

Application of Quantum Computation to High Energy Physics

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(本多正純)

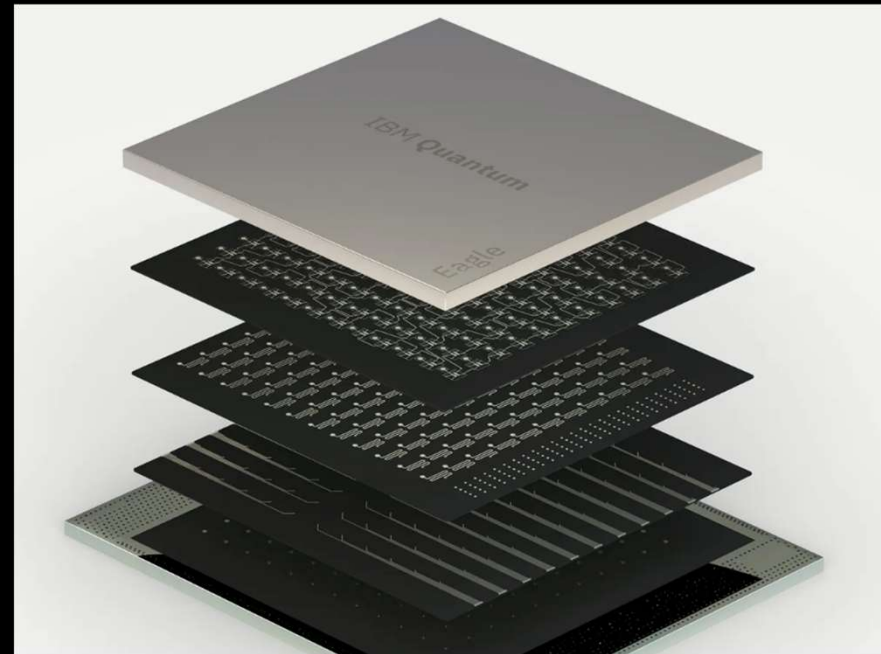
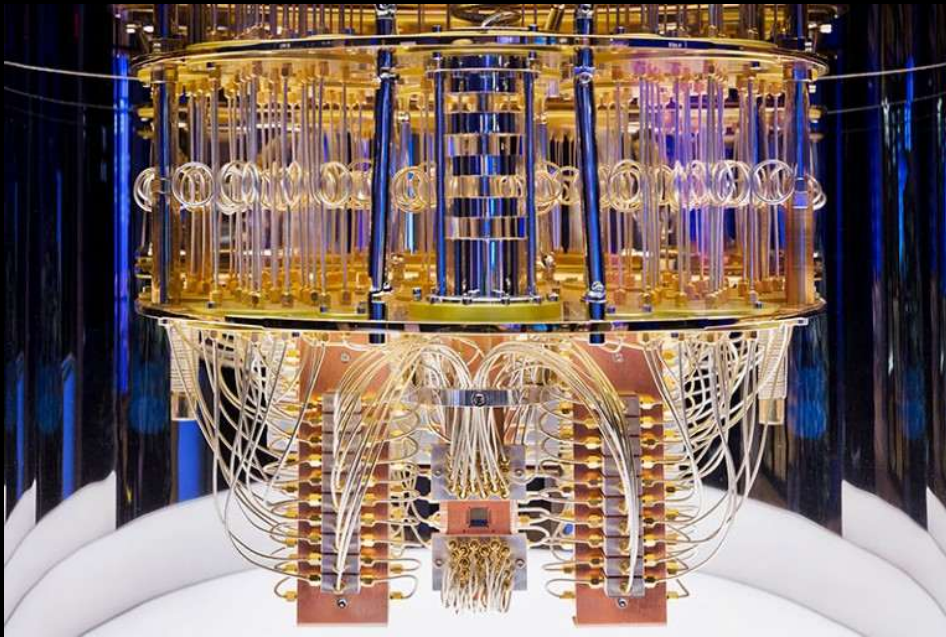
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Quantum computer sounds growing well...



Article

Evidence for the utility of quantum computing before fault tolerance

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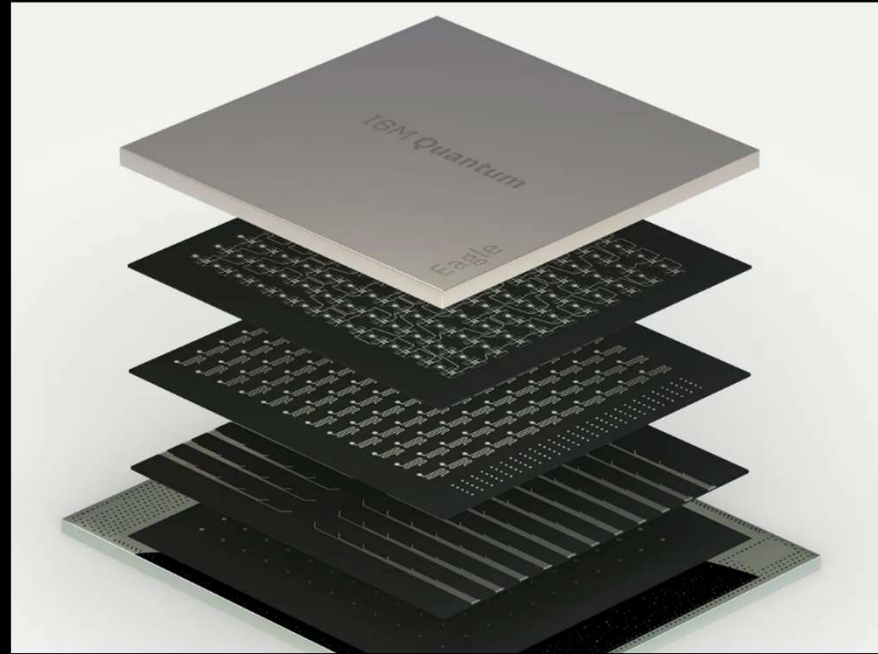
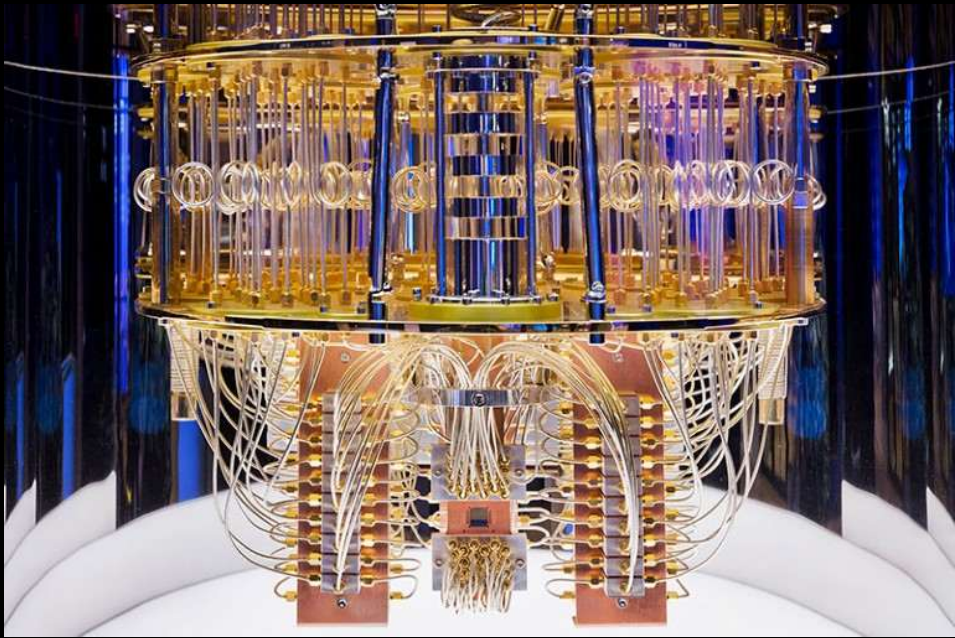
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Quantum computing promises to offer substantial speed-ups over its classical

Quantum computer sounds growing well...

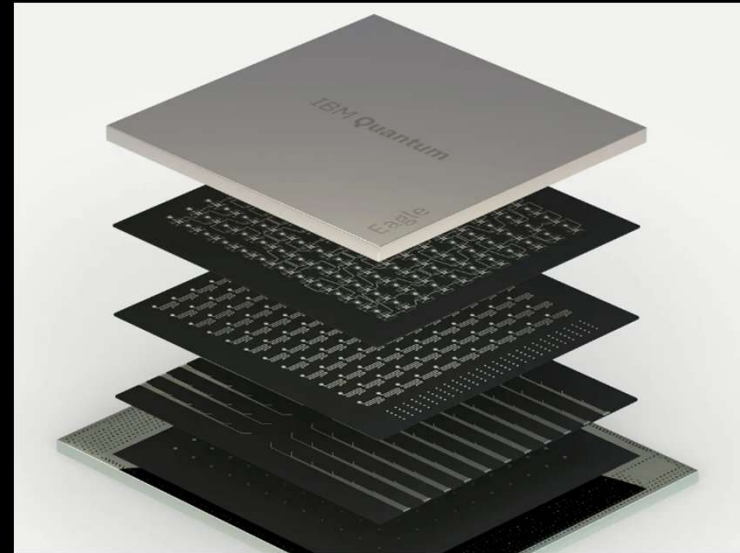
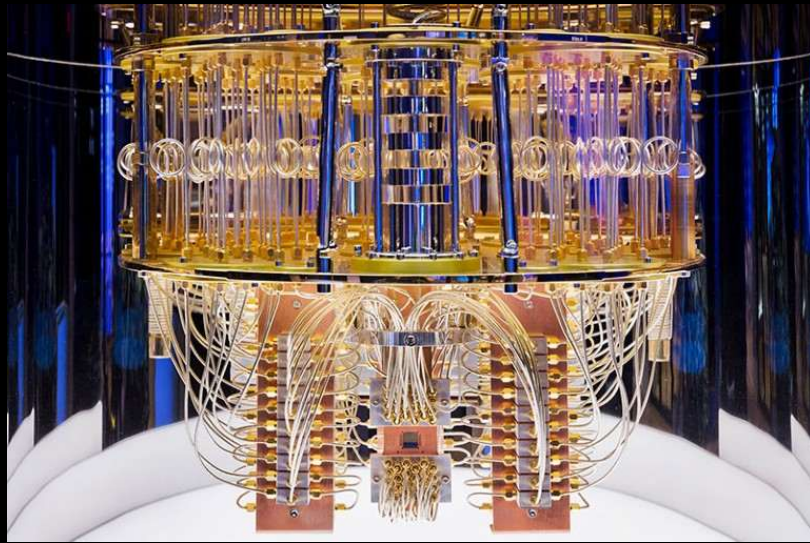


Article

Evidence for the utility of quantum computing before fault tolerance

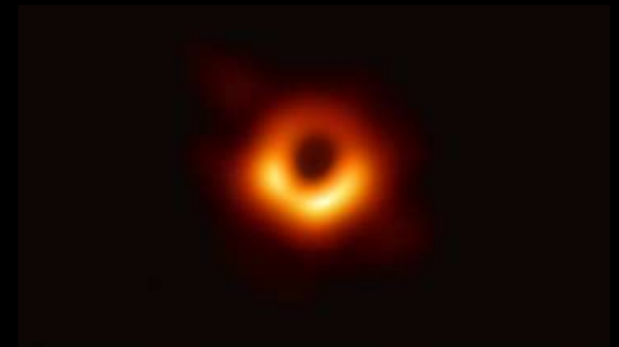
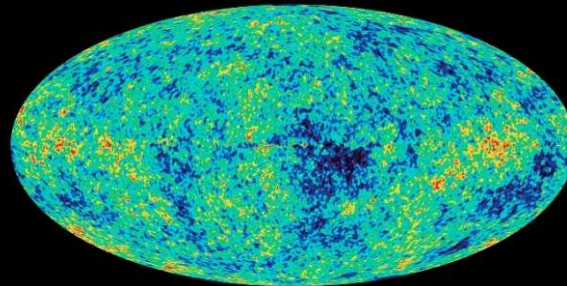
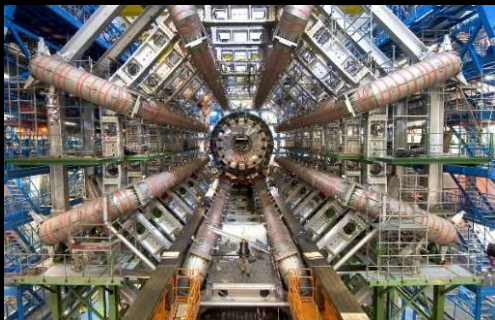
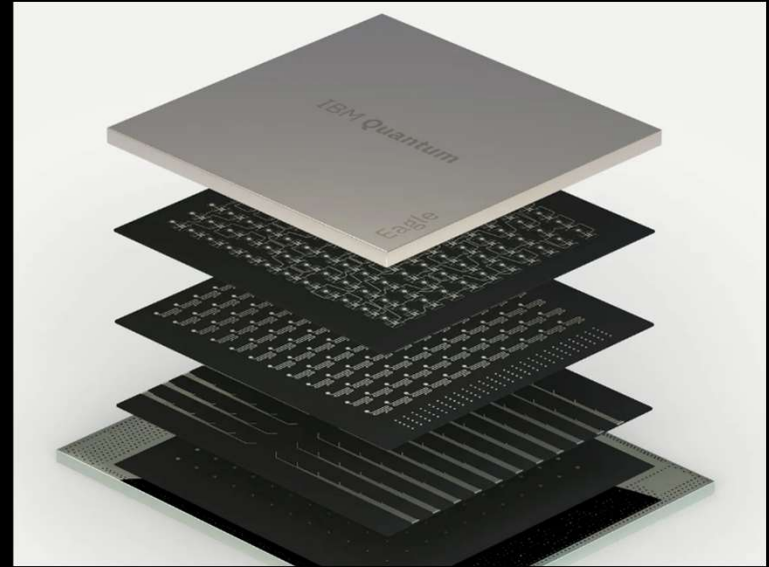
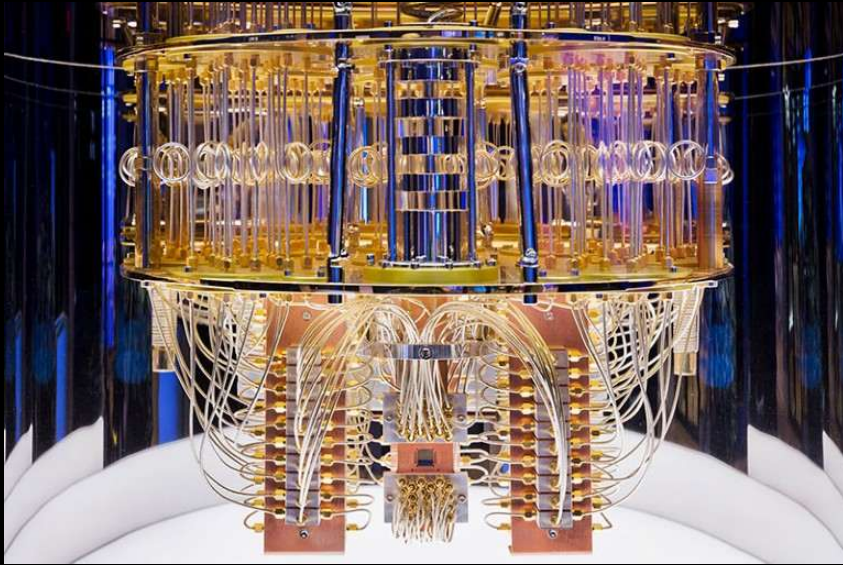
How can we use it for us?

Applications mentioned in media ?



etc...

In my mind...



etc...

What is meant by

“Application of Quantum Computation
to High Energy Physics” ??

In general, it is

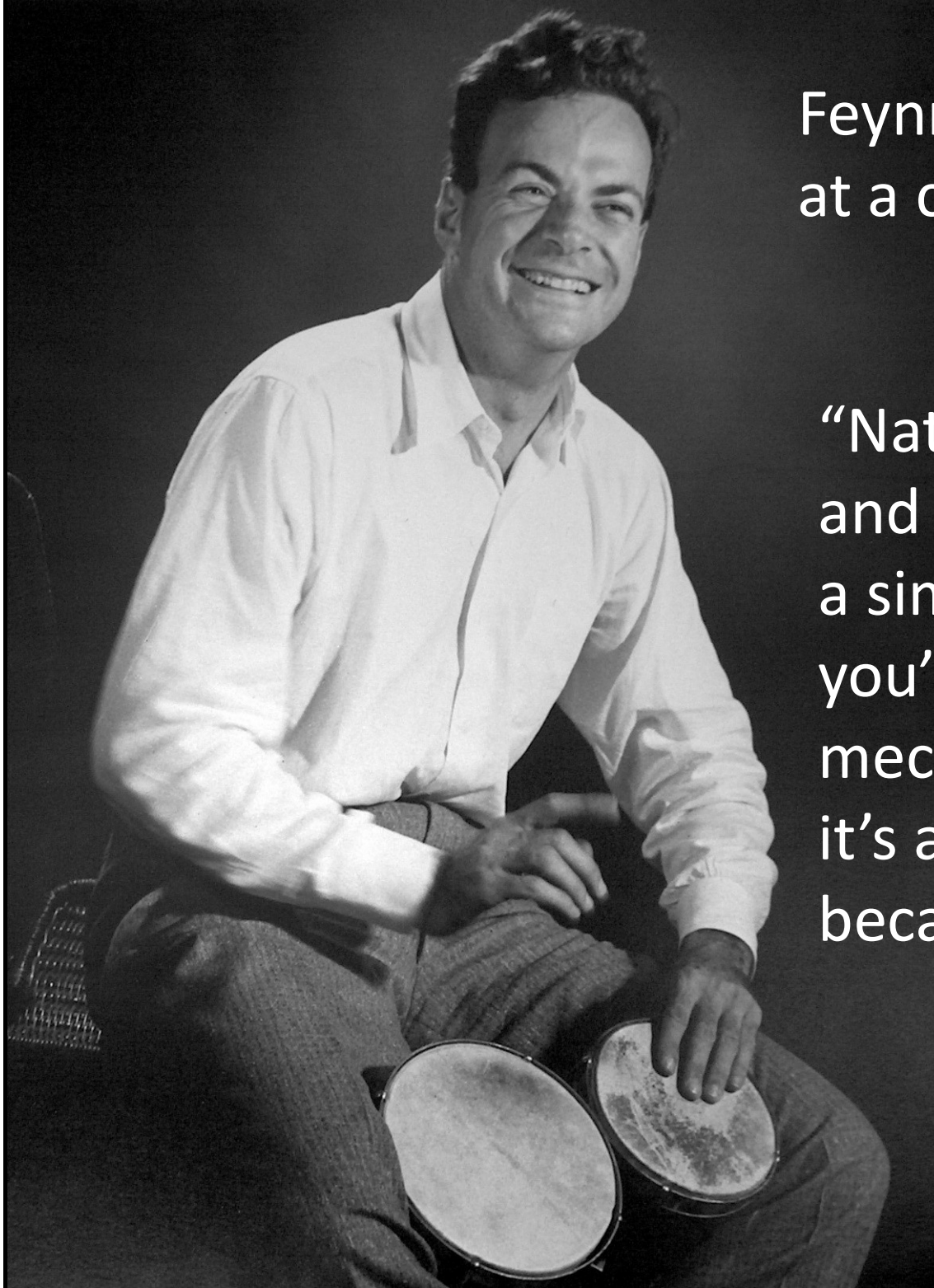
to replace (a part of) computations by quantum algorithm

Therefore,

physical meaning of **qubits** in quantum computer
depends on contexts

Here,

qubits = **states** in quantum system



Feynman as a keynote speaker
at a conference in MIT (1981):

“Nature isn’t classical, dammit,
and if you want to make
a simulation of Nature,
you’d better make it quantum
mechanical, and by golly
it’s a wonderful problem
because it doesn’t look so easy.”

Focus of this talk:

Application of Quantum Computation to Quantum Field Theory (QFT)

- Generic motivation:

simply would like to use powerful computers?

- Specific motivation:

Quantum computation is suitable for **operator** formalism

—→ Liberation from infamous **sign problem** in Monte Carlo?

(next slide)

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

- ① Discretize **Euclidean** spacetime by lattice:



& make **path integral** finite dimensional:

$$\int D\phi \mathcal{O}(\phi) e^{-S[\phi]} \quad \longrightarrow \quad \int d\phi \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

②

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

- ① Discretize **Euclidean** spacetime by lattice:



& make **path integral** finite dimensional:

$$\int D\phi \mathcal{O}(\phi) e^{-S[\phi]} \quad \longrightarrow \quad \int d\phi \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

- ② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a **probability**:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\#(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

problematic when Boltzmann factor **isn't** $R_{\geq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term — complex action
- chemical potential — indefinite sign of fermion determinant
- real time — “ $e^{iS(\phi)}$ ” *much worse*

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

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Examples w/ sign problem:

- topological term — complex action
- chemical potential — indefinite sign of fermion determinant
- real time — “ $e^{iS(\phi)}$ ” *much worse*

In **operator formalism**,

sign problem is absent from the beginning

Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞ -dim. Hilbert space
regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞ -dim. Hilbert space
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Technically, computers have to

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Quantum computers do this job?

Contents

0. Introduction

1. Quantum computation (QC)

2. Ising model

3. QC for QFT

4. Outlook

Qubit = Quantum Bit

Qubit = Quantum system w/ 2 dim. Hilbert space

Basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{“computational basis”}$$

Generic state:

$$\alpha|0\rangle + \beta|1\rangle \quad \text{w/} \quad |\alpha|^2 + |\beta|^2 = 1$$

Ex.) Spin 1/2 system:

$$|0\rangle = |\uparrow\rangle, \quad |1\rangle = |\downarrow\rangle$$

(We don't need to mind how it is realized as “users”)

Multiple qubits

2 qubits – 4 dim. Hilbert space:

$$|\psi\rangle = \sum_{i,j=0,1} c_{ij} |ij\rangle, \quad |ij\rangle \equiv |i\rangle \otimes |j\rangle$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

N qubits – 2^N dim. Hilbert space:

$$|\psi\rangle = \sum_{i_1, \dots, i_N=0,1} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle,$$

$$|i_1 i_2 \dots i_N\rangle \equiv |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$$

Rule of the game

Do something interesting by a combination of

1. action of Unitary operators:

$$|\psi\rangle \quad \text{---} \boxed{U} \text{---} \quad U|\psi\rangle$$

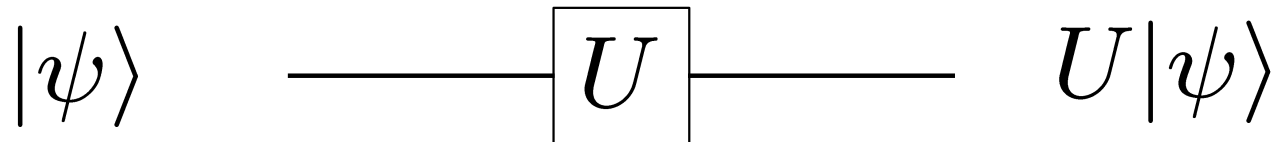
&

2.

Rule of the game

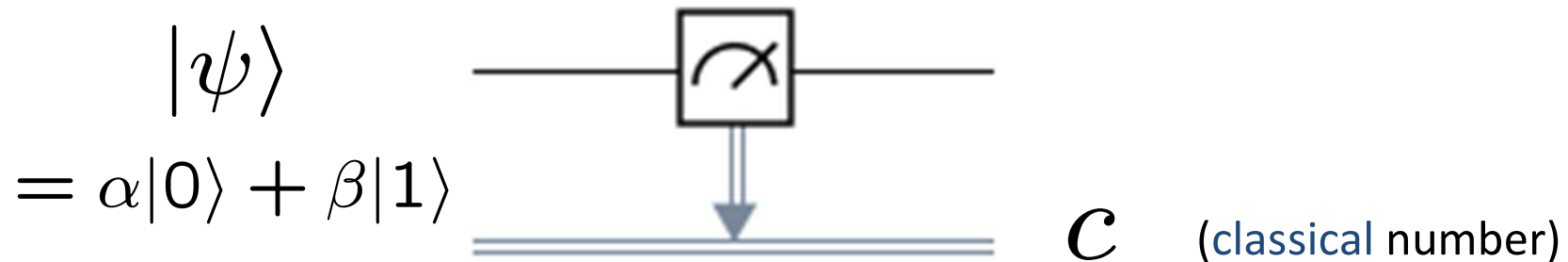
Do something interesting by a combination of

1. action of Unitary operators:



&

2. measurements:



$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Unitary gates used here

X, Y, Z gates: (just Pauli matrices)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

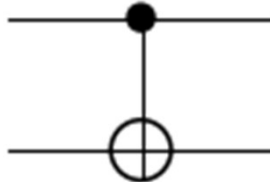
X is “**NOT**”: $X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$

R_X, R_Y, R_Z gates:

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \quad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \quad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

Controlled X (NOT) gate:

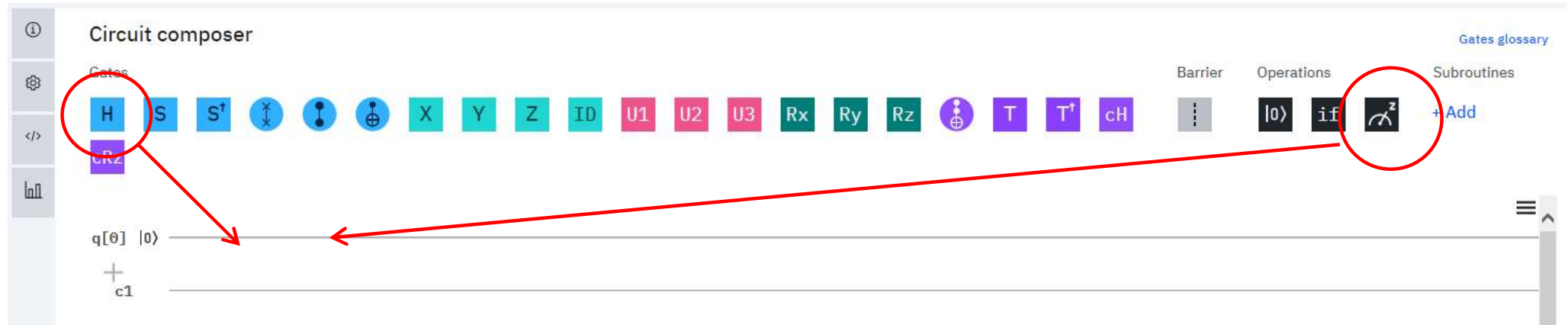
$$\begin{cases} CX|00\rangle = |00\rangle, & CX|01\rangle = |01\rangle, \\ CX|10\rangle = |11\rangle, & CX|11\rangle = |10\rangle \end{cases}$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$


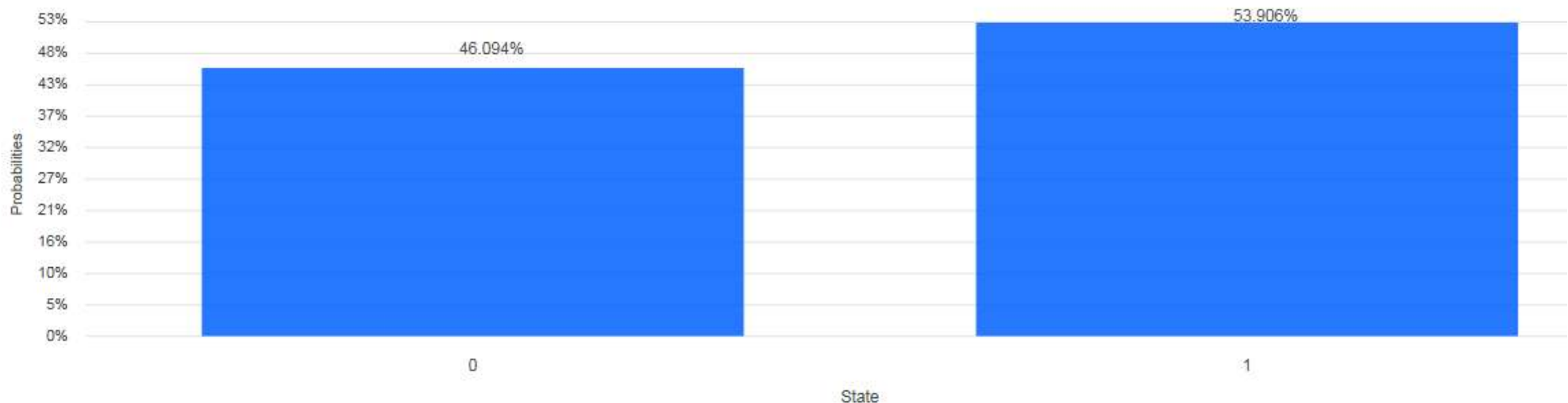
Atmosphere (?) of using quantum computer...

Suppose we'd like to measure the state: $H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$

Screenshot of IBM Quantum:



Output of 1024 times measurements (“shots”) :



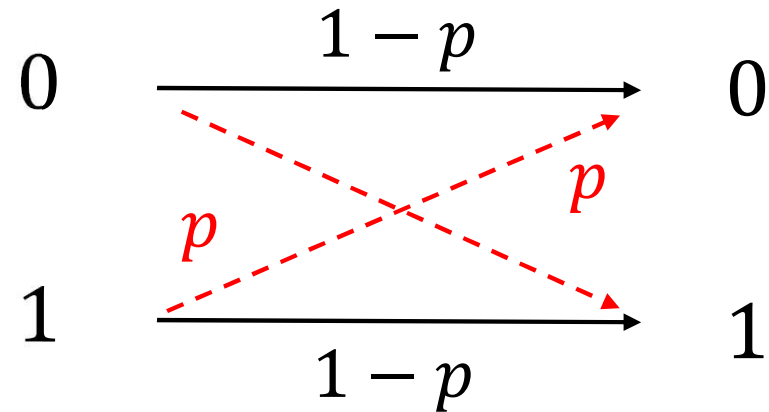
Idea: express physical quantities in terms of “probabilities”
& measure the “probabilities”

Errors in classical computers

Computer interacts w/ environment → error/noise

Errors in classical computers

Computer interacts w/ environment $\Rightarrow$ error/noise

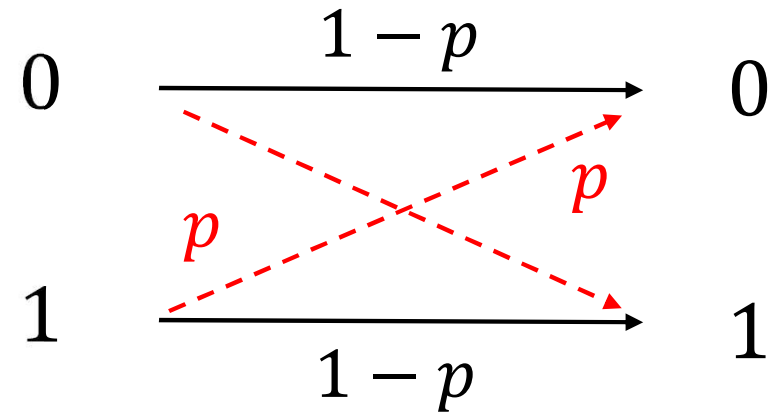


Suppose we send a bit but have “error” in probability p

A simple way to correct errors:

Errors in classical computers

Computer interacts w/ environment $\Rightarrow$ error/noise



Suppose we send a bit but have “error” in probability p

A simple way to correct errors:

① Duplicate the bit (**encoding**): $0 \rightarrow 000$, $1 \rightarrow 111$

② Error detection & correction by “**majority voting**”:

$001 \rightarrow 000$, $011 \rightarrow 111$, etc...

$\Rightarrow P_{\text{failed}} = 3p^2(1 - p) + p^3$ (improved if $p < 1/2$)

Errors in quantum computers

Computer interacts w/ environment ➡ **error/noise**

Unknown unitary operators are multiplied:

(in addition to decoherence & measurement errors)

$$|\psi\rangle \xrightarrow[\text{error!}]{\text{error!}} U|\psi\rangle$$

not only bit flip!

Errors in quantum computers

Computer interacts w/ environment → **error/noise**

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We need to include “quantum error corrections”
but it seems to require a **huge** number of qubits

~ major obstruction of the development

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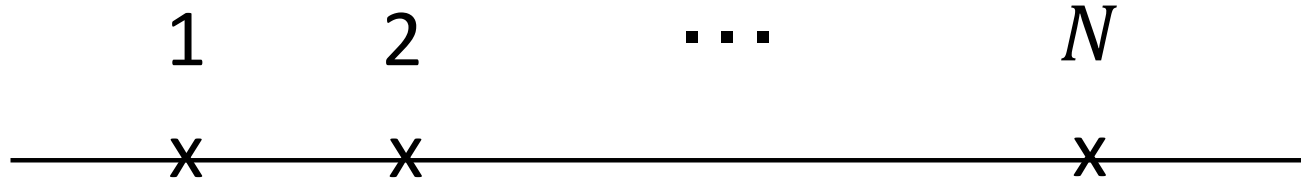
1. Quantum computation (QC)

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The (1+1)d transverse Ising model



Hamiltonian (w/ open b.c.):

$(X_n, Y_n, Z_n: \sigma_{1,2,3} \text{ at site } n)$

$$\hat{H} = -J \sum_{n=1}^{N-1} Z_n Z_{n+1} - h \sum_{n=1}^N X_n$$

Let's construct **the time evolution op.** $e^{-i\hat{H}t}$

Time evolution operator

Time evolution of any state is studied by acting the operator

$$e^{-i\hat{H}t} = e^{-i(H_X + H_{ZZ})t}$$

where

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1Z_2$$

How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

Step 1: Suzuki-Trotter decomposition:

Time evolution operator

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How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

Step 1: Suzuki-Trotter decomposition:

($\exists$ higher order improvements)

$$\begin{aligned} e^{-i\hat{H}t} &= \left(e^{-i\hat{H}\frac{t}{M}} \right)^M \\ &\simeq \left(e^{-iH_X\frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}} \right)^M + \mathcal{O}(1/M) \end{aligned} \quad (M: \text{large positive integer})$$

Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ} \frac{t}{M}} \right)^M$$

The **1st** one is trivial:

$$e^{-iH_X \frac{t}{M}} = e^{-i\frac{ht}{M} X_2} e^{-i\frac{ht}{M} X_1} = \overset{\text{acting on qubit 2}}{R_X^{(2)}\left(\frac{2ht}{M}\right)} \overset{\text{acting on qubit 1}}{R_X^{(1)}\left(\frac{2ht}{M}\right)}$$

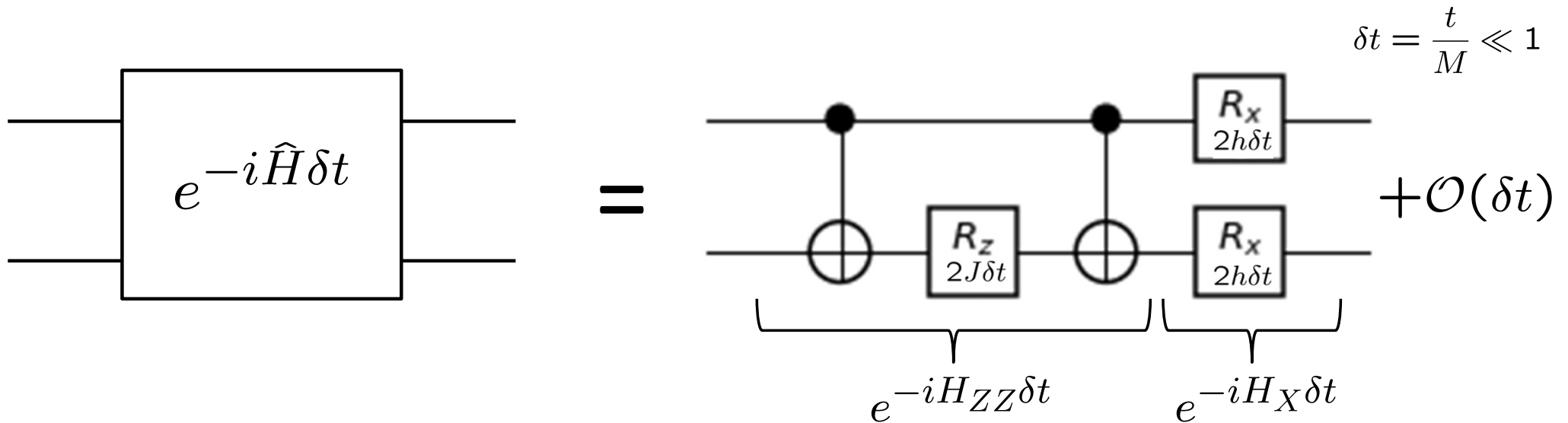
The **2nd** one is nontrivial:

$$e^{-iH_{ZZ} \frac{t}{M}} = e^{-i\frac{Jt}{M} Z_1 Z_2} = \cos \frac{Jt}{M} - i Z_1 Z_2 \sin \frac{Jt}{M}$$

One can show

$$e^{-i\frac{Jt}{M} Z_1 Z_2} = CX R_Z^{(2)}\left(\frac{2Jt}{M}\right) CX$$

“Computational cost” for large size system



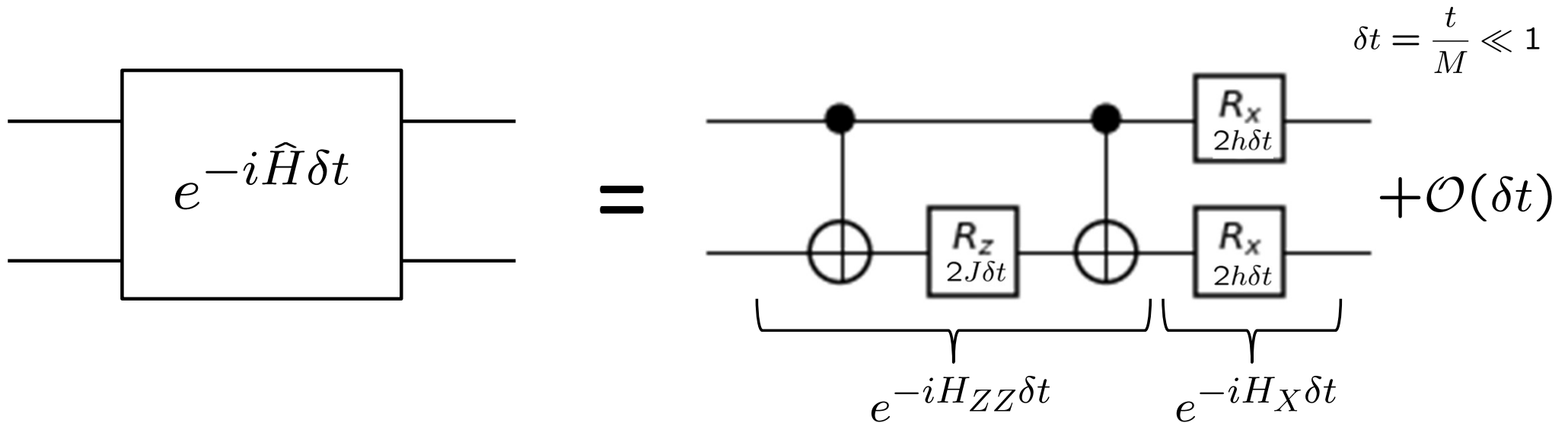
Classical computer (naively)

multiplications of matrices to vectors w/ sizes = 2^N

exponentially large steps

Quantum computer

“Computational cost” for large size system

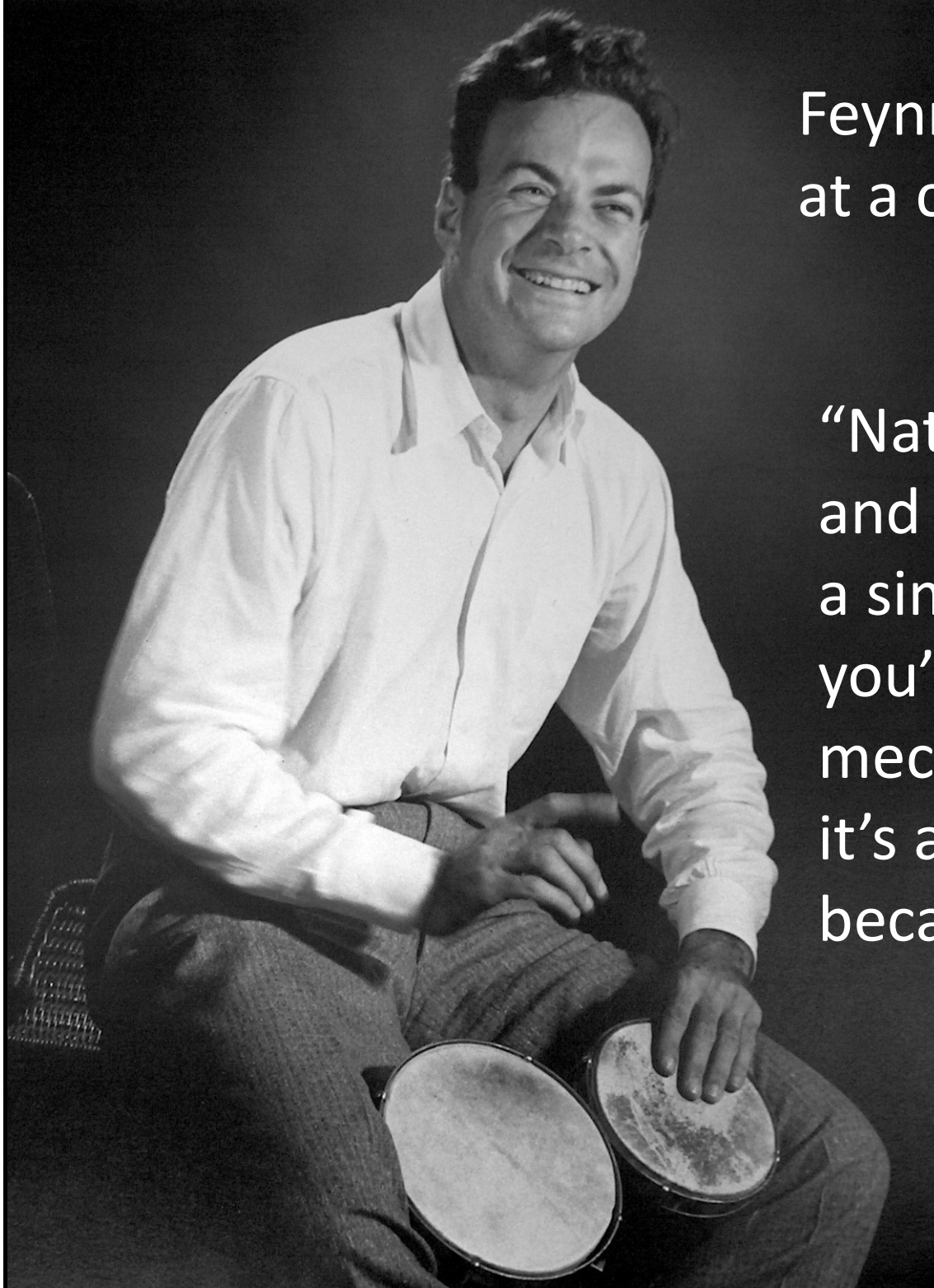


Classical computer (naively)

multiplications of matrices to vectors w/ sizes = 2^N
exponentially large steps

Quantum computer

time evolution = $O(NM)$ experimental operations
polynomial steps



Feynman as a keynote speaker
at a conference in MIT (1981):

“Nature isn’t classical, dammit,
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“Regularization” of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

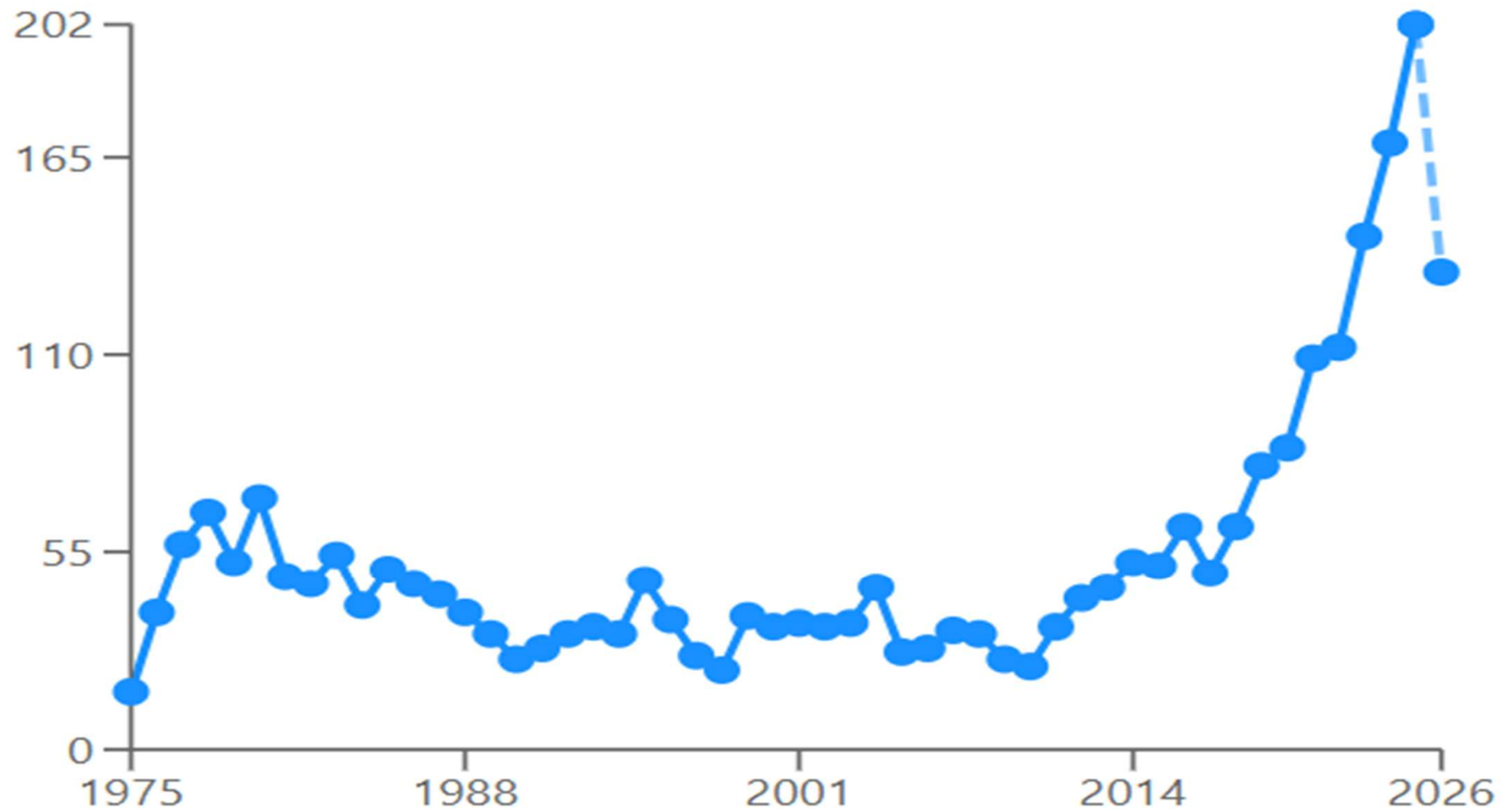
————→ Make it finite dimensional!

- **Fermion** is easiest (up to doubling problem)
 - Putting on spatial lattice, Hilbert sp. is finite dimensional
- **scalar**
 - Hilbert sp. at each site is ∞ dimensional
(need truncation or additional regularization)
- **gauge field** (w/ kinetic term)
 - no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
 - ∞ dimensional Hilbert sp. in higher dimensions

Citation history of “Hamiltonian Formulation of Wilson's Lattice Gauge Theories” by Kogut-Susskind

(totally 2794 at this moment)

Citations per year



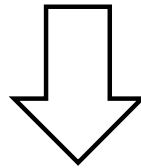
(1+1)d free Dirac fermion

Continuum:

$$H = \int dx \left[-i \bar{\psi} \gamma^1 \partial_1 \psi + m \bar{\psi} \psi \right]$$

$$\psi(x) = \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} \quad \begin{aligned} \gamma^0 &= \sigma_3, \\ \gamma^1 &= i\sigma_2 \end{aligned}$$

$$= \int dx \left[-i(\psi_u^\dagger \partial_1 \psi_d + \psi_d^\dagger \partial_1 \psi_u) + m(\psi_u^\dagger \psi_u - \psi_d^\dagger \psi_d) \right]$$



Lattice (w/ N sites and spacing a):

“Staggered fermion” [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{array}{l} \longrightarrow \text{odd site} \\ \longrightarrow \text{even site} \end{array}$$

$$H = -\frac{i}{2a} \sum_{n=1}^{N-1} (\chi_n^\dagger \chi_{n+1} - \chi_{n+1}^\dagger \chi_n) + m \sum_{n=1}^N (-1)^n \chi_n^\dagger \chi_n$$

$$\{\chi_m, \chi_n^\dagger\} = \delta_{mn}, \quad \{\chi_m, \chi_n\} = 0$$

Jordan-Wigner transformation

$$\{\chi_m, \chi_n^\dagger\} = \delta_{mn}, \quad \{\chi_m, \chi_n\} = 0$$

This is satisfied by the operator:

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - iY_n}{2} \left(\prod_{i=1}^{n-1} -iZ_i \right) \quad (X_n, Y_n, Z_n: \sigma_{1,2,3} \text{ at site } n)$$

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Then the system is mapped to the spin system:

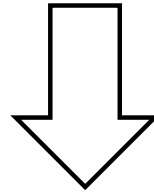
$$\hat{H} = \frac{w}{2} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=1}^N (-1)^n Z_n$$

Now we can apply quantum algorithms to QFT!

Pure Maxwell theory

Continuum:

$$\mathcal{H} = \frac{1}{2} E_i^2 + \frac{1}{2} B_i^2 \quad \partial_i E^i = 0$$



Lattice:

$$\mathcal{H} = \frac{a^d}{2} \sum_{\mathbf{n}, i} L_{\mathbf{n}, i}^2 + \text{Re} \sum_{\text{plaquette}} \sum_{i < j} \prod_{P \in \text{plaquette}} U_P$$

$$[U_{\mathbf{m}, i}, L_{\mathbf{n}, j}] = i \delta_{ij} \delta_{\mathbf{m}, \mathbf{n}}$$

Gauss law:

$$\sum_i (L_{\mathbf{n} + \mathbf{e}_i, i} - L_{\mathbf{n}, i}) = 0$$

Ex. (1+1)d pure Maxwell theory w/ θ

Continuum:

$$\mathcal{L} = \frac{1}{2g^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} \quad \xrightarrow{\quad \Pi = \frac{1}{g^2} \dot{A} + \frac{\theta}{2\pi} \quad} \quad \mathcal{H} = \frac{1}{2} \left(\Pi - \frac{\theta}{2\pi} \right)^2$$

Lattice:

$$H = \frac{g^2 a}{2} \sum_n \left(L_n + \frac{\theta}{2\pi} \right)^2 \quad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$

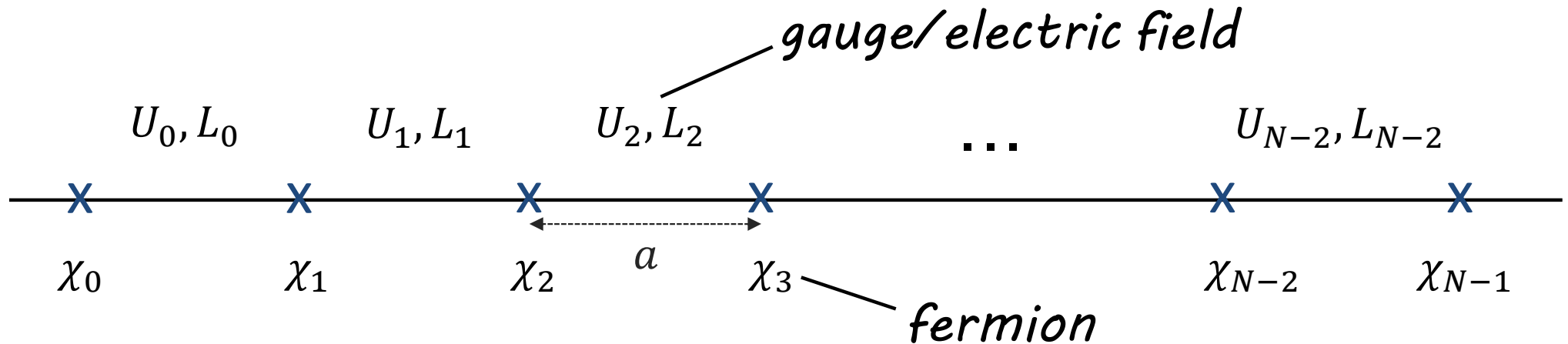
Gauss law:

$$L_{n+1} - L_n = 0$$

$$\left\{ \begin{array}{l} \text{▪ open b.c.} \\ L_n = L_{n-1} = L_{n-2} = \cdots = L_1 = (b.c.) \\ \text{▪ p.b.c.} \\ L_n = L_{n-1} = \cdots = L_1 = \cdots = L_{n+1} = L_n \end{array} \right.$$

one d.o.f. remains

Charge- q Schwinger model (1+1d QED)



$$H = J \sum_{n=0}^{N-2} \left(L_n + \frac{\theta_0}{2\pi} \right)^2 - i w \sum_{n=0}^{N-2} \left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \text{h.c.} \right] + m \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n$$

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Physical states are subject to **Gauss law**:

$$(L_n - L_{n-1}) |\text{phys}\rangle = q \left[\chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2} \right] |\text{phys}\rangle$$

" $\nabla \cdot \vec{E}(x)$ "
" $\rho(x)$ "

Schwinger model as qubits

1. Take **open b.c.** & solve **Gauss law**:

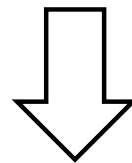
$$L_n = L_{-1} + q \sum_{j=1}^n \left(\chi_j^\dagger \chi_j - \frac{1 - (-1)^j}{2} \right) \quad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

3. Map to spin system: $\chi_n = \frac{X_n - iY_n}{2} \left(\prod_{i=1}^{n-1} -iZ_i \right)$ ($X_n, Y_n, Z_n: \sigma_{1,2,3}$ at site n)

"Jordan-Wigner transformation"

[Jordan-Wigner'28]



$$H = J \sum_{n=0}^{N-2} \left[q \sum_{i=0}^n \frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi} \right]^2 + \frac{w}{2} \sum_{n=0}^{N-2} [X_n X_{n+1} + Y_n Y_{n+1}] + \frac{m}{2} \sum_{n=0}^{N-1} (-1)^n Z_n$$

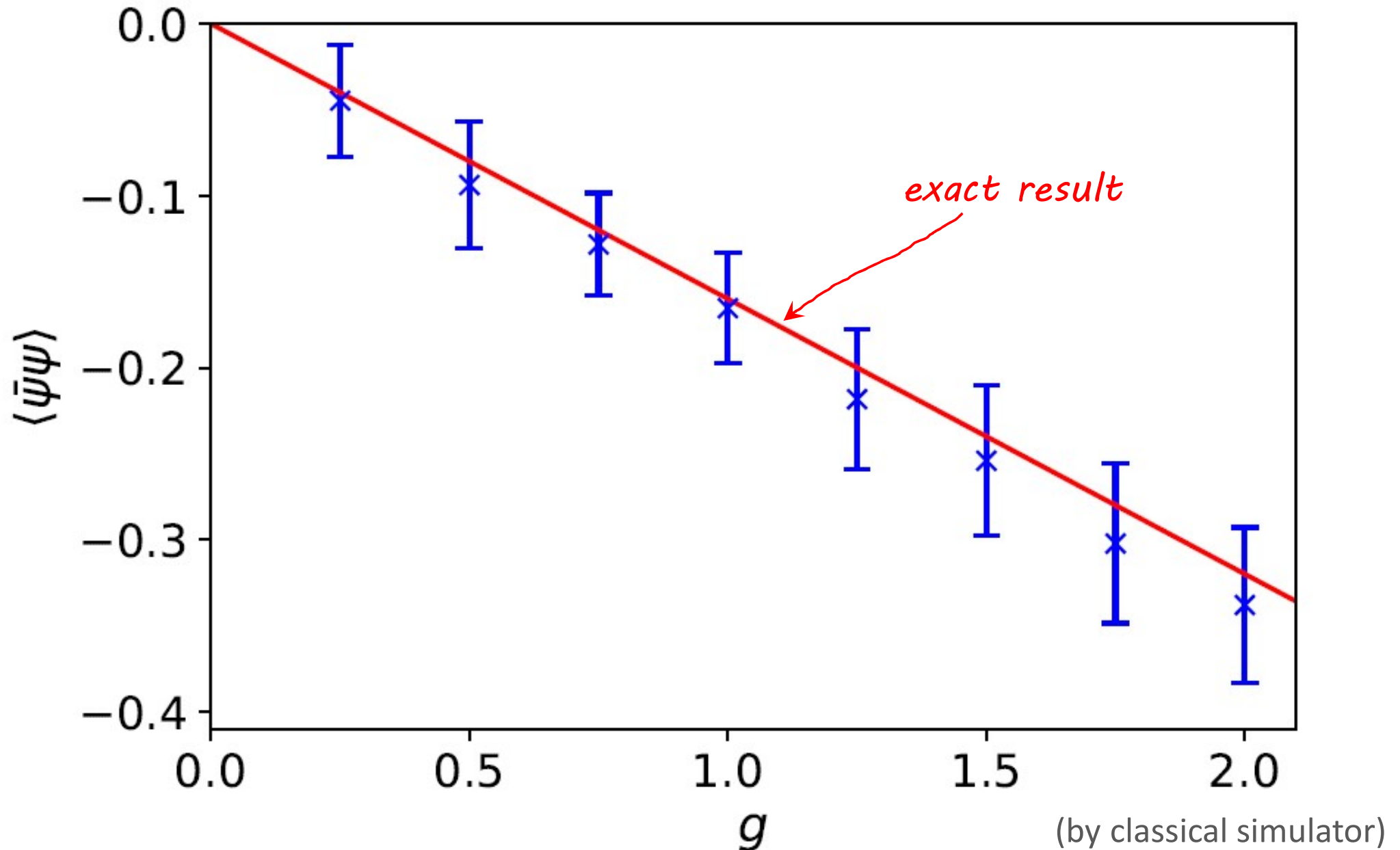
Qubit description of the Schwinger model !!

Ground state expectation value in **massless** case

$T = 100, \delta t = 0.1, N_{\max} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

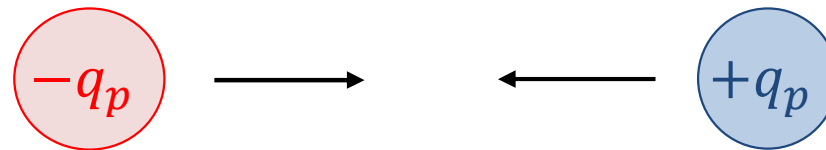
(after continuum limit)



Screening versus Confinement

Let's consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2 g^2}{2} x ?$$

Coulomb law in 1+1d
||
confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

▪ massless case:

$$V(x) = \frac{q_p^2 g^2}{2\mu} (1 - e^{-q\mu x}) \quad \text{screening} \quad \mu \equiv g/\sqrt{\pi}$$

▪ massive case:

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

▪ massless case:

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▪ massive case:

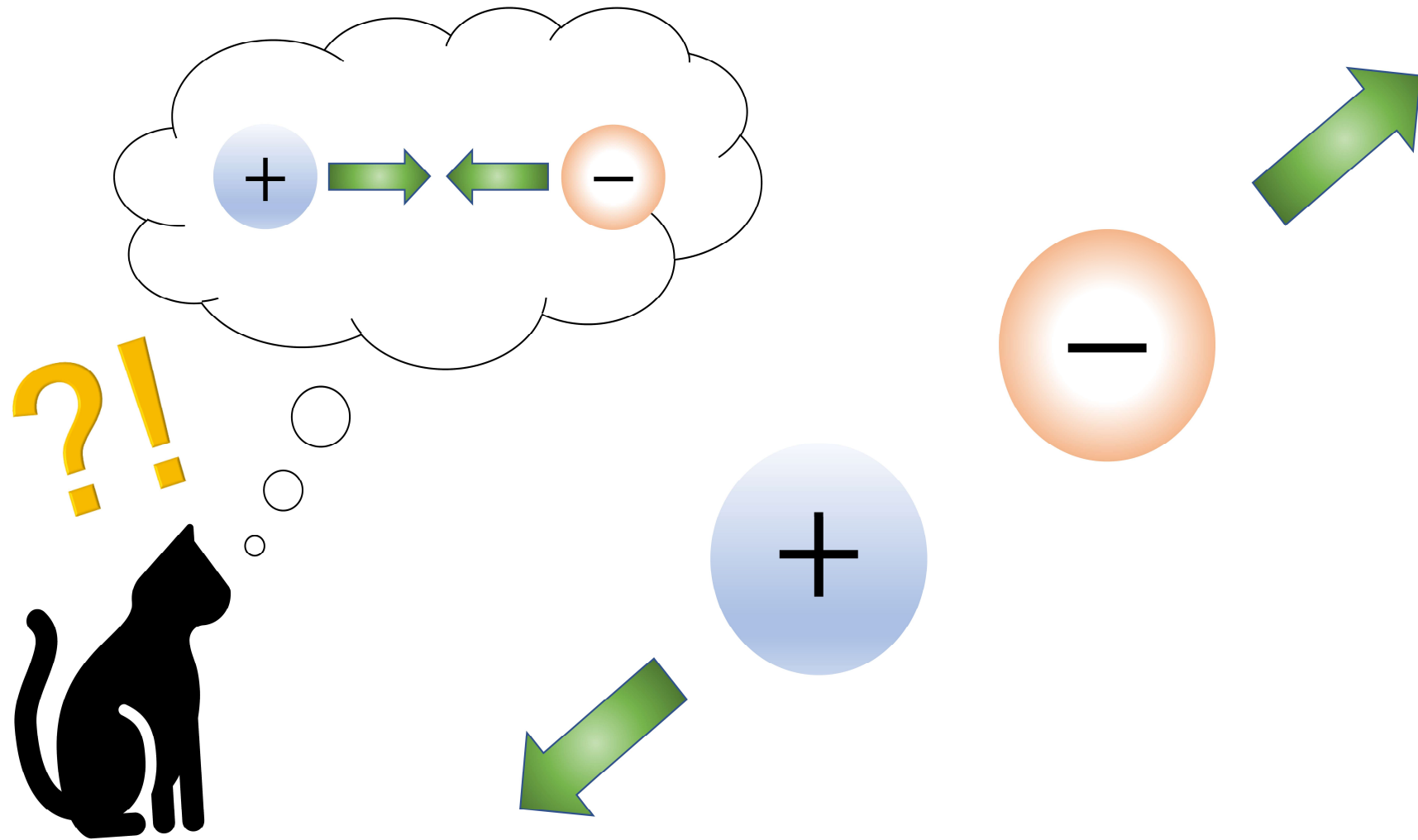
[cf. Misumi-Tanizaki-Unsal '19]

$$\Sigma \equiv ge^{\gamma}/2\pi^{3/2}$$

$$V(x) \sim mq\Sigma \left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right) \right) x \quad (m \ll g, |x| \gg 1/g)$$

$$\left\{ \begin{array}{ll} = \text{Const.} & \text{for } q_p/q = \mathbf{Z} \quad \text{screening} \\ \propto x & \text{for } q_p/q \neq \mathbf{Z} \quad \text{confinement?} \\ & \text{but sometimes negative slope!} \end{array} \right.$$

That is, as changing the parameters...



Let's explore this aspect by quantum simulation!

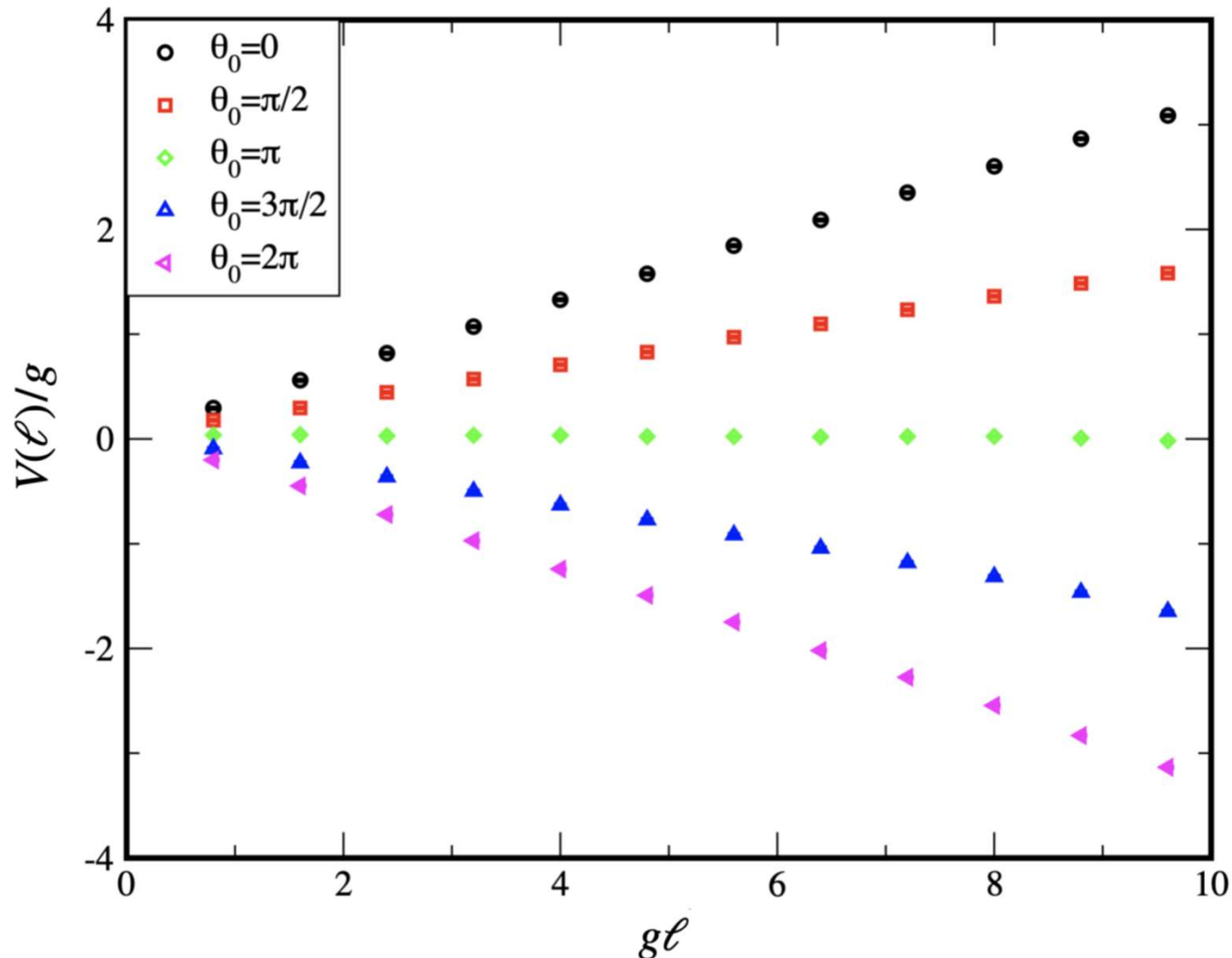
Positive / negative string tension

(by classical simulator)

[MH-Itou-Kikuchi-Tanizaki '21]

[cf. MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$



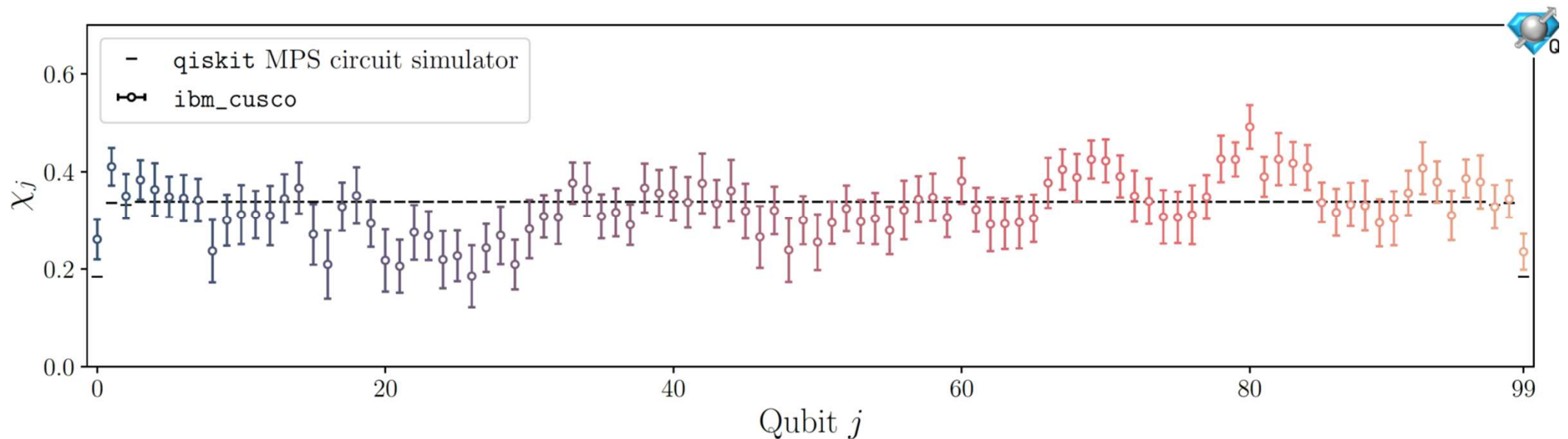
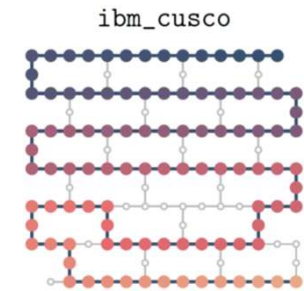
Sign(tension) changes as changing θ -angle!!

100 qubit simulation of Schwinger model

(127-qubit device: **ibm_cusco** w/ error mitigation)

[Farrel-Illa-Ciavarella-Savage '23]

Ground state exp. of local chiral condensate :



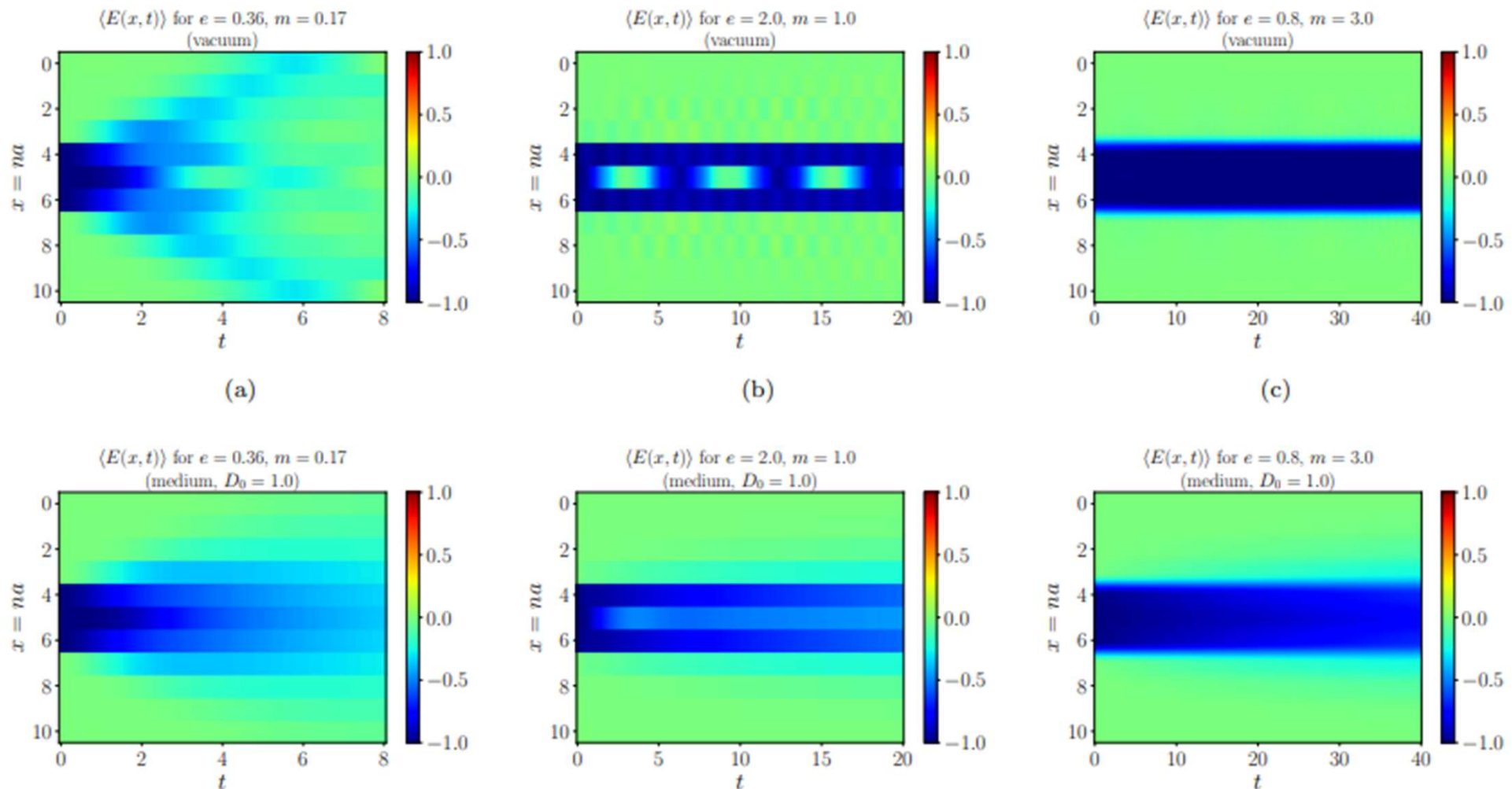
Lindblad dynamics w/ heat bath & String breaking

Setup:

[Lee-Mulligan-Ringer-Yao '23]

Schwinger model in scalar field environment w/ Yukawa int.

Energy densities in vacuum v.s. medium: (by classical simulator)



Quantum Mpemba effect

[in preparation, Fujii-Fujimura-MH-Le]

Classical Mpemba effect:

(Sometimes) hotter initial state has faster cooling

Quantum Mpemba effect:



Quantum Mpemba effect

[in preparation, Fujii-Fujimura-MH-Le]

Classical Mpemba effect:

(Sometimes) hotter initial state has faster cooling



Quantum Mpemba effect:

(Sometimes) initial state w/ more broken symmetry has faster symmetry restoration

Quantum Mpemba effect

[in preparation, Fujii-Fujimura-MH-Le]

Classical Mpemba effect:

(Sometimes) hotter initial state has faster cooling



Quantum Mpemba effect:

(Sometimes) initial state w/ more broken symmetry has faster symmetry restoration

It has been observed in spin chains & CFTs.

[cf. Joshi et al. '24, Benini-Godet-Singh '24, Fujimura-Shimamori '25, etc.]

How about non-CFTs such as Schwinger model?

Quantum Mpemba effect (cont'd)

Focus

[in preparation, Fujii-Fujimura-MH-Le]

Remaining $U(1)$ global sym. of the gauge sym.

Initial state

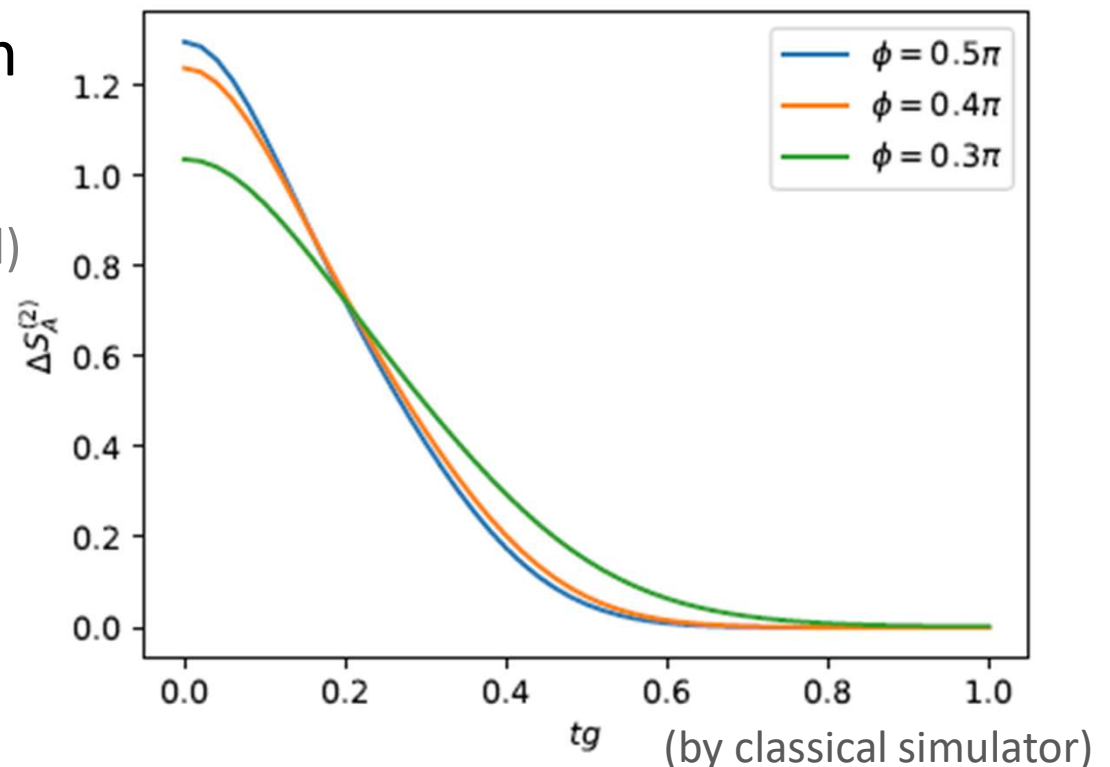
$$|\psi_1(\phi)\rangle_F = e^{-i\frac{\phi}{2} \sum_{n=1}^N Y_n} |00 \dots 0\rangle$$

Quantification of sym. breaking = Entanglement asymmetry

We propose a quantum algorithm
to efficiently compute Renyi
entanglement asymmetry: (skipped)

$$\Delta S_A^{(2)} = \log \text{Tr}_A [\rho_A^2] - \log \text{Tr}_A [\rho_{A,S}^2]$$

$$\rho_{A,S} := \sum_{q_A \in \mathbb{Z}} \Pi_{q_A} \rho_A \Pi_{q_A} ,$$



Some other applications

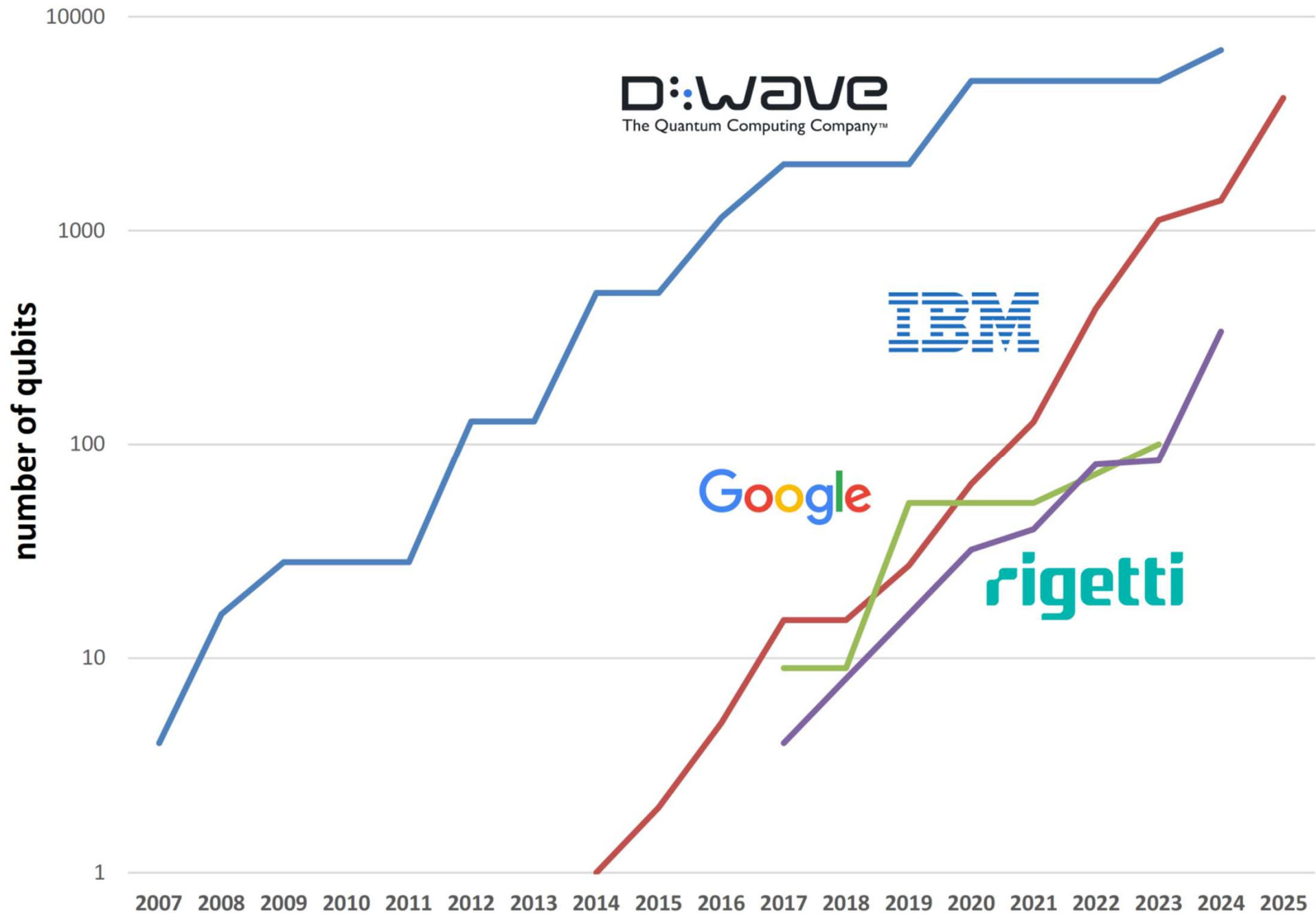
- Efficient simulation of (2+1)d $U(1)$ gauge th.
[Kane-Grabowska-Nachman-Bauer '22]
- Scattering [cf. Chai-Crippa-Jansen-Kuhn-Pascuzzi-Tacchino-Tavernelli '23]
- finding energy spectrum [in preparation, MH-Ghim]
- Local thermalization in 2+1d $SU(2)$ gauge theory
[Chen-Chen-Meher-Muller-Schafer-Yao '26]
- Chiral fermion [Hayata-Nakayama-Yamamoto '23]
- Formulations of Non-abelian gauge th.
[cf. Byrnes-Yamamoto '05, Buser-Gharibyan-Hanada-MH-Liu '20 Zache-Gonzalez-Cuadra-Zoller '23, Hayata-Hidaka '23]
- Conformal bootstrap [Bao-Liu '18]
- quantum machine learning [Nagano-Miessen-Onodera-Tavernelli-Tacchino-Terashi '23, Hammed-Nojiri-Yamazaki '24, etc...]
- String/M-theory [Gharibyan-Hanada-MH-Liu '20]

etc...

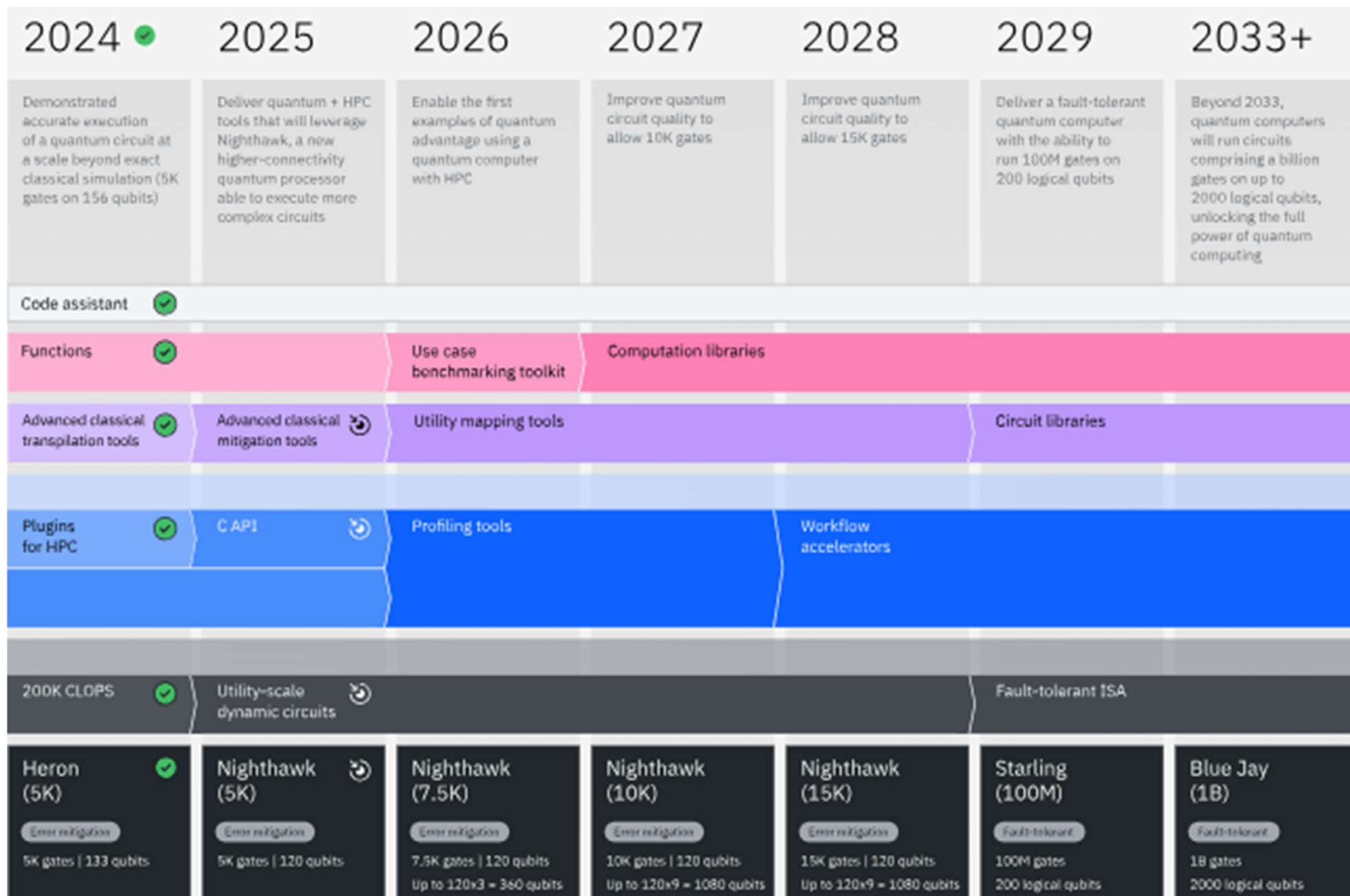


Outlook

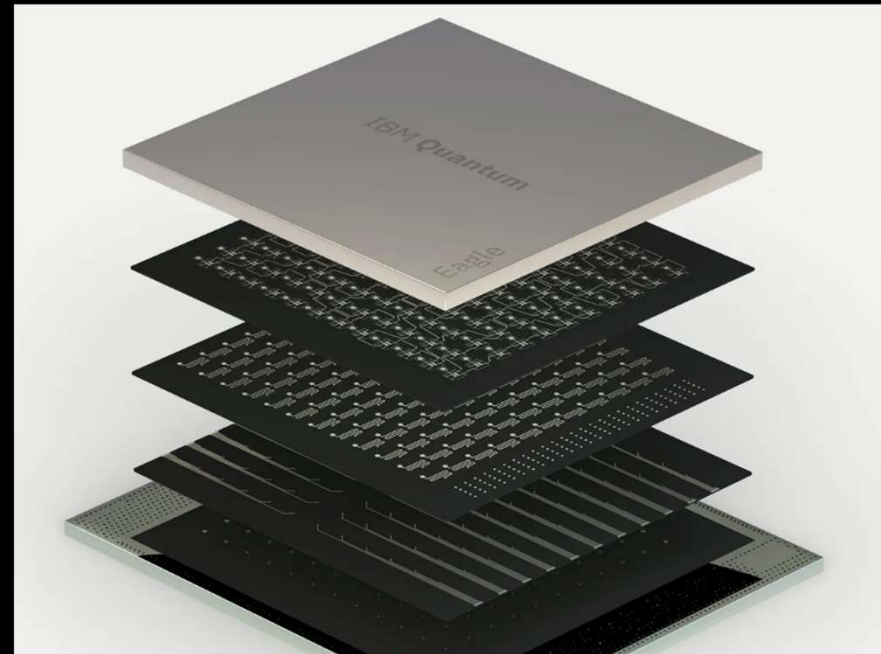
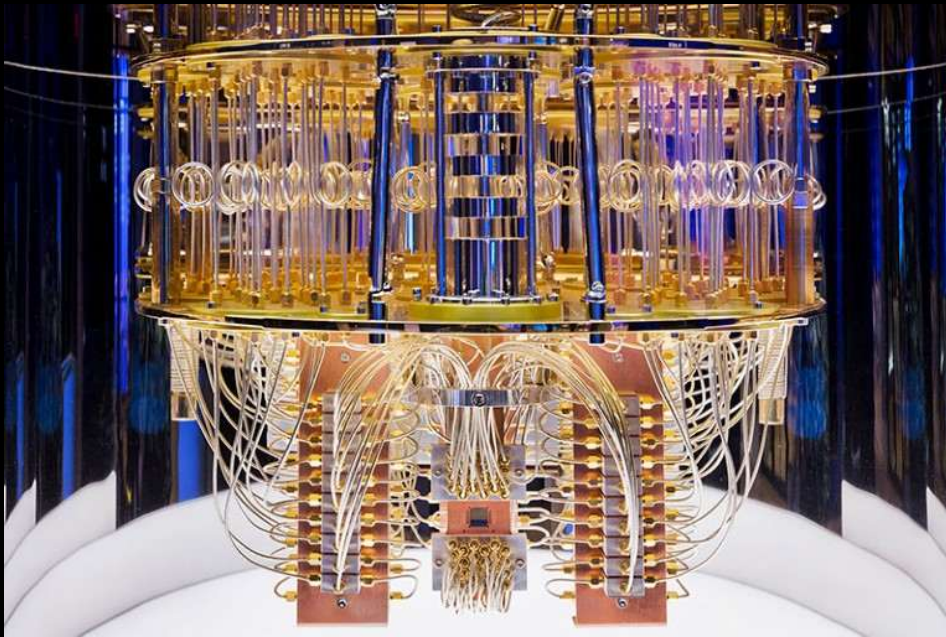
“Quantum” Moore’s law?



Cf. IBM's roadmap



The challenge by IBM's 127-qubit device



Article

Evidence for the utility of quantum computing before fault tolerance

<https://doi.org/10.1038/s41586-023-06096-3>

Received: 24 February 2023

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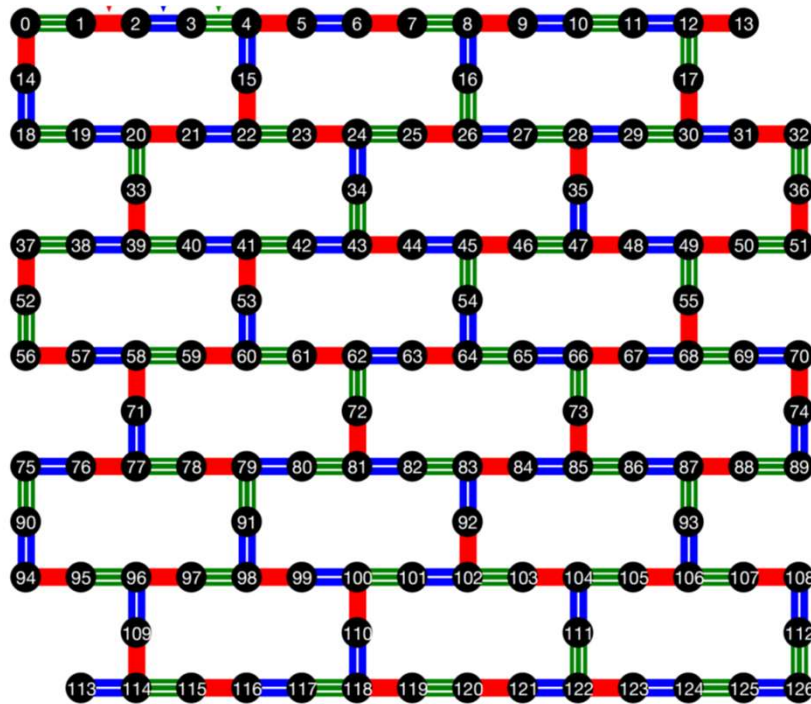
Published online: 14 June 2023

Youngseok Kim^{1,6}✉, Andrew Eddins^{2,6}✉, Sajant Anand³, Ken Xuan Wei¹, Ewout van den Berg¹, Sami Rosenblatt¹, Hasan Nayfeh¹, Yantao Wu^{3,4}, Michael Zaletel^{3,5}, Kristan Temme¹ & Abhinav Kandala¹✉

Quantum computing promises to offer substantial speed-ups over its classical

The challenge by IBM's 127-qubit device (cont'd)

Task: **time evolution** of **Ising model** on a lattice
w/ shape = the qubit config. of the device



$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j + h \sum_i X_i,$$

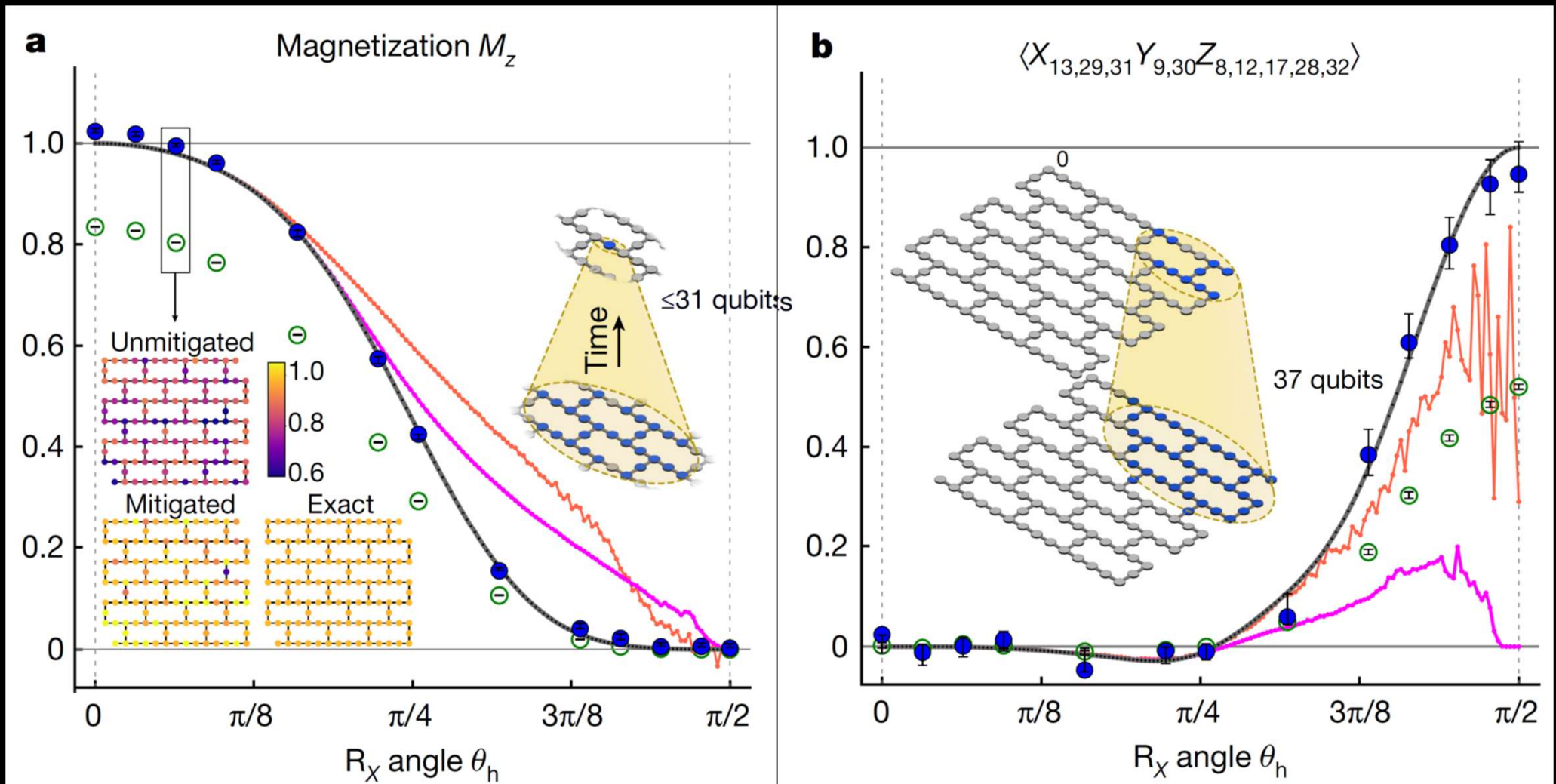
$$|\psi(t)\rangle := e^{-iHt} |00 \dots 0\rangle$$

$$\langle \psi(t) | \mathcal{O} | \psi(t) \rangle$$

Strategy: Suzuki-Trotter approximation
+ error mitigation by extrapolation

The challenge by IBM's 127-qubit device (cont'd)

○ Unmitigated ● Mitigated — MPS ($\chi = 1,024$; 127 qubits) — isoTNS ($\chi = 12$; 127 qubits) — Exact



“Quantum supremacy”?

But...

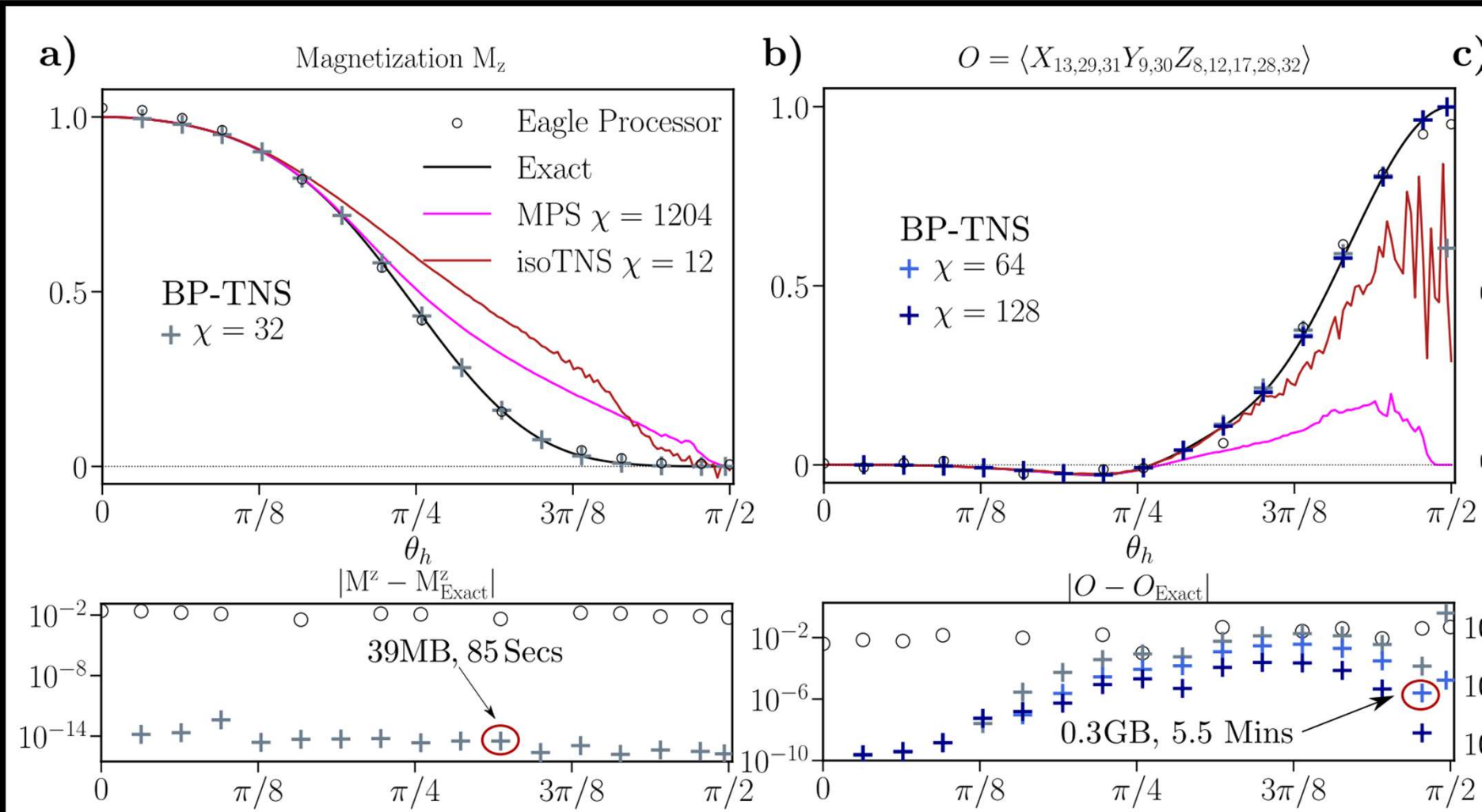
arXiv > quant-ph > arXiv:2306.14887

Quantum Physics

[Submitted on 26 Jun 2023]

Efficient tensor network simulation of IBM's kicked Ising experiment

Joseph Tindall, Matt Fishman, Miles Stoudenmire, Dries Sels



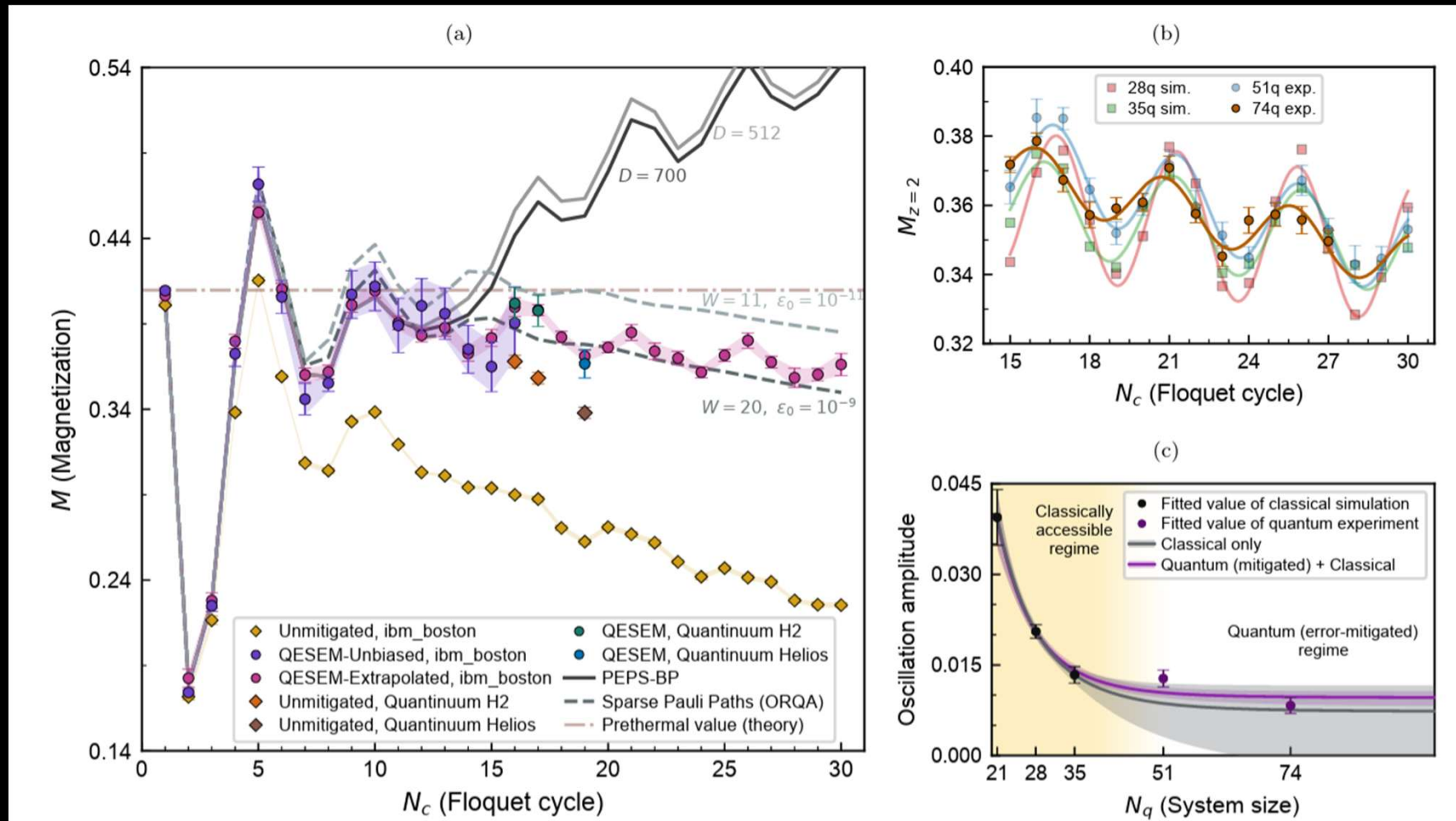
The latest challenge (using IBM & Quantinuum's devices)

Problem: time evolution in Floquet Ising model

$$U_F = \prod_{r=1}^3 \left[e^{-i\frac{\theta_{zz}}{2} C_r} e^{-i\frac{\theta_z}{2} Z_\Sigma} e^{-i\frac{\theta_x}{2} X_\Sigma} \right],$$

$$X_\Sigma = \sum_i X_i, \quad Z_\Sigma = \sum_i Z_i, \quad C_r = \sum_{\langle ij \rangle \in E_r} Z_i Z_j$$

[Leviatan et al., July 2026]



Applications to astro./cosmo. so far

- Inflation [Liu-Li '20] [work in progress, MH-Hong]
- Neutrino physics [Balentekin-Cervia-Patwardhan-Rrapaj-Siwach '23 etc.]
- Dark sector showers [Chigusa-Yamazaki '22, Bauer-Chigusa-Yamazaki '23]
- Boltzmann eq. [Yamazaki-Uchida-Fujisawa-Yoshida '23]
- Fluid dynamics [Uchida-Miyamoto-Yamazaki-Fujisawa-Yoshida '24]
- Detection of gravitational wave
[Gao-Hayes-Croke-Messenger-Veitch '21, Miyamoto-Mossas-Yamamoto-Kuroyanagi-Nesseris]
- Wheeler-De Witt eq. [Joseph-Varela-Watts-White-Feng-Hassan-McGuigan, Czelusta-Mielczarek '21] etc...
- Wormholes [Nature, Jafferis-Zlokapa-Lykken-Kolchmeyer-Davis-Lauk-Neven-Spiropulu '23]
[work in progress, Hasegawa-MH]

Thanks!