The Entangled State from a cosmological Euclidean Wormhole

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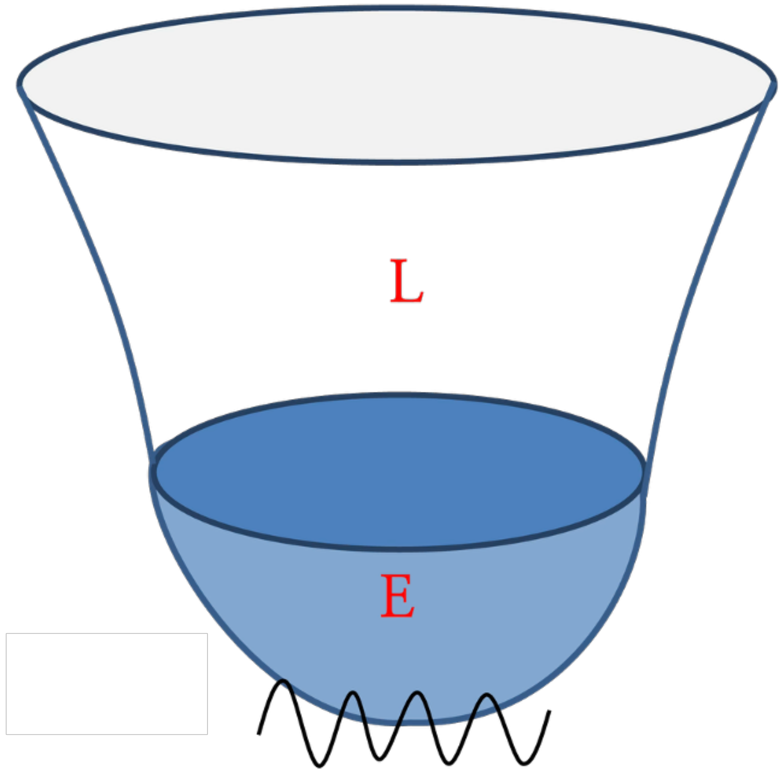
In collaboration with: Pisin Chen (NTU), Kuan-Nan Lin (APCTP), Dong-han Yeom (PNU)

arXiv: 2505.02807, 2608.xxxxx
(2404.15450)

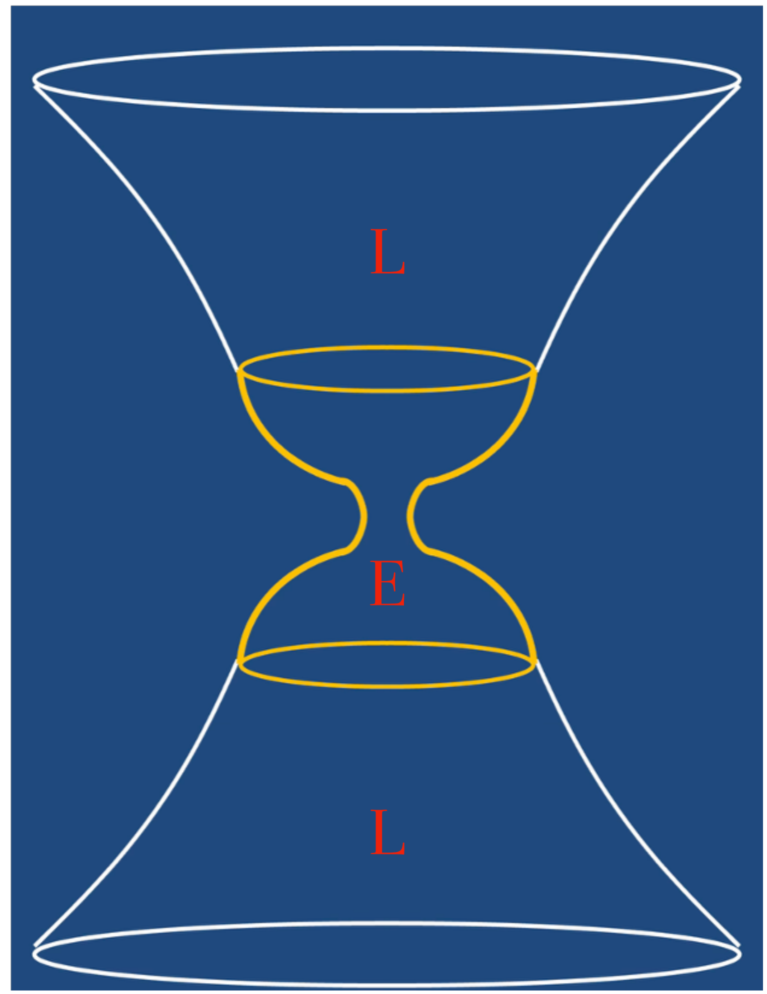
2026/8/7

Quantum crossroads: Interdisciplinary, hybrid workshop at the interface of open quantum systems, high-energy physics, and quantum science

dS (NB) instanton ⇒ Euclidean vacuum



Euclidean wormhole ⇒ ???



Outline

- de Sitter instanton & The Euclidean (Bunch-Davies) vacuum
- The Entangled State from a cosmological Euclidean Wormhole
- Summary

Quantum Cosmology – Euclidean path integral approach

The Wheeler-DeWitt equation: $\hat{H}\Psi = 0$

The Euclidean path integral is a solution to the Wheeler-DeWitt equation at the formal level:

$$\Psi\left[h_{\mathrm{f}}, \Phi_{\mathrm{f}}; h_{\mathrm{i}}, \Phi_{\mathrm{i}}\right] = \int \mathscr{D}g_{\mu\nu}\mathscr{D}\Phi \; e^{-S_{\mathrm{E}}\left[g_{\mu\nu}, \phi\right]}$$

Quantum Cosmology – Euclidean path integral approach

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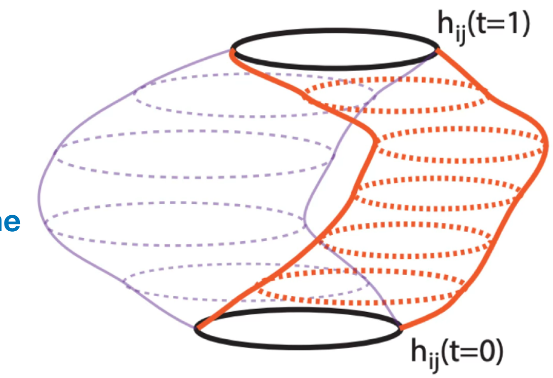
Steepest-descent approximation for practical computations:

$$\Psi\left[h_{\mathrm{f}}, \Phi_{\mathrm{f}}; h_{\mathrm{i}}, \Phi_{\mathrm{i}}\right] \approx \exp\left(-I[\bar{g}_{\mu\nu}, \bar{\phi}]\right) \int \mathscr{D}\phi\, e^{-I_m\left[\bar{g}_{\mu\nu}, \bar{\phi}\,;\,\phi\right]}$$

The **on-shell** solution
(**instanton**)

perturbations
related to QF in curved spacetime

$I_m\left[\bar{g}_{\mu\nu}, \bar{\phi}\,;\,\phi\right]$ is is the quadratic action of the scalar field perturbations $\phi = \Phi - \bar{\phi}$



A figure from Lehners (2023)

de Sitter instanton

the Euclidean action: $t = -i\tau$

$$S_E = \int \sqrt{+g}\, d^4x \left[\frac{R}{16\pi} + \frac{1}{2}\left(\partial_\mu \Phi\right)^2 + U(\Phi) \right],$$

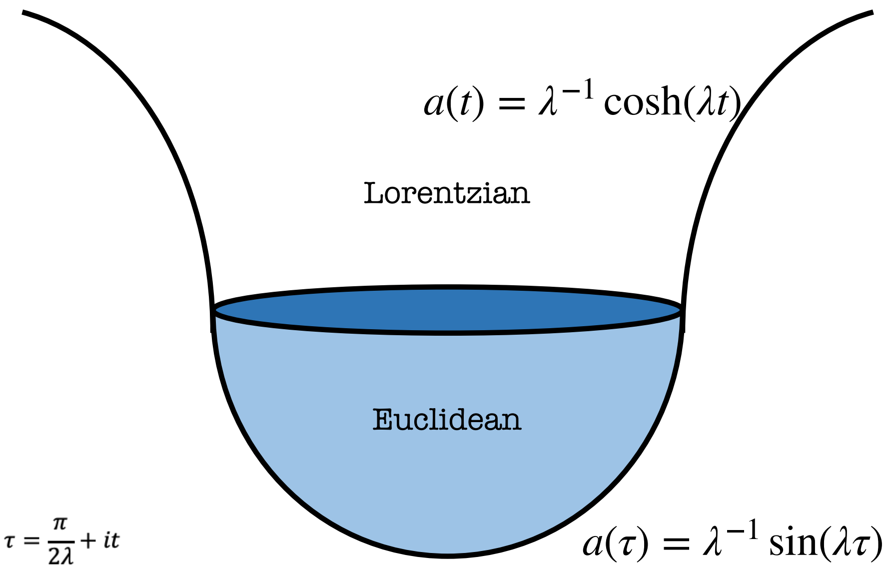
a homogenous, isotropic, and closed metric:

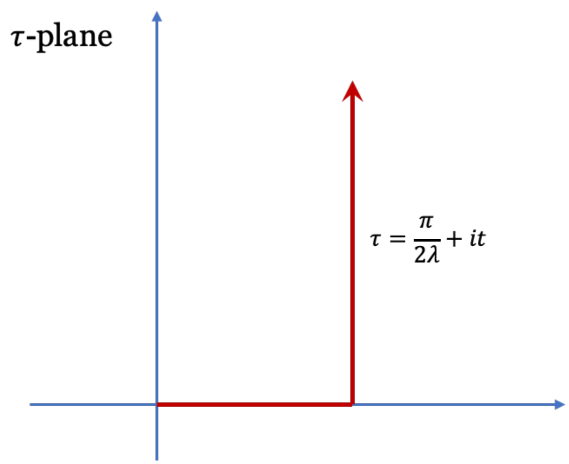
$$ds_E^2 = \sigma^2 \left(d\tau^2 + a^2(\tau) d\Omega_3^2 \right),$$

$$d\Omega_3^2 = d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta d\varphi^2)$$

no field, just a cosmological constant

$$\lambda = \sqrt{\Lambda/3}$$





de Sitter instanton and the Euclidean vacuum

J. J. Halliwell and S. W. Hawking, PRD (1985)
R. Laflamme, PLB (1987)
J. J. Halliwell arXiv: 0909.2566

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$$\Psi\left[h_{\mathrm{f}}, \Phi_{\mathrm{f}}; h_{\mathrm{i}}, \Phi_{\mathrm{i}}\right] \approx \exp\left(-I[\bar{g}_{\mu\nu}, \bar{\phi}]\right)\int \mathscr{D}\phi\, e^{-I_m\left[\bar{g}_{\mu\nu}, \bar{\phi}\,;\,\phi\right]}$$

instanton

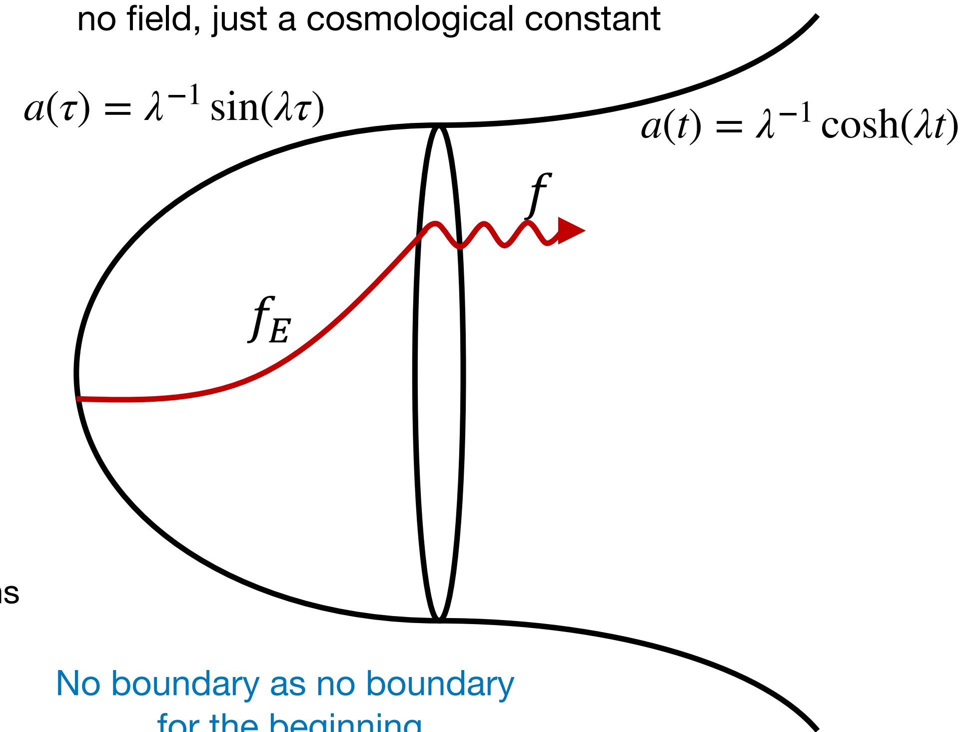
$$\phi(\mathbf{x}, \tau) = \sum_{nlm} f_{nlm}(\tau) Q^n_{lm}(\mathbf{x})$$

$Q^n_{lm}(\mathbf{x})$:three-sphere harmonics. $f_{nlm}(\tau)$: Euclidean mode solutions

$$f''_{nlm} + 3\frac{a'}{a}f'_{nlm} - \left(m^2 + \frac{n^2-1}{a^2}\right)f_{nlm} = 0$$

the dS instanton

no field, just a cosmological constant



No boundary as no boundary
for the beginning

$$\Psi\left[h_{\rm f},\Phi_{\rm f};h_{\rm i},\Phi_{\rm i}\right]\approx\exp\left(-I[\bar g_{\mu\nu},\bar\phi]\right)\underline{\int\mathscr{D}\phi\; e^{-I_m\left[\bar g_{\mu\nu},\bar\phi\,;\,\phi\right]}}$$

$$\phi(\mathbf{x},\tau)=\sum_{nlm}f_{nlm}(\tau)Q^n_{lm}(\mathbf{x})$$

$$\psi_n=\int\mathscr{D}f_n\exp\Big[I_n(a,f_n)\Big]\propto\exp\Big[-I_{n(cl)}(a,f_n)\Big]=\exp\Big[\frac{-1}{2}a^3f_nf_n'\Big|_{bdy}\Big]$$

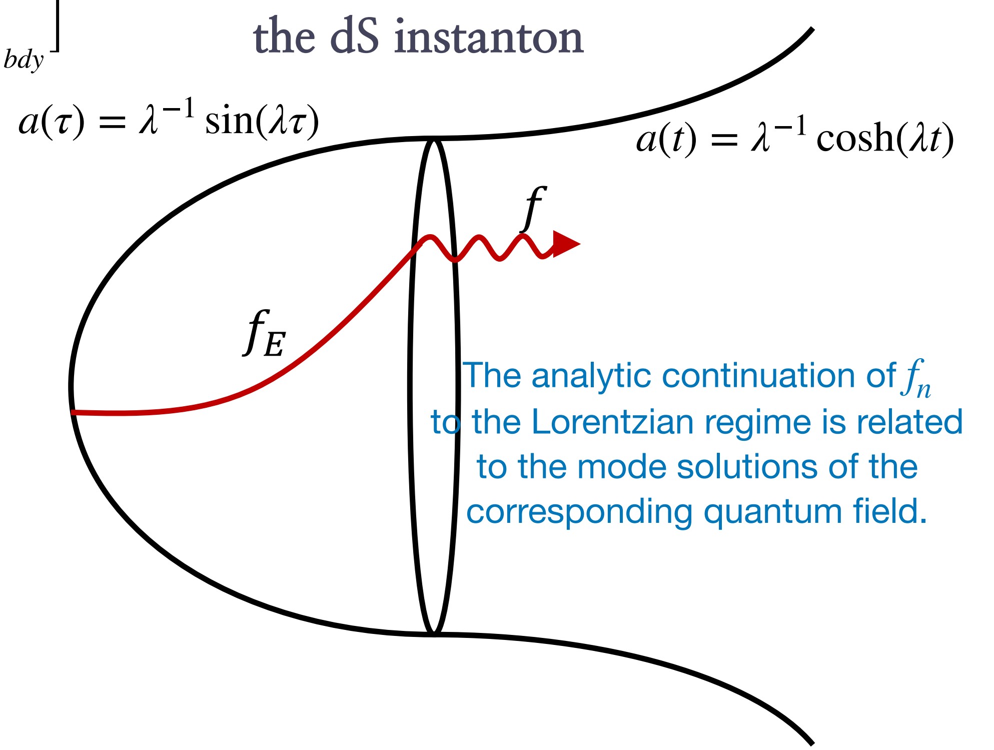
a quadratic action

$$\psi_n(\tilde f_n)\propto\exp\Big[\frac{-1}{2}a^3f_nf_n'\Big|_{\tau=\frac{\pi}{2\lambda}}\Big]=\exp\left[\frac{-1}{2}\left(a^3\frac{f_n'}{f_n}\Big|_{\tau=\frac{\pi}{2\lambda}}\right)\tilde f_n^2\right]$$

one boundary

$\tilde f_n$ denotes the quantity at the $\tau=\dfrac{\pi}{2\lambda}$ boundary

$$f_n''+3\frac{a'}{a}f_n'-\left(m^2+\frac{n^2-1}{a^2}\right)f_n=0$$



$$\psi_n(\tilde{f}_n) \propto \exp\left[\frac{-1}{2}a^3 f_n f_n'\Big|_{\tau=\frac{\pi}{2\lambda}}\right] = \exp\left[\frac{-1}{2}\left(a^3\frac{f_n'}{f_n}\Big|_{\tau=\frac{\pi}{2\lambda}}\right)\tilde{f}_n^2\right]$$

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$$f_n'' + 3\frac{a'}{a}f_n' - \left(m^2 + \frac{n^2-1}{a^2}\right)f_n = 0$$

$$f_n(\tau) = A_n g_n(\tau) + B_n d_n(\tau)$$

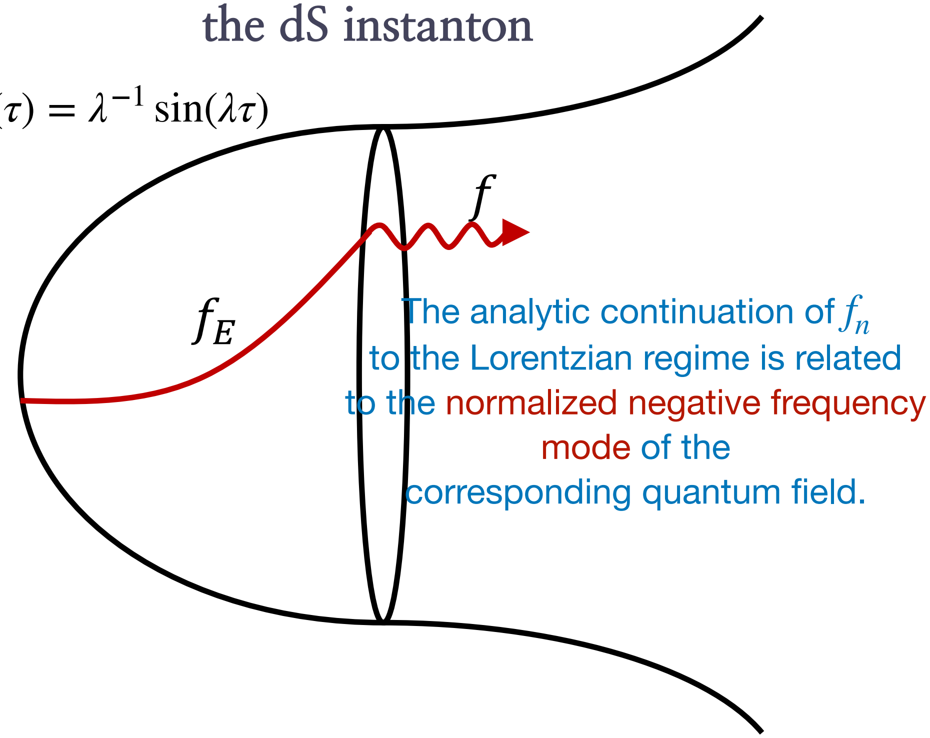
related to the QF in the Lorentzian spacetime

$$\hat{\mathscr{F}}_n(t) = y_n(t)\hat{a}_n + y_n^*(t)\hat{a}_n^\dagger$$

$$\psi_n(\mathscr{F}_n) \propto \exp\left[\frac{i}{2}a^3(t)\frac{\dot{y}_n^*}{y_n^*}\mathscr{F}_n^2\right]$$

the dS instanton

$a(\tau) = \lambda^{-1}\sin(\lambda\tau)$



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$\tilde{f}_n$ denotes the quantity at the $\tau = \dfrac{\pi}{2\lambda}$ boundary

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$$f_n(\tau) = A_n g_n(\tau) + B_n d_n(\tau)$$

The issue of regularity of at $a(0) = 0$

There is only one **unique** solution that is not divergent there

The analytic continuation of this special mode solution defines

Euclidean (Bunch-Davies) vacuum

related to the QF in the Lorentzian spacetime

$$\hat{\mathscr{F}}_n(t) = y_n(t)\hat{a}_n + y_n^*(t)\hat{a}_n^\dagger$$

$$\psi_n(\mathscr{F}_n) \propto \exp\left[\frac{i}{2}a^3(t)\frac{\dot{y}_n^*}{y_n^*}\mathscr{F}_n^2\right]$$

the dS instanton

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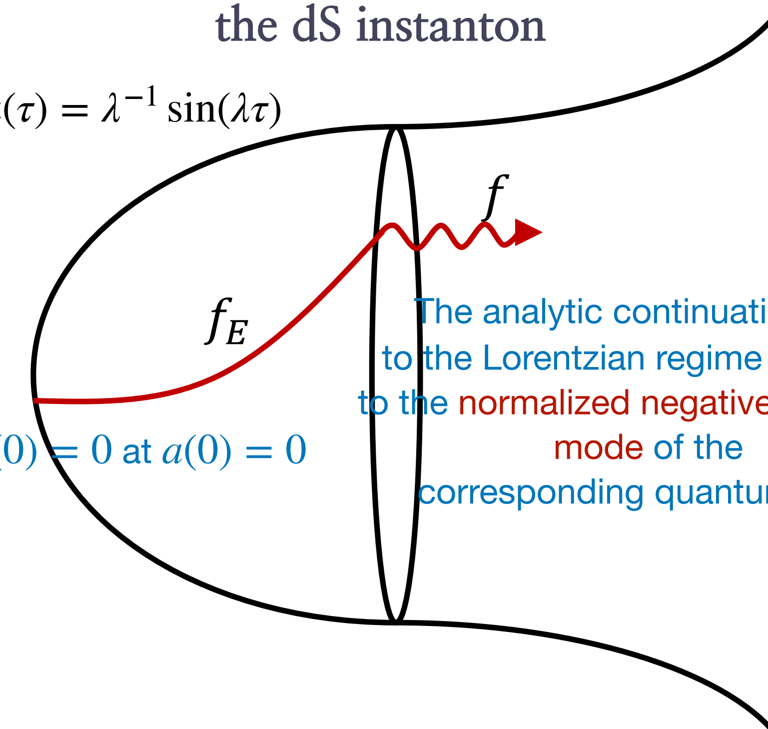
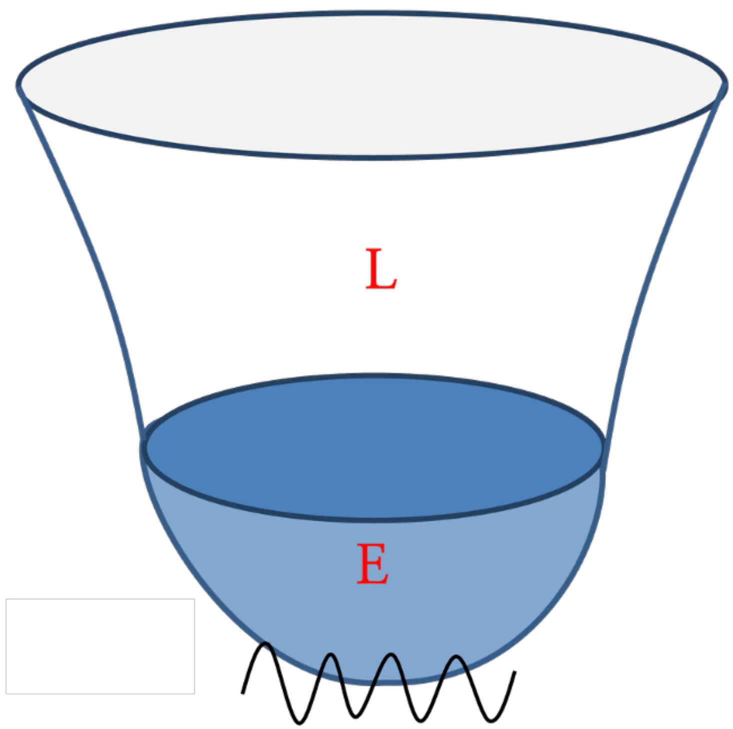


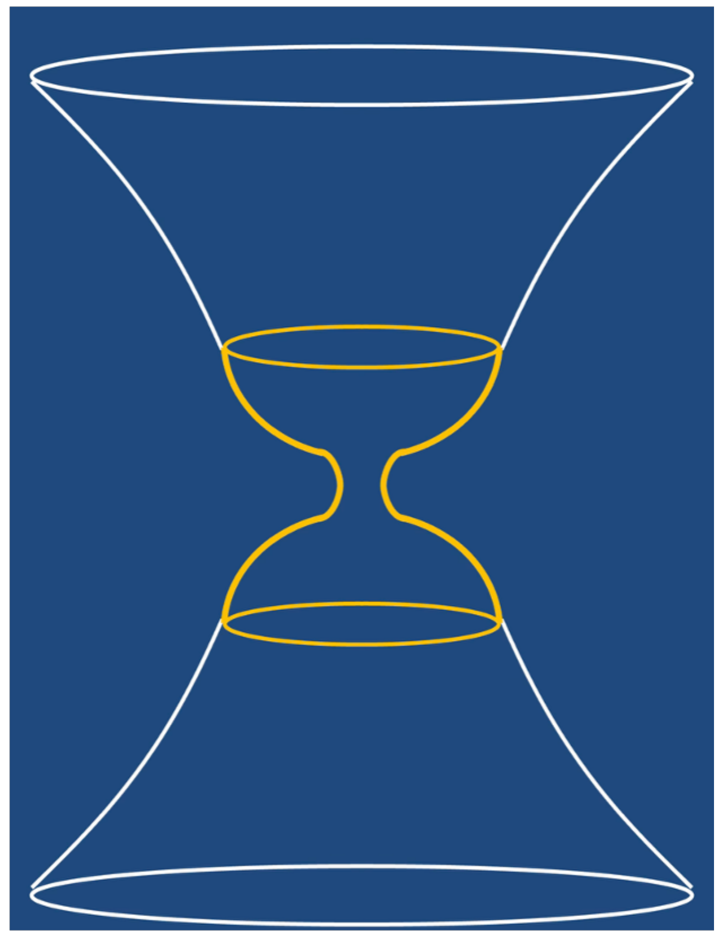
$g_n(0) = 0$ at $a(0) = 0$

The analytic continuation of f_n to the Lorentzian regime is related to the normalized negative frequency mode of the corresponding quantum field.

dS (NB) instanton ⇒ Euclidean vacuum

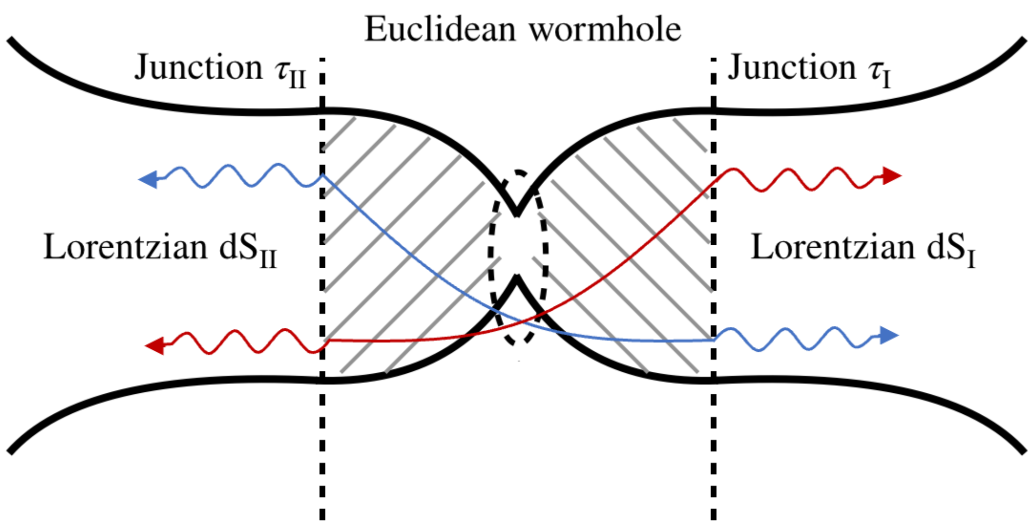


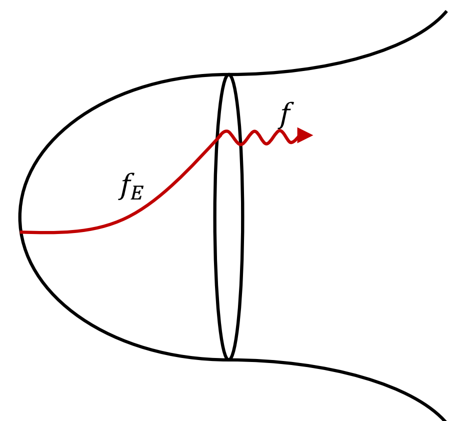
Euclidean wormhole ⇒ ???

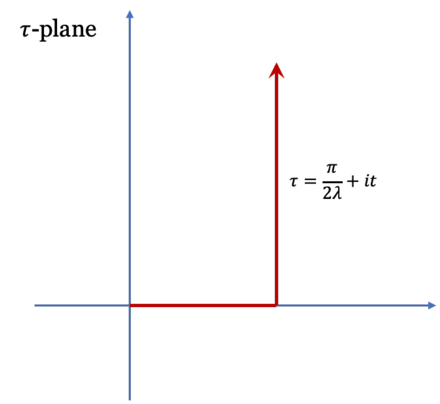


The analytic continuation of modes

e-Print: 2505.02807
P. Chen, **WL**, K. Lin, D. Yeom







Klebanov-Susskind-Banks (KSB) wormhole:
cut and paste two de Sitter instantons

The analytic continuation of modes

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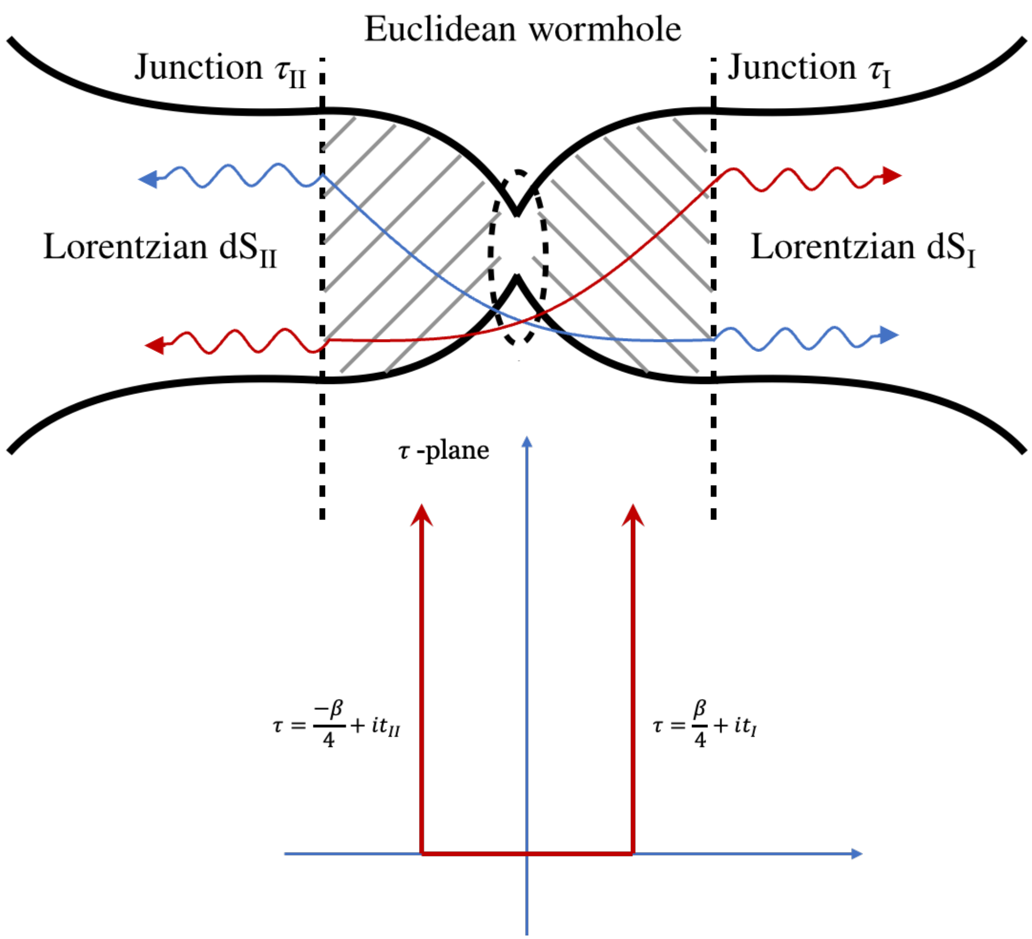
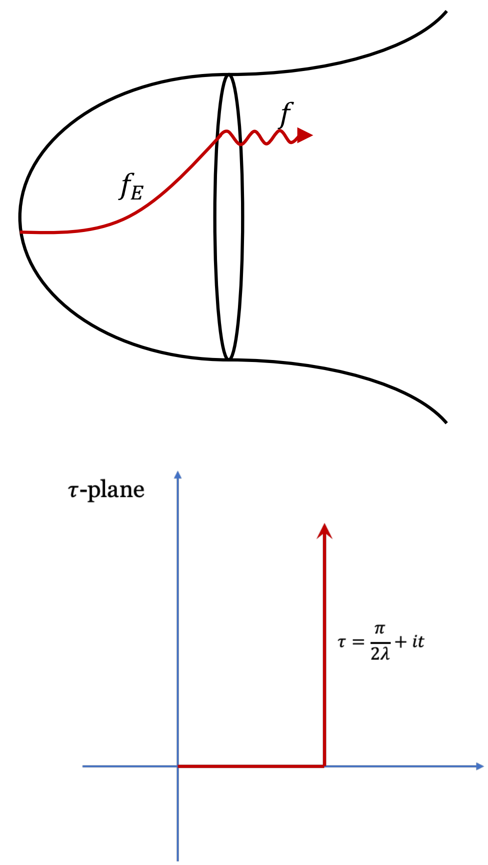
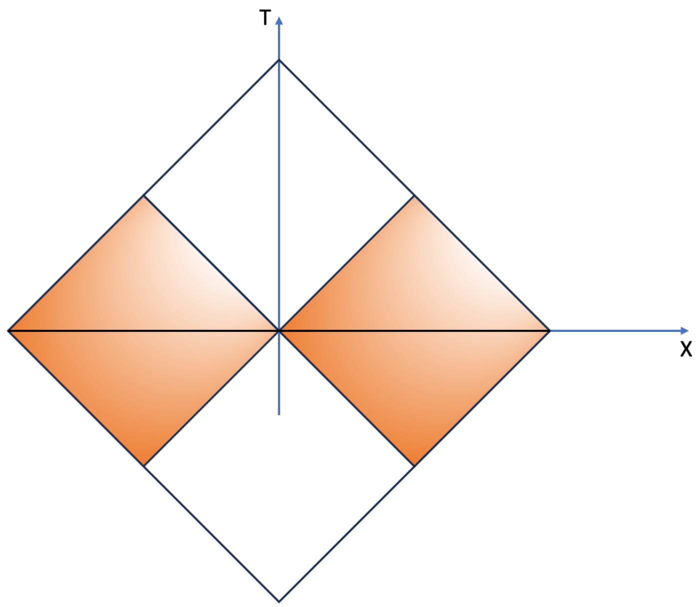


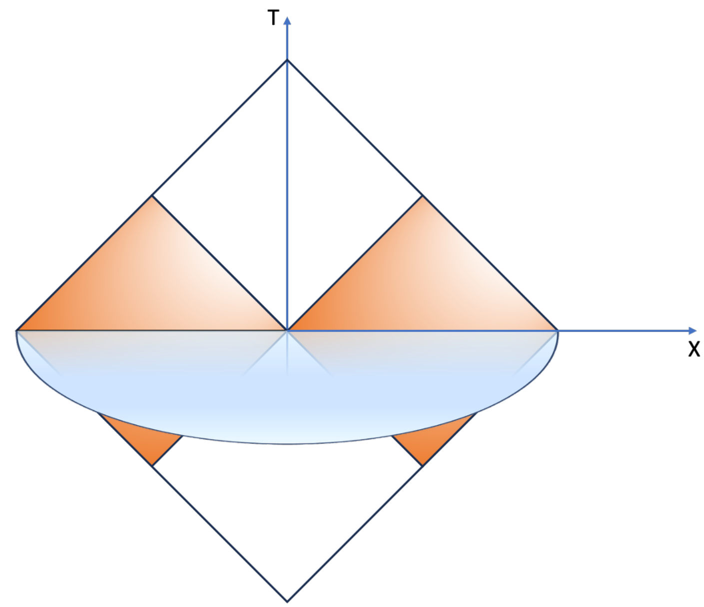


The Unruh effect and the thermofield double state

W.G.Unruh; PRD (1976)
W. Israel; PLA (1976)
R. Laflamme; NPB (1989)
S. Carroll: Spacetime and Geometry



The Penrose diagram of the
Minkowski spacetime and the Rindler wedges



the blue area represents half of the Euclidean region
(a 3D figure)

The Unruh effect and the thermofield double state

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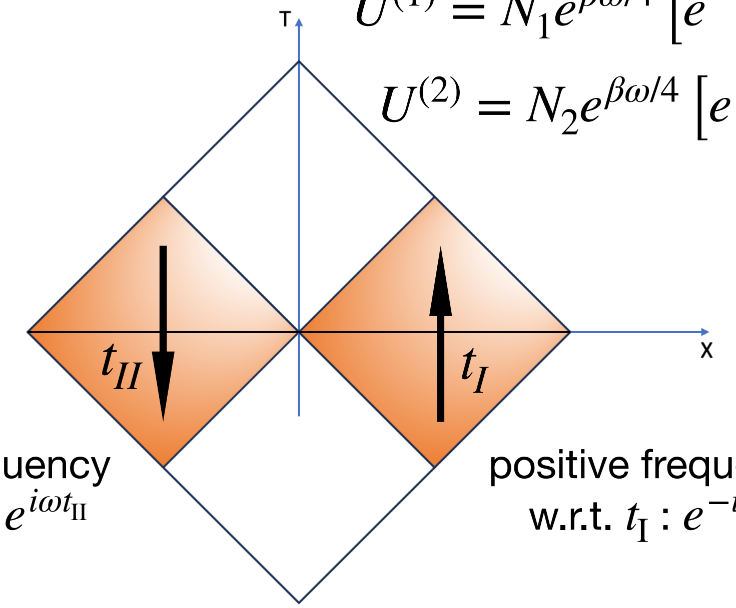
$$U^{(1)} = N_1 e^{\beta\omega/4}\left[e^{-i\omega t_{\mathrm{I}}} + e^{-\beta\omega/2} e^{-i\omega t_{\mathrm{II}}}\right],$$

$$U^{(2)} = N_2 e^{\beta\omega/4}\left[e^{i\omega t_{\mathrm{II}}} + e^{-\beta\omega/2} e^{i\omega t_{\mathrm{I}}}\right],$$



positive frequency
w.r.t. $-t_{\mathrm{II}}$: $e^{i\omega t_{\mathrm{II}}}$

positive frequency
w.r.t. t_{I} : $e^{-i\omega t_{\mathrm{I}}}$

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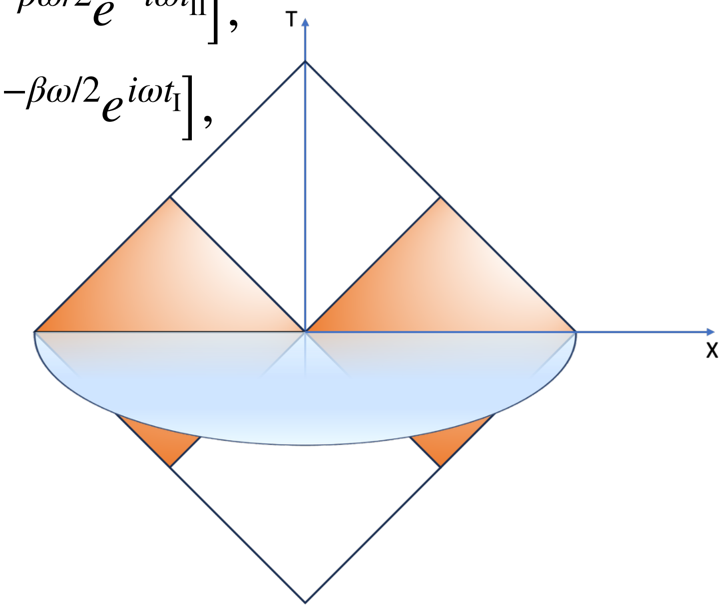
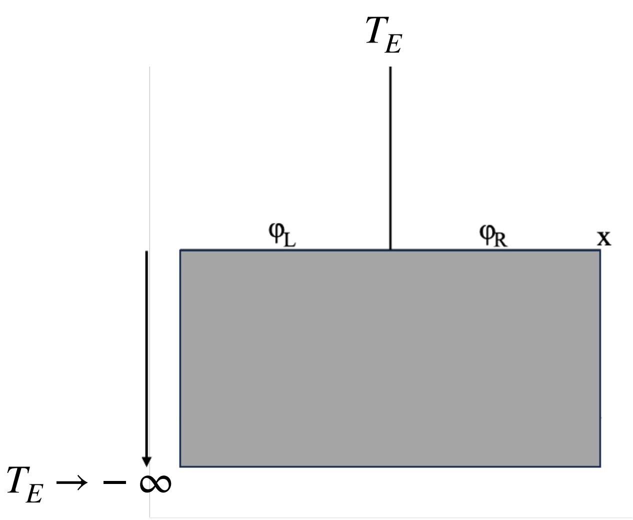
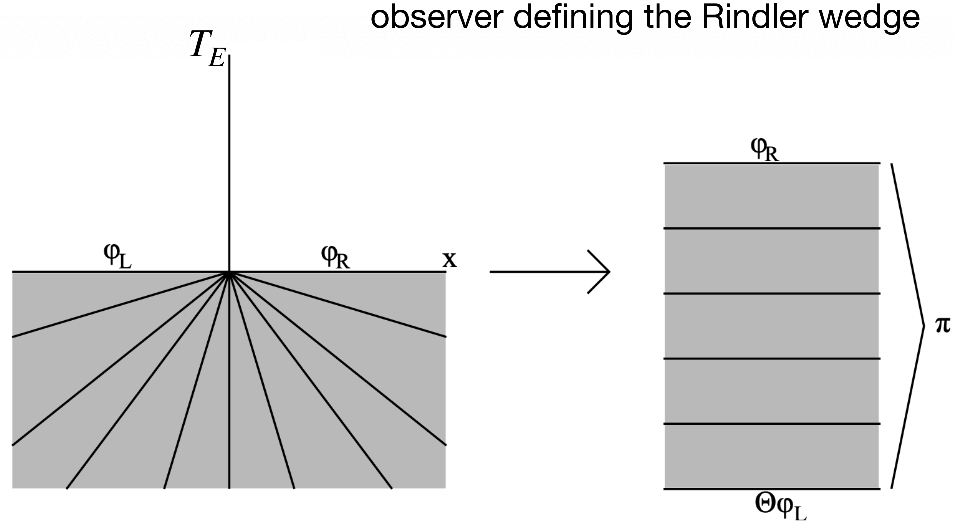


the blue area represents half of the Euclidean region
(a 3D figure)

$$\langle \phi_L, \phi_R \,|\, \mathrm{vac} \rangle \propto \int_{T_E \to -\infty;\, \phi=(0,0)}^{T_E=0;\, \phi=(\phi_L,\phi_R)} \mathscr{D}\phi\, e^{-S_E} = \int_{\kappa t_E=0;\, \phi=\phi_L}^{\kappa t_E=\pi;\, \phi=\phi_R} \mathscr{D}\phi\, e^{-S_E}$$

κ : the magnitude of the 4-acceleration of the observer defining the Rindler wedge





A figure from Harlow's lecture notes (2014) arXiv:1409.1231

The Euclidean SHO example:

R. Laflamme; NPB (1989)
Geometry and Thermofields

$$S_{\mathrm{E}} = \frac{m}{2}\int_{\tau_{\mathrm{II}}}^{\tau_{\mathrm{I}}} d\tau \left[\dot{x}^2(\tau) + \omega^2 x^2(\tau)\right]$$

$$K_E(x_f, x_0; \tau_f, \tau_0) = \int_{x(\tau_0)=x_0}^{x(\tau_f)=x_f} \mathscr{D}x\, e^{-S_E/\hbar} = \left(\frac{-1}{2\pi\hbar}\frac{\partial^2 S_{E(cl)}}{\partial x_f \partial x_i}\right)^{1/2} \exp\left(\frac{-1}{\hbar}S_{E(cl)}\right)$$

(works for free fields)

$$\frac{S_{\mathrm{E}(cl)}}{m\omega} = \frac{(1+z^2)(x_{\mathrm{I}}^2 + x_{\mathrm{II}}^2) - 4z x_{\mathrm{I}} x_{\mathrm{II}}}{2(1-z^2)};\ z = e^{-\beta\omega/2}$$

$S_{E(cl)}$ is the on-shell action

The Mehler Formula:

$$\frac{1}{\sqrt{1-z^2}}\exp\left[\frac{4xyz - (1+z^2)(x^2+y^2)}{2(1-z^2)}\right] = e^{-(x^2+y^2)/2}\sum_{\mu=0}^{\infty}\frac{z^\mu}{2^\mu \mu!}H_\mu(x)H_\mu(y),$$

H_μ is the Hermite polynomial

$$\Psi(x_I, x_{II}) \propto e^{-S_{\mathrm{E}(cl)}} \propto \sum_{\mu=0}^{\infty} e^{-\mu\beta\omega/2}\frac{e^{-\beta\omega/4}}{2^\mu \mu!}e^{-m\omega x_{\mathrm{I}}^2/2} \times H_\mu\left(\sqrt{m\omega}x_{\mathrm{I}}\right) e^{-m\omega x_{\mathrm{II}}^2/2} H_\mu\left(\sqrt{m\omega}x_{\mathrm{II}}\right) \propto \sum_{\mu=0}^{\infty} e^{-\beta E_\mu/2}\,\phi_\mu(x_I)\,\phi_\mu(x_{II}),$$

partial trace over subsystem II : $\rho(x_I, x_I') = \int_{-\infty}^{\infty} dx_{II}\,\Psi(x_I, x_{II})\,\Psi^*(x_I', x_{II}) = \sum_\mu e^{-\beta E_\mu}\,\phi_\mu(x_I)\,\phi_\mu(x_I')$

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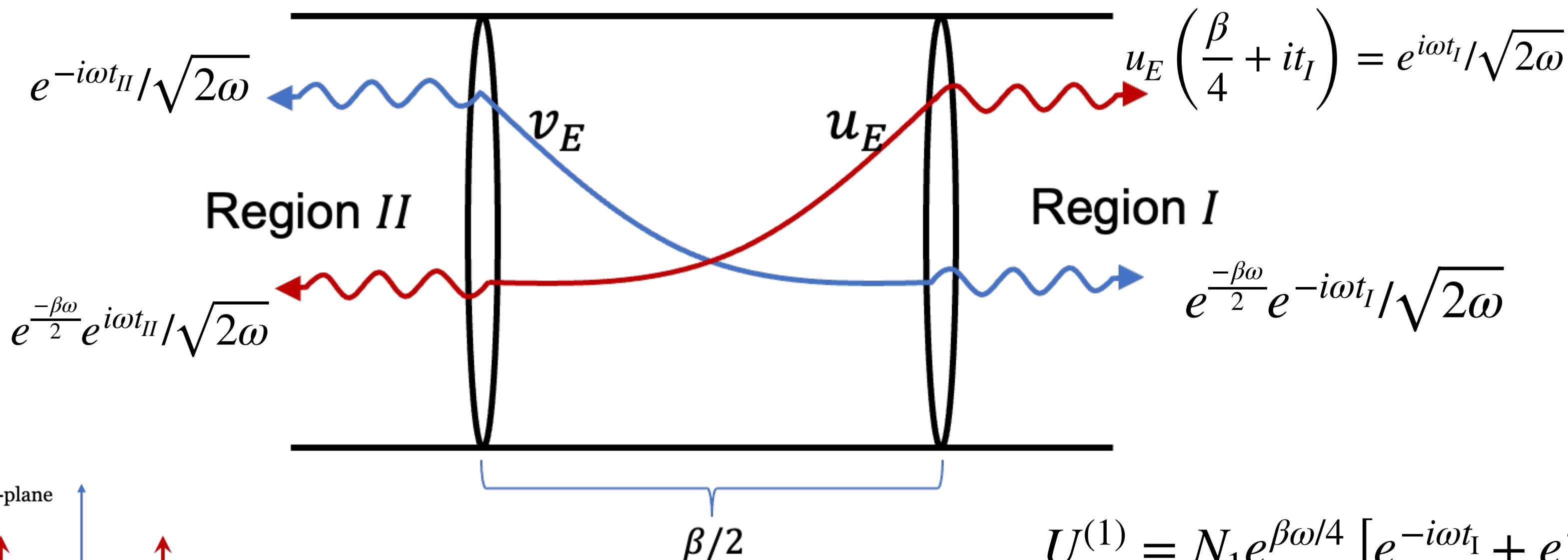
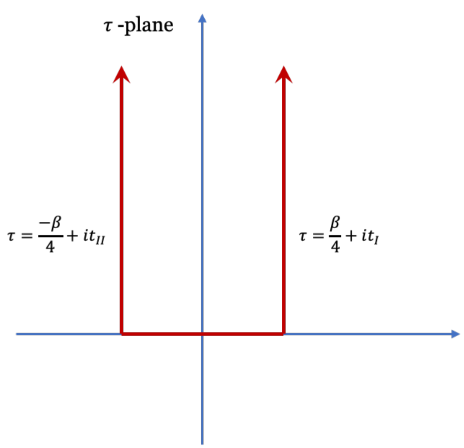
(In preparation)





$$U^{(1)} = N_1 e^{\beta\omega/4}\left[e^{-i\omega t_{\rm I}} + e^{-\beta\omega/2}e^{-i\omega t_{\rm II}}\right]$$

$$U^{(2)} = N_2 e^{\beta\omega/4}\left[e^{i\omega t_{\rm II}} + e^{-\beta\omega/2}e^{i\omega t_{\rm I}}\right],$$

$$u_E(\tau) = e^{\frac{-\beta\omega}{4}}e^{\omega\tau}/\sqrt{2\omega}$$

$$v_E(\tau) = e^{\frac{-\beta\omega}{4}}e^{-\omega\tau}/\sqrt{2\omega}$$

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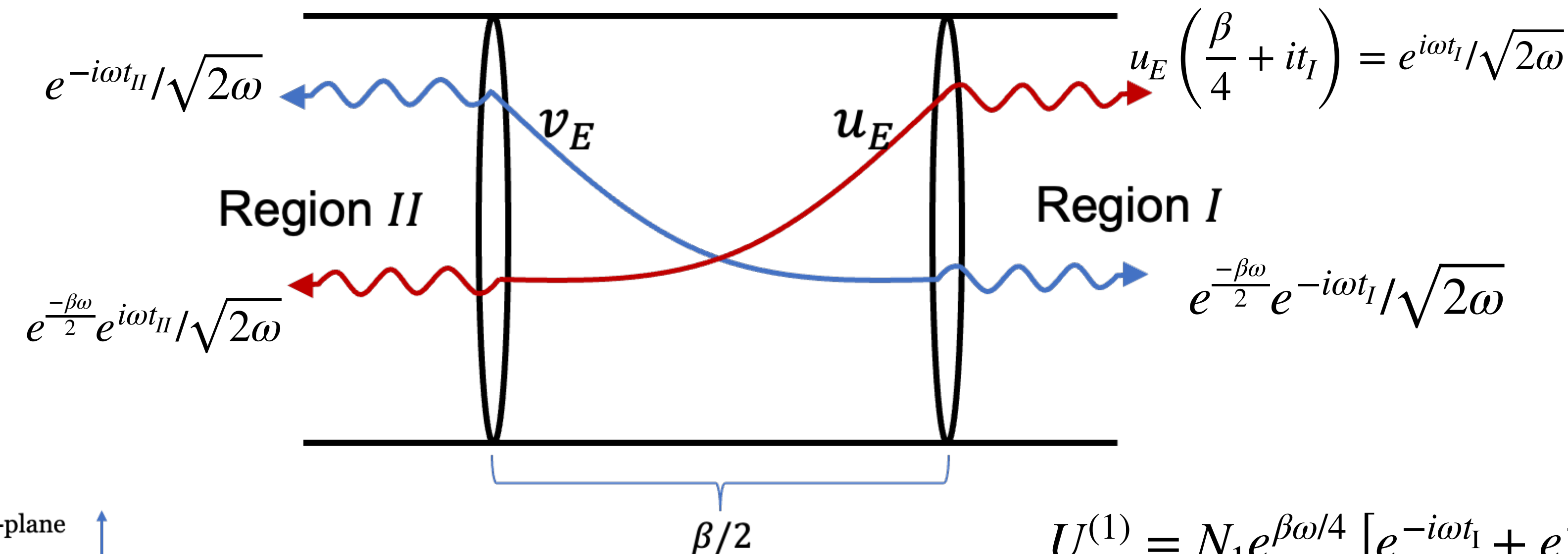

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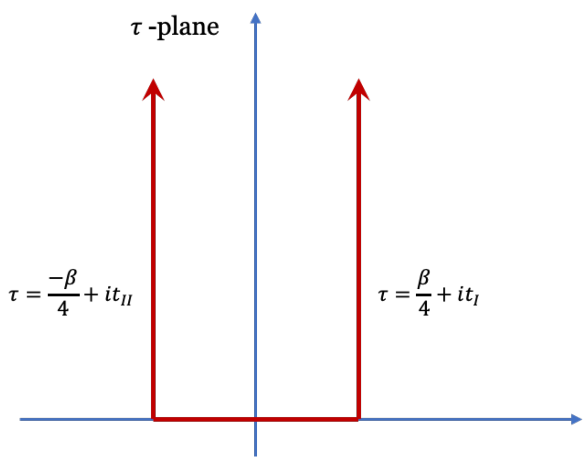
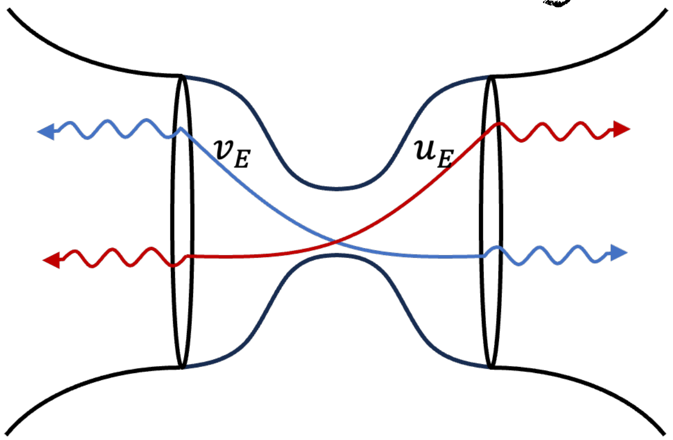


$$u_E(\tau) = e^{\frac{-\beta\omega}{4}} e^{\omega\tau}/\sqrt{2\omega}$$

$$v_E(\tau) = e^{\frac{-\beta\omega}{4}} e^{-\omega\tau}/\sqrt{2\omega}$$

Unique

A general formalism of the entangled state from a cosmological Euclidean wormhole



P. Chen, **WL**, K. Lin, D. Yeom (In preparation)

$$\Psi\left[h_{\mathrm{f}}, \Phi_{\mathrm{f}}; h_{\mathrm{i}}, \Phi_{\mathrm{i}}\right] \approx \exp\left(-I[\bar{g}_{\mu\nu}, \bar{\phi}]\right) \int \mathscr{D}\phi \, e^{-I_m\left[\bar{g}_{\mu\nu}, \bar{\phi}\,;\,\phi\right]}$$

instanton

$$\phi(\mathbf{x}, \tau) = \sum_{nlm} f_{nlm}(\tau) Q^n_{lm}(\mathbf{x})$$

The perturbations on a Euclidean wormhole as Euclidean parametric oscillator $\chi''_{nlm} - \omega_n^2(\tau)\chi_{nlm} = 0$

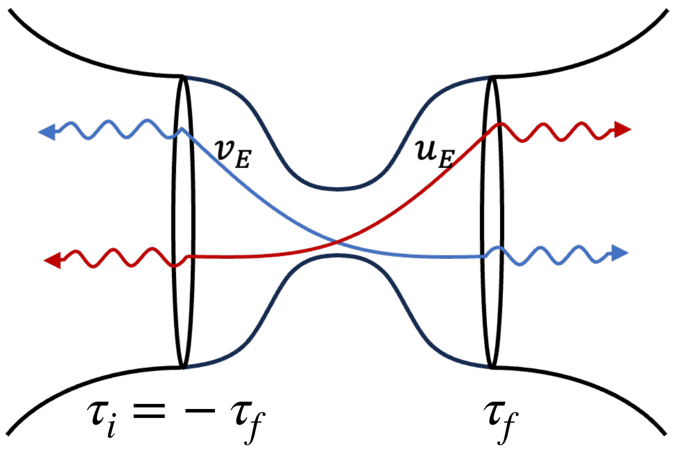
$$K_E(q_f, q_i; \tau_f, \tau_i) = \int_{q(\tau_i)=q_i}^{q(\tau_f)=q_f} \mathscr{D}q \, e^{-S_E/\hbar} = \sqrt{\frac{W_E}{2\pi\hbar P_E}} \exp\left[\frac{-1}{2\hbar P_E}\Big((u_{Ei}v'_{Ef} - u'_{Ef}v_{Ei})q_f^2 + (u_{Ef}v'_{Ei} - u'_{Ei}v_{Ef})q_i^2 - 2W_E q_i q_f\Big)\right]$$

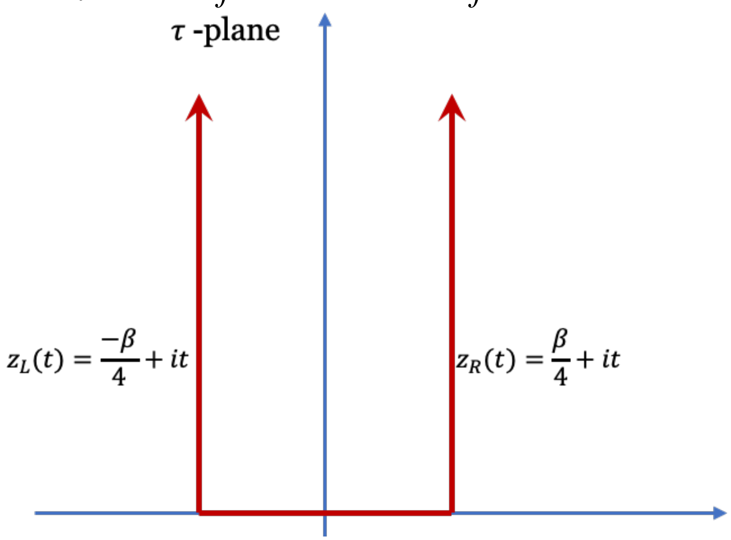
$$P_E \equiv u_{Ef}v_{Ei} - u_{Ei}v_{Ef} \qquad W_E \equiv u'_E v_E - u_E v'_E$$

What is the condition of the mapping between $K_E(q_f, q_i; \tau_f, \tau_i)$ and the Mehler Formula?

$$\frac{1}{\sqrt{1-z^2}} \exp\left[\frac{4xyz - (1+z^2)(x^2+y^2)}{2(1-z^2)}\right] = e^{-(x^2+y^2)/2} \sum_{\mu=0}^{\infty} \frac{z^\mu}{2^\mu \mu!} H_\mu(x) H_\mu(y),$$

A general formalism of the entangled state from a cosmological Euclidean wormhole



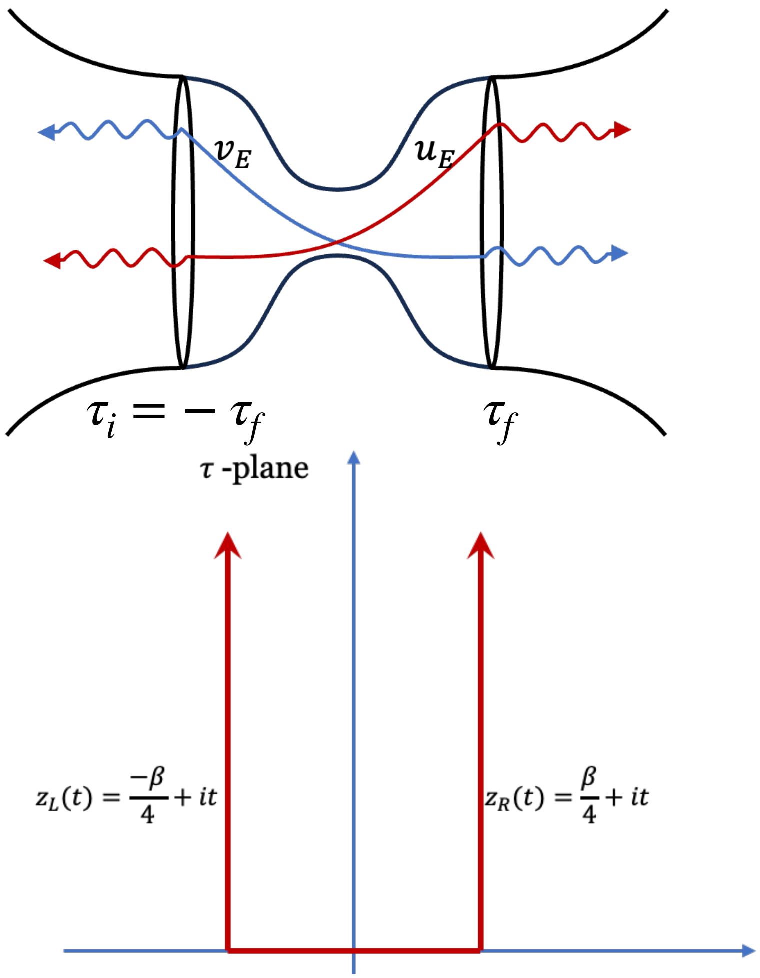


$$\chi''_{nlm} - \omega_n^2(\tau)\chi_{nlm} = 0$$

$$\omega_n^2(\tau) = \frac{3}{4}\frac{(a')^2}{a^2} + \frac{3}{2}\frac{a''}{a} + \frac{n^2-1}{a^2} + \frac{1}{2}\frac{d^2V}{d\bar{\phi}^2}$$

$\omega_n^2(\tau)$ is an analytic even function and real-valued on the contour: $\omega_n^2(z_L(t)) = \omega_n^2(z_R(t))$

A general formalism of the entangled state from a cosmological Euclidean wormhole



$$\chi''_{nlm} - \omega_n^2(\tau)\chi_{nlm} = 0$$

$$\omega_n^2(\tau) = \frac{3}{4}\frac{(a')^2}{a^2} + \frac{3}{2}\frac{a''}{a} + \frac{n^2-1}{a^2} + \frac{1}{2}\frac{d^2V}{d\bar{\phi}^2}$$

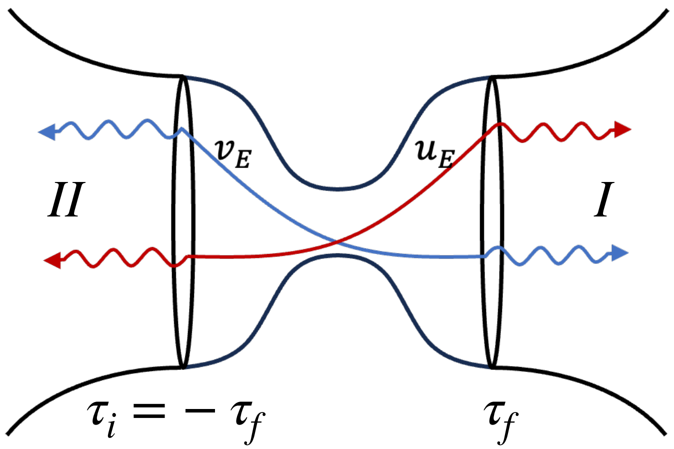
$\omega_n^2(\tau)$ is an analytic even function and real-valued on the contour: $\omega_n^2(z_L(t)) = \omega_n^2(z_R(t))$

ansatz: $v_E(\tau) = u_E(-\tau)$

not even nor odd function;
analytic and real-valued when $\tau \in \mathbf{R}$;

$$v_E\left(\frac{\mp\beta}{4} + it\right) = u_E\left(\frac{\beta}{4} \mp it\right) = u_E^*\left(\frac{\beta}{4} \pm it\right)$$

A general formalism of the entangled state from a cosmological Euclidean wormhole



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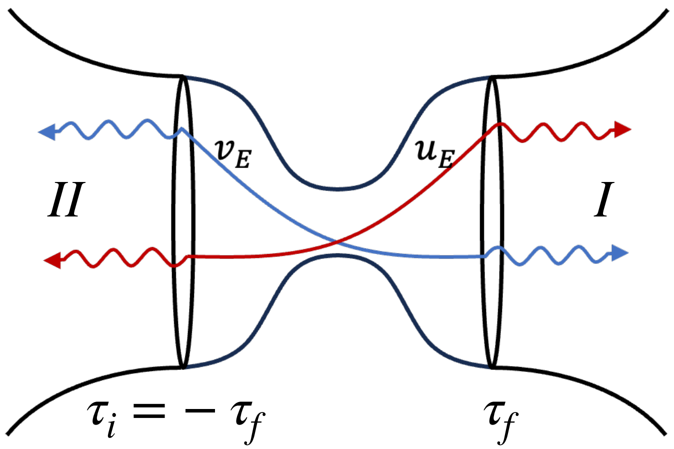
A general formalism of the entangled state from a cosmological Euclidean wormhole

P. Chen, **WL**, K. Lin, D. Yeom
(In preparation)



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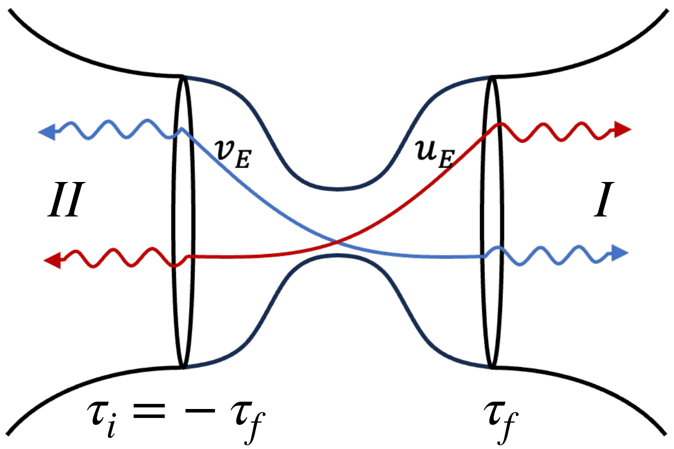

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➡ vacuum is uniquely defined

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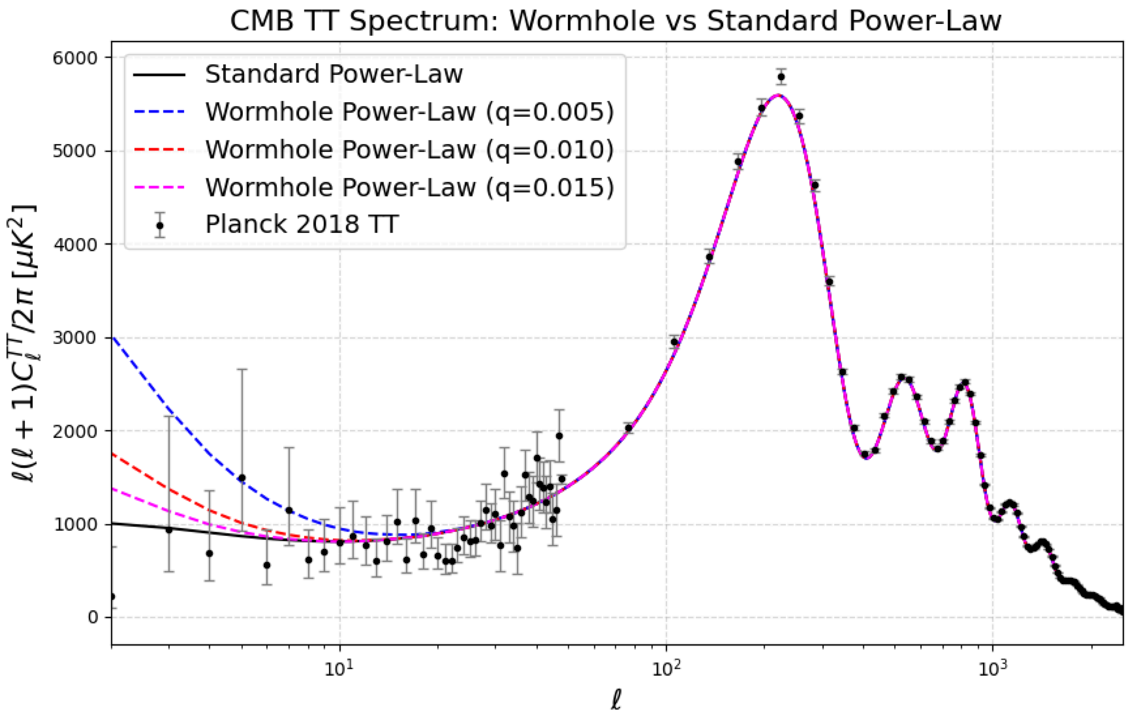

P. Chen, **WL**, K. Lin, D. Yeom
e-Print: 2505.02807

$$P(n) = P_{\mathrm{BD}}(n)\left(1 + 2|\beta_n|^2\right)$$

$$U_i^{(1)} = \alpha_i u_i^I + \beta_i \bar{u}_i^{II},$$
$$U_i^{(2)} = \alpha_i u_i^{II} + \beta_i \bar{u}_i^{I},$$

$$|\Omega\rangle = \prod_i |\Omega_i\rangle = \prod_i \frac{1}{|\alpha_i|} \sum_{\mu=0}^{\infty} \left(\frac{\bar{\beta}_i}{\bar{\alpha}_i}\right)^{\mu} |\mu_i\rangle_{\mathrm{I}} |\mu_i\rangle_{\mathrm{II}},$$

Summary

- The perturbations on a Euclidean wormhole instanton leads to entangled initial state of the perturbations between the two Lorentzian universes.

- The relation is a generalization of the thermofield double state. Due to the non-trivial Euclidean geometry, the entangled initial state on one side deviates from a thermal distribution. This formalism selects a unique vacuum.

- This pair-universe entanglement enhances the long-wavelength modes corresponding to the low l range of the CMB power spectrum.

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Thank you!