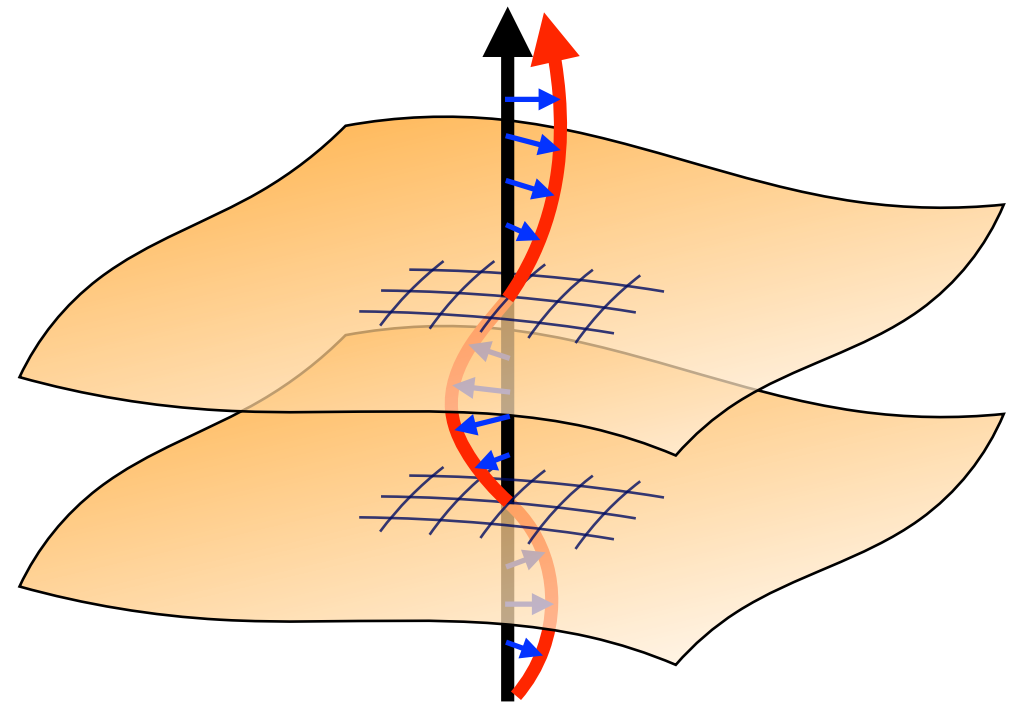
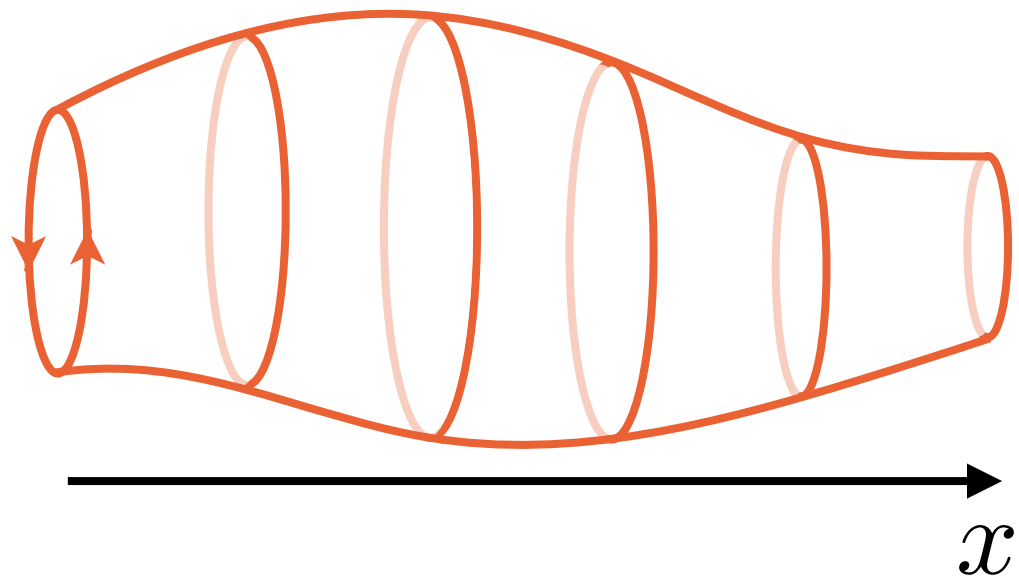


A Field-Theoretical Perspective on **Hydrodynamics**



Masaru Hongo

Niigata University/RIKEN iTHEMS

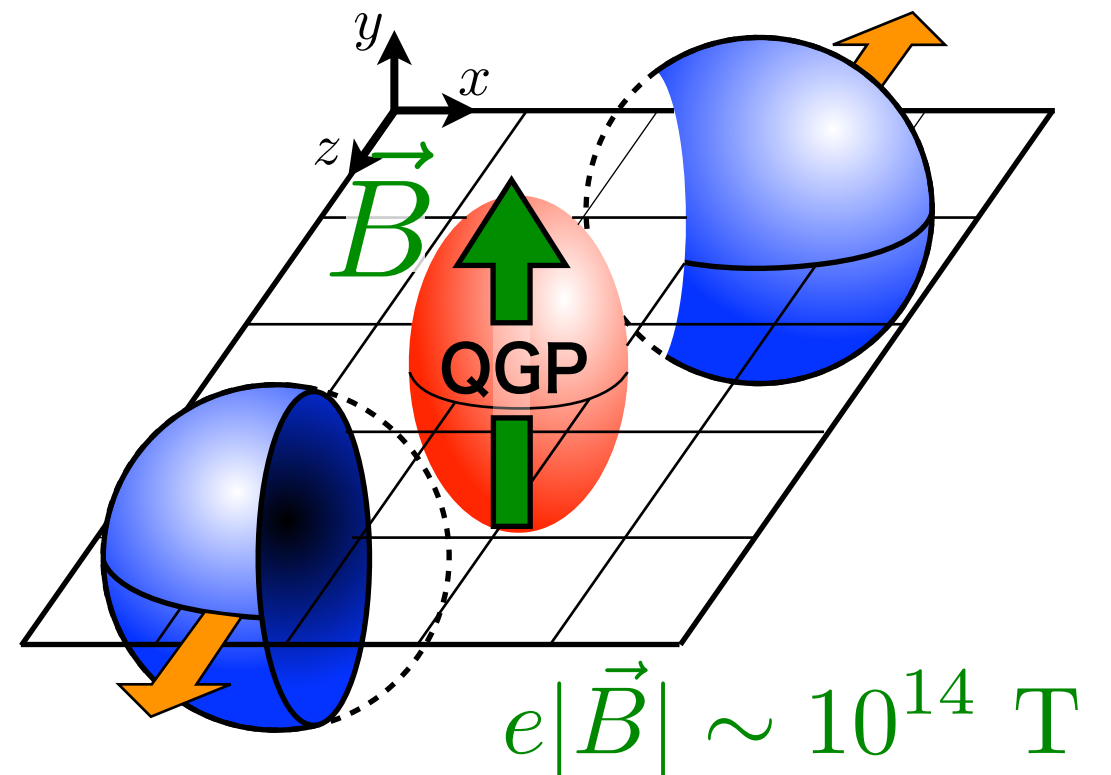
Quantum crossroads, Kavli IPMU, 2026/8/6

Where am I from?

From Niigata, Japan



From hep, nucl, stat-phys, ...



◆ Main Research Interests

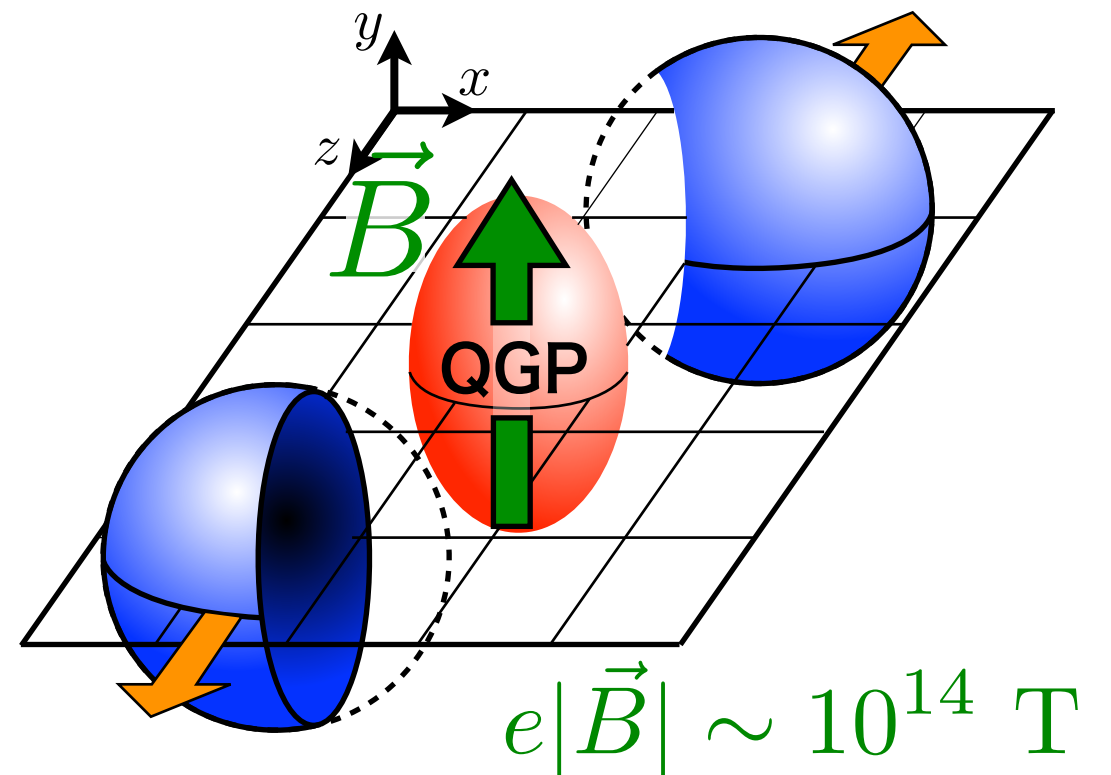
- Hydro from QFT & thermodynamics
- Effective field theory of NG modes
- Quantum few-body physics

Where am I from?

From Niigata, Japan



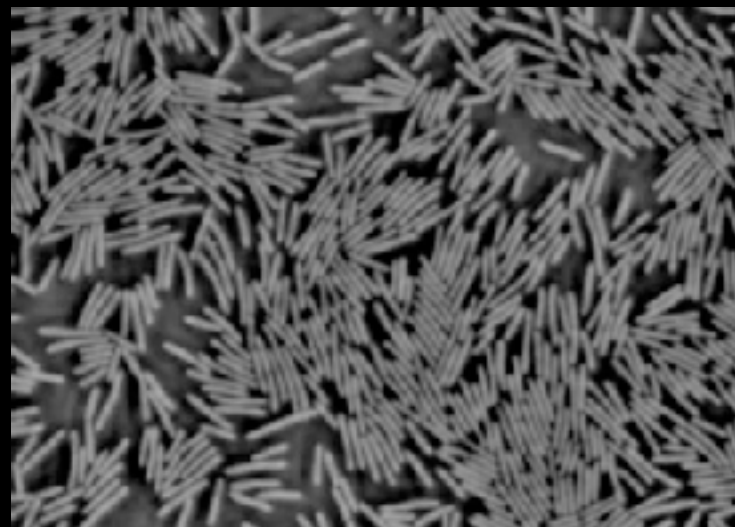
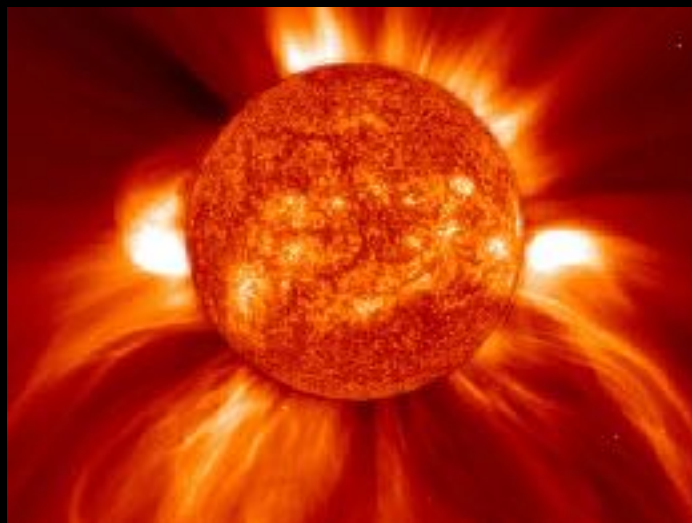
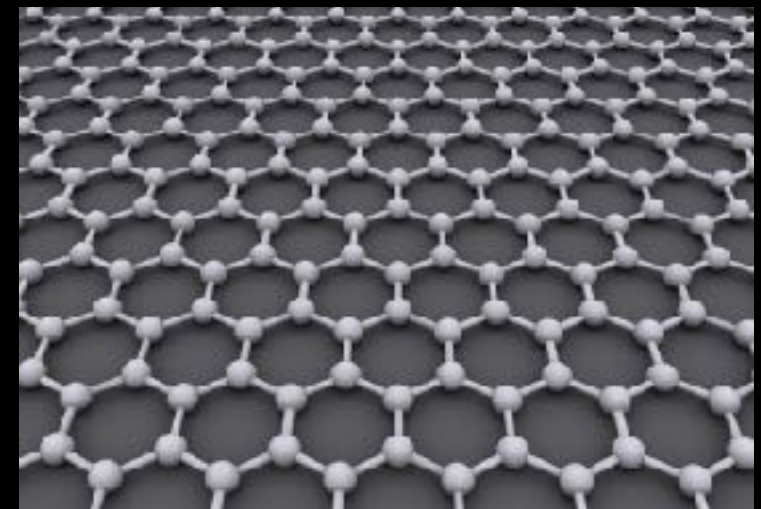
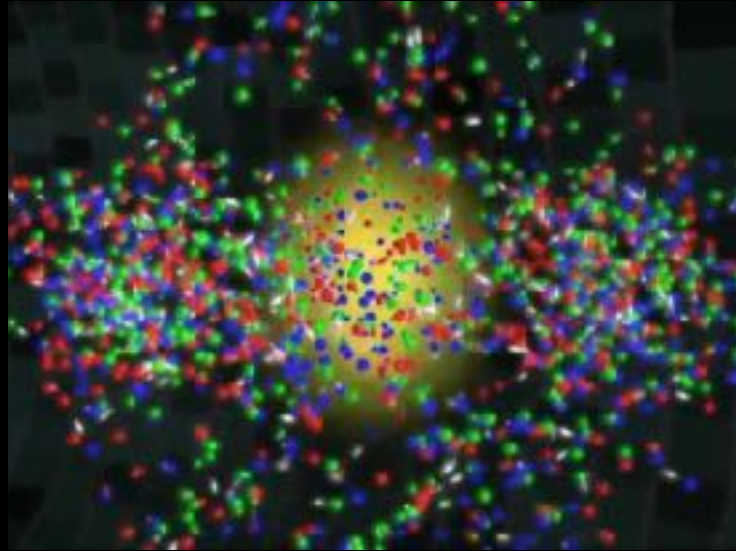
From hep, nucl, stat-physics, ...



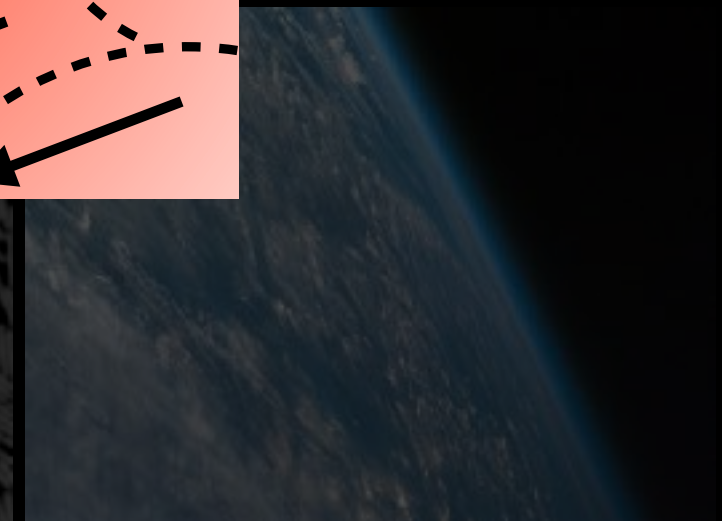
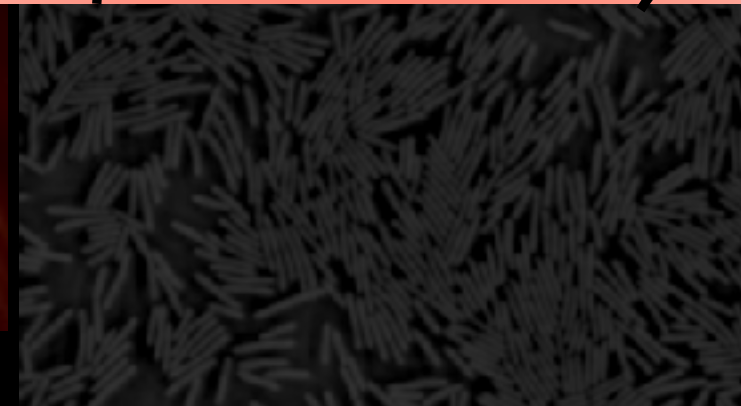
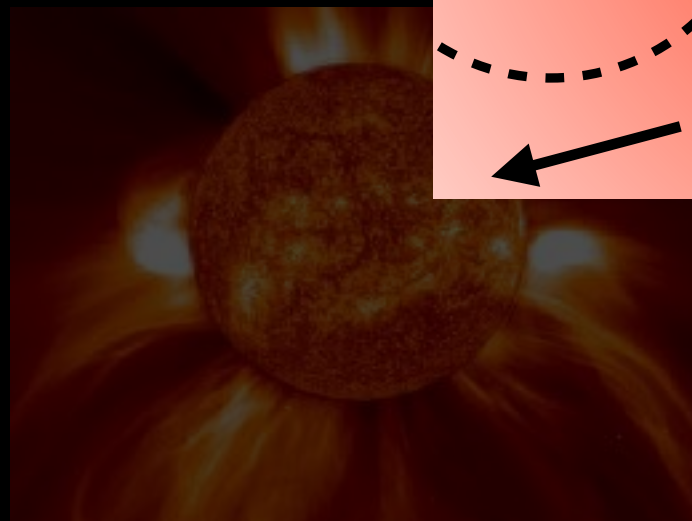
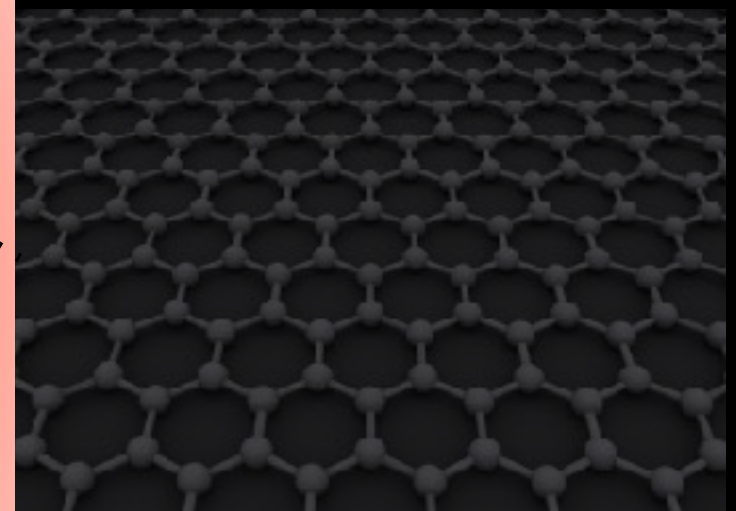
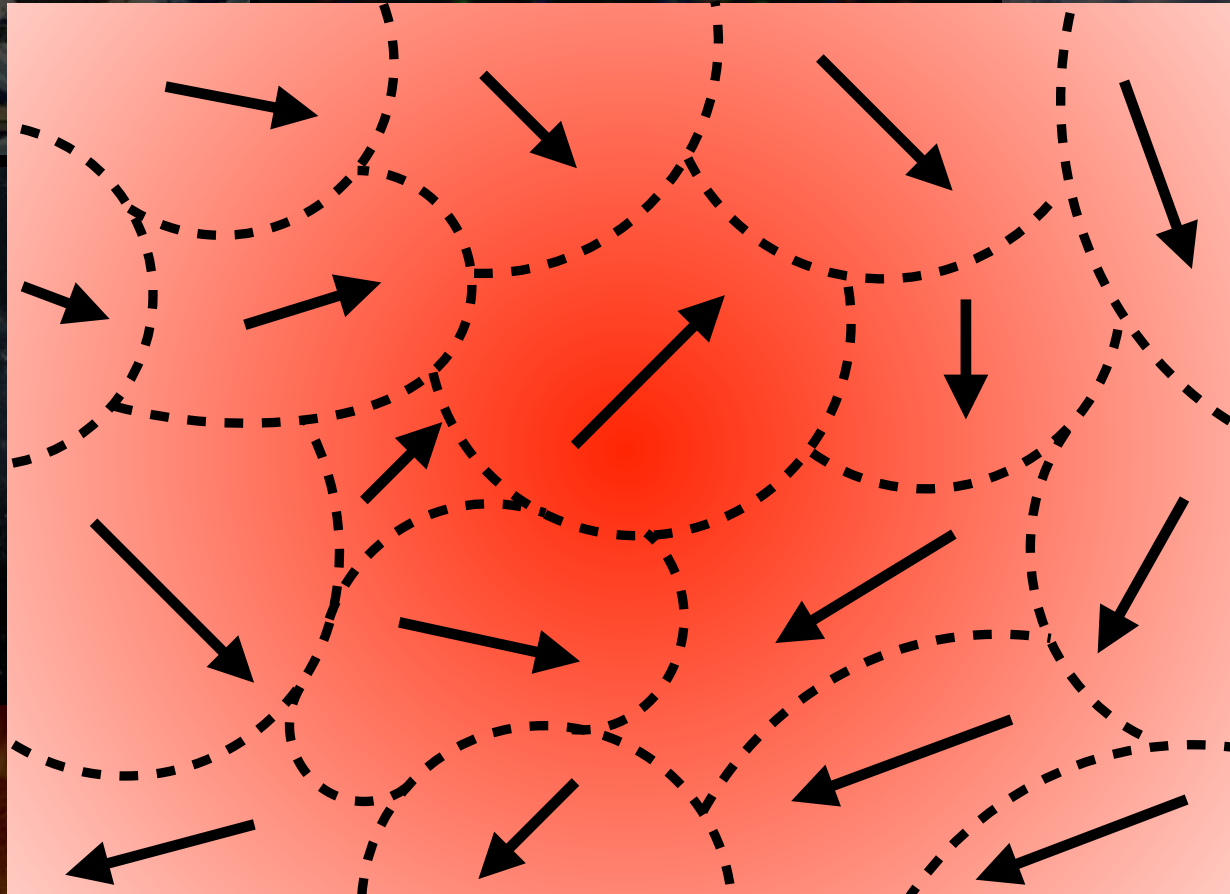
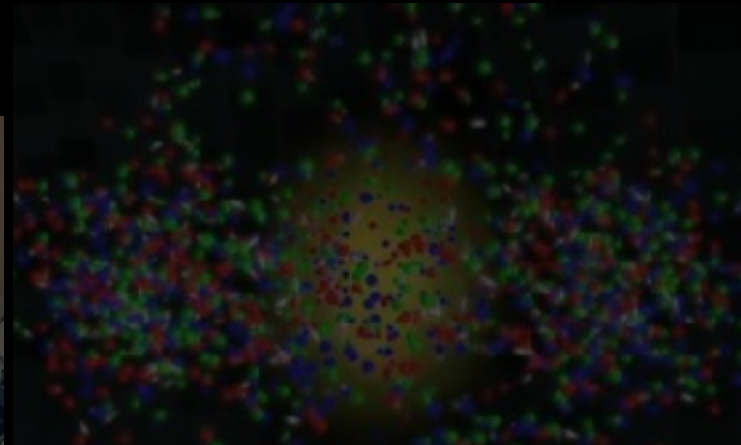
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- Hydro from QFT & thermodynamics
- Effective field theory of NG modes
- Quantum few-body physics

What hydrodynamics does is



What hydrodynamics does is



Hydrodynamics is

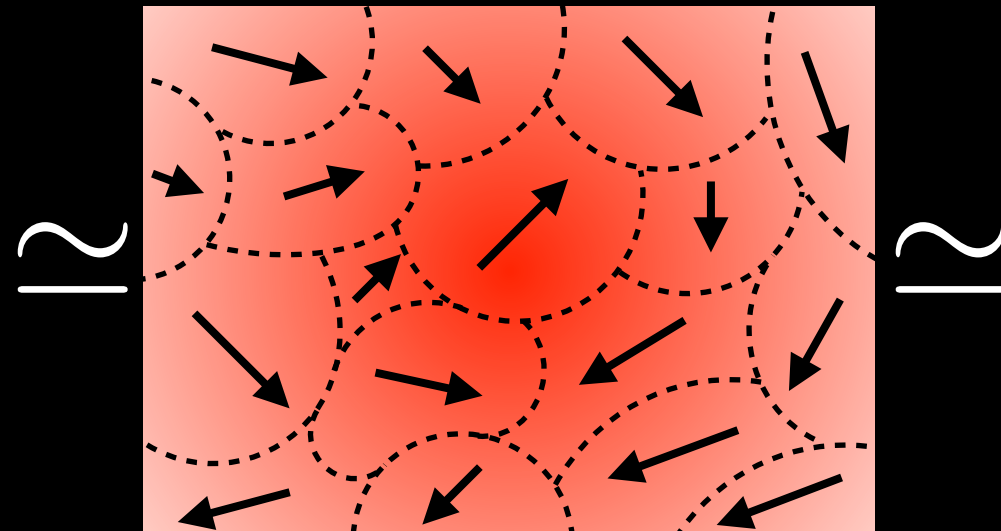
- **Effective theory** for **macroscopic dynamics**
- **Universal description**, not depending on details
- Only **conserved quantity = symmetry**

Quark-Gluon Plasma

10^{-12} cm

$T \sim 10^{12}$ K

Hydro: $\{\beta(x), \vec{v}(x)\}$



Neutron Star

10 km

$\rho \sim 10^{12}$ kg/cc

Hydrodynamics is

How can we derive

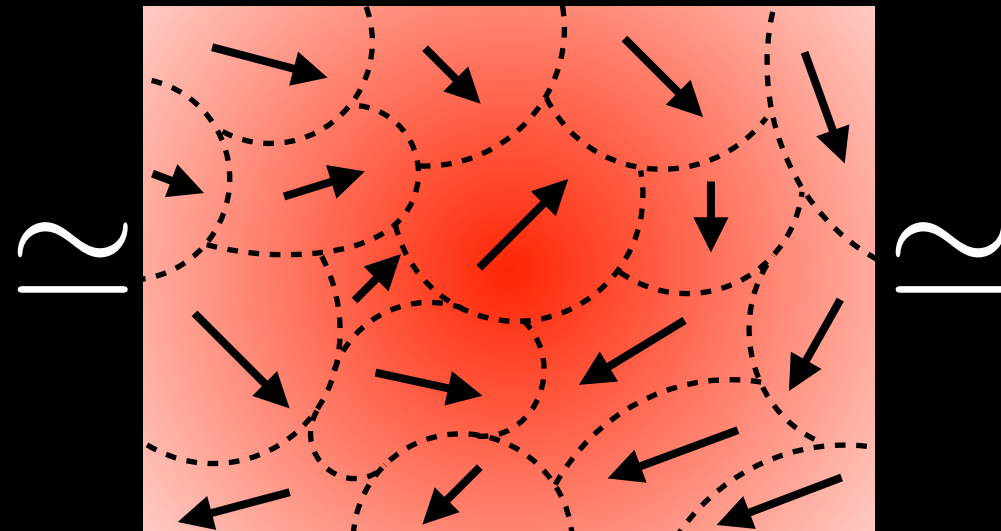
hydrodynamics from micro?

Quark-Gluon Plasma

10^{-12} cm

$T \sim 10^{12}$ K

Hydro: $\{\beta(x), \vec{v}(x)\}$

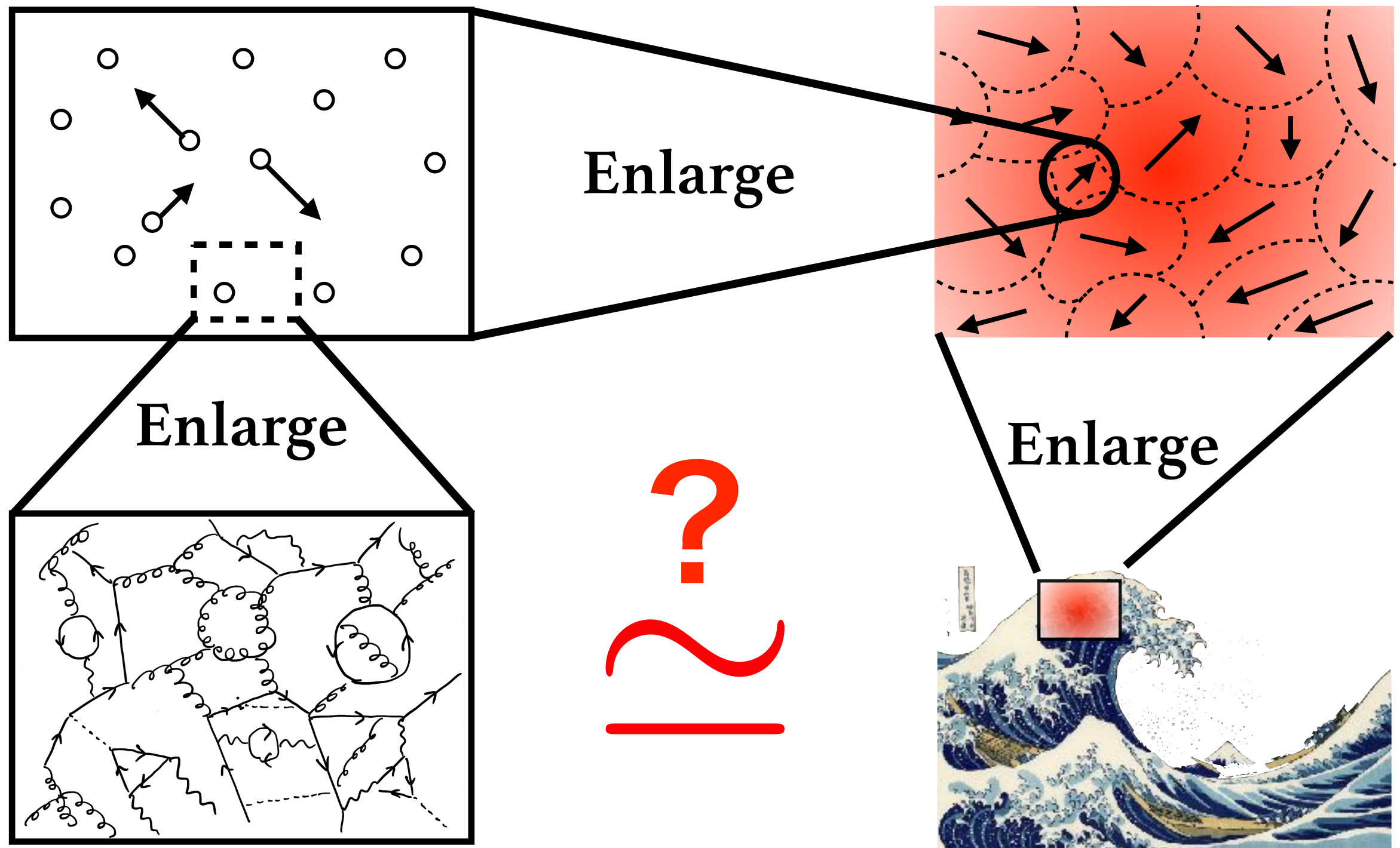


Neutron Star

10 km

$\rho \sim 10^{12}$ kg/cc

Micro(QFT) and Macro(Hydro)

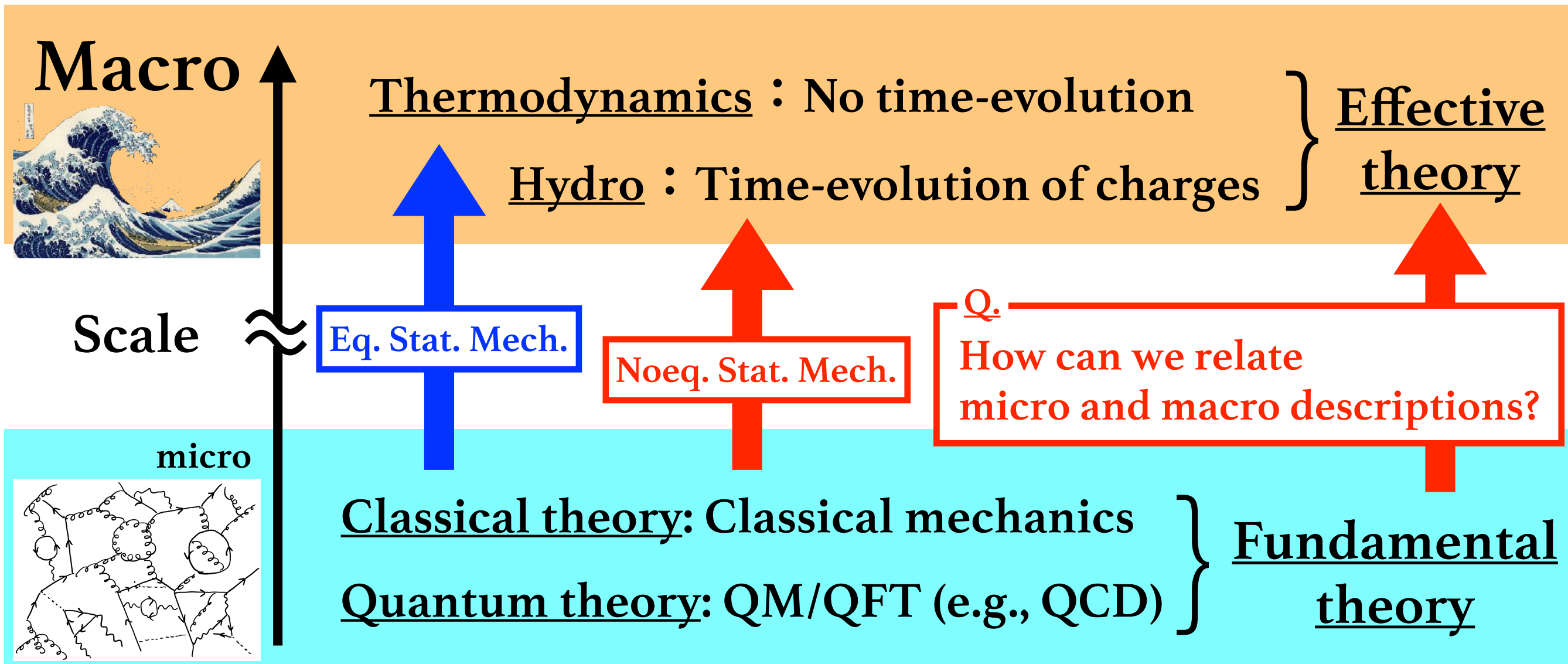


Micro description: QFT

Macro description: hydro

Hydro as **Nonequilibrium**

Q. What is the problem?



A. I would like to relate **effective theory** and **fundamental theory**!

(Macro) (Micro)

「Nonlinear」 hydrodynamics

◆ Nonrelativistic Navier-Stokes eq. = **Nonlinear**!

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \Delta \mathbf{v} + \left(\frac{\zeta}{\rho} + \frac{1}{3} \frac{\eta}{\rho} \right) \nabla (\nabla \cdot \mathbf{v}) + \mathbf{K}$$

(1) Conventional hydrodynamics

- EOM = Nonlinear **deterministic** PDE!
- Dynamics of **Avg.** of conserved charge densities

In terms of QFT

Classical Yang-Mills
equation of motion
is **nonlinear**!

(2) Fluctuating hydrodynamics

- EOM = Nonlinear **stochastic** PDE!
- Dynamics of ***n*-pt fcn.** of charge densities
(Nonlinear hydro fluctuation)

Quantum fluctuation
coming from
nonlinear interaction
of Yang-Mills theory

Outline: Hydro from QFT

Part 1. **Conventional** hydrodynamics [Ref. Hayata-Hidaka-MH-Noumi, (2015), MH .(2017)]

Part 2. **Fluctuating** hydrodynamics [Ref. Crossley-Glorioso-Liu, (2017) Delacretaz, 2606.02391]

Outline: Hydro from QFT

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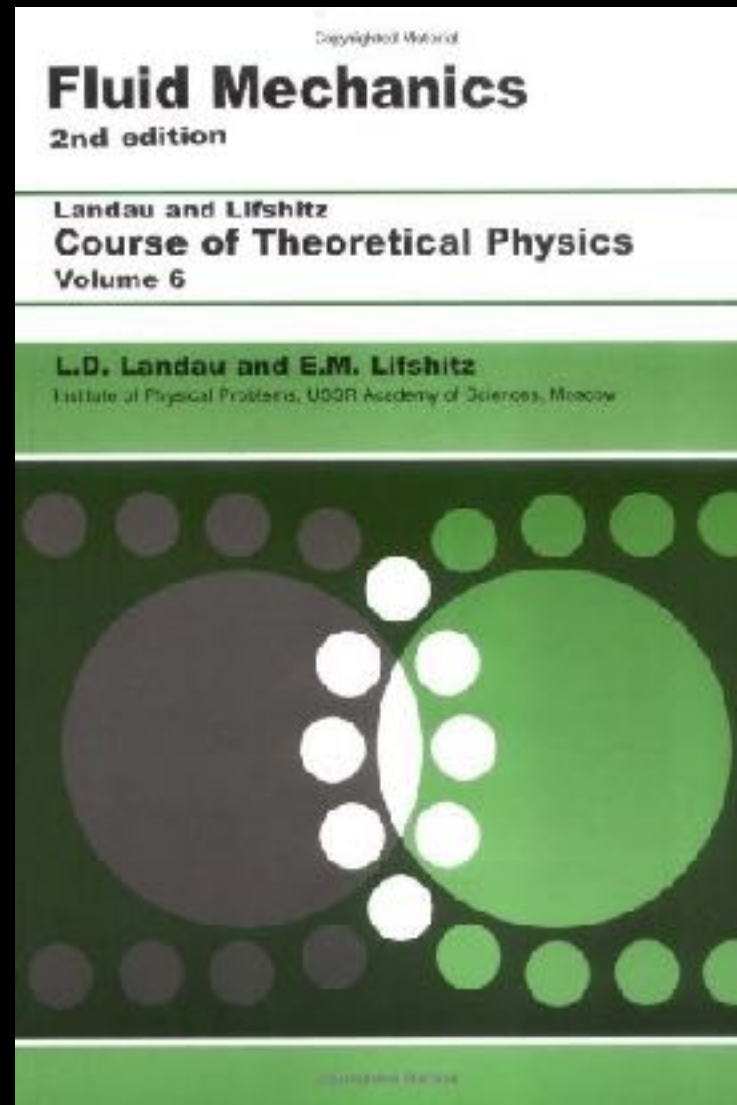
Part 1. Question part

What is hydro ?

Main Question in Part I

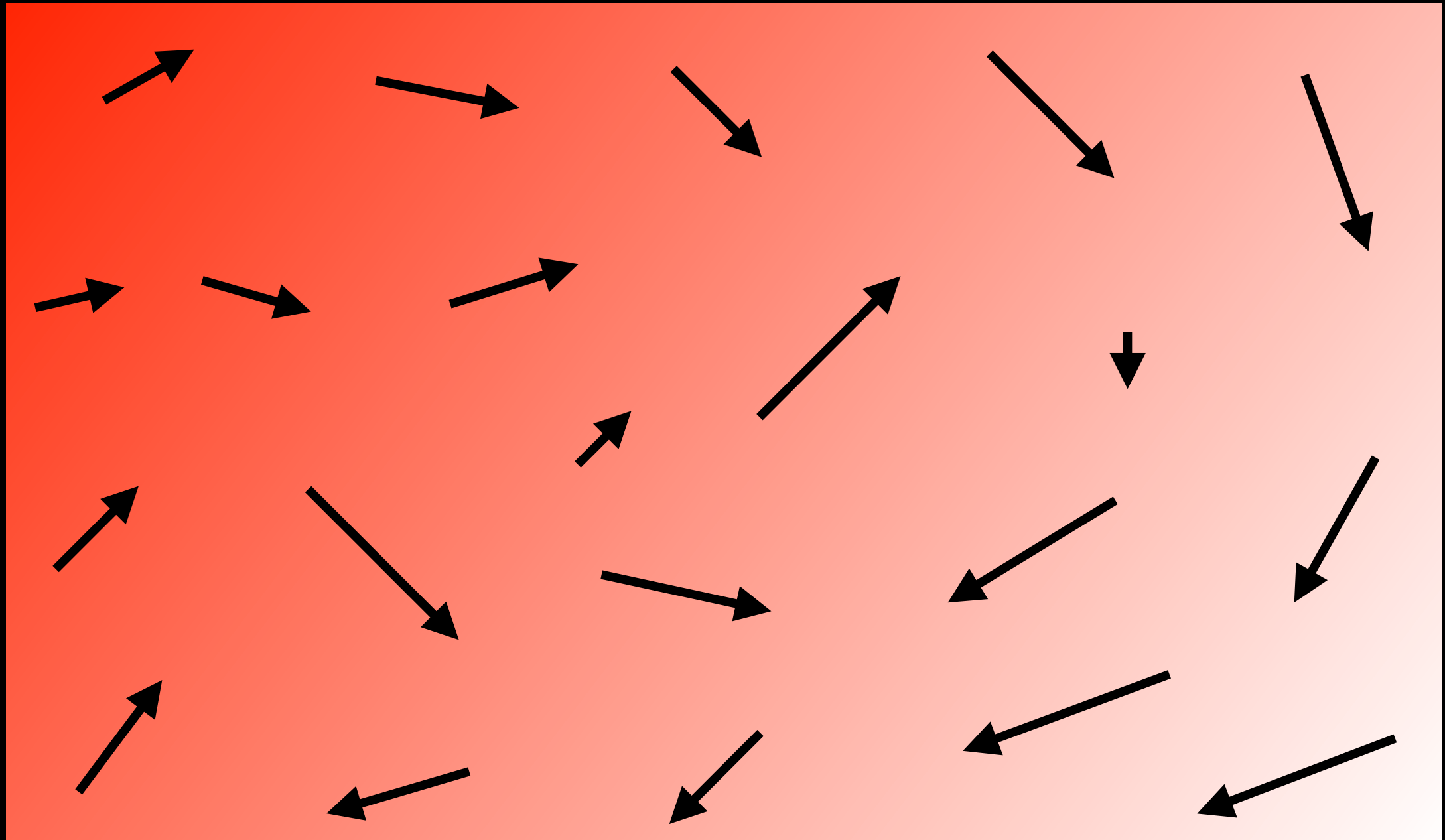
Q. Why $T^{\mu\nu} = (e + p)u^\mu u^\nu + pg^{\mu\nu} + \dots$?

Answer 1.



Answer2. Part I of My talk

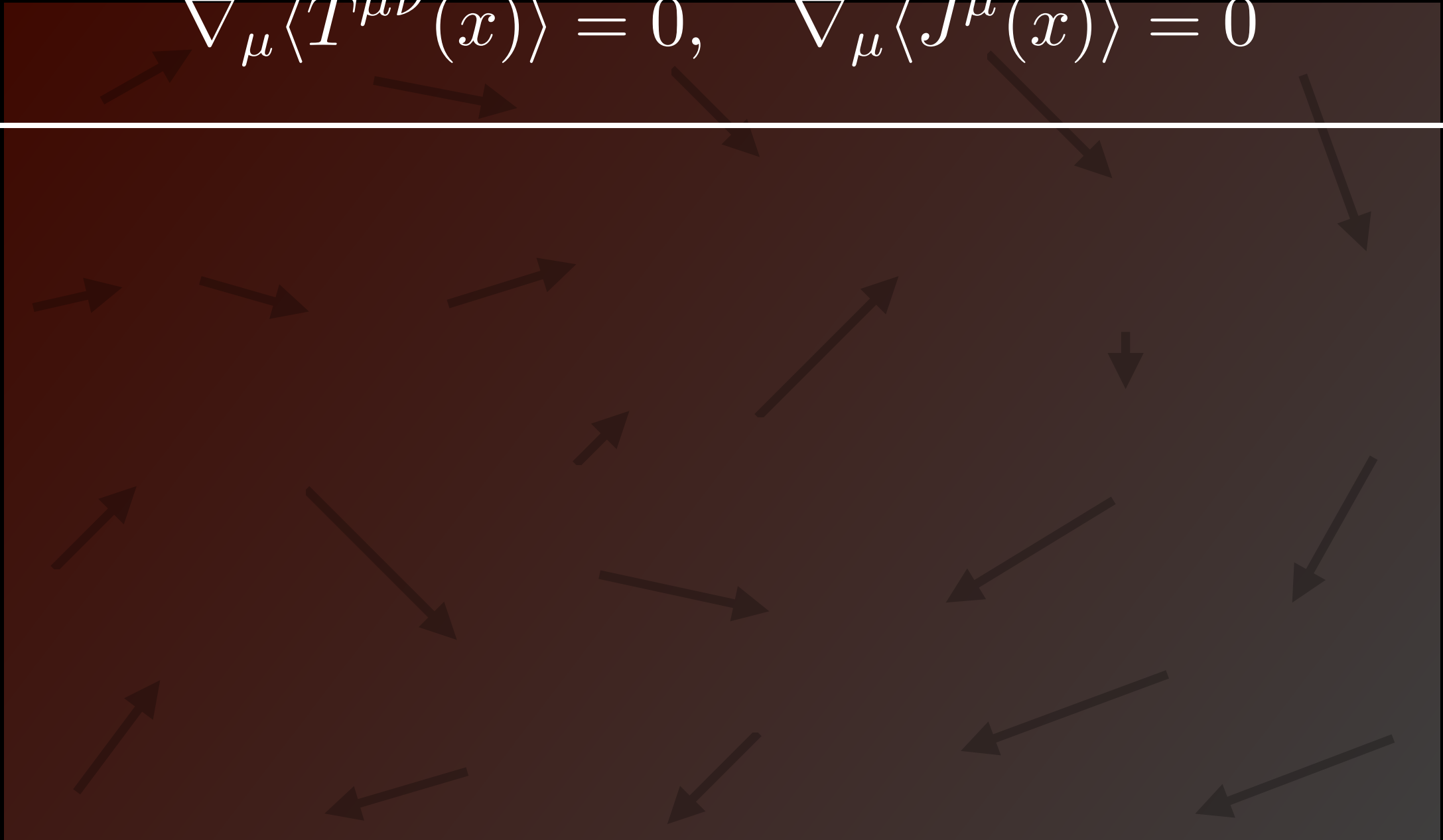
Hydrodynamic equation?



Hydrodynamic equation?

Conservation laws

$$\nabla_{\mu} \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_{\mu} \langle \hat{J}^{\mu}(x) \rangle = 0$$



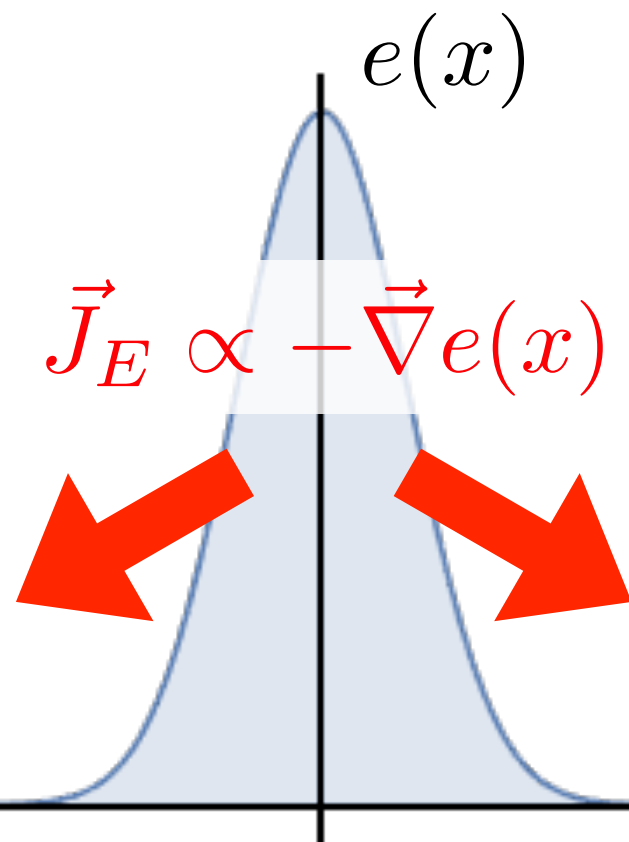
Prototype: Energy diffusion

◆ Bulding blocks of hydrodynamic equation

(1) Conservation law: $\partial_t e + \vec{\nabla} \cdot \vec{J}_E = 0$

(2) Constitutive relation: $\vec{J}_E = -D_E \vec{\nabla} e$

(3) Physical properties: Value of diffusion constant D_E

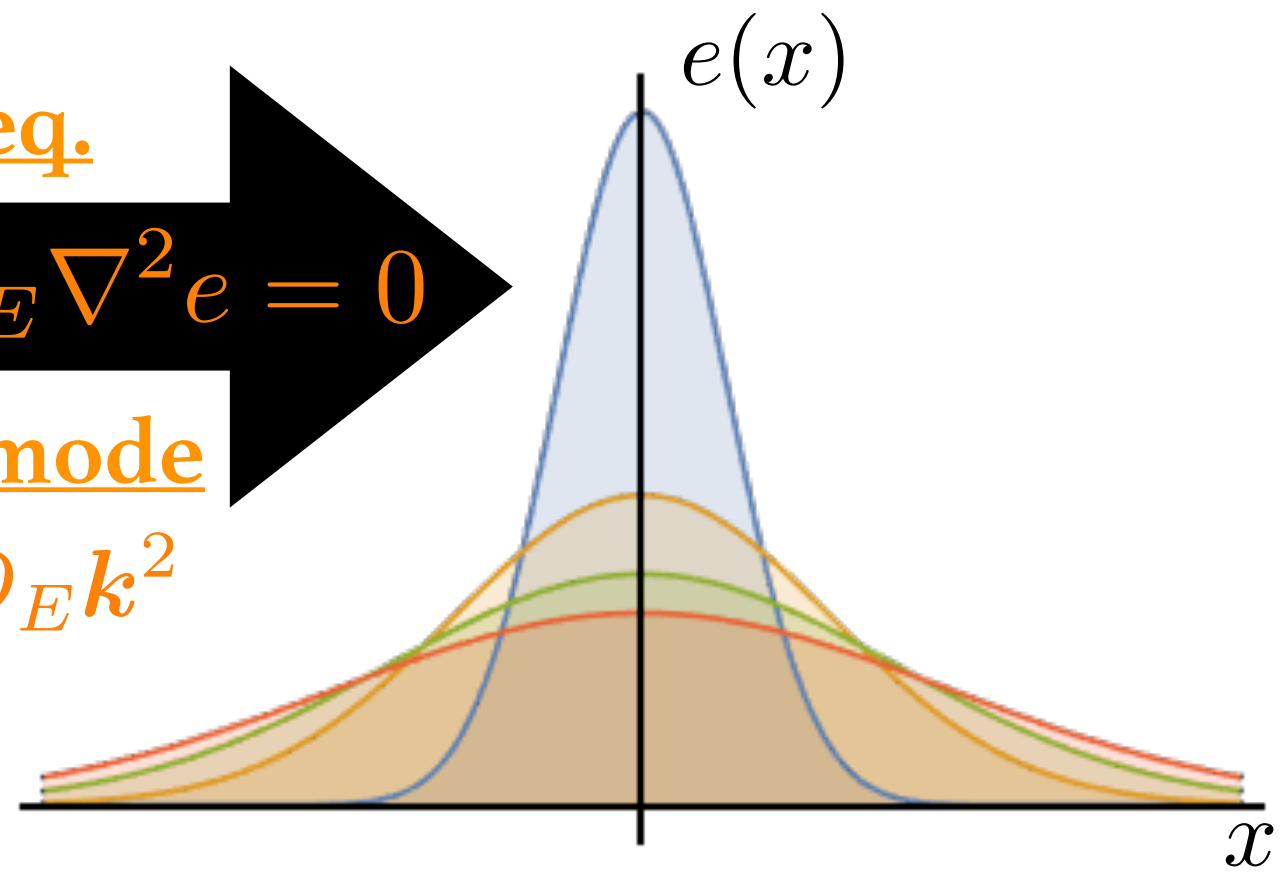


Diffusion eq.

$$\partial_t e - D_E \nabla^2 e = 0$$

Diffusion mode

$$\omega = -iD_E k^2$$



Hydrodynamic equation?

Given

Conservation laws

$$\nabla_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_\mu \langle \hat{J}^\mu(x) \rangle = 0$$



Constitutive relations

Avg. of current op. is determined by charge density!

$$\langle \hat{T}^{\mu\nu}(x) \rangle = T^{\mu\nu}[T^{0\nu}, J^0] = T^{\mu\nu}[\beta^\nu, \nu]$$

$$\langle \hat{J}^\mu(x) \rangle = J^\mu[T^{0\nu}, J^0] = J^\mu[\beta^\nu, \nu]$$



Physical properties

Equation of State (static): $p = p[T^{0\nu}, J^0] = p[\beta^\nu, \nu]$

Transport coeff. (dynamic): $L_i = L_i[T^{0\nu}, J^0] = L_i[\beta^\nu, \nu]$

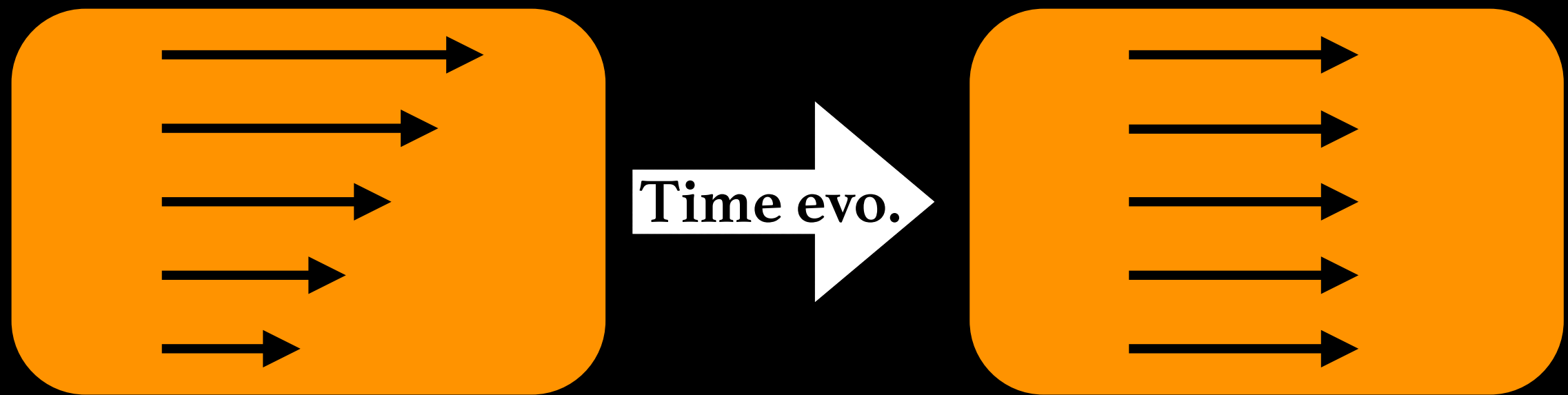
Non-equilibrium

Statistical Mechanics in a Nutshell

(4 pages)

How to treat dissipation?

Ex. Relaxation by shear viscosity

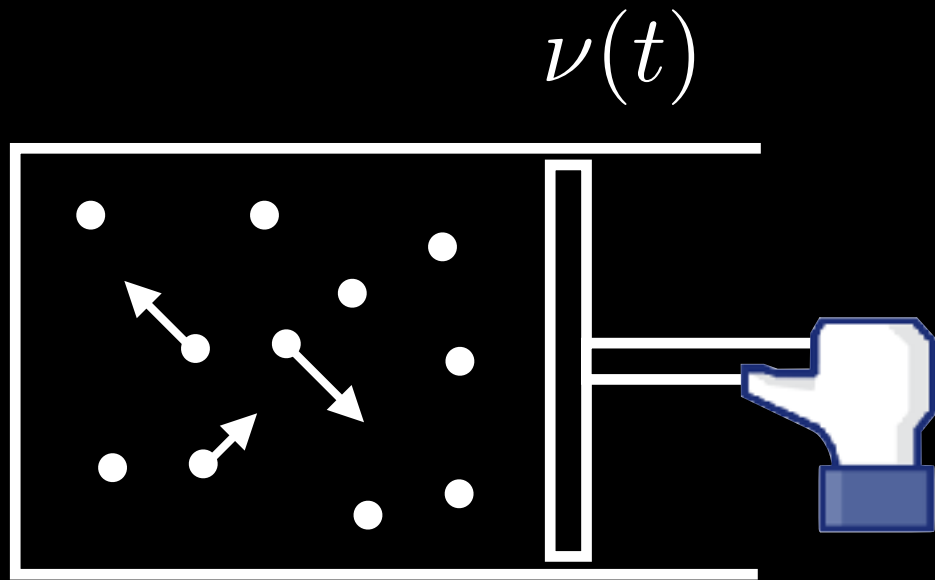


Friction-like phenomena between velocity gradient

From the 2nd law **Entropy** ↗

How the 2nd law is derived for equilibrium?

2nd law: Kelvin's principle



$$\Gamma = (\vec{r}_1, \dots, \vec{r}_N, \vec{p}_1, \dots, \vec{p}_N)$$

$$\mathcal{H}(\Gamma) = \sum_i \frac{\vec{p}_i^2}{2m} + V(\vec{r}_1, \dots, \vec{r}_N; \nu)$$

Consider a operation s.t.

$$\nu(t) : \underbrace{\nu(0)}_{\nu_0} = \underbrace{\nu(\tau)}_{\nu_1}$$

When the system evolves as $\Gamma \rightarrow \Gamma_t$ with the Hamilton EoM,

work defined by $W(\Gamma) \equiv H_{\nu_1}(\Gamma_t) - H_{\nu_0}(\Gamma)$

satisfies $\langle W(\Gamma) \rangle \geq 0$ for equilibrium state Γ

Derivation of Kelvin's principle

For **equilibrium state** Γ $\langle W(\Gamma) \rangle \geq 0$ is satisfied

For canonical ensemble $\ni \Gamma$

$$\int d\Gamma \frac{1}{Z} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} W(\Gamma) \geq 0$$

Proof.

$$\begin{aligned} \frac{1}{Z_{\beta, \nu_0}} \int d\Gamma e^{-\beta W(\Gamma)} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} &= \frac{1}{Z_{\beta, \nu_0}} \int d\Gamma e^{-\beta (H_{\nu_1}(\Gamma_t) - H_{\nu_0}(\Gamma))} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} \\ &= \frac{1}{Z_{\beta, \nu_0}} \int d\Gamma_t \left| \frac{d\Gamma_t}{d\Gamma} \right| e^{-\beta H_{\nu_1}(\Gamma_t)} = \frac{Z_{\beta, \nu_1}}{Z_{\beta, \nu_0}} = 1 \end{aligned}$$

$$\langle e^{-\beta W(\Gamma)} \rangle = 1 \geq 1 - \beta \langle W(\Gamma) \rangle \quad \longrightarrow \quad \boxed{\langle W(\Gamma) \rangle \geq 0}$$

Supplement: Jarzynski equality

Recalling $Z_{\beta,\nu} = e^{-\beta F(\beta,\nu)}$ leads to

[Jarzynski, 1997]

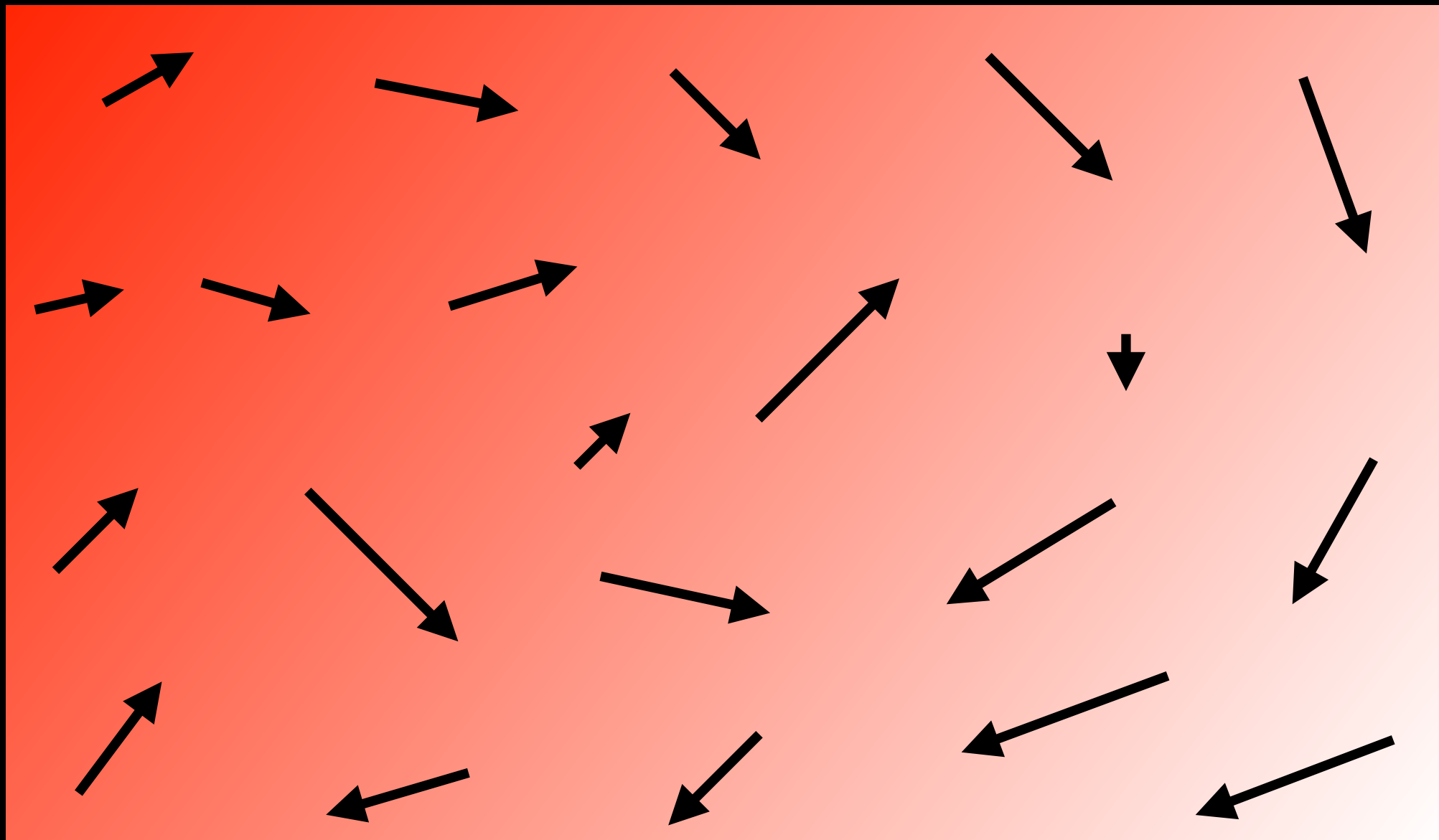
$$\langle e^{-\beta W(\Gamma)} \rangle = e^{-\beta(F(\beta,\nu_1) - F(\beta,\nu_0))}$$

Proof.

$$\begin{aligned} \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma e^{-\beta W(\Gamma)} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} &= \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma e^{-\beta(H_{\nu_1}(\Gamma_t) - H_{\nu_0}(\Gamma))} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} \\ &= \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma_t \left| \frac{d\Gamma_t}{d\Gamma} \right| e^{-\beta H_{\nu_1}(\Gamma_t)} = \frac{Z_{\beta,\nu_1}}{Z_{\beta,\nu_0}} \end{aligned}$$

Lesson from simple exercise

Prepare an **appropriate initial ensemble**
which describes **local thermodynamics**



Part 1. **Answer** part

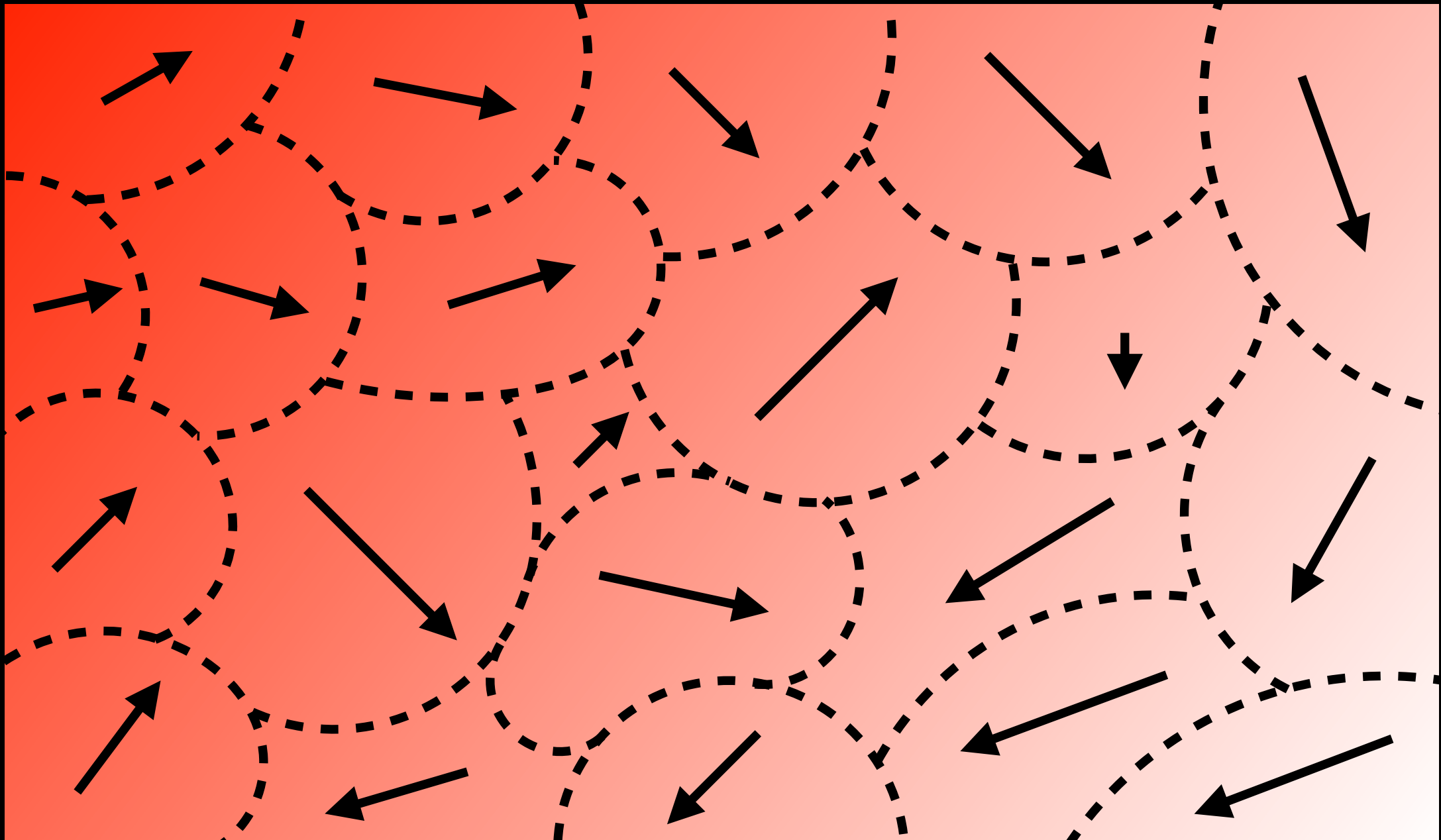
Deriving hydro from **QFT**

[Refs. Hayata-Hidaka-MH-Noumi, PRD(2015), MH Ann. Phys.(2017)]

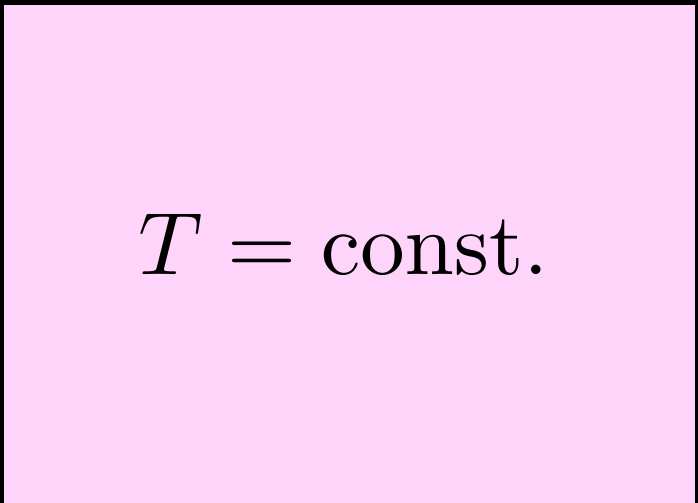
Lesson from noneq. stat-phys

Prepare an **appropriate initial ensemble!**

→ Quench from **initial local thermal state**



How to describe local thermal equil.

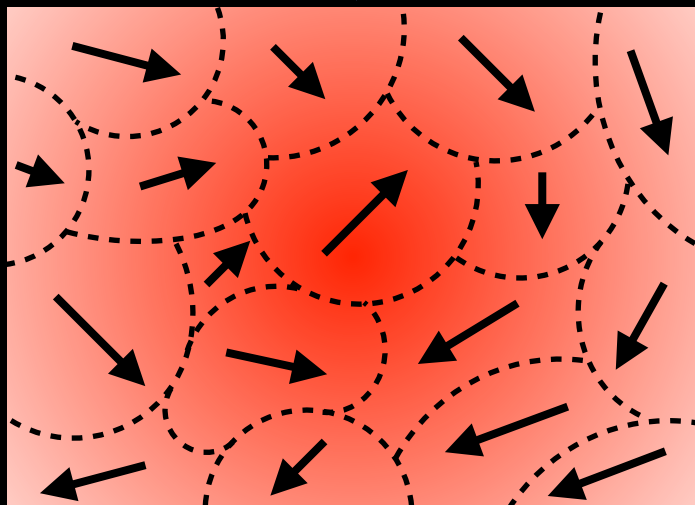

$$T = \text{const.}$$

Global thermal equilibrium:

Gibbs distribution:

$$\hat{\rho}_G = e^{-\beta \hat{H} - \Psi[\beta]}, \quad \Psi[\beta] \equiv \log \text{Tr} e^{-\beta \hat{H}}$$

Localize



$$\{\beta(x), \vec{v}(x)\}$$

Local thermal equilibrium:

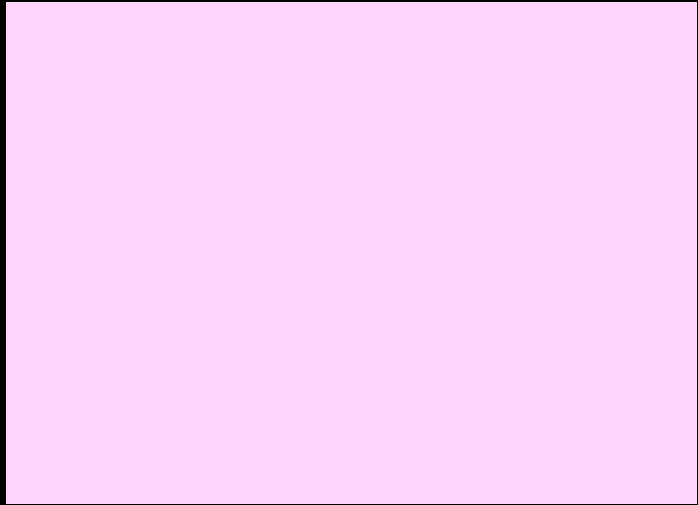
Local Gibbs (LG) distribution:

$$\hat{\rho}_{LG} = e^{-\hat{K} - \Psi[\beta^\mu(x), \nu(x)]}$$

$$\hat{K} = - \int d^3x \left(\beta^\mu(x) \hat{T}^0_\mu(x) + \nu(x) \hat{J}^0(x) \right)$$

What is Local Gibbs distribution?

Gibbs distribution



What is the state with maximizing
information entropy: $S(\hat{\rho}) = -\text{Tr} \hat{\rho} \log \hat{\rho}$

under constraints: -----

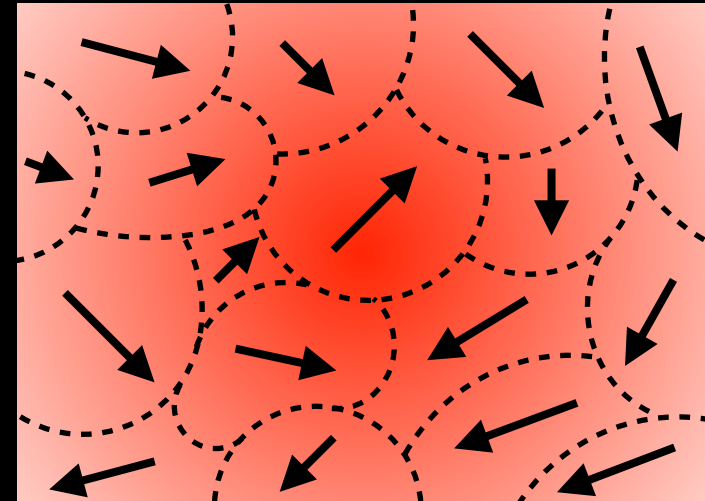
$$\langle \hat{H} \rangle = E = \text{const.}, \quad \langle \hat{N} \rangle = N = \text{const.}$$

Answer:

$$\hat{\rho}_G = e^{-\beta \hat{H} - \nu \hat{N} - \Psi[\beta, \nu]}$$

Lagrange multipliers: $\Lambda^a = \{\beta, \nu = \beta \mu\}$

Local Gibbs distribution



What is the state with maximizing
information entropy: $S(\hat{\rho}) = -\text{Tr} \hat{\rho} \log \hat{\rho}$

under constraints: -----

$$\langle \hat{T}_{\mu}^0(x) \rangle = p_{\mu}(x), \quad \langle \hat{J}^0(x) \rangle = n(x)$$

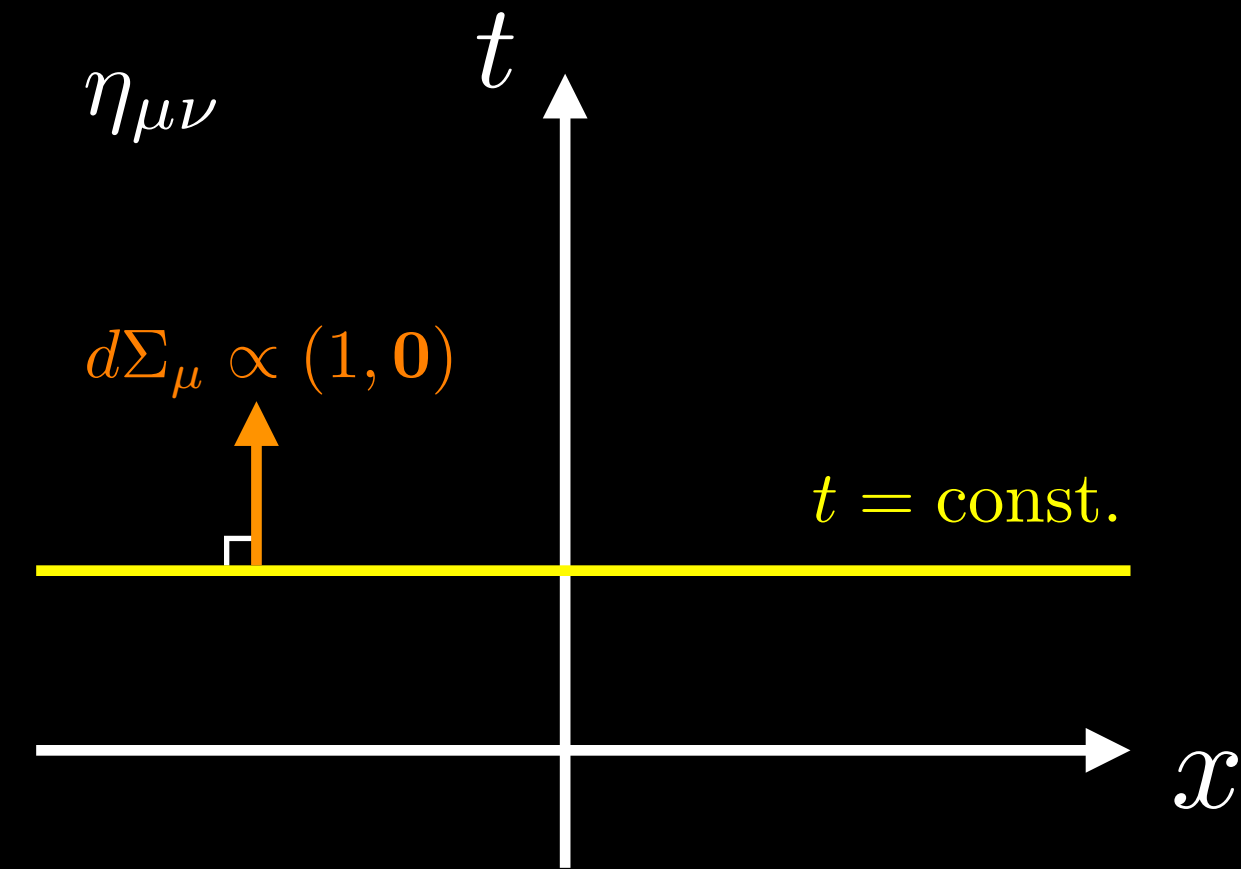
Answer:

$$\hat{\rho}_{LG} = e^{-\int d^{d-1}x (\beta^{\mu} \hat{T}_{\mu}^0 + \nu \hat{J}^0) - \Psi[\beta^{\mu}, \nu]}$$

Lagrange multipliers: $\lambda^a(x) = \{\beta^{\mu}(x), \nu(x)\}$

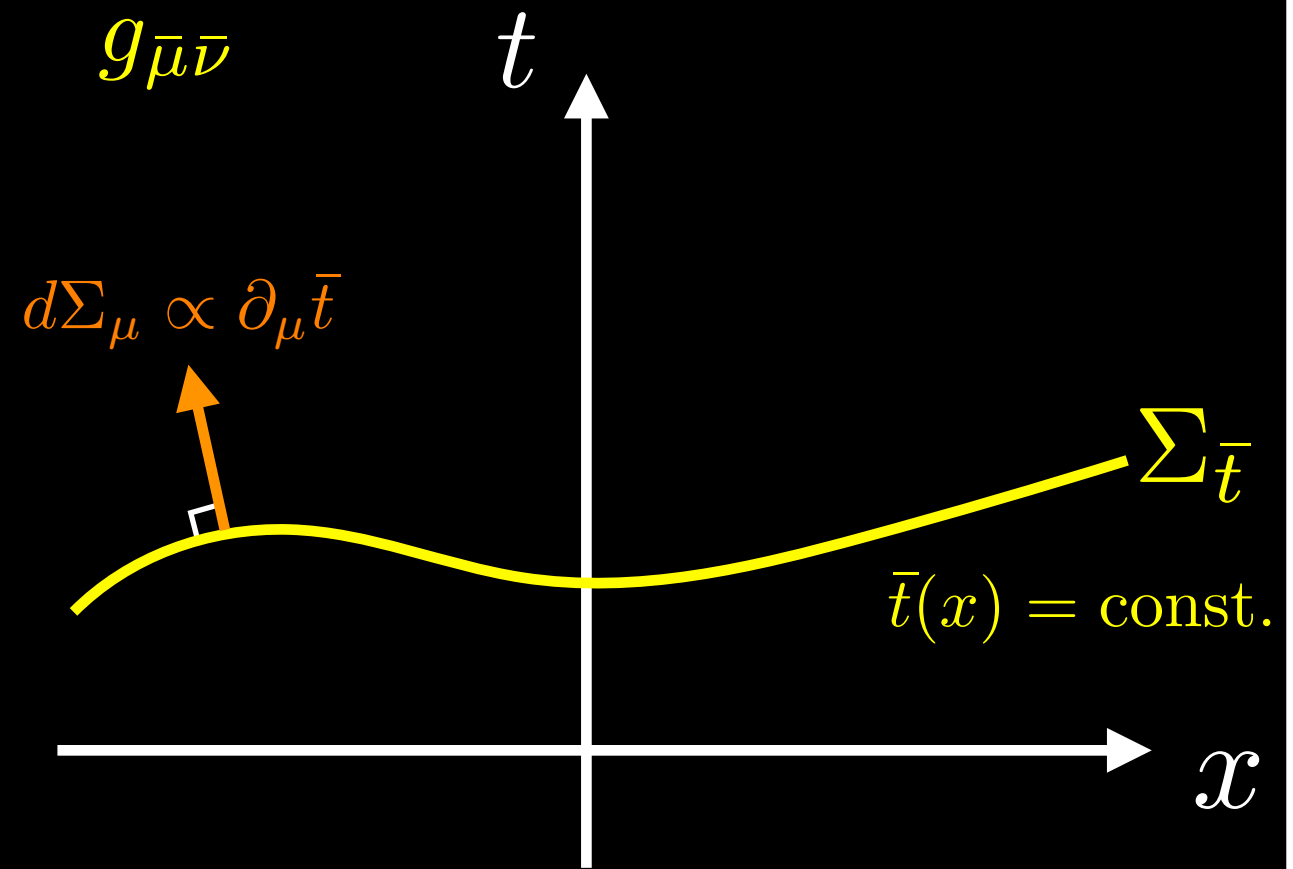
Introducing background metric

Flat spacetime



$$\hat{K} = - \int d^3x \left(\beta^\mu(x) \hat{T}_\mu^0(x) + \nu(x) \hat{J}^0(x) \right)$$

Curved spacetime



$$\hat{K} = - \int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}_\mu^\nu(x) + \nu(x) \hat{J}^\nu(x) \right)$$

- ➔ {
- ① Formulation becomes manifestly covariant
 - ② Background metric plays a role as external field coupled to $T^{\mu\nu}$

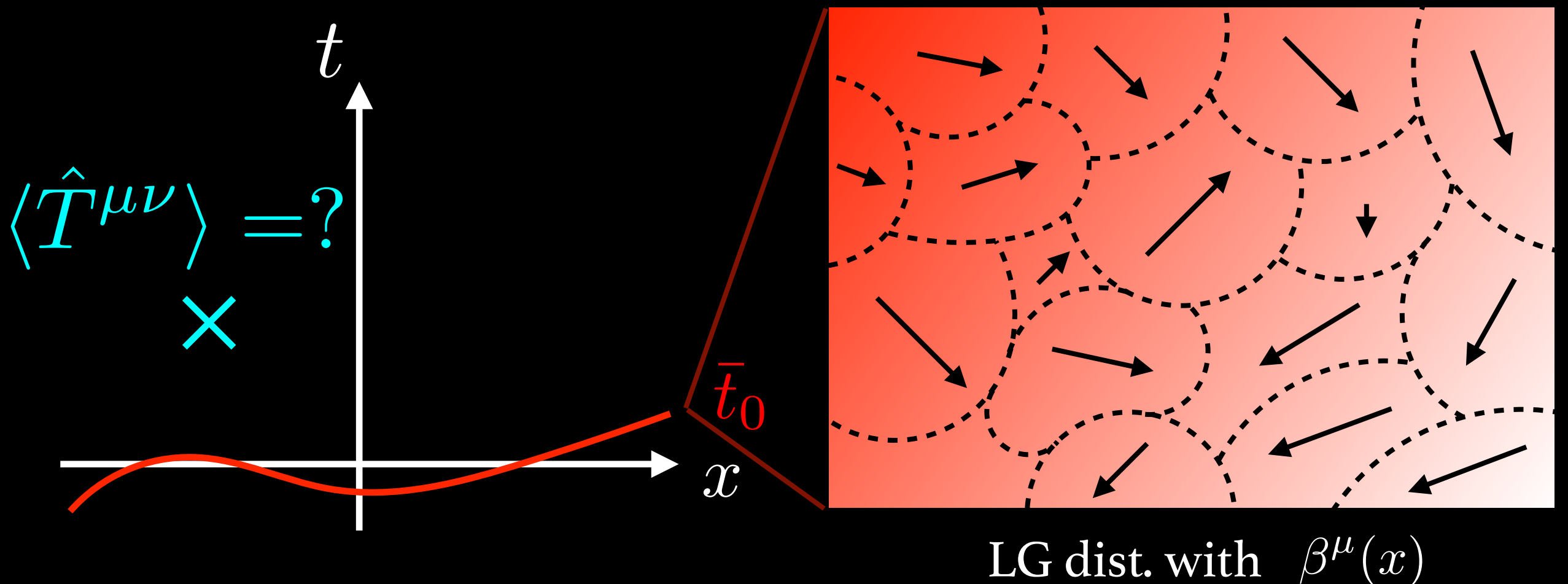
Dissipative part

Difficulty of problem

[Sasa PRL (2014), Hayata, Hidaka, MH, Noumi PRD (2015)]

Initial density operator: $\hat{\rho}(\bar{t}_0) = \hat{\rho}_{LG}[\bar{t}_0; \lambda] \equiv \exp \left[-\hat{S}[\bar{t}_0; \lambda] \right]$

Taking Heisenberg picture: $\langle \hat{\mathcal{O}}(x) \rangle = \text{Tr} \hat{\rho}(\bar{t}_0) \hat{\mathcal{O}}(x)$

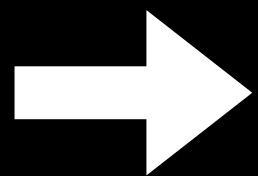


➡ Naive perturbation breaks down due to time evolution!!

Renormalized/optimized perturbation

When we cannot solve problem exactly

& naive perturbation **breaks down**



Choosing an appropriate “**Variational State**”

& reorganizing perturbation often works!!

= **Renormalized/optimized** perturbation

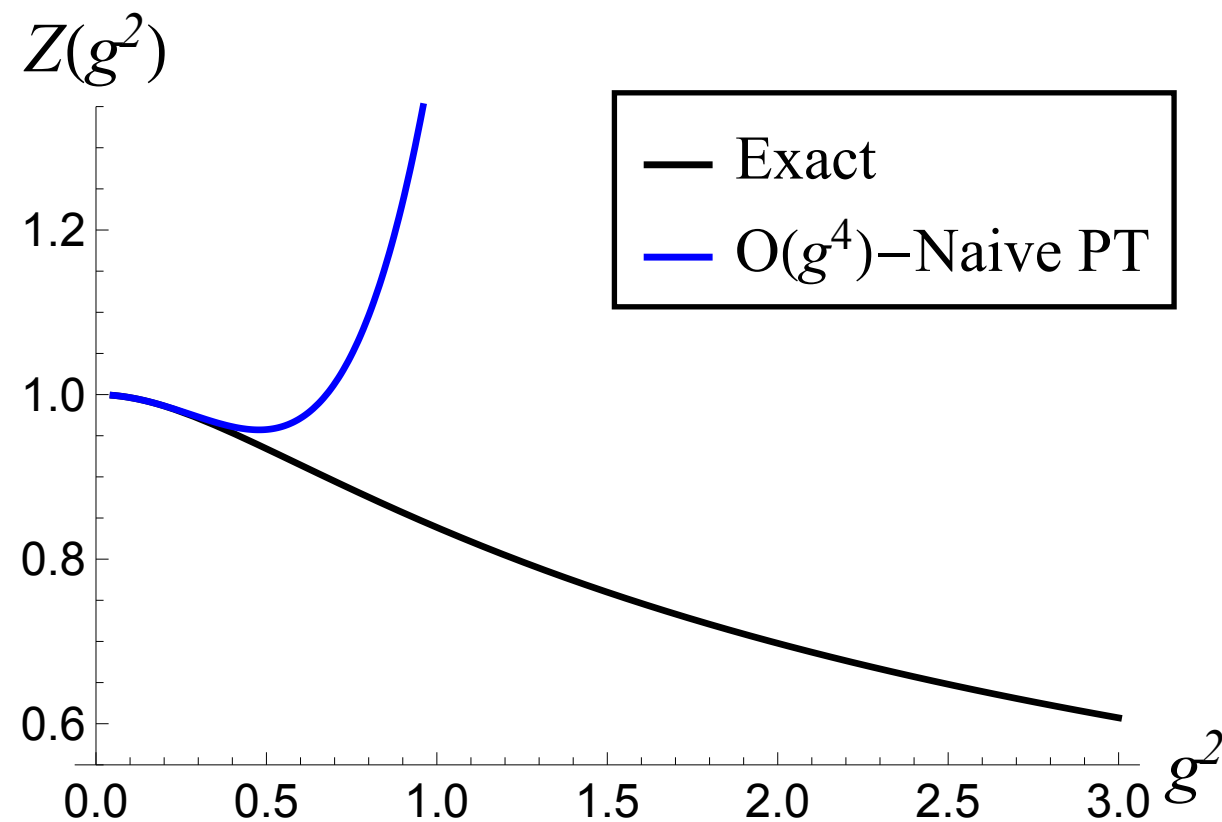
Idea of RPT/OPT: 1-dim. integral

$$Z(g^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - \frac{g^2}{8}x^4} = \frac{1}{\sqrt{2\pi g^2}} e^{\frac{1}{4g^2}} K_{1/4} \left(\frac{1}{4g^2} \right)$$

◆ **Naive** perturbation (Expansion w.r.t. g^2)

$$Z(g^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{g^2}{8}x^4 \right)^n = 1 - \frac{3}{8}g^2 + \frac{105}{128}g^4 + \dots$$

divergent!!



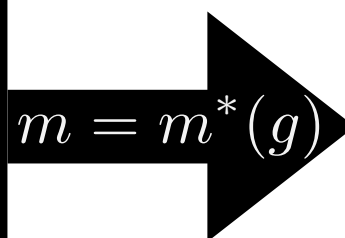
Idea of RPT/OPT: 1-dim. integral

$$Z(g^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - \frac{g^2}{8}x^4} = \frac{1}{\sqrt{2\pi g^2}} e^{\frac{1}{4g^2}} K_{1/4} \left(\frac{1}{4g^2} \right)$$

◆ Optimized/renormalized perturbation

$$\begin{aligned} Z(g^2) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{m^2}{2}x^2 + \frac{m^2-1}{2}g^2 - \frac{g^2}{8}x^4} \\ &\quad \text{“Ground state”} \quad \text{Perturbative} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{m^2}{2}x^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\delta m^2}{2}g^2 - \frac{g^2}{8}x^4 \right)^n \\ &\simeq \frac{1}{m} + \frac{\delta m^2}{2m^3} - \frac{3g^2}{8m^5} \equiv Z_{\text{LO}}(g^2, m) \end{aligned}$$

Condition to fix m : $\frac{\partial Z_{\text{LO}}(g^2, m)}{\partial m} = 0$



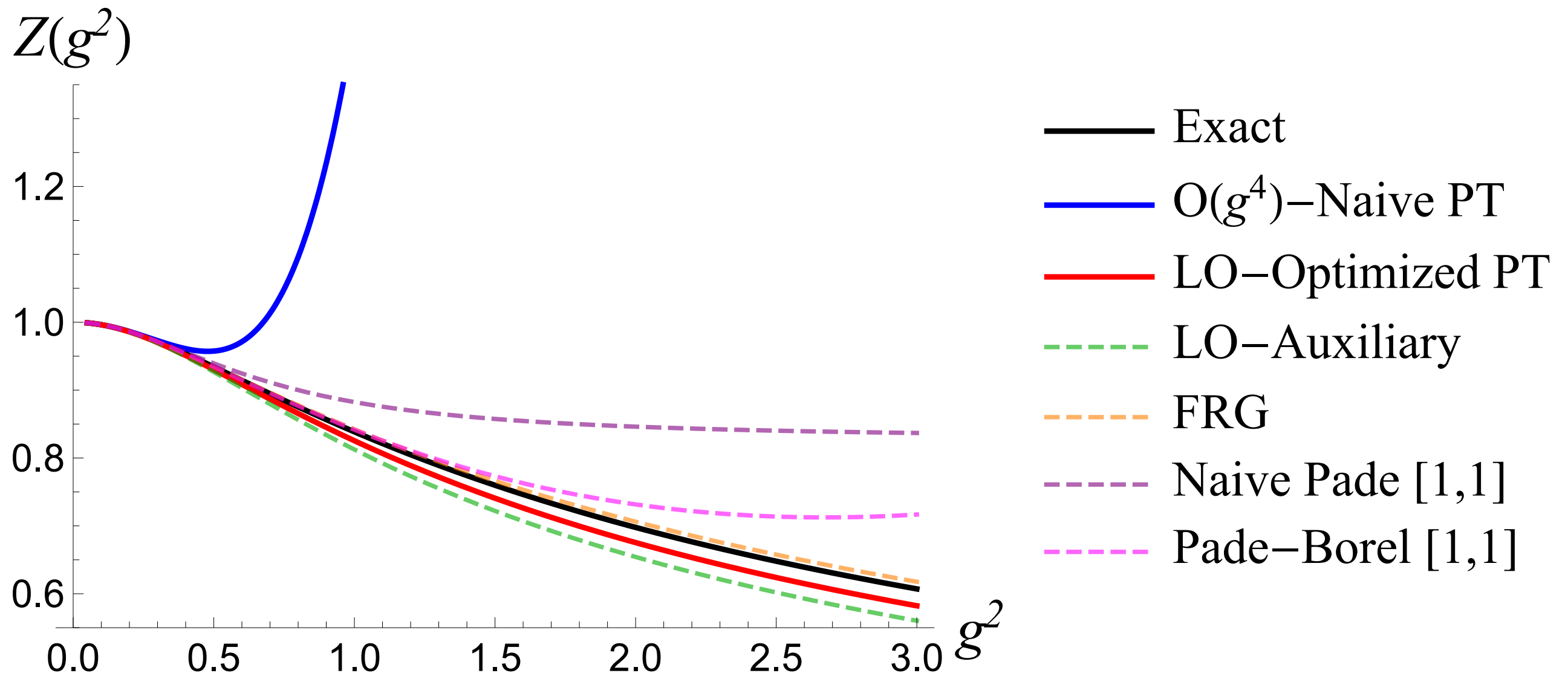
$$Z(g^2) \simeq Z_{\text{LO}}(g^2, m^*(g))$$

(Principle of Minimal Sensitivity)

Idea of RPT/OPT: 1-dim. integral

$$Z(g^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - \frac{g^2}{8}x^4} = \frac{1}{\sqrt{2\pi g^2}} e^{\frac{1}{4g^2}} K_{1/4} \left(\frac{1}{4g^2} \right)$$

◆ Comparison of **naive** and **optimized** perturbation



Renormalized/optimized perturbation

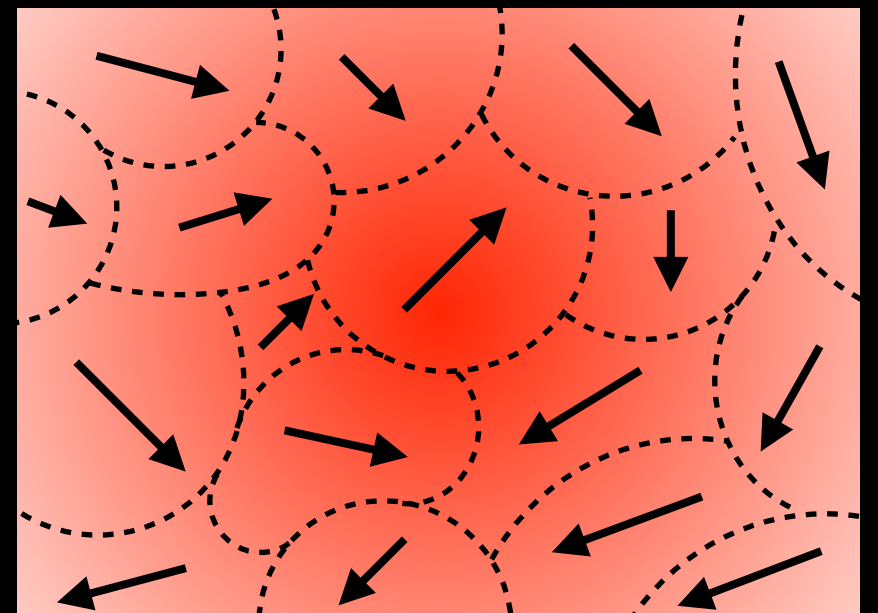
When we cannot solve problem exactly

& naive perturbation **breaks down**

➔ Choosing an appropriate “**Variational State**”
& reorganizing perturbation often works!!

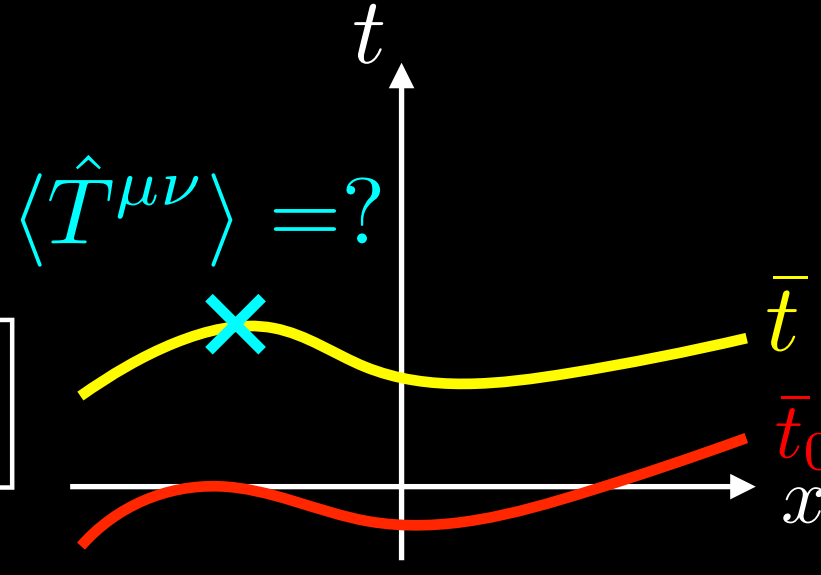
= **Renormalized/optimized** perturbation

In the case of hydrodynamics,
an appropriate “**Variational State**” is
Local thermal equilibrium!!



RPT/OPT for Time evolution

$$\begin{aligned}
 \hat{\rho}(\bar{t}_0) &= \hat{\rho}_{\text{LG}}[\bar{t}_0; \lambda] \equiv \exp \left[-\hat{S}[\bar{t}_0; \lambda] \right] \\
 &= \exp \left[\underbrace{-\hat{S}[\bar{t}; \lambda]}_{\text{LG dist. with } \lambda(\bar{t}) \text{ ("Variational state")}} + \underbrace{(\hat{S}[\bar{t}; \lambda] - \hat{S}[\bar{t}_0; \lambda])}_{\hat{\Sigma}[\bar{t}, \bar{t}_0] : \text{Deviation from LG ("Perturbation")}} \right]
 \end{aligned}$$



 $\langle \hat{T}^{\mu\nu} \rangle = ?$

$$= \hat{\rho}_{\text{LG}}[\bar{t}; \lambda] \hat{U}[\bar{t}, \bar{t}_0], \quad \hat{U}[\bar{t}, \bar{t}_0] \equiv T_\tau \exp \left(\int_0^1 d\tau \hat{\Sigma}_\tau[\bar{t}, \bar{t}_0] \right)$$

$$\langle \hat{\mathcal{O}}(x) \rangle = \langle \hat{U} \hat{\mathcal{O}}(x) \rangle_{\bar{t}}^{\text{LG}}$$

Condition to determine parameters

$$\langle \hat{\mathcal{O}}(x) \rangle = \langle \hat{U} \hat{\mathcal{O}}(x) \rangle_{\bar{t}}^{\text{LG}} \leftarrow \text{Parameter ?}$$

Fastest Apparent Convergence (FAC)

Corrections for conserved charges should be minimized!

$$\langle \hat{U} \delta \hat{\mathcal{J}}_a^{\bar{0}}(x) \rangle_{\bar{t}}^{\text{LG}} = 0 \Leftrightarrow \langle \hat{\mathcal{J}}_a^{\bar{0}}(x) \rangle = \langle \hat{\mathcal{J}}_a^{\bar{0}}(x) \rangle_{\bar{t}}^{\text{LG}}$$

➡ Determine new parameters $\beta^\mu(x)$, $\nu(x)$ on $\Sigma_{\bar{t}}$

➡ Avg. of current is calculable !! (in principle)

Entropy production and 2nd law

Entropy production operator

$$\begin{aligned}\hat{\Sigma}[\bar{t}, \bar{t}_0] &\equiv \hat{S}[\bar{t}; \lambda] - \hat{S}[\bar{t}_0; \lambda] & (\delta\hat{\mathcal{O}} \equiv \hat{\mathcal{O}} - \langle\hat{\mathcal{O}}\rangle_{\bar{t}}^{\text{LG}}) \\ &= - \int_{\bar{t}_0}^{\bar{t}} d^4x \left[(\nabla_\mu \beta^\nu) \delta \hat{T}^\mu_\nu + (\nabla_\mu \nu) \delta \hat{J}^\mu \right]\end{aligned}$$

- Quantum fluctuation theorem & 2nd law of thermodynamics

$$\langle \hat{\Sigma}[\bar{t}, \bar{t}_0] \rangle \geq 0 \Leftrightarrow \langle \hat{S}[\bar{t}, \lambda] \rangle - \langle \hat{S}[\bar{t}_0, \lambda] \rangle \geq 0$$

- $\hat{\Sigma}$ is proportional to derivatives: $\langle \hat{\mathcal{O}}(x) \rangle = \langle \hat{U} \hat{\mathcal{O}}(x) \rangle_{\bar{t}}^{\text{LG}}$

“Expansion of $\hat{U} = T_\tau e^{\int_0^1 d\tau \hat{\Sigma}_\tau[\bar{t}, \bar{t}_0]}$ ” = Derivative expansion

Dissipative derivative expansion

$$\begin{aligned}\langle \hat{T}^{\mu\nu}(x) \rangle &= \langle \hat{U} \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}} \\ &= \langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}} + \boxed{\int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau[\bar{t}, \bar{t}_0] \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}}} + \dots \\ &\equiv \langle \delta \hat{T}^{\mu\nu}(x) \rangle_{(0,1)}\end{aligned}$$

$$\begin{aligned}\langle \hat{J}^\mu(x) \rangle &= \langle \hat{U} \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}} \\ &= \langle \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}} + \boxed{\int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau[\bar{t}, \bar{t}_0] \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}}} + \dots \\ &\equiv \langle \delta \hat{J}^\mu(x) \rangle_{(0,1)}\end{aligned}$$

Constitutive relation & Kubo formula

Constitutive relations

$$\langle \delta \hat{T}^{\mu\nu}(x) \rangle_{(0,1)} = -\frac{\zeta}{\beta} h^{\mu\nu} h^{\rho\sigma} \nabla_\rho \beta_\sigma - 2\frac{\eta}{\beta} h^{\mu\rho} h^{\nu\sigma} \nabla_{\langle\rho} \beta_{\sigma\rangle}$$

$$\langle \delta \hat{J}^\mu(x) \rangle_{(0,1)} = -\frac{\kappa}{\beta} h^{\mu\nu} \nabla_\nu \nu \quad (h^{\mu\nu} n_\nu = 0)$$

Kubo formulae

$$\zeta = \beta(x) \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta} \hat{p}(x') e^{-\hat{K}\tau} \tilde{\delta} \hat{p}(x) \rangle_{\bar{t}}^{\text{LG}}$$

$$\eta = \frac{\beta(x)}{(d+1)(d-2)} \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta} \hat{\pi}^{\mu\nu}(x') e^{-\hat{K}\tau} \tilde{\delta} \hat{\pi}^{\rho\sigma}(x) \rangle_{\bar{t}}^{\text{LG}} h_{\mu\rho}(x) h_{\nu\sigma}(x)$$

$$\kappa = \frac{\beta(x)}{d-1} \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta} \hat{J}^\mu(x') e^{-\hat{K}\tau} \tilde{\delta} \hat{J}^\nu(x) \rangle_{\bar{t}}^{\text{LG}} h_{\mu\nu}(x)$$

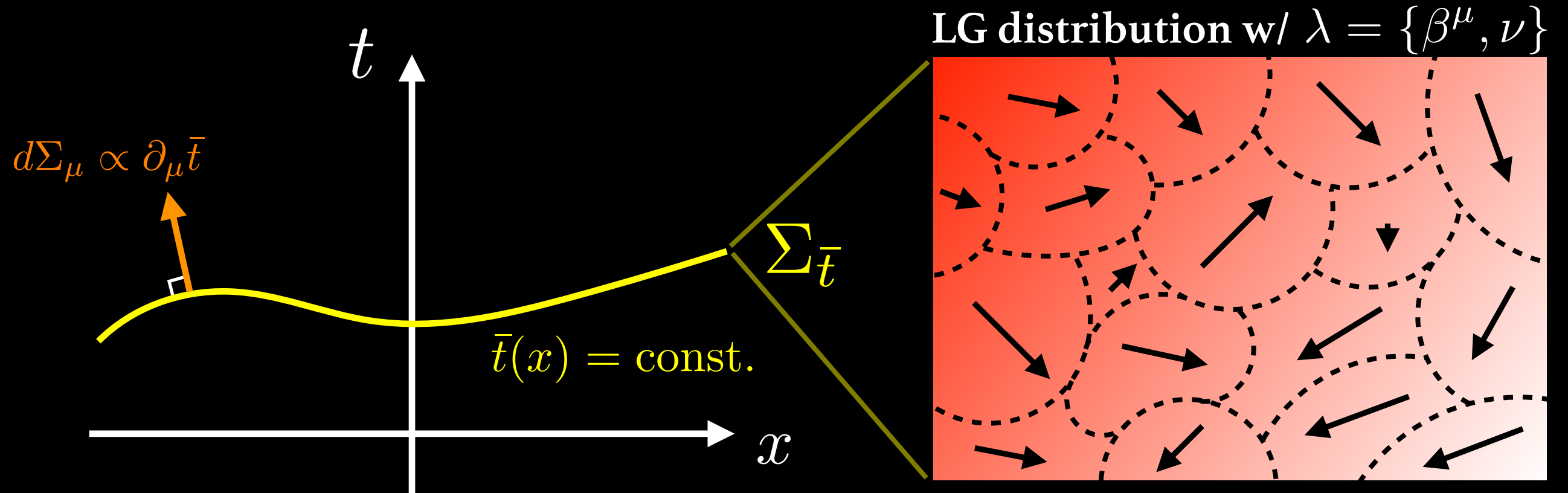
Non-dissipative part

$$\begin{aligned}
 \langle \hat{T}^{\mu\nu}(x) \rangle &= \langle \hat{U} \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}} \\
 &= \boxed{\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}}} + \boxed{\int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau[\bar{t}, \bar{t}_0] \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}}} + \dots \\
 &\quad \text{??} \qquad \qquad \qquad \equiv \langle \delta \hat{T}_{(1)}^{\mu\nu}(x) \rangle
 \end{aligned}$$

$$\begin{aligned}
 \langle \hat{J}^\mu(x) \rangle &= \langle \hat{U} \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}} \\
 &= \boxed{\langle \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}}} + \boxed{\int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau[\bar{t}, \bar{t}_0] \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}}} + \dots \\
 &\quad \text{??} \qquad \qquad \qquad \equiv \langle \delta \hat{J}_{(1)}^\mu(x) \rangle
 \end{aligned}$$

Preliminaries for Non-dissipative part

Local thermodynamic functional



Massieu-Planck functional

$$\Psi[\bar{t}; \lambda] = \log \text{Tr} \exp \left[- \int d^3 \bar{x} \sqrt{-g} \left(\beta^{\bar{\mu}}(\bar{x}) \hat{T}^{\bar{0}}_{\bar{\mu}}(\bar{x}) + \nu(\bar{x}) \hat{J}^{\bar{0}}(\bar{x}) \right) \right]$$

Variation formula in “hydrostatic gauge”

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t}; \lambda], \quad \langle \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta A_\mu(x)} \Psi[\bar{t}; \lambda]$$

Q. How can we calculate $\Psi \equiv \log Z$?

Thermal QFT in a Nutshell

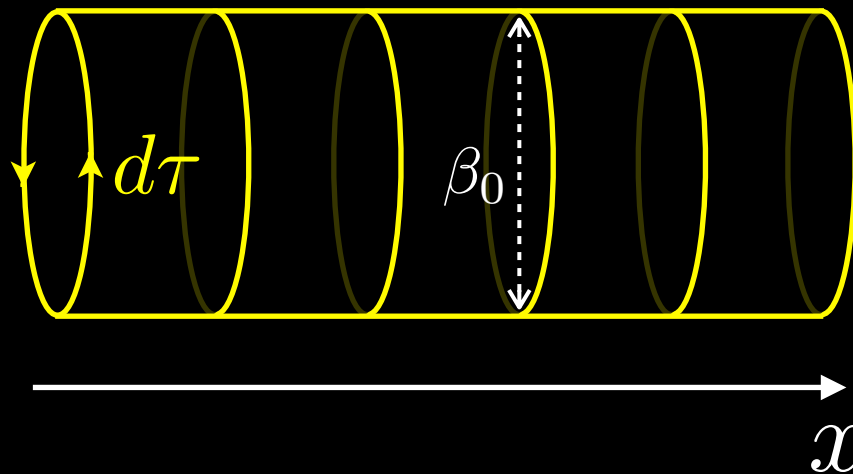
Global equil. β_0

$$T = \text{const.}$$

Path int.

Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the
flat spacetime
with size β_0

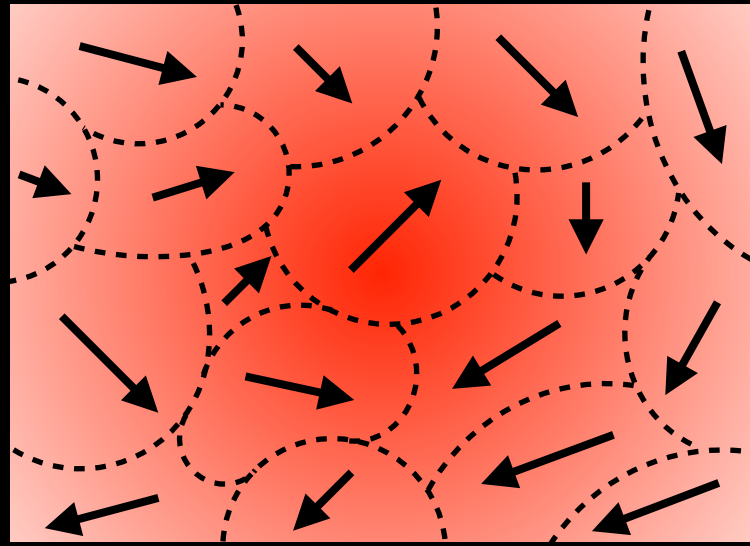
Gibbs dist.:
$$\hat{\rho}_G = \frac{e^{-\beta(\hat{H} - \mu\hat{N})}}{Z} = e^{-\beta(\hat{H} - \mu\hat{N}) - \Psi[\beta, \nu]}$$

Thermodynamic potential with Euclidean action

$$\begin{aligned} \Psi[\beta, \nu] &= \log \text{Tr} e^{-\beta(\hat{H} - \mu\hat{N})} = \log \int d\varphi \langle \pm\varphi | e^{-\beta(\hat{H} - \mu\hat{N})} | \varphi \rangle \\ &= \log \int_{\varphi(\beta) = \pm\varphi(0)} \mathcal{D}\varphi e^{+S_E[\varphi]}, \quad S_E[\varphi] = \int_0^\beta d\tau \int d^3x \mathcal{L}_E(\varphi, \partial_\mu\varphi) \end{aligned}$$

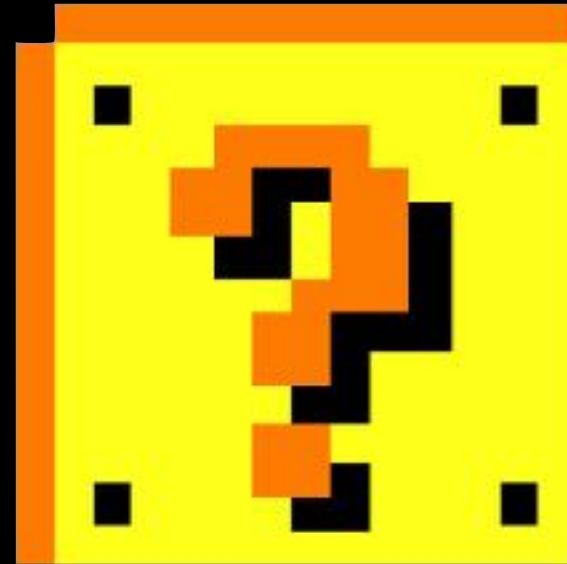
QFT for local thermal equilibrium?

Local equil. $\{\beta(x), \vec{v}(x)\}$



Path int.

Local Thermal QFT



Case study: Scalar field

$$\mathcal{L} = -\frac{g^{\bar{\mu}\bar{\nu}}}{2}\partial_{\bar{\mu}}\phi\partial_{\bar{\nu}}\phi - V(\phi)$$

$$\longrightarrow \hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} = \partial^\mu \hat{\phi} \partial^\nu \hat{\phi} + g^{\mu\nu} \mathcal{L}(\hat{\phi}, \partial_\rho \hat{\phi})$$

$$\begin{aligned} \Psi[\bar{t}; \lambda] &= \log \text{Tr} \exp \left[- \int d^{d-1} \bar{x} \sqrt{-g} \beta^\mu(x) \hat{T}^{\bar{0}}_{\mu}(x) \right] \\ &= \log \int \mathcal{D}\phi \exp (S_E[\phi, \beta^\mu]) = \log \int \mathcal{D}\phi \exp (S_E[\phi, \tilde{g}]) \end{aligned}$$

$$\begin{aligned} S[\phi, \beta^\mu] &= \int_0^{\beta_0} d\tau \int d^3 \bar{x} \sqrt{-g} e^\sigma u^{\bar{0}} \left[-\frac{e^{-2\sigma}}{2u^{\bar{0}}u_{\bar{0}}} (i\dot{\phi})^2 - \frac{-e^{-\sigma} u^{\bar{i}}}{u^{\bar{0}}u_{\bar{0}}} (i\dot{\phi}) \partial_{\bar{i}} \phi - \frac{1}{2} \left(\gamma^{\bar{i}\bar{j}} + \frac{u^{\bar{i}}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \right) \partial_{\bar{i}} \phi \partial_{\bar{j}} \phi - V(\phi) \right] \\ &= \int_0^{\beta_0} d\tau \int d^3 \bar{x} \sqrt{-\tilde{g}} \left[-\frac{\tilde{g}^{\bar{\mu}\bar{\nu}}}{2} \partial_{\bar{\mu}} \phi \partial_{\bar{\nu}} \phi - V(\phi) \right] \quad (e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0) \end{aligned}$$

Ψ in terms of Thermal metric

$$\Psi[\bar{t}; \lambda] = \log \int \mathcal{D}\phi \exp (S_E[\phi, ; \tilde{g}])$$

Thermal metric

$$\tilde{g}_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} -e^{2\sigma} & e^\sigma u_{\bar{j}} \\ e^\sigma u_{\bar{i}} & \gamma_{\bar{i}\bar{j}} \end{pmatrix}$$

$$(e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0)$$

Inverse thermal metric

$$\tilde{g}^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} \frac{e^{-2\sigma}}{u^{\bar{0}}u_{\bar{0}}} & -\frac{e^{-\sigma}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \\ -\frac{e^{-\sigma}u^{\bar{i}}}{u^{\bar{0}}u_{\bar{0}}} & \gamma^{\bar{i}\bar{j}} + \frac{u^{\bar{i}}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \end{pmatrix}$$

Interpretation of above result

$\Psi[\bar{t}; \lambda]$ is described by QFT in "curved spacetime" s. t.

$$d\tilde{s}^2 = -e^{2\sigma} (d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma} u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}} u_{\bar{j}}, \quad d\tilde{t} = -i d\tau)$$

Local Thermal QFT

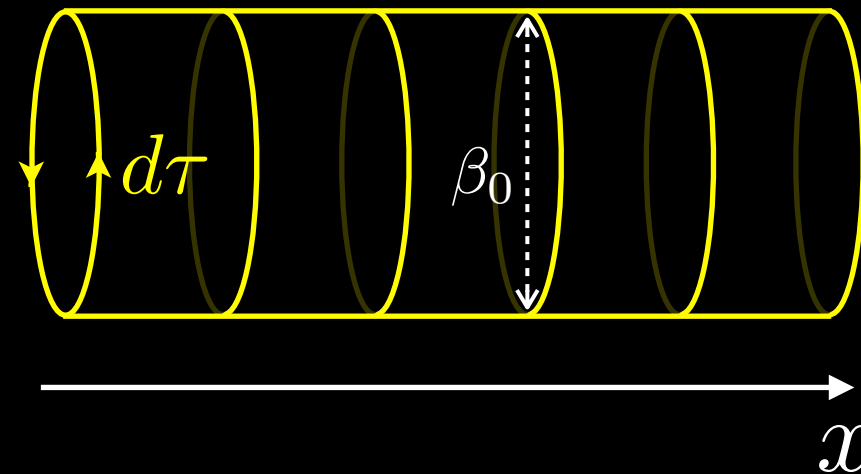
Global equil. β_0

$$T = \text{const.}$$

Path int.

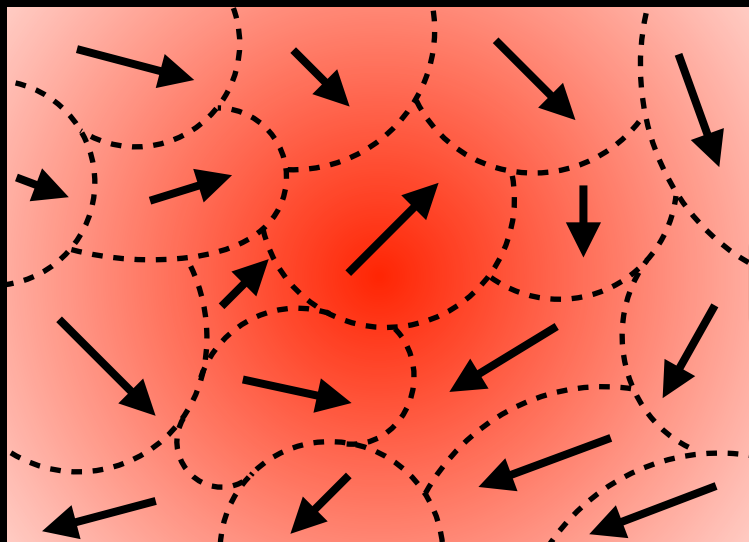
Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the
flat spacetime
with size β_0

Local equil. $\{\beta(x), \vec{v}(x)\}$

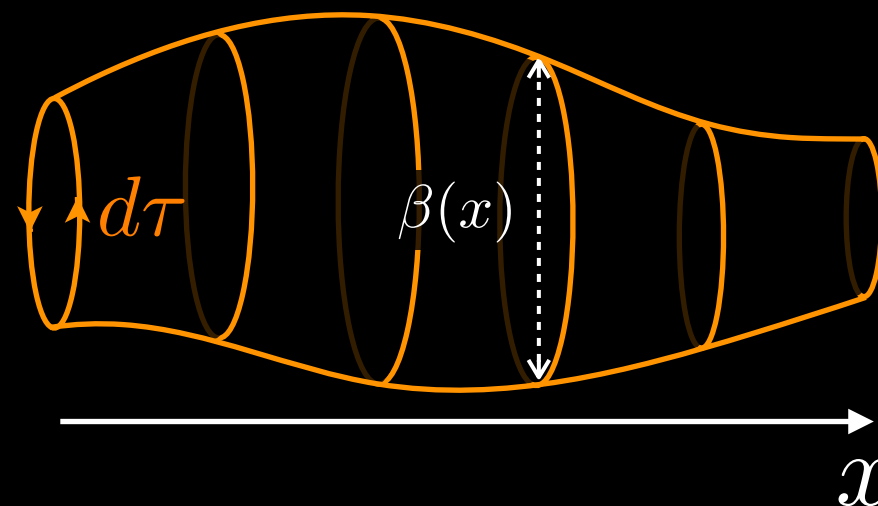


Path int.

Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]

[MH, Ann. Phys. (2017)]



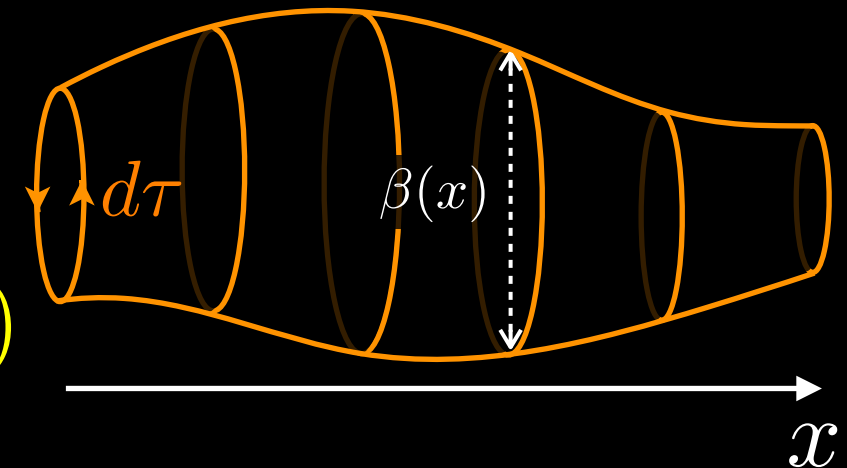
QFT in the
“curved spacetime”
with “line element”

$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

Symmetry of Local Thermal QFT

$$d\tilde{s}^2 = -e^{2\sigma} (d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv -e^{-\sigma} u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}} u_{\bar{j}}, \quad d\tilde{t} \equiv -i d\tau)$$



Symmetry of “curved spacetime”

(1) Spatial diffeomorphism :

Physics does not depend on the choice of Spatial coordinate system!!

(2) Imaginary time tran. & Kaluza-Klein gauge :

Parameters λ (e.g. T) does not depend on imaginary time!!

cf. “Hydrostatic” partition function

Banerjee et al.(2012), Jensen et al.(2012), MH (2017)

$$\Psi[\lambda] = \log \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{S[\psi, \bar{\psi}, \tilde{e}]} \text{ should respect the above symmetries!!}$$

Spatial diffeomorphism symmetry



(Mercator projection)



(Mollweide projection)

$$\bar{x} \longrightarrow \bar{x}'(\bar{x})$$

$$\Psi^{(0)}[\beta^\mu, \nu] = \int \mathcal{D}\phi e^{-S[\phi, \tilde{g}]} \ni$$



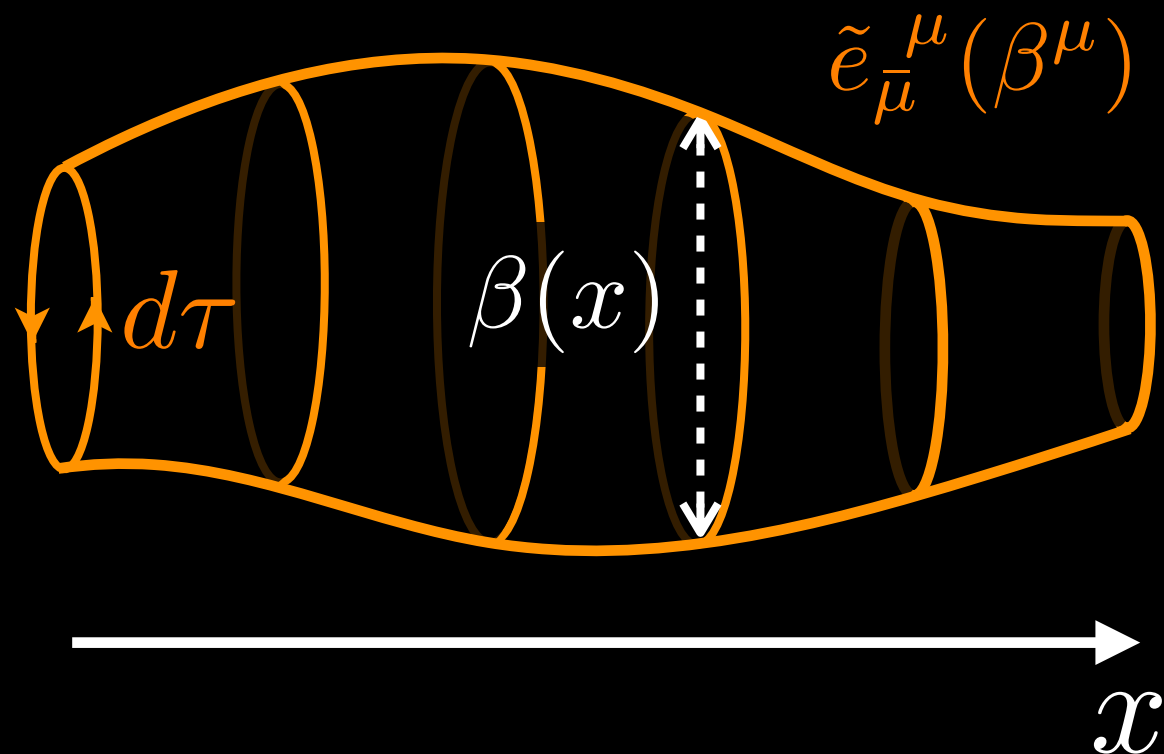
$$d^3\bar{x}\sqrt{\gamma}, \dots$$



$$\sqrt{\gamma}, \gamma_{ij}, \dots$$

Kaluza-Klein gauge symmetry

$$d\tilde{s}^2 = -e^{2\sigma} (d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}} \quad (d\tilde{t} = -i d\tau)$$



Parameters λ don't depend on **imaginary time $\mathcal{T}$** .

“Kaluza-Klein” gauge tr.

$$\begin{cases} \tilde{t} \rightarrow \tilde{t} + \chi(\bar{x}) \\ \bar{x} \rightarrow \bar{x} \\ a_{\bar{i}}(\bar{x}) \rightarrow a_{\bar{i}}(\bar{x}) - \partial_{\bar{i}} \chi(\bar{x}) \end{cases}$$

$$\Psi[\lambda] = \log \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{S[\psi, \bar{\psi}, \tilde{e}]} \ni$$

$$(f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}})$$



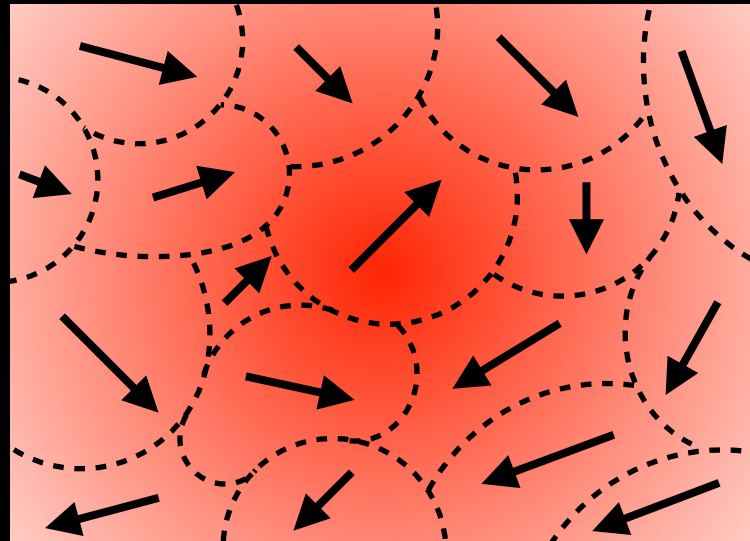
$$f^{\bar{i}\bar{j}} f_{\bar{i}\bar{j}}, \dots$$



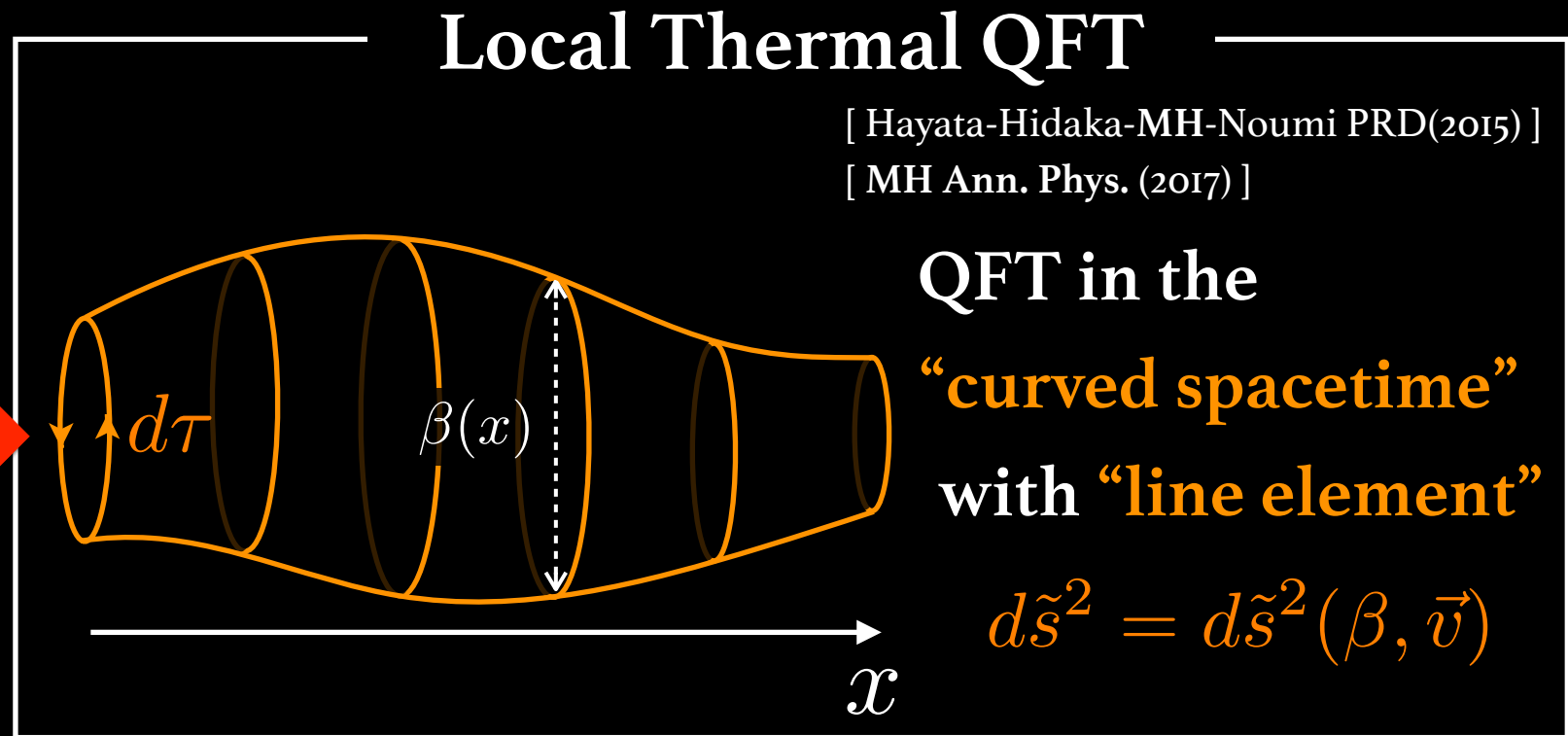
$$a_{\bar{i}}, a_{\bar{i}} a^{\bar{i}}, \dots$$

Short Summary: Local Thermal QFT

Local equil. $\{\beta(x), \vec{v}(x)\}$



Path int.



$$\Psi[\bar{t}; \lambda] \equiv \log \text{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu_\mu(x) + \nu(x) \hat{J}^\nu(x) \right) \right]$$

① $\Psi[\lambda]$ plays a role as the generating functional: $\langle \hat{T}^{\mu\nu}(x) \rangle^{\text{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\lambda]$

② $\Psi[\lambda]$ is written in terms of **QFT in curved spacetime**

$$d\tilde{s}^2 = -e^{2\sigma} (d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

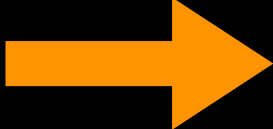
Q. How can we calculate $\Psi \equiv \log Z$?

A. Derivative expansion based on
symmetry of $\Psi = \log Z$!

Double derivative expansion

(1) Dissipative derivative expansion

$$\hat{\rho}_{\text{LG}}(\bar{t}_0) = \hat{\rho}_{\text{LG}}(\bar{t}) \left(1 + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \dots \right)$$

 Dissipative correction (viscosity etc.)

(2) Non-dissipative derivative expansion

$$\Psi[\beta^\mu, \nu] = \Psi^{(0)}[\beta^\mu, \nu] + \Psi^{(1)}[\beta^\mu, \nu, \partial] + \mathcal{O}(\partial^2) + \dots$$

Derivative expansion of Ψ

(2) Derivative expansion of ψ

$$\Psi[\beta^\mu, \nu] = \boxed{\Psi^{(0)}[\beta^\mu, \nu]} + \boxed{\Psi^{(1)}[\beta^\mu, \nu, \partial]} + \mathcal{O}(\partial^2) + \dots$$

$\simeq \beta p$ $= 0$ **Parity-even system**
 Symmetry property

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t}; \lambda] = T_{(0)}^{\mu\nu}[\lambda(x)] + \boxed{T_{(1)}^{\mu\nu}[\lambda(x), \nabla \lambda(x)]} + \dots$$

$$\langle \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta A_\mu(x)} \Psi[\bar{t}; \lambda] = J_{(0)}^\mu[\lambda(x)] + \boxed{J_{(1)}^\mu[\lambda(x), \nabla \lambda(x)]} + \dots$$

$= 0$

Parity-even case

$$\mu_R = \mu_L$$

Recipe for Masseiu-Planck fcn.

[Banerjee et al.(2012), Jensen et al.(2012)]

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi, \tilde{g}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda, \partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- **Building blocks** : $\lambda = \{e^\sigma, a_{\bar{i}}, \mu, A_{\bar{i}}\}$
- **Symmetry** : Spatial diffeo, Kaluza-Klein, Gauge

$$A_{\bar{i}} : \text{not Kaluza-Klein inv.} \longrightarrow \bar{A}_{\bar{i}} \equiv A_{\bar{i}} - \mu a_{\bar{i}}$$

- **Power counting scheme** : $\lambda = \mathcal{O}(p^0)$

$$f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1) \longrightarrow ff = \mathcal{O}(p^2)$$

$\Psi^{(0)} : \text{Order } \mathcal{O}(p^0)$

— Masseiu-Planck functional —

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi, \tilde{g}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda, \partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

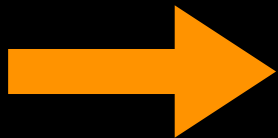
- **Building blocks** : $\lambda = \{e^\sigma, \cancel{a_i}, \mu, \cancel{\bar{A}_i}\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x} \sqrt{\gamma'} e^\sigma p(\beta, \mu)$$

— Perfect fluid —

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\text{LG}} = (e + p) u^\mu u^\nu + p \eta^{\mu\nu}$$

$$\langle \hat{J}^\mu(x) \rangle_{\bar{t}}^{\text{LG}} = n u^\mu$$



Derivation of Navier-Stokes eq.

Given

Conservation laws

$$\nabla_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_\mu \langle \hat{J}^\mu(x) \rangle = 0$$



Constitutive relations (1st order)

$$\begin{aligned} \langle \hat{T}^{\mu\nu}(x) \rangle &= (e + p)u^\mu u^\nu + pg^{\mu\nu} - \frac{\zeta}{\beta} h^{\mu\nu} h^{\rho\sigma} \nabla_\rho \beta_\sigma - 2\frac{\eta}{\beta} \nabla^{\langle\mu} \beta^{\nu\rangle} \\ \langle \hat{J}^\mu(x) \rangle &= nu^\mu - \frac{\kappa}{\beta} h^{\mu\nu} \nabla_\nu \nu \end{aligned}$$



Physical properties

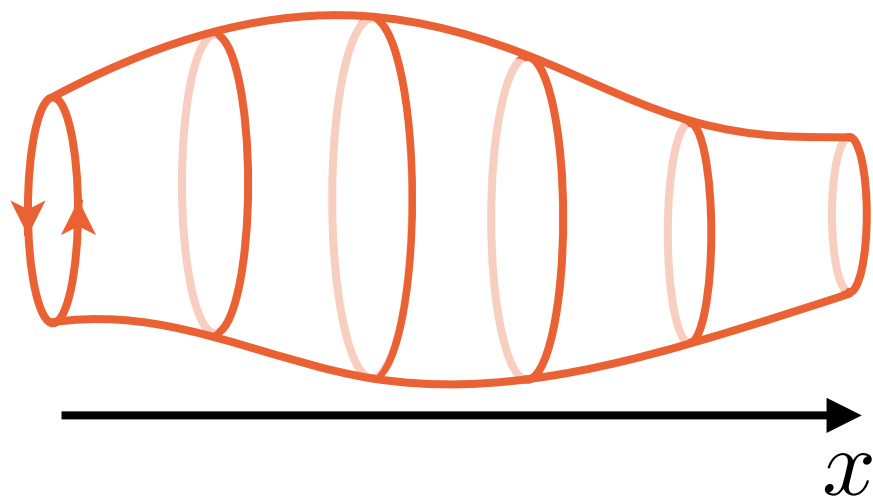
Static properties: $\Psi[\lambda] = \log \int \mathcal{D}\varphi_i e^{S_E[\varphi_i; \tilde{g}]} = \int d^3\bar{x} \sqrt{\gamma'} \beta p(\beta, \mu)$

Dynamic properties: $\zeta = \beta(x) \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta}\hat{p}(x') e^{-\hat{K}\tau} \tilde{\delta}\hat{p}(x) \rangle_{\bar{t}}^{\text{LG}} \text{ etc.}$

Outline: Hydro from QFT

Part 1. **Conventional** hydrodynamics [Ref. Hayata-Hidaka-MH-Noumi, (2015), MH .(2017)]

- Hydro = **Optimized perturbation** with respect to real-time dynamics
- Extention of **imaginary-time (Matsubara)** formalism:
Local thermal equilibrium = Quantum theory in “**curved spacetime**”



Part 2. **Fluctuating** hydrodynamics [Ref. Crossley-Glorioso-Liu, (2017) Delacretaz, 2606.02391]

Part 2. Question part

What is EFT of hydro?

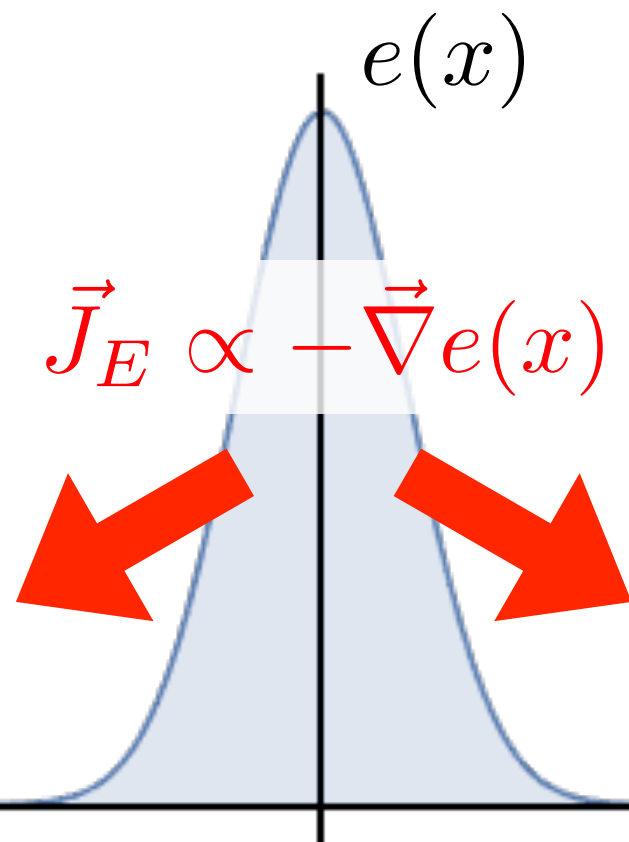
Prototype: Energy diffusion

◆ Bulding blocks of hydrodynamic equation

(1) Conservation law: $\partial_t e + \vec{\nabla} \cdot \vec{J}_E = 0$

(2) Constitutive relation: $\vec{J}_E = -D_E \vec{\nabla} e$

(3) Physical properties: Value of diffusion constant D_E

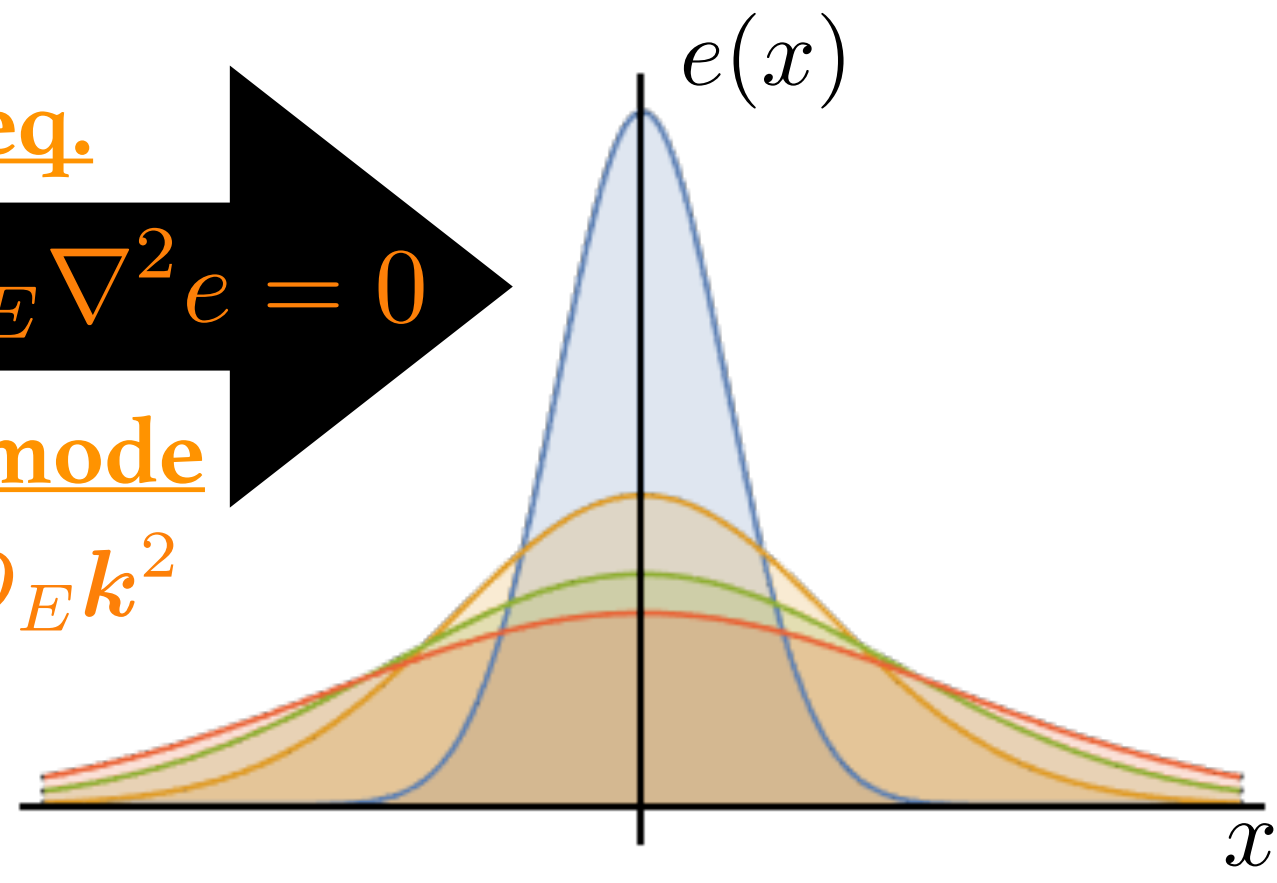


Diffusion eq.

$$\partial_t e - D_E \nabla^2 e = 0$$

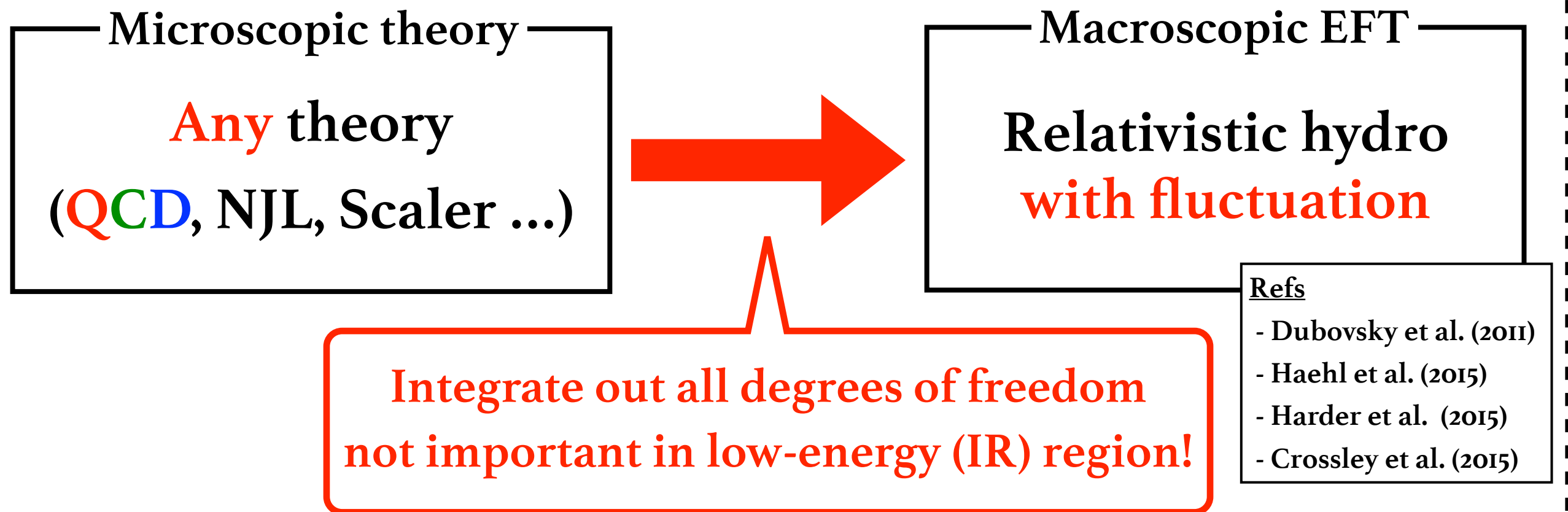
Diffusion mode

$$\omega = -iD_E k^2$$



EFT for hydrodynamics

◆ Hydrodynamics as EFT



not $T=0$, not equilibrium, but with dissipation!

➔ Real-time formalism!!

Goal of Part 2

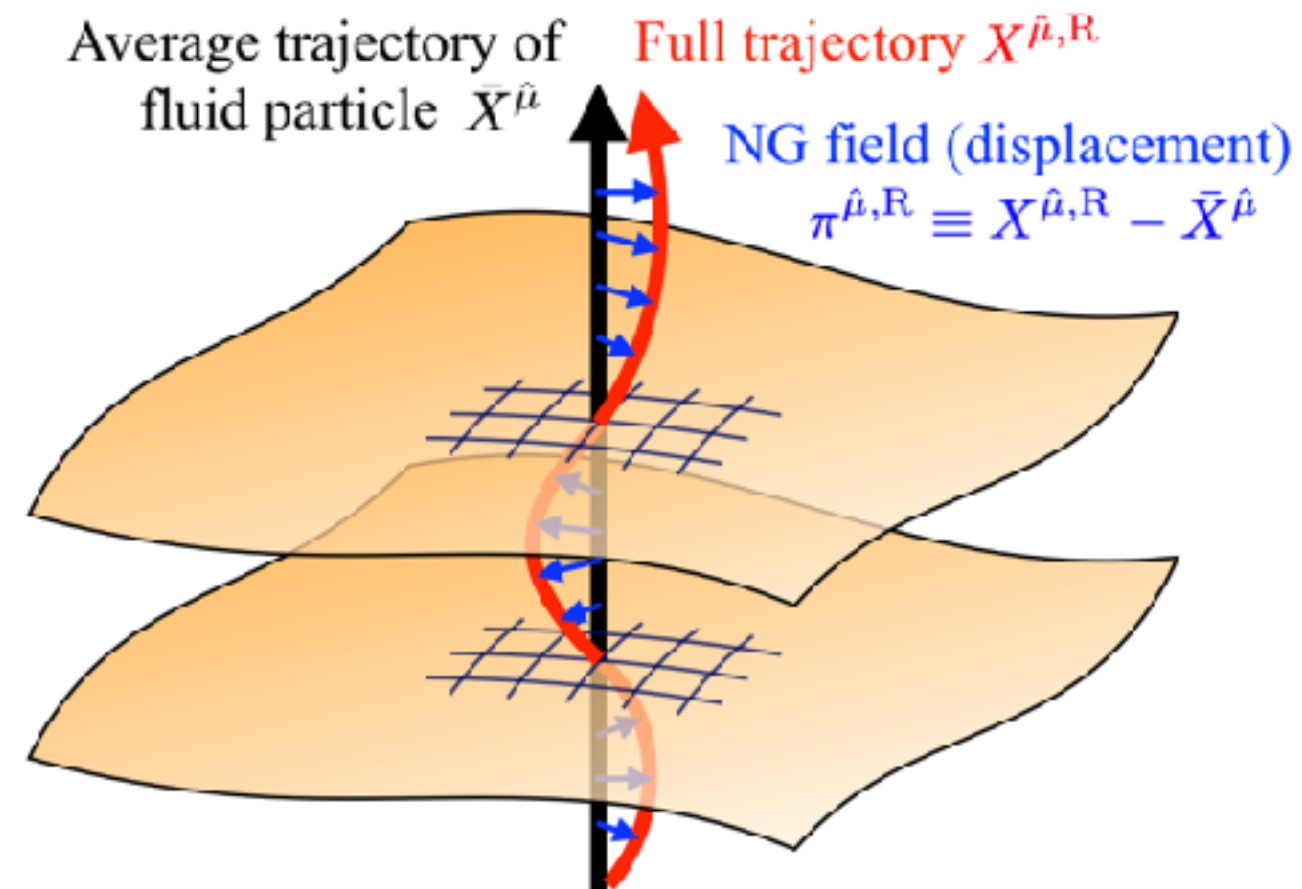
Effective Lagrangian (**real-time** formalism)

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\pi_{\text{hydro}} \exp(i\mathcal{S}_{\text{eff}}[\pi_{\text{hydro}}])$$

(corresponds to a construction of **chiral Lagrangian** in **QCD**)

Hydrodynamics is low-energy EFT of
a spacetime filling brane $X^\mu(\sigma^0, \sigma^i)$,
enjoying **emergent gauge symmetry**:

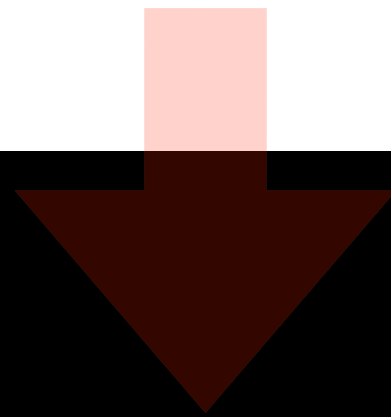
$$\begin{cases} \sigma^0 \rightarrow \sigma^0 + F^0(\sigma^0, \sigma^i) \\ \sigma^i \rightarrow \sigma^i + F^i(\sigma^i) \end{cases}$$



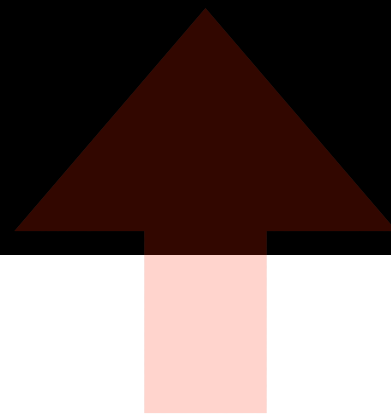
[See Crossley et al. arXiv: 1511.03646 [hep-th], MH et al. unpublished]

Part 2. Answer part
Nonequilibrium EFT

Top-down view from Keldysh

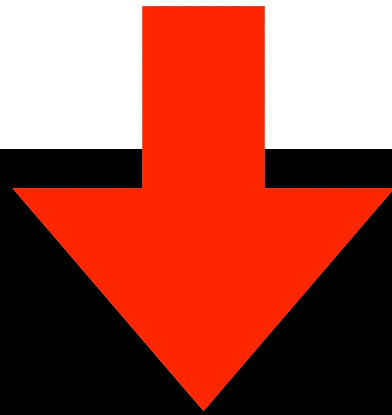


Hydrodynamics

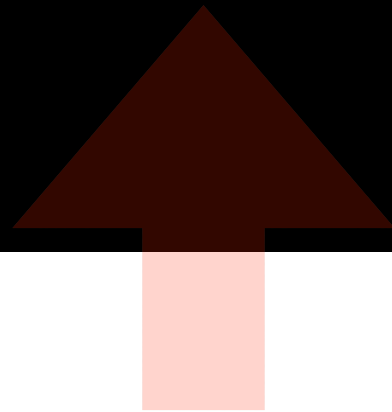


Bottom-up view from Langevin

Top-down view from Keldysh



Hydrodynamics



Bottom-up view from Langevin

Non-equilibrium

Effective Field Theory in a Nutshell

1. Basic formulation

2. Application to fluctuating hydro

See e.g.

Michael Crossley, Paolo Glorioso, Hong Liu [arXiv:1511.03646]

Luca Delacretaz [arXiv: 2606.02391]

Non-equilibrium

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Talks by Andrew and Toshifumi, ...

Schwinger-Keldysh in a Nutshell

QFT for T=0

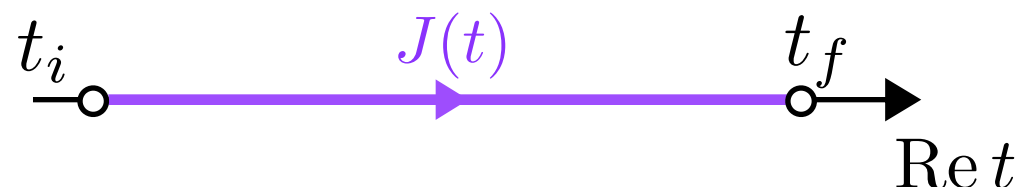
- Observable : **Transition amp.**

$$\langle \text{out} | \hat{\mathcal{O}} | \text{in} \rangle$$

$$\text{Ex. } S_{\beta\alpha} = \langle \beta | \hat{S} | \alpha \rangle$$

- Generating functional

$$Z[J] = \langle \text{vac} | \hat{U}_J | \text{vac} \rangle$$



Non-equilibrium QFT

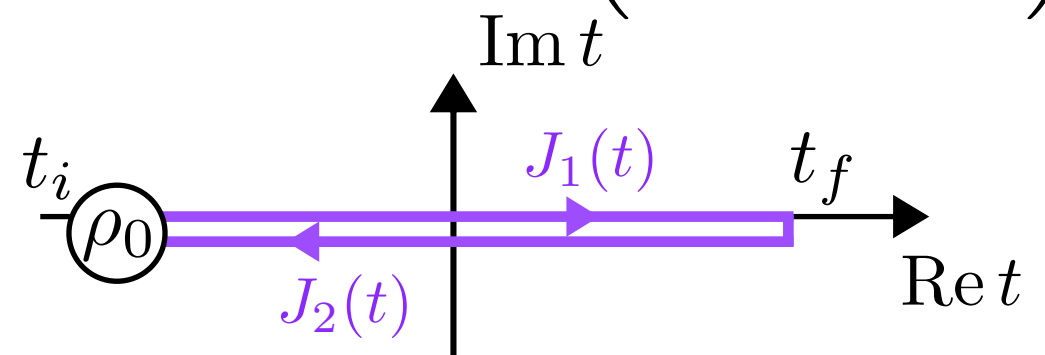
- Observable : **Average value**

$$\sum_{\text{in}} \langle \text{in} | \hat{\mathcal{O}} | \text{in} \rangle$$

$$\text{Ex. } \langle \hat{\varphi} \rangle = \text{Tr} (\hat{\rho}_0 \hat{\varphi})$$

- **CTP** generating functional

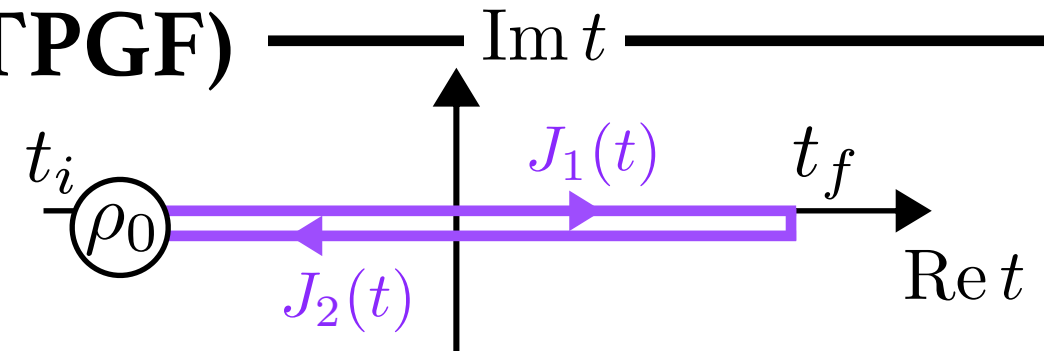
$$Z[J_1, J_2] = \text{Tr} \left(\hat{\rho}_0 \hat{U}_{J_2}^\dagger \hat{U}_{J_1} \right)$$



Properties of Closed-Time-Path GF

◆ CTP Generating Functional (CTPGF)

$$Z[J_1, J_2] = \text{Tr} \left(\hat{\rho}_0 \hat{U}_{J_2}^\dagger \hat{U}_{J_1} \right)$$



◆ Basic properties of CTPGF

① **Doubled** symmetry : $Z[j_1, j_2] = Z[j_1 + \delta_{\chi_1} j_1, j_2 + \delta_{\chi_2} j_2]$

② **Unitarity** condition : $Z[j_1 = j, j_2 = j] = 1$

③ **Reflectivity** condition : $Z[j_1, j_2]^* = Z[j_2, j_1]$

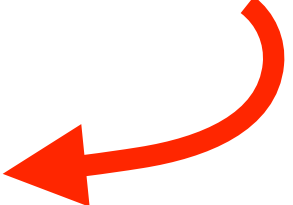
④ **KMS condition** (if initial state is **thermal equilibrium**) :

$$Z[j_1(x), j_2(x)] = Z[\epsilon_{PT_j} j_1(-x), \epsilon_{PT_j} j_2(-x - i\beta)]$$

Recipe for **effective action**

$$\begin{aligned} Z[j_1, j_2] &= \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 e^{i(S[\varphi_1; j_1] - S[\varphi_2; j_2])} \rho_0(\varphi) \\ &= \int \mathcal{D}\pi_1 \mathcal{D}\pi_2 e^{iS_{\text{eff}}[\pi_1, \pi_2; j_1, j_2]} \end{aligned}$$

“Integrate out UV mode
and keep NG mode”



◆ Constraints on $S_{\text{eff}}[\pi_1, \pi_2]$

- ①' **Doubled** symmetry : NG mode as **a nonlinear realization**
- ②' **Unitarity** condition : $S_{\text{eff}}[\pi_1 = \pi, \pi_2 = \pi] = 0$
- ③' **Reflectivity** condition : $S_{\text{eff}}[\pi_1, \pi_2]^* = -S_{\text{eff}}[\pi_2, \pi_1]$
- ③'' **Positivity** condition : $\text{Im } S_{\text{eff}}[\pi_1, \pi_2] \geq 0$

Dynamical **KMS** condition

$$Z[j_1(x), j_2(x)] = Z[\epsilon_{PT_j} j_1(-x), \epsilon_{PT_j} j_2(-x - i\beta)]$$

④” Dynamical **KMS condition** : in $\hbar \rightarrow 0$

$$S_{\text{eff}}[\pi_R, \pi_A] = S_{\text{eff}}[\tilde{\pi}_R, \tilde{\pi}_A]$$

under $\tilde{\pi}_R(x) = \pi_R(-x), \quad \tilde{\pi}_A(x) = \pi_A(-x) + i\beta \partial_{-t} \pi_R(-x)$

Non-equilibrium

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Stochastic charge diffusion

SWSSB: $U(1)_{\text{strong}} \times U(1)_{\text{weak}} \rightarrow U(1)_{\text{weak}}$

Doubled U(1) symmetry

◆ Effective Lagrangian for stochastic charge diffusion

$$\mathcal{L}_{\text{charge}} = -nD_0\pi^A - T\sigma\delta^{ij}D_i\pi^A\left(\partial_j\frac{\delta S_{\text{th}}}{\delta n} + e\beta E_j\right) + iT\sigma\delta^{ij}D_i\pi^AD_j\pi^A$$

with $D_\mu\pi^A = \partial_\mu\pi^A - eA_\mu^A$, $E_i \equiv \partial_i A_0^R - \partial_0 A_i^R$.

This Lagrangian enjoys **doubled** U(1) symmetry!

$$\left\{ \begin{array}{l} \text{- U(1)}_w \text{ symmetry: } n(x) \rightarrow n(x), \quad A_\mu^R(x) \rightarrow A_\mu^R(x) + \partial_\mu\alpha^R(x) \\ \text{- U(1)}_s \text{ symmetry: } \pi^A(x) \rightarrow \pi^A(x) + e\alpha^A(x), \quad A_\mu^A(x) \rightarrow A_\mu^A(x) + \partial_\mu\alpha^A(x) \end{array} \right.$$

U(1)_s symmetry tells us that

“Auxiliary field π^A is identified as Nambu-Goldstone (NG) field!!”

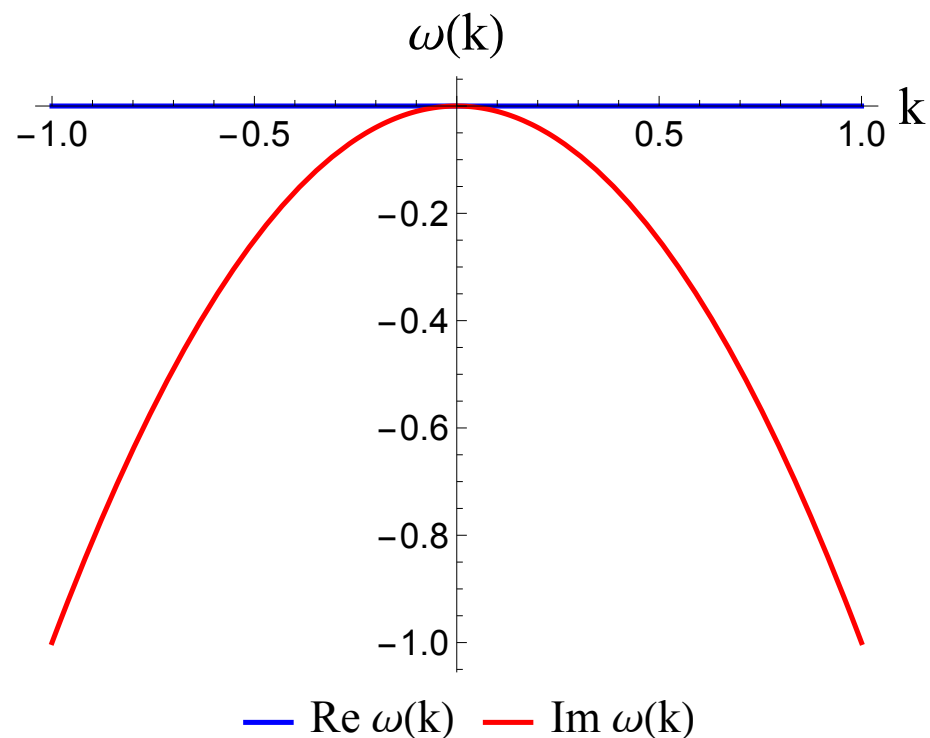
➡ Can we also describe U(1)_w symmetry using NG field??

Spectrum in linearized model

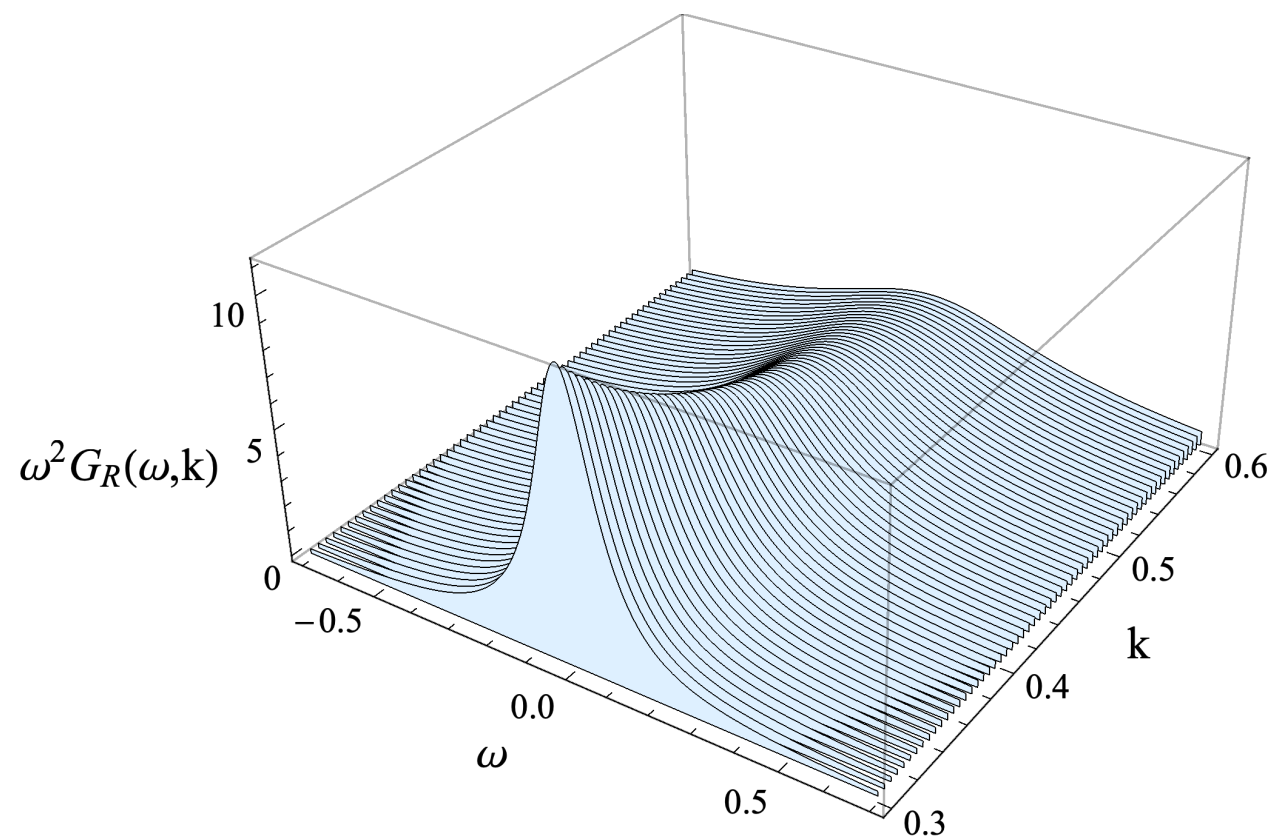
◆ Effective Lagrangian for stochastic charge diffusion

$$\begin{aligned}\mathcal{L}_{\text{charge}} &= -n\partial_0\pi^A + D\delta^{ij}\partial_i n\partial_j\pi^A + iT\sigma\delta^{ij}\partial_i\pi^A\partial_j\pi^A \\ &= \frac{i}{2} \begin{pmatrix} n & \pi^A \end{pmatrix} \begin{pmatrix} 0 & i(\partial_0 + D\nabla^2) \\ i(-\partial_0 + D\nabla^2) & -2T\sigma\nabla^2 \end{pmatrix} \begin{pmatrix} n \\ \pi^A \end{pmatrix}\end{aligned}$$

(a) Dispersion relation



(b) Symmetric Green's function



Z_2 symmetry from FDR

◆ Ward-Takahashi identity for Z_2 symmetry

$$Z[A_\mu^R, A_\mu^A] = Z[A_\mu^{R'}, A_\mu^{A'}], \quad \begin{cases} n(x) \rightarrow n'(x) = n(-x), \\ \pi^A(x) \rightarrow \pi^{A'}(x) = -\pi^A(-x) - i \frac{\delta S_{\text{th}}[n]}{\delta n(-x)}, \\ A_\mu^R(x) \rightarrow A_\mu^{R'}(x) = A_\mu^R(-x), \\ A_\mu^A(x) \rightarrow A_\mu^{A'}(x) = A_\mu^A(-x) - i\beta[\partial_0 A_\mu^R(-x) - \partial_\mu A_0^R(-x)] \end{cases}$$

Corollary 1. **Fluctuation theorem**

$$Z[A_\mu^R, A_\mu^A = i\beta E_\mu] = 1 = \int \mathcal{D}n_{\text{ini}} P_{\text{eq}}[n_{\text{ini}}] \langle \exp(-\Sigma) \rangle_\xi \quad \text{with} \quad \Sigma = \int d^4x e \beta E_\mu \check{J}^\mu(\check{n}_{\text{os}})$$

Corollary 2. **The 2nd law of thermodynamics**

$$\int \mathcal{D}n_{\text{ini}} P_{\text{eq}}[n_{\text{ini}}] \langle \Sigma \rangle_\xi \geq 0.$$

Corollary 3. **Fluctuation-dissipation relation**

$$2 \text{Im} G^R(\omega, \mathbf{k}) = \frac{\omega}{T} G^{\text{RR}}(\omega, \mathbf{k})$$

Redundant description

NG field for $U(1)_w$ symmetry can be introduced by

Josephson equation: $\mu = -D_0\pi^R$ with $D_\mu\pi^R \equiv \partial_\mu\pi^R - eA_\mu^R$.

◆ Effective Lagrangian for stochastic charge diffusion

$$\begin{aligned}\mathcal{L}_{\text{charge}} &= -nD_0\pi^A - T\sigma\delta^{ij}D_i\pi^A\left(\partial_j\frac{\delta S_{\text{th}}}{\delta n} + e\beta E_j\right) + iT\sigma\delta^{ij}D_i\pi^A D_j\pi^A \\ &= \chi D_0\pi^A D_0\pi^R - \sigma\delta^{ij}D_i\pi^A\partial_0 D_j\pi^R + iT\sigma\delta^{ij}D_i\pi^A D_j\pi^A\end{aligned}$$

$$\left\{ \begin{array}{l} \text{- } U(1)_w \text{ symmetry: } \pi^R(x) \rightarrow \pi^R(x) + e\alpha^R(x), \quad A_\mu^R(x) \rightarrow A_\mu^R(x) + \partial_\mu\alpha^R(x), \\ \text{- } U(1)_s \text{ symmetry: } \pi^A(x) \rightarrow \pi^A(x) + e\alpha^A(x), \quad A_\mu^A(x) \rightarrow A_\mu^A(x) + \partial_\mu\alpha^A(x) \end{array} \right.$$

The description with π^R has **gauge symmetry (redundancy)**

$$\pi^R(t, \boldsymbol{x}) \rightarrow \pi^R(t, \boldsymbol{x}) + \alpha(\boldsymbol{x})$$

Matching condition

◆ Effective Lagrangian for stochastic charge diffusion

$$\mathcal{L}_{\text{charge}} = \chi D_0 \pi^A D_0 \pi^R - \sigma \delta^{ij} D_i \pi^A \partial_0 D_j \pi^R + iT \sigma \delta^{ij} D_i \pi^A D_j \pi^A$$

Low-energy coefficients: $\left\{ \begin{array}{l} \text{- Charge susceptibility: } \chi \\ \text{- Electric conductivity: } \sigma \end{array} \right.$

◆ Matching condition

$$\lim_{\mathbf{k} \rightarrow 0} G_{\text{RA}}^{00}(\omega = 0, \mathbf{k}) = i^{-1} e^2 \chi, \quad \lim_{\omega \rightarrow 0} \frac{1}{\omega} G_{\text{RA}}^{ij}(\omega, \mathbf{k} = 0) = e^2 \sigma \delta^{ij}$$

Using the retarded Green's function: $G_{\text{R}}(\omega, \mathbf{k}) = i G_{\text{RA}}(\omega, \mathbf{k})$

$\lim_{\omega \rightarrow 0} \frac{1}{\omega} G_{\text{RA}}^{ij}(\omega, \mathbf{k} = 0) = e^2 \sigma \delta^{ij}$ is identified as **Green-Kubo formula!**

As usual, consideration of nonlinear term leads to renormalization of $\{\chi, \sigma\}$

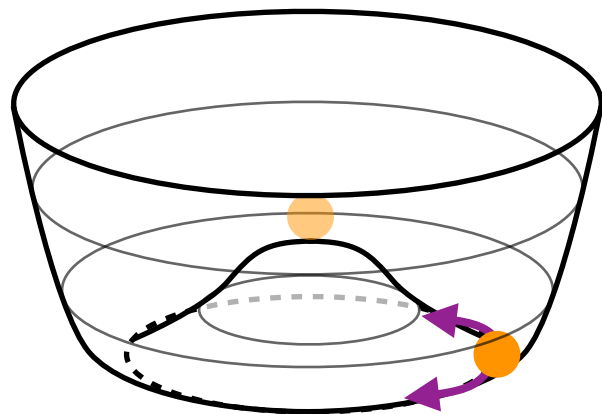
Stochastic dynamics in **superfluid** phase

SSB: $U(1)_{\text{strong}} \times U(1)_{\text{weak}} \rightarrow$ no unbroken symmetry

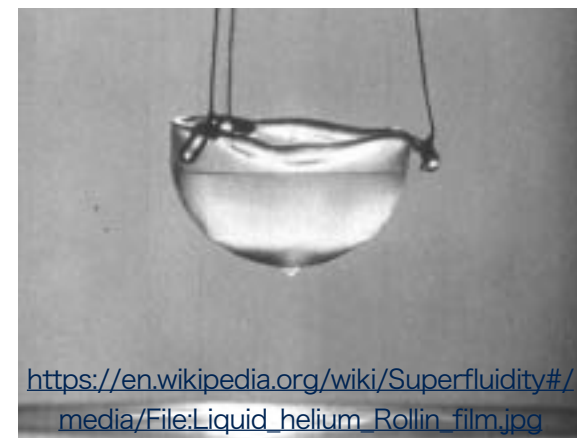
What is **superfluid**?

◆ Superfluid = fluid with spontaneous symmetry breaking

Micro : Selecting vacuum



Macro : Superfluid



◆ How to characterize the superfluid phase (*generalized rigidity*)

(1) Nonvanishing order parameter for a conserved charge $\hat{Q}_a$

$$\exists \hat{\Phi}(x) \text{ such that } \langle [i\hat{Q}_a, \hat{\Phi}_i(x)] \rangle \equiv \text{Tr}(\hat{\rho}[i\hat{Q}_a, \hat{\Phi}_i(x)]) \neq 0$$

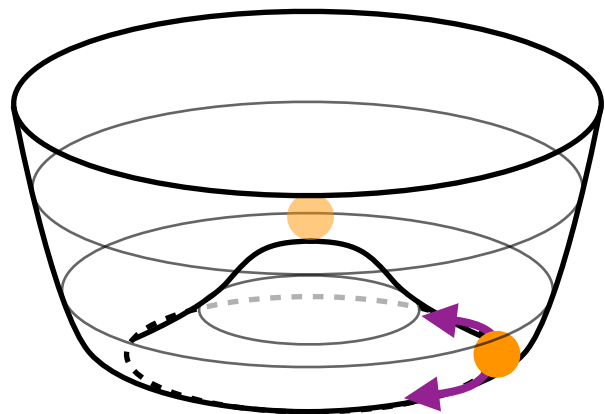
(2) Nonvanishing current-current correlation

$$\lim_{k \rightarrow 0} G_{\text{RA}}^{ij}(\omega = 0, \mathbf{k}) = e^2 \chi f^2 \neq 0$$

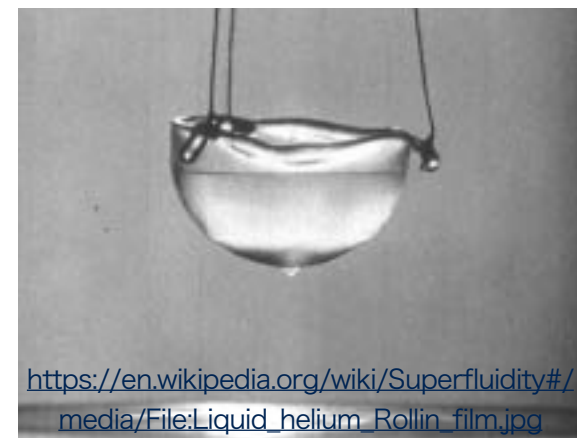
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(2) Nonvanishing current-current correlation

$$\lim_{k \rightarrow 0} G_{\text{RA}}^{ij}(\omega = 0, \mathbf{k}) = e^2 \chi f^2 \neq 0$$

To construct **superfluid** EFT

◆ Effective Lagrangian for stochastic charge diffusion

$$\mathcal{L}_{\text{charge}} = \chi D_0 \pi^A D_0 \pi^R - \sigma \delta^{ij} D_i \pi^A \partial_0 D_j \pi^R + iT \sigma \delta^{ij} D_i \pi^A D_j \pi^A$$

◆ Symmetries and redundancy

- $U(1)_w$ symmetry: $\pi^R(x) \rightarrow \pi^R(x) + e\alpha^R(x)$, $A_\mu^R(x) \rightarrow A_\mu^R(x) + \partial_\mu \alpha^R(x)$,

- $U(1)_s$ symmetry: $\pi^A(x) \rightarrow \pi^A(x) + e\alpha^A(x)$, $A_\mu^A(x) \rightarrow A_\mu^A(x) + \partial_\mu \alpha^A(x)$

~~- **Gauge symmetry (redundancy):** $\pi^R(t, \mathbf{x}) \rightarrow \pi^R(t, \mathbf{x}) + \alpha(\mathbf{x})$~~

Nonvanishing current-current correlation $\lim_{k \rightarrow 0} G_{RA}^{ij}(\omega = 0, \mathbf{k}) = e^2 \chi f^2 \neq 0$

➡ $\chi f^2 \delta^{ij} D_i \pi^A D_j \pi^R$ should be present!!

This term respects $U(1)$ symmetry, but not **gauge symmetry**!

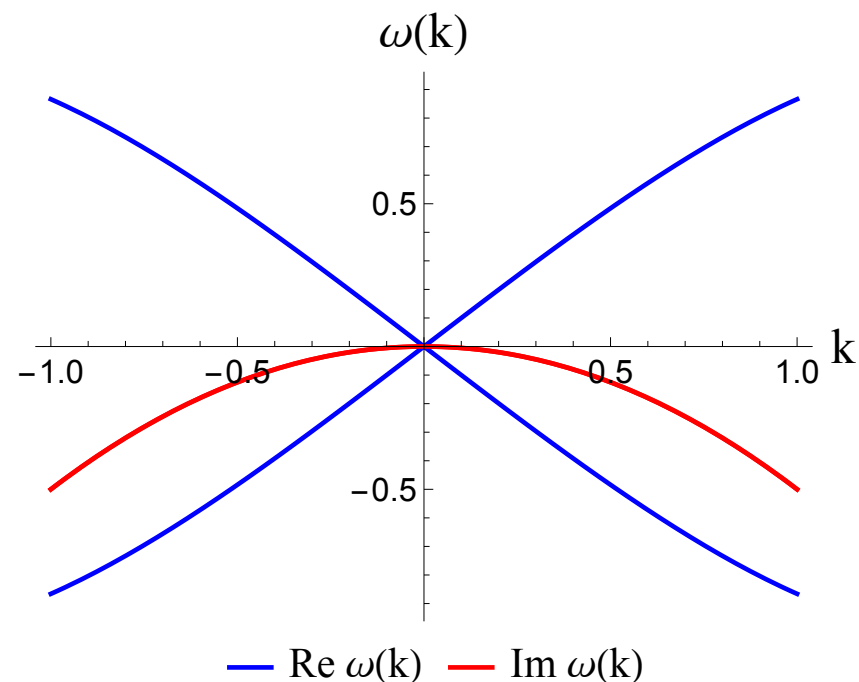
(The superfluid phase **loses** the above gauge symmetry)

Superfluid EFT at finite T

◆ Effective Lagrangian for superfluid ($T \neq 0$)

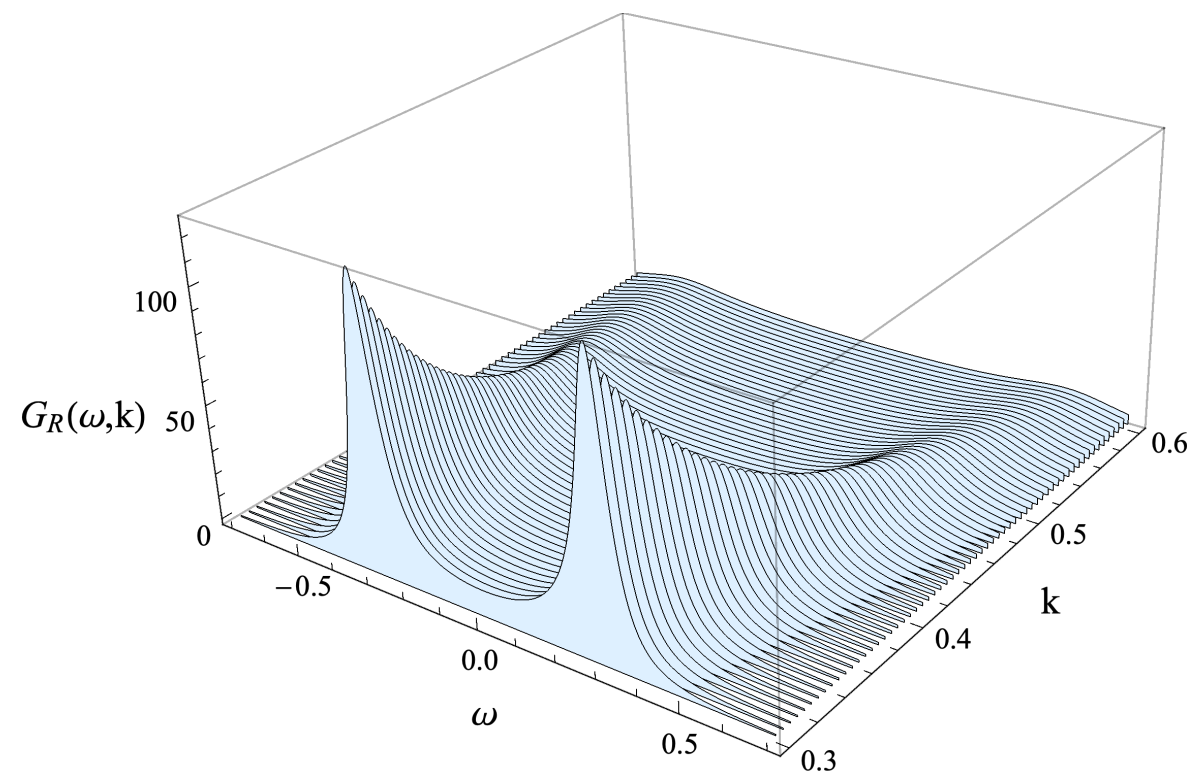
$$\begin{aligned}\mathcal{L}_{\text{NG}} &= \chi D_0 \pi^A D_0 \pi^R - \chi f^2 \delta^{ij} D_i \pi^A D_j \pi^R - \sigma \delta^{ij} D_i \pi^A \partial_0 D_j \pi^R + iT \sigma \delta^{ij} D_i \pi^A D_j \pi^A \\ &= \frac{i}{2} \begin{pmatrix} \pi^R & \pi^A \end{pmatrix} \begin{pmatrix} 0 & i\chi[\partial_0^2 + D \nabla^2 \partial_0 - f^2 \nabla^2] \\ i\chi[\partial_0^2 - D \nabla^2 \partial_0 - f^2 \nabla^2] & -2T\sigma \nabla^2 \end{pmatrix} \begin{pmatrix} \pi^R \\ \pi^A \end{pmatrix}\end{aligned}$$

(a) Dispersion relation



$$\omega = \frac{\pm \sqrt{4f^2 k^2 - D^2 k^4} - i D k^2}{2} \simeq \pm c_s |k| - \frac{i}{2} D k^2 + O(k^3)$$

(b) Symmetric Green's function



Superfluid EFT at finite T

◆ Effective Lagrangian for superfluid ($T \neq 0$)

$$\mathcal{L}_{\text{NG}} = \chi D_0 \pi^{\text{A}} D_0 \pi^{\text{R}} - \chi f^2 \delta^{ij} D_i \pi^{\text{A}} D_j \pi^{\text{R}} - \sigma \delta^{ij} D_i \pi^{\text{A}} \partial_0 D_j \pi^{\text{R}} + iT \sigma \delta^{ij} D_i \pi^{\text{A}} D_j \pi^{\text{A}}$$

◆ Symmetries

- $U(1)_{\text{w}}$ symmetry: $\pi^{\text{R}}(x) \rightarrow \pi^{\text{R}}(x) + e\alpha^{\text{R}}(x)$, $A_{\mu}^{\text{R}}(x) \rightarrow A_{\mu}^{\text{R}}(x) + \partial_{\mu}\alpha^{\text{R}}(x)$,
- $U(1)_{\text{s}}$ symmetry: $\pi^{\text{A}}(x) \rightarrow \pi^{\text{A}}(x) + e\alpha^{\text{A}}(x)$, $A_{\mu}^{\text{A}}(x) \rightarrow A_{\mu}^{\text{A}}(x) + \partial_{\mu}\alpha^{\text{A}}(x)$
- WT identity for Z_2 symmetry: $Z[A_{\mu}^{\text{R}}, A_{\mu}^{\text{A}}] = Z[A_{\mu}^{\text{R}'}, A_{\mu}^{\text{A}'}]$

◆ Matching condition for $\{\chi, \sigma, f^2\}$

$$\lim_{\mathbf{k} \rightarrow 0} G_{\text{RA}}^{00}(\omega = 0, \mathbf{k}) = i^{-1} e^2 \chi, \quad \lim_{\omega \rightarrow 0} \frac{1}{\omega} G_{\text{RA}}^{ij}(\omega, \mathbf{k} = 0) = e^2 \sigma \delta^{ij}, \quad \lim_{\mathbf{k} \rightarrow 0} G_{\text{RA}}^{ij}(\omega = 0, \mathbf{k}) = e^2 \chi f^2$$

In the similar manner, we can construct the EFT for **non-Abelian** superfluid based on **the coset construction** (Callan-Coleman-Wess-Zumino method)!

Fluctuating hydrodynamics

Extension to relativistic hydro

◆ Fluctuating relativistic hydrodynamics

$$\nabla_\mu \check{T}^{\mu\nu} = 0 \quad \text{with} \quad \begin{cases} \check{T}^{\mu\nu} = (\check{e} + \check{p}) \check{u}^\mu \check{u}^\nu + \check{p} g_{\text{R}}^{\mu\nu} - \bar{T} \eta^{\mu\nu\rho\sigma} \nabla_\rho \check{\beta}_\sigma + \check{\xi}^{\mu\nu}, \\ \langle \check{\xi}^{\mu\nu}(t, \mathbf{x}) \check{\xi}^{\rho\sigma}(t', \mathbf{x}') \rangle_\xi = 2\bar{T} \eta^{\mu\nu\rho\sigma} \delta(t - t') \delta^{(3)}(\mathbf{x} - \mathbf{x}'), \\ \eta^{\mu\nu\rho\sigma} = \eta \left(\frac{1}{2} (\Delta^{\mu\rho} \Delta^{\nu\sigma} + \Delta^{\mu\sigma} \Delta^{\nu\rho}) - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\rho\sigma} \right) + \zeta \Delta^{\mu\nu} \Delta^{\rho\sigma} \end{cases}$$

◆ Path-integral formula for generating functional

$$Z[g_{\mu\nu}^{\text{R}}, g_{\mu\nu}^{\text{A}}] = \int \mathcal{D}T_{\text{ini}}^{0\mu} P_{\text{fluc}}[\delta T_{\text{ini}}^{0\mu}] \int \mathcal{D}T^{0\mu} \mathcal{D}\pi_\mu^{\text{A}} \mathcal{J} e^{i\mathcal{S}_{\text{fluid}}[T^{0\mu}, \pi_\nu^{\text{A}}; g_{\mu\nu}^{\text{R}}, g_{\mu\nu}^{\text{A}}]}$$

$$\mathcal{S}_{\text{fluid}} = \int d^4x \sqrt{-g_{\text{R}}} \left[-\frac{1}{2} T^{\mu\nu} D_\mu \pi_\nu^{\text{A}} + \frac{i}{4} T \eta^{\mu\nu\rho\sigma} D_\mu \pi_\nu^{\text{A}} D_\rho \pi_\sigma^{\text{A}} \right]$$

$$\text{with} \quad \begin{cases} T^{\mu\nu} \equiv (e + p) u^\mu u^\nu + p g_{\text{R}}^{\mu\nu} - \bar{T} \eta^{\mu\nu\rho\sigma} \nabla_\rho \beta_\sigma, \\ D_\mu \pi_\nu^{\text{A}} \equiv \nabla_\mu \pi_\nu^{\text{A}} + \nabla_\nu \pi_\mu^{\text{A}} - g_{\mu\nu}^{\text{A}}. \end{cases}$$

π_μ^{A} is NG field with $\text{Diff}_{\text{R}} : \pi_\mu^{\text{A}} \rightarrow \pi_\mu^{\text{A}} + \xi_\mu^{\text{A}}, \quad g_{\mu\nu}^{\text{A}} \rightarrow g_{\mu\nu}^{\text{A}} + \nabla_\mu \xi_\nu^{\text{A}} + \nabla_\nu \xi_\mu^{\text{A}}$

What is NG field $\pi^{\mu,R}$

From elastic theory, we know

NG field for a solid = displacement vector

◆ Lagrangian specification of flow

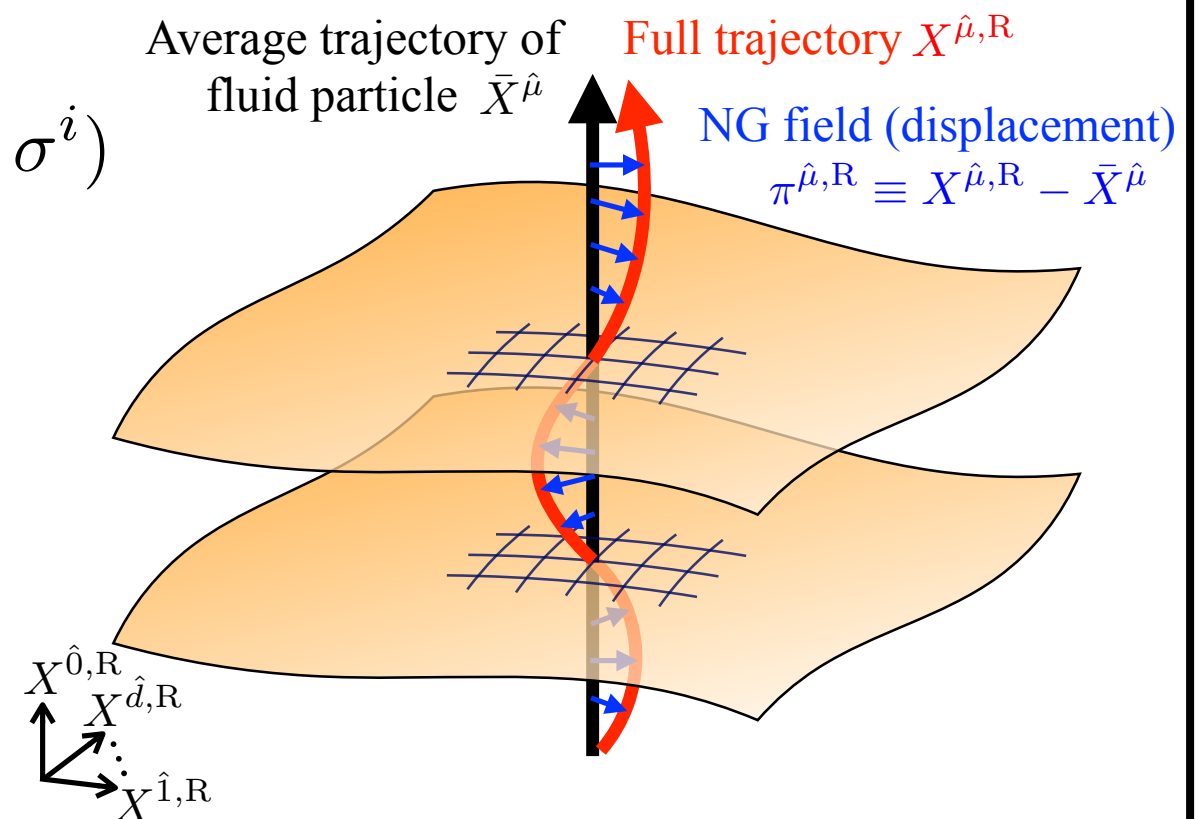
(I) Assign a label to each fluid particle: (σ^0, σ^i)

(2) Introduce a fluid particle position:

$$X^{\mu,R}(\sigma^0, \sigma^i) \text{ satisfying } \beta_0 \frac{\partial X^{\mu,R}}{\partial \sigma^0} = \beta^\mu$$

(3) Introduce displacement (NG) field:

$$X^{\mu,R} = \bar{X}^\mu + \pi^{\mu,R}$$



Gauge symmetry:

$$X^{\mu,R}(\sigma^0, \sigma^i) \rightarrow X^{\mu,R}(\sigma^0, \sigma^i) + F^\mu(\sigma^i)$$

(redundancy)

$$\text{or } \sigma^0 \rightarrow \sigma^0 + F^0(\sigma^0, \sigma^i), \quad \sigma^i \rightarrow \sigma^i + F^i(\sigma^i)$$

Worldvolume EFT for hydro

◆ Physical interpretation of gauge symmetry

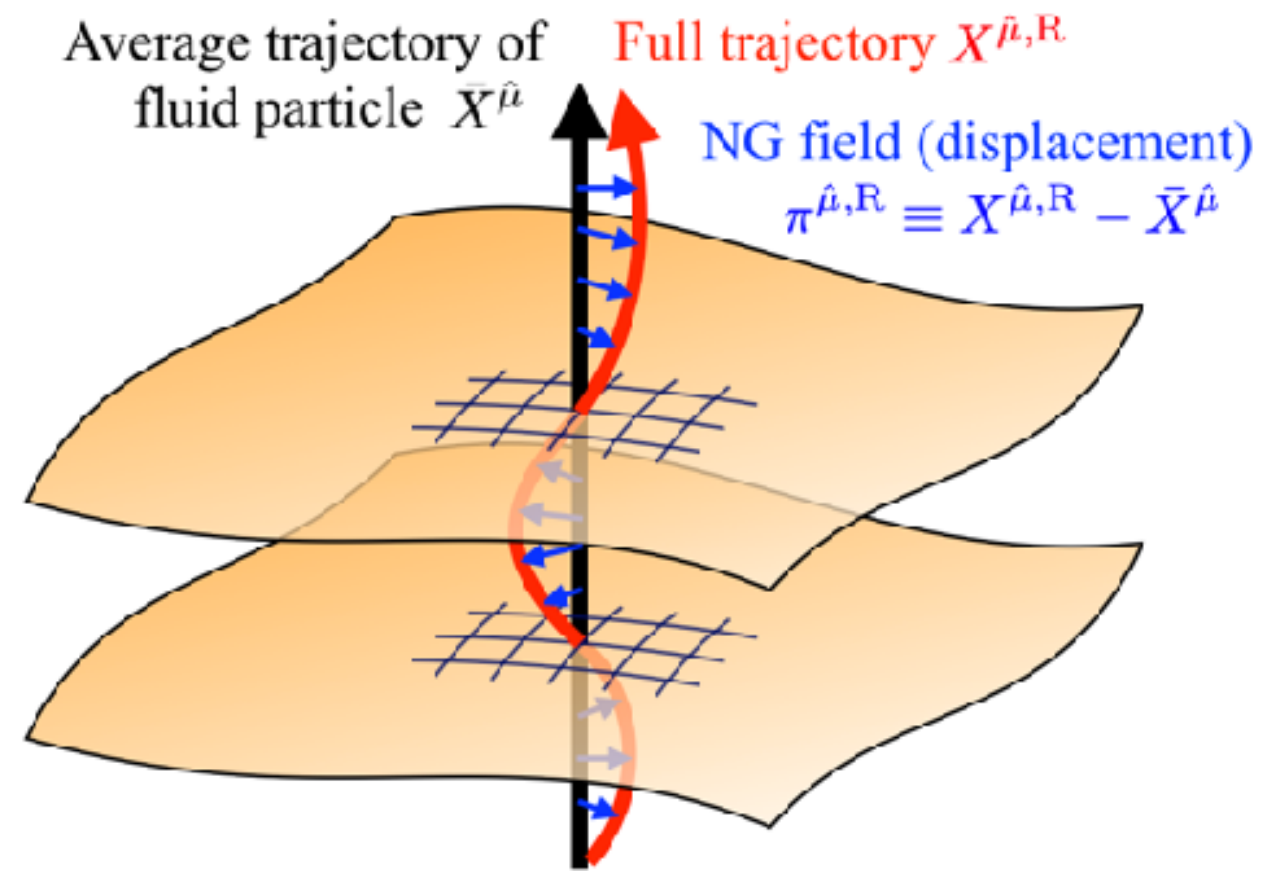
Gauge symmetry: $X^{\mu,R}(\sigma^0\sigma^i) \rightarrow X^{\mu,R}(\sigma^0\sigma^i) + F^{\mu}(\sigma^i)$
(redundancy) or $\sigma^0 \rightarrow \sigma^0 + F^0(\sigma^0, \sigma^i), \sigma^i \rightarrow \sigma^i + F^i(\sigma^i)$

Dynamics of the fluid is **unchanged**

- if we **reparameterize the time** of each fluid particle, and
- if we **reshuffle the label** of the fluid particles

Hydrodynamics is low-energy EFT of a spacetime filling brane $X^{\mu}(\sigma^0, \sigma^i)$, enjoying **emergent gauge symmetry**:

$$\begin{cases} \sigma^0 \rightarrow \sigma^0 + F^0(\sigma^0, \sigma^i) \\ \sigma^i \rightarrow \sigma^i + F^i(\sigma^i) \end{cases}$$



Kaluza-Klein gauge symmetry

◆ Induced metric for worldvolume

Choosing $\sigma^\mu = x^\mu$, we introduce

$$G_{\mu\nu}^R(x) \equiv \frac{\partial X^{\hat{\mu},R}(x)}{\partial x^\mu} \frac{\partial X^{\hat{\nu},R}(x)}{\partial x^\nu} g_{\hat{\mu}\hat{\nu}}^R(X^R(x))$$

Redundancy: $X^{\mu,R}(\sigma^0 \sigma^i) \rightarrow X^{\mu,R}(\sigma^0 \sigma^i) + F^\mu(\sigma^i)$

can be regarded as (noncompact) **Kaluza-Klein gauge symmetry**

➡ Normal fluid is described by $G_{00}^R = \beta^\mu \beta^\nu g_{\mu\nu}^R / (\beta_0)^2 = -\beta^2(x) / (\beta_0)^2$

◆ Variation formula for a nondissipative part

$$T_{(0)}^{\mu\nu}(x) = \frac{2}{\beta_0 \sqrt{-G^R}} \frac{\delta \Psi[G_{\mu\nu}^R]}{\delta G_{\mu\nu}^R(x)} \quad \text{with} \quad \begin{cases} \Psi_{\text{fluid}} = \int d^3x \beta_0 \sqrt{-G} p(\beta_0 \sqrt{-G_{00}}), \\ \Psi_{\text{solid}} = \int d^3x \beta_0 \sqrt{-G} p(\beta_0 \sqrt{-G_{00}}, G_{ij}), \end{cases}$$

Summary of **Worldvolume EFT**

◆ Effective Lagrangian for a dissipative fluid/solid

$$\mathcal{S}_{\text{fluid}} = \int d^4x \sqrt{-G_R} \left[-\frac{1}{2} H_{\mu\nu}^A \frac{2}{\beta_0 \sqrt{-G_R}} \frac{\delta \Psi}{\delta G_{\mu\nu}^R} - \frac{1}{4} T \eta^{\mu\nu\rho\sigma} (\mathcal{L}_\beta G_{\rho\sigma}) H_{\mu\nu}^A + \frac{i}{4} T \eta^{\mu\nu\rho\sigma} H_{\mu\nu}^A H_{\rho\sigma}^A \right]$$

(1) Dynamical **degrees of freedom** = $\{X^{\mu,R}, \pi_{\nu,R}\}$

$$G_{\mu\nu}^R(x) \equiv \frac{\partial X^{\hat{\mu},R}(x)}{\partial x^\mu} \frac{\partial X^{\hat{\nu},R}(x)}{\partial x^\nu} g_{\hat{\mu}\hat{\nu}}^R(X^R(x)), \quad H_{\mu\nu}^A \equiv g_{\mu\nu}^A - \nabla_\mu \pi_\nu^A - \nabla_\nu \pi_\mu^A$$

(2) Diffeomorphism is **nonlinearly realized** with NG fields $\{\pi^{\mu,R}, \pi_{\nu,R}\}$

(3) Thermodynamic functional Ψ controls whether **SSB occurs or not**

$$\Psi_{\text{fluid}} = \int d^3x \beta_0 \sqrt{-G} p(\beta_0 \sqrt{-G_{00}}), \quad \Psi_{\text{solid}} = \int d^3x \beta_0 \sqrt{-G} p(\beta_0 \sqrt{-G_{00}}, G_{ij})$$

(4) Z_2 symmetry implements **the fluctuation-dissipation relation**

(5) **Matching condition** = Linear response (e.g. **Green-Kubo**) formula

Goal of today's talk

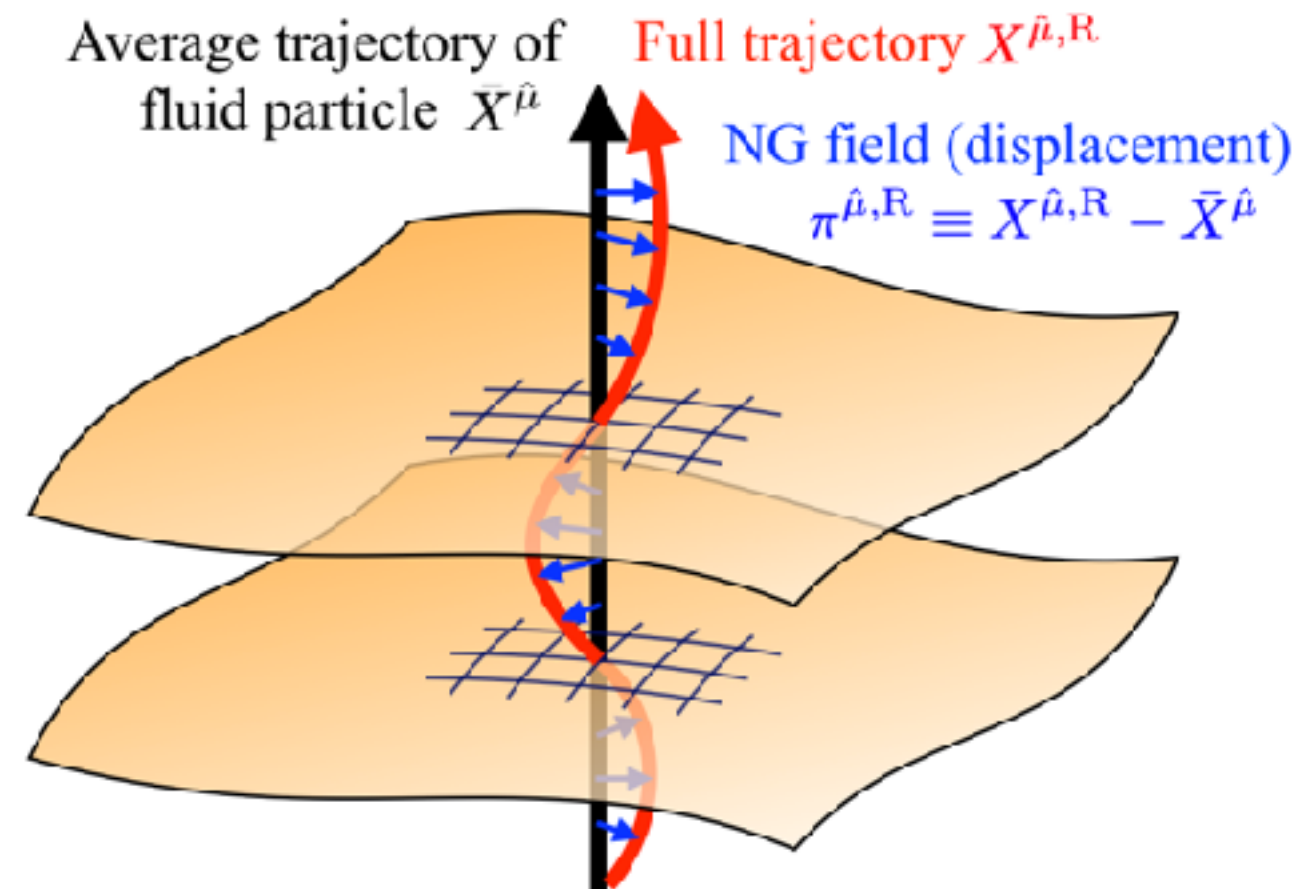
2. Effective Lagrangian (**real-time** formalism)

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\pi_{\text{hydro}} \exp(i\mathcal{S}_{\text{eff}}[\pi_{\text{hydro}}])$$

(corresponds to a construction of **chiral Lagrangian** in **QCD**)

Hydrodynamics is low-energy EFT of a spacetime filling brane $X^\mu(\sigma^0, \sigma^i)$, enjoying **emergent gauge symmetry**:

$$\begin{cases} \sigma^0 \rightarrow \sigma^0 + F^0(\sigma^0, \sigma^i) \\ \sigma^i \rightarrow \sigma^i + F^i(\sigma^i) \end{cases}$$



[See Crossley et al. arXiv: 1511.03646 [hep-th], MH et al. unpublished]

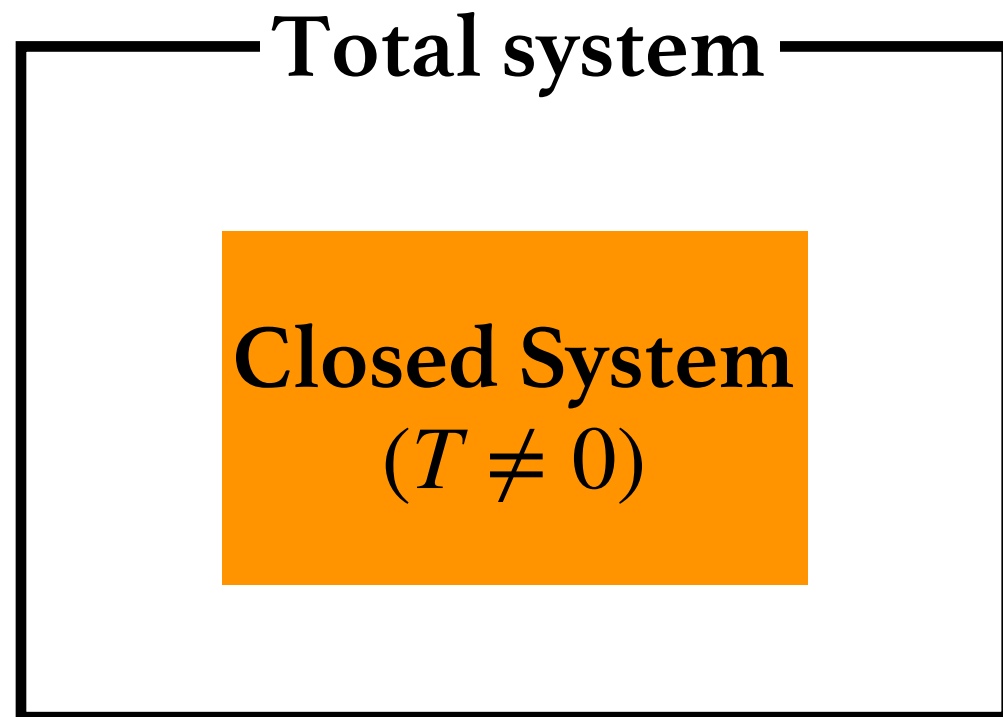
Beyond Hydrodynamics

Diffusive NG modes in open system

NG modes at $T \neq 0$ and beyond?

Q. How NG modes behave in a more general setup?

Closed Systems at $T \neq 0$



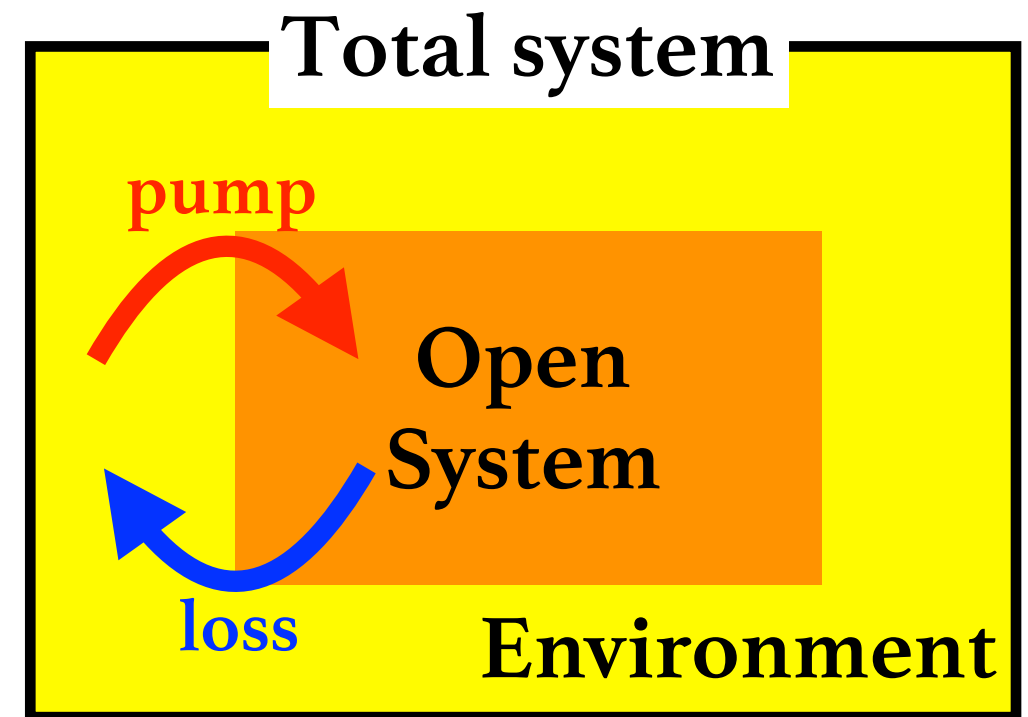
- Type-A: Diffusive propagation
- Type-B: Diffusive precession

Landry, 1912.12301, 2008.11725

Akyuz-Goon-Penco, 2306.17232

Delacrétaz et al.: 2111.13459, MH et al, 2411.08016

Noneq. open system



- Type-A: Diffusion (+ gapped damp.)
- Type-B: Diffusive precession

Minami-Hidaka, 1509.05042, 1907.08241

MH-Kim-Noumi-Ota, 1805.06240, 1907.08609

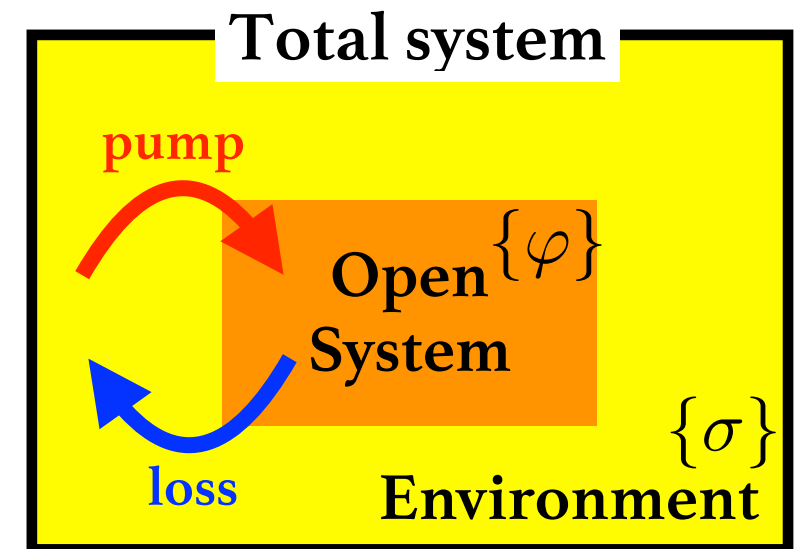
Sieberer-et al, 1512.00637, Akyuz-Penco: 2503.22840

Doubled symmetry structure

◆ Dynamical degrees of freedom

$$\text{Total system (G-inv.)} = \text{System } \{\varphi\} + \text{Environment } \{\sigma\}$$

$$S_{\text{tot}}[\varphi, \sigma] = S_{\text{sys}}[\varphi] + S_{\text{env}}[\sigma] + S_{\text{int}}[\varphi, \sigma]$$



◆ Doubled symmetry structure in total & open systems

$$\begin{aligned} Z &= \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 e^{i(S_{\text{tot}}[\varphi_1, \sigma_1] - S_{\text{tot}}[\varphi_2, \sigma_2])} \rho_0(\varphi, \sigma) \\ &= N \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 e^{i(S_{\text{sys}}[\varphi_1] - S_{\text{sys}}[\varphi_2]) + i\Gamma[\varphi_1, \varphi_2]} \rho_0(\varphi) \end{aligned}$$

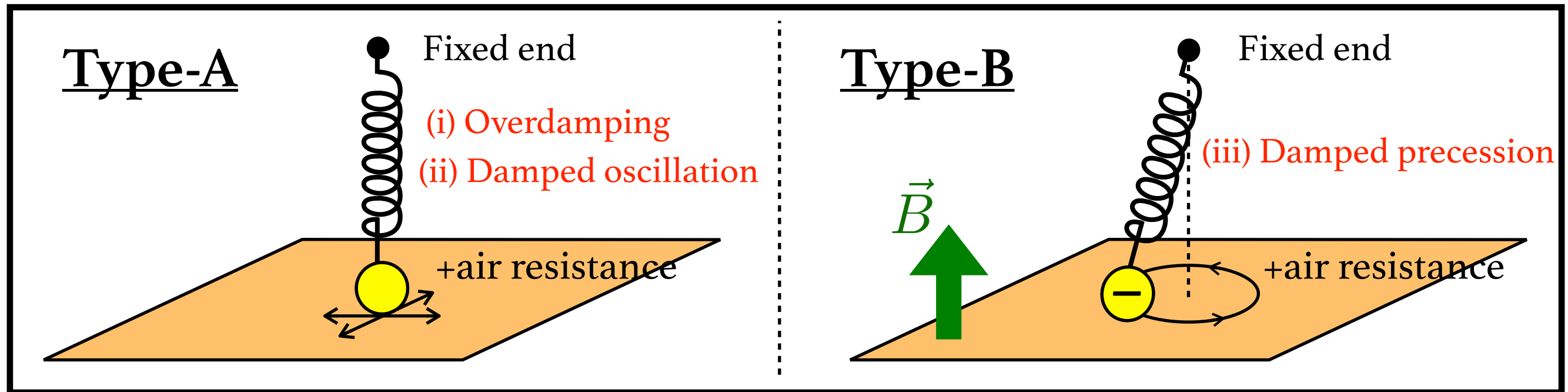
$S_{\text{tot}}[\varphi_1, \sigma_1] - S_{\text{tot}}[\varphi_2, \sigma_2]$ is invariant under $G_1 \times G_2 = \text{strong } G_R \times \text{weak (diag.) } G_A$

$S_{\text{sys}}[\varphi_1] - S_{\text{sys}}[\varphi_2] + \Gamma[\varphi_1, \varphi_2]$ can be only invariant under G_A

➡ We can consider the SSB of G_A (weak) symmetry in open systems!

NG mode in open system

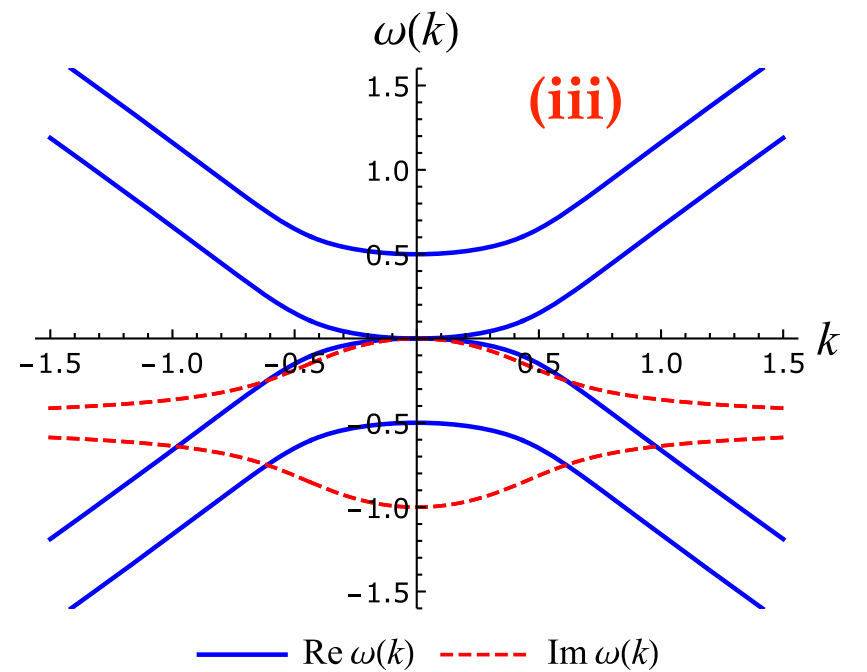
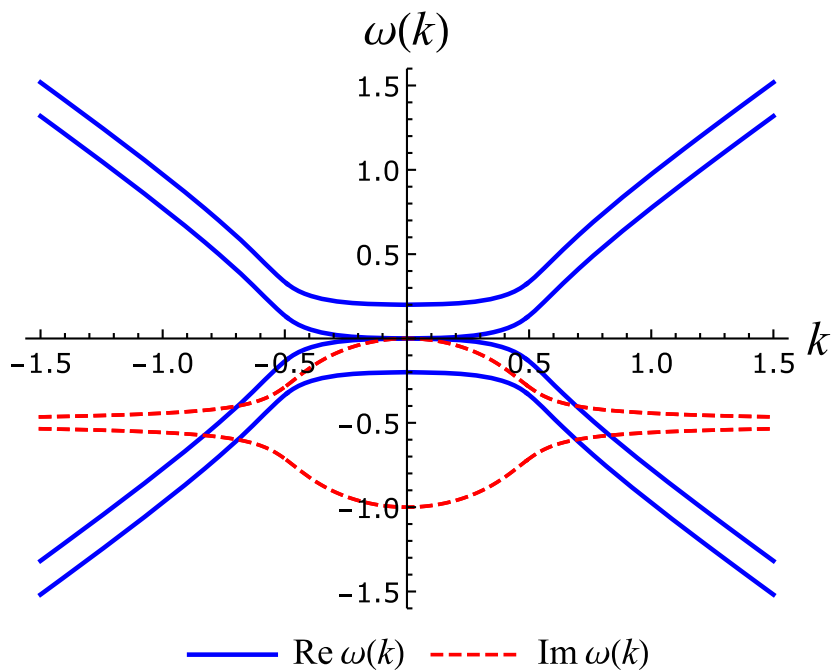
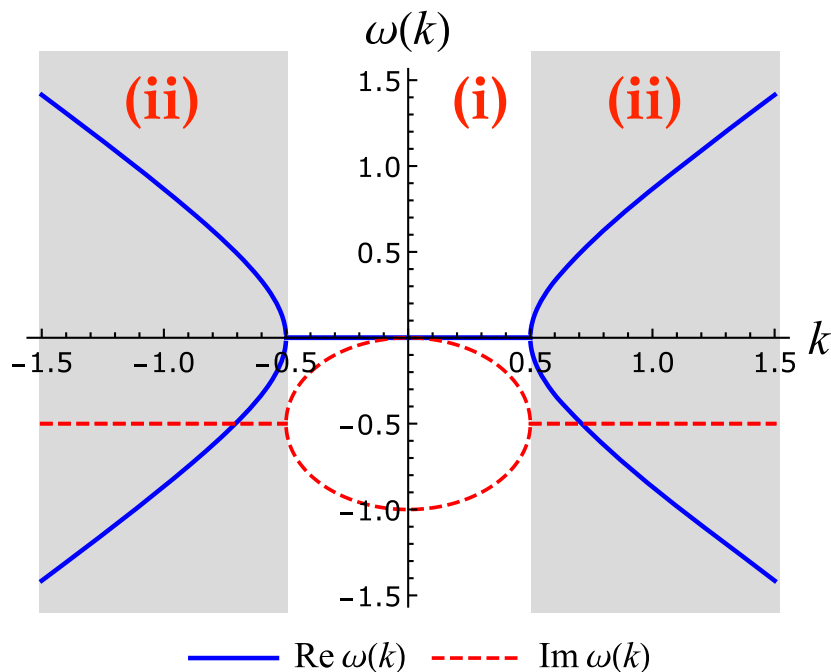
Minami-Hidaka, 1509.05042, 1907.08241, MH-Kim-Noumi-Ota, 1805.06240, 1907.08609 Sieberer-et al, 1512.00637, Akyuz-Penco: 2503.22840



(a) Type-A NG mode: $(f, \gamma, m) = (1.0, 1.0, 0.0)$

(b) Type-B NG mode: $(f, \gamma, m) = (1.0, 1.0, 0.2)$

(c) Type-B NG mode: $(f, \gamma, m) = (1.0, 1.0, 0.5)$



$\leftarrow$ Type-A limit

 $\longleftrightarrow$

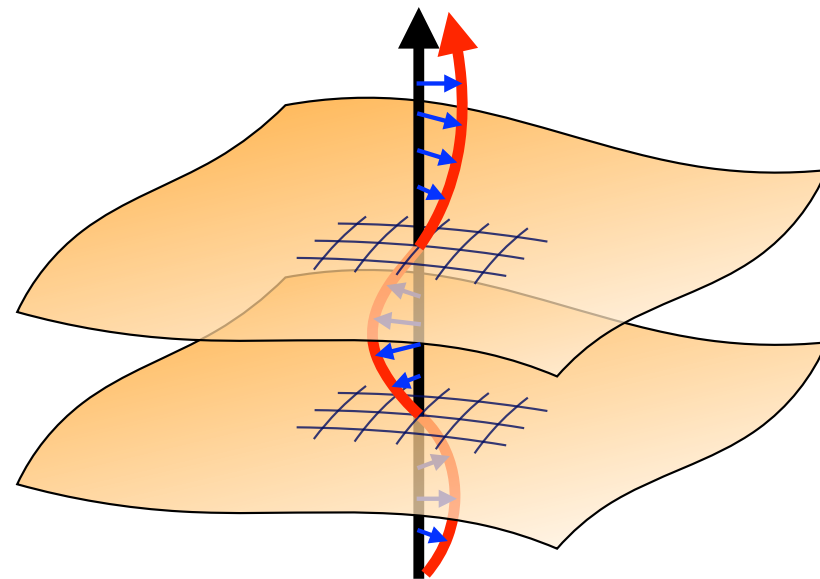
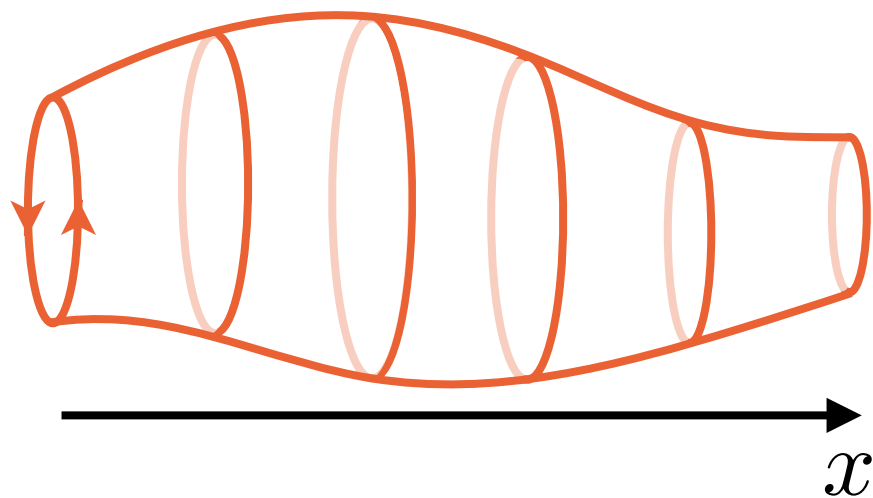
 Type-B limit $\rightarrow$

ex: Driven-dissipative BEC Dissipative ferri magnon Dissipative ferro magnon

Summary: Hydro from QFT

Part 1. Conventional hydrodynamics [Ref. Hayata-Hidaka-MH-Noumi, (2015), MH .(2017)]

- Hydro = **Optimized perturbation** with respect to real-time dynamics
- Extention of **imaginary-time (Matsubara)** formalism:
Local thermal equilibrium = Quantum theory in “**curved spacetime**”



Part 2. Fluctuating hydrodynamics [Ref. Crossley-Glorioso-Liu, (2017) Delacretaz, 2606.02391]

- How to formulate low-energy **EFT for nonequilibrium systems**
- Effective Lagrangian for **nonlinearly fluctuating hydrodynamics**
(cf. Chiral perturbation in QCD)