Holographic construction of instantons for dissipative quantum tunneling via a probe string

中央大学 CHUO UNIVERSITY — Knowledge into Action —

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Abstract:

Dissipative quantum tunneling, as described by the Caldeira-Leggett model, is suppressed due to a non-local term in the effective action arising from coupling to a thermal bath. In this work, we study this phenomenon holographically using a probe string in an AdS$_5$-Schwarzschild black hole background, with a boundary condition at the string endpoint that realizes a double-well potential in the dual field theory. We numerically construct **instanton--anti-instanton configurations** and examine **the tunneling rate** by evaluating the on-shell Nambu-Goto action as a function of temperature and friction. We report on the construction of this holographic setup and the analysis of the tunneling rate at finite temperature, comparing with conventional quantum mechanical tunneling.

Review: Quantum tunneling and dissipation

❑ Instanton method

- Consider a particle trapped by a one-dimensional double-well potential and the Euclidean action.

$$V(x) = a(x^2 - x_0^2)^2, \qquad S_{\mathrm{E}} = \int_{-\tau_0/2}^{\tau_0/2} \mathrm{d}\tau \left[\frac{m}{2}\left(\frac{\mathrm{d}x}{\mathrm{d}\tau}\right)^2 + V(x)\right]$$

- A non-trivial solution (instanton solution):

$$x_{\mathrm{inst}}(\tau) = x_0 \tanh\frac{\omega(\tau-\tau_c)}{2} \qquad \text{with} \qquad \omega = \sqrt{8ax_0^2/m}$$

- Transition amplitude under the dilute-instanton approximation:

$$\langle x_0|\, e^{-H\tau_0/\hbar}\, |x_0\rangle = N \int_{x_i=x_0}^{x_f=x_0} \mathcal{D}x\, e^{-S_{\mathrm{E}}/\hbar} = \sqrt{\frac{m\omega}{\pi\hbar}}\, e^{-\frac{\omega\tau_0}{2}} \cosh\left(K e^{-S_{\mathrm{inst}}/\hbar}\tau_0\right)$$

where the on-shell action is $S_{\mathrm{inst}} = \dfrac{m^2\omega^3}{12a} = \dfrac{4}{3}x_0^3\sqrt{2ma}$

- In the limit of $\tau_0 \to \infty$, lower energy states become dominant and the two lowest energy levels are given by

$$E_{\pm} = \frac{\hbar\omega}{2} \pm \underline{\hbar K e^{-S_{\mathrm{inst}}/\hbar}}$$

Tunneling amplitude

The two-level system with the off-diagonal element $H_{\mathrm{eff}} = \begin{pmatrix} E_0 & \Delta \\ \Delta & E_0 \end{pmatrix} \Longrightarrow \begin{pmatrix} E_0 - \Delta & 0 \\ 0 & E_0 + \Delta \end{pmatrix}$

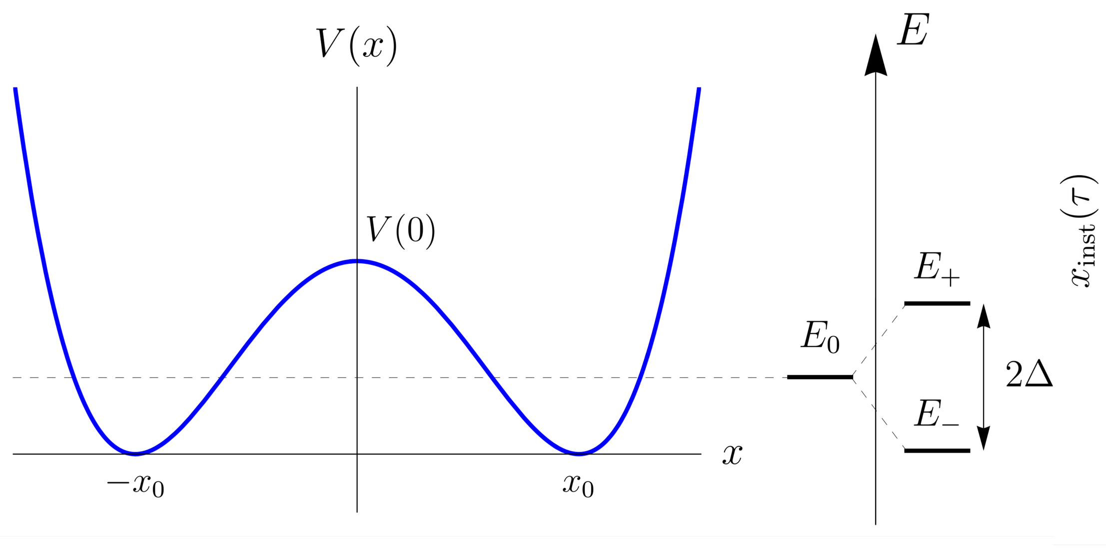


Fig.1 Double-well potential and energy splitting

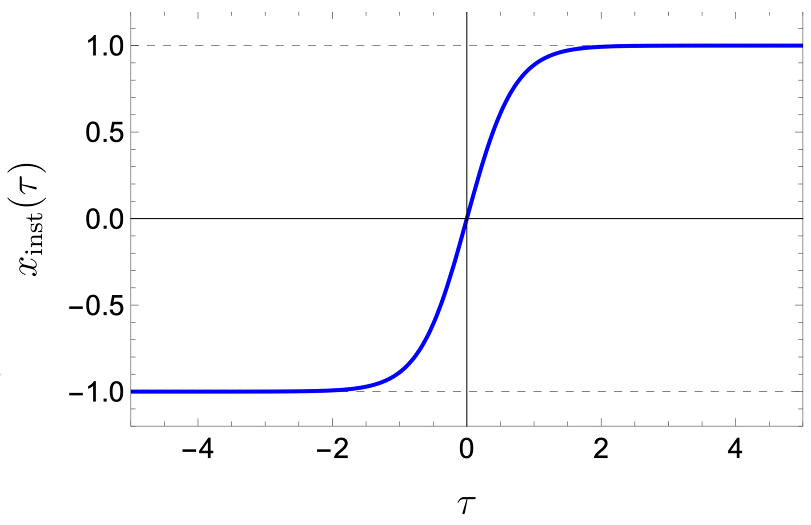


Fig. 2 Instanton solution

❑ Dissipative quantum tunneling (Caldeira-Leggett model)

$$H = \frac{p^2}{2M} + V(x) + \sum_j \left[\frac{p_j^2}{2m_j} + \frac{m_j\omega_j^2}{2}\left(q_j - \frac{c_j x}{m_j\omega_j^2}\right)^2\right].$$

Integrating out the environment degree of freedom:

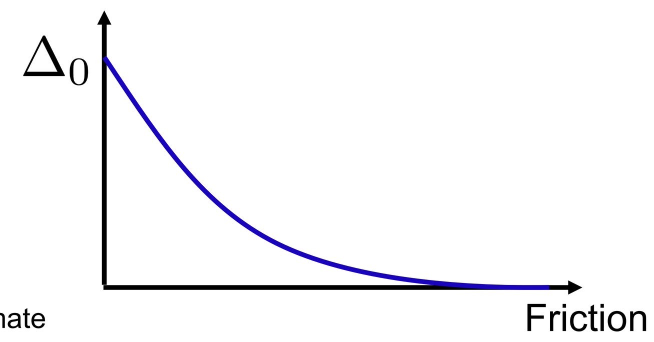
$$S_{\mathrm{eff}} = S_0 + S_{\mathrm{diss}}, \qquad S_{\mathrm{diss}} = \frac{1}{2}\int d\tau\, d\tau'\, K(\tau-\tau')\,[x(\tau) - x(\tau')]^2.$$

Non-local friction term

where $K(\tau) = \dfrac{1}{\pi}\displaystyle\int_0^\infty \mathrm{d}\omega\, J(\omega) e^{-\omega|\tau|}$

Tunneling amplitude: exponential suppression

$$\Delta \propto e^{-S_{\mathrm{eff}}/\hbar},$$
$$S_{\mathrm{eff}} > S_0 \;\Rightarrow\; \Delta_{\mathrm{diss}} < \Delta_0.$$



*Weak-dissipation semiclassical estimate

Holographic construction of instantons

❑ Holographic setup

- Euclidean-AdS$_5$ geometry: $\tau \sim \tau + \beta$

$$\mathrm{d}s_{\mathrm{E}}^2 = \frac{r^2}{L^2}\left(f(r)\mathrm{d}\tau^2 + \mathrm{d}\vec{x}^2\right) + \frac{L^2}{r^2}\frac{\mathrm{d}r^2}{f(r)}.$$

L: AdS radius

$$f(r) = 1 - \frac{r_{\mathrm{H}}^4}{r^4}$$

$$T = 1/\beta = r_{\mathrm{H}}/(\pi L^2)$$

Dimensionless coordinates

$$\rho \equiv \int_{r_{\mathrm{H}}}^{r} \frac{\mathrm{d}r'}{\sqrt{r'^2 f(r')}} = \frac{1}{2}\mathrm{arctanh}\left[f(r)^{1/2}\right]$$

$$\theta \equiv 2r_{\mathrm{H}}\tau/L^2, \quad \vec{X} \equiv 2r_{\mathrm{H}}\vec{x}/L^2$$

$$\mathrm{d}s_{\mathrm{E}}^2 = L^2\mathrm{d}\rho^2 + \frac{L^2}{4\sqrt{1-\tanh^2(2\rho)}}\left(\tanh^2(2\rho)\,\mathrm{d}\theta^2 + \mathrm{d}\vec{X}^2\right)$$

- Nambu-Goto action for a probe string:

$$S_{\mathrm{NG}} = +T_{\mathrm{s}}\int \mathrm{d}^2\sigma \sqrt{\det\gamma_{ab}} \qquad \gamma_{ab} = \frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}g_{\mu\nu} \text{ : induced metric}$$

- Imposing a double-well potential at the boundary

$$S_{\mathrm{gauge}} = \int \mathrm{d}x^\mu A_\mu = \int \mathrm{d}\tau\, \Phi(X) = \int \mathrm{d}\tau\, a\left(\frac{L^2}{2r_{\mathrm{H}}}\right)^4 (X^2 - X_0^2)^2$$

- Variation of the total boundary action:

$$\delta S_{\mathrm{tot}} = \delta S_{\mathrm{NG}} + \delta S_{\mathrm{gauge}} = \int_0^{2\pi}\mathrm{d}\theta \left[\underline{T_{\mathrm{s}}\frac{g_{\theta\theta}g_{xx}X'}{\sqrt{\gamma}} + 4a\left(\frac{L^2}{2r_{\mathrm{H}}}\right)^5 X(X^2 - X_0^2)}\right]\delta X\Bigg|_{\rho=\rho_c}$$

Our boundary condition

- The parameters in the system: (mass, friction coefficient, temperature)

$$m = T_{\mathrm{s}}\int_0^\Lambda \mathrm{d}r\sqrt{\gamma}\Bigg|_{X=\mathrm{const.}} = T_{\mathrm{s}}\Lambda, \quad \gamma = \frac{\pi}{2}\sqrt{2\lambda}T^2 = T_{\mathrm{s}}\left(\frac{r_{\mathrm{H}}}{L}\right)^2, \quad T = \frac{r_{\mathrm{H}}}{\pi L^2}$$

(a, X_0) determine the shape of the potential.

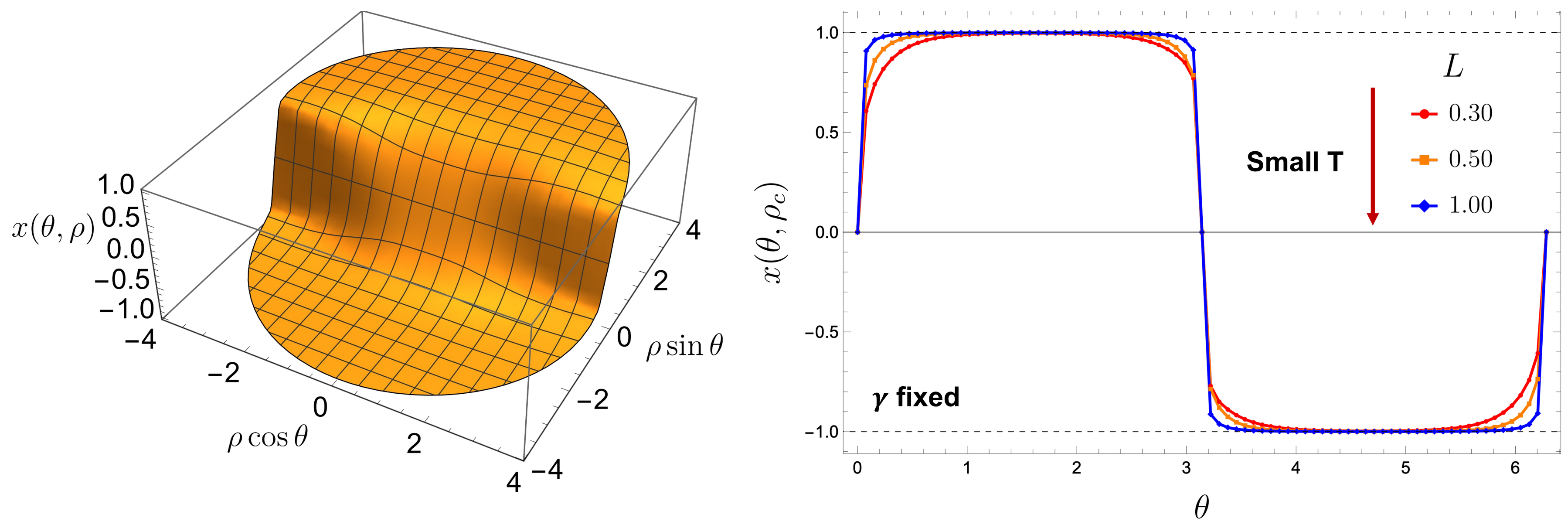


Fig.3 The numerical bulk solution (left) and the boundary profile (right)

❑ Numerical results

- Instanton—anti-instanton solution is numerically obtained (Fig. 3).
- Temperature (controlled by L) with γ fixed changes the profile.

❑ Comparison with quantum mechanics

- The on-shell action increases as a increases.
  - Tunneling amplitude is **suppressed** by higher potential wall.
  - But **larger amplitude** compared to QM! (assisted by N=4 SYM gauge fields?)
- The on-shell action increases as T increases.
  - Tunneling amplitude is **suppressed** by higher heat bath temperature.

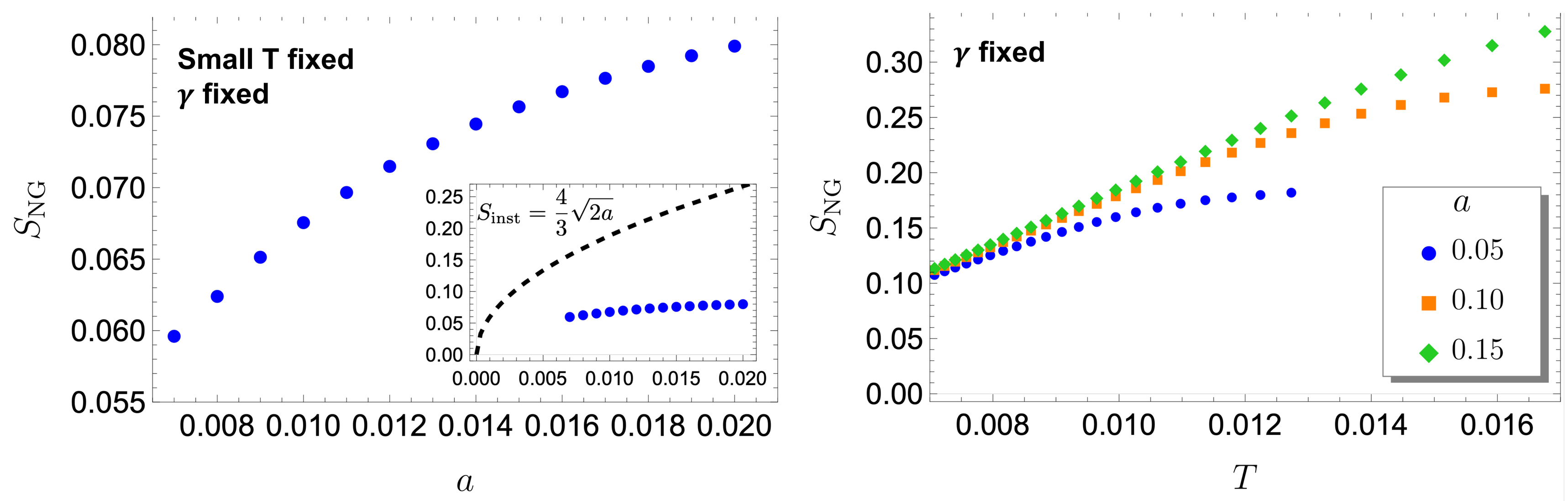


Fig.4 The on-shell action as a function of a (left) and temperature (right)

Summary

- Holographically construct instanton—anti-instanton solutions.
- Evaluate the tunneling rate from the on-shell Nambu-Goto action.
- Background gauge field enhances quantum tunneling.
- In contrast, finite temperature suppresses quantum tunneling.