Quantum Crossroads @ Kavli-IPMU, 2026/08/06

Organizing Non-Equilibrium Photon Dynamics from Symmetry Principles

Based on: JHEP 06 (2026) 051

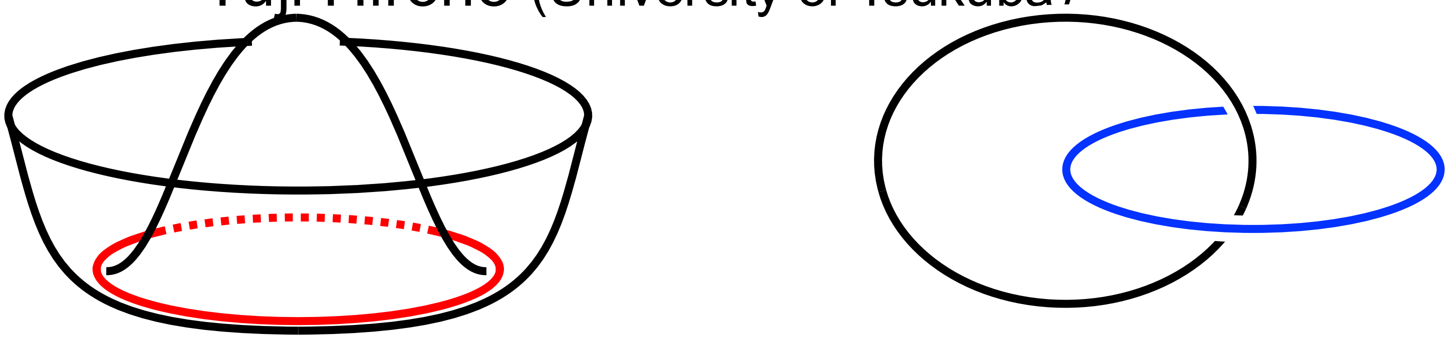
Speaker: Genki Yoshimura

(The University of Osaka, University of Tsukuba)

Collaborators: Yukinao Akamatsu

(National Institute of Technology, Matsue College)

Yuji Hirono (University of Tsukuba)



Effective Field Theory (EFT)

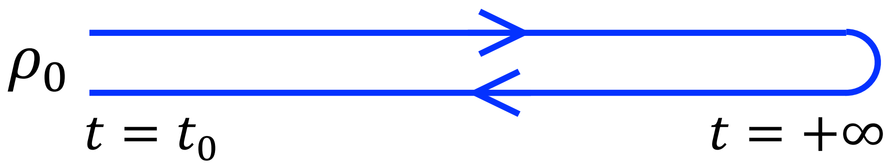
Organizing effective dynamics from symmetry principles
Universal description independent of microscopic details

Example: Chiral Symmetry Breaking $\rightarrow$ Chiral EFT

EFT on Schwinger–Keldysh contour
organizes non-equilibrium dynamics from symmetries

Example: Global U(1) $\rightarrow$ Charge Diffusion

Crossley et al, JHEP 2015.



Higher-Form Symmetry

Gaiotto et al, 1412.5148

A generalization of conventional global symmetry

New symmetry principles for EFTs

Local

charged operators

$$\phi(x) \longmapsto \phi'(x)$$

×

x

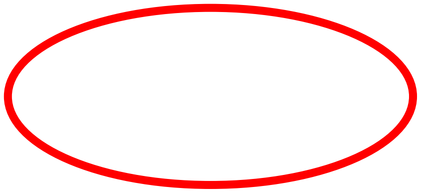
"0-form symmetry"

Nambu–Goldstone modes
e.g., Pions

1-dimensional (line)

charged operators

$$W(C) \longmapsto W'(C)$$



C

1-form symmetry

Nambu–Goldstone modes
e.g., **Photons**

Non-Equilibrium EFT
×
Higher-Form Symmetry

Q. How does higher-form symmetry constrain local-equilibrium dynamics?

- What dissipative effects emerge?
- Does the number of low-energy modes change?
- How are the dispersion relation modified?

Outline

Introduction & Motivation

How to Construct Effective Lagrangian

1. Identify IR-DoF based on SSB
2. Organize Constraints
3. Construct Effective Lagrangian

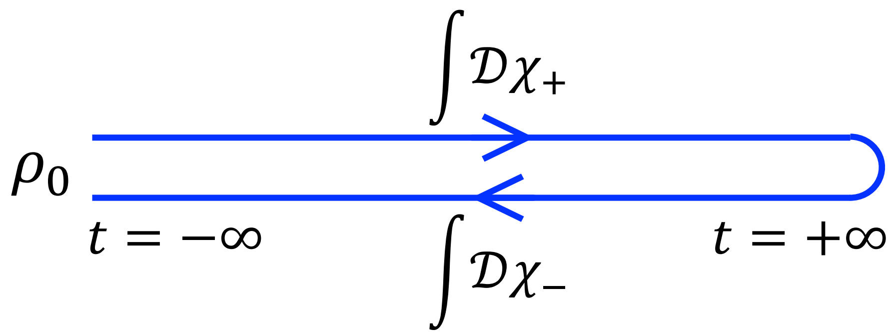
Physical Properties

Doubled DoF in SK-EFT

Review: Glorioso, Liu, 1805.09331

χ: Low-energy effective DoF

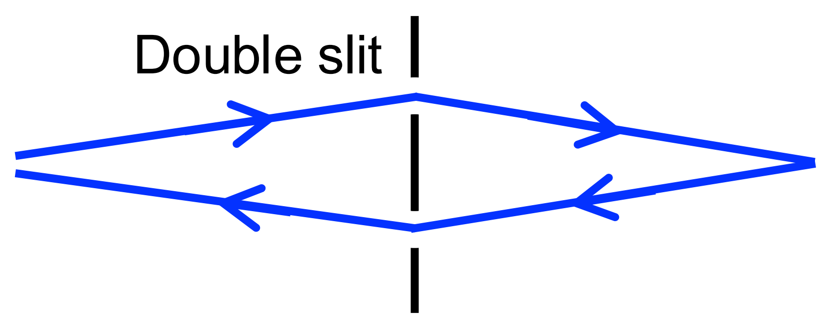
Time evolution of density matrix



$$\langle \mathcal{O} \rangle = \int \mathcal{D}\chi_+ \mathcal{D}\chi_- \, \mathcal{O} \, \exp(\, \mathrm{i} I_{\mathrm{eff}}[\chi_+, \chi_-] \,)$$

Classical: $\chi_r = \dfrac{\chi_+ + \chi_-}{2}$

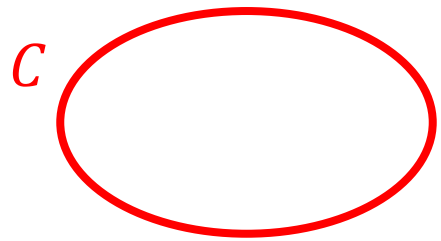
Fluctuation: $\hbar \chi_a = \chi_+ - \chi_-$



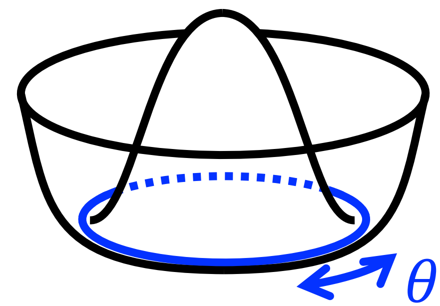
Doubled Symmetry in SK-EFT

$\mathrm{U}(1)$ 1-form SSB in SK: $\quad \mathrm{U}(1)_+ \times \mathrm{U}(1)_- \longrightarrow \mathbf{1}$

Non-locality from 1-form



$\mathrm{U}(1)_\pm$ SSB



Broken direction: $W_\pm(C) = e^{\mathrm{i}\theta}$

Parametrize by Local NG fields

$$W_\pm(C) = \exp\left[\mathrm{i}\oint_C A_{\pm\mu}(x)\,\mathrm{d}x^\mu\right]$$

Settings of EFT

- **DoF**: $\{A_+, A_-\}$

  Classical: $A = \dfrac{A_+ + A_-}{2}$ Fluctuation: $\hbar a = A_+ - A_-$

- **Spacetime**: 3+1 dimension, uniform, isotropic

  ∗ Lorentz symmetry is absent

- **Symmetry**: $\mathrm{U}(1)_+ \times \mathrm{U}(1)_-$ 1−form $\longrightarrow$ 1

  Invariants under symmetry

	Electric	Magnetic
Classical	$\boldsymbol{E} \coloneqq -\boldsymbol{\nabla} A^0 - \dot{\boldsymbol{A}}$	$\boldsymbol{B} \coloneqq \boldsymbol{\nabla} \times \boldsymbol{A}$
Fluctuation	$\boldsymbol{e} \coloneqq -\boldsymbol{\nabla} a^0 - \dot{\boldsymbol{a}}$	$\boldsymbol{b} \coloneqq \boldsymbol{\nabla} \times \boldsymbol{a}$

- **Nonequilibrium constraints**: KMS, Unitarity

Dynamical KMS Symmetry

Sieberer et al, PRB 2015

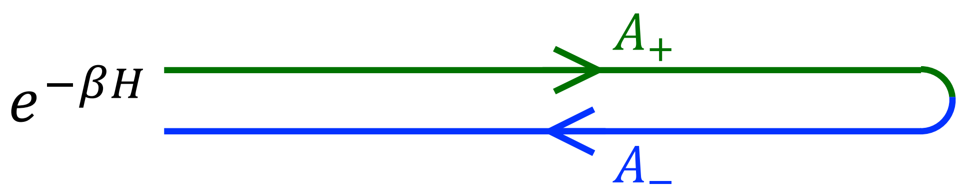
- Emergent symmetry characterizing local equilibrium
- Combination of KMS condition and time-reversal symmetry

KMS condition: $\langle X(t_1)\, Y(t_2)\rangle_\beta = \langle Y(t_2 - \mathrm{i}\beta/2)\, X(t_1 + \mathrm{i}\beta/2)\rangle_\beta$

time-reversal sym. Θ: $\langle YX\rangle_\beta = \left\langle \Theta X^\dagger Y^\dagger \Theta^{-1}\right\rangle_\beta$

dynamical KMS sym: $\langle X(t_1)\, Y(t_2)\rangle_\beta = \left\langle \tilde{X}(t_1)\, \tilde{Y}(t_2)\right\rangle_\beta$

generating functional: $e^{W[J_+,J_-]} \propto \left\langle U^\dagger(+\infty,-\infty;J_-)U(+\infty,-\infty;J_+)\right\rangle_\beta$



Dynamical KMS Transformation

Sieberer et al, PRB 2015

$$\tilde{A}_\pm(t_1) \coloneqq \eta_A A_\pm(-t_1 \pm \mathrm{i}\hbar\beta/2)$$

- $\eta_A = \pm 1$ from time-reversal Θ
- Imaginary time shift from KMS

In classical lim. $\hbar \longrightarrow 0$

$$\widetilde{\boldsymbol{E}} = \pm \boldsymbol{E} \qquad \tilde{\boldsymbol{e}} = \pm\left(\boldsymbol{e} + \mathrm{i}\beta\dot{\boldsymbol{E}}\right)$$
$$\widetilde{\boldsymbol{B}} = \pm \boldsymbol{B} \qquad \widetilde{\boldsymbol{b}} = \pm\left(\boldsymbol{b} + \mathrm{i}\beta\dot{\boldsymbol{B}}\right)$$

Construction of Effective Lagrangian

✓ 1-form sym. ✓ Uniform & isotropic ✓ Local equilibrium

Allowed terms (higher-derivatives omitted)

$\boldsymbol{e}\cdot\boldsymbol{E}$ $\boldsymbol{b}\cdot\boldsymbol{B}$ $\boldsymbol{e}\cdot\dot{\boldsymbol{B}}-\boldsymbol{b}\cdot\dot{\boldsymbol{E}}$ $\boldsymbol{e}^2+\mathrm{i}\beta\boldsymbol{e}\cdot\dot{\boldsymbol{E}}$ $\boldsymbol{b}^2+\mathrm{i}\beta\boldsymbol{b}\cdot\dot{\boldsymbol{B}}$

Unitarity conditions

- Involve at least one a
- a^{2n+1}: Real, a^{2n}: Pure imaginary
- $\mathrm{Im}(\mathcal{L}_{\mathrm{eff}}[a,A])\geq 0$

Effective Lagrangian GY, Akamatsu, Hirono, 2601.00605

$$\mathcal{L}_{\mathrm{eff}}[A,a]=\epsilon\boldsymbol{e}\cdot\boldsymbol{E}-\mu^{-1}\boldsymbol{b}\cdot\boldsymbol{B}+\gamma\left(\boldsymbol{e}\cdot\dot{\boldsymbol{B}}-\boldsymbol{b}\cdot\dot{\boldsymbol{E}}\right)$$
$$+\mathrm{i}\frac{\tau_E}{\beta}\boldsymbol{e}\cdot(\boldsymbol{e}+\mathrm{i}\beta\dot{\boldsymbol{E}})+\mathrm{i}\frac{\tau_B}{\beta}\boldsymbol{b}\cdot(\boldsymbol{b}+\mathrm{i}\beta\dot{\boldsymbol{B}})$$

Outline

Introduction & Motivation

How to Construct Effective Lagrangian

Physical Properties

1. Overview of Effective Lagrangian
2. Fluctuation-Dissipation Relation
3. Dispersion Relation
4. Entropy Production

Overview of Effective Lagrangian

$$\mathcal{L}_{\text{eff}}[A,a] = \epsilon \boldsymbol{e}\cdot\boldsymbol{E} - \mu^{-1}\boldsymbol{b}\cdot\boldsymbol{B} + \gamma\left(\boldsymbol{e}\cdot\dot{\boldsymbol{B}} - \boldsymbol{b}\cdot\dot{\boldsymbol{E}}\right) + \mathrm{i}\frac{\tau_E}{\beta}\boldsymbol{e}\cdot\left(\boldsymbol{e}+\mathrm{i}\beta\dot{\boldsymbol{E}}\right) + \mathrm{i}\frac{\tau_B}{\beta}\boldsymbol{b}\cdot\left(\boldsymbol{b}+\mathrm{i}\beta\dot{\boldsymbol{B}}\right)$$

Maxwell Part

ϵ : Electric permittivity μ : Magnetic permeability

Magneto-Electric Cross Coupling (Parity violating)

γ : Coefficient

Fluctuation & Dissipation Part

$\tau_{E,B} \geq 0$: Relaxation times

Fluctuation–Dissipation Relation (FDR)

Langevin EoM. $\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{D} = 0 \\ \dot{\boldsymbol{D}} - \boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{0} \end{cases}$ Bianchi id. $\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \\ \dot{\boldsymbol{B}} + \boldsymbol{\nabla} \times \boldsymbol{E} = \boldsymbol{0} \end{cases}$

Electric flux density $\boldsymbol{D} \coloneqq \epsilon \boldsymbol{E} - \tau_E \dot{\boldsymbol{E}} + \gamma \dot{\boldsymbol{B}} + \boldsymbol{\xi}_E$

Magnetic field $\boldsymbol{H} \coloneqq \mu^{-1} \boldsymbol{B} + \tau_B \dot{\boldsymbol{B}} + \gamma \dot{\boldsymbol{E}} + \boldsymbol{\xi}_B$

White Gaussian noise $\left\langle \xi^i_{E,B}(x) \xi^j_{E,B}(y) \right\rangle = \dfrac{2\tau_{E,B}}{\beta} \delta^{ij} \delta^{(4)}(x-y)$

$\boldsymbol{D}$ lags $\boldsymbol{E}$ $\Longrightarrow$ Dielectric loss

$\boldsymbol{B}$ lags $\boldsymbol{H}$ $\Longrightarrow$ Magnetic loss

Relaxation time $\propto$ Noise strength^2 —— FDR

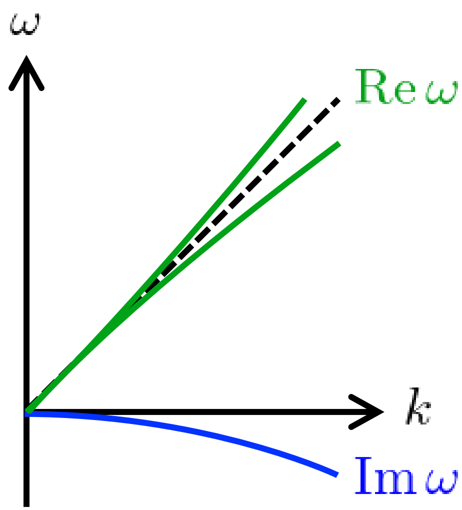
Dispersion Relation

Helicity eigenmode

$$\boldsymbol{E}(t,\boldsymbol{x}) = E_0 \begin{pmatrix} 1 \\ \pm \mathrm{i} \\ 0 \end{pmatrix} \exp(-\mathrm{i}\omega t + \mathrm{i}kz), \quad \boldsymbol{B}(t,\boldsymbol{x}) = \frac{E_0}{v} \begin{pmatrix} \mp \mathrm{i} \\ 1 \\ 0 \end{pmatrix} \exp(-\mathrm{i}\omega t + \mathrm{i}kz)$$

v : Speed of light in medium

$$\omega(k) = \pm vk + \left(\pm\gamma v - \mathrm{i}\frac{\tau_E + \tau_B}{2} \right) v^2 k^2 + \mathcal{O}(k^3)$$



Linear dispersion: NG Photon in medium

Entropy Current

$$\begin{cases} s^0 = \frac{1}{2}\beta(\epsilon \boldsymbol{E}^2 - \mu^{-1}\boldsymbol{B}^2) - \beta \boldsymbol{E}\cdot\overline{\boldsymbol{D}} - \beta A^0 \nabla\cdot\overline{\boldsymbol{D}} \approx -\beta \times \text{(Energy density)} \\ \boldsymbol{s} = -\beta \boldsymbol{E}\times\overline{\boldsymbol{H}} - \beta A^0\left(\dot{\overline{\boldsymbol{D}}} - \nabla\times\overline{\boldsymbol{H}}\right) \approx -\beta \times \text{(Energy flux)} \end{cases}$$

Entropy production $\langle \partial_\mu s^\mu \rangle \propto$ Energy dissipation rate

$$\partial_\mu s^\mu[A] = \beta \dot{A}^0 \nabla\cdot\overline{\boldsymbol{D}} + \beta \dot{\boldsymbol{A}}\cdot\left(\dot{\overline{\boldsymbol{D}}} - \nabla\times\overline{\boldsymbol{H}}\right) + \beta\left(\tau_E \dot{\boldsymbol{E}}^2 + \tau_B \dot{\boldsymbol{B}}^2\right)$$

$$\overline{\boldsymbol{D}} \coloneqq \epsilon(\boldsymbol{E} - \tau_E \dot{\boldsymbol{E}} + v^2\gamma\dot{\boldsymbol{B}})$$
$$\overline{\boldsymbol{H}} \coloneqq \mu^{-1}(\boldsymbol{B} + \tau_B\dot{\boldsymbol{B}} + \gamma\dot{\boldsymbol{E}})$$

Mean parts

Gauge dependent terms are suppressed up to thermal noise

Unitarity condition $\mathrm{Im}(\mathcal{L}_{\mathrm{eff}}) \geq 0$

$\Downarrow$

Non-decreasing $\langle \partial_\mu s^\mu \rangle \geq 0$

Conclusion

Nonequilibrium EFT for U(1) 1-form Symmetry

SSB:

$$\mathcal{L}_{\text{eff}}[A,a] = \epsilon \boldsymbol{e}\cdot\boldsymbol{E} - \mu^{-1}\boldsymbol{b}\cdot\boldsymbol{B} + \gamma\left(\boldsymbol{e}\cdot\dot{\boldsymbol{B}} - \boldsymbol{b}\cdot\dot{\boldsymbol{E}}\right) + \mathrm{i}\frac{\tau_E}{\beta}\boldsymbol{e}\cdot(\boldsymbol{e}+\mathrm{i}\beta\dot{\boldsymbol{E}}) + \mathrm{i}\frac{\tau_B}{\beta}\boldsymbol{b}\cdot(\boldsymbol{b}+\mathrm{i}\beta\dot{\boldsymbol{B}})$$

Unbroken:

$$\mathcal{L}_{\text{eff}}[a,A] = \boldsymbol{e}\cdot\left(\epsilon\boldsymbol{E}+\gamma\dot{\boldsymbol{B}}\right) - \boldsymbol{b}\cdot\left(\tau_B\dot{\boldsymbol{B}}+\gamma\dot{\boldsymbol{E}}+\kappa\boldsymbol{E}\times\dot{\boldsymbol{B}}\right) + \frac{\mathrm{i}}{\beta}\tau_B\boldsymbol{b}^2$$

- ✓ Symmetry-based approach for higher-form
- ✓ Unitarity and KMS symmetry are implemented
- ✓ Appliable to both of broken/unbroken phases

Applicability of EFTs is broadened

Thank You!

Unbroken Phase

SSB: $\mathrm{U}(1)_{+}^{[p]} \times \mathrm{U}(1)_{-}^{[p]} \longrightarrow 1$

Unbroken: $\mathrm{U}(1)_{+}^{[p]} \times \mathrm{U}(1)_{-}^{[p]} \longrightarrow \mathrm{U}(1)_{r}^{[p]}$

Dimension of G/H decreases

but r-a fields should make pair

—— Gauge redundancy appears

Unbroken Phase (0-form)

Gauging $\mathrm{U}(1)_r^{[0]}$: $I_{\mathrm{eff}}[\mathrm{d}\pi_r, \mathrm{d}\pi_a] \longmapsto I_{\mathrm{eff}}[A + \mathrm{d}\pi_r, \mathrm{d}\pi_a]$

Gauge transformation: $\begin{cases} A \longmapsto A + \mathrm{d}\lambda(t, \boldsymbol{x}) \\ \pi_r \longmapsto \pi_r - \lambda(t, \boldsymbol{x}) \end{cases}$

$A_0 = \mu$ has physical meaning in hydro-EFT

—— Gauge transformation preserving A_0

$$\pi_r \longmapsto \pi_r - \lambda'(\boldsymbol{x})$$

Building block for EFT: $\dot{\pi}_r$, $\boldsymbol{\nabla}\dot{\pi}_r$, $\dot{\pi}_a$, $\boldsymbol{\nabla}\pi_a$

Unbroken Phase (1-form)

Gauging $\mathrm{U}(1)_r^{[1]}$: $I_{\text{eff}}[\mathrm{d}A, \mathrm{d}a] \longmapsto I_{\text{eff}}[G + \mathrm{d}A, \mathrm{d}a]$

Gauge transformation: $\begin{cases} G \longmapsto G + \mathrm{d}\Lambda(t, \boldsymbol{x}) \\ A \longmapsto A - \Lambda(t, \boldsymbol{x}) \end{cases}$

Correspondent of chemical potential: G_{0i}

—— Gauge transformation preserving G_{0i}

$$\boldsymbol{E} \longmapsto \boldsymbol{E}$$
$$\boldsymbol{B} \longmapsto \boldsymbol{B} - \boldsymbol{\Lambda}_B(\boldsymbol{x})$$

Building block for EFT: $\boldsymbol{E}$, $\dot{\boldsymbol{B}}$, $\boldsymbol{e}$, $\boldsymbol{b}$

$$\mathcal{L}_{\text{eff}}[a, A] = \boldsymbol{e} \cdot \left(\epsilon \boldsymbol{E} + \gamma \dot{\boldsymbol{B}}\right) - \boldsymbol{b} \cdot \left(\tau_B \dot{\boldsymbol{B}} + \gamma \dot{\boldsymbol{E}} + \kappa \boldsymbol{E} \times \dot{\boldsymbol{B}}\right) + \frac{\mathrm{i}}{\beta} \tau_B \boldsymbol{b}^2$$

1-Form Symmetries in Electromagnetism

Generalized conservation laws

$\mathrm{U}(1)_\mathrm{E}$ $\partial_\nu F^{\mu\nu} = j^\mu$ $\longleftarrow$ Charged matter

$\mathrm{U}(1)_\mathrm{M}$ $\partial_\nu \widetilde{F}^{\mu\nu} = 0$

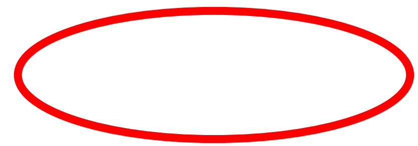
Systems	Plasma	Insulator $(j^\mu = 0)$
$\mathrm{U}(1)_\mathrm{E}$	explicitly broken	SSB
$\mathrm{U}(1)_\mathrm{M}$	unbroken	SSB
Nonequilibrium EFT	S. Vardhan, et al. 2408.12868	GY et al. (2026) 2601.00605
Phenomenology	“Magnetohydrodynamics”	Electromagnetism in media
Low-energy modes	Diffusion of magnetic flux $\omega = -\mathrm{i}Dk^2$	Electromagnetic waves $\omega = vk$

Identification of IR-DoF

NG modes: Oscillation of **broken direction** $W_\pm(C_1)$

Parametrize the oscillation.

Local NG fields $A_\pm(x)$ $\xrightarrow[\oint_{C_1} \mathrm{d}x^\mu]{\text{Integration}}$ $W_\pm(C_1)$



	c.f., 0-form	**1-form**
Parametrization	$\Omega(x) = \exp[\mathrm{i}\pi(x)]$	$W_\pm(C_1) = \exp\left[\mathrm{i}\oint_{C_1} A_{\pm\mu}(x)\,\mathrm{d}x^\mu\right]$
Symmetry Transformation	$\pi_\pm \longmapsto \pi_\pm + h_\pm$ where $\mathrm{d}h_\pm = 0$	$A_\pm \longmapsto A_\pm + \Lambda_\pm$ where $\mathrm{d}\Lambda_\pm = 0$

1-Form Symmetries in Electromagnetism

0–form conservation law: $\partial_\mu J^\mu = 0$

1–form conservation law: $\partial_\nu J^{\mu\nu} = 0$

e.g., Maxwell theory in vacuum $\partial_\nu F^{\mu\nu} = 0, \quad \partial_\nu \widetilde{F}^{\mu\nu} = 0$

$$\Longrightarrow \ \omega = \pm ck$$

Photon: Nambu–Goldstone (NG) modes from $U(1)_E$ and $U(1)_M$

D. Gaiotto, et al. (2015).

Symmetry	Transformation	SSB	Conservation
$U(1)_E$	$A \longmapsto A + \Lambda, \ \mathrm{d}\Lambda = 0$	$\langle W \rangle \neq 0$	$\partial_\nu F^{\mu\nu} = 0$
$U(1)_M$	$\widetilde{A} \longmapsto \widetilde{A} + \widetilde{\Lambda}, \ \mathrm{d}\widetilde{\Lambda} = 0$	$\langle T \rangle \neq 0$	$\partial_\nu \widetilde{F}^{\mu\nu} = 0$

cf) U(1) 0-form: $\pi \longmapsto \pi + \lambda, \ \mathrm{d}\lambda = 0$

Unitarity Conditions

Review: Glorioso, Liu, 1805.09331

UV theory should be unitary

Integrate out high-energy DoF

SK–EFT is not unitary
but imprints of unitarity still exist

- $\mathcal{L}_{\text{eff}}[a=0,A]=0$ Involve at least one a
- $\mathcal{L}_{\text{eff}}[a,A]=-\mathcal{L}^{*}_{\text{eff}}[-a,A]$ a^{2n+1}: Real, a^{2n}: Pure Imag
- $\mathrm{Im}(\mathcal{L}_{\text{eff}}[a,A])\geq 0$ Positive semidefinite

Higher Extension of Maurer-Cartan Form

Maurer-Cartan (MC) form describes change of broken direction and serves as a **building block** in the coset construction.

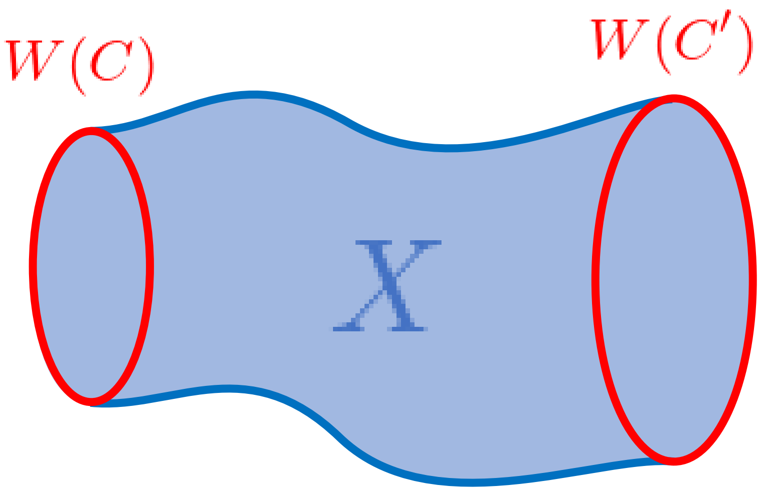
$\Omega(x)$: broken direction for 0-form symmetry (coset variable)

$$\Omega(x') = \exp\left(\mathrm{i}\int_x^{x'} \omega\right)\Omega(x)$$

Maurer-Cartan 1-form

$$\omega = \Omega^\dagger \mathrm{d}\Omega$$

$W(C)$: broken direction for 1-form symmetry (Wilson loop)



$$W(C') = \exp\left(\mathrm{i}\int_X F\right)W(C)$$

$$\partial X = C \cup (-C')$$

Maurer-Cartan 2-form

$$F = \mathrm{d}A$$

Dynamical KMS Transformation

$$A \longmapsto \Theta A \qquad \partial_t \longmapsto -\partial_t$$

$$a \longmapsto \Theta\left(a + \mathrm{i}\beta \dot{A}\right) \qquad \text{(e.g., } \Theta = \text{time reversal)}$$

$$\begin{aligned}
&\mathrm{KMS}[\boldsymbol{e} \cdot \boldsymbol{E}] - \boldsymbol{e} \cdot \boldsymbol{E} \\
&= \left(\boldsymbol{e} + \mathrm{i}\beta \dot{\boldsymbol{E}}\right) \cdot \boldsymbol{E} - \boldsymbol{e} \cdot \boldsymbol{E} \\
&= \partial_t \left(\frac{\mathrm{i}\beta}{2} \boldsymbol{E}^2\right)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{KMS}\left[\boldsymbol{e} \cdot \left(\boldsymbol{e} + \mathrm{i}\beta \dot{\boldsymbol{E}}\right)\right] - \boldsymbol{e} \cdot \left(\boldsymbol{e} + \mathrm{i}\beta \dot{\boldsymbol{E}}\right) \\
&= \left(\boldsymbol{e} + \mathrm{i}\beta \dot{\boldsymbol{E}}\right) \cdot \boldsymbol{e} - \boldsymbol{e} \cdot \left(\boldsymbol{e} + \mathrm{i}\beta \dot{\boldsymbol{E}}\right) \\
&= 0
\end{aligned}$$

Maxwell Equation in Nonequilibrium

To clarify thermal noise, introduce auxiliary field $\boldsymbol{\xi}_{E,B}$

$$\int \mathcal{D}a\, \mathcal{D}A\, \mathcal{O} \exp\left\{ \mathrm{i} \int \left[\boldsymbol{e} \cdot (\cdots)_{\boldsymbol{E},\boldsymbol{B}} - \boldsymbol{b} \cdot (\cdots)_{\boldsymbol{E},\boldsymbol{B}} + \frac{\mathrm{i}}{\beta} \left(\tau_E \boldsymbol{e}^2 + \tau_B \boldsymbol{b}^2 \right) \right] \right\}$$

$$\propto \int \mathcal{D}a\, \mathcal{D}A\, \mathcal{D}\boldsymbol{\xi}_E\, \mathcal{D}\boldsymbol{\xi}_B\, \mathcal{O} \exp\left\{ \mathrm{i} \int \left[\boldsymbol{e} \cdot \boldsymbol{D} - \boldsymbol{b} \cdot \boldsymbol{H} + \frac{\mathrm{i}\beta}{4} \left(\frac{1}{\tau_E} \boldsymbol{\xi}_E^2 + \frac{1}{\tau_B} \boldsymbol{\xi}_B^2 \right) \right] \right\}$$

White Gaussian noise $\left\langle \xi^i_{E,B}(x) \xi^j_{E,B}(y) \right\rangle = \dfrac{2\tau_{E,B}}{\beta} \delta^{ij} \delta^{(4)}(x-y)$

Electric flux density $\boldsymbol{D} := \epsilon \boldsymbol{E} - \tau_E \dot{\boldsymbol{E}} + \gamma \dot{\boldsymbol{B}} + \boldsymbol{\xi}_E$

Magnetic field $\boldsymbol{H} := \mu^{-1} \boldsymbol{B} + \tau_B \dot{\boldsymbol{B}} + \gamma \dot{\boldsymbol{E}} + \boldsymbol{\xi}_B$

Instability from Chiral Magnetic Effect

Is the constriction only with $\boldsymbol{E}, \boldsymbol{B}$ enough?
—— Unique exception: $\boldsymbol{a} \cdot \boldsymbol{B}$

- Parity-odd
- Induces electric current $\boldsymbol{j}_{\mathrm{e}} = \dfrac{\delta I_{\mathrm{eff}}}{\delta \boldsymbol{a}} \propto \boldsymbol{B}$

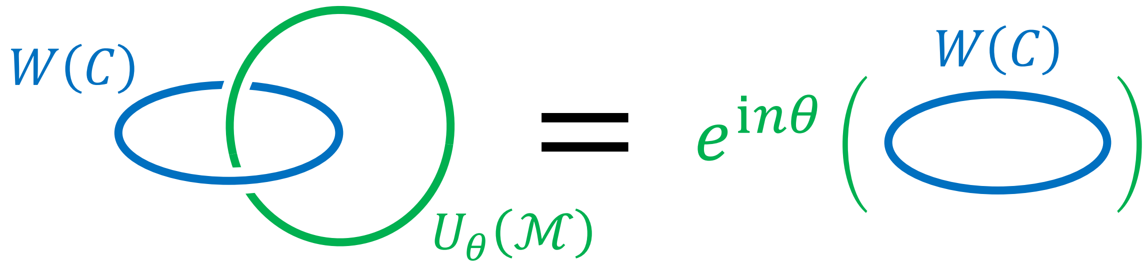
Includes **"Chiral Magnetic Effect"**: $\boldsymbol{j}_{\mathrm{e}} \propto \mu_5 \boldsymbol{B}$

$\boldsymbol{a} \cdot \boldsymbol{B}$ induces **instability**

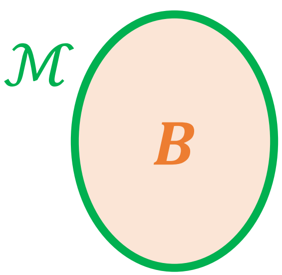
Dispersion (weak-field region) $\omega(k) = \pm \mathrm{i} \sqrt{\dfrac{\chi}{\epsilon}} k^{1/2}$ $\quad \mathrm{Im}\,\omega > 0$ Blow up

Nonlinear effect becomes essential for steady state

Ward-Takahashi Identity



$$\left\langle W(C)\exp\left(-\mathrm{i}\theta\int_{\mathcal{M}}\star j\right)\right\rangle = \int \mathcal{D}a\,\mathcal{D}A\,W(C)\exp\left(-\mathrm{i}\theta\int_{\mathcal{M}}\star j\right)e^{\mathrm{i}I_{\mathrm{eff}}}$$



$$= \int \mathcal{D}a\,\mathcal{D}A\,W(C)\exp\left(-\mathrm{i}\theta\int_{B}\mathrm{d}\star j\right)e^{\mathrm{i}I_{\mathrm{eff}}}$$

Stokes's theorem

$$\star\frac{\delta I_{\mathrm{eff}}}{\delta A} =: \mathrm{d}\star j$$

$$= \int \mathcal{D}a\,\mathcal{D}A\,W(C)\exp\left(-\mathrm{i}\theta\int_{B}\star\frac{\delta I_{\mathrm{eff}}}{\delta A}\right)e^{\mathrm{i}I_{\mathrm{eff}}}$$

$$= \int \mathcal{D}a\,\mathcal{D}A\,W(C)\exp\left(-\theta\int_{B}\star\frac{\delta}{\delta A}\right)e^{\mathrm{i}I_{\mathrm{eff}}}$$

$$= \int \mathcal{D}a\,\mathcal{D}A\,e^{\mathrm{i}I_{\mathrm{eff}}}\exp\left(\theta\int_{B}\star\frac{\delta}{\delta A}\right)W(C)$$

$$= \int \mathcal{D}a\,\mathcal{D}A\,e^{\mathrm{i}I_{\mathrm{eff}}}\exp\left(\theta\int_{B}\star\,\mathrm{i}\delta[C]\right)W(C)$$

$$= \int \mathcal{D}a\,\mathcal{D}A\,e^{\mathrm{i}I_{\mathrm{eff}}}\exp(\mathrm{i}\theta\,\mathrm{Link}[\mathcal{M},C])W(C)$$

$$= \exp(\mathrm{i}n\theta)\langle W(C)\rangle$$