# Non-Hermiticity and Non-Markovianity of open quantum systems

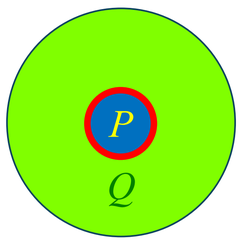
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## Introducing Non-Hermitian Hamiltonian

### Feshbach Formalism

$$H|\psi\rangle = E|\psi\rangle \quad\Rightarrow\quad H_{\rm eff}P|\psi\rangle = EP|\psi\rangle$$

$$H_{\rm eff} = PHP + PHQ\frac{1}{E-QHQ}QHP$$




### Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) eq.

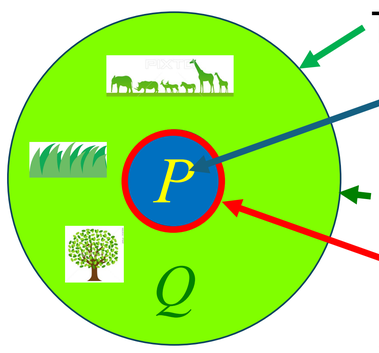
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H_0,\rho] + \gamma\left(L\rho L^\dagger - \frac{1}{2}\{L^\dagger L,\rho\}\right)$$
$$= -\mathrm{i}\left(H_{\rm nh}\rho - \rho H_{\rm nh}^\dagger\right) + \gamma L\rho L^\dagger$$
$$\Rightarrow\quad H_{\rm nh} := H_0 - \mathrm{i}\frac{\gamma}{2}L^\dagger L$$

- Born-Markov approximation → weak coupling
- Breaks time-reversal symmetry by hand
- Transient non-Hermiticity

### Three classes of open quantum systems

| Problem | System | Environment |
|---|---|---|
| **Easy** | No interaction | No interaction |
| **Intermediate** | Interaction | No interaction |
| **Difficult** | Interaction | Interaction |

## Feshbach Formalism (1958)




Total $H$

System $PHP$

Environment $QHQ$

coupling $PHQ + QHP$

Projection operators

$$P+Q=1$$
$$P^2=P,\quad Q^2=Q$$

$$H|\psi\rangle = E|\psi\rangle \quad\Rightarrow\quad H_{\rm eff}P|\psi\rangle = EP|\psi\rangle$$

$$PH(P+Q)|\psi\rangle = EP|\psi\rangle$$
$$QH(P+Q)|\psi\rangle = EQ|\psi\rangle$$
$$(PHP)P|\psi\rangle + (PHQ)Q|\psi\rangle = EP|\psi\rangle$$
$$(QHP)P|\psi\rangle + (QHQ)Q|\psi\rangle = EQ|\psi\rangle$$
$$\Rightarrow\quad Q|\psi\rangle = \frac{1}{E-QHQ}(QHP)P|\psi\rangle$$

$$(PHP)P|\psi\rangle + (PHQ)\frac{1}{E-QHQ}(QHP)P|\psi\rangle = EP|\psi\rangle$$

$$H_{\rm eff} = PHP + PHQ\frac{1}{E-QHQ}QHP$$

Strong coupling

## Resonant-state expansion

N. Hatano and G. Ordonez, J. Math. Phys. 55, 122106 (2014)

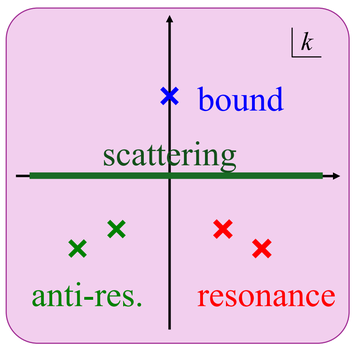
$$H_{\rm eff} = PHP + PHQ\frac{1}{E-QHQ}QHP$$

Infinite

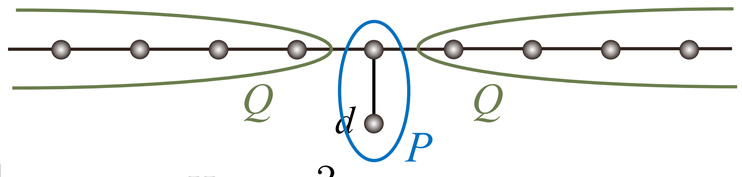
$\pm\mathrm{i}\eta$ Non-Hermiticity

### A new complete set

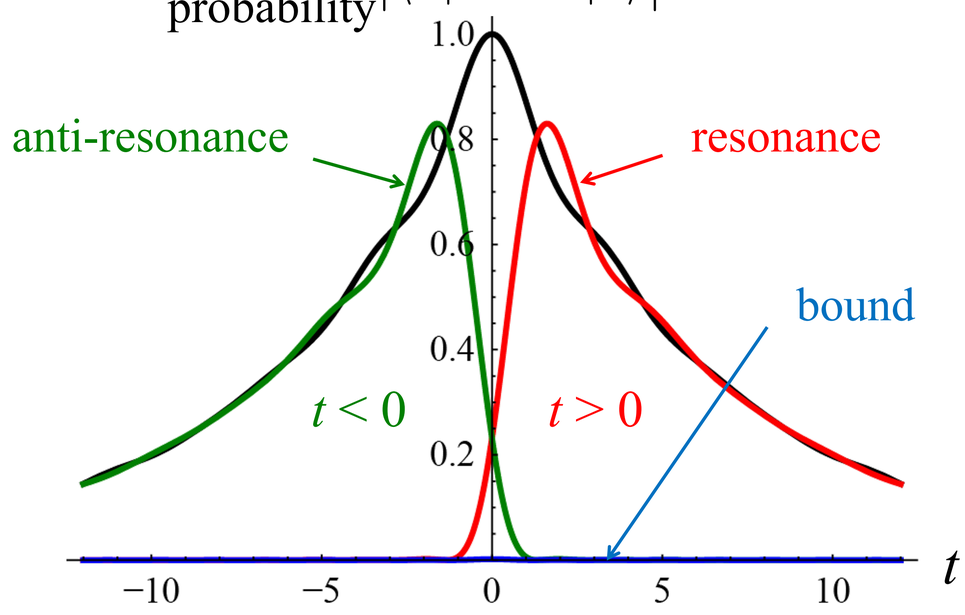
$$1 = P+Q \begin{cases} P = \sum_{\rm p.s.} P|\psi_n\rangle\langle\psi_n|P \\ Q = \int \mathrm{d}k\, Q|\phi_k\rangle\langle\phi_k|Q \end{cases}$$




G. Ordonez, N. Hatano, J. Phys. A: Math. Theor. **50** (2017) 405304



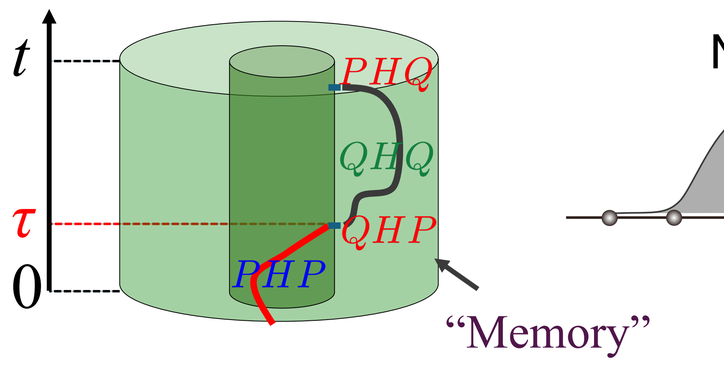

Survival probability $\left|\langle d|e^{-iHt}|d\rangle\right|^2$



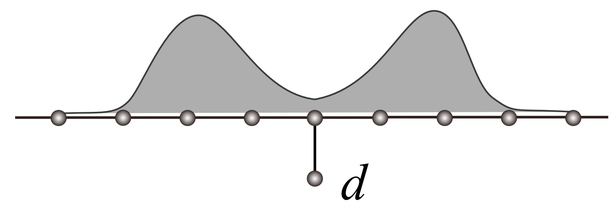

## Non-Markovianity of open quantum systems

$$\mathrm{i}\hbar\partial_t P|\Psi(t)\rangle = (PHP)P|\Psi(t)\rangle + \int_0^t d\tau (PHQ)e^{-\mathrm{i}QHQ(t-\tau)}(QHP)P|\Psi(\tau)\rangle$$

Memory kernel




Memory effect




Survival Probability $\left|\langle d|e^{-iHt}|d\rangle\right|^2$



Gonzalo Ordonez

$E = -2\cos k$

$\Rightarrow 1 - at^2$



Hiroki Nakabayashi

$E \propto \mathbf{k}^n,\quad \mathbf{k}\in\mathbb{R}^d$

$\Rightarrow 1 - t^{2-d/n}$



Takano Taira

$E = ck$

$\Rightarrow$ purely exp.



Hayato Kinkawa

$E = c|\mathbf{k}|,\quad \mathbf{k}\in\mathbb{R}^3$

(Photonic heat bath)

$\Rightarrow 1 - Ct^2$