

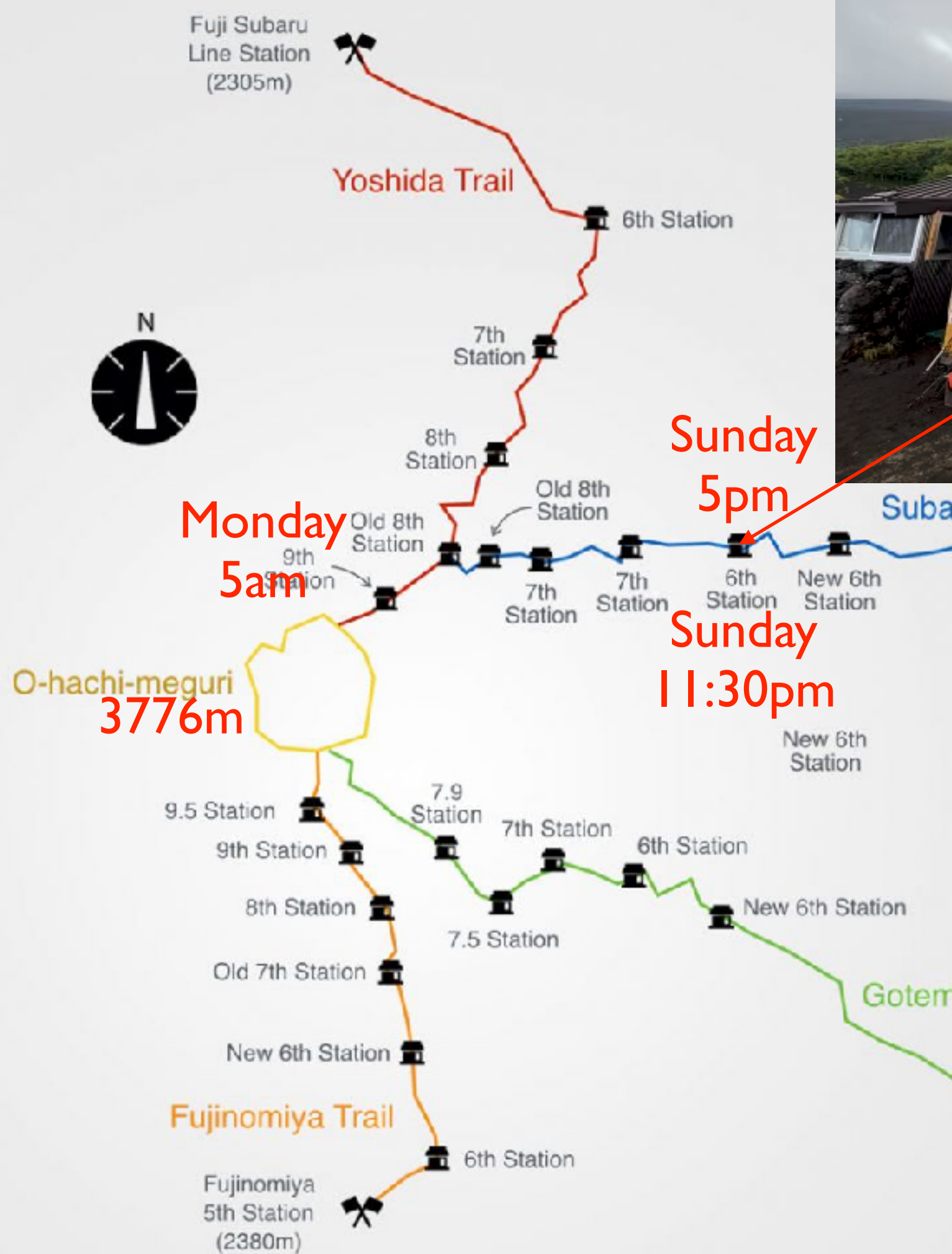
Exact Solutions to Gauge Theories and Generalized Symmetries



QUANTUM CROSSROADS

Hitoshi Murayama (Berkeley, Kavli IPMU)
Kavli IPMU, August 6, 2026









Closed

- My talk is about closed quantum systems in QFT
- no Schwinger-Keldysh
- but this group loves quantum and challenges
- “studying quantum properties of matter”
- hopefully it is still interesting

Exact Solutions to Gauge Theories and Generalized Symmetries



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Kavli IPMU, August 6, 2026

What Crazy Things Desperate Theorists Do



QUANTUM CROSSROADS

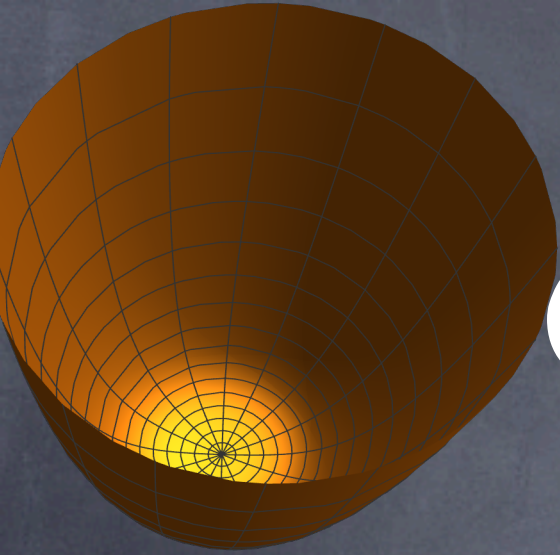
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Non-Perturbative

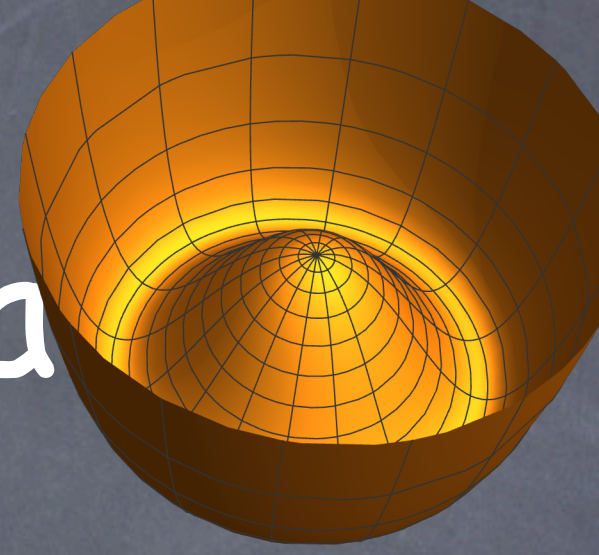
- We know perturbation theory is good only when couplings are small
- It is an asymptotic series that does not converge
- Strong interaction (QCD) is indeed strong
- confinement, chiral symmetry breaking
- any method to study it without perturbation?
- I propose to use supersymmetry

Crazy things

- approach the theory in unusual way
- $D=4-\varepsilon$ and expand in $\varepsilon=1$ to reach $D=3$
- $N_c \rightarrow \infty$ first and expand in $1/N_c=1/3, 1$
- $g \ll 1$ first and extrapolate to $g \gg 1$
- the key is continuity of universality class (i.e. symmetry of the ground state) and dynamics



Critical Phenomena



$T > T_c$

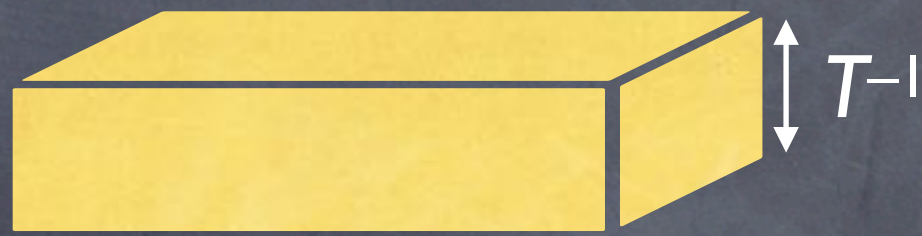
$T < T_c$

- condensed matter systems with $O(N)$ symmetry exhibit 2nd order phase transition

- Naive Landau theory: $V = \frac{1}{2}\mu^2\vec{\phi}^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2$
 $\mu^2 \propto (T - T_c) \quad \phi \propto (T_c - T)^{1/2} \leftarrow \nu$

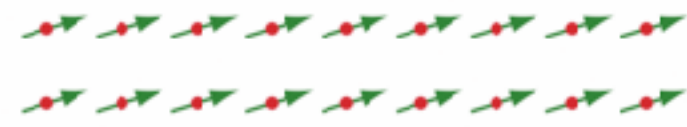
N	1	2	3	∞
ν	0.630	0.672	0.711	1

- doesn't work!



3D QFT

$T < T_c$



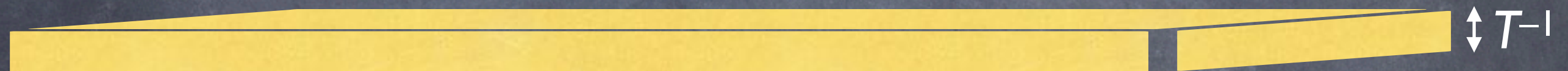
(b)

$T > T_c$



(c)

- finite temperature is periodic in the imaginary time direction $\tau = \hbar/kT = 1/T$
- Near the critical temperature T_c , correlation length becomes very long
- 4D becomes a 3D pancake



- Expand in Matsubara modes of "mass" = $n T$

$$\phi(\tau, \vec{x}) = T^{1/2} \sum_{n=-\infty}^{\infty} \phi_n(\vec{x}) e^{2\pi i n \tau k T}$$

$$\int d\tau (\partial_\tau \phi)^2 + (\vec{\nabla} \phi)^2 = \frac{1}{2} \vec{\nabla} \phi_0 \vec{\nabla} \phi_0 + \sum_{n=0}^{\infty} (2\pi n k T)^2 \phi_n^* \phi_n + \vec{\nabla} \phi_n^* \vec{\nabla} \phi_n$$

- only $n=0$ mode important at long distances

3D QFT

- 3D $\lambda\phi^4$ theory becomes very strong in the IR
- perturbation theory doesn't work
- what can we do?
- 4D $\lambda\phi^4$ theory is trivial in the IR
- $D=4-\varepsilon$ and expand in $\varepsilon=1$ to reach $D=3$
- Crazy!

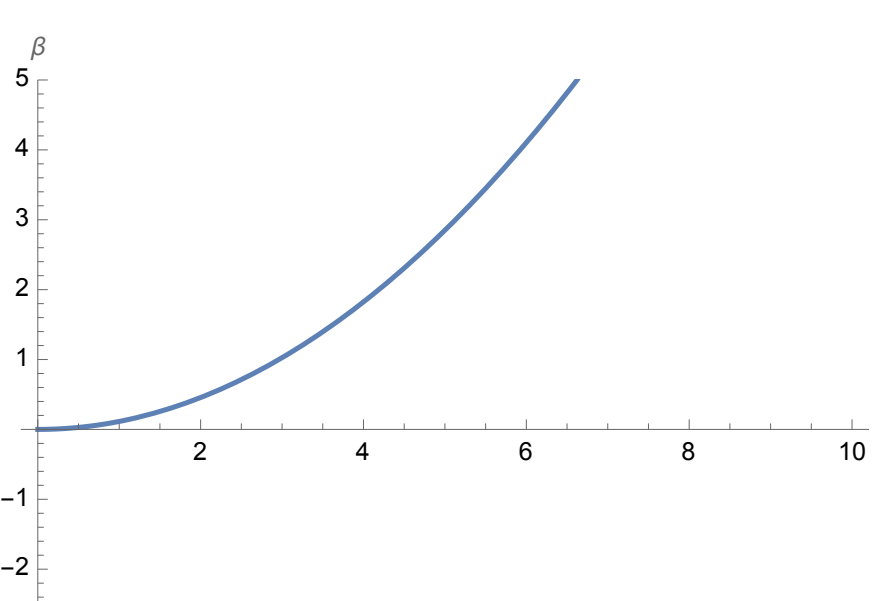
Wilson-Fisher fixed point (1971)

• renormalization group equation

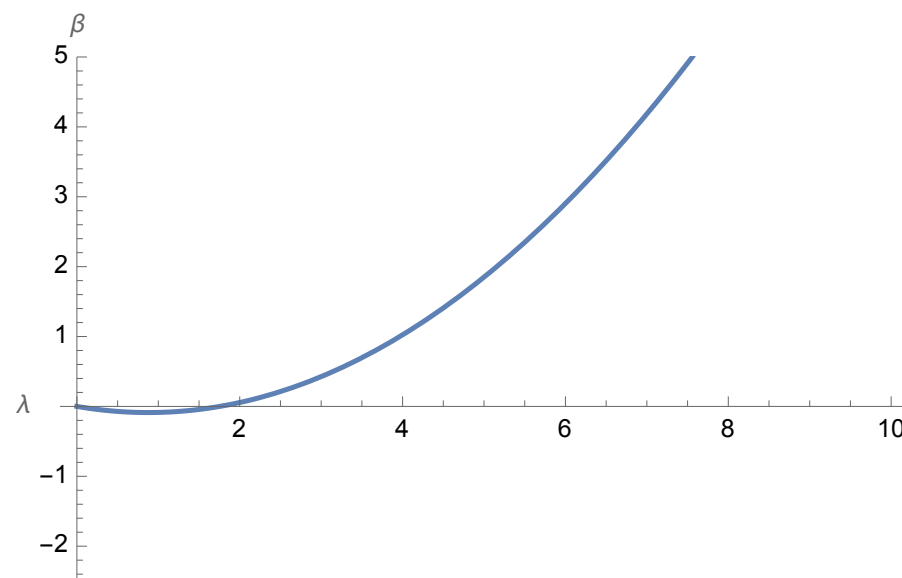
$$\mu \frac{d\lambda}{d\mu} = \beta(\lambda) = -\epsilon\lambda + (N+8)\frac{\lambda^2}{8\pi^2} + O(\lambda^3)$$

$$\lambda_* = \frac{8\pi^2}{N+8}\epsilon$$

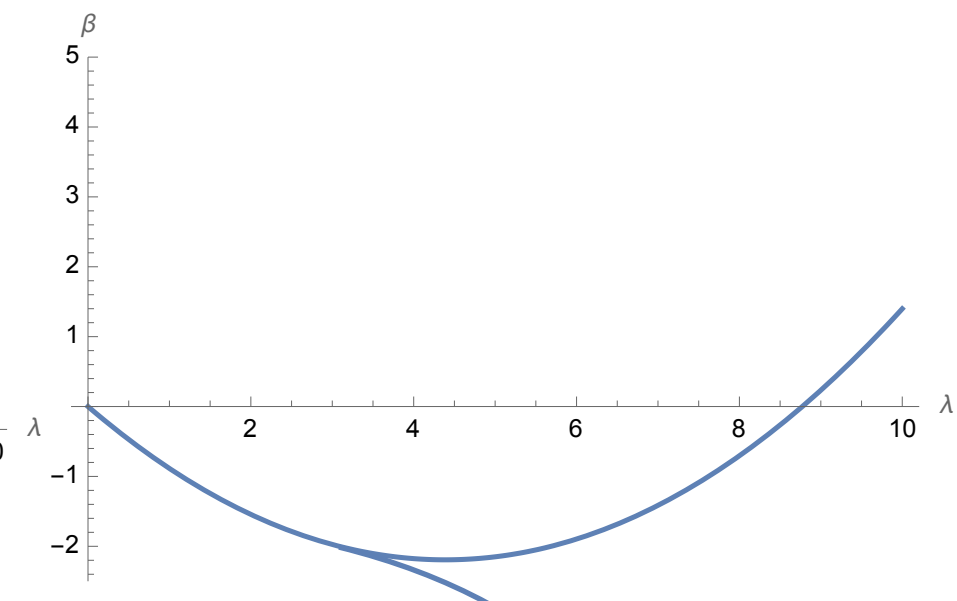
$N=1$



$D=4$



$D=3.8$



$D=3$

forge ahead: ε expansion

N	1	2	3	∞
ν	0.630	0.672	0.711	1
$\varepsilon=1$	0.630	0.674	0.705	

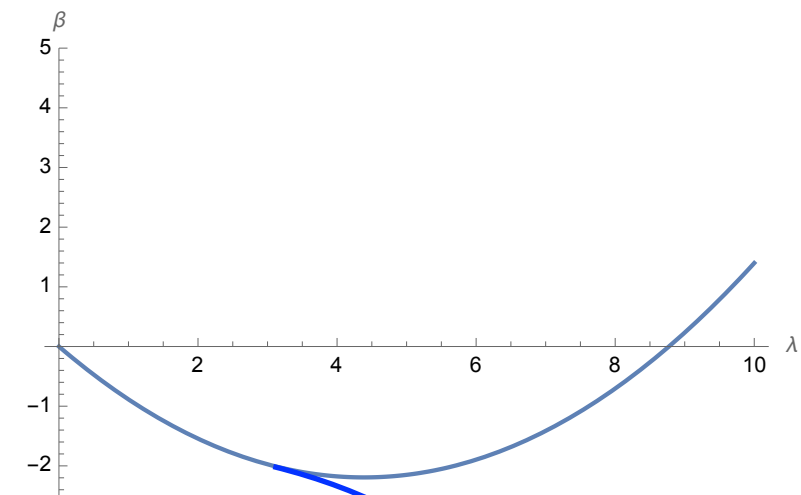
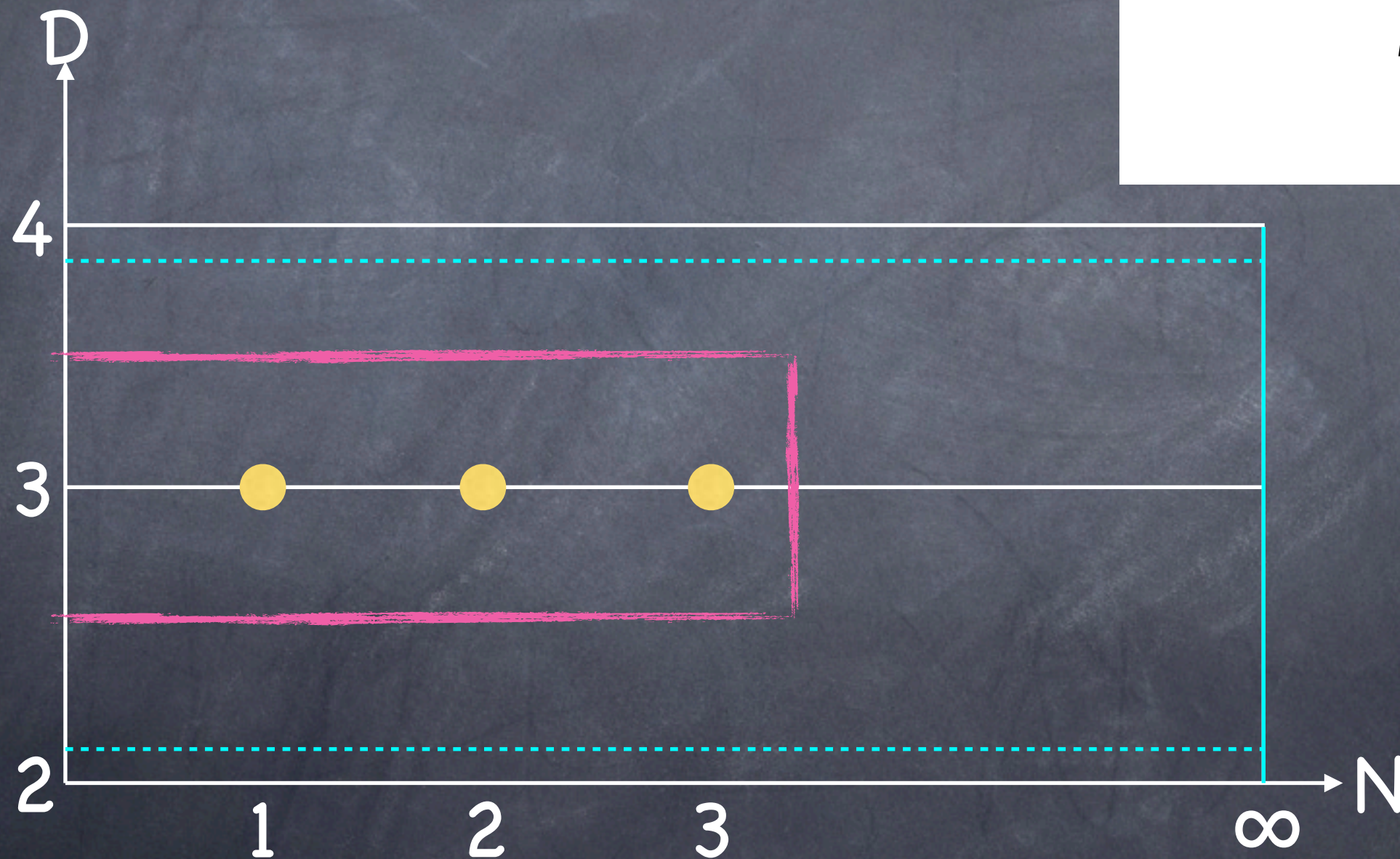
conformal
bootstrap

- perturbation in $\lambda \sim \varepsilon$
- resummation (Borel sum)
- works surprisingly well!

Zinn-Justin 1993

fails for some other universality classes

picture



$D=3$

no guarantee

- CP^{N-1} models $U(N)/U(N-1) \times U(1)$
- $D=2$, $N=\infty$: asymptotically free, confinement, mass gap, very much like Yang-Mills!
- But $N=2$ is free massless fermions (Bethe ansatz, Haldane conjecture in anti-ferromagnet, WZW model)
- it still goes a lot of mileage from $N=\infty$ to 3

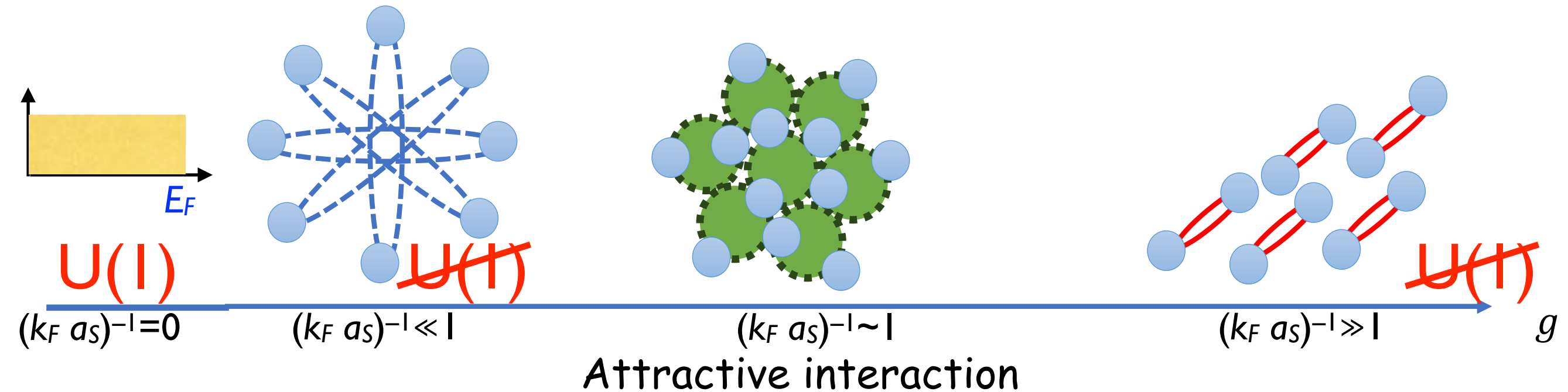
Main point

- very few methods to study strongly coupled systems, e.g., QCD
- non-perturbative effects $e^{-8\pi^2/g^2\hbar}$ are essential
- supersymmetry makes exact analytic studies of non-perturbative effects possible
- small anomaly-mediated SUSY breaking still allows for exact solutions

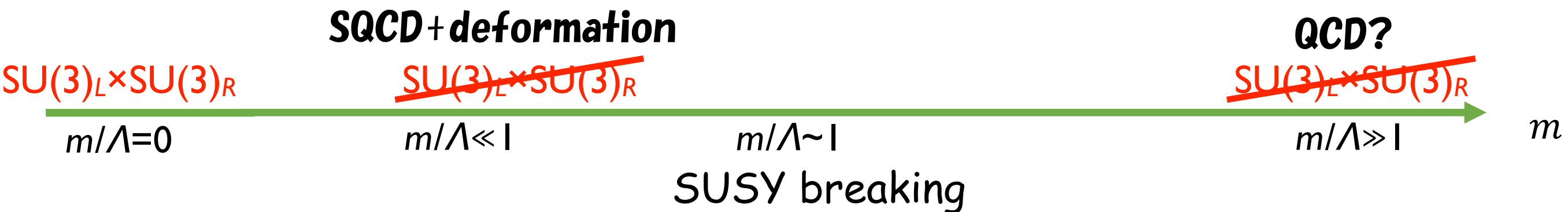


only for S-wave, not for P-wave

BCS-BEC cross over



SQCD&AMSB (ASQCD)



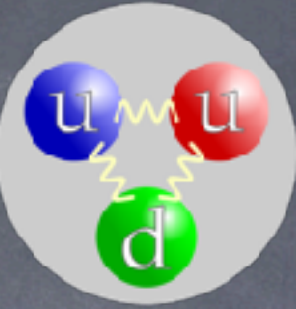
wasn't planned

- I got interested in the possibility that dark matter, axion, etc may be composite
- had to know more non-perturbative physics
- immediately hit limits of our understanding
- became a desperate theorist
- tried crazy things

Concrete results

- Chiral Symmetry Breaking
- Confinement (when appropriate)
- U(1) problem $m_{\eta'}^2 > 0$
- Chiral Lagrangian (Hidden Local Symmetry) with **Wess-Zumino-Witten term**
- **Large N_c Scaling** $f_\pi^2 \propto O(N_c)$ etc
- quark condensates $\langle \bar{q}q \rangle$, gluon condensates $\langle G_{\mu\nu} G^{\mu\nu} \rangle$
- **Witten's formula on topological susceptibility** $\chi = \frac{\partial^2 E}{\partial \theta^2}$
- **0^+ hadron mass spectrum**
- finite electromagnetic corrections
- **symmetry breaking in chiral gauge theories**

Do we understand QCD?



- When we first learn about quarks, we get told we can never see them

Scam?

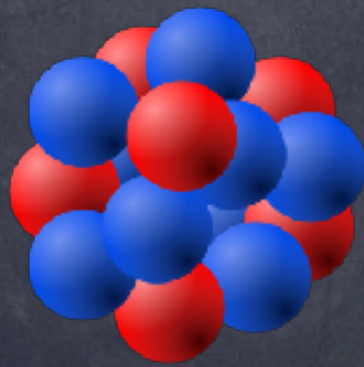
Confinement $V = \alpha' r$!

asymptotic freedom

can't prove confinement

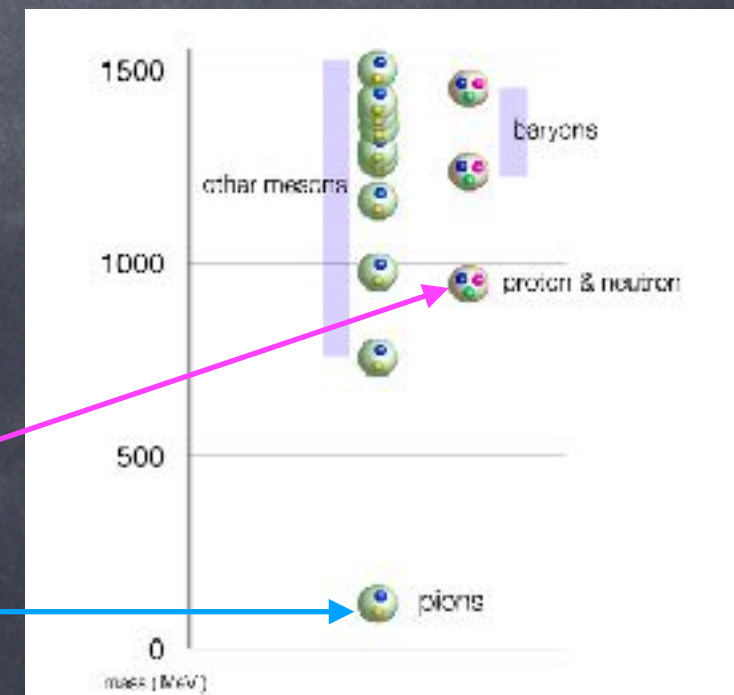
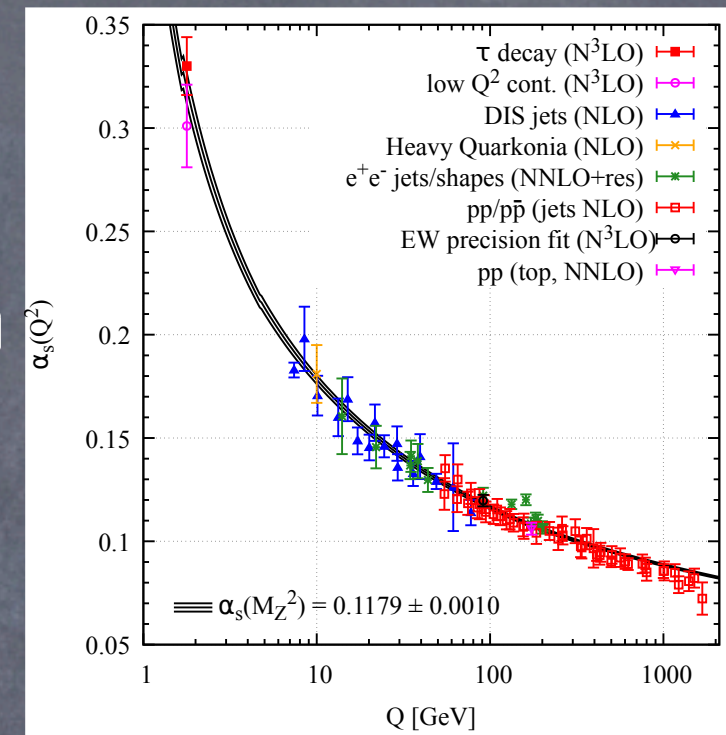
- Another puzzle:** proton and pion are made of same quarks

why pion $\approx$ massless $\ll$ proton?



$$p = uud$$

$$\pi^+ = u\bar{d}$$

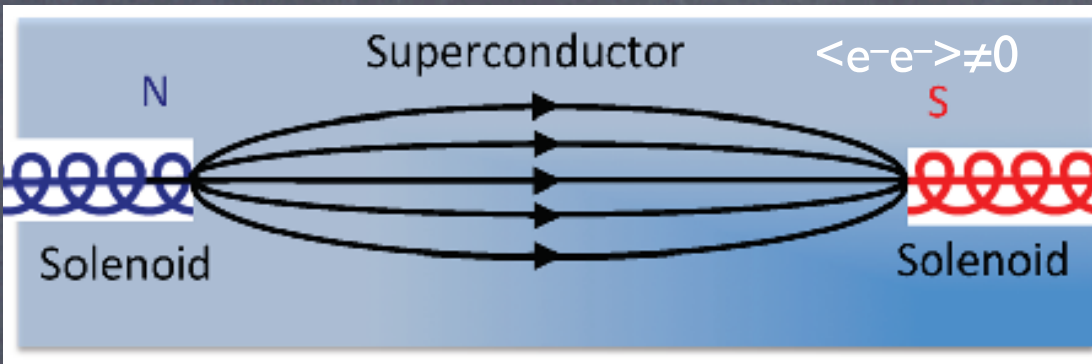


Elements in the Universe

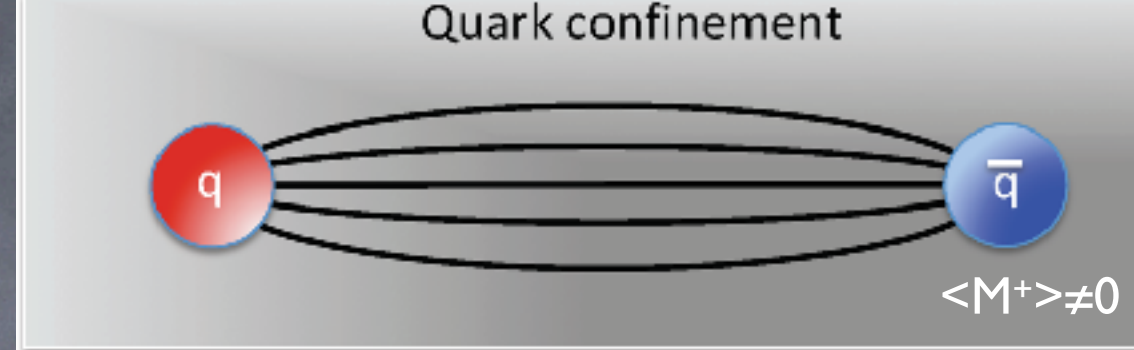
Atomic Number → ← Symbol ← Name ← Atomic Weight ← Electrons per shell State of matter (color of name) Subcategory in the metal-metalloid-nonmetal trend (color of background) Unknown chemical properties																	
1 IA H Hydrogen 1.008 1	2 IIA He Helium 4.003 2																
3 Li Lithium 6.94 2-1	4 Be Beryllium 9.012 2-2																
11 Na Sodium 22.990 2-1	12 Mg Magnesium 24.305 2-2																
19 K Potassium 39.098 2-1	20 Ca Calcium 40.078 2-2	21 Sc Scandium 44.956 2-2	22 Ti Titanium 47.867 2-2	23 V Vanadium 50.942 2-3	24 Cr Chromium 51.996 2-3	25 Mn Manganese 54.938 2-3	26 Fe Iron 55.845 2-3	27 Co Cobalt 58.933 2-3	28 Ni Nickel 58.693 2-3	29 Cu Copper 63.546 2-3	30 Zn Zinc 65.38 2-3	31 Ga Gallium 69.723 2-3	32 Ge Germanium 72.63 2-3	33 As Arsenic 74.922 2-3	34 Se Selenium 78.96 2-3	35 Br Bromine 79.904 2-3	36 Kr Krypton 83.798 2-3
37 Rb Rubidium 85.468 2-3	38 Sr Strontium 87.62 2-3	39 Y Yttrium 88.906 2-3	40 Zr Zirconium 91.224 2-3	41 Nb Niobium 92.906 2-3	42 Mo Molybdenum 95.94 2-3	43 Tc Technetium (98) 2-3	44 Ru Ruthenium 101.07 2-3	45 Rh Rhodium 102.91 2-3	46 Pd Palladium 106.42 2-3	47 Ag Silver 107.87 2-3	48 Cd Cadmium 112.41 2-3	49 In Indium 114.82 2-3	50 Sn Tin 118.71 2-3	51 Sb Antimony 121.76 2-3	52 Te Tellurium 127.6 2-3	53 I Iodine 126.90 2-3	54 Xe Xenon 131.29 2-3
55 Cs Cesium 132.905 2-3	56 Ba Barium 137.327 2-3	57-71 Lanthanides	72 Hf Hafnium 178.49 2-3	73 Ta Tantalum 180.948 2-3	74 W Tungsten 183.84 2-3	75 Re Rhenium 186.21 2-3	76 Os Osmium 190.23 2-3	77 Ir Iridium 192.22 2-3	78 Pt Platinum 195.08 2-3	79 Au Gold 196.97 2-3	80 Hg Mercury 200.59 2-3	81 Tl Thallium 204.38 2-3	82 Pb Lead 207.2 2-3	83 Bi Bismuth 208.98 2-3	84 Po Polonium (209) 2-3	85 At Astatine (210) 2-3	86 Rn Radon (222) 2-3
87 Fr Francium (223) 2-3	88 Ra Radium (226) 2-3	89-103 Actinides	104 Rf Rutherfordium (261) 2-3	105 Db Dubnium (268) 2-3	106 Sg Seaborgium (266) 2-3	107 Bh Bohrium (264) 2-3	108 Hs Hassium (277) 2-3	109 Mt Meitnerium (276) 2-3	110 Ds Darmstadtium (281) 2-3	111 Rg Roentgenium (282) 2-3	112 Cn Copernicium (285) 2-3	113 Nh Nihonium (284) 2-3	114 Fl Flerovium (289) 2-3	115 Mc Moscovium (290) 2-3	116 Lv Livermorium (293) 2-3	117 Ts Tennessine (294) 2-3	118 Og Oganesson (294) 2-3
57 La Lanthanum 138.91 2-3	58 Ce Cerium 140.12 2-3	59 Pr Praseodymium 140.91 2-3	60 Nd Neodymium 144.24 2-3	61 Pm Promethium (145) 2-3	62 Sm Samarium 150.36 2-3	63 Eu Europium 151.96 2-3	64 Gd Gadolinium 157.25 2-3	65 Tb Terbium 158.93 2-3	66 Dy Dysprosium 162.50 2-3	67 Ho Holmium 164.93 2-3	68 Er Erbium 167.26 2-3	69 Tm Thulium 168.93 2-3	70 Yb Ytterbium 173.05 2-3	71 Lu Lutetium 174.97 2-3			
89 Ac Actinium 227 2-3	90 Th Thorium 232.04 2-3	91 Pa Protactinium 231.04 2-3	92 U Uranium 238.03 2-3	93 Np Neptunium 237 2-3	94 Pu Plutonium 244 2-3	95 Am Americium 243 2-3	96 Cm Curium 247 2-3	97 Bk Berkelium 247 2-3	98 Cf Californium 251 2-3	99 Es Einsteinium 252 2-3	100 Fm Fermium 257 2-3	101 Md Mendelevium 258 2-3	102 No Nobelium 259 2-3	103 Lr Lawrencium 262 2-3			

Unless pions are light, we wouldn't exist!

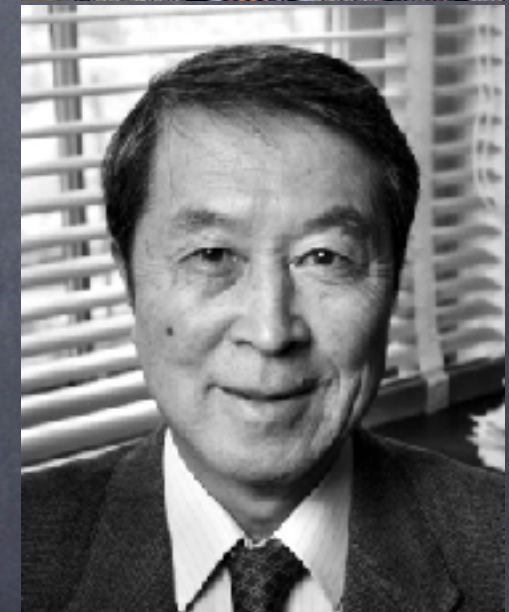
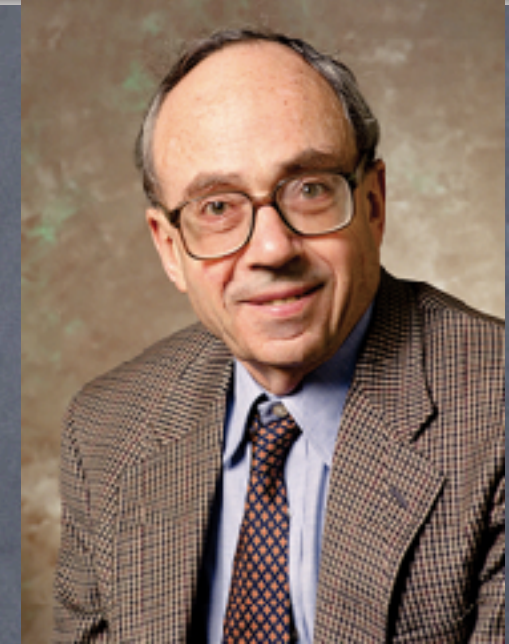
Credit: Wikimedia Commons



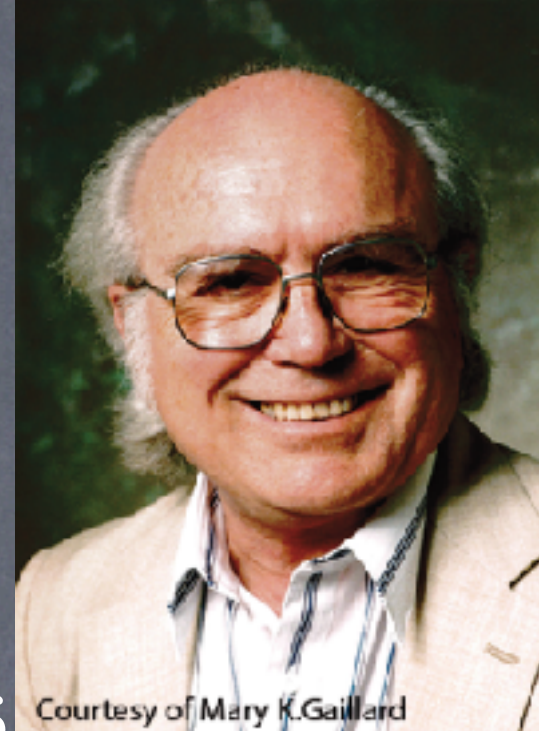
ideas



- Qualitative ideas help us
- Confinement**
 - dual Meissner effect (Mandelstam)
 - assume** monopole condensation
 - quarks confined by electric flux tube
 - but where are monopoles in QCD?
- Chiral symmetry breaking** (Nambu)
 - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
 - assume** broken to $SU(N_f)_V \times U(1)_B$ $\langle q^i \bar{q}_j \rangle = v^3 \delta_j^i \neq 0$
 - pion = Nambu-Goldstone boson = massless
 - but does QCD really break chiral symmetry?

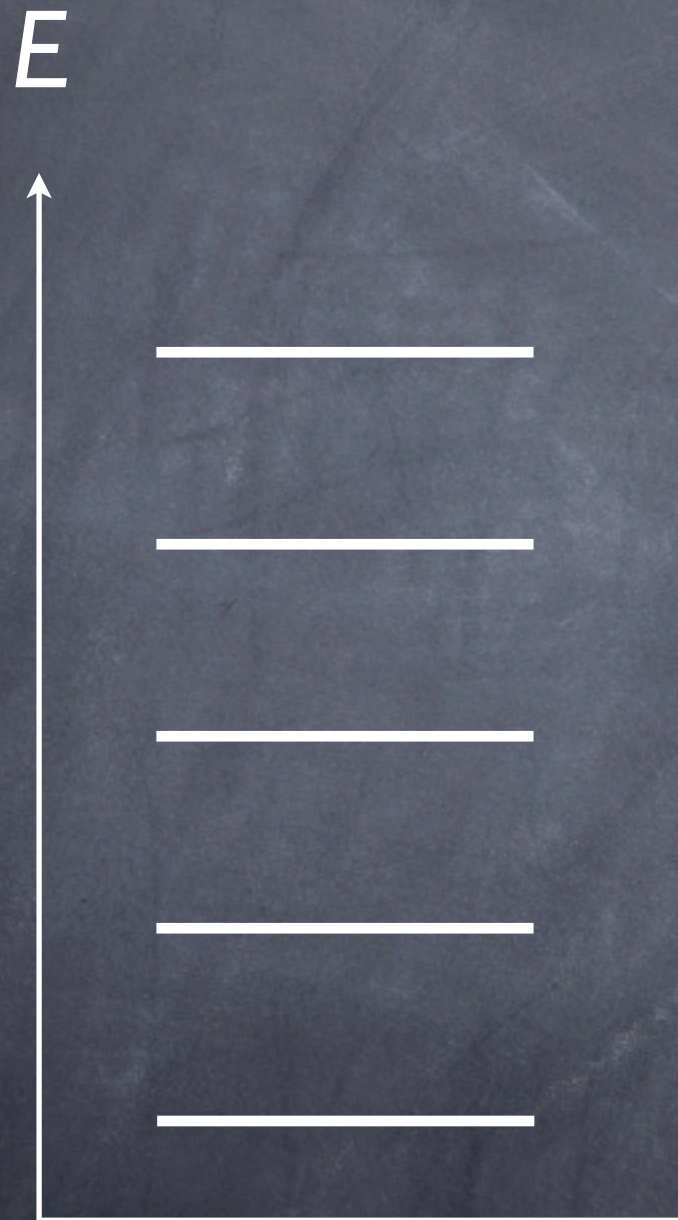


Progress with Supersymmetry

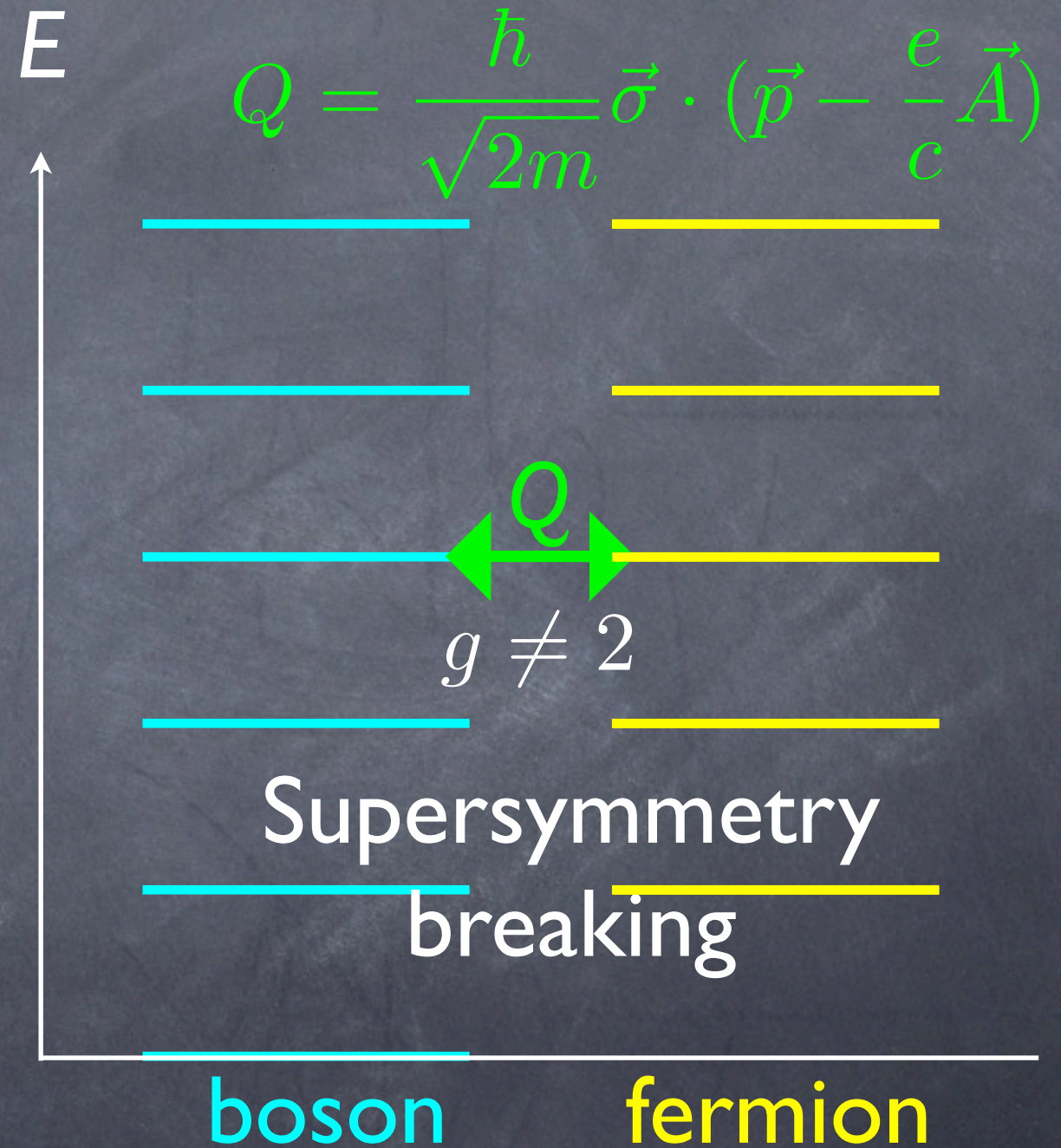


- symmetry between bosons and fermions
- **Confinement** (Seiberg-Witten)
 - $N=2$ $SU(2)$ Supersymmetric Gauge Theory
 - $SU(2) \rightarrow U(1)$: magnetic monopoles!
 - special points = massless monopoles
 - $N=1$ perturbation: monopole condensation!
- **Chiral symmetry breaking**
 - $N=2$ doesn't have chiral symmetry
 - $N=1$ (Seiberg) has too unusual phases
- doesn't look like the real world!

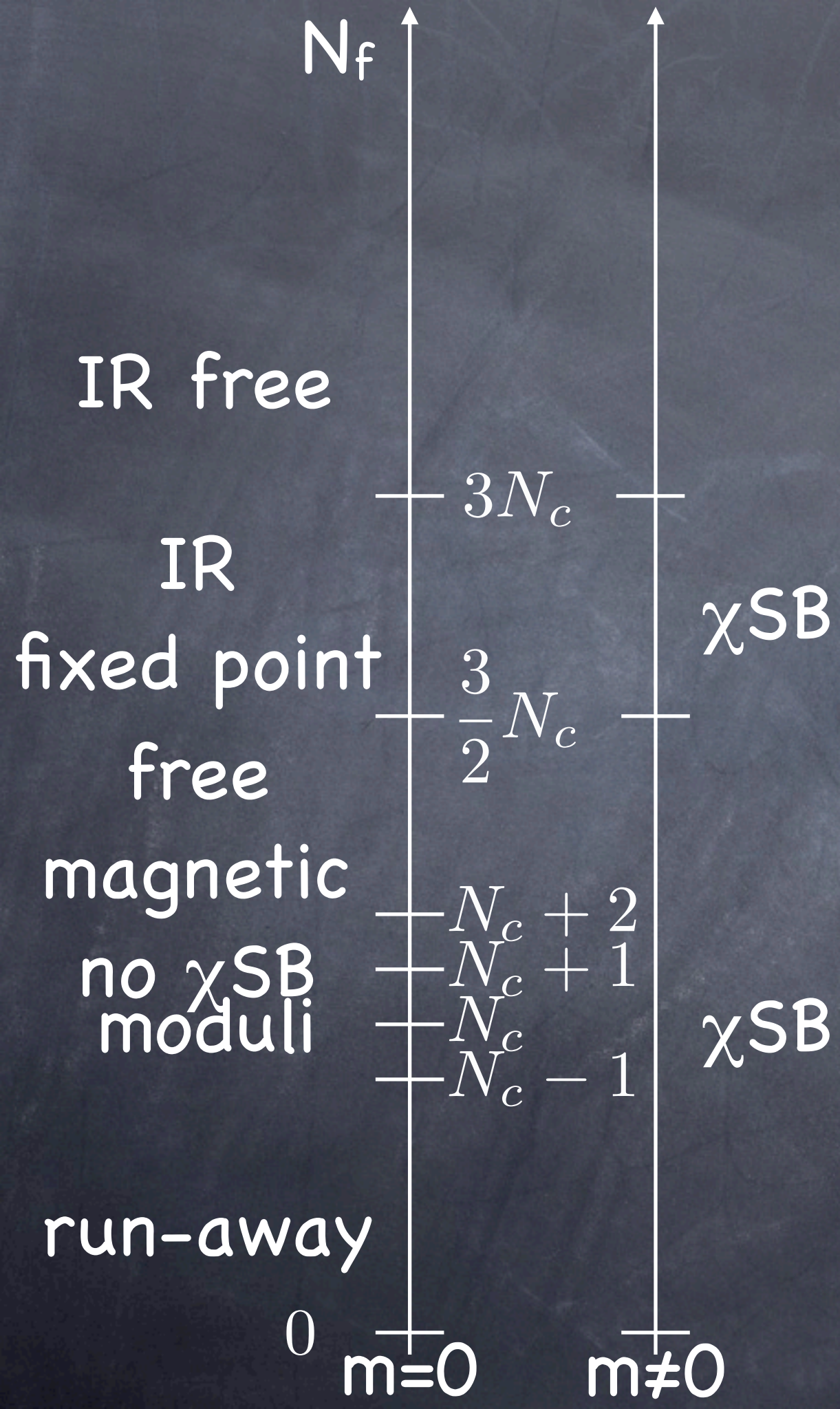
Landau levels $H = Q^2$



$$H = \frac{\hbar^2}{2m} \left(\vec{p} - \frac{e}{c} \vec{A} \right)^2$$



$$H = \frac{\hbar^2}{2m} \left(\vec{p} - \frac{e}{c} \vec{A} \right)^2 - g \frac{e}{mc} \vec{s} \cdot \vec{B}$$



χ SB

χ SB

$m = \infty$

Λ

Anomaly Mediation of SUSY Breaking (AMSB)

- Tree-level piece on dimensionful parameters

$$V_{\text{AMSB}} = -m \left(\phi \frac{\partial W}{\partial \phi} - 3W \right)$$

- loop-level piece from running

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

Randall, Sundrum (1998)
Giudice, Luty, HM, Rattazzi (1998)



ometimes convenient to introduce a complex gravitino mass, defi

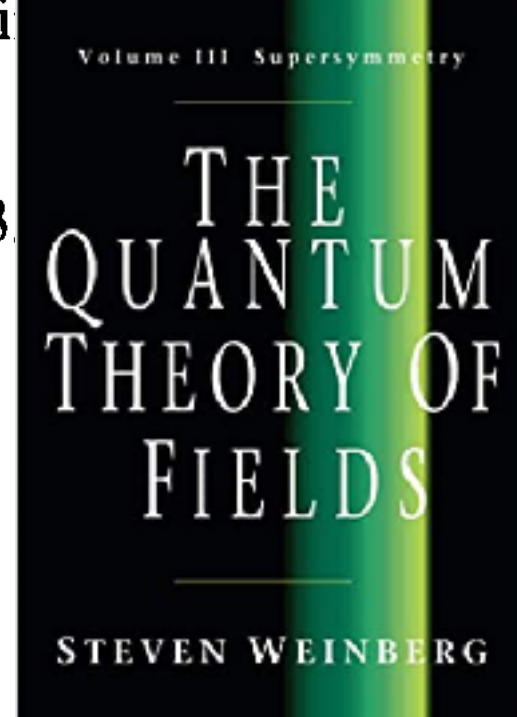
$$\tilde{m}_g \equiv \frac{2\kappa}{3} (\langle s \rangle + i\langle p \rangle), \quad (31.3)$$

bsolute magnitude is the physical gravitino mass (31.3.19).

31.4 Anomaly-Mediated Supersymmetry Breaking

In Section 28.3 the possibility was raised that supersymmetry may be broken in some sort of hidden sector of superfields that do not carry the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the standard model, and communicated to observable particles gravitationally. In this section we will deal with one class of supersymmetry-breaking effects in the minimum supersymmetric standard model, those of first order in $\kappa \equiv \sqrt{8\pi G}$. This includes the gaugino masses and the parameters A_{ij} and B in the Lagrangian density (28.4.1). Other supersymmetry-breaking effects such as squark and slepton squared masses are of second order in κ , and will be taken up in Section 31.7, when we consider gravity-mediated supersymmetry breaking using the general supergravity formalism described in Section 31.6.

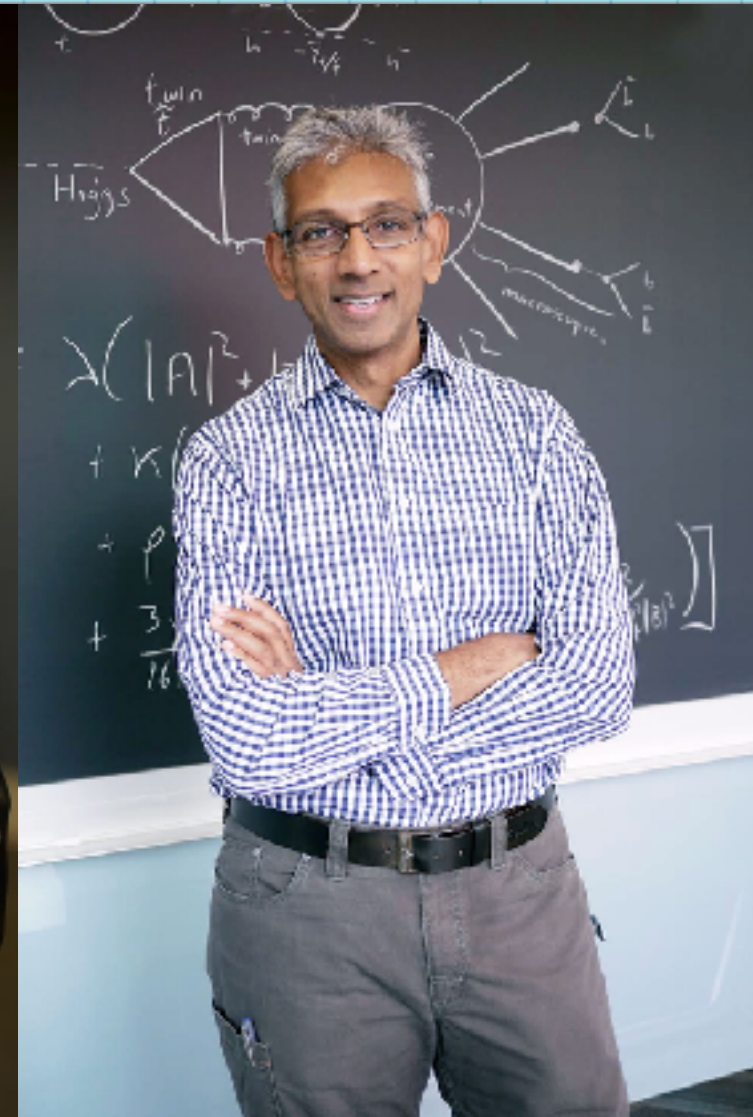
We can find the effects of gravity-mediated supersymmetry breaking to first order in κ by simply replacing the component fields of the gravitational supermultiplet in the interaction (31.1.34) with their expectation values. The only ones of these component fields that can acquire non-vanishing vacuum expectation values from the spontaneous breakdown



Saved by time differences

- worked in summer 1998 at CERN to come up with theory of supersymmetry **before LHC discovers it**
- left CERN with a half-finished draft
- Giudice, Rattazzi@CERN
- Luty@Maryland

1669 citations



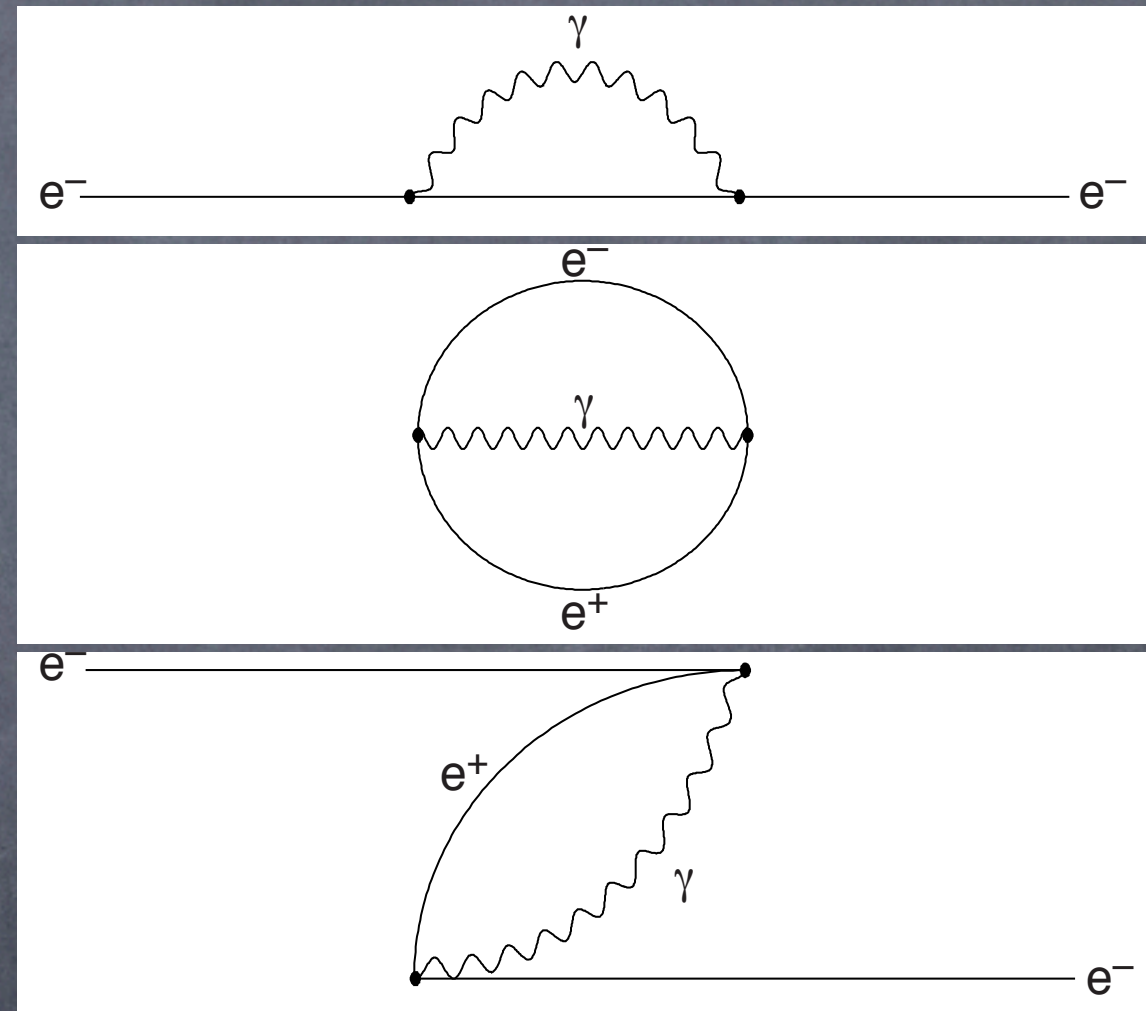
Electron mass is natural by doubling #particles

- Electron creates a force to repel itself

$$\Delta m_e c^2 \sim \frac{e^2}{r_e} \sim \text{GeV} \frac{10^{-17} \text{cm}}{r_e}$$

- 10^{-4} fine-tuning?
- quantum mechanics and anti-matter

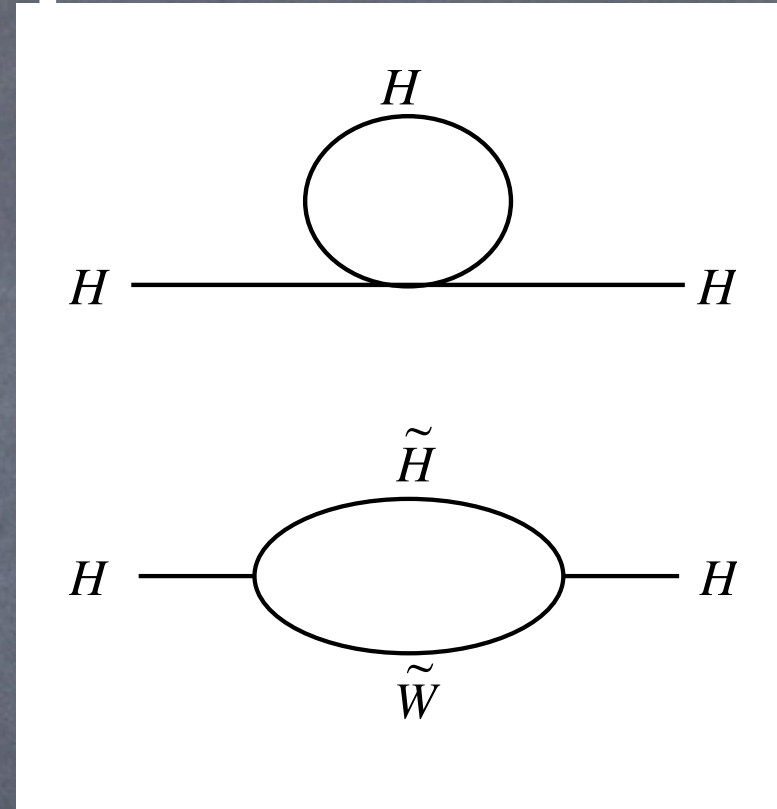
⇒ only 10% of mass even
for Planck-size $r_e \sim 10^{-33} \text{cm}$



$$\Delta m_e \sim m_e \frac{\alpha}{4\pi} \log(m_e r_e)$$

Higgs mass is natural by doubling #particles?

- Higgs also repels itself
- Double #particles again
⇒ superpartners
- only log sensitivity to UV
- Standard Model made
consistent up to higher
energies

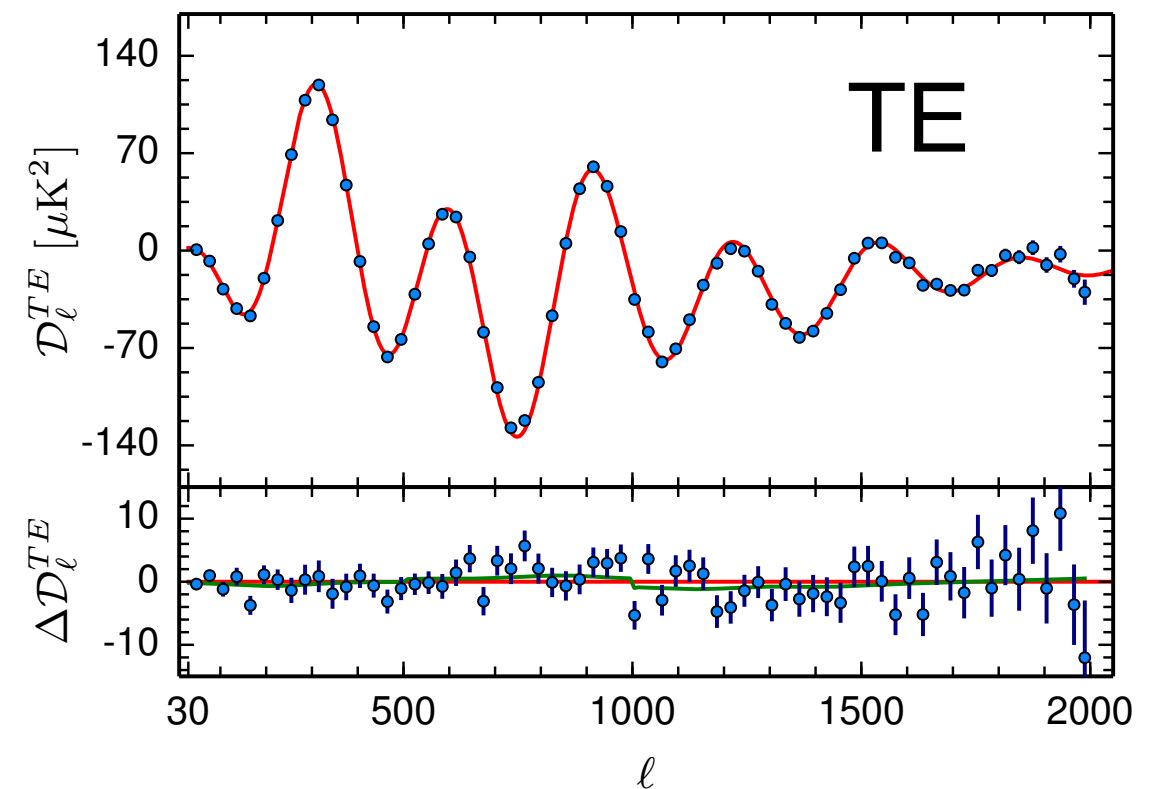
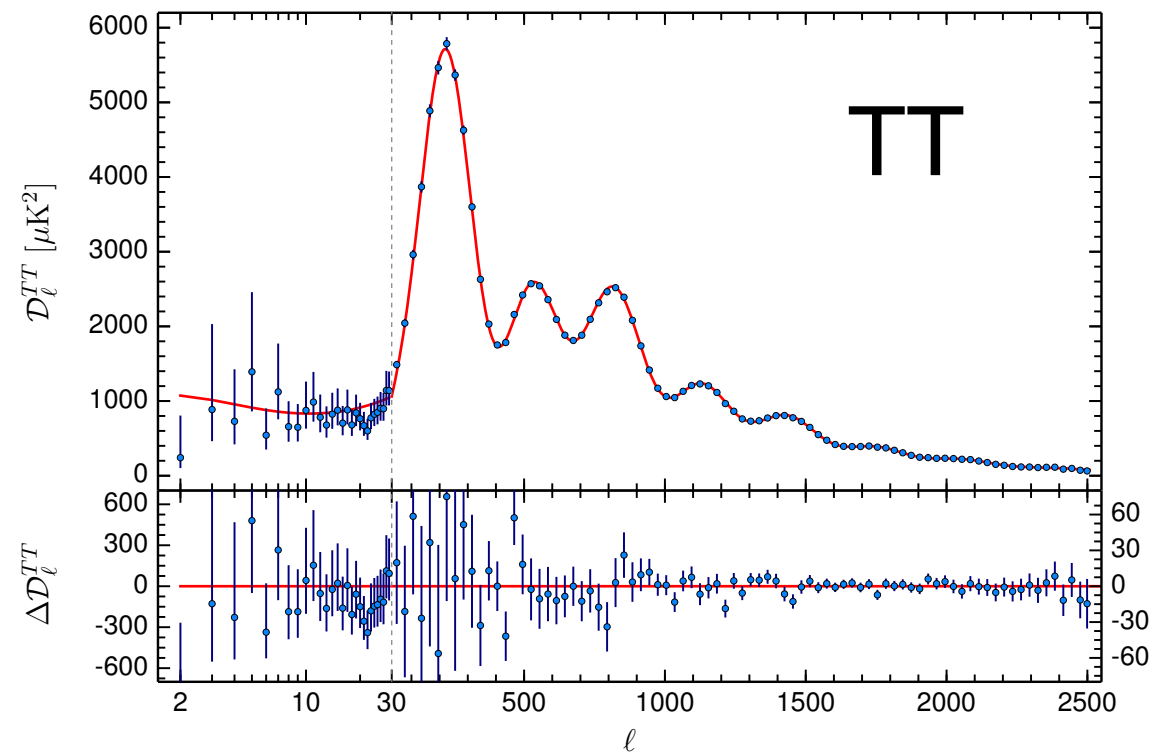
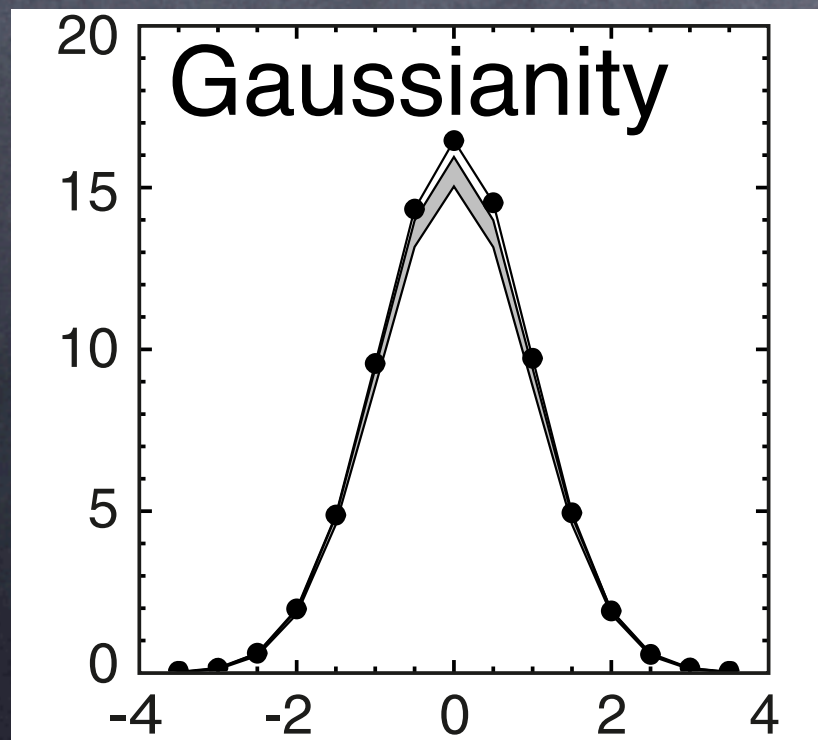


$$\Delta m_H^2 \sim \frac{\alpha}{4\pi} m_{SUSY}^2 \log(m_H r_H)$$

➔ I still take it seriously

Naturalness works!

- Why is the Universe big?
- Inflation
 - horizon problem
 - flatness problem
 - large entropy



ENGINEERING
**Machines That
Change Shape**

MEDICINE
**An Off Switch
for Cancer**

NEUROSCIENCE
**How to Reach
"Vegetative" Patients**

SCIENTIFIC AMERICAN

ScientificAmerican.com

IF SUPERSYMMETRY

CRISIS

DOESN'T PAN OUT,

IN

SCIENTISTS NEED A NEW WAY

PHYSICS

TO EXPLAIN THE UNIVERSE

?



MAY 2014

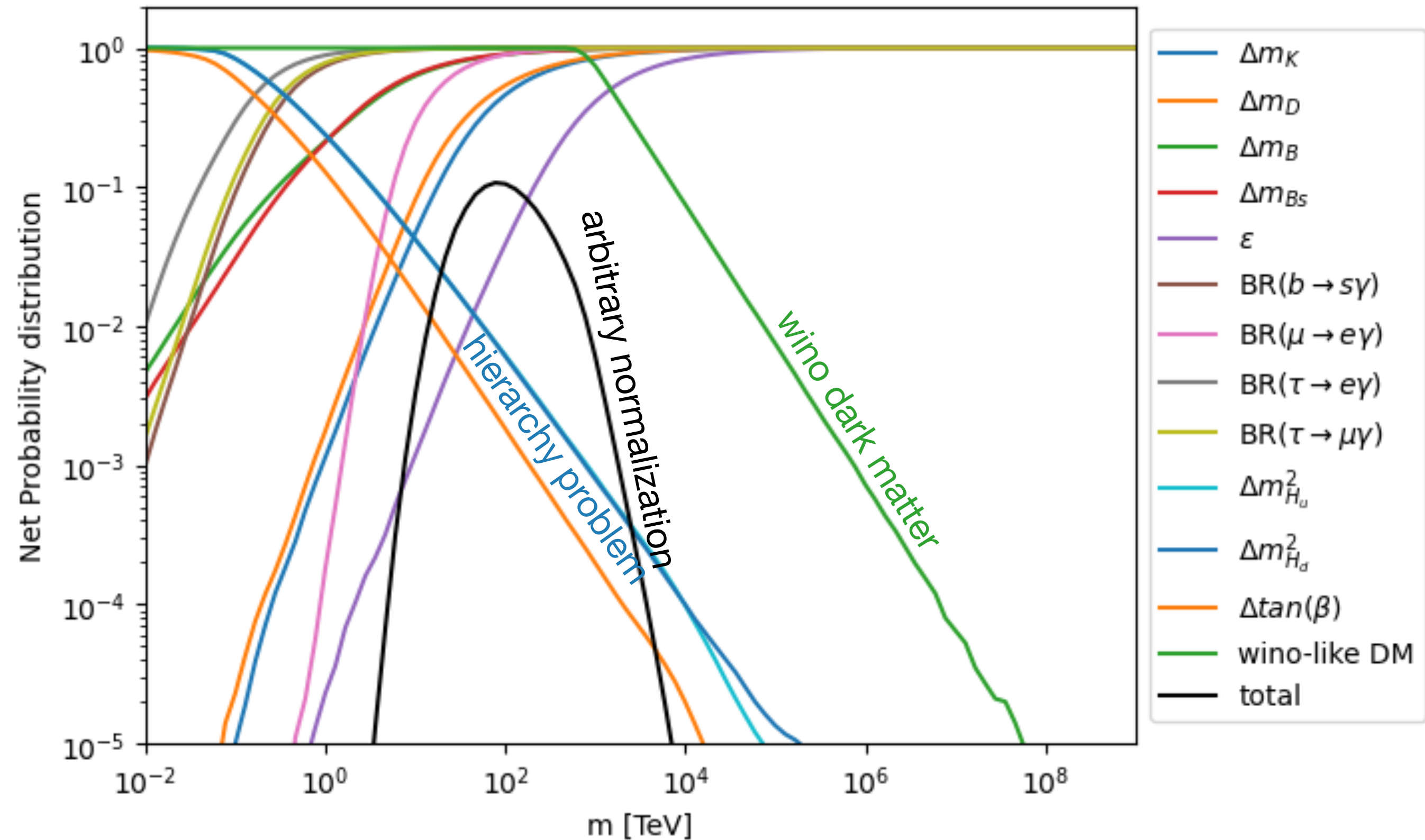


A person in a white shirt and blue pants stands in a vast, flat, yellowish-brown landscape under a clear blue sky. They are looking up at a wooden signpost that has two horizontal signs. The top sign says 'Multiverse' and the bottom sign says 'Naturalness'. The person's hand is raised towards the signpost.

Multiverse

Naturalness

Split [530543], Max Probability of 1.07e-01 at $m = 7.74e+01$ TeV



HM, Risshin Okabe, Gup Singh, Bethany Suter in preparation

Supersymmetry as a mathematical tool

$$N_f < N_c$$

run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)} \quad M^{ij} = \delta^{ij} \phi^2$$

$$V = 2N_f \left| \frac{\Lambda^{3N_c - N_f}}{\phi^{N_c + N_f}} \right|^{\frac{2}{N_c - N_f}} - m(3N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{\frac{1}{N_c - N_f}} + c.c.$$

$$M_{ij} = \delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$$\langle \bar{q}_i q_j \rangle = m M_{ij} \neq 0$$

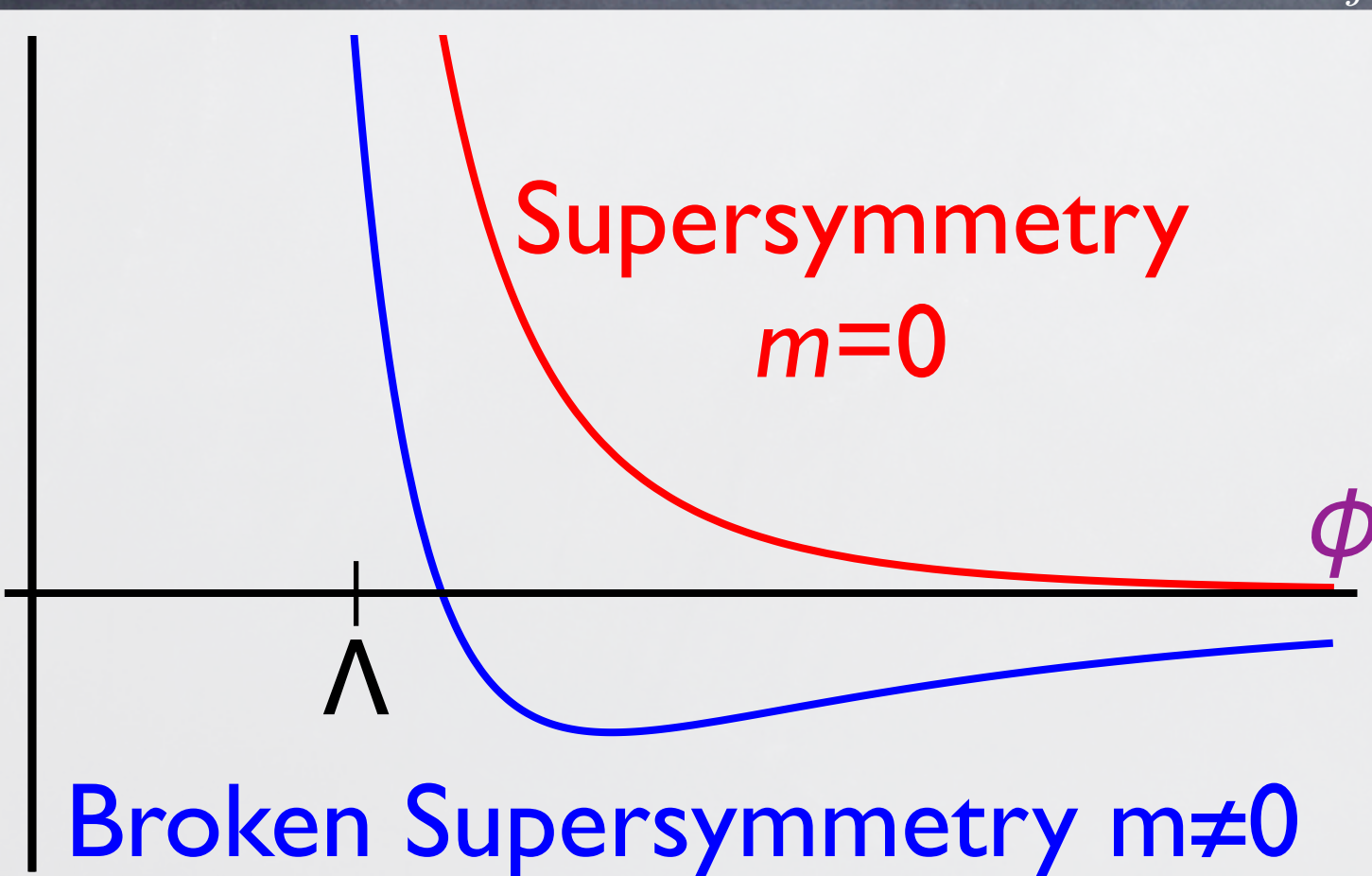
$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

Proving χ SB!

mesino loop $\rightarrow$ WZW term

$N_f=1$ special

no NGB, gapped



U(1) Problem

- Classically $U(N_f) \times U(N_f)$ global symmetry
- We don't see U(1) NGB in data
- U(1) broken by anomaly $\partial_\mu j_1^\mu = \frac{N_f}{8\pi^2} \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} \neq 0$
- Does it lead to a finite mass?

$$m_{\eta'}^2 = \frac{2(3-x)^2 x}{(1+x)(1-x)^2} m^2 = 18m^2 x + \mathcal{O}(x^2) \quad x = \frac{N_f}{N_c}$$

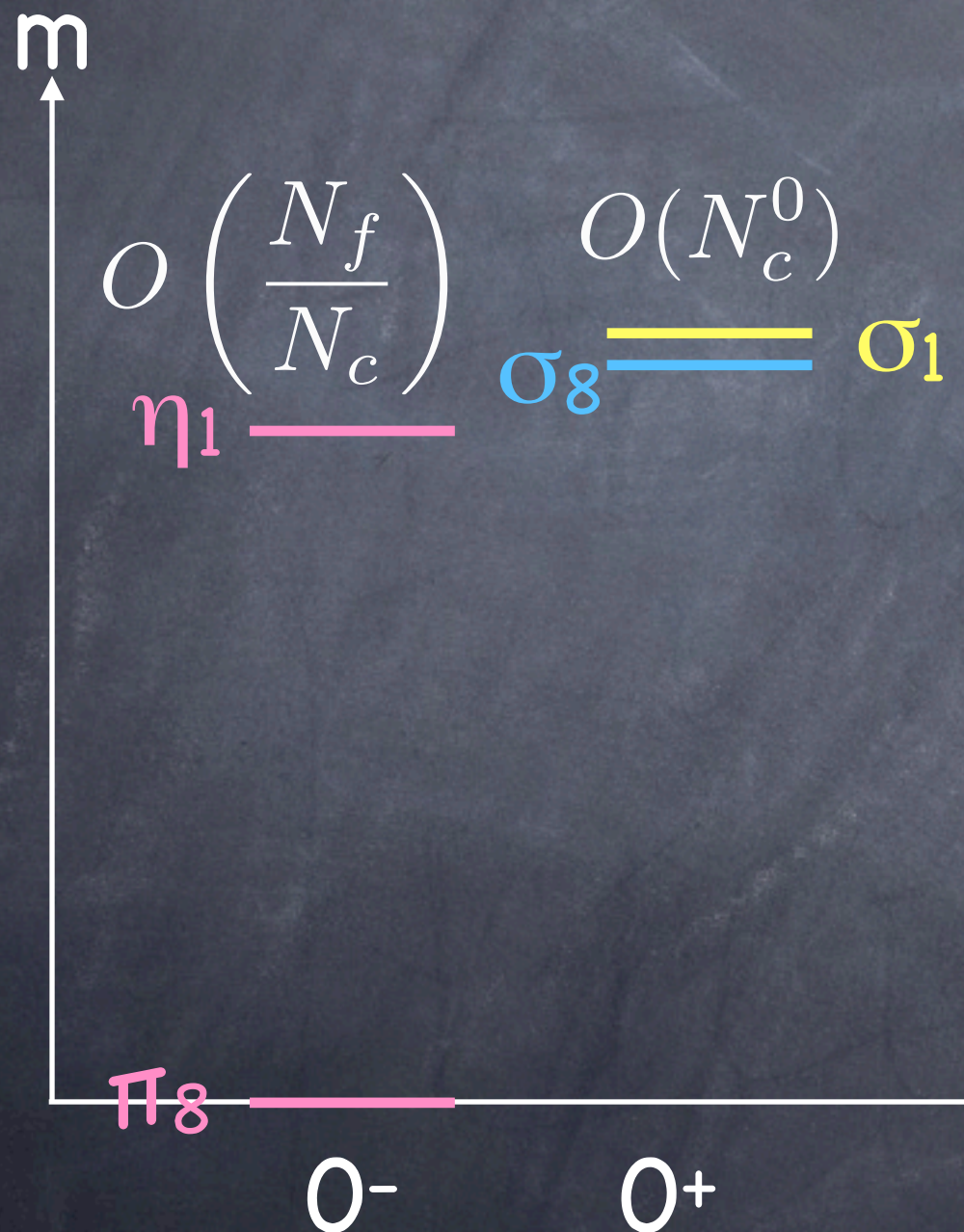
Witten's formula $f_\pi^2 m_{\eta'}^2 = 2N_f \left(\frac{\partial^2 \mathcal{E}}{\partial \theta^2} \right)_{\theta=0}^{\text{no quarks}}$

$$f_\pi^2 m_{\eta'}^2 = 4 \frac{x(3-x)}{(1-x)^2} \left(\frac{3-x}{1+x} \frac{m}{\Lambda} \right)^x m \Lambda^3 \xrightarrow{x \rightarrow 0} 12x m \Lambda^3 \quad 2N_f \left(\frac{\partial^2 \mathcal{E}}{\partial \theta^2} \right)_{\theta=0}^{\text{no quarks}} = 12x m \Lambda^3$$

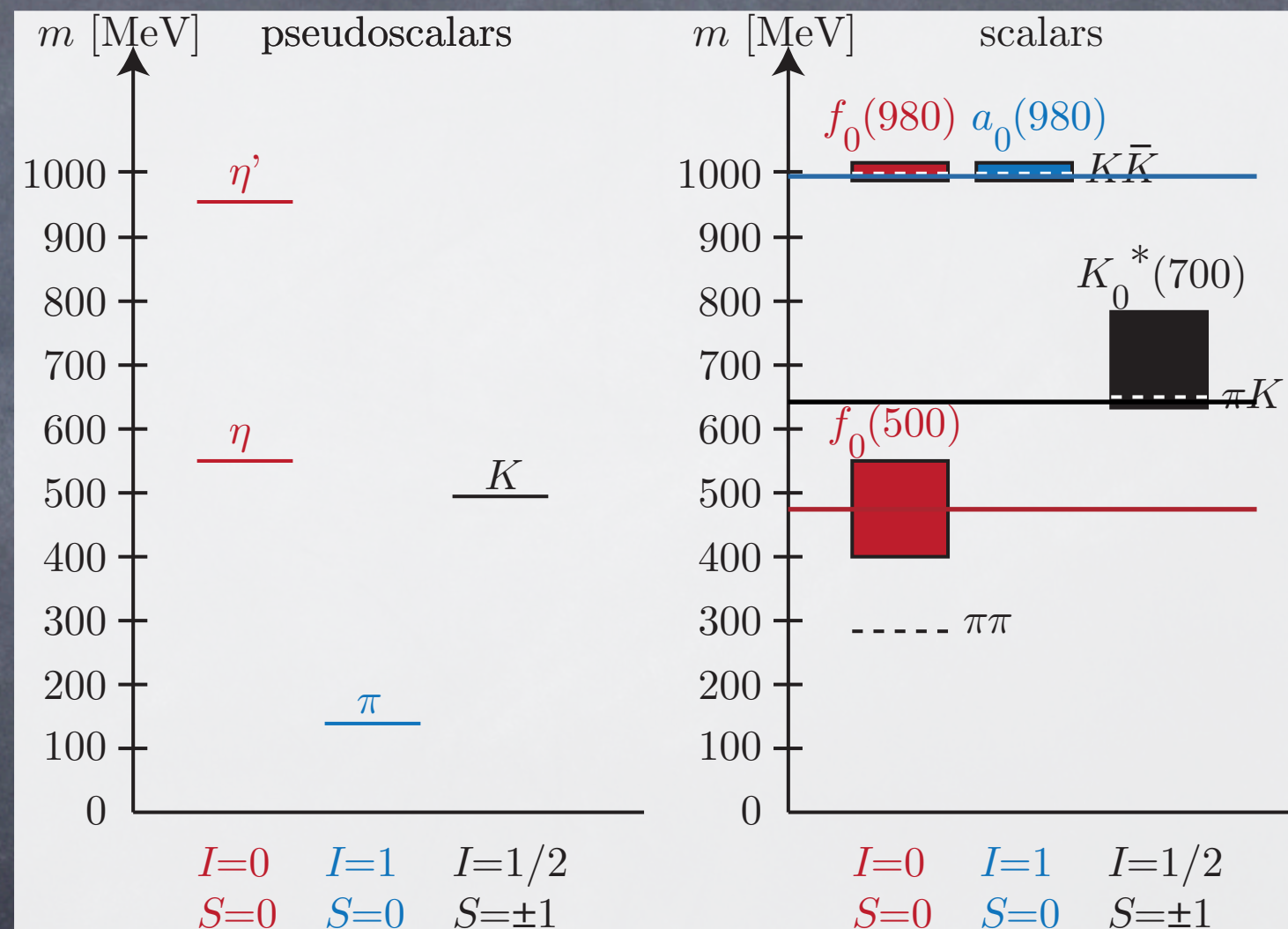


Scalar spectrum

Found $c' < 0$



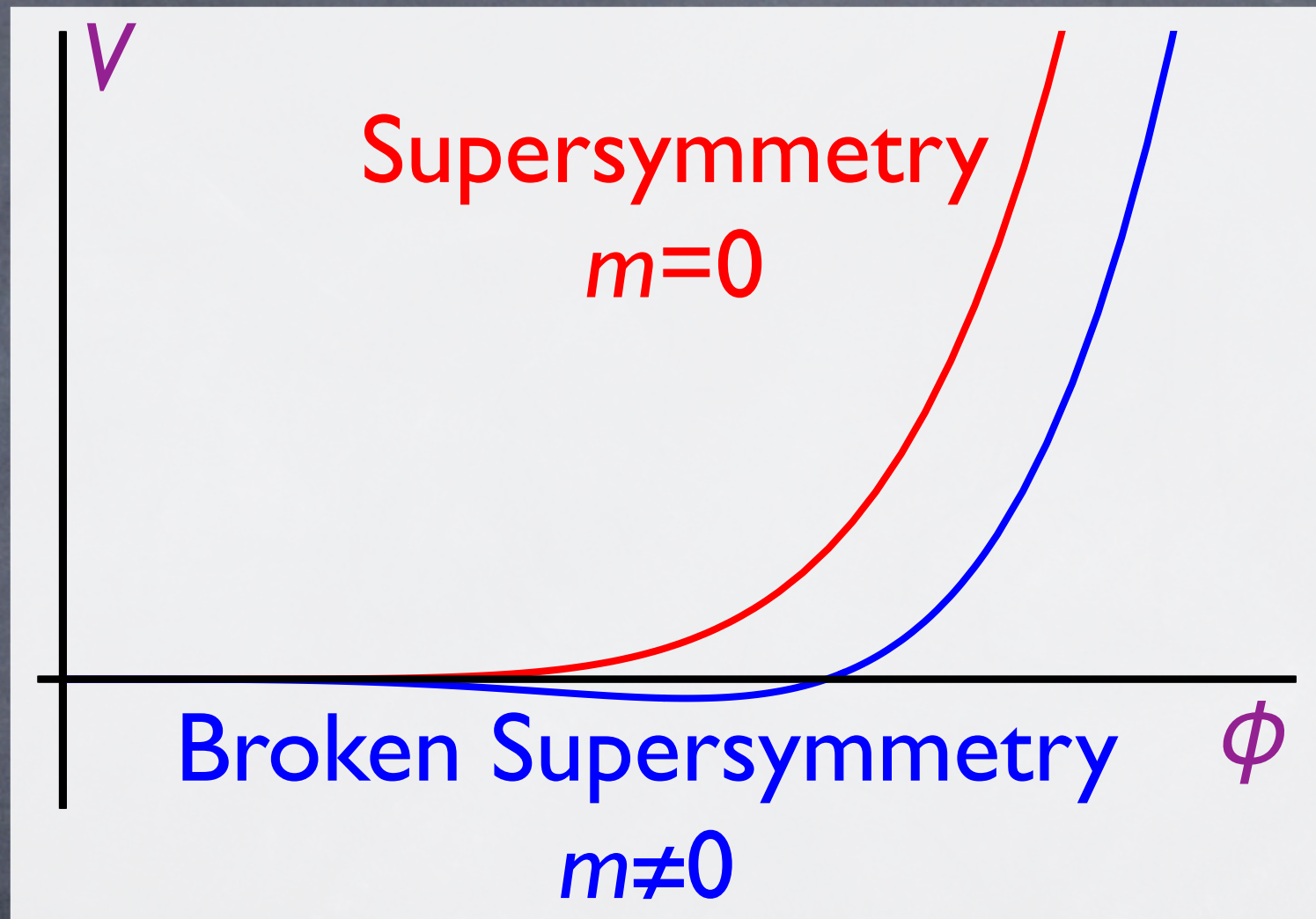
$$m_\pi^2 = c(m_{q_i} + m_{q_j}) \quad m_\sigma^2 = m_8^2 + c'(m_{q_i} + m_{q_j})$$



$$c' = -2.2c$$

$SU(N_c)$ w/ $N_c < N_f < 3N_c/2$

- “Confinement without χ_{SB} ” (Seiberg)

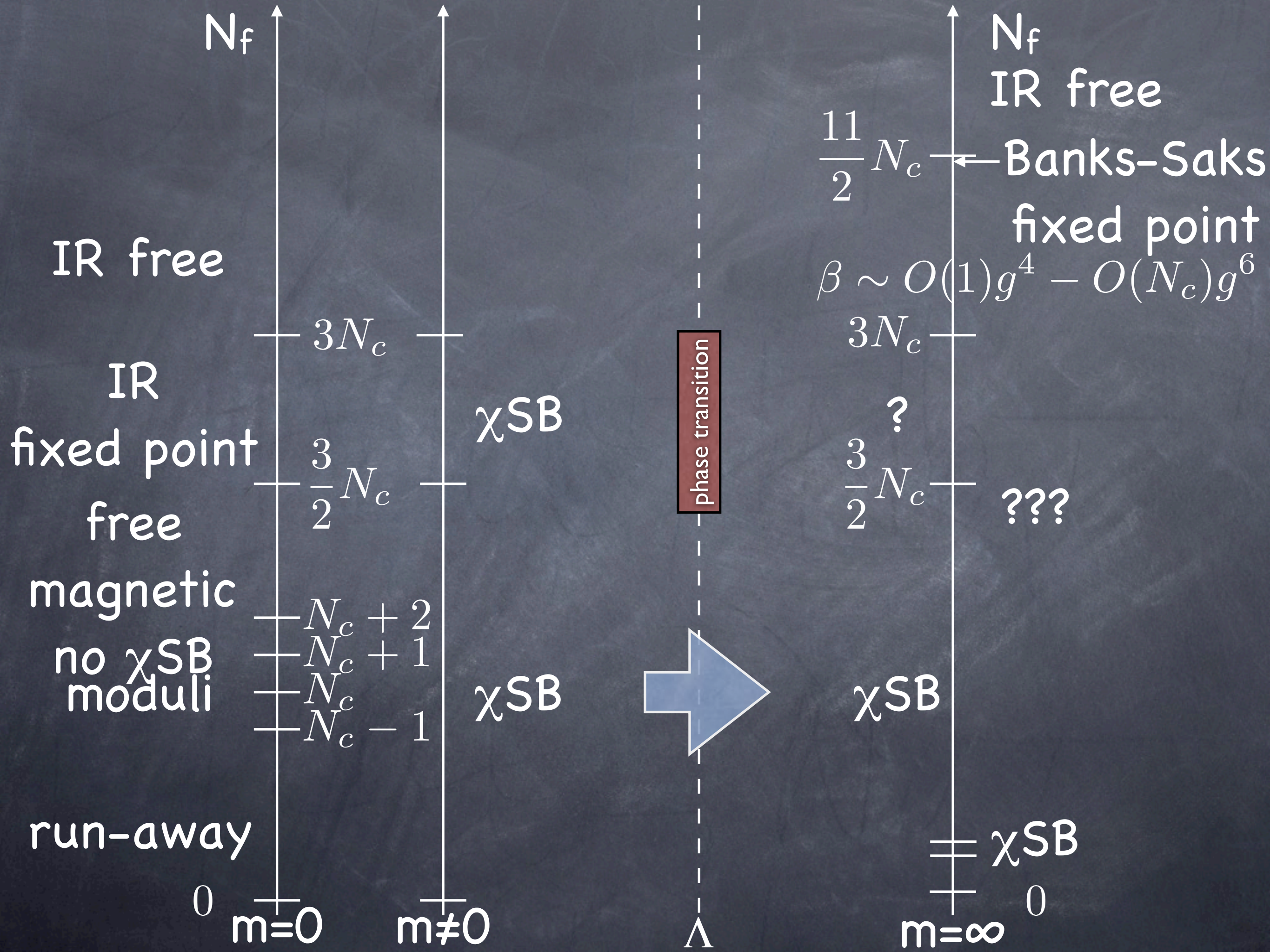


$$\bar{Q}_i Q_j = \phi \delta_{ij} \neq 0$$

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

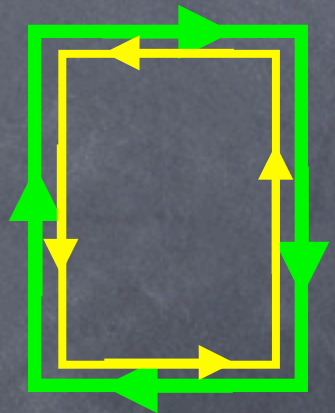
- χ_{SB} !

- massless pions



confinement vs screening

- We've derived χ SB in $SU(N_c)$ QCD
 - strictly speaking, it has no confinement
 - massless quarks in the fundamental rep can screen any color charges
- Wilson loop is perimeter law $\langle \text{Tr} e^{i \oint A dx} \rangle \propto e^{-L}$
- $SO(N_c)$ QCD with quarks in vector rep
 - cannot screen Z_2 center (e.g. spinor rep)
 - rigorous definition of confinement $\langle \text{Tr} e^{i \oint A dx} \rangle \propto e^{-A}$
 - can we see an interplay with χ SB?
- Complete classification of t-confining theories



$$N_f = N_c - 2$$

- for $M^{ij} = Q^i Q^j \neq 0$ with rank $M = N_f$, $SO(N_c)$ is broken to $SO(2)$

- Coulomb branch $u = \det M$
- two singularities $V \approx - \left(\frac{\lambda^2}{16\pi^2} \right)^4 m^4$
- $u = \det M = 0$

- dyons: $q_i^\pm$ $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$

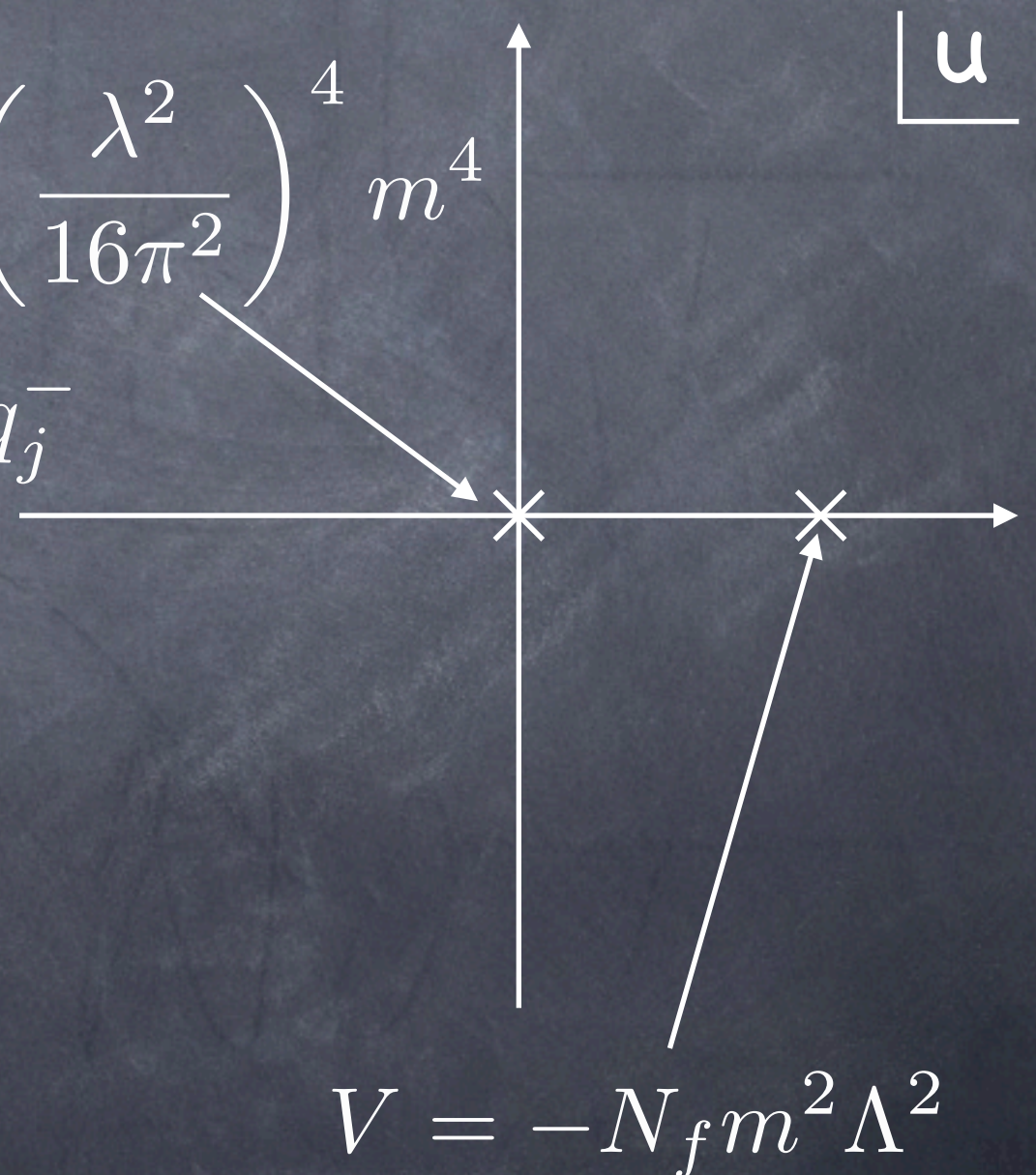
- $u = \det M = \Lambda^{2N_f}$

- monopoles:

$$W = (u - \Lambda^{2N_f}) E^+ E^-$$

$$|E^\pm| = (m\Lambda)^{1/2}$$

- both monopoles and meson condense!



complete list of “t-confining” theories

theory	SUSY	With AMSB
(A) G	$\mathbb{Z}_{2h} \rightarrow \mathbb{Z}_2$	no symmetries
(B) $\text{Spin}(k) + (k-4)V^i$	$\mathbb{Z}_{2k-8} \times SU(k-4) \times U(1)_R$ $\rightarrow \mathbb{Z}_{k-4} \times SU(k-4) \times U(1)_R$	$\mathbb{Z}_{2k-8} \times SU(k-4)$ $\rightarrow \mathbb{Z}_{k-4} \times SO(k-4)$
(C) $\text{Spin}(k) + (k-3)V^i$	$\mathbb{Z}_{2k-6} \times SU(k-3) \times U(1)_R$ no breaking	$\mathbb{Z}_{2k-6} \times SU(k-3)$ $\rightarrow \mathbb{Z}_{k-3} \times SO(k-3)$
(D) $SU(6) + A^{ijk}$	$\mathbb{Z}_6 \times U(1)_R \rightarrow \mathbb{Z}_2 \times U(1)_R$	$\mathbb{Z}_6 \rightarrow 1$
(E) $Sp(k) + A^{ij}$	$\mathbb{Z}_{2k-2} \times U(1)_R \rightarrow \mathbb{Z}_{k-1} \times U(1)_R$	$\mathbb{Z}_{2k-2} \rightarrow \mathbb{Z}_2$
(F) $SU(2k) + A^{ij} + \tilde{A}_{ij}$	$\mathbb{Z}_{4k-4} \times U(1)_V \times U(1)_R$ $\rightarrow \mathbb{Z}_{2k-2} \times U(1)_V \times U(1)_R$	$\mathbb{Z}_{4k-4} \times U(1)_V$ $\rightarrow \mathbb{Z}_2 \times U(1)_V$
(G) $\text{Spin}(12) + 2S$	$\mathbb{Z}_{16} \times SU(2) \times U(1)_R$ $\rightarrow \mathbb{Z}_8 \times SU(2) \times U(1)_R$	$\mathbb{Z}_{16} \times SU(2) \rightarrow SO(2)$

one-form symmetry
useful to define
criteria for confinement

Table 2. Summary of symmetry breaking patterns in SUSY and non-SUSY vacua

Riku Ishikawa, HM, Shota Saito, to appear soon

Finite T , μ

- Bai, de Lima, Stolarski, 2506.18964
- Arthur Platschorre, to appear
- Still learning how to do it correctly

Near-SUSY

Chiral gauge theories

with Csaba Csáki, Dan Kondo, Andrew Goh, Jacob Leedom,
Zhiyao Lu, Pablo Quilez, Shota Saito, Gurpreet Singh, Bethany
Suter, Cameron Sylber, Ofri Telem, Jason Wong, Bea Noether

Nature is chiral

- Standard model is a chiral gauge theory
- Nature taught us
 - abelian theory (QED)
 - non-abelian confining theory (QCD)
 - non-abelian broken theory (EW)
- There can be more
 - dark matter, axion, dark energy, inflation
 - likely that some of them are chiral
- need to know what chiral gauge theories do!

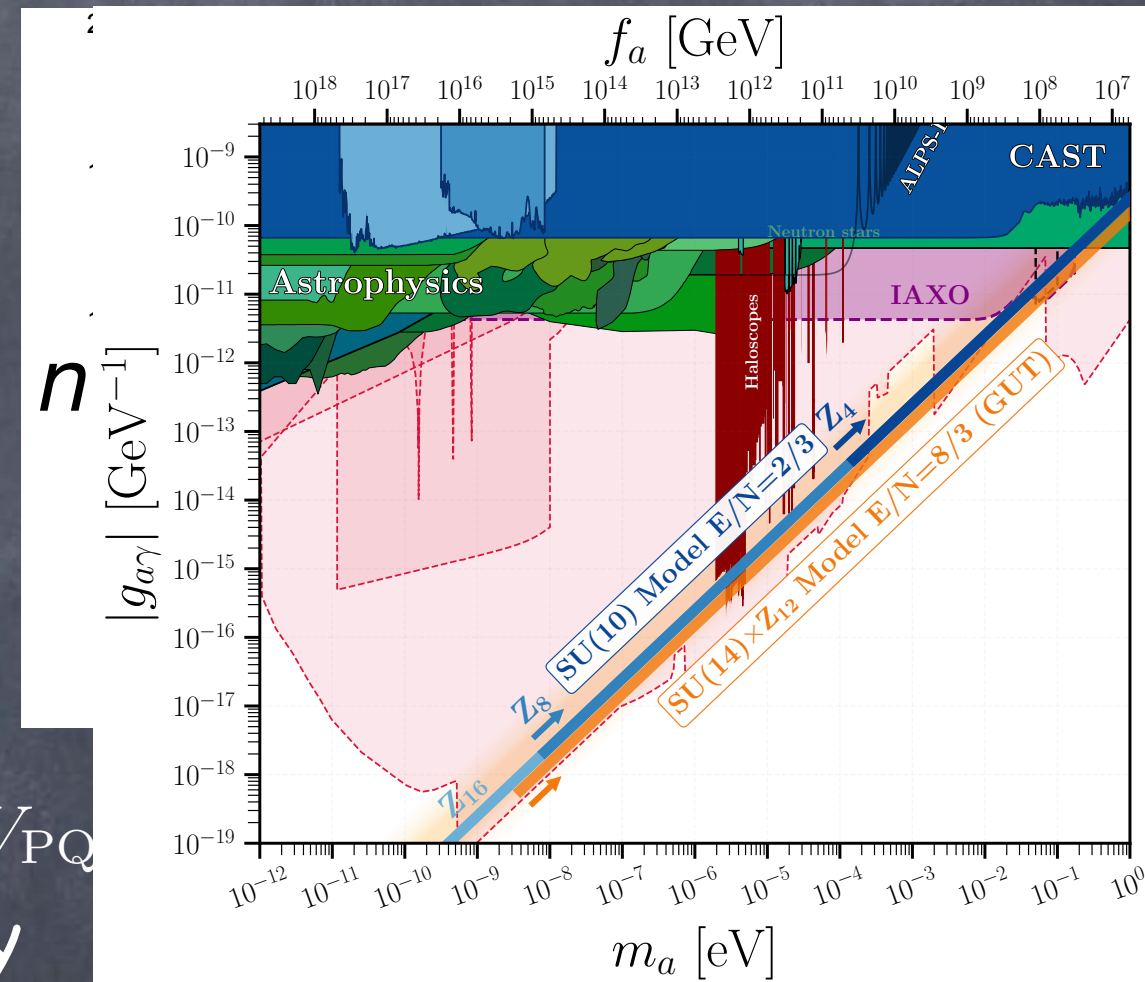
UV theory		SUSY + AMSB		Tumbling	
N_F	G_{global}	H_{global}	$m_f = 0$	H_{global}	$m_f = 0$
SU(N_C) + A($\square$) + ($N_C - 4 + N_F$) $\tilde{F}(\bar{\square})$ + $N_F F(\square)$, $N_C = \text{Odd}$ [15, 20]					
0	SU($N_C - 4$) $\times$ U(1)	Sp($N_C - 5$) $\times$ U(1)	$\square + 1$	SU($N_C - 4$) $\times$ U(1)	$\square\square$
1	SU($N_C - 3$) $\times$ U(1) ²	Sp($N_C - 3$) $\times$ U(1)	$\square$	SU($N_C - 4$) $\times$ U(1)	$\square\square$
2	SU($N_C - 2$) $\times$ SU(2) $\times$ U(1) ²	Sp($N_C - 3$) $\times$ U(1) ²	$\square$	SU($N_C - 4$) $\times$ SU(2) $\times$ U(1)	($\square\square$, 1)
SU(N_C) + A($\square$) + ($N_C - 4 + N_F$) $\tilde{F}(\bar{\square})$ + $N_F F(\square)$, $N_C = \text{Even}$ [15, 20]					
0	SU($N_C - 4$) $\times$ U(1)	Sp($N_C - 4$)	none	SU($N_C - 4$) $\times$ U(1)	$\square\square$
1	SU($N_C - 3$) $\times$ U(1) ²	Sp($N_C - 4$) $\times$ U(1)	none	SU($N_C - 4$) $\times$ U(1)	$\square\square$
2	SU($N_C - 2$) $\times$ SU(2) $\times$ U(1) ²	Sp($N_C - 4$) $\times$ SU(2) $\times$ U(1)	none	SU($N_C - 4$) $\times$ SU(2) $\times$ U(1)	($\square\square$, 1)
SU(N_C) + S($\square\square$) + ($N_C + 4$) $\tilde{F}(\bar{\square})$ [18]					
0	SU($N_C + 4$) $\times$ U(1)	SO($N_C + 4$)	none	SU($N_C + 4$) $\times$ U(1)	$\square$
SO(10) + $N_F \psi(\mathbf{16})$ [19]					
1	none	none	none	none	none
2	SU(2)	SU(2)	none	SO(2)	none
3	SU(3)	SO(3)	none	SO(3)	none
$E_6 + N_F \psi(\mathbf{27})$ [This Work]					
1	none	none	none	none	none
2	SU(2)	none	none	SO(2)	none
3	SU(3)	SU(3)	$\square\square\square$	SO(3)	none
		U(1) $\times$ U(1)	$S_{iii}(i = 1, 2, 3)$		
		none	none		

New BSM model building tools

- SUSY + AMSB appears useful to gain insight into strong gauge dynamics
- new possibilities:
 - massless composite fermions composite axion
 - light composite scalars Gherghetta, HM, Quilez
 - new symmetry breaking patterns
- also many (in principle) testable predictions on dynamics
- look forward to future developments on lattice

Axion Quality Problem

- there have been arguments that quantum gravity doesn't respect global symmetries
- string theory: no global symmetries $\phi = f_a e^{ia/f_a}$
- Is $U(1)_{PQ}$ broken at Planck scale? V_{PQ}
- can shift the axion minimum away from $\theta_{QCD}=0$
- $\theta_{QCD} < 10^{-10}$, $f_a > 10^9$ GeV would require $n \geq 9$
- can write models of composite axion using the exact SUSY dynamics to avoid the quality problem (Gherghetta, HM, Noether, Quilez)



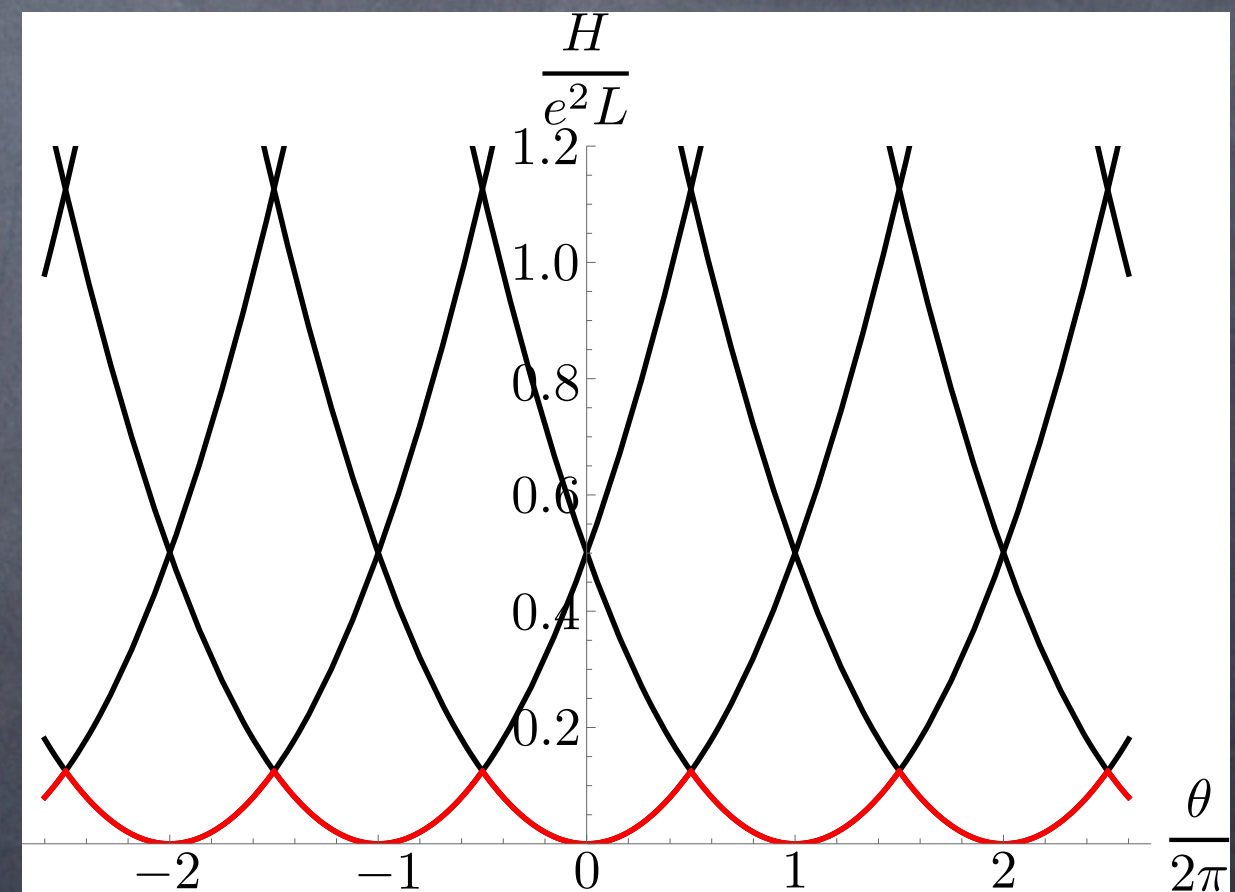
2D Yang–Mills Theories

HM, Bea Noether, Digvijay Varier, Ruoyao Zheng
to appear soon

Compact U(1) in 1+1D

- Many similarities with pure Yang-Mills in 4D
 - theta term
 - confinement
- Can be solved exactly
 - Electric dipole moment
 - partition function with full θ -dependence
 - deconfinement at $\theta=\pi$

$$H = \frac{e^2}{2L} \left(-i \frac{d}{da} - L \frac{\theta}{2\pi} \right)^2 = \frac{e^2 L}{2} \left(n - \frac{\theta}{2\pi} \right)^2$$



Path Integral

$$z = \frac{1}{2\pi} (-ig^2 LT) \frac{\theta}{2\pi}$$

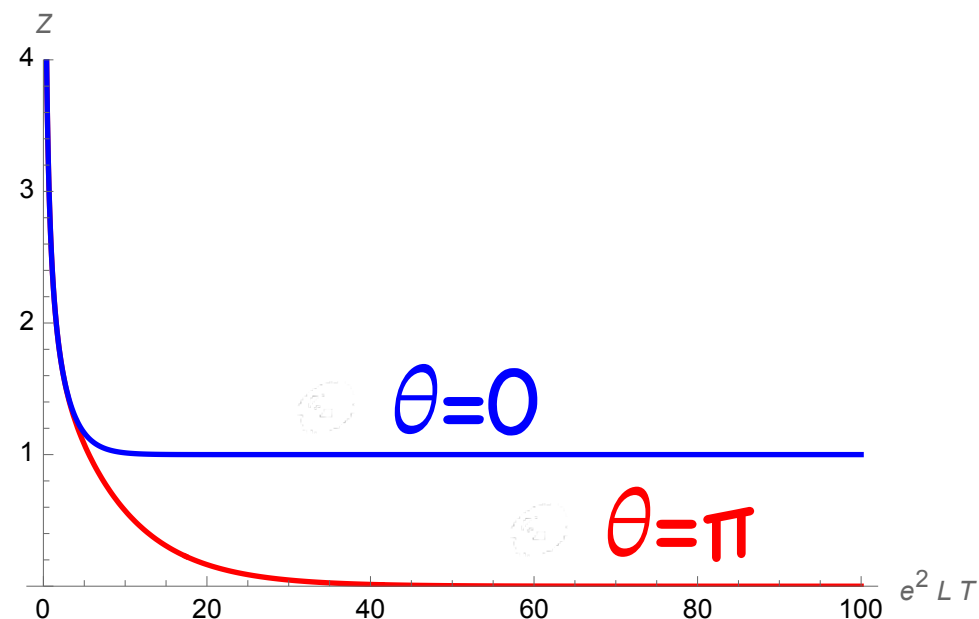
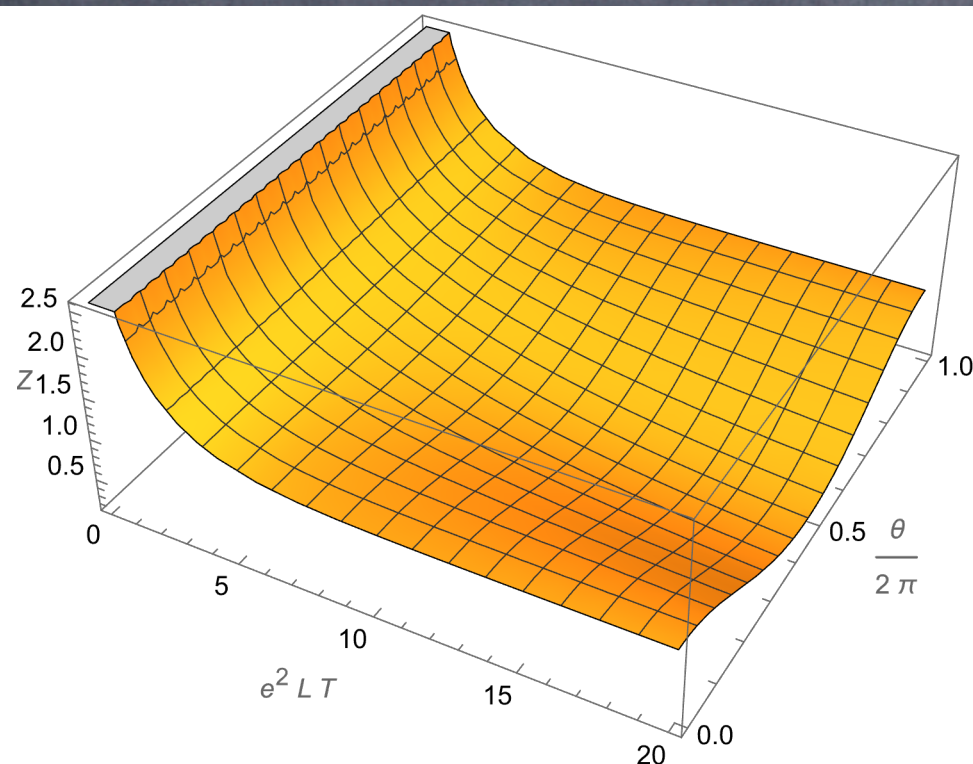
$$\tau = \frac{1}{2\pi} (ig^2 LT)$$

$$Z = \sum_n e^{-\beta \frac{e^2 L}{2} \left(n - \frac{\theta}{2\pi}\right)^2} = e^{-\frac{1}{2} g^2 LT \left(\frac{\theta}{2\pi}\right)^2} \vartheta(z; \tau)$$

• modular transformation of Jacobi elliptic theta functions (Poisson summation)

$$Z = \left(\frac{2\pi}{e^2 \beta L} \right)^{1/2} \sum_n e^{-\frac{1}{2e^2 L \beta} (2\pi n)^2 + i n \theta}$$

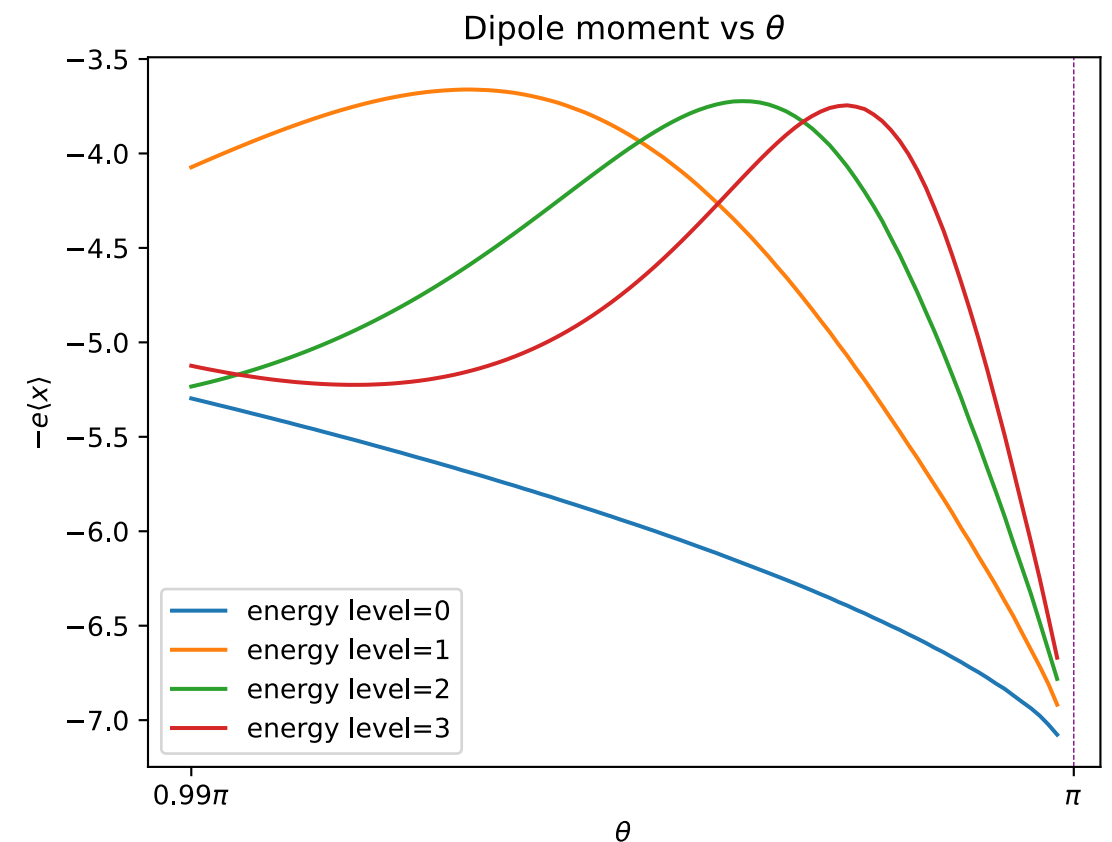
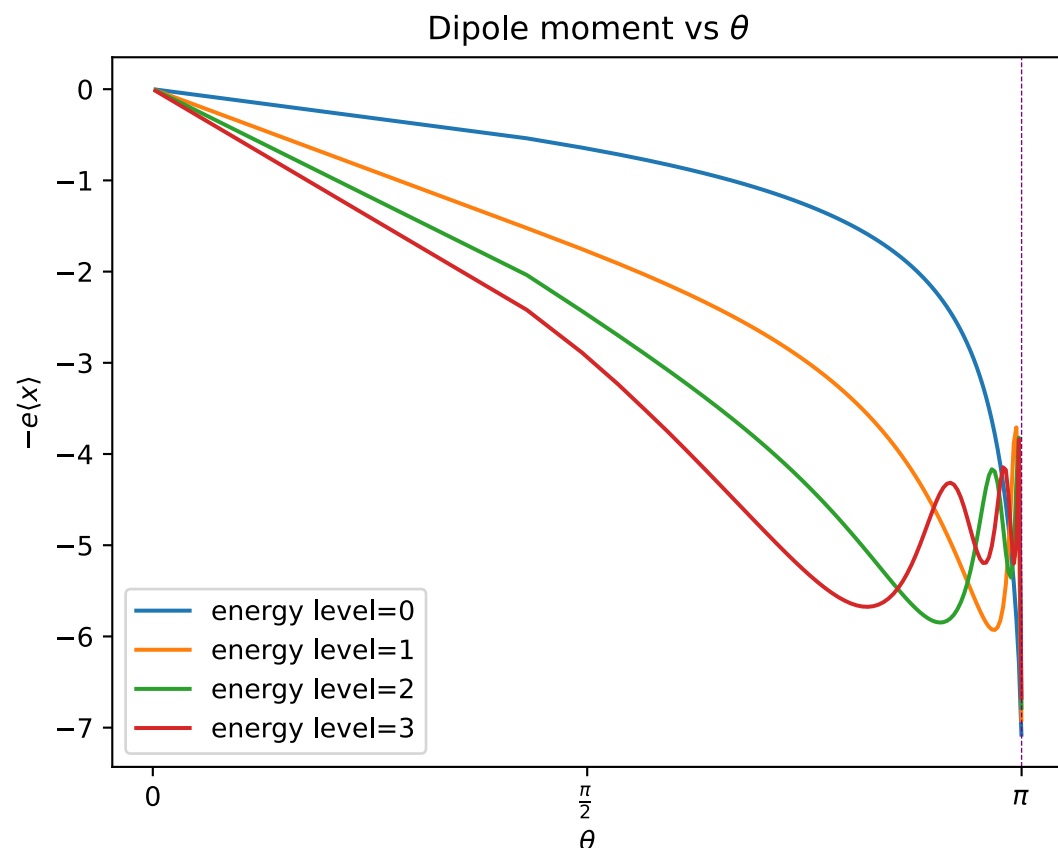
• equivalent to summation over instantons



Electric Dipole Moments

$$H = \frac{1}{2\mu} p^2 + \frac{1}{2} e^2 |x| + e^2 \frac{\theta}{2\pi} x$$

- θ violates P and C but not CP
- EDM is induced by θ
- deconfinement at $\theta=\pi$, EDM diverges



Pure YM in 1+1D

- Exact solutions go back to Migdal (1975):
 - wave functions are characters
 - energy eigenvalues are 2nd order Casimirs
- Consider $S^1 = [0, L]$
- wave functions are characters for irreps

$$H = \frac{1}{2} g^2 L T^a T^a$$

$$E_R = \frac{1}{2} g^2 L C_2(R)$$

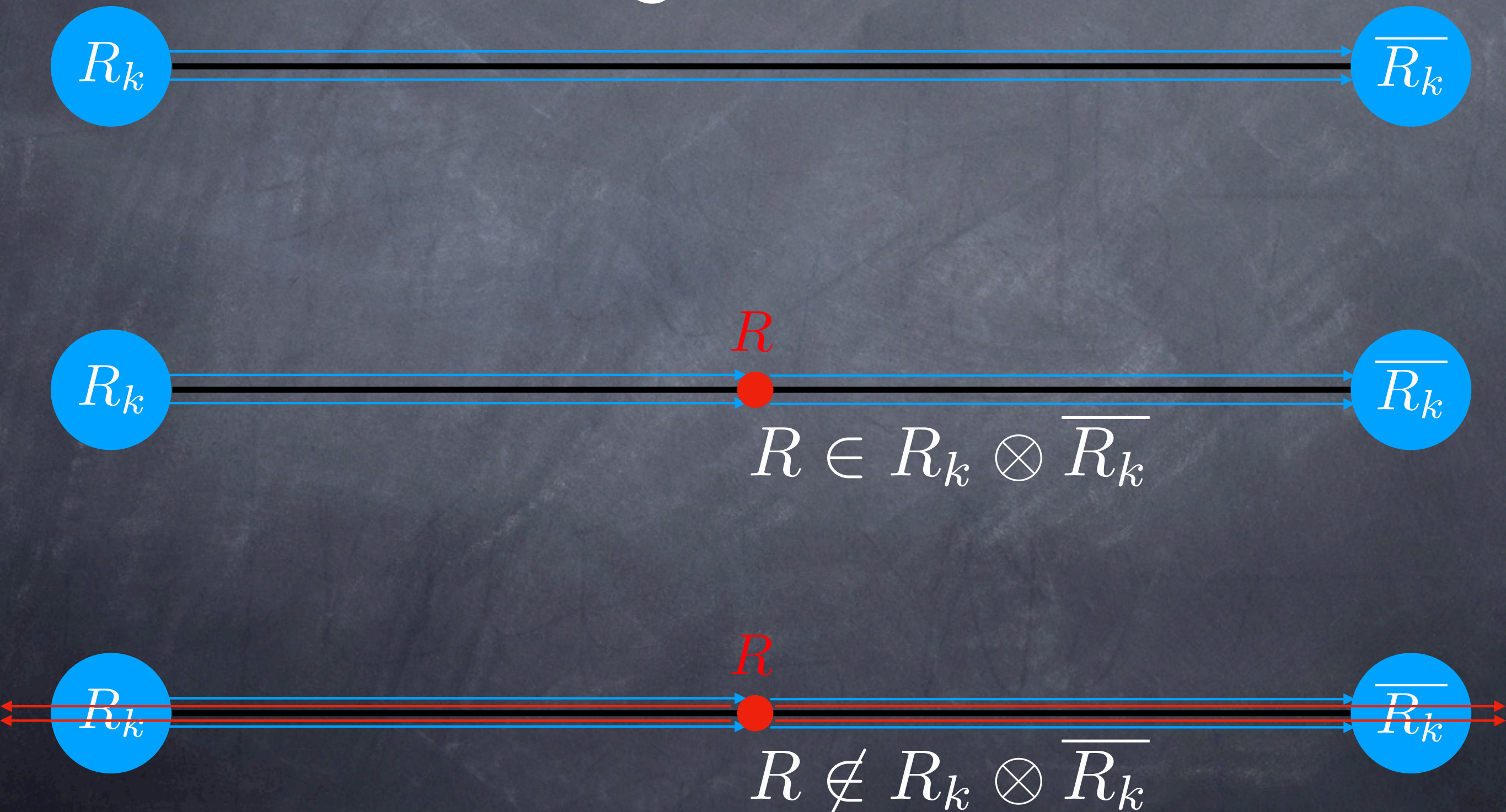
- Ground state is singlet
- all non-trivial representations confined

Pure YM in 1+1D

- $SU(N)/Z_N$: N different definitions due to Z_N -valued Stiefel-Whitney classes $w_2(SU(N)/Z_N)$
Ahanory, Seiberg, Tachikawa (2015)
- canonical: subspace of Hilbert space with $k \bmod N$ boxes of Young tableaux (N -ality)
- cast to path integral using Poisson summation
- corresponding path integrals are weighted by Stiefel-Whitney class w_2 :
 $e^{2\pi i k w_2 / N}$ (odd N) or $e^{2\pi i (k+m) w_2 / N}$ (even $N=2m$)
- without weighting by w_2 , adjoint confined for odd N , not for even N

Sloppily, $\theta=\pi$ confines but not other θ

Interpretation of the ground state



Conclusions

- it is good sometimes to be crazy
- exact results near SUSY
 - near-SUSY to non-SUSY cross over (?)
 - many non-perturbative effects reproduced
- Mysteries about chiral gauge theories
 - we don't know what they do
 - results do not agree with past conjectures
- nature may use them! e.g. dark matter, axion
- curious behavior of 2D YM
- Looking forward to the rest of the meeting!