Quantum crossroads
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Time-like Entanglement and Holography

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Grant-in-Aid for Transformative Research Areas A
Extreme Universe

References

- Based on the collaboration with
  Jonathan Harper (Case Western),
  Taishi Kawamoto (Kyoto U.), **Ryota Maeda** (YITP, Kyoto),
  and **Namami Nakamura** (YITP, Kyoto) Traversable wormhole

[1] JHEP 04 (2025), 086, arXiv:2502.03531,
[2] PRD 113(2026) no.12, 126017, arXiv:2512.13800.

- Based on the collaboration with
  Kosei Fujiki, **Michitaka Kohara**, **Javier Moreno**,
  and **Kotaro Shinmyo** (YITP, Kyoto),

[3] arXiv: 2608.01729.

De Sitter Holography

① Introduction

● The fact that black holes (BHs) possess entropy

→ A fundamental key feature that distinguishes the theory of gravity from other physical theories.

● BH entropy is a **coarse-grained entropy**, like thermal entropy.

→ does not distinguish between pure and mixed states.

● Then what is the **fine-grained entropy** in gravity ?

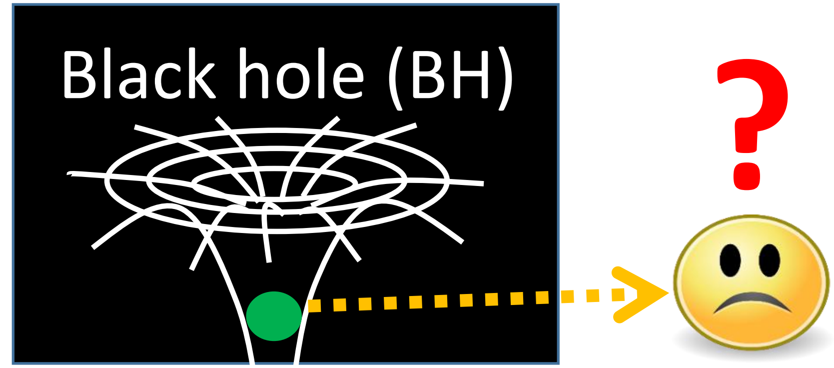
→ Holographic entanglement entropy !

(e.g. crucial to resolve BH information problem)

Black hole Entropy

[Bekenstein, Hawking, 1972-1976]

$$S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}$$



A_{BH} = Surface Area ⇒ **Geometry**

G_N = Newton constant ⇒ **Gravity**

$\hbar$ = Planck constant ⇒ **Quantum Mechanics**

k_B = Boltzmann const. ⇒ **Stat.Mech., Quantum Info.**

Quantum Gravity!

- BH is **thermodynamical** !
- BH entropy is proportional to the **area** !
- BH has the entropy even in **classical gravity** !

Holography !

BH entropy leads us to the idea of holography:

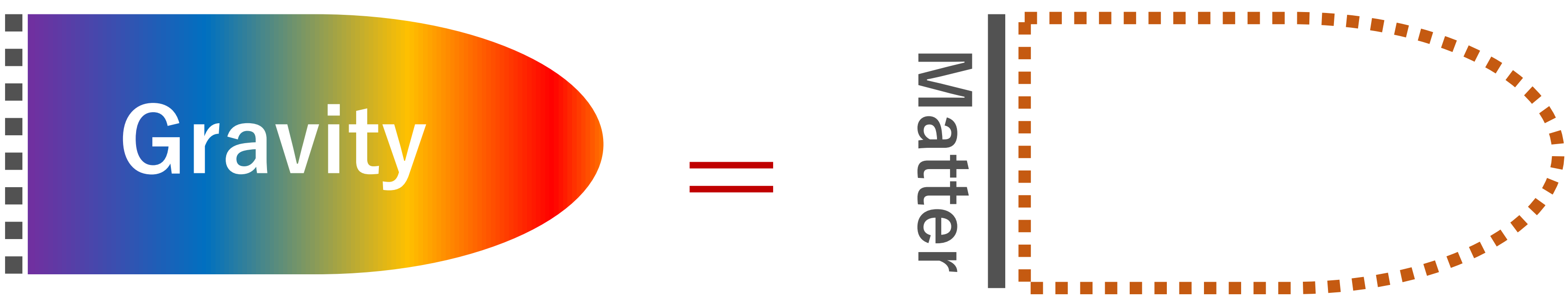
BH Entropy $\quad \boxed{S_{BH} = \frac{A_{BH}}{4G_N}}$ $\Rightarrow$ **Degrees of freedom in Gravity ∝ Area**

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂ M



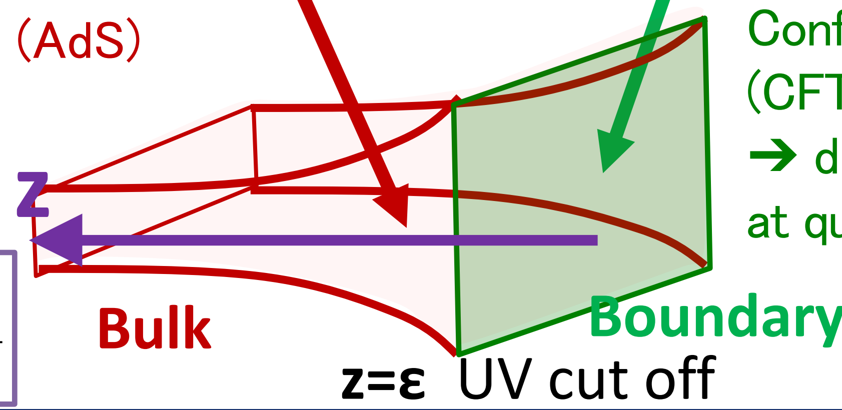
BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

Gauge/Gravity Duality: best known example of holography

Gauge/Gravity Duality (AdS/CFT) – [Maldacena 1997]

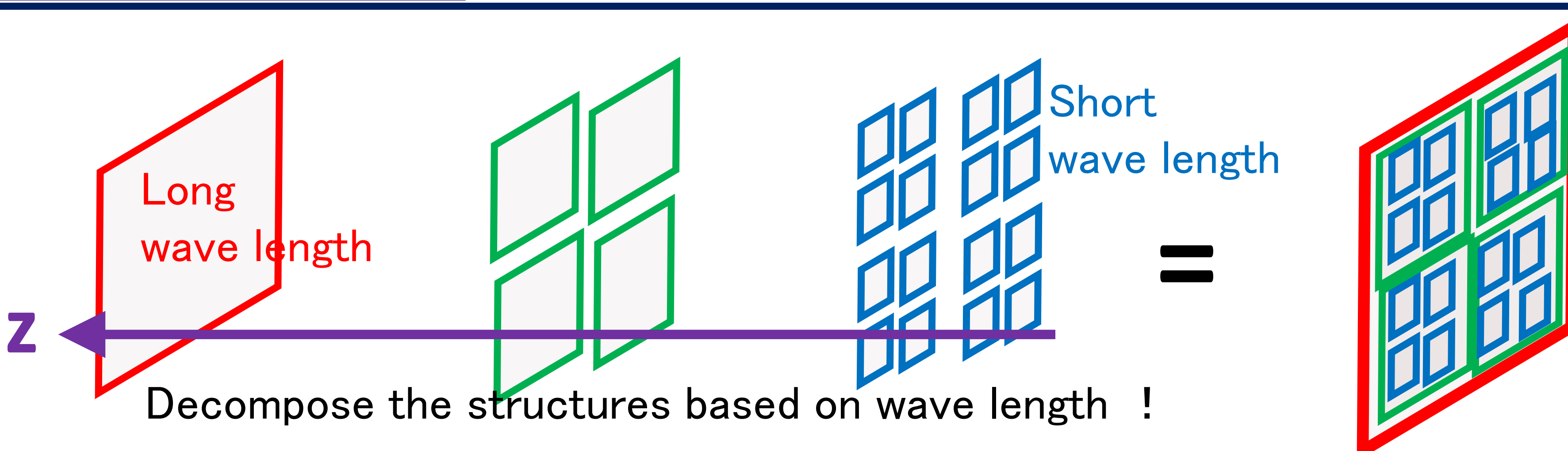
(Quantum) Gravity on d+1 dim. Anti-de Sitter Space **=** **d dim. Gauge theory (or Conformal field theory)**

Anti-de Sitter space (AdS)
➔ Universe with negative curvature



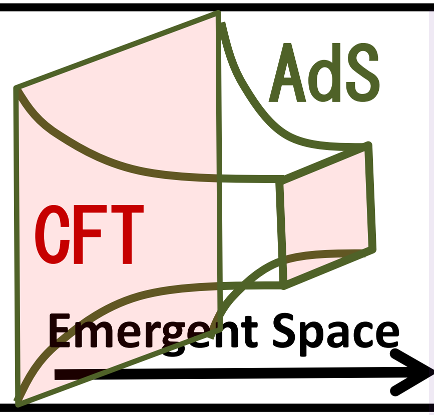
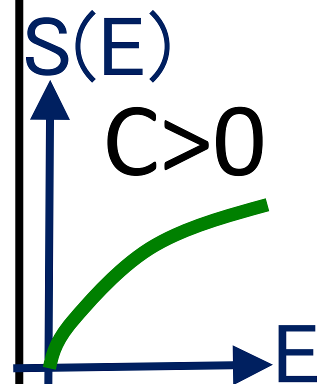
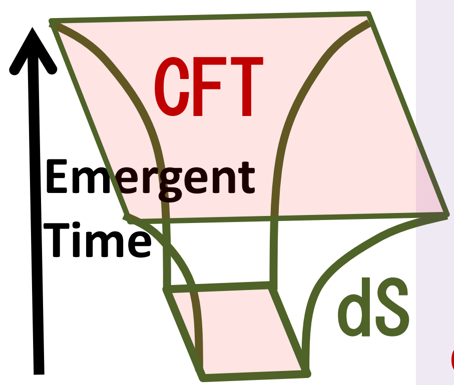
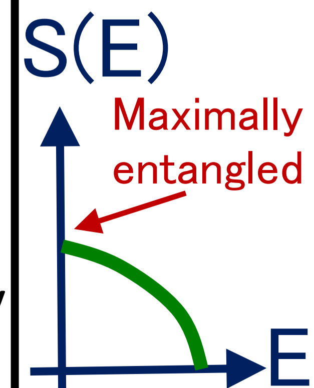
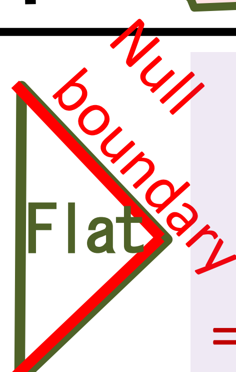
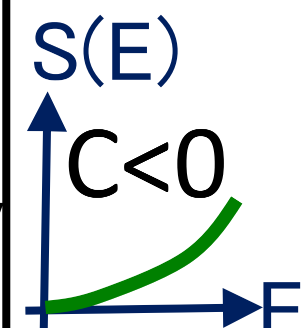
Conformal Field Theory (CFT)
➔ d dim. gapless system at quantum critical point

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d-1} dx_i^2}{z^2}$$



Decompose the structures based on wave length !

Classification of Maximally Symmetric Spacetimes and Holography

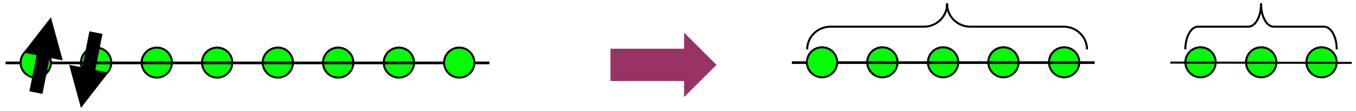
Type	Holography		Central charge	
Anti de Sitter Space AdS $\Lambda<0$		**AdS/CFT** Gravity in d+1 dim. AdS = d dim. CFT on $R^{1,d}$	For d=2, $C=\frac{3R_{AdS}}{2G_N}$ Unitary CFT	
de Sitter Space dS $\Lambda>0$		**dS/CFT** Gravity in d+1 dim. dS = d dim. CFT on R^{d} [No time !]	For d=2, $C=i\frac{3R_{dS}}{2G_N}$ Non-Unitary CFT	
Flat $\Lambda=0$		**Flat Space Holography** Gravity in d+1 dim. Flat space = d dim. CCFT on Null $\times$ S^{d-1}	For d=3, $C=i\infty$ Non-Unitary CFT	

Fine-Grained Entropy ➔ Entanglement Entropy (EE)

An amount of quantum entanglement (for pure states) is measured by **Entanglement Entropy (EE)** .

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin−chain



We introduce the reduced density matrix ρ_A

by tracing out B: $\rho_A = \mathrm{Tr}_B\left[|\Psi_{tot}\rangle\langle\Psi_{tot}| \right].$

The entanglement entropy (EE) S_A is defined by

$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]}$$

$\propto$ # of Bell Pairs between A and B

Measurement of EE in Experiments

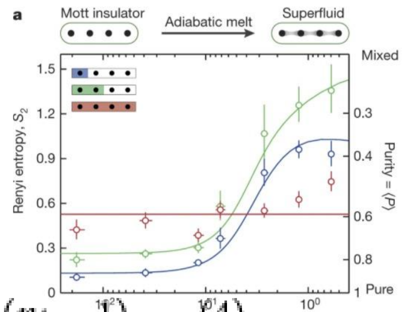
Ex.1: EE from Ultracold bosonic atoms

Published: 02 December 2015

Measuring entanglement entropy in a quantum many-body system

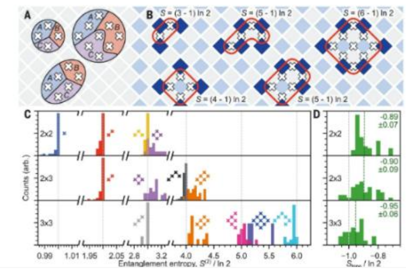
Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli & Markus Greiner

Nature 528, 77–83 (2015) | Cite this article



$$H = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i(n_i-1)$$

Ex3. Topological EE in superconducting qubits



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RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER, Y.-J. LIU, A. SMITH, C. KNAPP, M. NEWMAN, C. JONES, Z. CHEN, C. QUINTANA, X. MI, [...], AND P. ROUSHAN +88 authors

Ex2: EE from Trapped-ion quantum simulator

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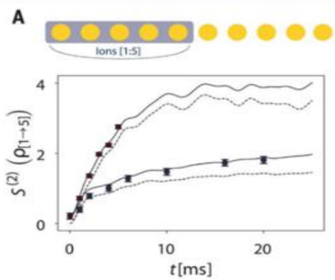
REPORT

Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS | Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-263 • DOI: 10.1126/science.aau4963

$$H_{XY} = \hbar\sum_{i<j} J_{ij}\left(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\right) + \hbar B\sum_j \sigma_j^z$$

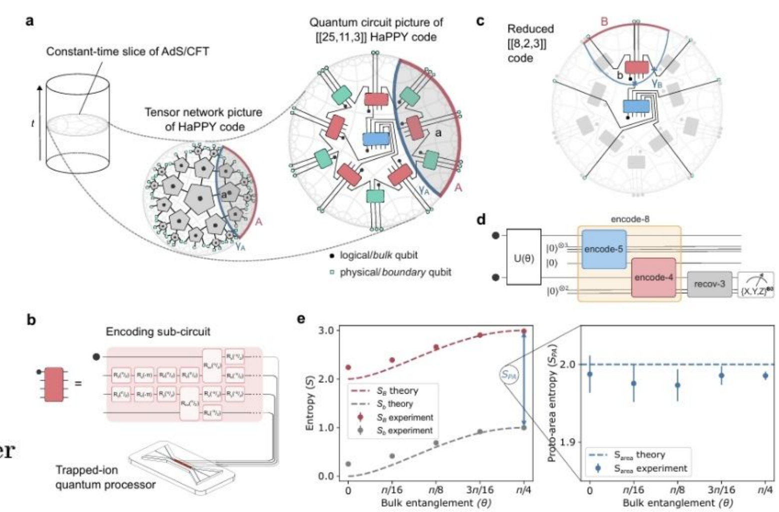


Ex4. Holographic EE from Trapped-ion

arXiv:2607.12047

Observation of gravity-like signatures in holographic codes on a quantum computer

Debopriyo Biswas,[1,*] Gong Cheng,[2,*] Krishnanand Karthikeyan,[2,*] Diana Muñoz-Valencia,[3] Vincent P. Su,[4] Hrant Gharibyan,[4] Daiwei Zhu,[5] Grant Salton,[5] Evgeny Epifanovsky,[5] Martin Roetteler,[5] Christopher Monroe,[1,5] John Preskill,[6] Norbert M. Linke,[3,1] ChunJun Cao,[2] and Crystal Noel[1]



Holographic Entanglement Entropy

[Ryu–TT 2006, Hubeny–Rangamani–TT 2007]

A generic Lorentzian asymptotic AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

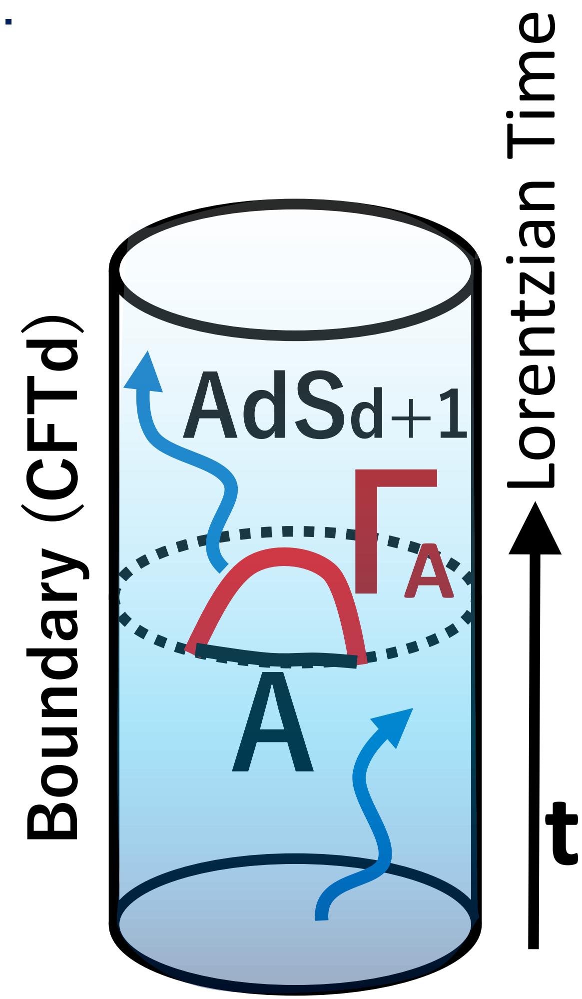
The time–dependent entanglement entropy

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|] \Rightarrow S_A(t).$$

is computed from an extremal surface area:

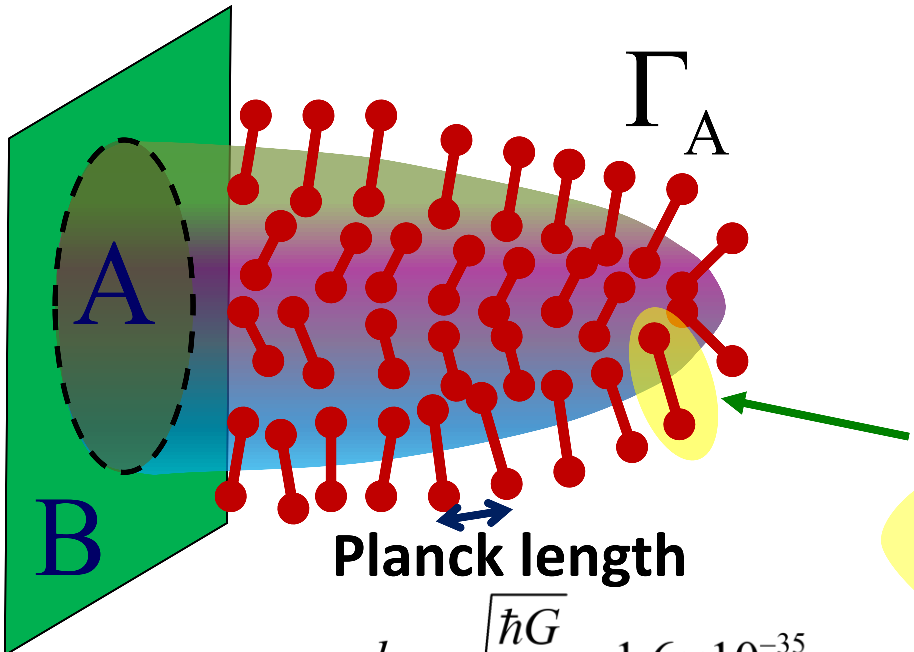
$$\boxed{S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]}$$

$$\partial A = \partial\Gamma_A \ \text{ and } \ A \sim \Gamma_A \ .$$



➔ Emergent Spacetime from Quantum Entanglement

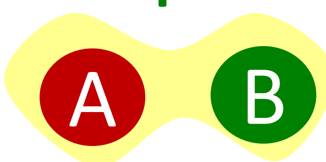
HEE implies ∃ a Bell pair for each Planck length area !



$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6\times 10^{-35} m$$

$$\boxed{S_A = \frac{\mathrm{Area}(\Gamma_A)}{4 l_{pl}^{D-1}}}$$

~10^{65} qubits per 1cm^2 !

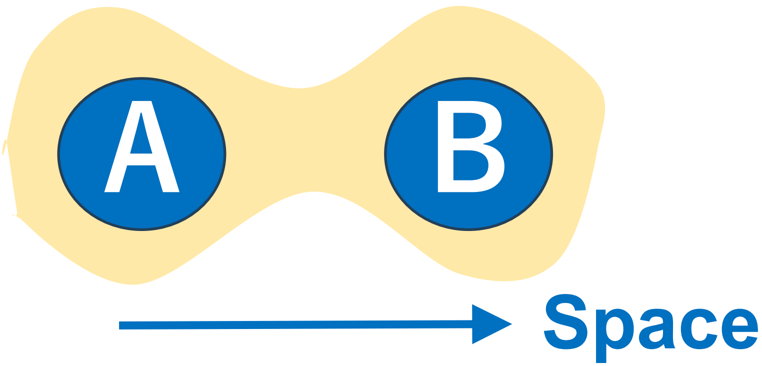


= Planck scale mini Universe

Spacetime may emerge from entangled Qubits !

[Swingle, Raamsdonk 2009, ⋯.]

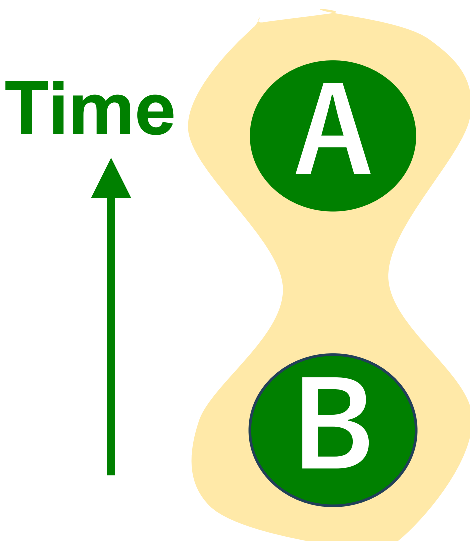
Quantum Entanglement ⇒ Space-like correlation



⇓

Emergent Space in Gravity

Question: How about time ?



What ? ⇒ Time-like correlation
(Causal Influence)

⇓

Emergent time in Gravity ?

In this talk, we generalize EE to time-like situations.

Contents

② Pseudo Entropy and Holography

[Nakata–Taki–Tamaoka –Wei–TT, 2020]

(2-1) Definition of Pseudo (Renyi) Entropy

For two quantum states $|\psi\rangle$ and $|\varphi\rangle$, we define the *transition matrix*:

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

We decompose the Hilbert space as $H_{tot} = H_A \otimes H_B$.

Introduce the reduced transition matrix: $\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right].$

Pseudo Entropy $\boxed{S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].}$

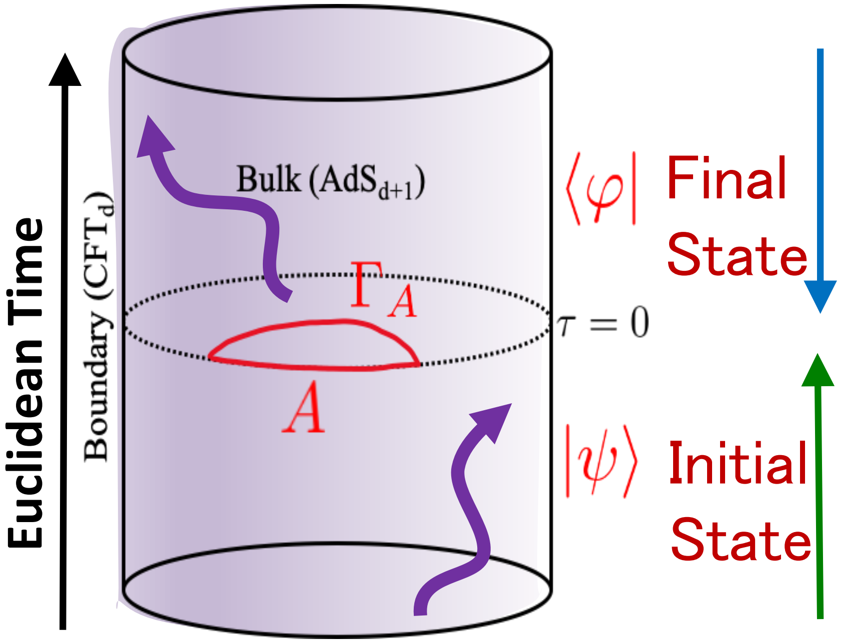
Renyi Pseudo Entropy $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].$

Generally, entropies for non-Hermitian density matrices are called pseudo entropy.

(2-2) Holographic Pseudo Entropy (HPE) Formula

In **Euclidean time dependent setups**, the minimal surface area coincides with the pseudo entropy.

$$S\left(\tau_A^{\psi|\varphi}\right) = \mathrm{Min}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

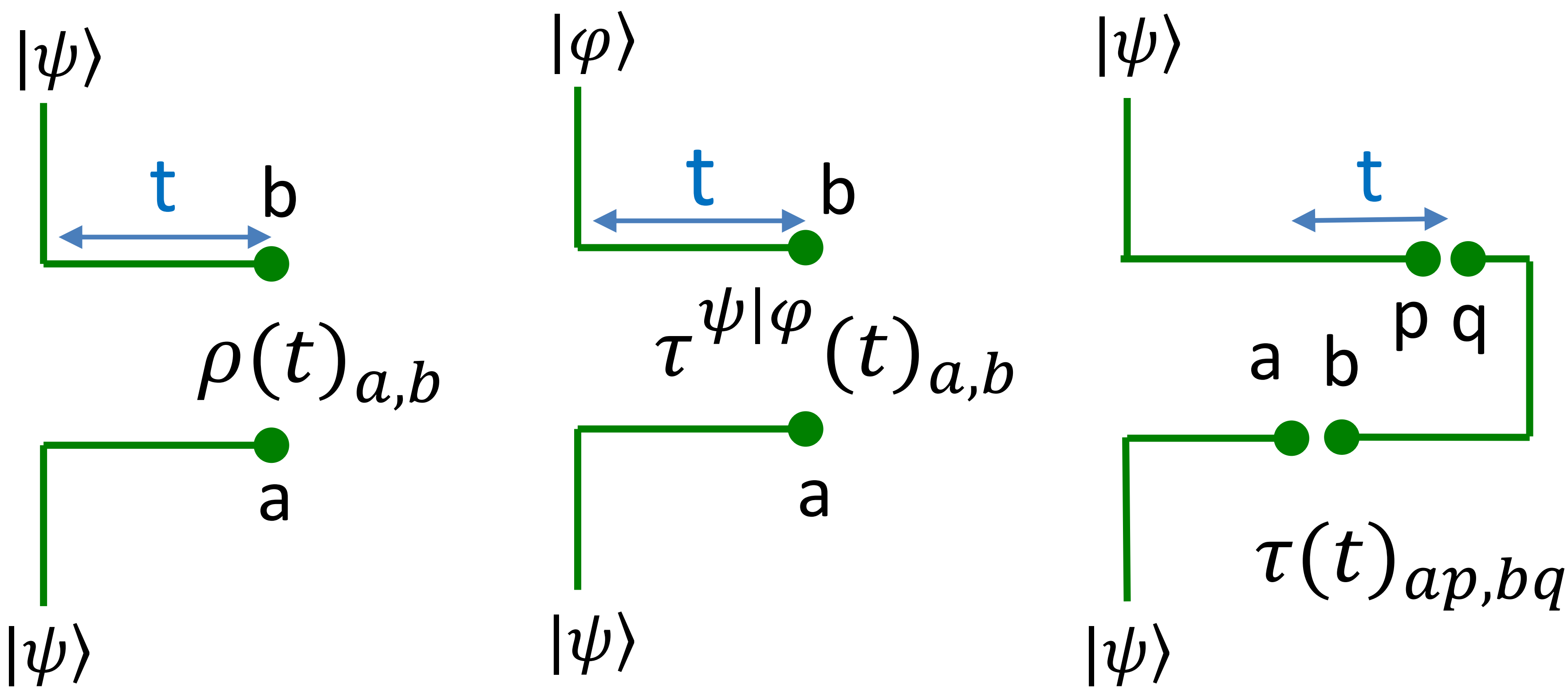


◆We can generalize this to a time-dependent setups.
In such general cases, the HPE is given by extremizing the area.

◆The HPE is expected to describe general setups where the "density matrices" are non-Hermitian. ➡ Time-like entanglement

Comments on Integration Contours

[1] Holographic EE **[2] Pseudo entropy** **[3] Time-like EE**

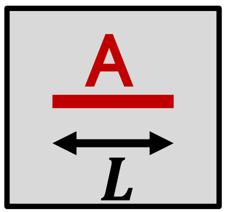


③ Time-like Entanglement Entropy

[Doi–Harper–Mollabashi–Taki–TT 2022]

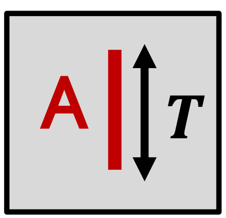
Consider a time-like version of entanglement entropy by rotating the subsystem A into a time-like one:

CFT on an infinite line



$$S_A = \frac{C}{3}\log\left[\frac{L}{\varepsilon}\right]$$

$$L \to iT$$

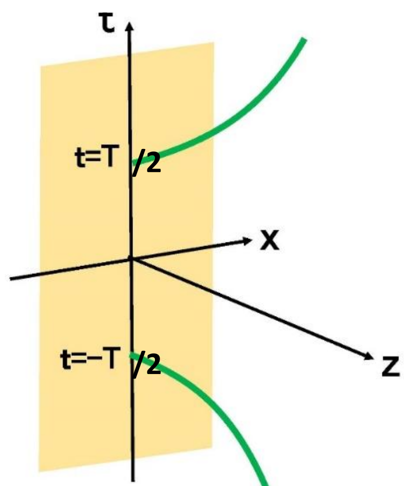


$$S_A = \frac{C}{3}\log\left[\frac{T}{\varepsilon}\right] + \frac{\pi}{6} iC$$

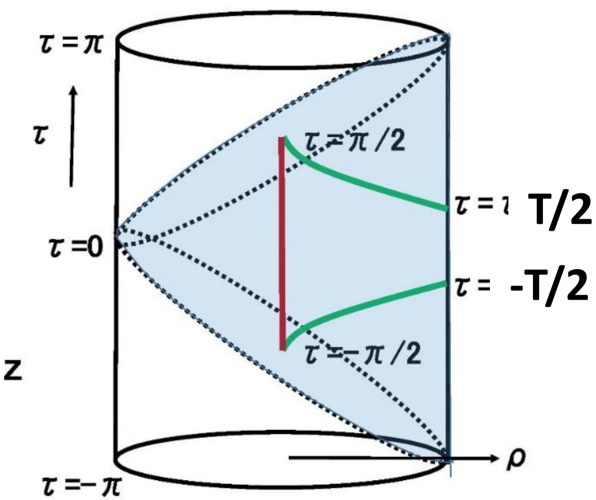
Imaginary part !

Holographic calculation

Poincare AdS

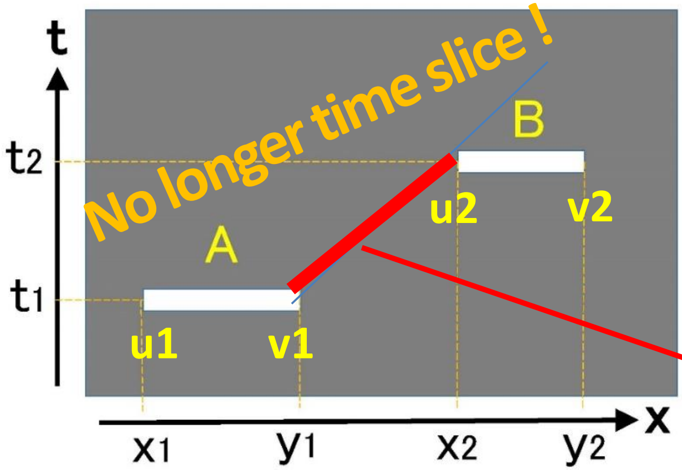


Global AdS



[For a systematic analysis of hol TEE, refer to Heller–Ori–Sereantes 2023 ➜ Use of complex extremal surfaces]

We can find an essentially same phenomenon in a more standard setup of entanglement entropy for double intervals:



e.g. Free Dirac fermion CFT c=1

If this interval is time-like, entropy gets complex valued !

$$S_{AB} = \frac{c}{6}\log\frac{|v_1-u_1|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_1-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_1|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|u_1-u_2|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|v_1-v_2|^2}{\epsilon^2}.$$

The imaginary part of TEE is explained by the time-like geodesic in AdS.

ρ_{AB} is not Hermitian ⟺ A and B are causally connected

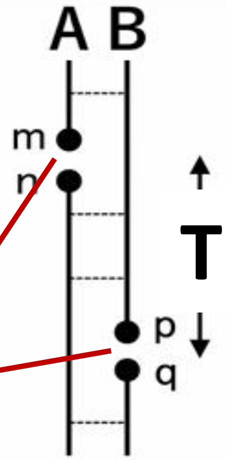
[Kawamoto−Maeda−Nakamura−TT 25, refer also to Parzygnat−Fullwood 22]

A Toy Example: Coupled Harmonic Oscillators

$$H = \frac{1}{\sqrt{1-\lambda^2}}\left[a^\dagger a + b^\dagger b + \lambda(a^\dagger b^\dagger + ab) + 1 - \sqrt{1-\lambda^2}\right].$$

$$\lambda = \tanh 2\theta$$

$$(\rho_{AB})^{m\,p}_{n\,q} =$$



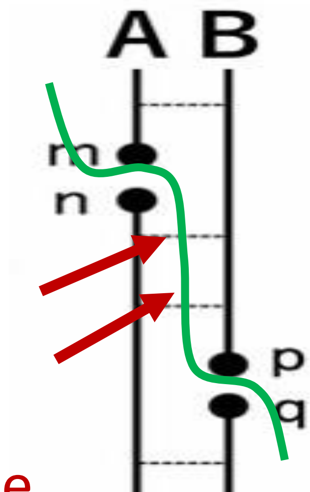
$$\boxed{[\rho_{AB}]^{a_2,b_2}_{a_1,b_1} = \langle\Psi_0|_{12}\cdot(|b_2\rangle\langle b_1|)_2\cdot\mathcal{P}e^{-i\int_{t_1}^{t_2}dtH_{12}(t)}\cdot(|a_2\rangle\langle a_1|)_1\cdot|\Psi_0\rangle_{12}}$$

$$\rho_{AB}^\dagger \neq \rho_{AB}$$

$$S^{(2)}_{AB} = \log\left[\frac{1+e^{-2iT}+(1-e^{-2iT})\cosh 4\theta}{2}\right].$$

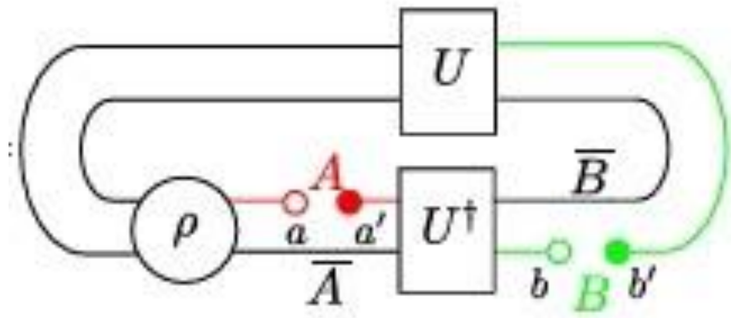
$$S(\rho_{AB}) \neq 0 \qquad \rho_{AB} = \text{mixed}$$

Purification
needs extra
Hilbert space



Recently, a clear theorem was shown by [Milekhin-Adamska-Preskill 2025]

$$\left(\rho_{\mathrm{AB}}\right)_{ab}^{a'b'} =$$



$$\langle [O_A(0), O_B(t)] \rangle = \mathrm{Tr}[(\rho_{\mathrm{AB}} - \rho_{AB}^{\dagger}) O_A O_{\mathrm{B}}]$$

Interactions between A and B

$$\boxed{\frac{1}{\dim H_A} \left\| \rho_{\mathrm{AB}} - \rho_{AB}^{\dagger} \right\|_2 \leq \frac{\left| \langle [O_A(0), O_B(t)] \rangle \right|}{\left\| O_A \right\|_2 \cdot \left\| O_B \right\|_2} \leq \left\| \rho_{\mathrm{AB}} - \rho_{AB}^{\dagger} \right\|_2}$$

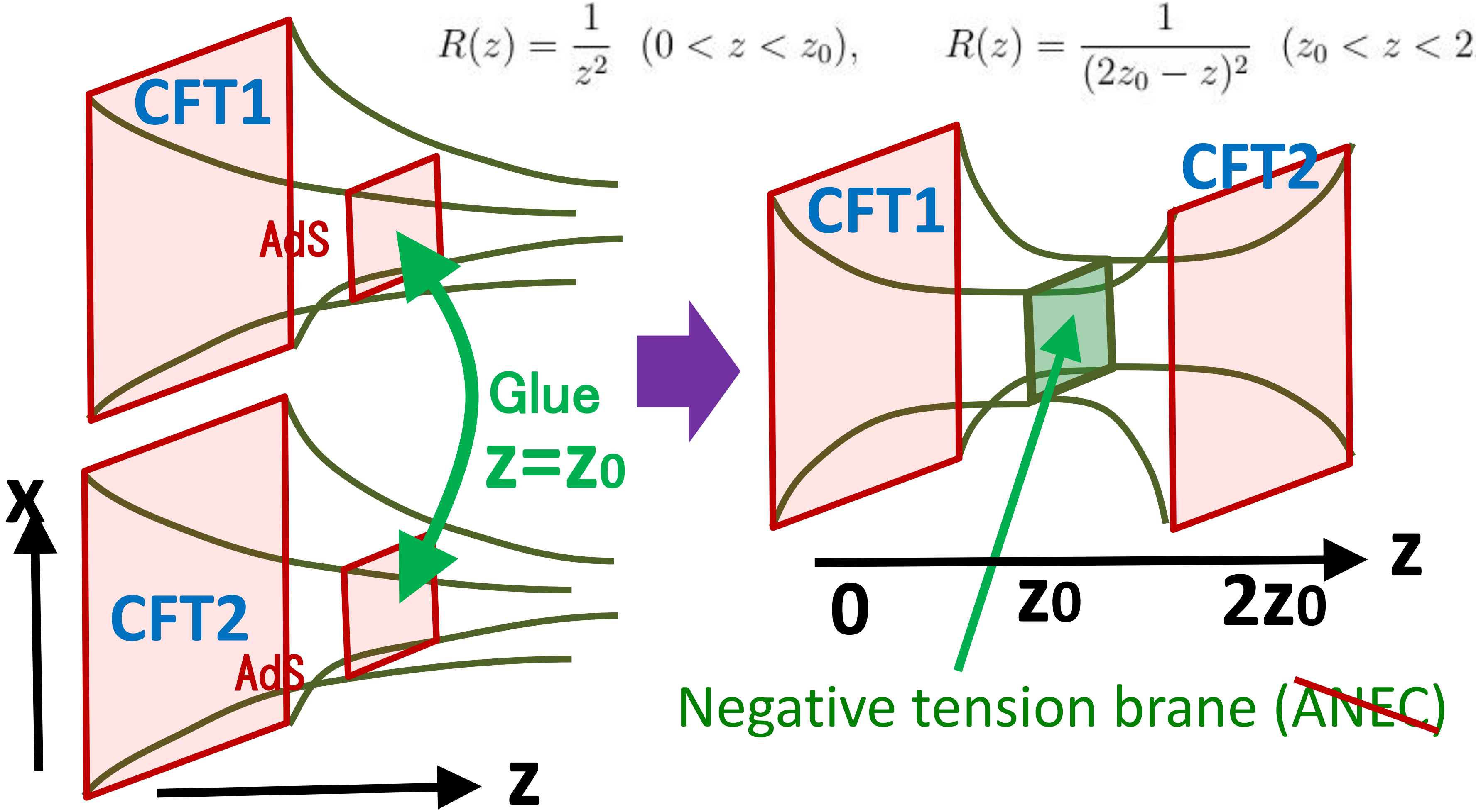
④ Traversable Wormhole and Time-like Entanglement

(4-1) General argument

Consider a simple model of traversable AdS wormhole:

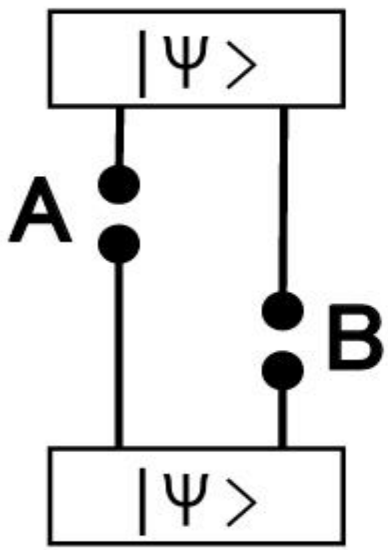
$$ds^2 = R(z)\left(dz^2 + \sum_{i=0}^{d-1} dx_i^2\right),$$

$$R(z) = \frac{1}{z^2} \quad (0 < z < z_0), \qquad R(z) = \frac{1}{(2z_0 - z)^2} \quad (z_0 < z < 2z_0).$$



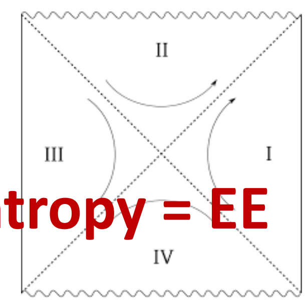
Two constructions of AdS Traversable wormhole

Non-traversable
=Eternal BH
[Maldacena 2001]
Thermofield double



$\rho_{AB}^{\dagger} = \rho_{AB}$

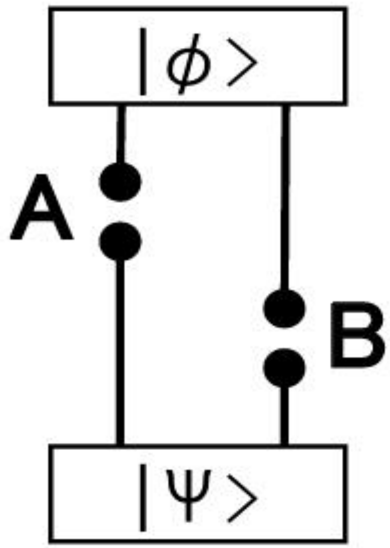
$S(\rho_{AB}) = 0$



BH entropy = EE

Traversable

[Kawamoto-Maeda
-Nakamura-TT 2025]
Model A (Janus)

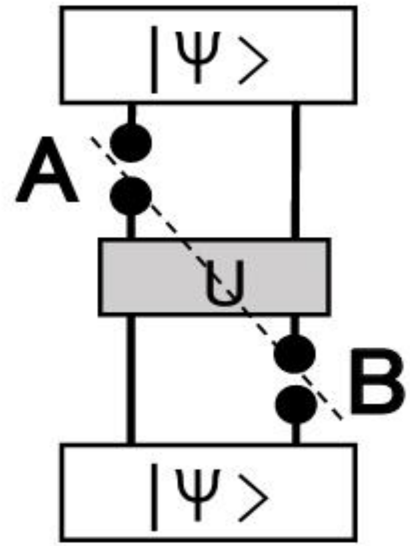


$\rho_{AB}^{\dagger} \neq \rho_{AB}$

$S(\rho_{AB}) = 0$

- ◆ No interactions between A and B
- ◆ H is non-hermitian

[Gao-Jafferis-Wall 2016]
Model B (Double trace)



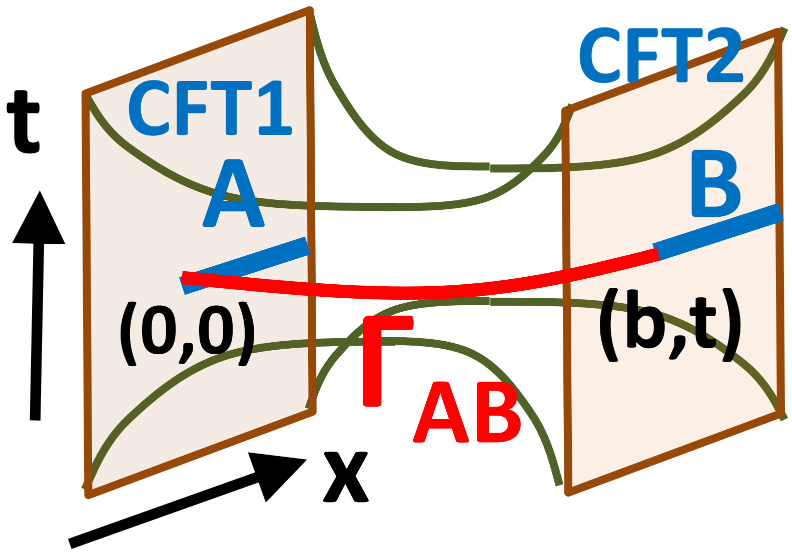
$\rho_{AB}^{\dagger} \neq \rho_{AB}$

$S(\rho_{AB}) \neq 0$

- ◆ ∃Interactions between A and B
- ◆ H is hermitian

Pseudo entropy (Time-like entanglement entropy)

How does S_{AB} look like ?



When $b^2 - t^2 + 4z_0^2 > 0$,

$$S_{AB} = \frac{c}{3}\log\left[\frac{b^2-t^2+4z_0^2}{4\varepsilon z_0}\right].$$

When $b^2 - t^2 + 4z_0^2 < 0$,

$$S_{AB} = \frac{c}{3}\log\left[\frac{t^2-b^2-4z_0^2}{4\varepsilon z_0}\right] + \frac{\pi}{3}ic.$$

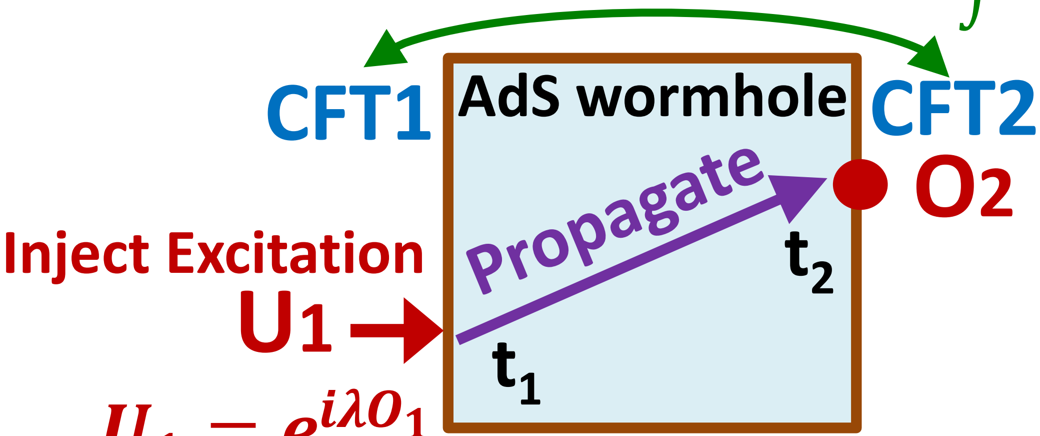
Γ_{AB} can be time-like in a traversable wormhole.

⇒ S_{AB} becomes complex valued because $\rho_{AB}^{\dagger} \neq \rho_{AB}$.
Thus, S_{AB} should be regarded as pseudo entropy.

(4-2) Double trace deformation of External BH (Model B)

Start with $|TFD\rangle = N\sum_n e^{-\beta H_{tot}/4}\,|n\rangle_1|n\rangle_2$

➜ Add interactions $H_{12} = \int dxdy\lambda(x,y)O_1(x)O_2(y)$



$H_{tot}=H_1+H_2+H_{12}$

$U_1 = e^{i\lambda O_1}$

$$\rho_2 = \mathrm{Tr}\left[e^{-it_2H_{tot}}\left(e^{it_1H_{tot}}U_1e^{-it_1H_{tot}}\right)|TFD\rangle\langle TFD|\left(e^{it_1H_{tot}}U_1^{\dagger}e^{-it_1H_{tot}}\right)e^{it_2H_{tot}}\right].$$

$$\langle O_2\rangle = \mathrm{Tr}[O_2\rho_2] \approx \langle TFD|O_2|TFD\rangle + i\lambda\langle TFD|[O_1(t),O_2]|TFD\rangle + O(\lambda^2)$$

Non-vanishing due to the interactions ↔ $\boxed{\rho_{AB}^{\dagger} \neq \rho_{AB}}$

(4-3) Wormhole via Janus deformation (Model A)

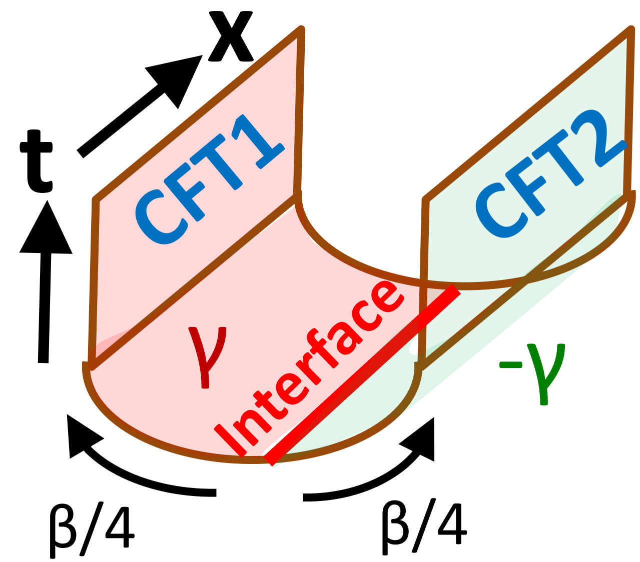
[Harper-Kawamoto-Maeda-Nakamura-TT 2026]

Janus deformation = asymmetric exactly marginal perturbations in a pair of CFTs

[Bak-Gutperle-Hirano 03]

$$S_{\mathrm{CFT1}} = S^{(0)}_{\mathrm{CFT}} + \gamma \int dx^d O_1(x)$$

$$S_{\mathrm{CFT2}} = S^{(0)}_{\mathrm{CFT}} - \gamma \int dx^d O_2(x)$$



- ◆We consider the TFD state of the doubled CFT for d=2.
- ◆In the standard Janus deformation, γ is real valued.
  We will extend γ to imaginary values.

Explicit construction from Janus deformation

We start with 3D Janus BH solutions in [Bak-Gutperle-Hirano 2007].

The model is given by the 3d gravity action [See also Nakaguchi-Ogawa -Ugajin 2014 for HEE analysis]

$$I = \frac{1}{16\pi G_N}\int d^3x \left[R - g^{ab}\partial_a\phi\partial_b\phi + 2\right].$$

The solution ansatz looks like

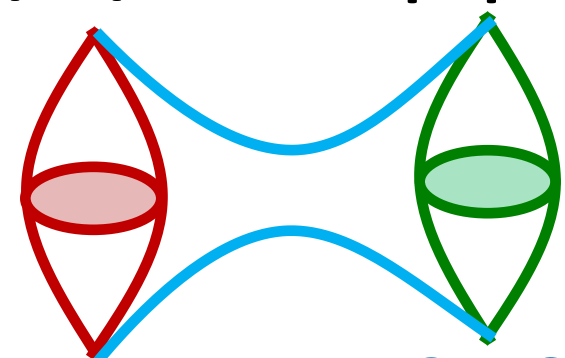
$$ds^2 = f(\mu)(d\mu^2 + ds^2_{AdS2}), \qquad \phi = \phi(\mu).$$

$$ds^2_{AdS2} = -d\tau^2 + r_0^2\cos^2\tau d\theta^2$$

γ is Janus deformation Parameter.

$$\frac{d\phi(\mu)}{d\mu} = \frac{\gamma}{\sqrt{f(\mu)}},$$

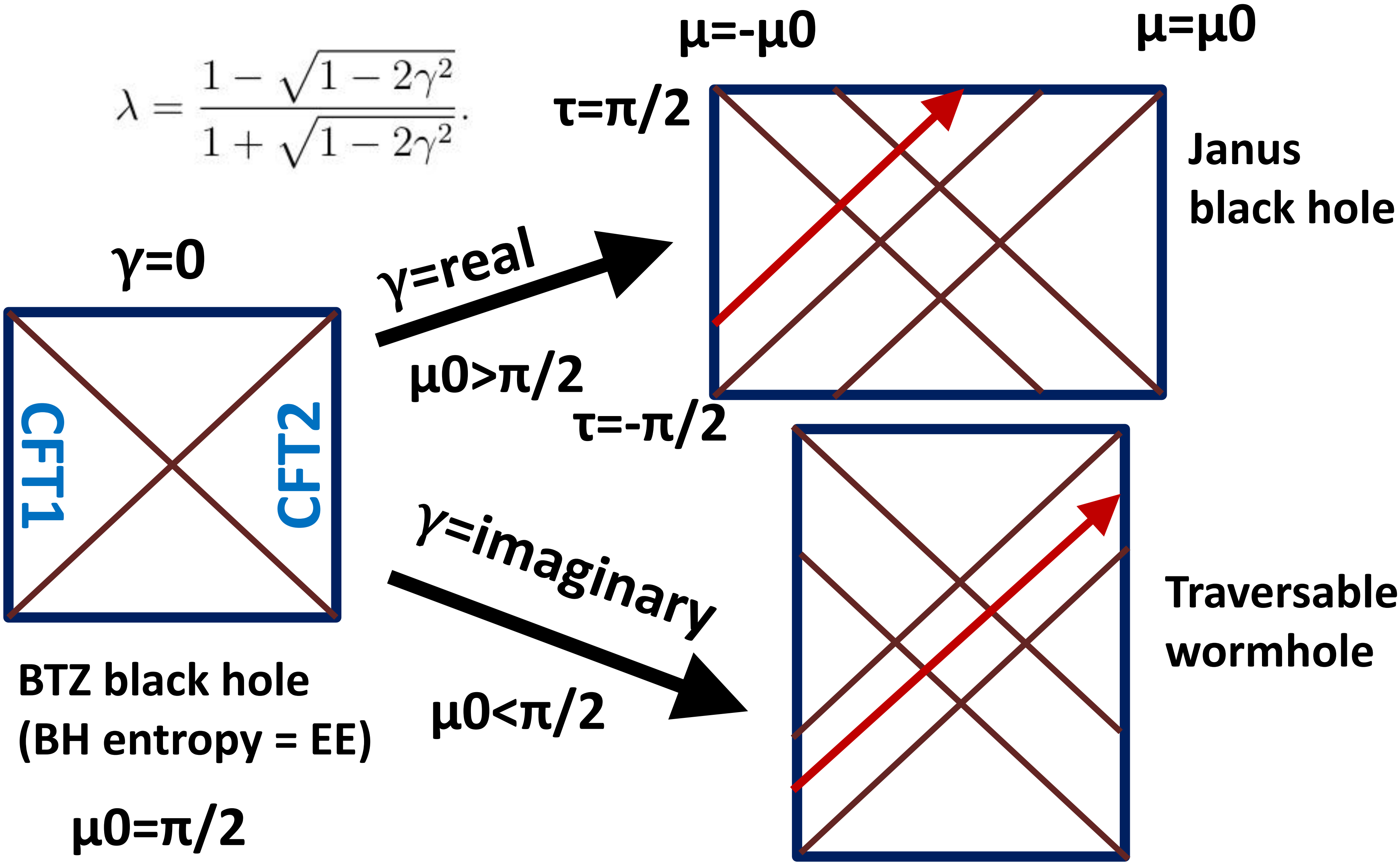
$$\frac{df(\mu)}{d\mu} = \sqrt{f(4f^2 - 4f + 2\gamma^2)}.$$



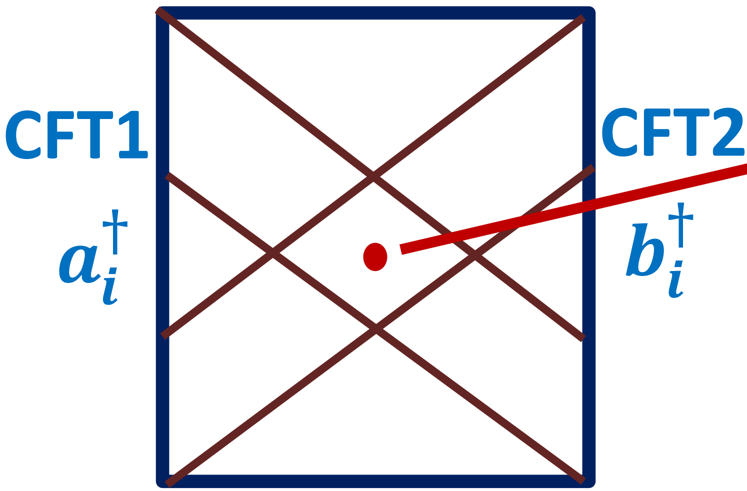
We now extend this solution to imaginary γ.

$$\mu_0 = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-\lambda x^2)}},$$

$$\lambda = \frac{1-\sqrt{1-2\gamma^2}}{1+\sqrt{1-2\gamma^2}}.$$



Entanglement between CFT1 and CFT2



$$S_1 = \frac{\pi}{6} c r_0 \sqrt{1+\sqrt{1-2\gamma^2}}$$

Area of throat

Monotonically decreasing function of γ^2

In the dual CFT, this is dual to the PE/EE in the deformed TFD state:

$$|\mathrm{TFD}(\beta,\gamma)\rangle = \tilde{\mathcal{N}} \exp\left[\sum_{i=1}^{\infty} e^{-\frac{\beta}{2}E_i}\left(\sin 2\theta\ a_i^\dagger b_i^\dagger + \cos 2\theta\left((a_i^\dagger)^2 - (b_i^\dagger)^2\right)\right)\right]|0\rangle$$

$$\langle\mathrm{TFD}(\beta,\gamma)| = \tilde{\mathcal{N}}\langle 0|\exp\left[\sum_{i=1}^{\infty} e^{-\frac{\beta}{2}E_i}\left(\sin 2\theta\ a_i b_i + \cos 2\theta\left((a_i)^2 - (b_i)^2\right)\right)\right] \quad \theta \equiv \frac{\pi}{4} + \gamma$$

S_1 becomes its maximum at θ=π/4 (i.e. no deformation) and decreases as γ^2 gets larger. For imaginary γ, it increases. This is consistent with the gravity dual.

Why traversable without interactions ?

The Hamiltonians H_1 and H_2 of CFT1 and CFT2 for γ=imaginary becomes non-Hermitian:

$$H_1 = H_0 + \gamma V, \quad H_2 = H_0 + \gamma^* V, \ \text{such that} \ H_1^\dagger = H_2$$

They have different eigen-vectors with complex eigen-values:

$$H_1 |n_+\rangle = E_n |n_+\rangle, \qquad H_2 |n_-\rangle = E_n^* |n_-\rangle,$$

$$\langle n_+ | H_2 = \langle n_+ | E_n^*, \qquad \langle n_- | H_1 = \langle n_- | E_n \ ,$$

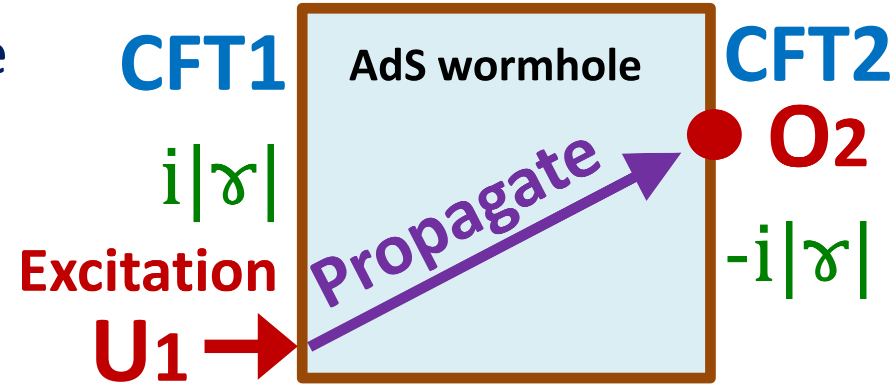
where we introduce their (Hermitian) conjugations by $|n_\pm\rangle^\dagger = \langle n_\pm|$.

They satisfy $\langle n_\mp | m_\pm \rangle = \delta_{n,m}, \qquad \sum_n |n_\pm\rangle\langle n_\mp| = 1.$

This motivates us to define the modified conjugation ‡ by

$$\boxed{\langle n_+ | O | m_- \rangle^* = \langle m_+ | O^\ddagger | n_- \rangle .}$$

The initial and final TFD state look like



$$\left|\mathrm{TFD}\right\rangle = \sum_n e^{-\frac{\beta}{4}(H_1+H_2)} \left|n_+\right\rangle_1 \left|n_+\right\rangle_2 \quad ,$$

$$\left\langle\overline{\mathrm{TFD}}\right| = \sum_n \left\langle n_-\right|_1 \left\langle n_-\right|_2 e^{-\frac{\beta}{4}(H_1+H_2)} \quad .$$

The density matrix $\rho = \left|\mathrm{TFD}\right\rangle\left\langle\overline{\mathrm{TFD}}\right|$ is not Hermitian $\rho^{\dagger} \neq \rho$.

However, it satisfies $\rho^{\ddagger} = \rho$, implying ‡ is good for the conjugation.

An observer in CFT2 probes the state:

$$\rho_2 = e^{-it_2 H_-^{(2)}} \mathrm{Tr}_1 \left[(e^{-it_1 H_+^{(1)}} e^{i\alpha O^{(1)}} e^{it_1 H_+^{(1)}}) |TFD\rangle\langle\overline{TFD}| (e^{-it_1 H_+^{(1)}} e^{-i\alpha O^{(1)\ddagger}} e^{it_1 H_+^{(1)}}) \right] e^{it_2 H_-^{(2)}} .$$

When the CFT1 is excited by $U_1 = e^{i\alpha O^{(1)}}$, the CFT2 observer sees

$$\begin{aligned} \langle O^{(2)}(t_2)\rangle &= \mathrm{Tr}[\rho_2 O^{(2)}] \\ &\simeq \langle\overline{TFD}|O^{(2)}|TFD\rangle + i\alpha\langle\overline{TFD}| \underline{\left(O^{(2)}(t_2)O^{(1)}(t_1) - O^{(1)\ddagger}(t_1)O^{(2)}(t_2)\right)} |TFD\rangle, \end{aligned}$$

We have $[O_1,O_2]=0$, but this is non-vanishing !

⑤ De Sitter Holography and Time-like Entanglement

A Sketch of dS/CFT [Strominger 2001, Witten 2001, Maldacena 2002,⋯.]

d+1 dim. Lorentzian de−Sitter spacetime

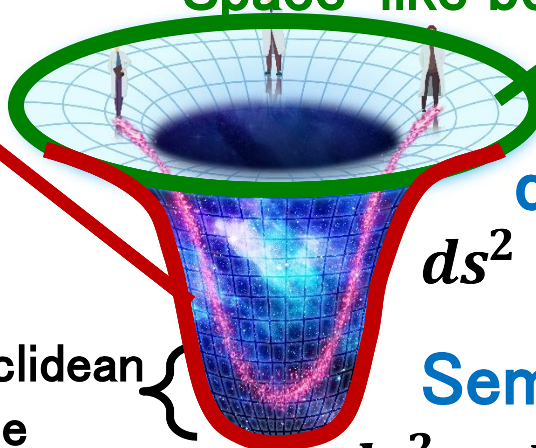
Dual

Euclidean d. dim CFT on S^d

Space−like bdy

Time

$t=\infty$



Lorentzian time

Euclidean time

de Sitter

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta\, d\Omega^2)$$

Time emerges from Euclidean CFT !

$$\Psi\,[\text{dS gravity}] = Z\,[\text{CFT}]$$

Why dS/CFT is much more difficult than AdS/CFT ?

[1] Dual Euclidean CFTs should be exotic and non-unitary !

A "standard" Euclidean CFTs is dual to gravity on hyperbolic space.

e.g. dS3/CFT2 ➔ ***Imaginary valued*** central charge $c \approx i\frac{3L_{dS}}{2G_N}$!

Unusual conjugation: $(L_n)^\dagger = (-1)^{n+1}\widetilde{L_n}$ [Doi-Ogawa-Shimyo-Suzuki-TT 2024]

[2] Time should emerge from Euclidean CFT !

From a usual Euclidean CFT, a space-like direction will emerge as RG scale.

How does a ***time-like direction emerge*** from a Euclidean CFT ?

[3] "Entanglement entropy" looks complex valued !

Extremal surfaces in dS which end on its boundary are ***time-like*** !

➜ Necessity of time-like EE (or pseudo entropy)

Non-unitary CFT dual of 3 dim. dS

[Hikida–Nishioka–Taki–TT, 2021]

Large c limit of SU(2)$_k$ × SU(2)$_{-k}$ WZW model (a 2dim. CFT)

= Einstein Gravity on 3 dim. de Sitter (radius L_{ds})

Level

$$k \approx -2 + \frac{4iG_N}{L_{dS}}$$

Central charge

$$c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$$

$$\boxed{Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E}}}$$

CFT partition function

De Sitter Entropy

This non-unitary CFT is equivalent to the Liouville CFT

at $b^{-2} \approx \pm\frac{i}{4G_N}$. $\quad I_{CFT}[\phi] = \int d^2x \left[\frac{1}{4\pi}(\partial_a\varphi\partial_a\varphi) + \underline{\mu e^{2b\varphi}}\right].$

[Hikida–Nishioka–Taki–TT, 2022]

complex !

The same Liouville CFT appears in [Verlinde–Zhang 2024] via DSSYK.

→Why two different holographic constructions lead to the same CFT ?

Holographic Entanglement Entropy in dS3/CFT2 ?

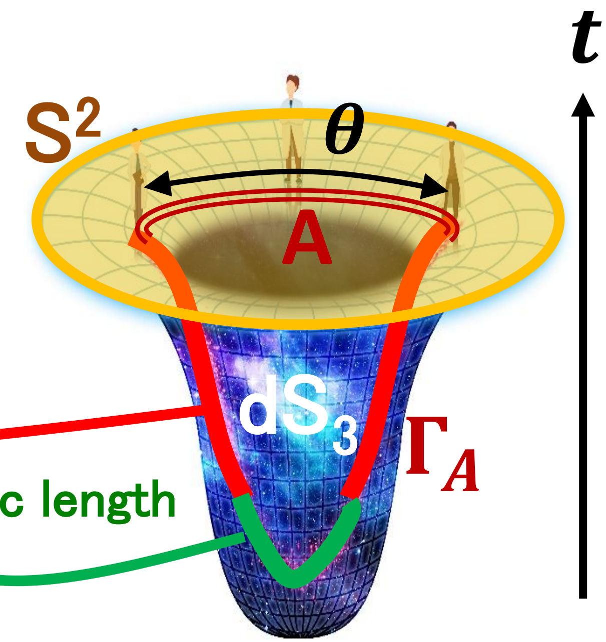
[Doi–Harper–Mollabashi–Taki–TT 2022]

In dS3/CFT2, the geodesic Γ_A becomes time-like and we find:

$$\boxed{S_A = \frac{\mathrm{L}(\Gamma_A)}{4G_N} = i\frac{c_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right) + \underbrace{\frac{c_{dS}}{6}\pi}_{S_{dS}/2}.}$$

Time-like geodesics length
➔imaginary part !

Space-like geodesic length
➔Real part



Complex valued entropy !

➜ More properly, this should be regarded as pseudo entropy.

Note: the emergent time coordinate = imaginary part of PE.

But, the de Sitter space has two boundaries !

Future Boundary t=∞

Past Boundary t=-∞

We can send signals between boundaries !

∃Causal influence !

Can we understand this as quantum entanglement ?

This has been called Cosmic ER=EPR [Cotler-Strominger 2023].

We argue that it is explained by **time-like entanglement** and that the dual CFT lives on a **Lorentzian torus**.

[Fujiki-Kohara-Moreno-Shinmyo-TT 2026]

dS/CFT as time-like entangled TFD state

[Fujiki-Kohara-Moreno-Shinmyo-TT 2026]

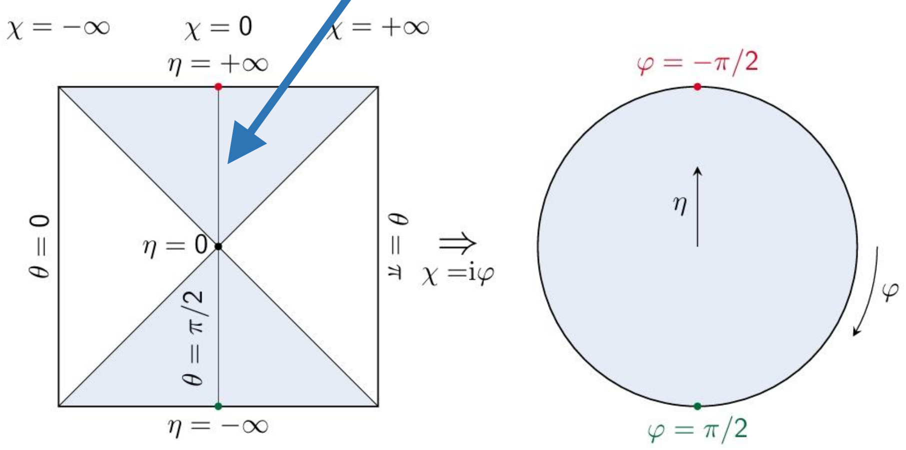
Consider the three dimensional de Sitter space (dS3):

$$ds^2 = \begin{cases} -\,dt^2 + \cosh^2 t\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) & \text{(global coordinates)} \\ -\,d\eta^2 + \cosh^2\eta\, d\phi^2 + \sinh^2\eta\, d\chi^2 & \text{(hyperbolic slicing)} \end{cases}$$

and perform the following coordinate transformation: $\chi = i\varphi,$

This leads to the following spacetime with "two times":

$$\boxed{ds^2 = -d\eta^2 + \cosh^2\eta d\phi^2 - \sinh^2\eta d\varphi^2,}$$



The asymptotic boundary $\eta=\infty$
Is given by a Lorentzian torus.

$$\varphi \sim \varphi + 2\pi, \quad \phi \sim \phi + 2\pi.$$

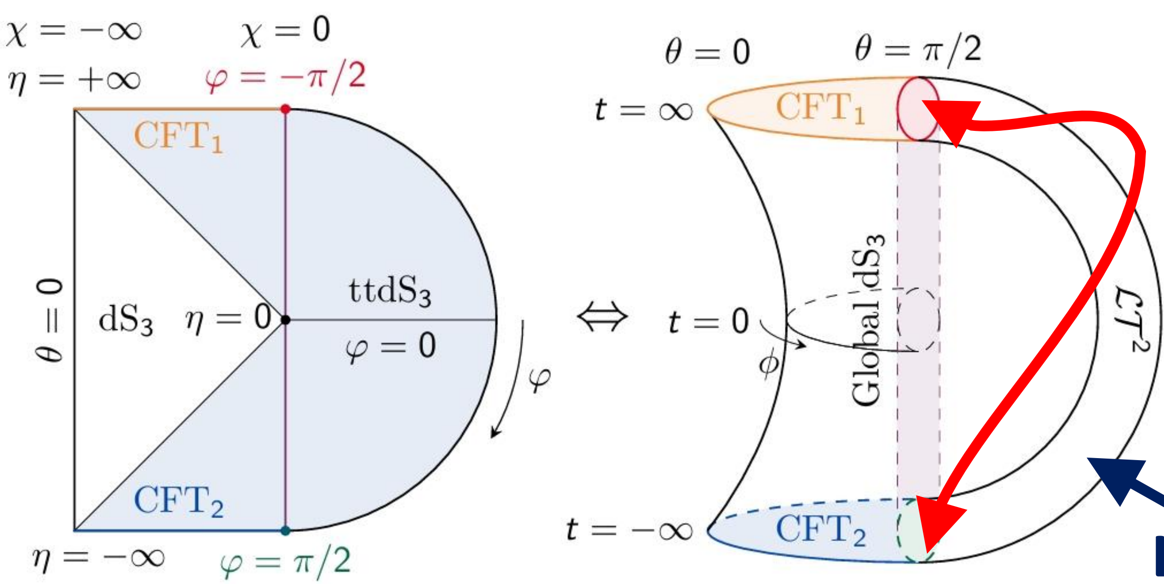
<u>Our claim</u>: The CFT dual to dS lives on a Lorentzian torus (φ, ϕ).

➜ The quantum state is described by the TFD state with imaginary temperature: $\boldsymbol{\beta = 2\pi i}$,

$$|TFD\rangle = \sum_n e^{-\beta E_n/2}\, |n\rangle_1 |n\rangle_2 \ ,$$

$$\langle\overline{TFD}| = \sum_n e^{-\beta E_n/2}\, \langle n|_1 \langle n|_2 \ .$$

Not hermitian conjugation !



Future (CFT1)
and Past (CFT2)
are **time-like entangled with each other !**

Lorentzian Torus

Note: Time-like entanglement in CFT explains the causal influence in dS.

Derivation of dS entropy

- If we apply the Cardy formula for 2d CFTs: $S_{CFT} = \frac{2\pi^2 c}{3\beta}$,

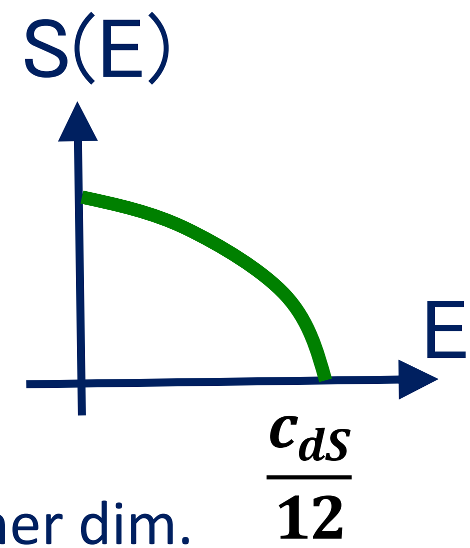
with the gravity dual values $c = \frac{3i}{2G_N}$, $\beta = 2\pi i.$ ←Lorentzian torus

then we can reproduce the known dS entropy: $S_{dS} = \frac{2\pi}{4G_N}$.

- If we look this analysis more closely, the density of CFT states is

$$S(E) = \log \rho(E) \approx 2\pi \sqrt{\frac{c_{dS}}{3}\left(\frac{c_{dS}}{12} - E\right)},$$

where we set $c_{dS} \equiv \frac{3}{2G_N}$.



This analysis can be generalized to dS3 BH and higher dim.

⑥ Conclusions

Key ideas

Is gravity the fastest "quantum computer" ?

Holography

➔New insights into quantum matter, quantum computation and quantum cryptography

Universe = Collection of Qubits

Does gravitational spacetime emerge from qubits？

Emergence

➔New approach to quantum gravity

In this talk we discussed a time-like extension of entanglement.

- Time-like EE has a clear gravity dual via holography.
- It is a useful when "non-hermitian density matrices" appear:

e.g. wormholes, de Sitter spaces, open systems, etc.

Imaginary part of Time-like EE➔ Emergence of Time, Traversability

Future and Past infinity of dS ➔ Time-like Entanglement

Thank you !

Basic Properties of Pseudo Entropy (PE)

- In general, $\tau_A^{\psi|\varphi}$ is not Hermitian. Thus PE is complex valued.

(More generally, we call $S(\tau_A)$ pseudo entropy when τ_A is not hermitan.)

- If either $|\psi\rangle$ or $|\varphi\rangle$ has no entanglement (i.e. direct product state) , then
$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0.$$

- We can show $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \left[S^{(n)}\left(\tau_A^{\varphi|\psi}\right)\right]^\dagger.$

- We can show $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = S^{(n)}\left(\tau_B^{\psi|\varphi}\right).$

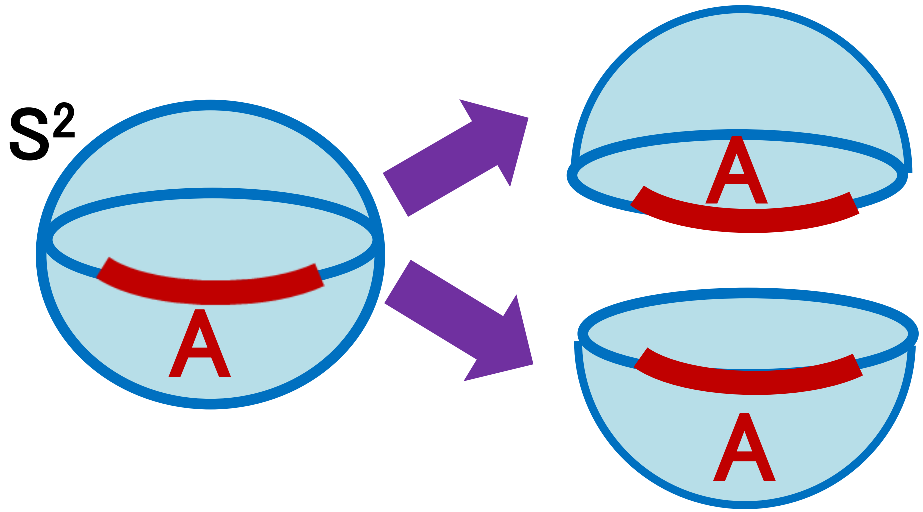
→"SA=SB"

This implies a local holographic formula !

This entropy is more properly considered as pseudo entropy (PE).

This is because the reduced density matrix $\boldsymbol{\rho_A}$ **is not Hermitian !**

$\rho_A =$



$$\langle\varphi| \sim \int_{S^2_+} D\varphi\, e^{-I_{CFT}[\varphi]}$$

Different states !

$$|\psi\rangle \sim \int_{S^2_-} D\varphi\, e^{-I_{CFT}[\varphi]}$$

2D CFT on the space with the metric: $h_{ab} = e^{2\phi}\delta_{ab},$ $\rightarrow$ $\rho_A \neq \rho_A^\dagger$

$$I_{CFT}[\phi] = i\frac{C_{ds}}{24\pi}\int d^2x[(\partial_a\phi)^2 + e^{2\phi}].$$

Note: the emergent time coordinate = imaginary part of PE.