Open Systems, Black Holes & Holography

An Applied Gravity Perspective

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Workshop & the Question I Want to Ask

We have seen Open Q-Systems arise across many areas of physics: condensed matter, cosmology, hydro, quantum info., and beyond.

A typical setup is as follows:

$$\text{Sys.} + \text{Env.} \quad \longrightarrow \quad \text{Integrate out env.} \quad \longrightarrow \quad \text{Effective action}$$

Tools: Master equations, Lindbladians, Schwinger–Keldysh (SK) formalism, Feynman–Vernon Influence functional etc.

My talk: $\boxed{\textbf{SK formalism in AdS/CFT Holography}}$

But, What is AdS/CFT? Why use it? How does it help?

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AdS/CFT for Our Purpose

What is AdS/CFT?

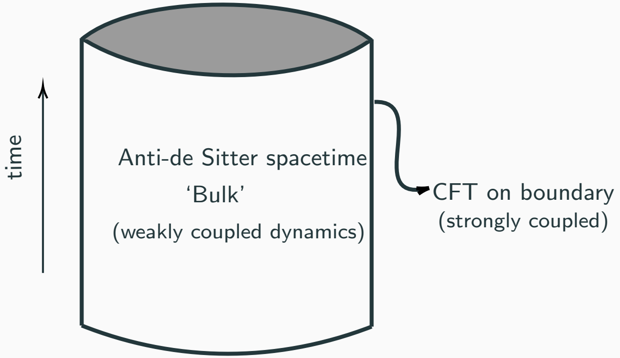


Figure 1: AdS/CFT Correspondence.

Problems in strongly coupled systems $\rightarrow$ simple problems in gravity.

Why?

For our purposes, this holographic duality is a powerful computational tool.

Suppose the **environment** is strongly interacting. Integrating out such a bath is generally difficult.

Holography provides an alternative description. The idea is simple:

$$\boxed{\text{Strongly coupled bath}} \quad \Longrightarrow \quad \boxed{\text{Gravitational dual}}$$

Compute in gravity and translate the answer back to the boundary theory using AdS/CFT dictionary. [Hartnolll-Lucas-Sachdev]

How?

A minimal AdS/CFT dictionary:

$$\text{Boundary operator } (O) \iff \text{Bulk field } (\phi)$$

$$Z_{\text{bdy}}[J] = Z_{\text{grav}}\left[\phi\big|_{\text{bdy}} = J\right] \simeq \exp\left\{i\, S_{\text{on-shell}}\left[\phi\big|_{\text{bdy}} = J\right]\right\}$$

$$\boxed{n\text{-point Boundary correlators} \propto \left.\frac{\delta^n S_{\text{on}-\text{shell}}}{\delta J^n}\right|_{J=0}.}$$

[GKPW dictionary]

One more thing:

$$\boxed{\text{Thermal state of boundary} \iff \text{Black hole in AdS}}$$

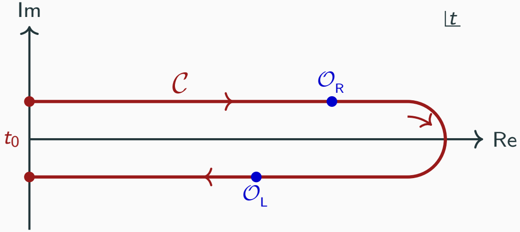
Black holes describe thermal baths. Dissipation/fluctuations are encoded in black-hole physics (via QNMs/Hawking radiation).

SK formalism & its gravity dual

Schwinger–Keldysh (SK) formalism

Many names: Schwinger-Keldysh, in-in, Closed Time Path/Contour. [Rammer, Kamenev]

In SK formalism, mixed states are being evolved: using forward and backward time evolution. [Schwinger '61, Keldysh '64]



Then, SK two-point Green's function is a 2×2 matrix:

$$G_{\mathcal{C}} = \begin{pmatrix} G_{\mathrm{RR}} & G_{\mathrm{RL}} \\ G_{\mathrm{LR}} & G_{\mathrm{LL}} \end{pmatrix} = \begin{pmatrix} G_F & G_< \\ G_> & G_{\bar{F}} \end{pmatrix} = -i \begin{pmatrix} \langle \mathbf{T}\{O_1 O_2\}\rangle & \langle O_2 O_1 \rangle \\ \langle O_1 O_2 \rangle & \langle \overline{\mathbf{T}}\{O_1 O_2\}\rangle \end{pmatrix} .$$

These Green's functions are not independent of each other,

$$G_F + G_{\bar{F}} = G_< + G_> \ , \tag{1}$$

In *average-difference* basis, the operators/fields/sources are defined as,

$$\mathcal{O}_a \equiv \frac{\mathcal{O}_{\mathrm{R}} + \mathcal{O}_{\mathrm{L}}}{2} \ , \qquad \mathcal{O}_d \equiv \mathcal{O}_{\mathrm{R}} - \mathcal{O}_{\mathrm{L}} \ , \tag{2}$$

Notation Alert!

Then, the SK two-point Green's functions as:

$$\begin{aligned} G_{dd} &= 0 \ , \\ G_{ad} &= -i\,\Theta_{t_2>t_1}\,\langle[\mathcal{O}_2, \mathcal{O}_1]\rangle \equiv G_{\mathrm{ret}} \ , \\ G_{da} &= -i\,\Theta_{t_1>t_2}\,\langle[\mathcal{O}_1, \mathcal{O}_2]\rangle \equiv G_{\mathrm{adv}} \ , \\ G_{aa} &= -\frac{i}{2}\,\langle\{\mathcal{O}_1, \mathcal{O}_2\}\rangle \equiv G_{\mathrm{kel}} \ , \end{aligned} \tag{3}$$

Generalisation to arbitrary *n*-point functions, fermions, gauge fields etc.

SK generating functional

SK generating functional is given by,

$$
\begin{aligned}
Z_{\mathrm{SK}}[J_{\mathsf{R}}, J_{\mathsf{L}}] &= \mathrm{Tr}\left[U[J_{\mathsf{R}}]\,\hat{\rho}_{\mathrm{initial}}\,(U[J_{\mathsf{L}}])^{\dagger}\right] \\
&= \left\langle \exp\left\{ i \int \mathrm{d}^d x \left[J_{\mathsf{R}}(x)\mathcal{O}_{\mathsf{R}}(x) - J_{\mathsf{L}}(x)\mathcal{O}_{\mathsf{L}}(x)\right] \right\} \right\rangle
\end{aligned}
$$

The corresponding path integral

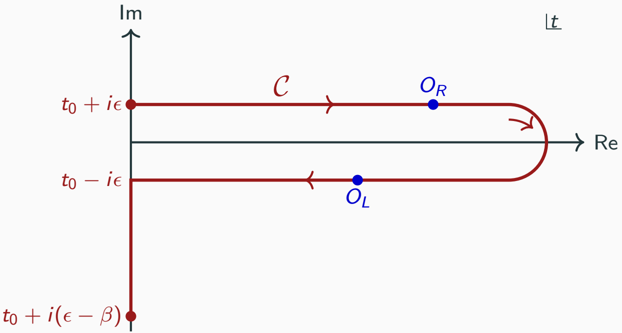
$$
\int D[\phi] \exp\left\{ iS + i\int J_{\mathsf{R}}\mathcal{O}_{\mathsf{R}} - i\int J_{\mathsf{L}}\mathcal{O}_{\mathsf{L}} \right\}
$$

where $S = \int_C dt\, L(t)$.

Relation with Feynman-Vernon influence functional, `[Lecture by Andrew Tolley]`

Take initial state to be thermal, $\rho_\beta = e^{-\beta H}$. Such a state amounts to an imaginary time evolution with the compact circle of size β.

Then, SK contour at finite temperature is given by,



SK contour at finite temperature -II

Everything follows similarly when the initial state is thermal. However, there are now KMS constraints (coming from β periodicity).

2-point KMS condition: $\langle A(t-i\beta)\ B(0)\rangle_\beta = \langle B(0)\ A(t)\rangle_\beta$. It relates

$$G_{\rm kel} = (1+2n_B)[G_{\rm ret} - G_{\rm adv}]$$

Future-Past basis: A convenient basis for SK in the thermal case :

$$J_F \equiv -(1+n_B)J_{_\text{R}} + n_B J_{_\text{L}}\ , \qquad J_P(\omega) = -n_B(J_{_\text{R}} - J_{_\text{L}})\ .$$

A natural question

Question:

What is gravitational dual to the SK formalism?.

A geometry whose bdy limit is SK contour and the on-shell action evaluated on the geometry gives SK correlators.

Immediate idea would be take the **two copies of BH in AdS**. right direction.

Several prescriptions have been proposed over the past two decades. The latest one called **grSK geometry**. [Crossley-Glorioso-Liu '18], [Jana-Loganayagam-Rangamani 2020]

grSK prescription: Glue two copies of the black hole exterior along their future horizons.

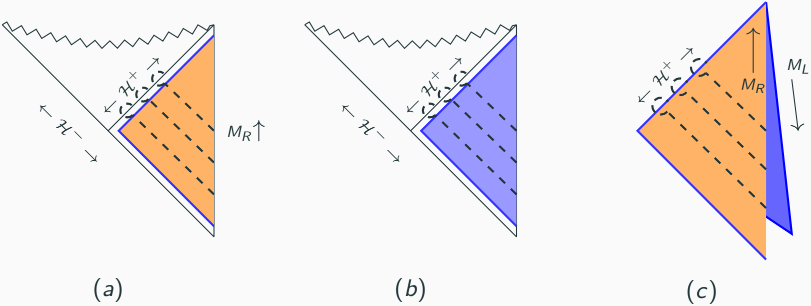


Figure 2: Figures (a) and (b) show two copies of the AdS black hole, respectively. Figure (c) represents the grSK geometry.

grSK geometry - II

[1] Take AdS blackbrane

$$\mathrm{d}s^2 = -r^2 f\,\mathrm{d}v^2 + 2\mathrm{d}v\,dr + r^2\,\mathrm{d}\mathbf{x}_{d-1}^2\ , \qquad f = 1 - \left(\frac{r_+}{r}\right)^d\ .$$

where r_+ is its horizon radius.

[2] Promote r to complex radial plane and pick a codim=1 slice, parametrized by ζ (analog of tortoise coordinate):

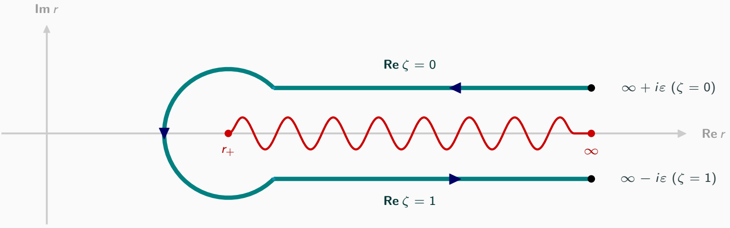


Figure 3: At fixed v, the radial contour parametrized by ζ, with $\frac{d\zeta}{dr} = \frac{2}{i\beta\ r^2 f}$.

Remarks on grSK

grSK prescription has been quite successful:

— SK correlators, with manifest *microscopic unitarity* and *thermality*.

— Generalised to system with $\mu \neq 0$ or charged BHs.

— Constructing *local* open EFT for holographic baths,

— Derive non-linear holographic FDTs,

— Exterior field theory or Scattering against Hawking radiation.

Still no derivation of it from gravitational PI.

A simple setup

Probing the grSK geometry

A massless self-interacting massless scalar field in the grSK geometry,

$$S = -\oint_{\mathrm{grSK}} \left[\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{n!}\phi^n\right] . \tag{4}$$

where λ is the interaction parameter.

Using Green's functions, solve this system perturbatively,

$$\phi = \phi_{(0)} + \lambda\phi_{(1)} + \ldots \ , \tag{5}$$

with boundary conditions,

$$\lim_{\zeta\to 0,1} \phi(\zeta, k) = J_{\mathrm{L},\mathrm{R}}(k) \tag{6}$$

where $J_{\mathrm{L},\mathrm{R}}$ are the left and the right boundary sources respectively.

Solving for the Dynamics

Use CPT: $G^{\text{out}}(\zeta, k) = e^{-\beta\omega\zeta} G^{\text{in}}(\zeta, -k)$, to find the solution.

Substituting the solution into the action yields the on-shell action or boundary SK generating functional,

$$S^{\text{grSK}}_{\text{on-shell}} \sim \ln Z^{\text{bdy}}_{\text{SK}} \sim S^{\text{bdy}}_{\text{IF}}$$

On-shell action naturally gives rise to an exterior field theory, incorporating:

- **Dissipation**: particles falling into the black hole (BH).
- **fluctuation**: as manifested through Hawking radiation.

[Loganayagam-Martin '24, Martin-SKS '24]

Thus, SK correlators can be obtained from a field theory living in the BH exterior.

Why Only Exterior?

Note G^{in} is analytic in r while G^{out} is not. Because of the prefactor $e^{-\beta\omega\zeta}$.

$$\oint_{\text{grSK}} [G^{\text{in}} \ldots] = \, , 0$$
$$\oint_{\text{grSK}} e^{-\beta\omega\zeta} F(r) = (1 - e^{-\beta\omega}) \int_{r_+}^{\infty} dr\, r^{d-1}\, e^{-\beta\omega\zeta} F(r)$$

where F is analytic in r.

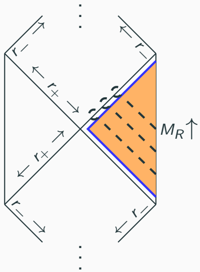
$$\boxed{\text{grSK results} \Longrightarrow \text{Exterior results} + \text{statistical factors}}$$

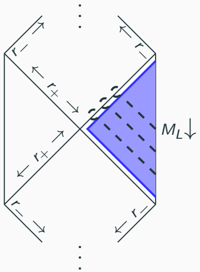
Also, we develop exterior diagrammatics for this on-shell action. For charged black holes, see [SKS 2501.17852]

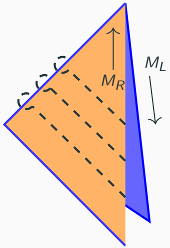
Aside: We only obtain exterior theory for non-derivative interactions in the bulk.

Extra: Charges BH or RNSK geometry

$$f(r) = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}\ , \qquad \mathcal{A}_M\, \mathrm{d}x^M = -\mu \left(\frac{r_+}{r}\right)^{d-2} \mathrm{d}v\ .$$







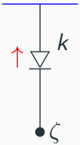
SK correlators from Exterior field Theory

Feynman rules : Propagators

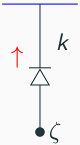
Consider $|\Phi|^4$ theory: $S = \oint_{\mathrm{RNSK}} \left(\frac{1}{2}|D\Phi|^2 + \frac{\lambda}{4}|\Phi|^4\right]$

Ingredients of Feynman diagrams: propagators, vertices, etc.

The boundary-to-bulk propagators are as follows.

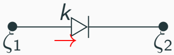


$\equiv \frac{G^{\text{in}}(r,k)}{1+n_k} J_F(k)$,

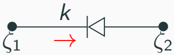


$\equiv G^{\text{out}}(r,k) J_P(k)$,

The bulk-to-bulk propagators are



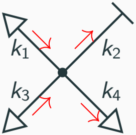
$\equiv -i\mathbb{G}_{\text{ret}}(\zeta_2|\zeta_1, k)$,



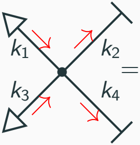
$\equiv -i\mathbb{G}_{\text{adv}}(\zeta_2|\zeta_1, k)$.

Feynman rules : Vertices

Two examples of vertices:



$$= -i\lambda \frac{n_{-k_2,1}}{n_{k_{134},-1}} \ ,$$

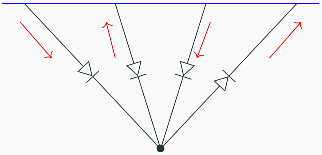


$$= -i\lambda \frac{\mathfrak{n}_{-k_2,1}\,\mathfrak{n}_{-k_4,1}}{n_{k_{13},-2}} \ ,$$

where $n_{k,\alpha} \equiv \frac{1}{e^{\beta(k^0 - \alpha\, \mu q)} - 1}$ with $k_{12\ldots m} \equiv k_1 + k_2 + \ldots k_m$.

Integrate bulk points over BH exterior, with symmetry factors.

An example: Tree-level contact diagram in $|\Phi|^4$ theory with $(J_F)^3 J_P$ terms,



$$\sim \ -\frac{\lambda}{2}\frac{1}{n_{k_4}} \int_{r_+}^{\infty} dr\ r^{d-1} \left[\bar{G}_{\rm in}(r,k_1) G_{\rm in}(r,k_2) \bar{G}_{\rm in}(r,k_3) G_{\rm out}(r,k_4)\right]$$

Comments on exterior field theory

grSK geometry: compute real-time/SK correlators to any order using exterior diagrammatics.

Some of other applications:

- The study of *local* open EFTs.
- Exterior field theory of Hawking radiation (or Scattering against Hawking radiation).

Average-difference basis: $S_{\rm os}[J_a, J_d]$ gives stochastic EOM with J_a and iJ_d identifying stochastic field and noise, respectively. Here, coefficients related by FDTs.

Holographic FDTs

FDT – A Quick Review

Take Brownian particle described by the Langevin equation:

$$\ddot{q} + \gamma\,\dot{q} = \mathfrak{f}\,\eta(t)\ , \tag{7}$$

where γ, η, and $\mathfrak{f}$ represent dissipation, noise, and fluctuation strength.

Then Fluctuation-Dissipation Theorem (FDT) relates noise to dissipation as:

$$\mathfrak{f} = \frac{2}{\beta}\,\gamma\ . \tag{8}$$

– 2-point KMS condition $\rightarrow$ linear FDT. [Rammer 2007]

– For few non-linear FDTs = KMS + "additional inputs". [Chakrabarty et. al. 2018-19]

KMS v/s FDT: KMS relations constrain microscopic correlations, while FDTs relate macroscopic dissipative coefficients to fluctuation strength.

FDTs in holography

Illustration: ϕ^n theory and its on-shell action $S_{\rm os}$ at contact level,

$$S_{\rm os}^{(\rm contact)} = -\lambda \int d^d x \left\{ \sum_{k=1}^{4} \theta_k \frac{J_a^{n-k}}{(n-k)!} \frac{(iJ_d)^k}{k!} - \sum_{k=1}^{n-1} \bar{\theta}_k \frac{J_a^{n-k}}{(n-k)!} \frac{(iJ_d)^{k-1}}{(k-1)!} \partial_t (iJ_d) \right\} + \mathcal{O}\left(\partial_t^2\right)$$

The corresponding stochastic EOM is nothing but a non-linear generalisation of Langevin equation [Jana-Loganayagam-Rangamani 2020]:

$$\left(-\partial_t^2 + \nabla^2 + \gamma \partial_t\right) \Phi + \sum_{k=1}^{n-1} \frac{\eta^k}{k!} \left(\theta_k + \bar{\theta}_k \partial_t\right) \Phi^{n-k} = f\, \eta \ , \tag{9}$$

with **non-linear FDTs** given as

$$\frac{2}{\beta}\gamma = f \ , \qquad \frac{2}{\beta}\bar{\theta}_k + \theta_{k+1} + \frac{1}{4}\theta_{k-1} = 0 \ . \tag{10}$$

Extended for charged BH or holographic FDTs at $\mu \neq 0$. See [SKS 2501.17852]

Conclude

Summary

We discussed the gravity dual to the SK formalism (grSK geometry).

It allows to calculate SK correlators to any orders, $\langle \overline{\mathbf{T}}(O_1 ... O_m)\mathbf{T}(O_{m+1} ... O_n)\rangle$.

Thus, it gives influence phase (to any order) for holographic baths.

We reduced all computations to exterior field theory and developed exterior diagrammatics/rules for $S_{\rm os}, Z_{\rm SK}, S_{\rm IF}$.

computation of (non)-linear holographic FDTs (to any order).

Outlook

Fluctuating fluid/gravity correspondence using gravitational dual to Schwinger-Keldysh.

What about grav. PI derivation of grSK geometry.

Bulk loop corrections to holographic FDTs.

Another way to obtain exterior field theory. Maybe canonical (opertor-based)?

grSK anologue of non-equilibrium steady states? Sudden quenches?

Thank You!

Extras

Holographic linear FDT at $\mu \neq 0$

SK generating functional v/s Stochastic EOM. We will work with former.

The on-shell action at quadratic order in the *average-difference* basis:

$$S_{(2)} = \int \frac{d^d k}{(2\pi)^d} \left[K_{\rm ret}\, \bar{J}_d(-k) J_a(k) + K_{\rm adv}\, \bar{J}_a(-k) J_d(k) + K_{\rm kel}\, \bar{J}_d(-k) J_d(k) \right] \,.$$

These correlators obey the two-point KMS condition,

$$K_{\rm kel} = \frac{1}{2} \coth\left(\frac{\beta(\omega - \mu q)}{2}\right) \left[K_{\rm ret} - K_{\rm adv} \right] \,,$$

leads to linear FDR, as expected.

Recap: Linear FDT at $\mu = 0$

The 2-point KMS condition (at $\mu = 0$) in high-temperature ($\beta \to 0$) limit

$$K_{\text{kel}} = \frac{2}{\beta}\, \omega\, K_{\text{ret}} \ .$$

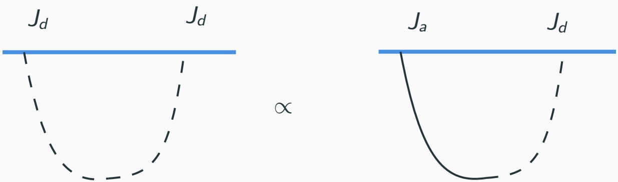


Figure 4: Schematic representation of linear FDR.

Non-linear FDTs at small but finite density

The on-shell action at quartic order (from contact diagrams),

$$S_{(4)} = \int d^d x \, \mathcal{L}_0 + \int d^d x \, \mathcal{L}_1 + \mathcal{O}\left((\beta \partial_t)^2 \right) \ , \tag{11}$$

where the terms $\mathcal{L}_0$ and $\mathcal{L}_1$ are:

$$\mathcal{L}_0 = \sum_{r,s=0}^{2} \mathcal{G}_{r,s} \ \left[\bar{J}_a\right]^r [J_a]^s \left[\bar{J}_d\right]^{2-r} [J_d]^{2-s} \ ,$$

$$\mathcal{L}_1 = \sum_{r=0}^{1} \sum_{s=0}^{2} \left\{ \mathcal{G}_{\dot{r},s} \left(\partial_t \bar{J}_a\right) + \mathcal{H}_{\dot{r},s} \left(\partial_t \bar{J}_d\right) \right\} \left[\bar{J}_a\right]^r [J_a]^s \left[\bar{J}_d\right]^{1-r} [J_d]^{2-s} + h.c \ .$$

The relationships between these coefficients lead to non-linear FDRs, e.g

$$\left(\mathcal{G}_{\dot{0},0} + \mathcal{G}_{\dot{0},0}\right) - \left(\mathcal{H}_{\dot{1},0} + \mathcal{H}_{0,\dot{1}}\right) - 2i\beta \mathcal{G}_{0,0} + \frac{i\beta}{12} \left(\mathcal{G}_{2,0} + \mathcal{G}_{1,1} + \mathcal{G}_{0,2}\right) = 0 \ . \tag{12}$$

[SKS 2501.17852]

Non-linear FDT at $\mu = 0$

The quartic on-shell action at $\mu = 0$, is

$$S_{(4)} \propto \int d^d x \left\{ \sum_{k=1}^{4} \theta_k \frac{J_a^{4-k}}{(4-k)!} \frac{(iJ_d)^k}{k!} - \sum_{k=1}^{3} \bar{\theta}_k \frac{J_a^{4-k}}{(4-k)!} \frac{(iJ_d)^{k-1}}{(k-1)!} \partial_t (iJ_d) \right\} ,$$

with non-linear FDR as

$$\frac{2}{\beta} \bar{\theta}_k + \theta_{k+1} + \frac{1}{4} \theta_{k-1} = 0 \ .$$

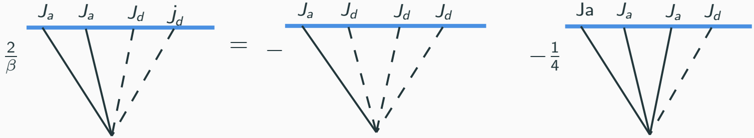


Figure 5: Schematic representation of a non-linear FDR.