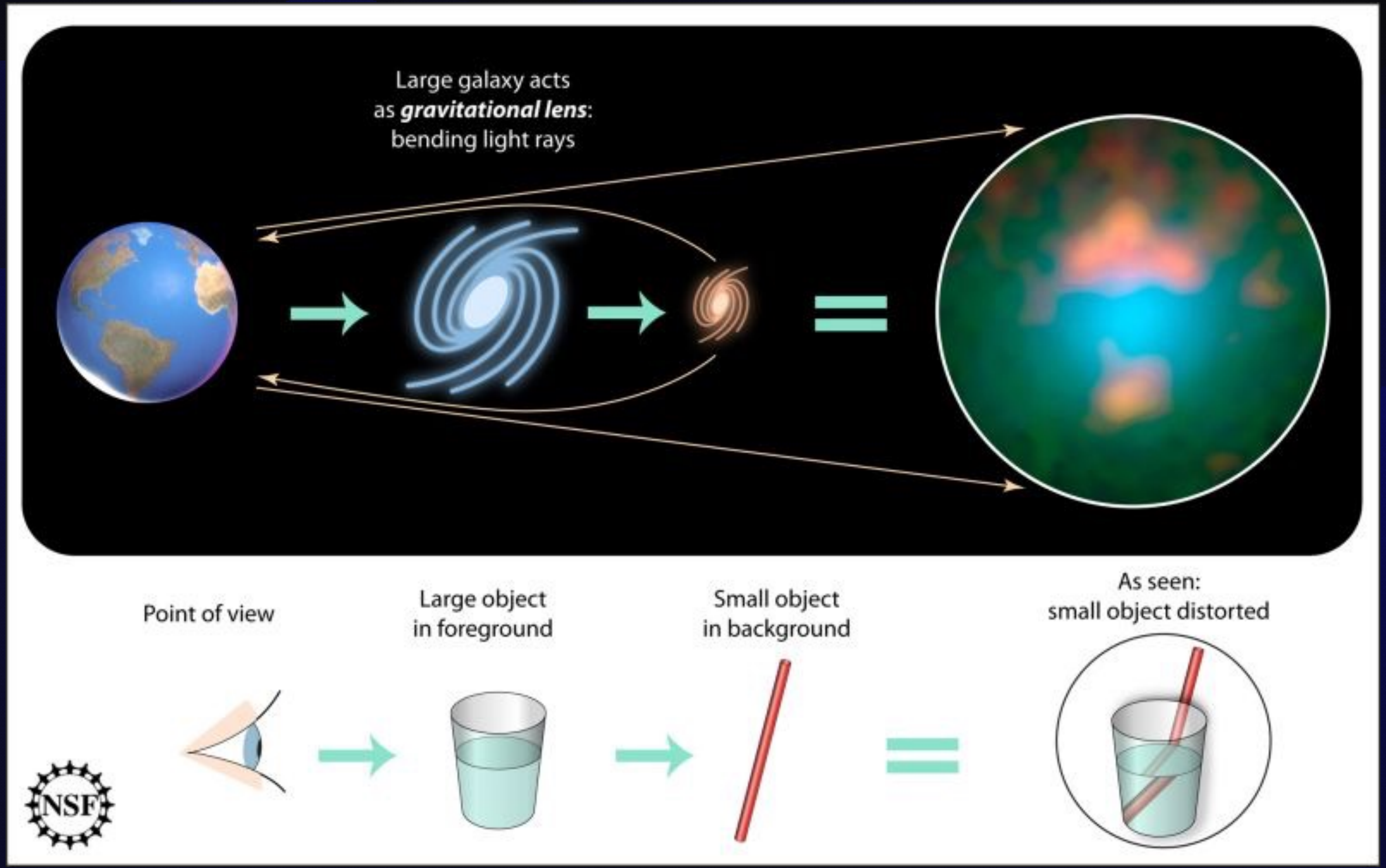


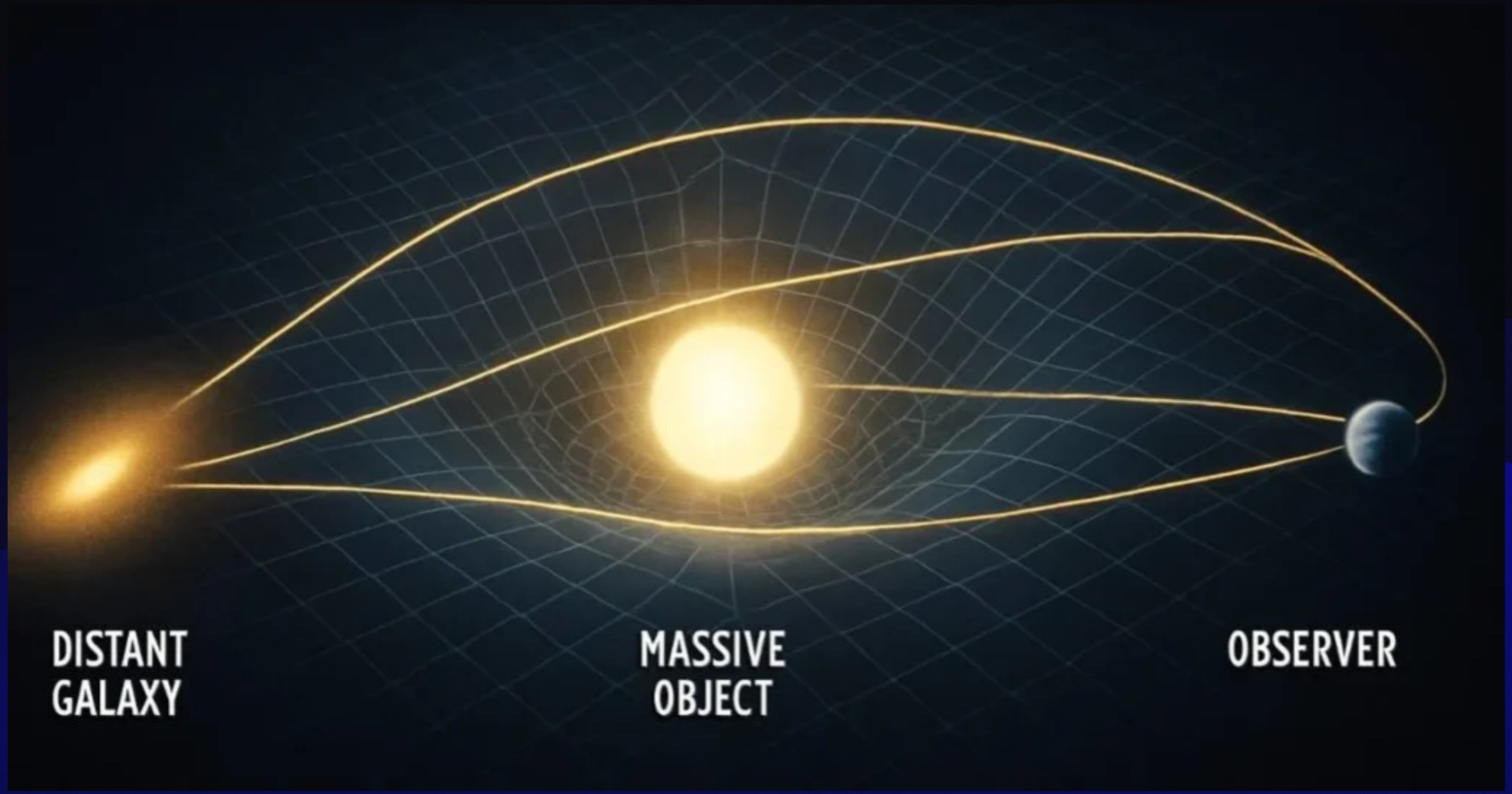
POSSIBLE INTERACTIONS

GRAVITATIONAL WAVE LENSING

GRAVITATIONAL WAVES' LENSING

Gravitational Lensing: Massive objects act as lenses





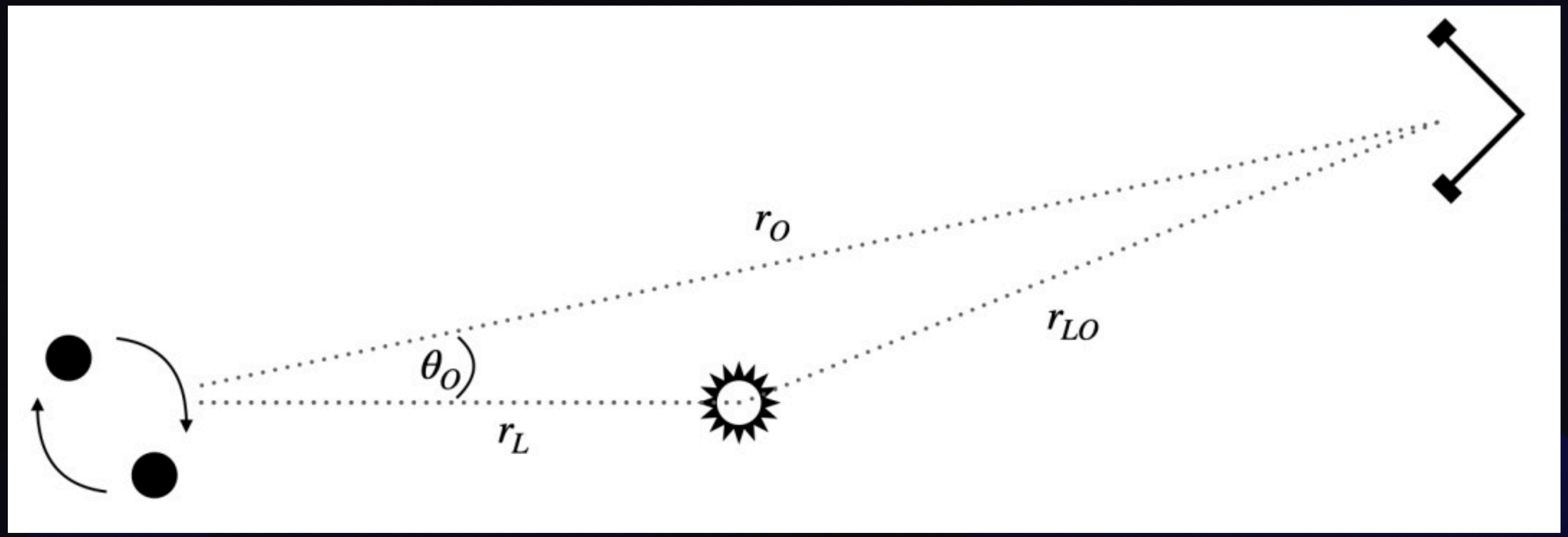
OPTICS REGIME

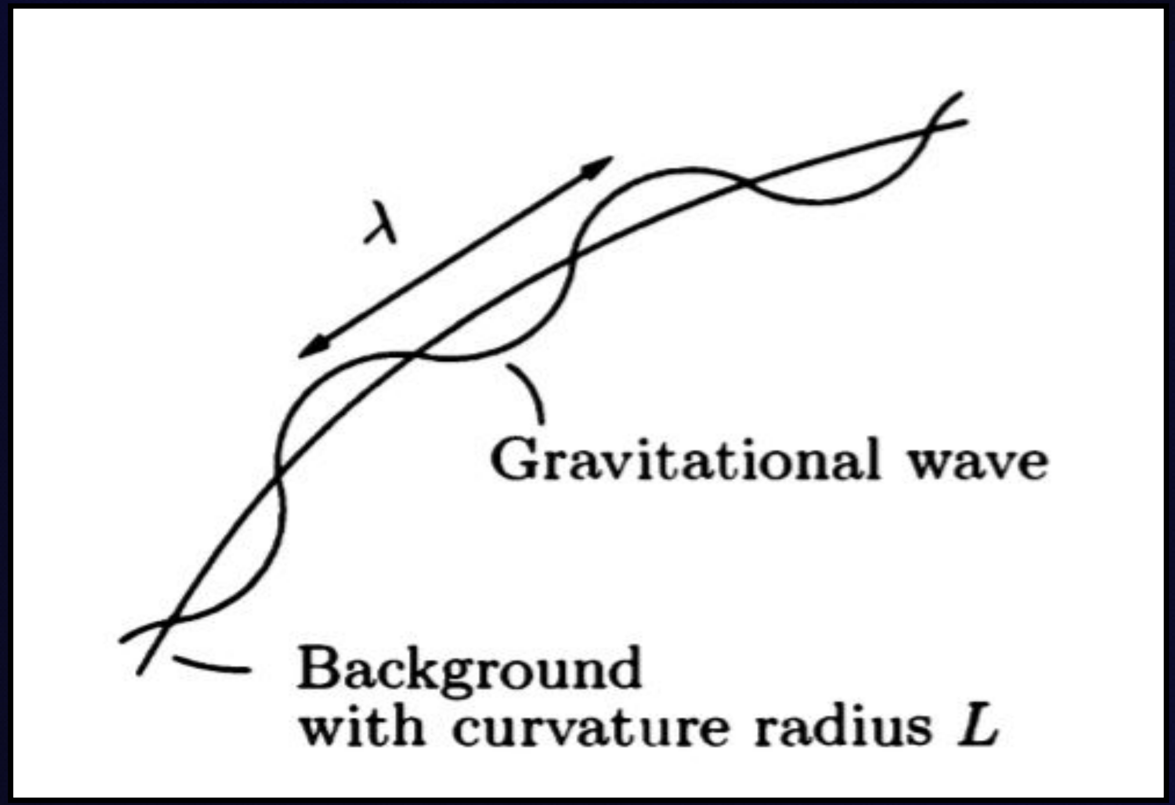
Depending on the particular setup of our system:
- GW wavelength
- compactness of the lens
- impact parameter

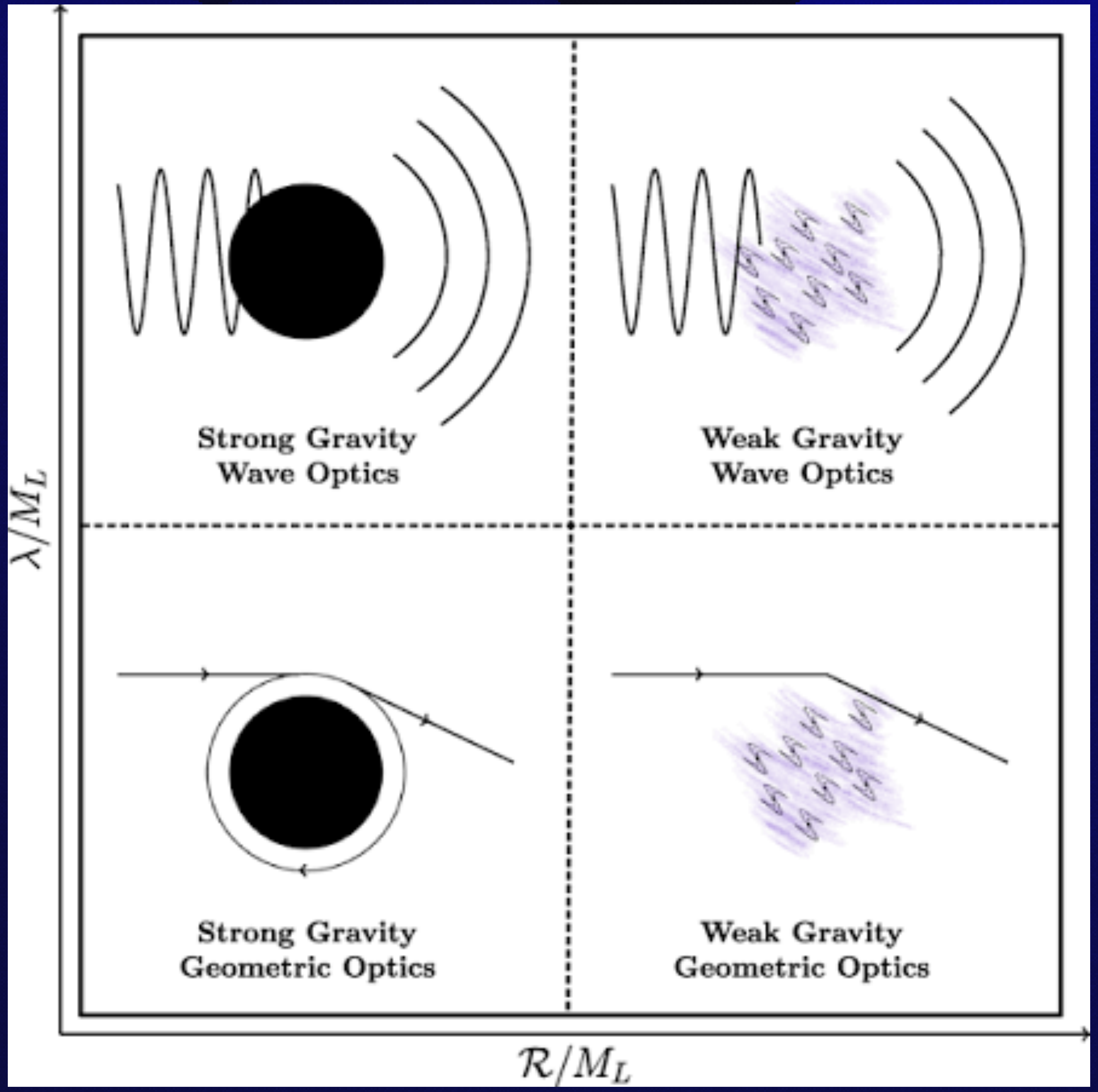
lensing can have different phenomenology

$$\omega \equiv \frac{L}{\lambda}$$

If GW wavelength
 is smaller when compared to the lens -> geometric optics regime
 is larger when compared to the lens -> **wave** optics







Geometric Optics (GO)

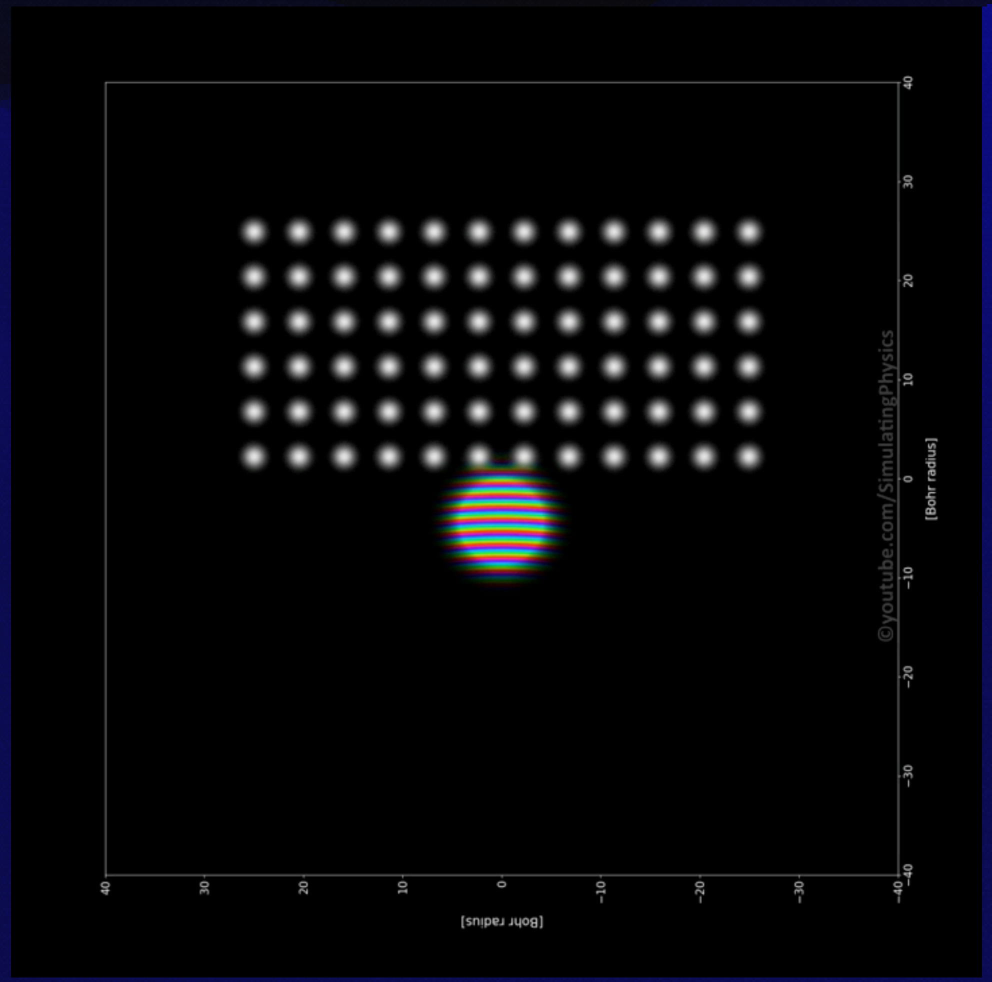
Focus on high frequency regime

- GWs follow trajectories: rays
- Conserved polarization
- Conservation law for gravitons

Wave Optics (WO)

Focus on lows frequency regime:

- No definition of trajectory
- Multiple paths contribute
- Wave effects become dominant



Different optical regimes are treated with different formalisms

THE HAMILTON ANALOGY

Is there a way in which we can generalize this analogy to study of geometric and wave optics?

THE HAMILTON ANALOGY

In the same way in which quantum mechanics approaches classical limit estremizing the action along one path when $\hbar$ -> 0, we reach geometric optics one when 1/ ω -> 0

WAVE OPTICS: STANDARD APPROACH

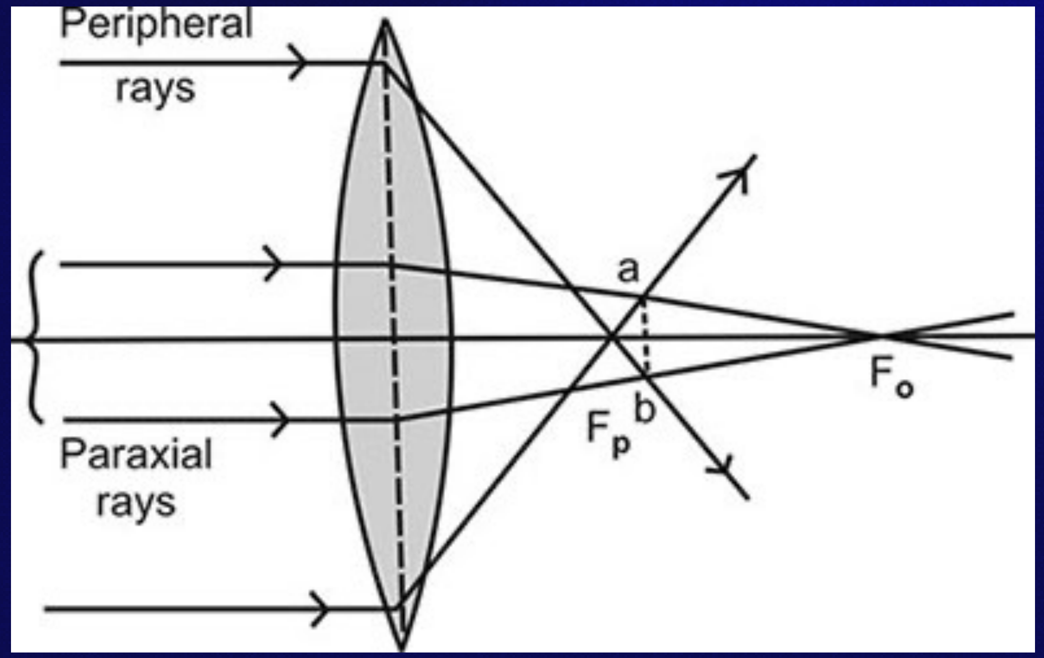
Nakamura & Deguchi '99

1. Polarization is conserved → scalar wave:

$$\Box \Psi = 0 \quad \rightarrow \quad \left[\nabla^2 + \omega^2 (1 - 4\alpha U)\right] \tilde{\Psi}(\mathbf{x}) = 0$$

2. Define the Amplification Factor: $F = \tilde{\Psi}/\tilde{\Psi}_{NL}$

KG → Schrödinger



3. Impose Eikonal and Paraxial approximations:

$$|\partial_r^2 F(\mathbf{x})| \ll |2i\omega\, \partial_r F(\mathbf{x})| \qquad sin\theta \simeq \theta$$

One finds a Schrödinger-like equation with $r \leftrightarrow t,\ \omega \leftrightarrow \hbar^{-1}$

$$F(\vec{r_o}) = \int \mathscr{D}\theta\, exp\left\{ i\omega \int_0^{r_o} dr \left[\frac{r^2}{2}|\dot{\theta}|^2 - 2U\right]\right\}$$

MULTIPLE LENSES SCENARIO

Finite number of additional lenses

⟶ Widely treated in GO: line-of-sight or kinetic description

⟶ Few cases in WO: trajectory-based techniques fail

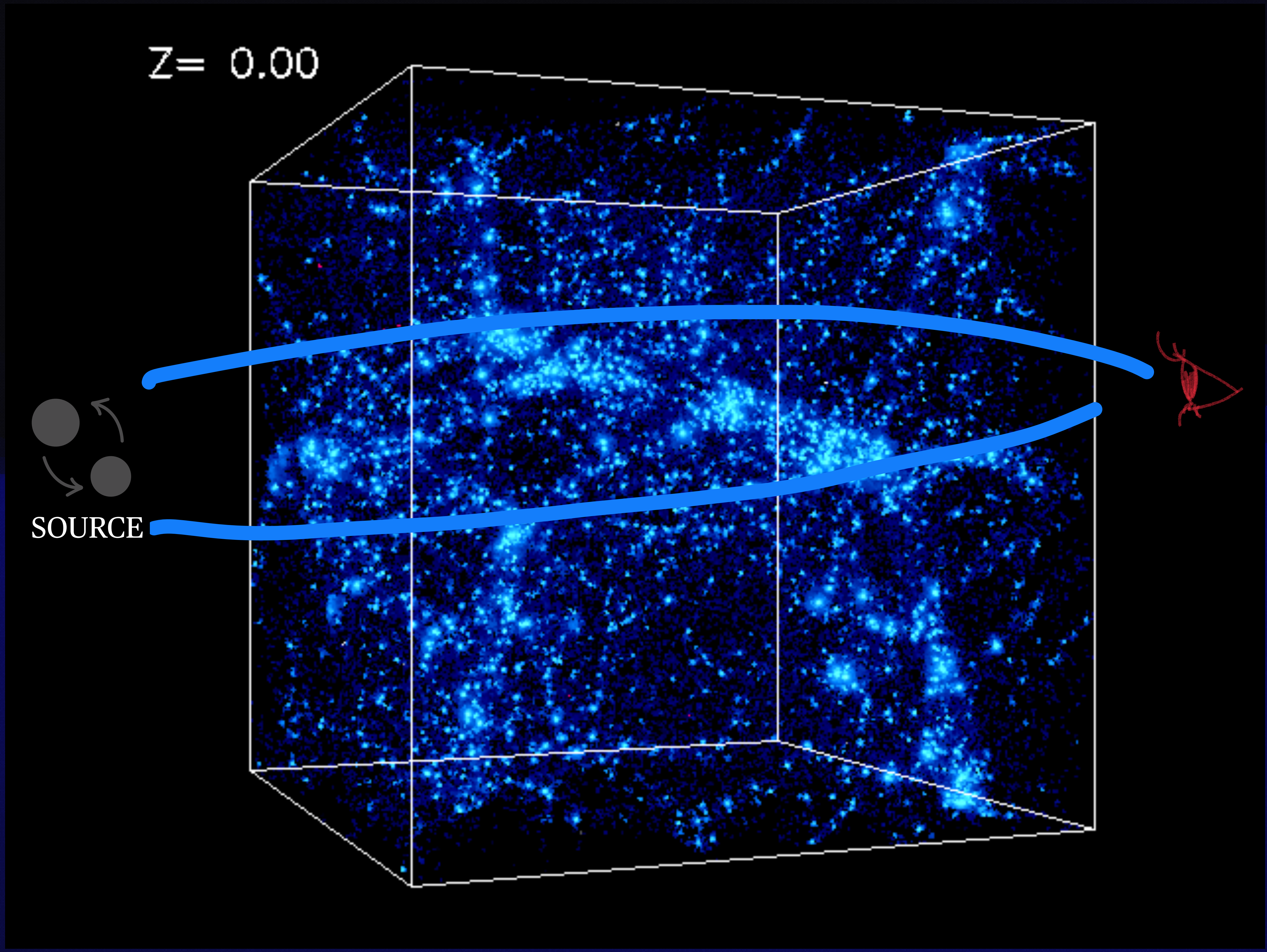
GOAL: Waves propagate through a **random distribution of weak lenses**

Observables are associated with **ensemble** of realizations of matter distribution

QUENCHED DYSORDER DESCRIPTION

In our work

Cosmological gravitational potential is treated as the frozen impurity potential in a disordered medium:
It modifies the propagation of the wave without acquiring an independent dynamics or exchanging energy with it



MULTIPLE LENSES SCENARIO: SET UP

Waves propagating in a Universe with static matter inhomogeneities

$$ds^2 = -(1 + 2\alpha\Phi(\boldsymbol{x}))dt^2 + (1 - 2\alpha\Phi(\boldsymbol{x}))dx^2$$

Treat GWs as a real scalar field obeying KG

$$\left[\partial_{\boldsymbol{x}}^2 - \left(1 - 4\alpha\Phi\left(\boldsymbol{x}\right)\right)\partial_t^2\right]\zeta(\boldsymbol{x}, t) = 0$$

Final wave is fully determined given the one realization of the gravitational potential

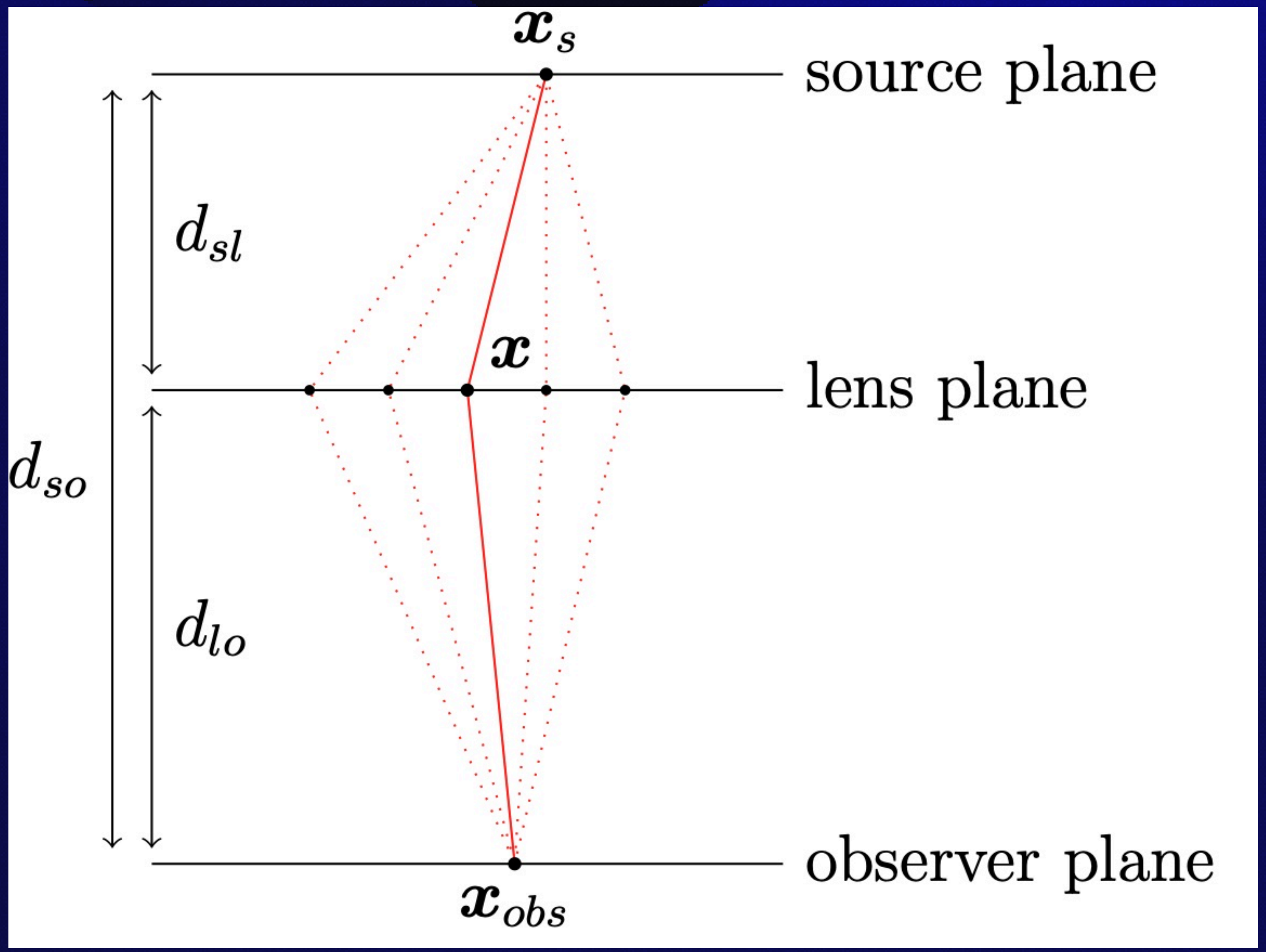
How can we obtain a statistical description of the wave propagation?

MULTIPLE LENSES SCENARIO: SET UP

Waves through a static random medium

1. Solve the EOM for a fixed realization of the gravitational potential
2. Average over the stochastic ensemble of lenses

☠ Explicit solutions are based on the *thin lens approximation*



We wish to describe an **extended lens distribution**

Formulate the problem at the level of field configurations

using the **averaged density matrix**

WHY DENSITY MATRIX DESCRIPTION

Gravitational waves here are classical solutions of a linear wave equation

↓

solutions can be superimposed coherently

↗ analogously to Young experiment

↘ coherence in terms of relative phases

↓ for example think to the two initial field configurations of the Schwinger Keldysh path integral

We use a language typical of **QUANTUM FIELD THEORY**

- No quantization in the fundamental sense!
- No $\hbar$ in the action
- Ω plays the role of a semiclassical scale: analogue of $\hbar$

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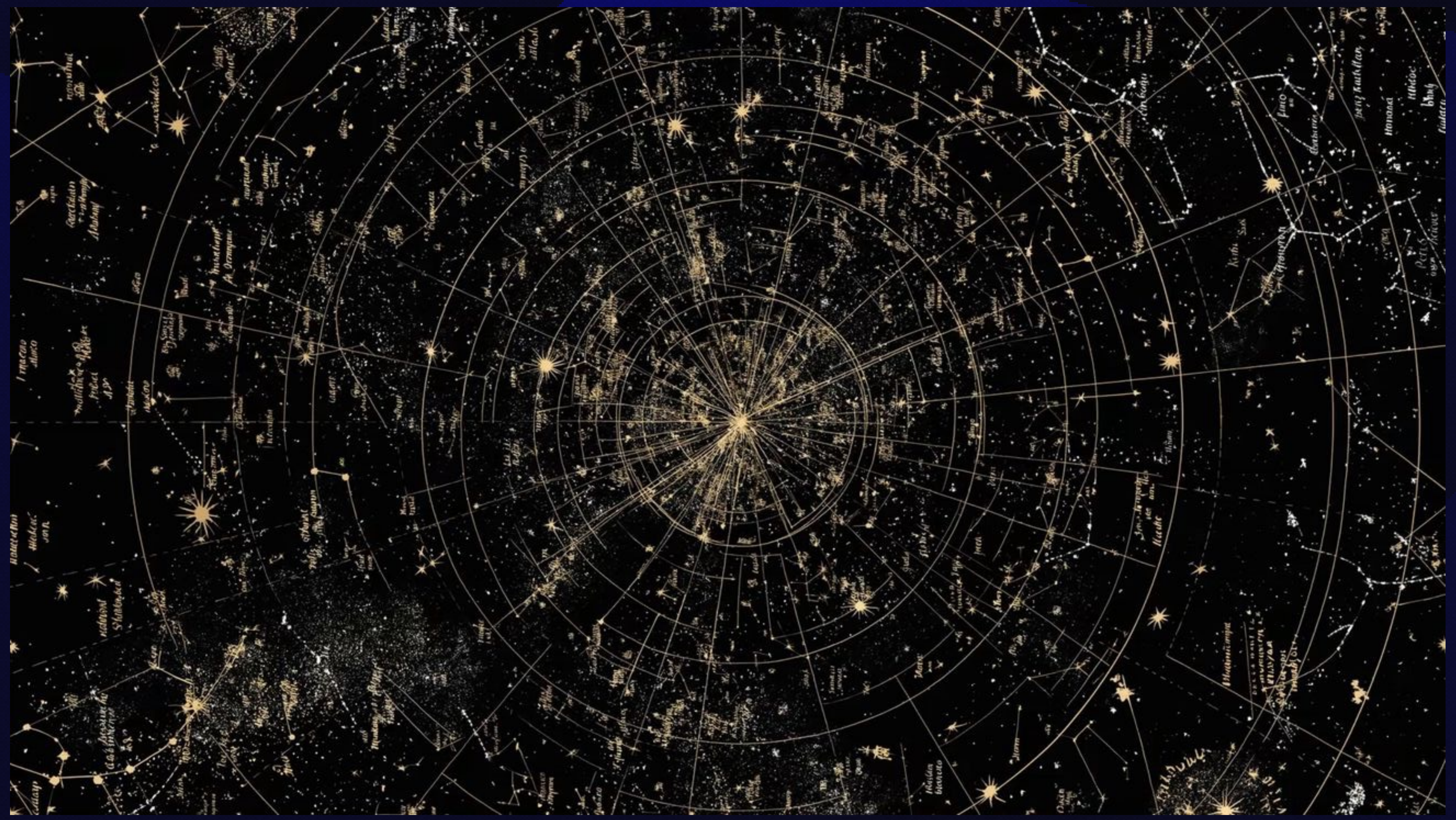
What do we expect?



- Single realization: paths interfere → **INTERFERENCE PATTERN**
- Different realizations:

  ✴️Phase relations fluctuate → interference pattern no longer aligned

  ✴️Ensemble averaging suppresses coherence of the density matrix

We consider a FIXED realization of the gravitational potential

$$S\left[\zeta,\Phi\right] = S_0\left[\zeta\right] + S_{int}\left[\zeta,\Phi\right]$$

Free propagation of the wave

$$S_0 = \int_{t_i}^{t_f} d^4x\, \partial_\mu\zeta(\boldsymbol{x},t)\partial^\mu\zeta(\boldsymbol{x},t)$$

Coupling term

$$S_{int} = \int_{t_i}^{t_f} d^4x\, \Phi(\boldsymbol{x})\partial_t\zeta(\boldsymbol{x},t)\partial_t\zeta(\boldsymbol{x},t)$$

CONVENTIONS

- Indices raised/lowered with Minkowski
- $\int_{t_i}^{t_f} d^4x = \int_{t_i}^{t_f} dt \int d^3\boldsymbol{x}$

1. DENSITY MATRIX DESCRIPTION

State for a fixed realization of Φ

$$\underbrace{\hat{\rho}^{\Phi}(t) = \hat{\mathscr{U}}_{\Phi}(t,t_i)\hat{\rho}(t_i)\hat{\mathscr{U}}^{\dagger}_{\Phi}(t,t_i)}$$

Unitary evolution

$\longrightarrow$ $\hat{\rho}(t_i)$ independent on the realisation of Φ

Path integral representation of matrix elements time evolution operator

$$\mathscr{U}_{\Phi}(\zeta_f,\zeta_i;t_f,t_i) = \langle \zeta_f | \hat{\mathscr{U}}_{\Phi}(t_f,t_i) | \zeta_i \rangle = \int_{\zeta_i}^{\zeta_f} \mathscr{D}\zeta \, e^{i\Omega S[\zeta,\Phi]}$$

$\longrightarrow$ $S_0 + S_{int}$= system action evaluated between t_i and t_f

$\longrightarrow$ Backward and forward propagation: Schwinger-Keldysh formalism

2. DENSITY MATRIX DESCRIPTION

Ensemble evolution

$$\rho_{\rm av}(\zeta_f^{\pm}; t) = \underbrace{\int \mathscr{D}\Phi\, P[\Phi] \int d\zeta_i^{\pm} \mathscr{U}_{\Phi}(\zeta_f^{+}, \zeta_i^{+}; t, t_i)\rho(\zeta_i^{\pm}; t_i)\mathscr{U}_{\Phi}^{\dagger}(\zeta_f^{-}, \zeta_i^{-}; t, t_i)}_{\textbf{Non unitary}}$$

$\longrightarrow$ $P[\Phi]$ probability density functional of Φ

$\longrightarrow$ Same realization of Φ on both branches

Disorder average couples the 2 branches of evolution =

imprint of the random medium responsible for the emergence of non-unitary features

3. DENSITY MATRIX DESCRIPTION

Why density matrix evolution?

$$\rho_{\rm av}(\zeta_f^{\pm};t) = \int \mathcal{D}\Phi\, P[\Phi] \int d\zeta_i^{\pm} \mathcal{U}_{\Phi}(\zeta_f^{+},\zeta_i^{+};t,t_i)\rho(\zeta_i^{\pm};t_i)\mathcal{U}_{\Phi}^{\dagger}(\zeta_f^{-},\zeta_i^{-};t,t_i)$$

- Observables expectation value
- GW phase correlation

What we need to compute

DISORDER STATISTICS

We assume gravitational potential to be a **Gaussian random field**

$$P[\Phi] \propto \exp\left\{ -\frac{1}{2} \int d^3k\, \Phi(\boldsymbol{k}) \tilde{C}^{-1}(\boldsymbol{k}) \Phi(-\boldsymbol{k}) \right\}$$

$\longrightarrow$ Standard ΛCDM picture

$\longrightarrow$ Technically: enables exact Gaussian integral over $\boldsymbol{\Phi}$

$\longrightarrow$ $\tilde{C}^{-1}(\boldsymbol{k})$= Power spectrum of the gravitational potential generated by matter inhomogeneties

TIME EVOLUTION COMPUTATION

Background field method decomposition

$$\zeta(\boldsymbol{x},t) = \zeta_{cl}(\boldsymbol{x},t) + \frac{1}{\sqrt{\Omega}}\eta(\boldsymbol{x},t)$$

Implies

$$\mathcal{U}_{\Phi}(\zeta_f,\zeta_i;t_f,t_i) = \frac{\mathcal{N} e^{i\Omega S_{cl}[\zeta_i,\zeta_f,\Phi]}}{\sqrt{det'(K_0^{-1}K_{\Phi})}} \longrightarrow S_{cl} = S_0[\zeta_i,\zeta_f] + \alpha S_1[\zeta_i,\zeta_f,\Phi] + \alpha^2 S_2[\zeta_i,\zeta_f,\Phi]$$

$\downarrow$ Perturbative expansion of the classical action up to α^2

$\downarrow$ Normalisation factor

$\alpha \ll 1$ (weak field)

1. AVERAGE EVOLUTION

Doubling $\mathscr{U}_\Phi$

$$\rho_{\rm av}(\zeta_f^\pm;t) = \mathcal{N}\int d\zeta_i^\pm \rho(\zeta_i^\pm;t_i) e^{i\Omega[\delta S_0[\zeta_i^\pm,\zeta_f^\pm]+i\alpha^2\int\frac{d^3k}{(2\pi)^3}\tilde{C}(\boldsymbol{k})\delta\mathscr{A}_2[-\boldsymbol{k},\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]]} e^{-\frac{\alpha^2\Omega^2}{2}\int\frac{d^3k}{(2\pi)^3}\delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]\tilde{C}(\boldsymbol{k})\,\delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$$

Implies

$\longrightarrow \quad e^{i\Omega[\delta S_0[\zeta_i^\pm,\zeta_f^\pm]}$

$\longrightarrow \quad e^{i\Omega\alpha^2\int\frac{d^3k}{(2\pi)^3}\tilde{C}(\boldsymbol{k})\delta\mathscr{A}_2[-\boldsymbol{k},\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$

$\longrightarrow \quad e^{-\frac{\alpha^2\Omega^2}{2}\int\frac{d^3k}{(2\pi)^3}\delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]\tilde{C}(\boldsymbol{k})\,\delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$

2. AVERAGE EVOLUTION

$$\longrightarrow \quad \boxed{e^{i\Omega[\delta S_0[\zeta_i^\pm,\zeta_f^\pm]}}$$

Coherent free propagation

where $\delta S_0[\zeta_i^\pm, \zeta_f^\pm] = S_0[\zeta_i^+, \zeta_f^+] - S_0[\zeta_i^-, \zeta_f^-]$

$$\longrightarrow \quad e^{i\Omega\alpha^2 \int \frac{d^3k}{(2\pi)^3} \tilde{C}(\boldsymbol{k})\delta\mathscr{A}_2[-\boldsymbol{k},\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$$

$$\longrightarrow \quad e^{-\frac{\alpha^2\Omega^2}{2} \int \frac{d^3k}{(2\pi)^3} \delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]\tilde{C}(\boldsymbol{k})\, \delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$$

3. AVERAGE EVOLUTION

$\longrightarrow \quad e^{i\Omega[\delta S_0[\zeta_i^\pm,\zeta_f^\pm]}$

Coherent pure fase factor

$\longrightarrow \quad e^{i\Omega\alpha^2 \int \frac{d^3k}{(2\pi)^3} \tilde{C}(\boldsymbol{k})\delta\mathscr{A}_2[-\boldsymbol{k},\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$

where $\tilde{C}(\boldsymbol{k}) = [\tilde{C}(-\boldsymbol{k})]^*$ and $\delta\mathscr{A}_2[\boldsymbol{k}, -\boldsymbol{k}, \zeta_i^\pm, \zeta_f^\pm] = [\delta\mathscr{A}_2[-\boldsymbol{k}, \boldsymbol{k}, \zeta_i^\pm, \zeta_f^\pm]]^*$

$\longrightarrow \quad e^{-\frac{\alpha^2\Omega^2}{2} \int \frac{d^3k}{(2\pi)^3} \delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]\tilde{C}(\boldsymbol{k})\, \delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$

4. AVERAGE EVOLUTION

$$\longrightarrow \quad e^{i\Omega[\delta S_0[\zeta_i^\pm,\zeta_f^\pm]}$$

$$\longrightarrow \quad e^{i\Omega\alpha^2\int\frac{d^3k}{(2\pi)^3}\tilde{C}(\boldsymbol{k})\delta\mathscr{A}_2[-\boldsymbol{k},\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$$

$$\longrightarrow \quad \boxed{e^{-\frac{\alpha^2\Omega^2}{2}\int\frac{d^3k}{(2\pi)^3}\delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]\tilde{C}(\boldsymbol{k})\,\delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}}$$

where $\tilde{C}(\boldsymbol{k}) \geq 0$ and $\delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm] = [\delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]]^*$

Damping factor/dephasing

DECOHERENCE FACTOR

$$\longrightarrow \quad e^{-\frac{\alpha^2\Omega^2}{2}\int \frac{d^3k}{(2\pi)^3}\delta\mathscr{A}_1[\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]\tilde{C}(\boldsymbol{k})\,\delta\mathscr{A}_1[-\boldsymbol{k},\zeta_i^\pm,\zeta_f^\pm]}$$

Notice

$\longrightarrow$ For $\Omega \neq 0$ configurations with non vanishing mismatch are suppressed (**suppression enhanced for large value of Ω**)

$\longrightarrow$ If disorder has finite correlation length l_C **decoherence** is controlled by the overlap between this band and the momentum support of $\delta\mathscr{A}_1$= **efficient when GW resolves in homogeneities**

Ω LIMITS

$\longrightarrow$ For small Ω both the phase and the soppression factor become weeks, the coupling with static potential becomes ineffective = the propagation approaches the free case

Instead

$\longrightarrow$ For large Ω the phase oscillation becomes very rapid . The integral selects stationary phase configuration

INFLUENCE FUNCTIONAL APPROACH

$$\longrightarrow \quad \rho_{\rm av}(\zeta_f^+, \zeta_f^-; t) = \int d\zeta_i^+ \, d\zeta_i^- \, \rho(\zeta_i^+, \zeta_i^-; t_i) \int_{\zeta_i^+}^{\zeta_f^+} \mathscr{D}\zeta^+ \int_{\zeta_i^-}^{\zeta_f^-} \mathscr{D}\zeta^- \, e^{i\Omega\left(S_0[\zeta^+] - S_0[\zeta^-]\right)} \, \mathscr{F}[\zeta^+, \zeta^-]$$

Where

$$\longrightarrow \quad \mathscr{F}[\zeta^+, \zeta^-] = \exp\Bigg\{ -8\alpha^2\Omega^2 \int_0^T d^4x \int_0^T d^4y \times$$

$$\times [\partial_{t_x}\zeta^+(x)\partial_{t_x}\zeta^+(x) - \partial_{t_x}\zeta^-(x)\partial_{t_x}\zeta^-(x)] \, C(\mathbf{x}, \mathbf{y}) [\partial_{t_y}\zeta^+(y)\partial_{t_y}\zeta^+(y) - \partial_{t_y}\zeta^-(y)\partial_{t_y}\zeta^-(y)] \Bigg\}$$

= The average over the random potential induces an effective self-interaction for the field on the Schwinger--Keldysh contour. The **resulting theory** is **quartic and nonlocal in space, with coupling strength set by the disorder correlator.**

SUMMARY

→ Quenched disorder techniques are applied to study the GW propagation

→ Extension beyond geometric optics to the propagation of GW trough a distribution of lenses

→ The quantity $\delta\mathscr{A}_1$ measures the branches mismatch while the matter power spectrum selects average contributing momenta

→ Decoherence is stronger when GW wavelength is comparable to potential correlation length

→ Derivation is general and applies to any system with the same class of actions

FUTURE PROSPECTS

→ Extension of the computation to intensity correlation function (**Speckles pattern phenomenon**)