Symmetry and Simplicity outside Hydrodynamics

CUHK-SZ

Yi Yin

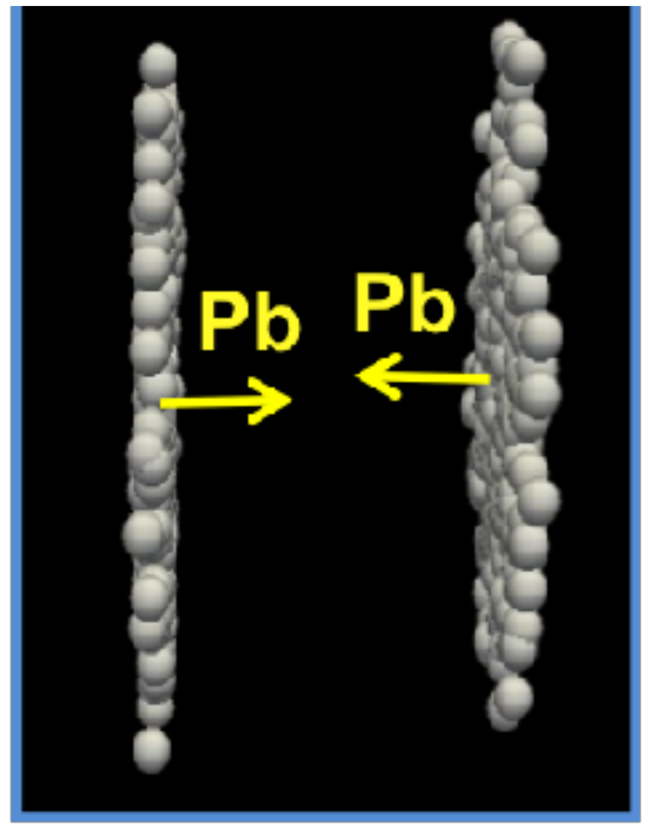
IPMU workshop, Aug. 6

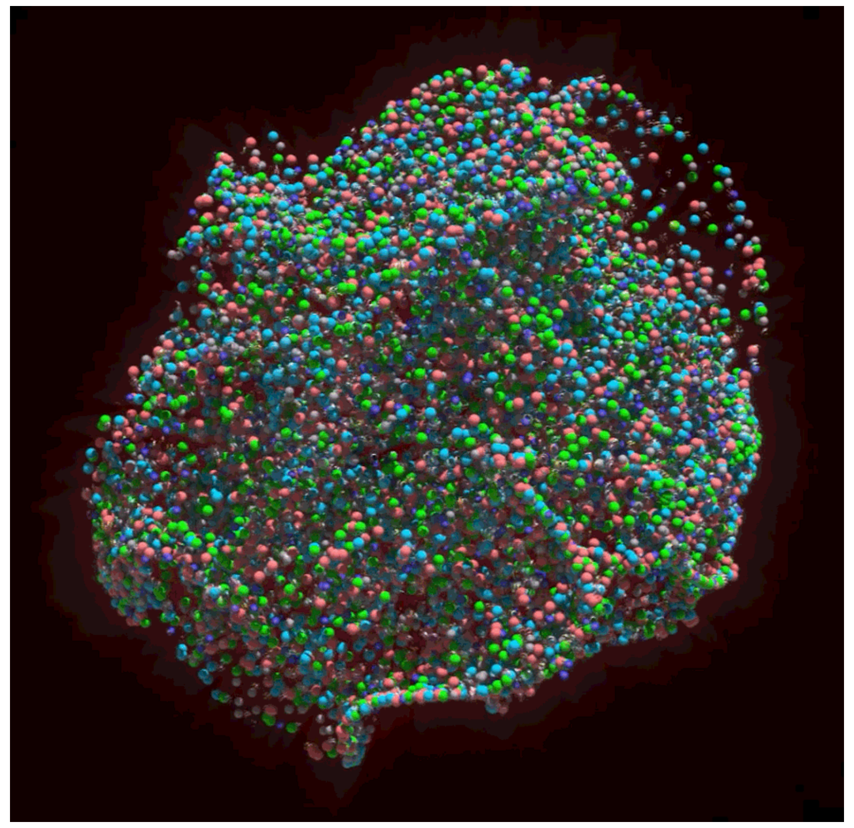
Heavy Ion Collisions (Little Bang)

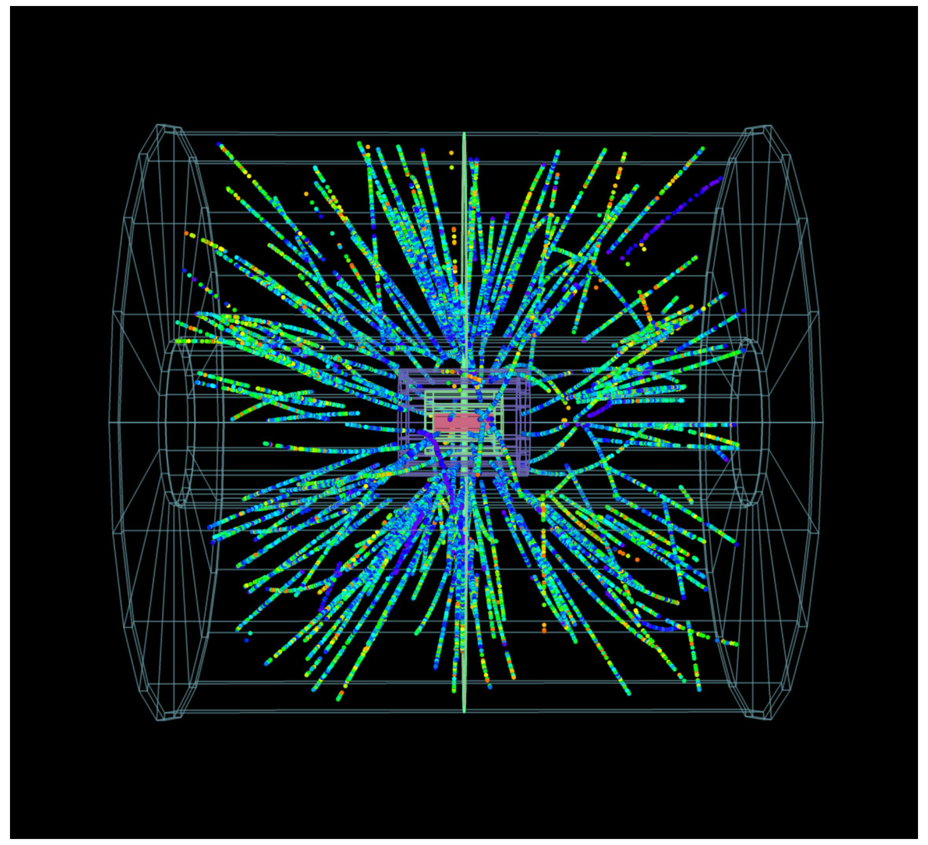
Image from CERN

Before collision *Quark matter* *Produced hadrons*

$10\ fm = 10^{-14} m$







$T \sim 300\ MeV \sim 10^{12} K$

- recreates hot quark matter (quark-gluon plasma, QGP) at RHIC and LHC

Quark Liquid

- In $T \to \infty$ limit, quark matter must be weakly coupled quark/gluon quasiparticles (asymptotic freedom)

- Those produced in heavy-ion collisions behave as a liquid with extraordinarily small shear viscosity (small "friction" in fluid flow):

$\eta/s \sim 1/(4\pi)$: seen strongly-coupled gauge theories, unitary fermi gas etc

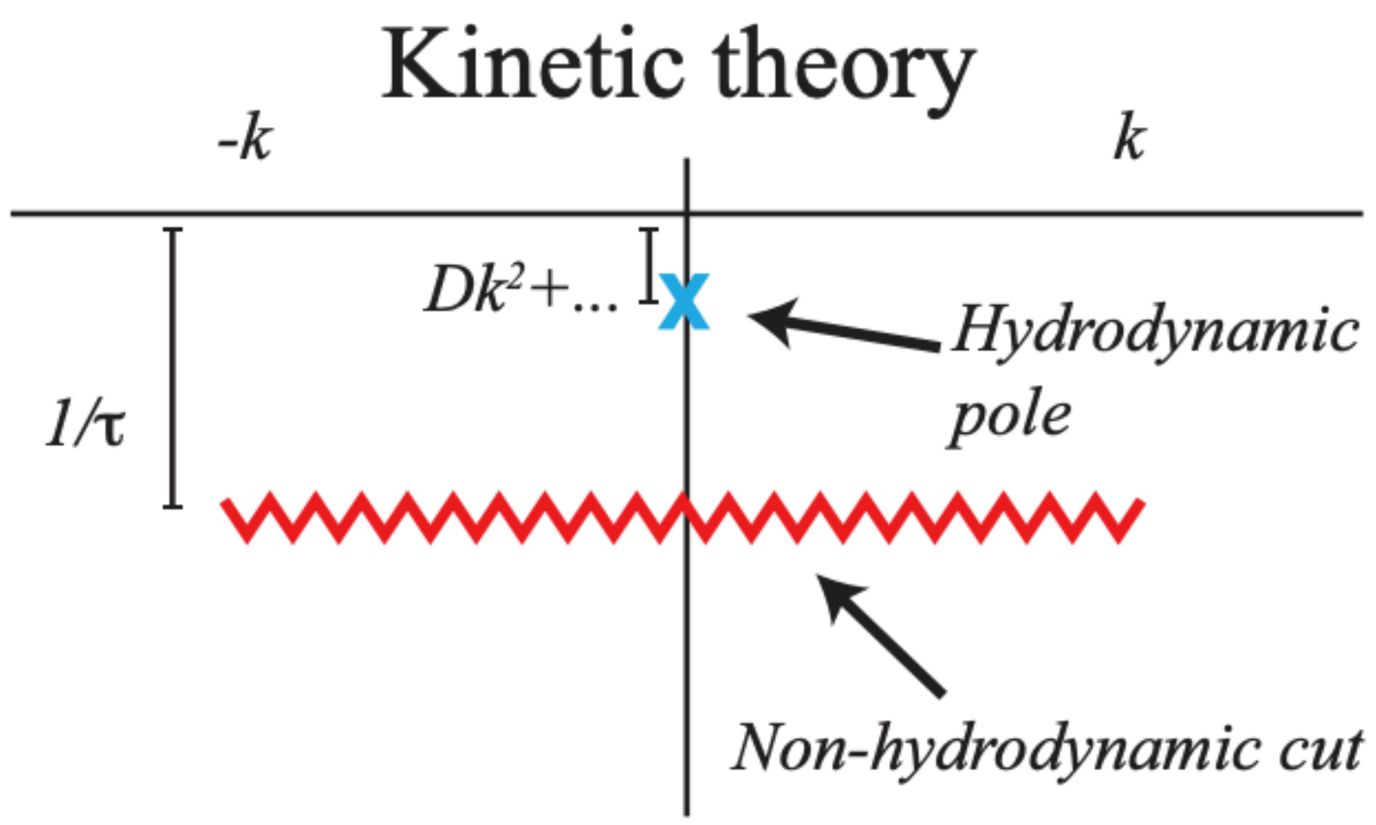
- Interdisciplinary connection to condensed matter physics, astrophysics; many exciting subjects meet here

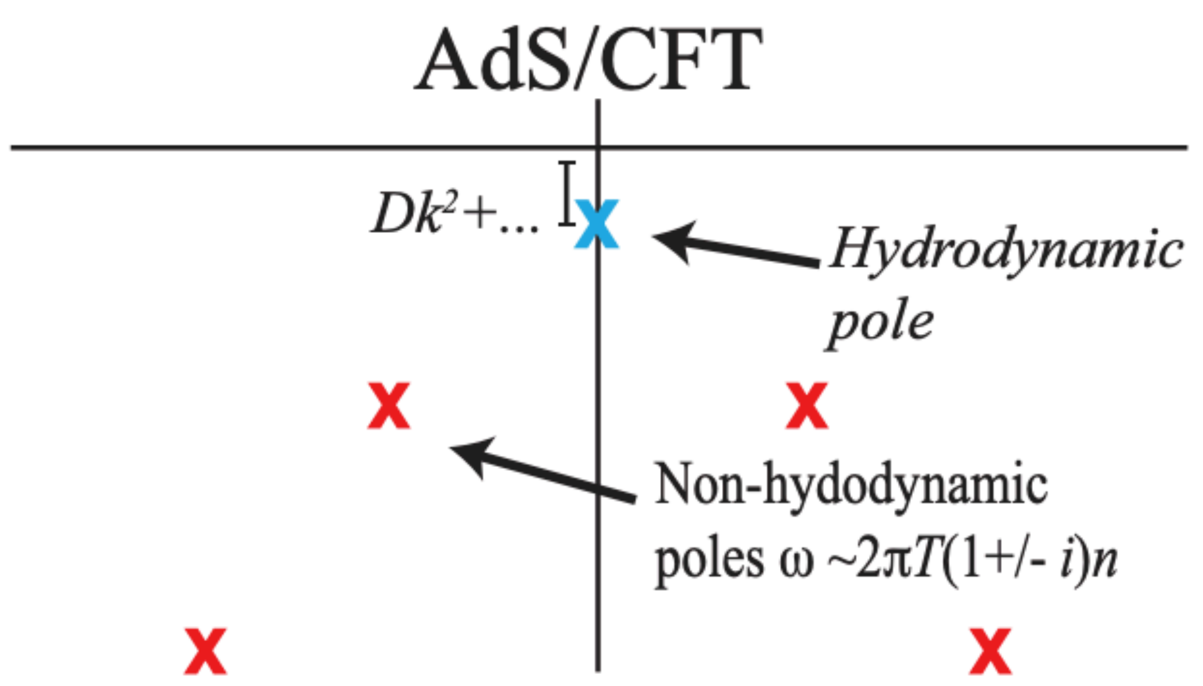
What Next?

- How does quark liquid emerge from asymptotic QCD?
- Strategy: draw general lessons by exploring QCD-like theories

Non-hydro. Excitations are Diverse

Fig. from Kurkela-Wiedemann-Wu, EPJC 19'





Holography: Einstein eqns around black hole background

- Common wisdom: "Hydro. are all alike; every nonhydro. sector is different in its own way."
- Similar situation in open systems: memory effects require keeping diverse environmental modes.
- However, we uncover principled and simple description beyond hydro

Outline

- Symmetry organize non-hydrodynamic modes
- Shift symmetry “localize” non-Markovian thermal fluctuations
- Only a few modes may dominate outside hydro. regime
- Outlook

Symmetry-based SK action for non-hydro

An-Brants-Heller-YY, 2511.11555

Schwinger-Keldysh EFT

e.g. Glorioso-Crossley-Liu, JHEP 17';

$$\mathrm{tr}(\rho_{\mathrm{mic}}\,\hat{O}) \approx \int \mathscr{D}\chi \mathscr{D}\chi_a \, e^{iI_{EFT}[\chi,\chi_a]} \, O[\chi]$$

χ: parameter fields for d.o.f.s at scale of interest

$$I[\chi,\chi_a] \sim E[\chi]\,\chi_a + F[\chi]\chi_a^2$$

E.o.M Fluctuations

- Symmetry constraint from both underlying microscopic theory and emergent properties of ρ_I (e.g. KMS relation and more)

EFT for Diffusion

Crossley et al, JHEP2015

- Describing dynamics of conserved charge density n associated with a global $U(1)_V$ in hydro. limit
- Parameter fields: phase (r-)field ψ and noise ψ^a
- Symmetry
  - Global $U(1)_V$ (for both r and a field):
  - The state of normal fluid (unbroken U(1)) is implemented as shift symmetry acting on r-field

$$\boxed{\psi(x) \to \psi(x) + \lambda(\vec{x})}$$

EFT for Diffusion

Crossley et al, JHEP2015

$$I[\psi, \psi^a; V] = \int_x \left[\chi\,(\partial_t\psi\,\partial_t\psi^a) + \sigma\,\partial_t(\vec{\partial}\psi)\cdot(\vec{\partial}\psi^a) \right] + \text{noise term}$$

fixed by KMS relation

$$n = \chi\,\partial_t\psi \qquad \vec{j} = \sigma\,\partial_t\vec{\partial}\psi = -\,D\,\vec{\partial}n$$

- E.o.M for $\psi \leftrightarrow$ diffusion equation

- Shift symmetry forbids sound-like mode

$$\cancel{(\vec{\partial}\psi)\cdot(\vec{\partial}\psi^a)} \longrightarrow \omega = \cancel{c}k - iDk^2$$

Can we extend symmetry-based construction beyond hydro. limit?

Reservation: hydro. modes are conserved density associated with symmetry. Non-hydro. modes are not obviously related to symmetry

"we invoke a procedure that is often useful in theoretical physics. …"It is more blessed to ask forgiveness than permission." The stratagem tells you to make clear-cut simplifying assumptions, work out their consequences, and check to see that you don't run into contradictions"

Wilczek, Physics Today, 2000

Symmetry-based Construction

An-Brants-Heller-YY, 2511.11555

- Assumption: relevant non-hydro. modes can be described using theories with multiple global U(1)s
  - one combination is microscopic $U(1)_V$; others are emergent
  - scenarios: 1) $U(1)$s are unbroken 2) maximally broken; 3) hybrid
- Consequence: action constructed with field content and shift-symmetry naturally identified in each scenario
- Check: 1) a unified framework for almost all known theories: kinetic theory, holography, Maxwell-Cattaneo; 2) generating new ones

Scenario 1: all U(1) are unbroken

- Fields: phase field φ_m , φ_m^a for each $U(1)$ labelled by m
- All U(1)s are unbroken → shift symmetry under $\varphi_m(x) \to \varphi_m(x) + \lambda_m(\vec{x})$

$$\mathscr{L} = \sum_m \chi_m \left[(\partial_t \varphi_m)(\partial_t \varphi_m^a) + \cancel{m^2(\varphi_m)^2} + (\mathbf{v}_m \cdot \nabla \partial_t \varphi_m)\, \varphi_m^a + \text{dissipation} \right]$$

$\mathbf{v}_m$: *m-dependent background vector*

$$\boxed{\partial_t(\chi_m \partial_t \varphi_m) + \mathbf{v}_m \cdot (\chi_m \partial_t \vec{\partial} \varphi_m) = \text{dissipation}}$$

Describing evolution of generalized density $f_m = \chi_m \partial_t \varphi_m$ labeled by m that "flow" with velocity $\mathbf{v}_m$

Connection to kinetic theory

Else et al 2402.14066; Delacretaz et al, 2203.05004

- View m as single particle momentum $m \to \vec{p}$

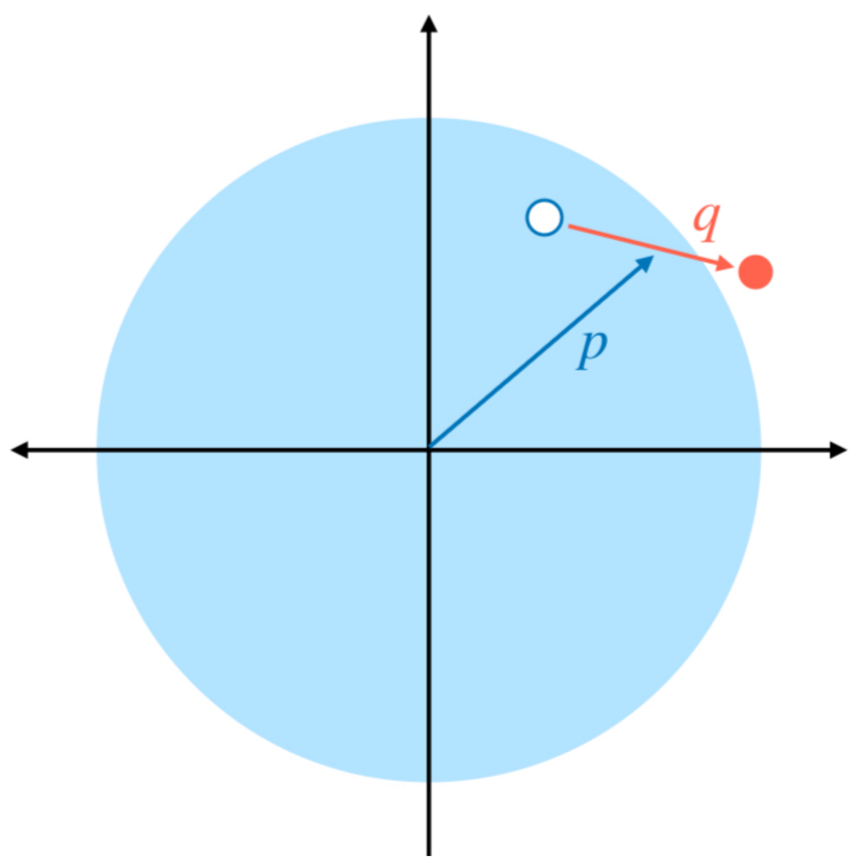
  $\chi_m \partial_t \varphi_m \to f(t, \vec{x}, \vec{p}), \ \mathbf{v}_m \to \mathbf{v}_p$

  $$\boxed{\partial_t f(t, \vec{x}, \vec{p}) + \vec{v} \cdot \vec{\partial} f(t, \vec{x}, \vec{p}) = \ldots}$$

- Origins of infinite $U(1)$: quasi-particles at different patches of Fermi surface (F.S.) can choose their phase independently

- Unbroken U(1)s ↔ Liouville theorem as conservation of phase space volume

- Application: Landau Fermi liquid, action for hard-thermal loop



Maximally Broken Case

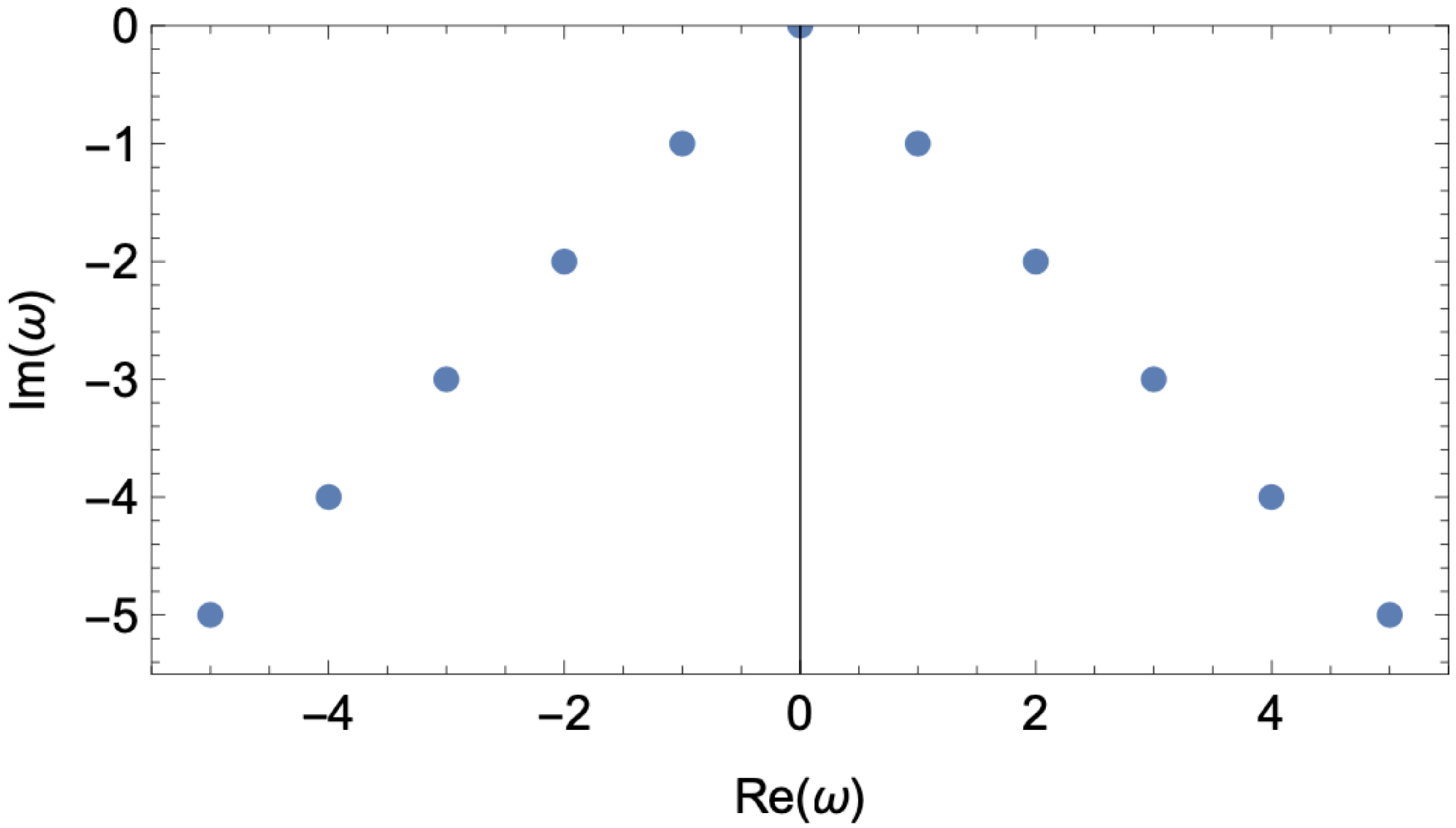
Spontaneous breaking

$$U(1)_1 \times \ldots U(1)_K \times U(1)_V \qquad \rightarrow \qquad U(1)_{\rm dia} \sim U(1)_V\,, \qquad n = 1,\ldots, K$$

- Fields: phase field φ_n +emergent gauge field A_n
- Common shift symmetry: $\varphi_n(x) \rightarrow \varphi_n(x) + \lambda(t,\vec{x})$;ensuring action depends on Goldstones of broken symmetries

Describing multiple emergent massive photons in medium

Modes



- Holo-like (Christmas tree) modes by tuning model parameters

Continuum limit $K \to \infty$

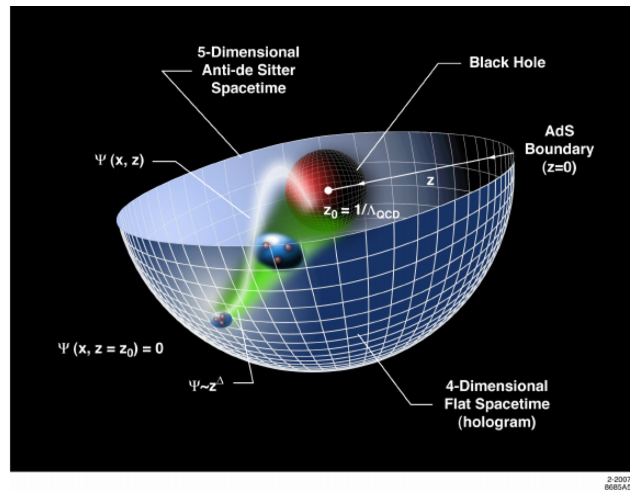
- 4+1d gauge theory in curved spacetime: "n" labels fifth coord. $\rho_n = na$

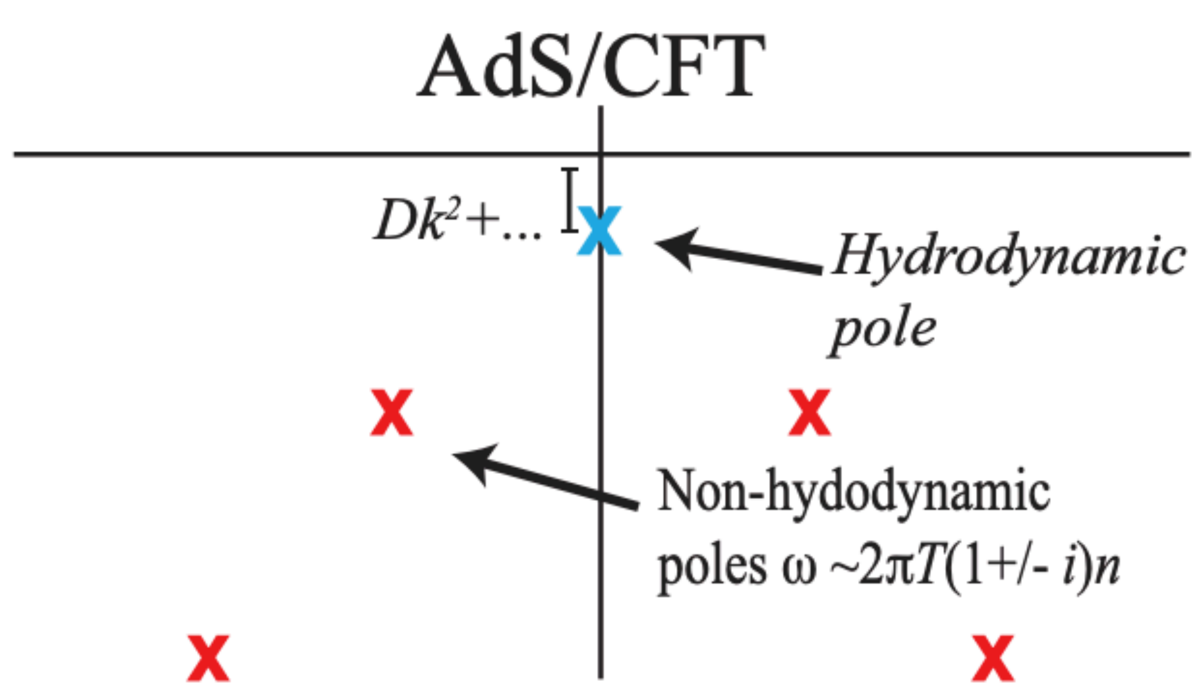
$$\mathscr{L} \sim \sqrt{-h}\, h^{MP} h^{NQ} F_{MN} F_{PQ}$$

  - Emergent photons lives on the slices along ρ

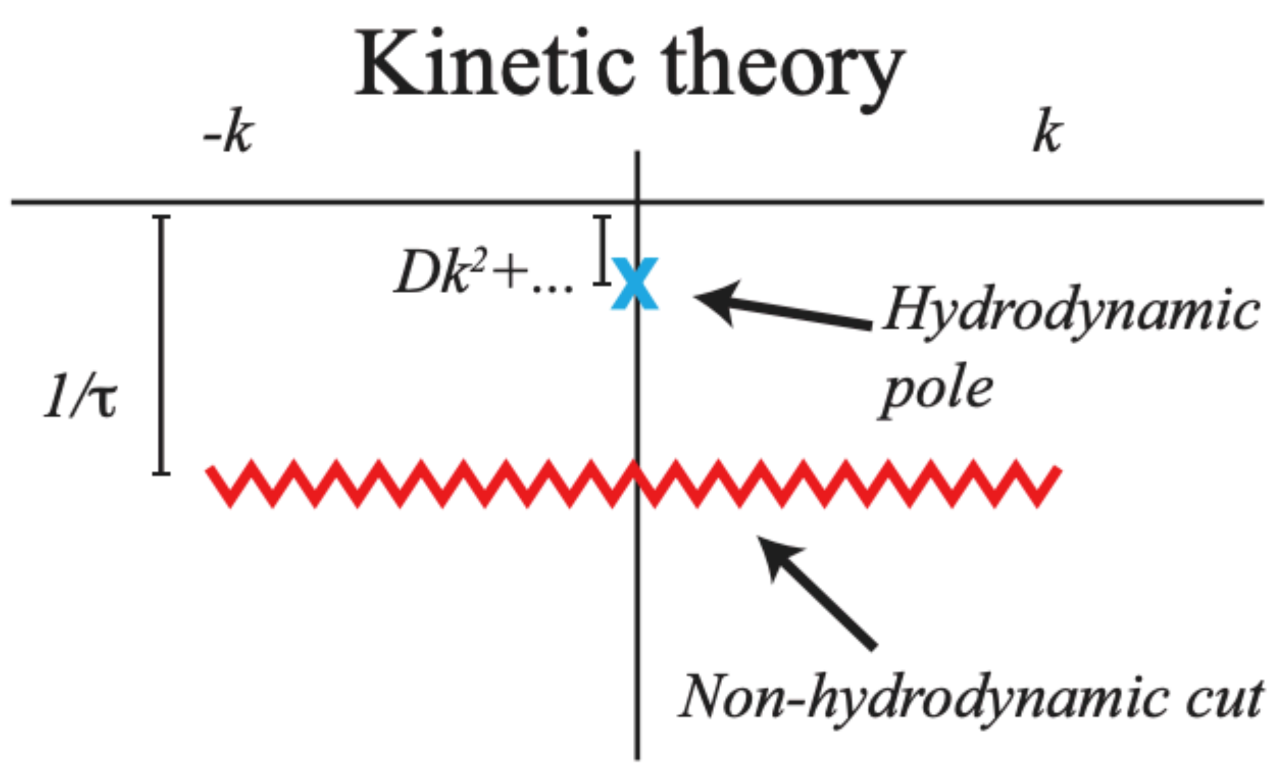
- Holo. model is a subclass of theories with breaking emergent symmetry

$$\boxed{V \text{ - - - - - - } A_K \quad \ldots \quad A_n \text{ - - - - - } A_{n-1} \qquad \text{ - - - - - } (A)^1}$$

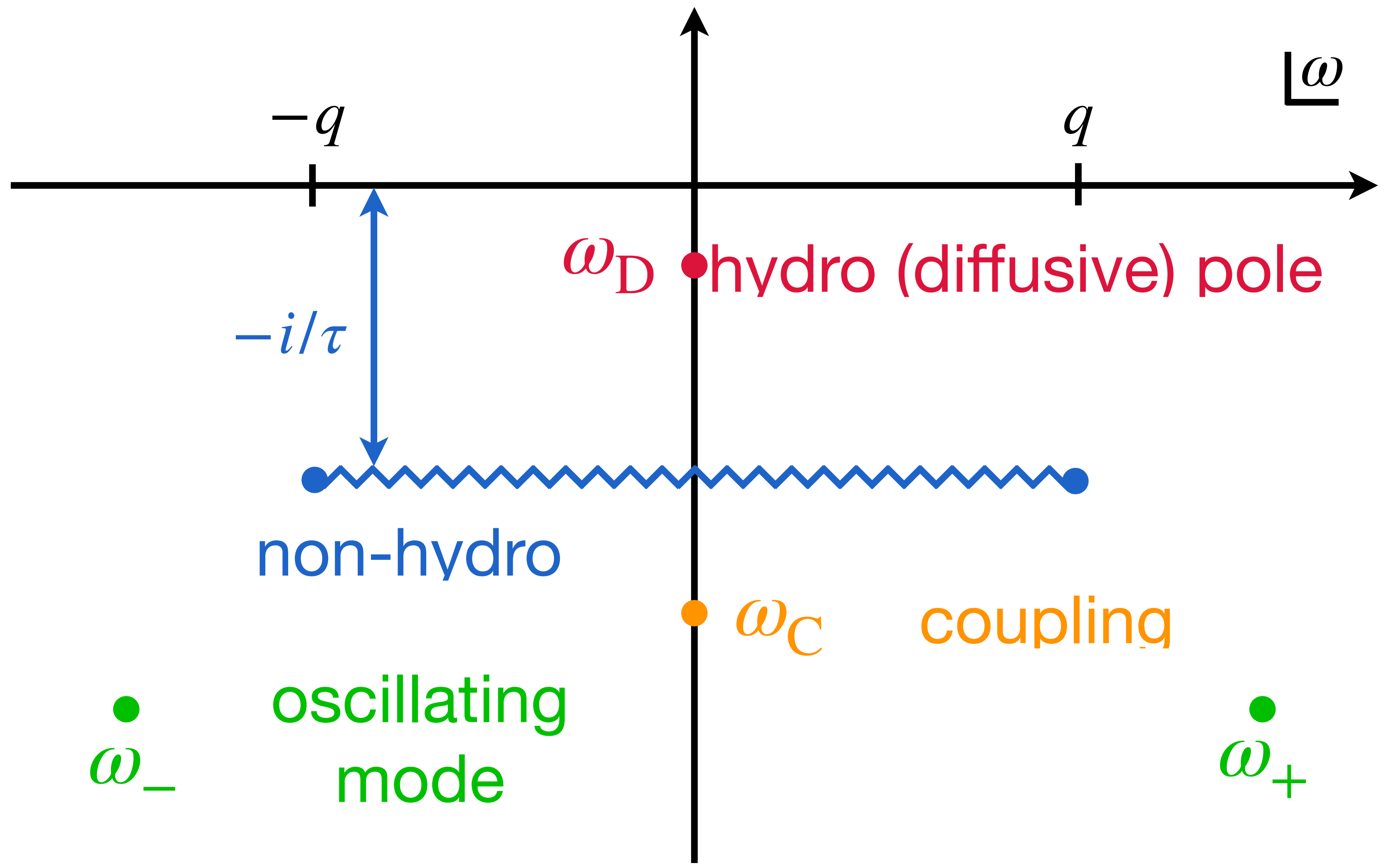




Maximally Broken U(1)



Unbroken U(1)



Mixture

The nonhydro. is diverse, but the diversity can be classified by the fate of emergent U(1) symmetry

Shift symmetry “localize” non-Markovian thermal fluctuations

“non-Markovian” Fluctuations

- Fluct-dissi part of the action for a generic field χ with damping rate γ

$$I_{\rm FD} = \int_\omega i\,\omega\,\gamma\,[\chi^a(-\omega)\chi(\omega) + \frac{1}{2}\coth\left(\frac{\omega}{2T}\right)|\chi^a(\omega)|^2]$$

- Markovian limit $\omega \ll T$: I_{FD} is local and obeys dynamical KMS symmetry
- But I_{FD} is non-local in general ω; singular at Matsubara mode $\omega = m(2\pi T)$
- Goal: restore locality by treating Matsubara modes dynamically

Assumptions

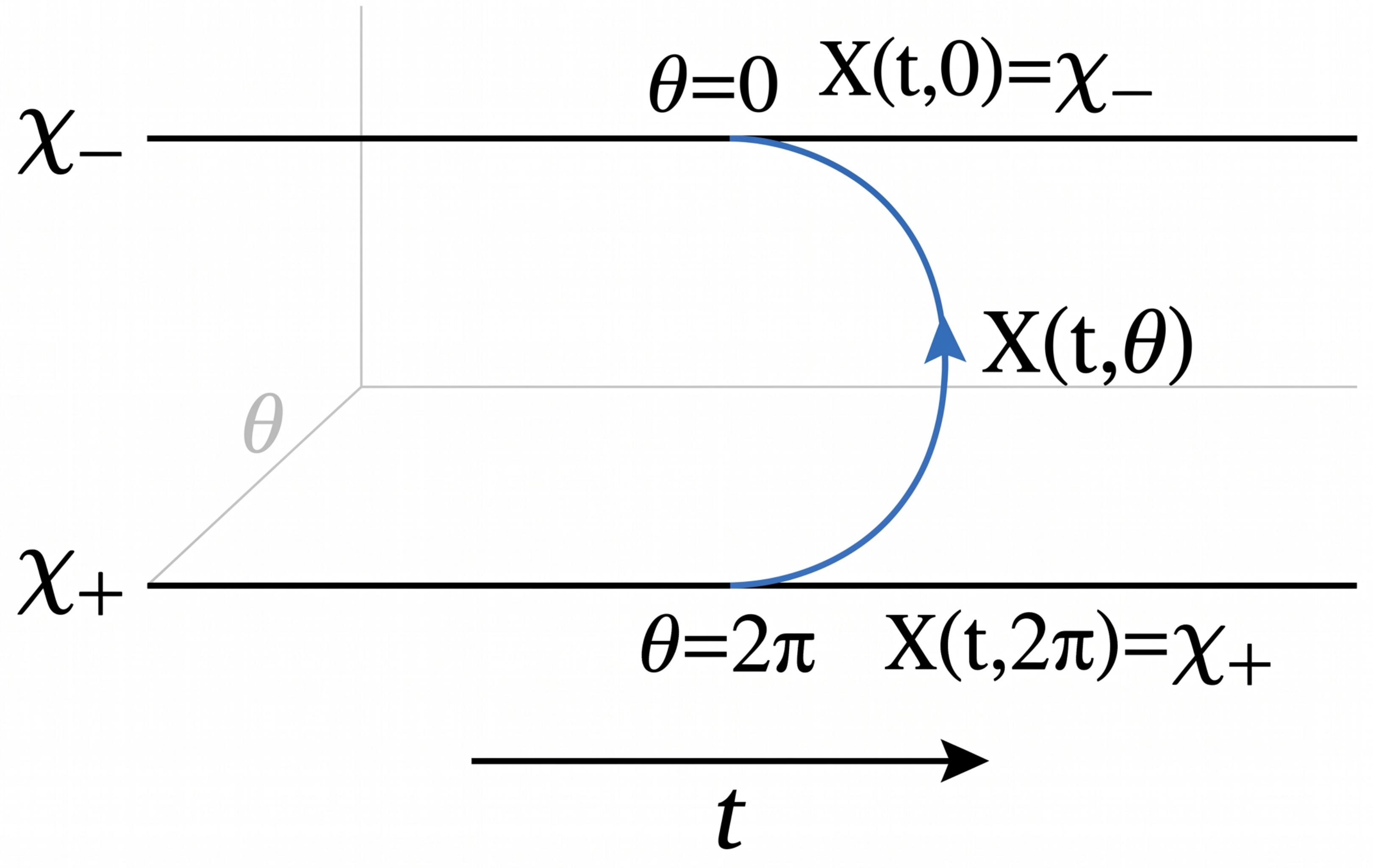
- Introducing "Matsubara fields" $X(x;\theta)$, $\theta \in (0,2\pi)$ on the circle connecting fields on the two SK contours

$$X(\theta=0)=\chi_- \,, \qquad X(\theta=2\pi)=\chi_+$$

- The action $\tilde{I}[X]$ is invariant shift

$$\delta_{\text{shift}} X(\theta,x)=\lambda(x)\,,$$

(no change on $\chi_a=\chi_+-\chi_-$)



An-Brants-Heller-YY, 2511.11555

see also Glorioso, Crossley, Liu 2018 in holography context

X organized by shift symmetry

$$\int \mathscr{D}X \exp\left(i \int dt \int_0^{2\pi} d\theta \; \left[ia(\partial_\theta X)^2 - b(\partial_\theta X\, \partial_t X) \right] \right)$$

Fluct-diss at general ω

$$= \exp\left(i \int_\omega i\,\omega\,\gamma\,[\chi^a(-\omega)\chi(\omega) + \frac{1}{2}\coth\left(\frac{\omega}{2T}\right) |\chi^a(\omega)|^2] \right)$$

$$a = 2\pi T\gamma \, , b = \gamma$$

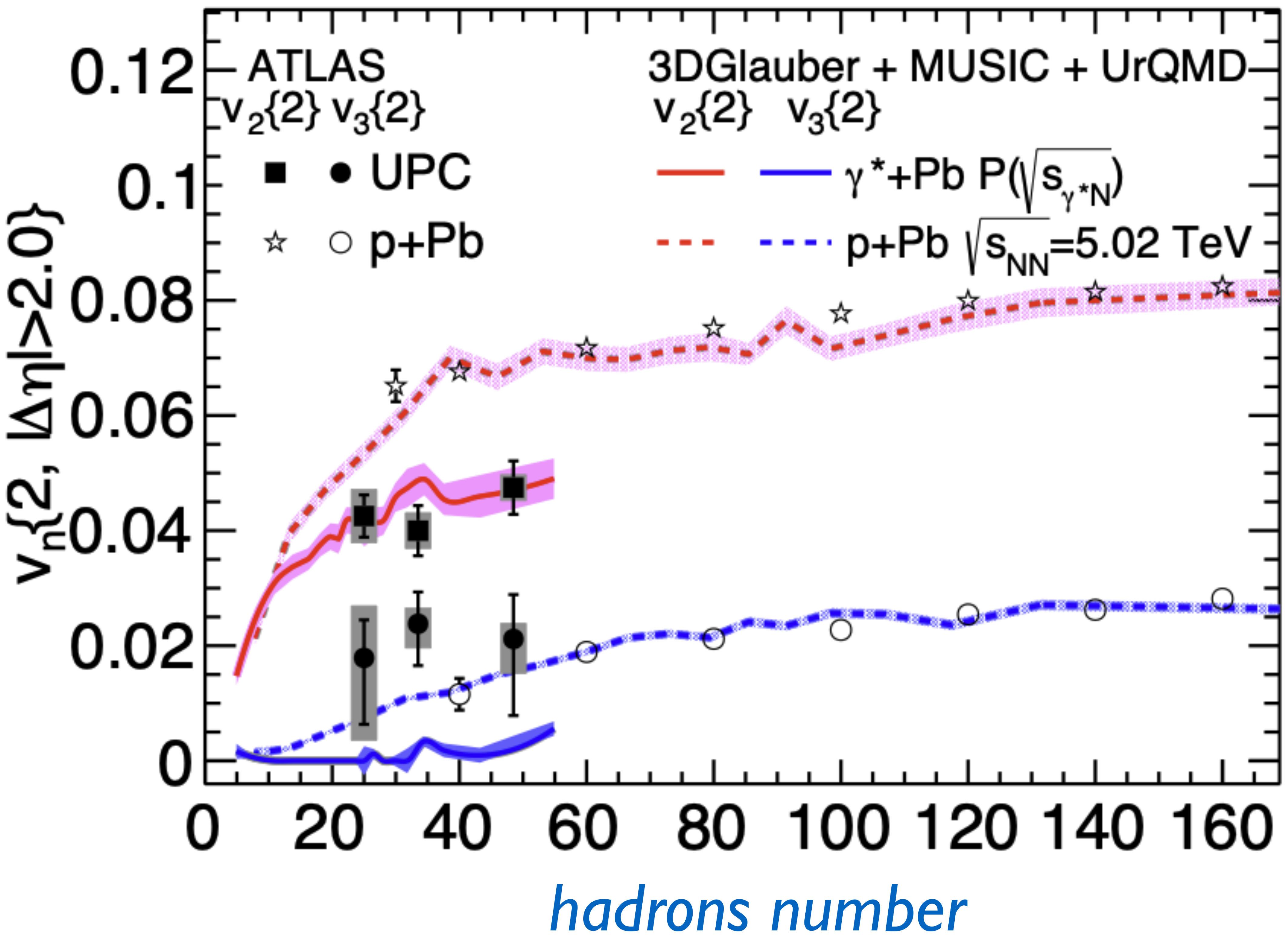
- So far: principled organization
- Next: simplification. Do we need all of the modes?

Simplification outside hydro. regime

Weiyao Ke and YY, PRL 23
Mu-feng Lei, Chen-ya Zhan, Zhen-kang Lu and YY, in preparation

Collectivity of produced hadrons

Wenbin Zhao, Chun Shen, Bjorn Schenke,, PRL 2022



Modeling with hydrodynamics coupled with a few non-hydro modes works well even for heavy-ion collisions systems with size comparable to l_{mfp}

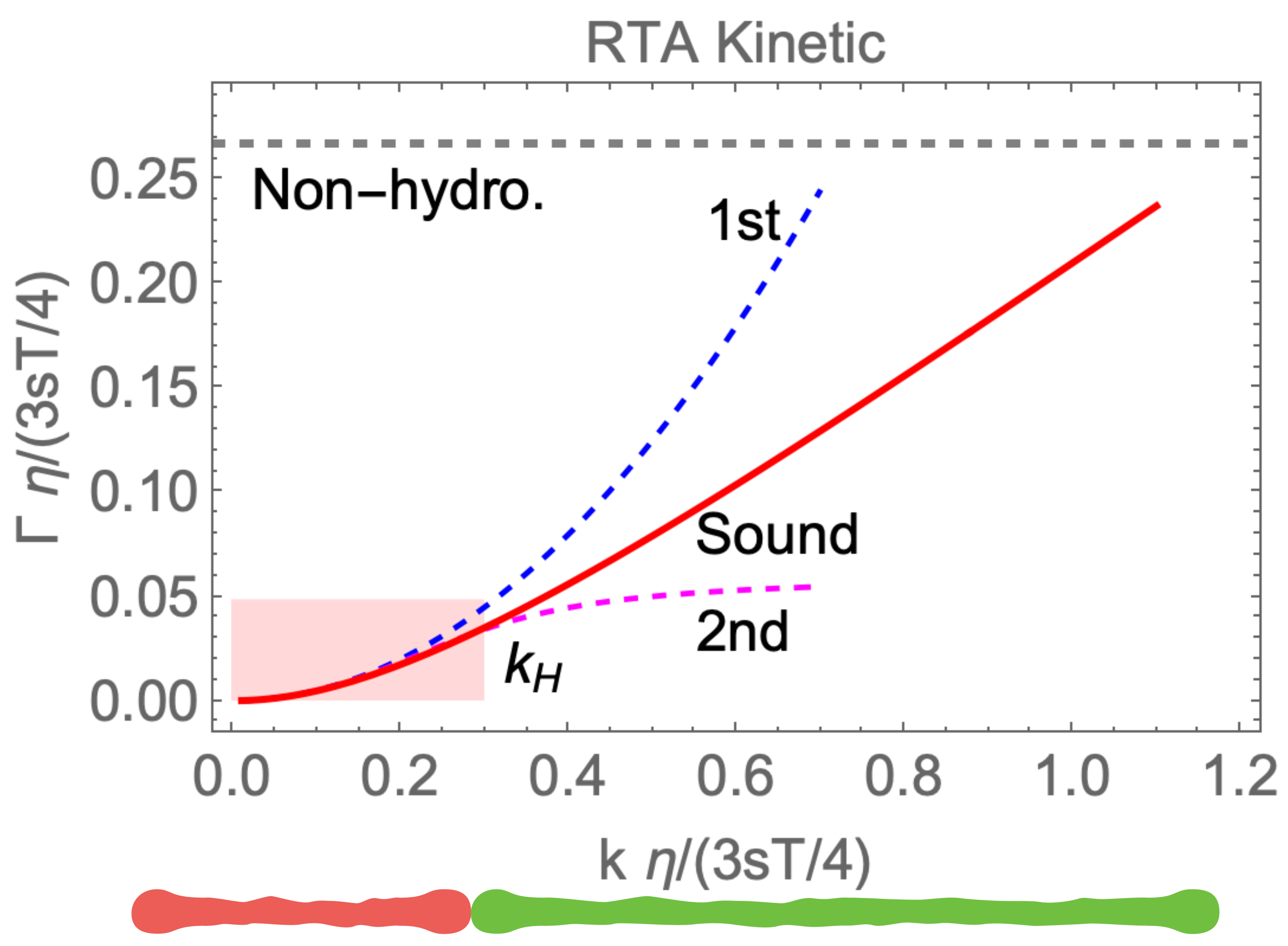
“Ground state modes”

Weiyao Ke and YY, PRL 23

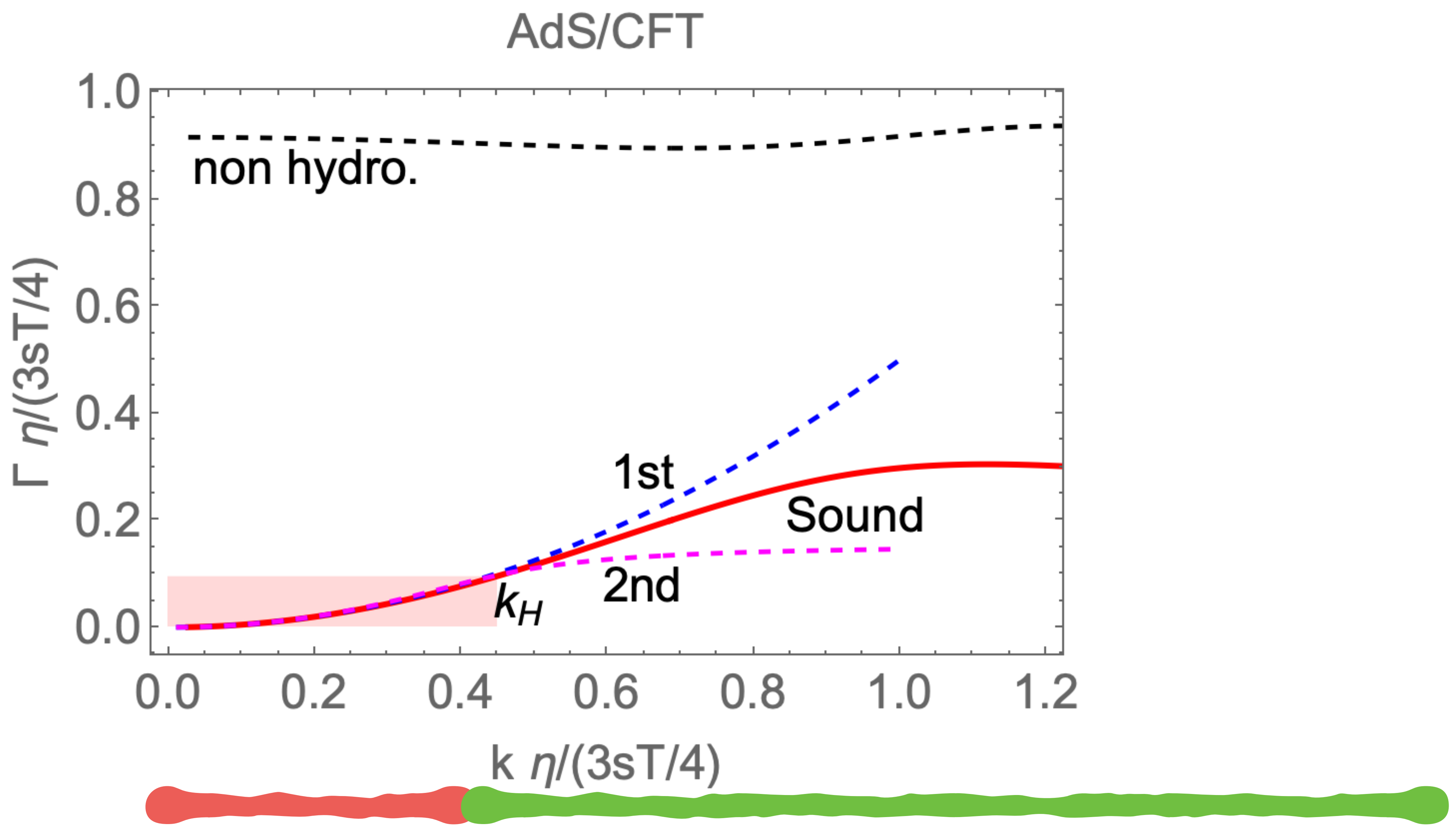
- Those with lowered damping rate than other modes
  - In hydro. regime, they are simply hydro. modes
  - but may exists even beyond and dominates the response (hypothesis)

Check: ground-state modes in sound channel

Romatschke, EPJC 16'.



Amado-Hoyos-Landsteiner-Montero, JHEP 08



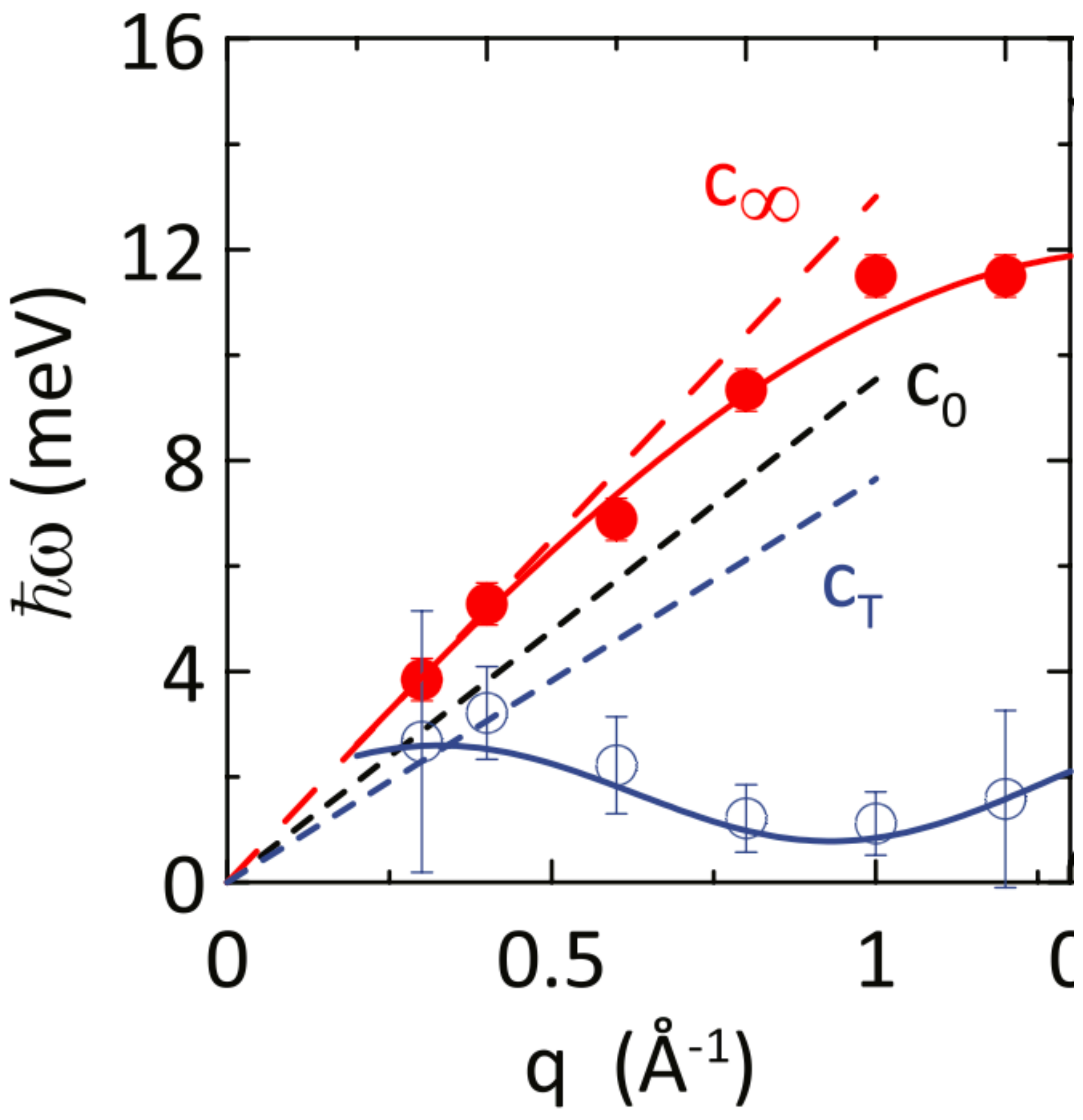
Hydro.

Hydro.

- Exists in QCD-like Plasmas

see also kinetic theory with full QCD collision integral, Xiaojian Du et al, PLB 23

High-frequency sound in solid liquid



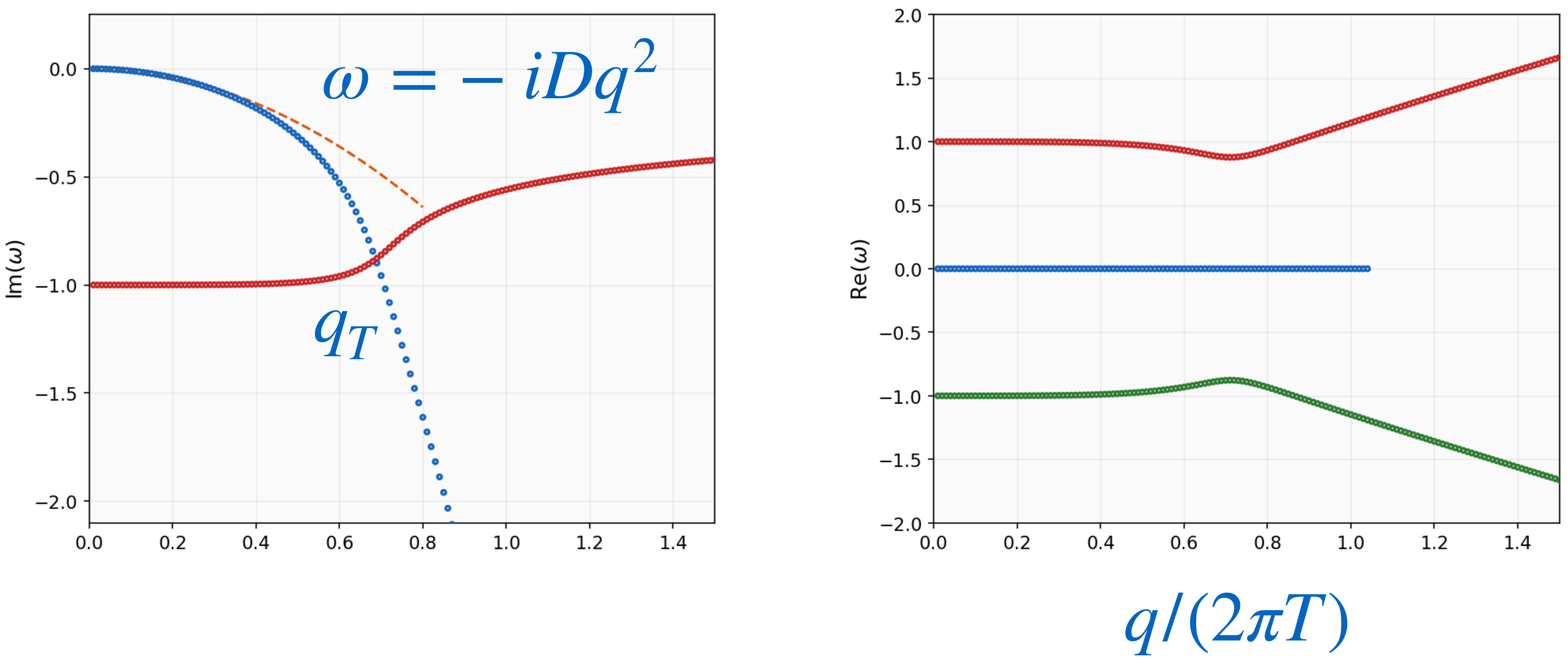
liquid Hg, Petrillo and Sacchetti ,Advances in Physics 21'; many other examples

- observed up to $q \sim$ inter-atom distance

Extended Hydro.

- Assumption: by coupling hydro. with a minimum set of non-hydro. modes, we can describe ground state modes in both hydro. and non-hydro. regime
- Check: $K = 1$ emergent broken U(1) vs holography ($K \to \infty$) in density channel

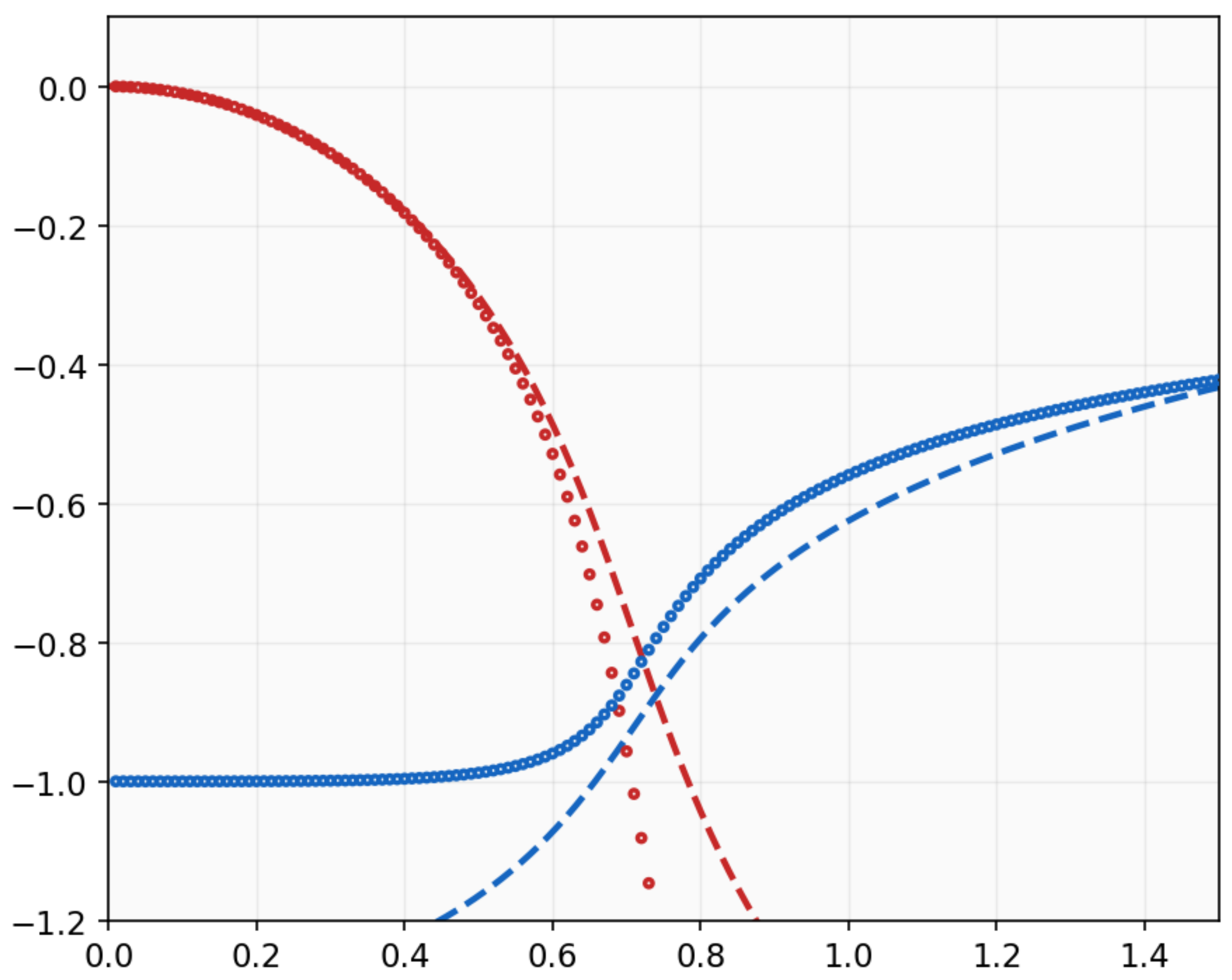
G.S. modes in Holography
$(K \to \infty)$



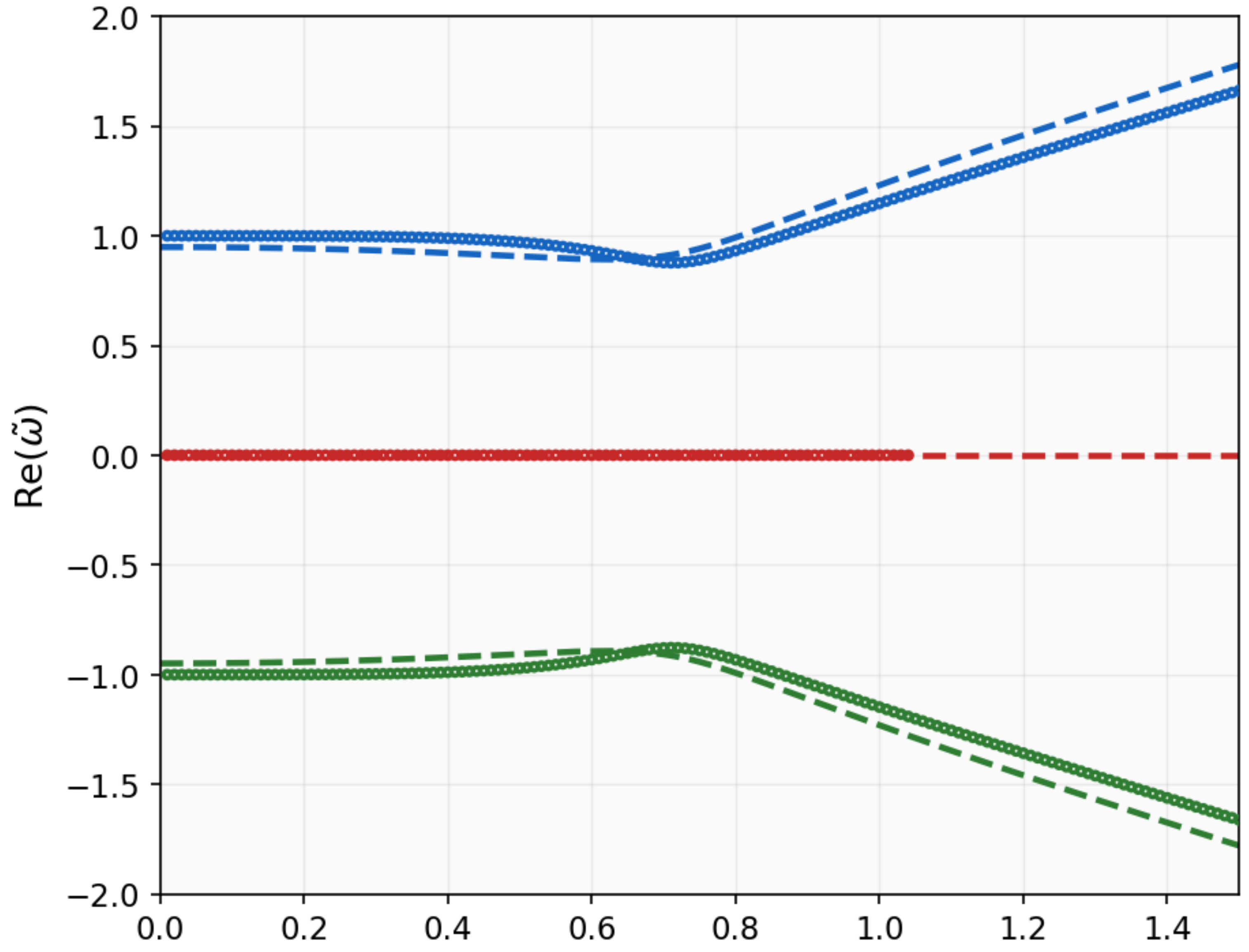
- $q < q_T$: a purely damping mode
- $q > q_T$: a pair of propagating modes

Solid $K \to \infty$ (AdS/CFT) vs dashed $K = 1$ (minimum)

$Im(\omega)$ vs $q/(2\pi T)$



$Re(\omega)\ q/(2\pi T)$



Multiple modes organized by symmetry

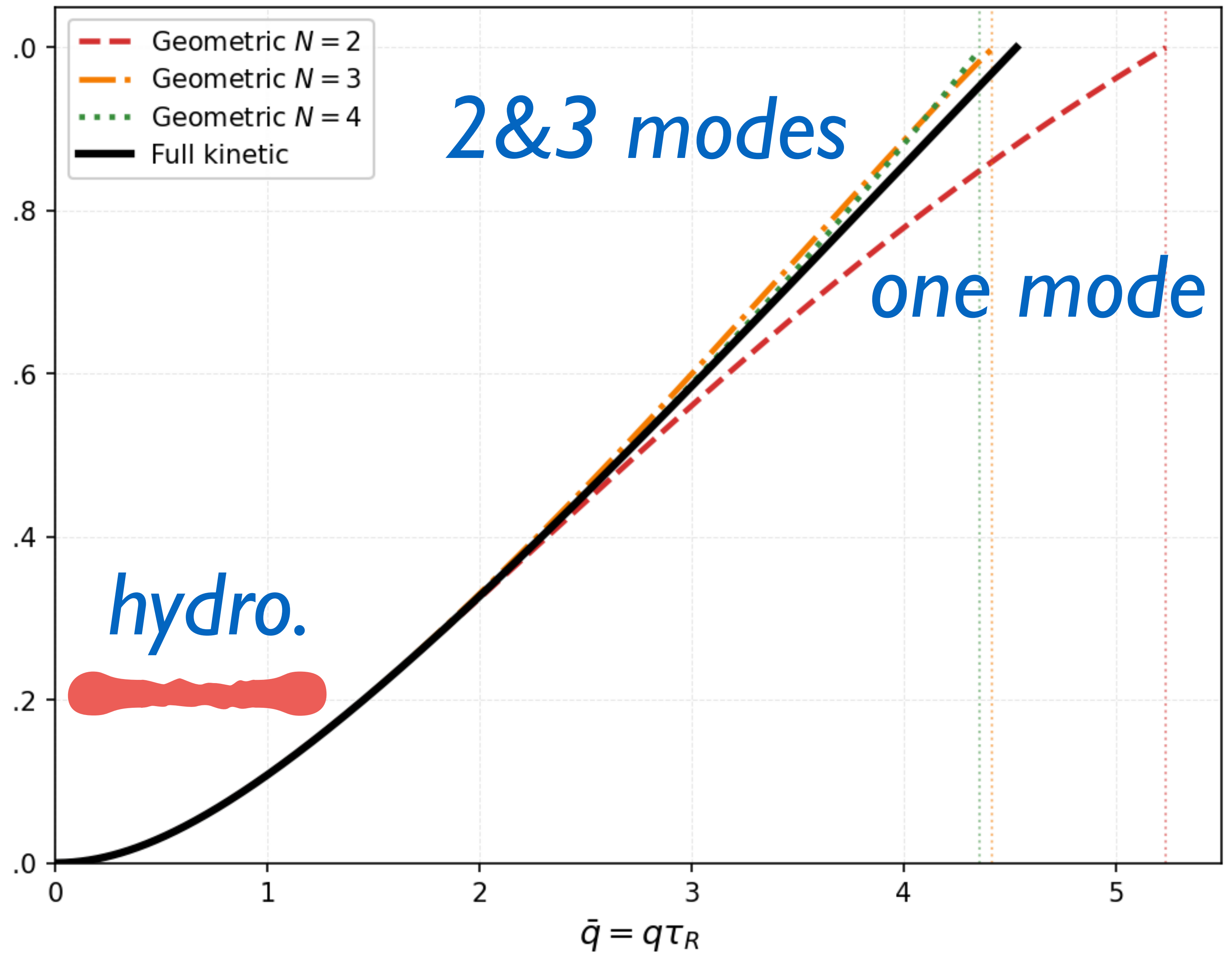
Integrating out $\downarrow$

One mode with same symmetry structure

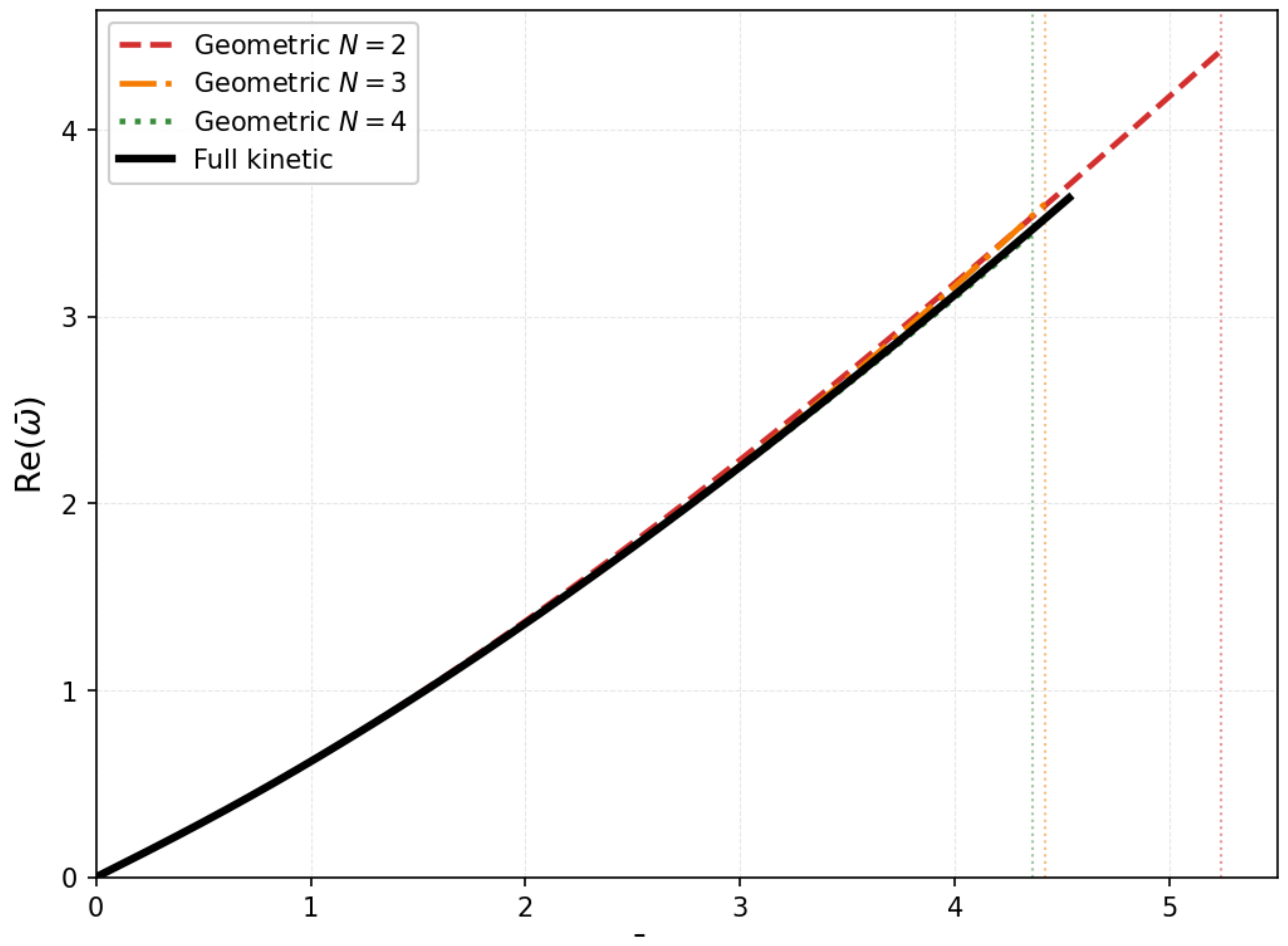
Symmetry may help understanding the simplicity

A Similar Story for Kinetic Theory

$Im(\omega)$ vs $q\tau_R$

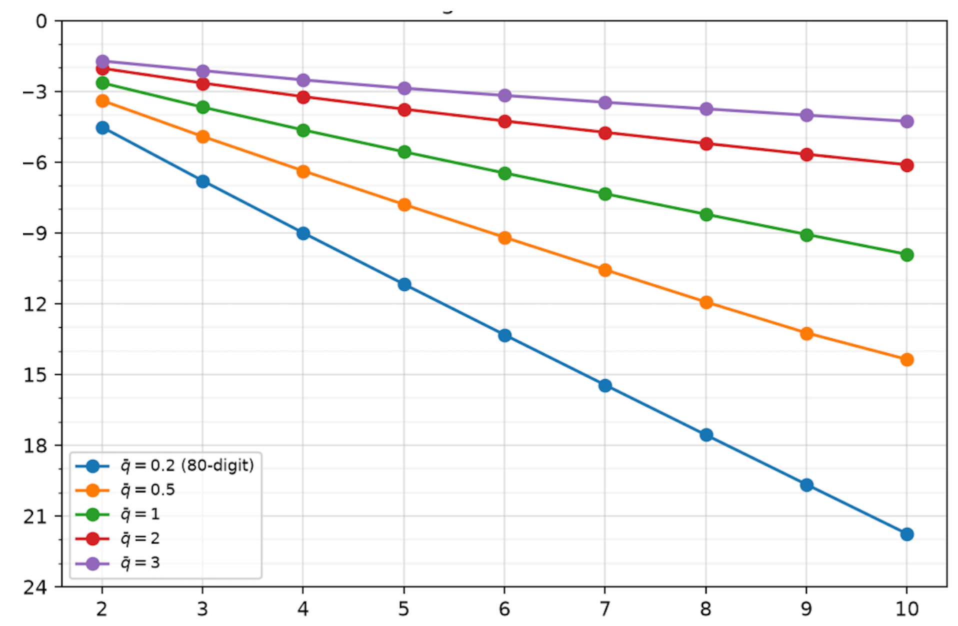


$Re(\omega)$ vs $q\tau_R$



G.S modes in sound channel in kinetic theory vs hydro coupled with a few modes

- Systematic improvement with increasing no. of modes



log of error vs mode #

Summary and outlook

Extension to emergent non-abelian symmetries

- Diffeomorphism: non-linear coupling among non-hydro. modes

Phonons as Goldstone Bosons

H. Leutwyler (University of Bern and CERN)

- $SU(N)$: e.g. kinetic theory with spin

Zonglin Mo, YY 2512.23960; PRD in press

Summary

- Emergent symmetries unify the description of diverse non-hydrodynamic excitations and non-Markovian thermal fluctuations
- Ground-stated modes: minimum extension of hydro. works surprisingly well and improvable
- Outlook: applications to quark matter, other open systems and neural network architectures

Back-up

Action

- Building block: $(\partial\varphi_n + A_n) - (\partial\varphi_{n'} + A_{n'})$ + "emergent" EM fields $\mathbf{E}$, $\mathbf{B}$

$$\boxed{\begin{aligned}\sum_n f_n^2\, \Delta\mathscr{A}^r_{t,n}\Delta\mathscr{A}^a_{t,n} - g_n^2\, \Delta\vec{\mathscr{A}}^r_n \cdot \Delta\vec{\mathscr{A}}^a_n \\ +\epsilon_n \vec{E}^r_n \cdot \vec{E}^a_n - \frac{1}{\mu_n}\vec{B}^r_n \cdot \vec{B}^a_n \\ +\sigma_n \vec{E}_n \cdot \vec{\mathscr{A}}^a\end{aligned}}$$

$$\mathscr{A}_n \equiv \partial\varphi_n + A_n$$

$$\Delta\mathscr{A}_n = \mathscr{A}_{n+1} - \mathscr{A}_n$$

Assuming nearest-neighbor coupling

Describing multiple emergent photons dissipated by Ohmic current

K broken U(1) case revisited

$$E_{K+1} = 0 \text{ - - - - - - - - - - - - - - - } E_n \text{ - - - - - - - } E_{n-1} \text{ - - - - - - - - - - - - - - - } E_1$$

- Multiple emergent E_n-field with nearest neighbour coupling

$$E_n = -\frac{g_n^2}{\epsilon_n(\omega^2 - u_n^2 q^2)}(E_{n+1} - E_n) + \frac{g_{n-1}^2}{\epsilon_n(\omega^2 - u_{n-1}^2 q^2)}(E_n - E_{n-1}) \,.$$

$$E_1 = \frac{g_1^2}{\epsilon_1(\omega^2 - u_1^2 q^2)} \left(E_2 - E_1\right) + \frac{\sigma}{(i\omega)\epsilon_1} E_1$$

Reduction

- Assuming integrating out $E_{n\geq 2}$ modes do not change symmetry structure, E.o.M reduces to $K=1$ broken emergent U(1) model

$$\boxed{E_1 = \frac{g_*^2}{\epsilon_*(\omega^2 - u_*^2 q^2)}\left(-E_1\right) + \frac{\sigma}{(i\omega)\epsilon_*} E_1}$$

- Here, g_*, u_* etc are frequency-independent parameters accounting for the effects of other modes

From NTFPs to (generalized) Hydrodynamics

A lot of citations the last years! For an easy intro/review see: B. Doyon, et. al Phys. Rev. X **15**, 010501 (2025)

Generalized Hydrodynamics $\leftrightarrow$ Hydro for **integrable systems**

$$\partial_t q_n + \partial_x j_n = 0$$

Follows same logic as usual hydrodynamics but taking into account all conserved quantities
$\rightarrow$ **local GGE instead of thermal state**

➡ Large/infinite number of extensive conserved quantities
➡ strongly restricts dynamics
(Gibbs ensemble $\rightarrow$ Generalized Gibbs ensemble)
➡ Long-lived quasiparticles

Gradient expansion of currents leads to simplified equations at Euler scale

$$\boxed{\partial_t \rho_\lambda + \partial_x \left(v^{\text{eff}}_\lambda [\rho] \rho_\lambda \right) = 0}$$

Generally: integrability is rare, but **near-integrability** has pretty large regime of applicability

$$\partial_t q_n + \partial_x j_n = \mathcal{I}_n \left[q \right]$$

Integrability breaking terms $\leftrightarrow$ generalized Boltzmann collision term $\mathcal{I}_n$

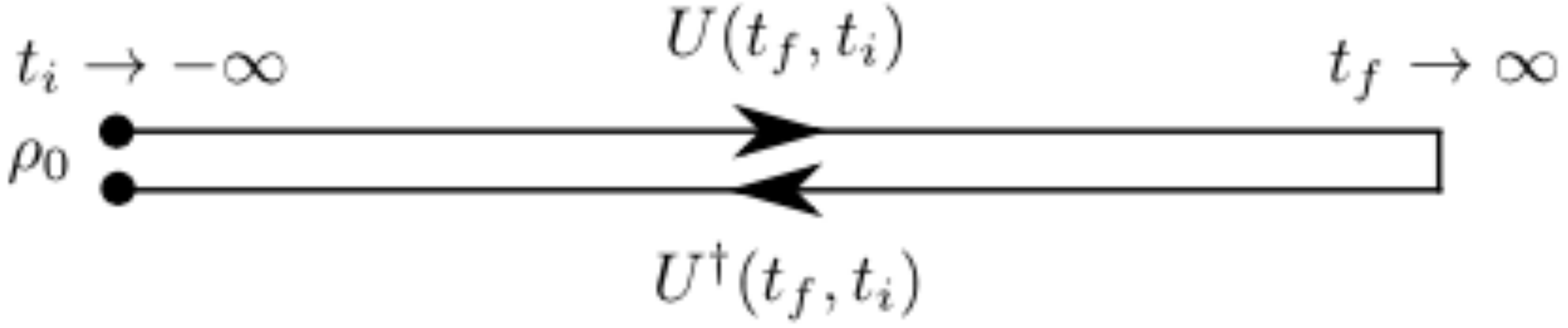
Can universality of the quasi-1D NTFP be phrased as universality of integrability breaking corrections?

Schwinger-Keldysh EFT

e.g. Glorioso-Crossley-Liu, JHEP 17';

- The evolution of density matrix represented as path integral on the SK contour

$$\rho(t_f) = U(t_f, t_i)\rho_I U^+(t_f, t_i)$$



r-field(average): $\phi_r \equiv \left(\phi_- + \phi_+\right)/2$ a-field(noise): $\phi_a \equiv \phi_- - \phi_+$

- EFT: action in terms of parameter fields χ that describe relevant d.o.f.s
  - Observables expressible in χ, $O(\chi)$ $\quad tr(\rho_{\mathrm{mic}}\,\hat{O}) \approx \int \mathscr{D}\chi \mathscr{D}\chi_a\, e^{iI_{EFT}[\chi,\chi_a]}\, O[\chi]$
- Constraints: unitarity, symmetry, emergent properties of ρ_I (e.g. KMS relation and more)

Example: Maxwell-Cattaneo Eqn and its action

- Promoting $\vec{j}$ as the dynamical field in addition to n

$$\partial_t \vec{j} = -\frac{1}{\tau_J}(\vec{j} - D\vec{\partial} n) \qquad\qquad \partial_t n + \vec{\partial} \cdot \vec{j} = 0$$

(Analogous to MIS equation for $T^{\mu\nu}$)

- Corresponding action are constructed phenomenologically (a la Martin–Siggia–Rose)

Mullins, Hippert,Noronha, PRD 23; Jain, Kovtun, JHEP 24'

- In general, one needs multiple non-hydro. fields whose descriptions would be complicated

Emergent Gauge Symmetry and Massive Vector Mesons

- Massive meson like ρ, a_1 can be treated as the vector boson of spontaneously breaking emergent local $SU(2)$

Bando et al, 1980s

- Generalization to finite (infinite) number of HLS (predecessor of AdS/QCD)

Son-Stephanov, 2004

Reproducing a host of hadronic phenomena with acceptable precision

Pion ~ hydro. modes

Massive meson ~ non-hydro. modes

Message

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